PONTIFICIA UNIVERSIDAD CATOLICA DE CHILE
ESCUELA DE INGENIERIA

EMPIRICAL ANALYSIS OF COPPER PRICE ESTIMATION USING A THREE-FACTOR NO-ARBITRAGE STOCHASTIC MODEL

MARIAVICTORIA DEL CARMEN ENBERG GAETE

Thesis submitted to the Office of Research and Graduate Studies in partial fulfillment of the requirements for the Degree of Master of Science in Engineering

Advisor:
GONZALO CORTÁZAR

Santiago de Chile, October 2021

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Members of the Committee:

GONZALO CORTÁZAR

TOMÁS REYES

HÉCTOR ORTEGA

DOMINGO MERY

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To my family for their unconditional support throughout my life.

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RESUMEN

Predecir el precio del cobre no es una tarea fácil. Productores, consumidores, inversionistas e incluso algunos gobiernos deben enfrentarse a este tema. Existen muchas fuentes de información y diferentes modelos para completarla, sin embargo, no hay un consenso claro de cuándo es mejor utilizar un modelo sofisticado y cuándo es mejor utilizar la información que se tiene disponible.

En esta tesis se utiliza un modelo de tres factores sin arbitraje calibrado utilizando contratos futuros y las expectativas de los analistas para probar su poder predictivo y compararlo con predicciones obtenidas utilizando los contratos futuros de la bolsa LME, de la bolsa COMEX y el consenso de los analistas para obtener una recomendación. Se utilizaron datos desde enero del 2010 a diciembre del 2020 con este propósito. Tres métricas fueron calculadas a partir de los resultados: *Relative* MSPE, RMSE y Dstat. A partir de estas se obtiene que el los futuros de la bolsa LME y el modelo obtienen resultados mejores que la predicción *no change* en casi todos los periodos pronosticados. Finalmente se realiza una recomendación para utilizar el modelo en lugar de los contratos futuros para obtener una mejor predicción, la regla es la siguiente: si el ratio del precio del contrato futuro dado por el modelo, o el precio spot esperado determinado por el modelo con respecto al precio spot actual está por encima de la mediana histórica, la predicción del modelo es más confiable, en cambio, si el ratio está por debajo de la mediana histórica se recomienda utilizar los contratos futuros de la bolsa LME.

Palabras Claves: predicción; precio del cobre; bolsa LME; contratos futuros; expectativas de los analistas.

ABSTRACT

Predicting the price of copper is not a straightforward task. Producers, consumers, investors and even some governments face this issue. There are many sources of information and different models to complete it, however, there is no clear consensus on when it is better to use a sophisticated model and when it is better to use the information that is available.

In this thesis, a three-factor no-arbitrage commodity pricing model calibrated using futures contracts and analysts' expectations is used to test its predictive power and compare it with predictions obtained using LME exchange futures contracts, COMEX exchange futures contracts and analysts' consensus to obtain a recommendation. Data from January 2010 to December 2020 were used for this purpose. Three metrics were calculated from the results: Relative MSPE, RMSE and Dstat. From these it is obtained that the LME futures and the model obtain better results than the no change forecast, a commonly used benchmark, in almost all the forecast periods.

Finally, a recommendation is made to use the model instead of the futures contracts to obtain a better prediction, the rule is as follows: if the ratio of the future contract price given by the model, or the expected spot price determined by the model with respect to the current spot price is above the historical median, the prediction of the model is more reliable, on the other hand, if the ratio is below the historical median it is recommended to use the LME futures contracts.

Keywords: Copper prices; Futures Prices; Expected Prices; Forecasting; LME; Analysts' Forecast.

1. INTRODUCTION

Copper has an important role nowadays since its wide use in several industries. Thus, forecasting copper prices is useful for many people for different purposes. Copper is the world's third most used metal (following iron and aluminum) because of its physical characteristics: versatility and conductivity (Wang et al, 2019). Movements in copper prices can be seen as an early indicator of global economic performance given the importance of copper in various industries such as transportation, telecommunications, construction, among others (Buncic & Moretto, 2015). Even for countries whose economies are strongly dependent on copper production like Chile and Zambia (Sánchez Lasheras et al, 2015) it’s important to have a prediction of copper price. For instance, in Chile this metal represents about a half of Chilean exports and nearly 45% of Foreign Direct Investment (Brown & Hardy, 2019).

The role of copper has evolved over time from being a commodity that is used as a primary input in the production process of final goods, to a financial asset that is held and traded for speculative purposes (Buncic & Moretto, 2015). Because of this change, there are more participants in the copper market, making more difficult to predict its price and its drivers. This participants can be producers, consumers, governments and investors (García & Kristjanpoller, 2019).

Copper is traded on the physical futures exchanges: the London Metal Exchange (LME), the New York Commodity Exchange (COMEX) and the Shanghai Futures Exchange (SHFE) (Sánchez Lasheras et al., 2015). Nonetheless, as reference price we will use the LME copper prices since this exchange provides appropriately located storage facilities to enable market participants to take or make physical delivery (Dooley & Lenihan, 2005; Watkins & McAleer, 2004). Besides it is the biggest futures exchange for copper handling more than half of the world trades (Li & Li, 2015).

Different methods have been used to estimate the spot price of copper. There is a wide variety of approaches in the models used, in the data to calibrate them and in the metrics to evaluate their performance.

The information of future contracts and analysts' expectations could be potentially useful for predicting the price of copper. To determine whether this information can be useful for forecasting, we compared the predictive power of futures contracts, analysts' consensus and the model developed by Cifuentes et al. (2020).

Although there are different forecasting models available, we chose this model because it combines futures contracts and analysts' expectations, both data available on Bloomberg. The objective of this study is to generate a recommendation for market participants interested in forecasting the price of copper at different horizons. In order to test this, we will analyze the selected model in different out-of-sample periods and establish metrics to measure its performance in comparison to alternative methods.

The remainder of this paper is organized as follows: Section 2 reviews the models used in recent years in the literature, Section 3 explains the model and how it is calibrated, Section 4 analyzes the data that will be used in our model and as an alternative method to predict the metal price, Section 5 mentions the metrics that will be used to test the forecasting power of the model, Section 6 exhibits the results, Section 7 discusses these results and, finally, Section 8 concludes.

2. LITERATURE REVIEW

Different methods have been used to estimate the future copper prices. The simplest method to estimate a future price of a commodity is using its current price. This can be modeled as the random walk model without drift, which implies that changes in the spot price are unpredictable, so the best forecast is simply the current spot price (Alquist, Kilian, & Vigfusson, 2013):

$$\hat{S}_{t+h|t} = S_t \tag{2.1}$$

where $\hat{S}_{t+h|t}$ is the prediction of the spot price in h periods and S_t is the current spot price. This prediction is called no change forecast. Additionally to this model, many more have been developed in the past years. The following is a review of some models developed in the last years and some metrics used to measure its performance.

Kriechbaumer et al. (2014) use a wavelet-ARIMA model for predicting monthly base metal prices. This method can be used with different wavelet transform types to fit the better based on the MAE and RMSE of the resulting forecast. In their case, they obtained an improved in forecasting accuracy compared to an ARIMA model, but their performance is not significantly different from a no change forecast.

Another approach to forecast a commodity price is using futures contracts, since they have as underlying the commodity price. In fact, the price of a futures contract at a certain time is given by the expected spot price under the risk neutral measure as stated in Cortazar, Kovacevic and Schwartz (2015). They propose a commodity pricing model to estimate the expected price of oil and copper, using their future contracts. This model consist of a two-factor Schwartz and Smith (2000) commodity model and they restricted the commodity pricing process to match the expected returns obtained from the CAPM and also zero expected returns. They obtained the price based on the future price and an approximation of the risk premium.

Instead of using the copper prices or its futures contract prices Buncic and Moretto (2015) use a dynamic model averaging and dynamic model selection approach to forecast copper prices. This method selects the predictor variables for a model, these were chosen from three different groups: (i) fundamentals, (ii) financialization and (iii) exchange rates and stock prices, which contained 18 factors. This way they identify the main drivers for the copper price and as the model is time varying, the selected factors change over time. To evaluate the forecast some metrics where calculated MSPE, relative MSPE, out-of-sample R^2, among others. The results show that this approach significantly outperforms the random walk benchmark for forecast horizons up to 6 months ahead.

Another approach to address this issue is using algorithms. In the case of Sánchez Lasheras et al. (2015) they propose two neural networks (multilayer perceptron neural network and Elman neural network) to predict the copper price and compare their results with an ARIMA model. With this method they obtained in terms of RMSE that the two neural networks perform better than the ARIMA model. Moreover, Chen, He and Zhang (2016) use a novel grey wave forecasting method to predict metal prices, this method improves the forecasting accuracy of time series with irregular fluctuation ranges. They get as a result a multi-step-ahead forecast and its performance is measure using RMSE. The forecast obtained from this model indicates that their forecasting method outperforms an ARMA model and a no change forecast. Among these algorithms, Liu et al. (2017) predict copper prices using a machine learning algorithm. This method uses variables that are correlated with copper prices such as gold, silver, crude oil, natural gas, lean hogs, coffee, the Dow Jones Index, and past copper prices. Their method allows them to obtained forecast in short and long horizon. They calculated the MAPE of their prediction and RMSE obtaining a 4% and 8% respectively. Following forecasting methods based on algorithms we find Dehghani and Bogdanovic (2018), who propose a bat algorithm to predict copper prices. Other three models are then compared, one of them was a time series and the others two were intelligence algorithms (particle swarm optimization and differential evolution). In terms of RMSE the bat algorithm was the best in the selected period. Then Dehghani (2018) uses an artificial neural network called gene expression programming to predict future copper prices. This method uses as input variables correlates with copper prices such as silver price, nickel price, aluminum price, OPEC crude oil price, WTI crude oil price, BRENT crude oil price and CLP exchange rate. The author compares the model with two alternatives a multivariate regression and times series, obtaining better results using his algorithm.

Recently, Alameer et al. (2019) use 10 input variables as predictors for the future fluctuations of copper prices with a hybrid model. They propose a model that employs a genetic algorithm to adjust the adaptive neuro-fuzzy inference system (ANFIS) parameters. Then they calculated the RMSE, MSE and MAE of their predictions and compared it with other models (SVM, ANFIS, ARIMA and GARCH) obtaining that the hybrid model performs better than the rest. On the same path of hybrid models, Wang et al., (2019) attempted to predict the copper spot prices by developing a hybrid predictive technique combining complex network and artificial neural network techniques. First, they transform the original price time series to a price volatility network. Secondly, after the original data are reconstructed, three artificial neural networks techniques are applied to forecast the future copper price.

Lastly, Cifuentes et al. (2020) proposed a non-stationary version of the canonical three-factor model formulation of Dai and Singleton (2002) using both futures prices and analysts' forecasts. The model follows closely the one developed for oil by Cortazar et al. (2018). As we mentioned before, we select this model to test its forecasting ability and test it against copper price approximation based on the data.

As mentioned before there are different approaches to predict copper spot price, they use a wide variety of input data since many factors influence its price and its movements and sometimes is hard to select few of them. This is shown by Guzmán and Silva (2018) who select fundamental and non-fundamental factors to explain copper price, the first has to do with the supply and demand of this commodity while the second factors are those different from supply and demand. For the periods analyzed they concluded that to explain movements in the price of copper it is necessary to consider fundamentals and macroeconomic and financial variables.

All these studies show us that this is an ongoing field, there is no consensus about the best model, data, or metrics to use when forecasting copper prices. To address this we chose the model developed by Cifuentes et al., (2020), and tested it in different out-of-sample periods and comparing it against the futures contracts and the analysts' consensus. We selected the futures contracts that reflect a detailed analysis of the market (Wets & Rios,

2015) and the analysts' consensus since it involve the analysts' studies about how the market could behave in the future. All the information was obtained from Bloomberg.

3. THE MODEL

In this section, we explain the model used to obtain the copper prices. This model was developed by Cifuentes et al. (2020) to obtain explicit expressions for risk premiums, futures prices and spot price expectations for copper. In our case we will use this model to predict copper prices and to test its prediction power against alternatives methods. We follow closely the description presented by the authors of the model.

3.1 Model definition

A non-stationary N-factor model is calibrated using both futures prices and analysts' expectations. This model is a version of the canonical $A_0(N)$ Dai and Singleton (2002) model with stochastic risk premiums as in Duffee (2002). Here we describe the N-factor term structure of a non-stationary version of the canonical model.

Let S_t be the spot price at time t, then:

$$\ln S_t = Y_t = h' x_t \tag{3.1}$$

$$dx_t = \left(-A x_t + \begin{bmatrix} b_1 \\ 0 \\ \vdots \\ 0 \end{bmatrix} \right) dt + dw_t \tag{3.2}$$

where h is an $n\ x\ 1$ vector of constants, x_t is an $n\ x\ 1$ vector of state variables, b_1 is a scalar, A is an $n\ x\ n$ upper triangular matrix with its first diagonal element being zero and the remaining elements all different and strictly positive. Let dw_t be an $n\ x\ 1$ vector of uncorrelated Brownian motions such that:

$$dw_t dw_t' = I\ dt \tag{3.3}$$

where I is an $n\ x\ n$ identity matrix.

Let Π_t be the commodity risk premium at time t and assume that:

$$\Pi_t = \lambda + \Lambda\, x_t \tag{3.4}$$

Hence the risk adjusted version of the model is:

$$Y_t = h' x_t \tag{3.5}$$

$$dx_t = \left(-(A+\Lambda)x_t + \begin{bmatrix} b_1 \\ 0 \\ \vdots \\ 0 \end{bmatrix} - \lambda \right) dt + dw_t^Q \tag{3.6}$$

where dw_t^Q is a Brownian motion under the risk neutral measure Q, λ is an $n\ x\ 1$ vector and Λ is an $n\ x\ n$ matrix that needs no additional condition.

As stated in Cox et al. (1981) the futures price can be written as the risk-adjusted expected spot price at time T:

$$F_t(\mathrm{T}) = E_t^{\mathrm{Q}}(\mathrm{S_T}) = e^{E_t^Q(Y_T) + \frac{1}{2}Var^Q(Y_T)} \tag{3.7}$$

with:

$$E_t^{\mathrm{Q}}(x_T) = e^{-(A+\Lambda)(T-t)}x_t + \left(\int_0^{T-t} e^{-(A+\Lambda)\tau} d\tau\right)(b-\lambda) \tag{3.8}$$

$$Cov^Q(x_T) = \int_0^{T-t} e^{-(A+\Lambda)\tau}(e^{-(A+\Lambda)\tau})' d\tau \tag{3.9}$$

Then, the expected price satisfies the following equations:

$$E_t(\mathrm{S_T}) = e^{E_t(Y_T)+\frac{1}{2}Var(Y_T)} \tag{3.10}$$

$$E_t(x_T) = e^{-A(T-t)}x_t + \left(\int_0^{T-\mathrm{t}} e^{-A\tau}d\tau\right)b \tag{3.11}$$

$$Cov(x_T) = \int_0^{T-t} e^{-A\tau}(e^{-A\tau})'d\tau \tag{3.12}$$

Finally, model implicit volatilities of future prices σ_F and expected prices σ_E may be determined as follows:

$$\sigma_F = \sqrt{h'e^{-(A+\Lambda)(T-t)}e^{-(A+\Lambda)(T-t)'}h} \tag{3.13}$$

$$\sigma_E = \sqrt{h'e^{-A(T-t)}e^{-A(T-t)'}h} \tag{3.14}$$

3.2 Model estimation

The state variables and the parameters of the model where estimated using the Kalman filter (Kalman, 1960). This method uses all the available data in each iteration to estimate the optimal value of the state variables, besides the number of observations can vary in each iteration. In every iteration the filter can be represented in two equations, the first is the measurement equation, which indicates the relationship between the observable variable vector z_t and the state variable vector x_t as follows:

$$z_t = \mathrm{H_t}x_t + d_t + v_t \qquad v_t \sim N(0, R_t) \tag{3.15}$$

where z_t is an $m_t\ x\ 1$ vector that contains logarithm of price observations (futures and expected spot prices) at time t. H_t is a $m_t\ x\ n$ matrix, x_t is an $n\ x\ 1$ vector, d_t is an $m_t\ x\ 1$ vector and v_t is a measurement error vector of $m_t\ x\ 1$ dimension with zero mean and covariance R_t. In the model, m_t depends on the number of observations at each time, this way the dimension of $z_t, H_t, d_t, v_t\ y\ R_t$ can vary in each iteration.

The expected spot prices, proxied by analysts' forecast are nosier than futures prices, so there will be two measurement errors. This way the matrix R_t is defined by:

$$R_t = \begin{bmatrix} {\sigma_f}^2 & \cdots & 0 & 0 & \cdots & 0 \\ \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\ 0 & \cdots & {\sigma_f}^2 & 0 & \cdots & 0 \\ 0 & \cdots & 0 & {\sigma_e}^2 & \cdots & 0 \\ \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\ 0 & \cdots & 0 & 0 & \cdots & {\sigma_e}^2 \end{bmatrix} \qquad (3.16)$$

The second equation is the transition equation. This equation describes the stochastic process that the state variables follow:

$$x_{t+1} = \bar{A}x_t + \bar{c} + w_t \qquad w_t \sim N(0, Q) \qquad (3.17)$$

where $\bar{A}$ is an $n\,x\,n$ matrix, and $\bar{c}$ is an $n\,x\,1$ vector. $\bar{A}$ and $\bar{c}$ represents the discretization of the process. In the above expression, w_t is a vector of random variables with zero mean and an $n\ x\ n$ covariance matrix Q.

The parameters of the model were calculated by maximum likelihood.

4. DATA

The data used consist of futures contract prices, analysts' expectations, consensus forecast and spot prices. The period goes from January 2010 to December 2020, the in sample and out-of-sample periods vary during the evaluation of the model performance.

4.1 Futures contracts

The futures contracts were used for two objectives, the first was to calibrate the model and the second was to obtain a prediction for copper price. For the first purpose, we only considered the ones traded in the LME, meanwhile for the second the contracts traded in COMEX and LME were used independently to forecast the copper spot price. The copper future data use in the calibration process are weekly prices considering those maturing at six-month intervals.

4.1.1 LME

The futures traded expire the current month and the following 123 months. There are more futures contract available than information on analysts' expectations, so the methodology proposed by Cifuentes et al. (2020) was followed to select the futures contracts used to calibrate the model. This methodology takes LME weekly futures prices for the contract closest to its maturity and those maturing every six months whereas for analysts' expectations a weekly average is calculated and used for that week. The futures contracts considered for the calibration are shown in the next figure:

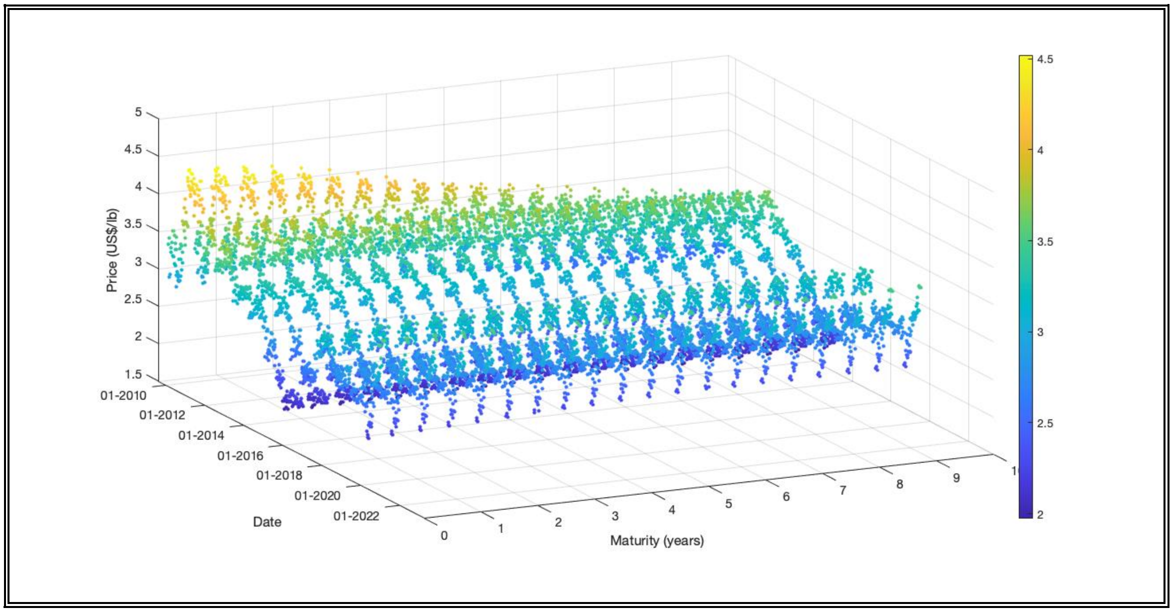

Figure 4-1: LME copper future prices available for the calibration

The prediction horizon tested goes from one month to 24 ahead, hence for each date that a forecast was done the future contracts that were considered were those available up to 24 months. The following table shows the amount of futures contract used in the calibration process for each period:

Table 4-1: LME Futures Contracts used in the calibration process

Years in sample	Amount of data
2010 – 2013	4160
2010 – 2014	5220
2010 – 2015	6260
2010 – 2016	7300
2010 – 2017	8340
2010 – 2018	9380
2010 – 2019	10420

The second objective of using this data was to obtain a prediction of the future price of copper based on the prices of the futures contracts with maturities closest to the date we wanted to predict. The futures contracts considered for this purpose are shown in the next figure:

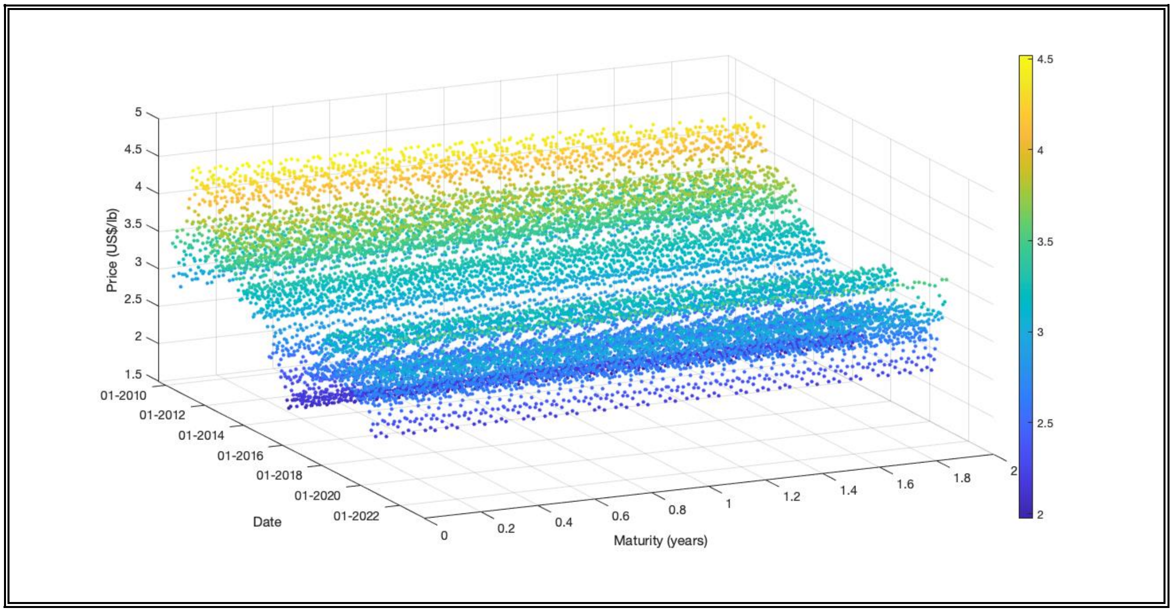

Figure 4-2: LME copper future prices up to 24 months

The following table shows the amount of futures contract available each year for this objective:

Table 4-2: LME Futures Contracts available up to 24 months

Year	Amount of data
2010	1243
2011	1243
2012	1245
2013	1245
2014	1266
2015	1243
2016	1244
2017	1244
2018	1245
2019	1243
Average	1246.1

4.1.2 COMEX

The futures contracts traded expire the current month, the next 23 months, and any March, May, July, September and December within a 60-month period that begins with the current month. These futures were used to get a prediction of the future price of copper based on the prices of the futures contracts with maturities closest to the date we wanted to predict. The futures contracts considered for this purpose are shown in the next figure:

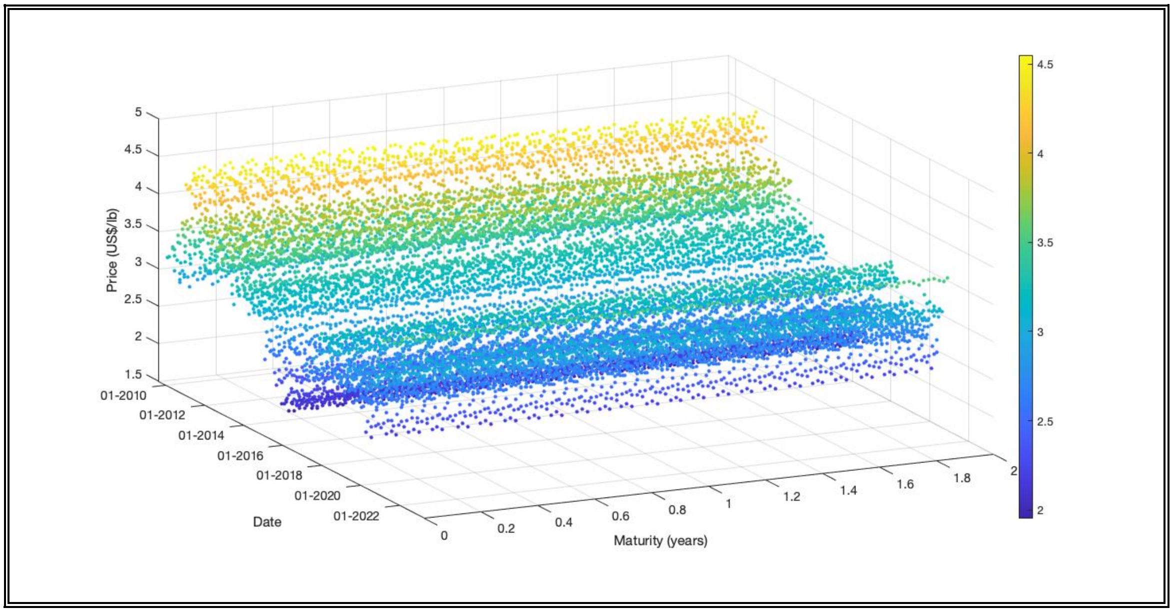

Figure 4-3: COMEX copper future prices up to 24 months

The amount of data available for predicting is shown in the next table:

Table 4-3: COMEX Futures Contracts available up to 24 months

Year	Amount of data
2010	1250
2011	1249
2012	1236
2013	1231
2014	1258
2015	1248
2016	1251
2017	1251
2018	1273
2019	1286
Average	1253.3

4.2 Analysts' forecast

Bloomberg has available the forecasts made by various analysts from different financial institutions. These forecasts are available when the analyst publishes them. There are two types of forecasts: quarterly and annual. These predictions are made for the average price on each quarter, or year, but following Cifuentes et al. (2020) we assume they represent the price in the middle of their time period. On one hand, quarterly forecasts are available for the current quarter and for the following five quarters. On the other hand, there are the annual forecasts that are valid for the year in which the forecast is made and the following four years. Each analyst can make a forecast for all or none these horizons. Figure 4-3 shows the analysts' expectation data used for the calibration of the model; the data available for each week was averaged as mentioned before:

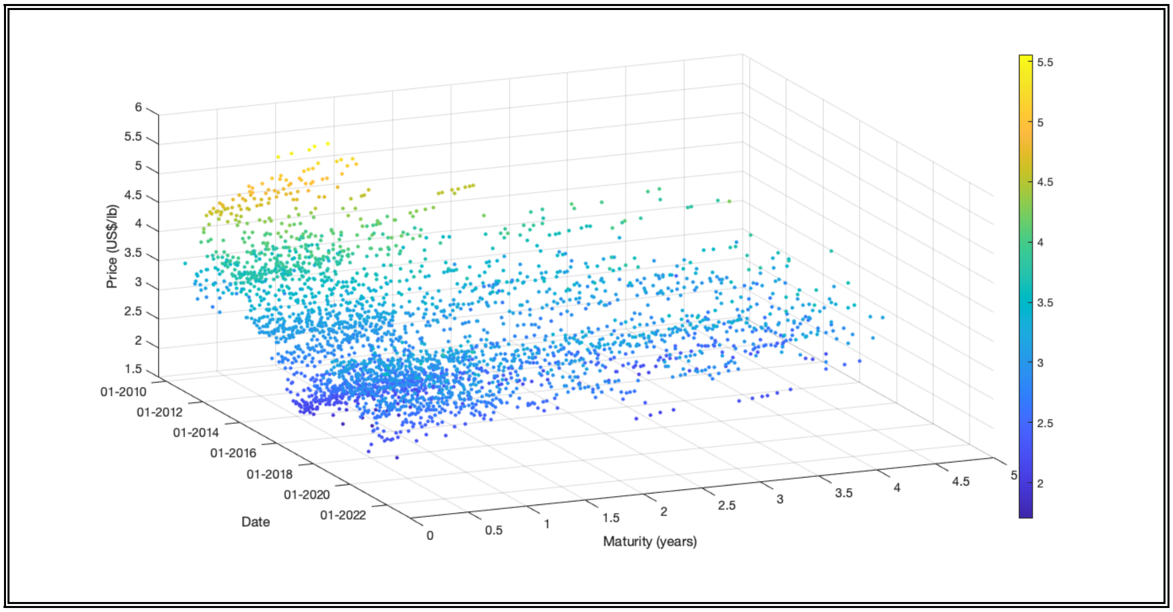

Figure 4-4: Analysts' expectations available in Bloomberg

A summary table of the data is shown below:

Table 4-4: Analysts' expectations available up to 24 months

Year	Amount of data
2010	240
2011	278
2012	344
2013	621
2014	711
2015	740
2016	783
2017	746
2018	561
2019	331
Average	535.5

In the next figure, we can see the difference between the amounts of data available for the calibration:

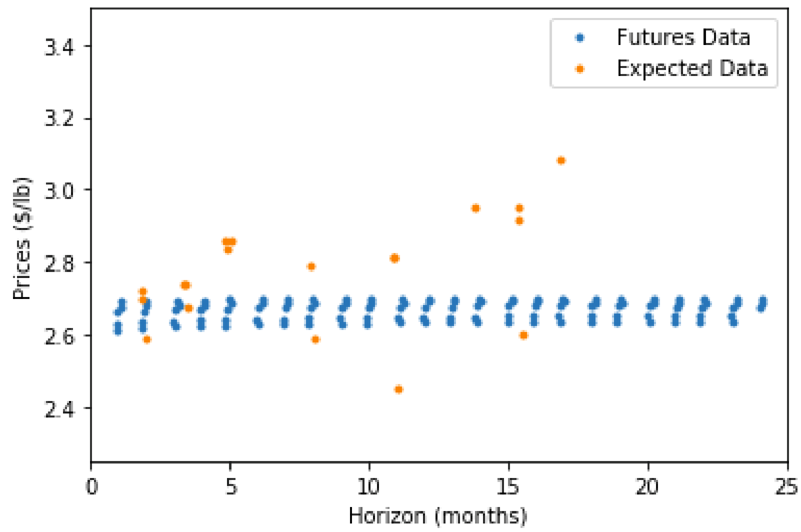

Figure 4-5: Futures and analysts' expected price data, third week, March 2017

As shown before there are more observations for futures prices than for analysts' expectations, this way the filtering process proposed by Cifuentes et al. (2020) is valuable for the model calibration. The Kalman filter can estimate the state variables and parameters without giving too much weight to the data that have the highest frequency.

4.3 Consensus

The consensus refers to the consensus forecast available in Bloomberg for copper prices. This is the median of the available analyst forecast on a certain date. As the analysts do their predictions for each quarter and year the consensus is available for those periods too. As in the case of analysts' expectations, the consensus was considered to represent the price in the middle of the period to be predicted.

Consensus was used in a similar way to futures contracts. With the available data, a forecast for the copper price was calculated using the consensus forecasts closest to the horizon to be forecast. The consensus considered for this purpose are shown in the next figure:

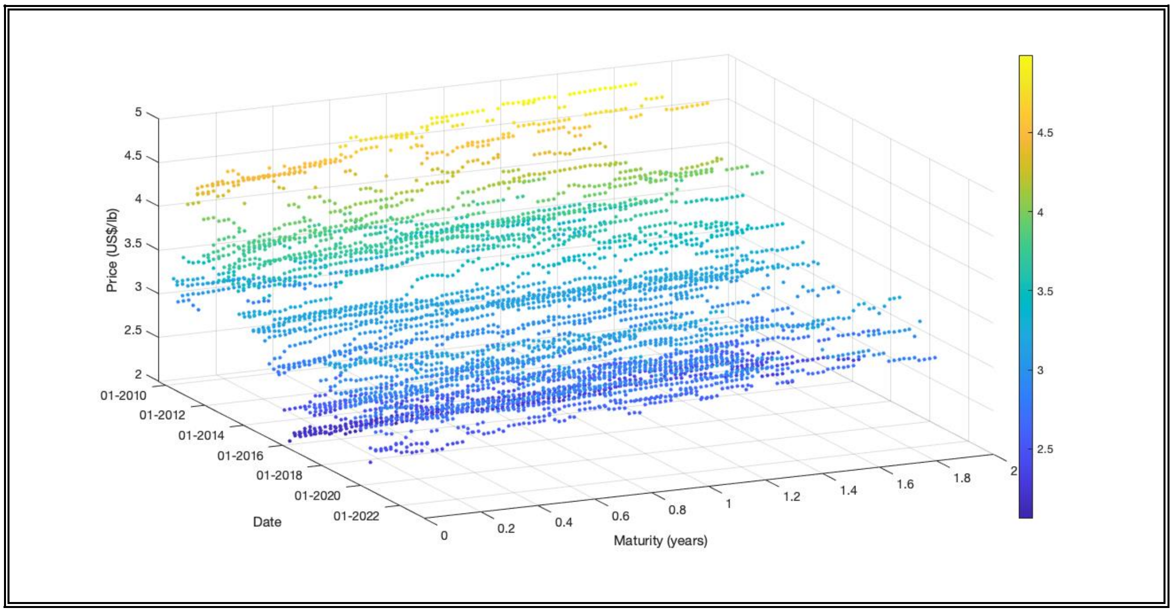

Figure 4-6: Consensus available in Bloomberg up to 24 months

The amount of data available for predicting is shown in the next table:

Table 4-5: Consensus available up to 24 months

Year	Amount of data
2010	411
2011	460
2012	473
2013	490
2014	458
2015	453
2016	466
2017	425
2018	391
2019	407
Average	443.4

4.4 Spot

The price attempted to be predicted corresponds to the spot price of copper on the LME exchange. The spot price predicted using the model, futures contracts, analysts' consensus and no change forecast corresponds to the price from January 2014 to December 2020. In the following figure we can see the spot price between January 2010 and December 2020:

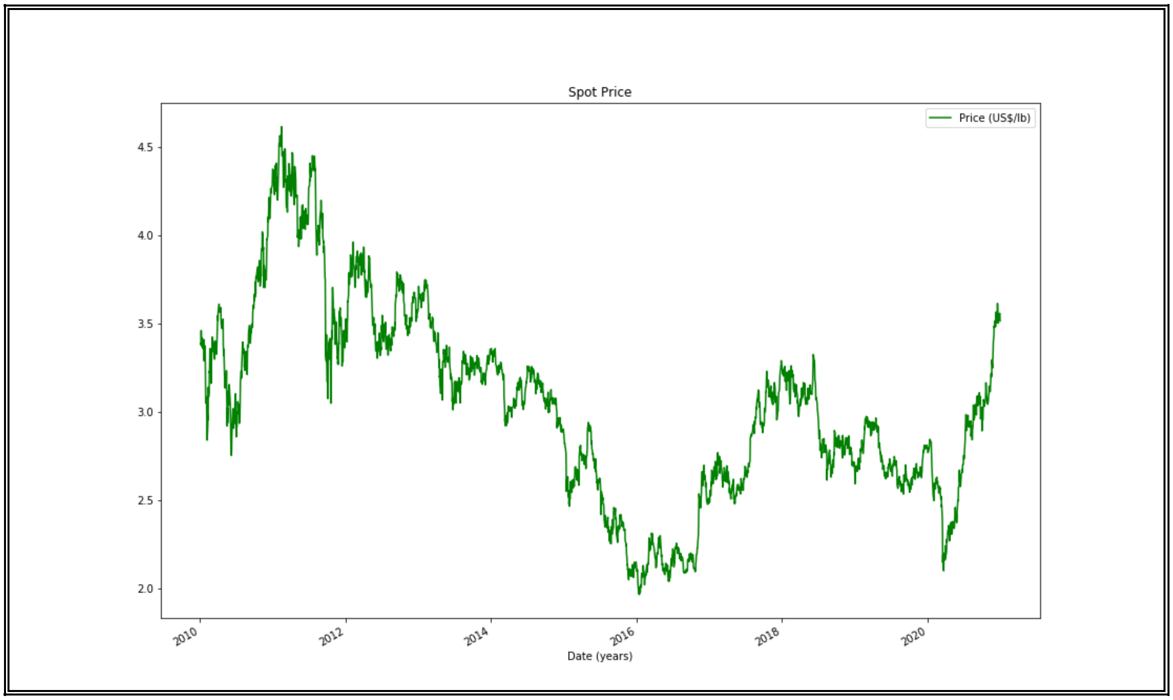

Figure 4-7: Copper spot price January 2014 to December 2020

As part of the analysis of the data, we plotted the spot price on each Wednesday versus the spot price of the previous week. Figure 4-8 shows this:

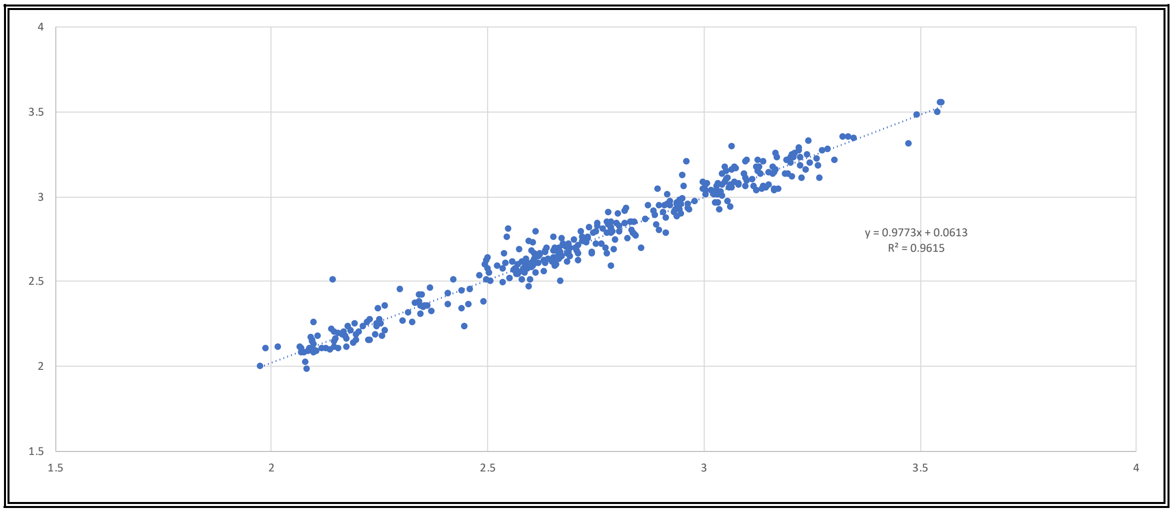

Figure 4-8: Copper spot price versus last week's price

As can be seen the spot's price for each consecutive week show a high correlation. Then we observed the return of the price using the current price and the price of the previous week and compare it with the return obtained on the week before. Figure 4-9 shows this relation:

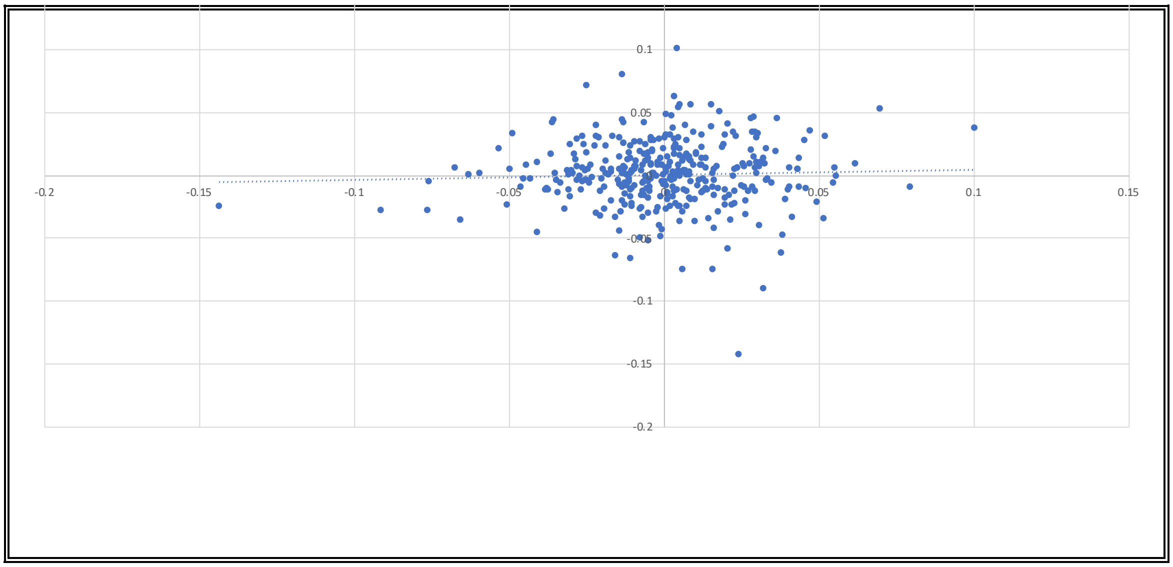

Figure 4-9: Weekly spot prices return versus last week return

In the previous figure we can see that the returns do not show a clear correlation[1].

[1] The correlation between the returns is 3.8%.

5. PERFORMANCE METRICS

In order to determine the accuracy of a model's prediction we established metrics that could measure the prediction error. Additionally, we used a benchmark prediction to compare its performance with the other predictions. The benchmark prediction was the random walk model without drift or, also called, no change forecast.

The performance metrics selected to compare the model with the other predictions are three: Relative Mean Squared Prediction Error, Root Mean Squared Error and Dstat.

5.1 Relative Mean Squared Prediction Error

This metric compares the mean squared prediction error of two models. In this case, the benchmark is the no change forecast. The squared prediction error is calculated as follows:

$$MSPE_i = \frac{1}{N}\sum_t^N \left(\mathrm{S}_{\mathrm{t+h|t}} - \hat{S}_{i,t+h|t}\right)^2 \qquad (5.2)$$

The Relative Mean Squared Prediction Error, according to Watson and Stock (2004) is defined as:

$$Relative\ MSPE_i = \frac{\sum_t^N \left(\mathrm{S}_{\mathrm{t+h|t}} - \hat{S}_{i,t+h|t}\right)^2}{\sum_t^N \left(\mathrm{S}_{\mathrm{t+h|t}} - \hat{S}_{0,t+h|t}\right)^2} \qquad (5.3)$$

where i is the forecasting model analyzed and i=0 refers to the no change benchmark model. If this indicator is less than 1, it means that the evaluated model is better than the reference model.

5.2 Root Mean Squared Error

This metric is used in several studies about copper forecasting (Alameer et al., 2019; Dehghani, 2018; Dehghani & Bogdanovic, 2018; Kriechbaumer et al., 2014; Liu et al., 2017; Sánchez Lasheras et al., 2015; Wets & Rios, 2015). This metric is calculated as follows:

$$RMSE^{h} = \sqrt{\frac{1}{N}\sum_{t}^{N}\left(\mathrm{S}_{\mathrm{t+h|t}} - \hat{S}_{i,t+h|t}\right)^{2}} \tag{5.4}$$

5.3 Dstat

This metric was used to measure the performance of directional prediction. Following the formulation of Yao and Tan (2000) this directional change statistic is calculated as:

$$D_{stat}^{h} = \frac{1}{N}\sum_{t\in T} a_{t,h} \tag{5.5}$$

where:

$$a_{t,h} = \begin{cases} 1 & if\ (S_{t+h} - S_t)\left(\hat{S}_{t+h} - S_t\right) > 0 \\ 0 & otherwise \end{cases} \tag{5.6}$$

A D_{stat}^{h} greater than 0.5 means that the obtained prediction is better than the no change model, which is expected to have a D_{stat}^{h} of 0.5 (Cortazar, Ortega, & Valencia, 2020).

6. RESULTS

In this section, we present the results of the prediction of copper spot price with the selected model, with the futures contract from LME and from COMEX, and with the analyst consensus.

6.1 Model

The model was calibrated several times and used to predict the copper prices in horizons up to 24 months. The first data set used to calibrate the model parameters was from 2010 to 2013. With this model, we forecast during 2014 for different horizons. Then one year was added to the calibration data set, the parameters were obtained again, and it was used to predict one year, and so on until the last data set that covered from 2010 to 2019.

6.1.1 Model fit

The model uses the available data of futures and analysts' expectation to jointly estimate the forecast (analyst expectations) and the futures curve. Figure 6-1 shows the forecast and the futures curves for the third week of March of 2015 and the data available for that date after filtering it. As can be seen that the forecast curve does not perfectly fit the available data. The reason for this is that the Kalman filter considers not only the data of that date but also the past data and their volatility. In the case of the futures curve this fits much better since the data is less volatile.

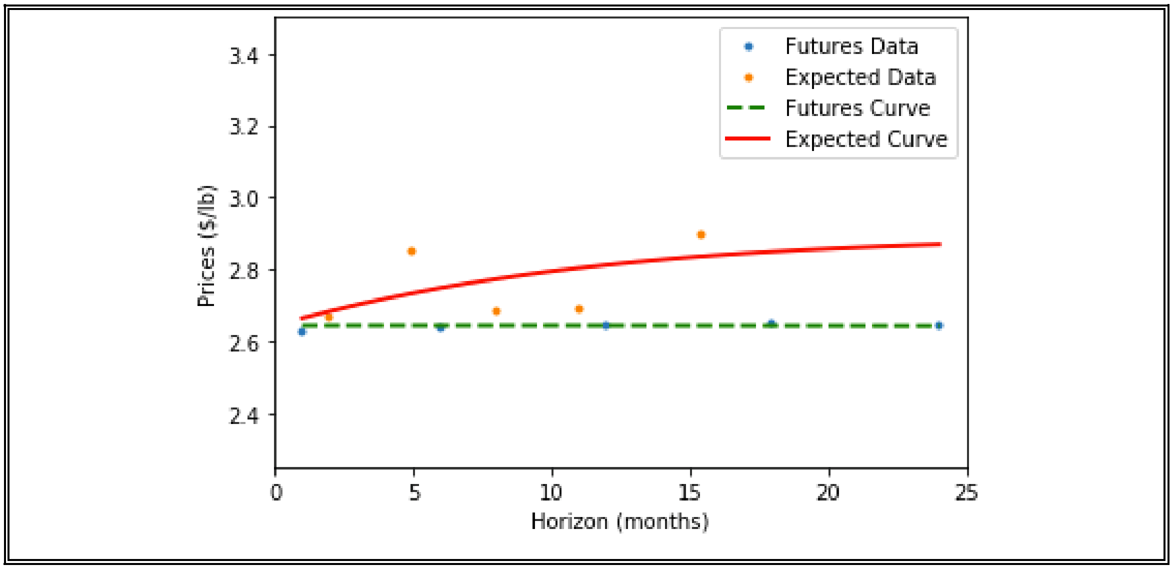

Figure 6-1: Forecast and futures curves and futures and analysts' expected price data, third week, March 2017

Table 6-1 computes the Mean Absolute Percentage Error (MAPE) for both curves, futures and forecast, for each calibration period. As can be seen the MAPE of analysts' forecast is greater than the futures data.

Table 6-1: Mean Absolute Percentage Error (MAPE) of forecast and futures curves in sample

Calibration Years	MAPE (%) between	
	Curve and Futures Prices Data	Curve and Analysts Expected Prices Data
2010-2013	0.21%	6.77%
2010-2014	0.21%	6.62%
2010-2015	0.19%	6.59%
2010-2016	0.18%	6.71%
2010-2017	0.18%	6.86%
2010-2018	0.18%	6.88%
2010-2019	0.17%	6.85%
Average	0.19%	6.76%
Standard Deviation	0.01%	0.12%

In the table above: the first column indicates the calibration period, the second indicates the MAPE of the futures curve and the futures contracts used in the calibration, and the third column indicates the MAPE of the expected curve and the analysts' expectations used in the calibration.

Table 6-2 computes the Mean Absolute Percentage Error (MAPE) for both curves, futures and forecast for each out-of-sample period. As in Table 6.1 the MAPE of analysts' forecast is greater than the futures data. As expected, the fit of both curves was worse than in sample.

Table 6-2: Mean Absolute Percentage Error (MAPE) of forecast and futures curves out-of-sample

Calibration Years	Year	MAPE (%) between	
		Curve and Futures Prices Data	Curve and Analysts Expected Prices Data
2010-2013	2014	0.29%	5.36%
2010-2014	2015	0.36%	6.25%
2010-2015	2016	0.11%	11.51%
2010-2016	2017	0.18%	8.13%
2010-2017	2018	0.20%	5.67%
2010-2018	2019	0.11%	7.14%
2010-2019	2020	0.13%	5.10%
Average		0.19%	7.02%
Standard Deviation		0.10%	2.25%

In the table above: the first column indicates the calibration period, the second indicates the year predicted, the third indicates the MAPE of the futures curve and the futures contracts used in the calibration in the predicted year, and the third column indicates the MAPE of the expected curve and the analysts' expectations used in the calibration in the predicted year.

6.1.2 Forecasting results

After the calibration process the model was evaluated through the metrics mentioned above. The Relative Mean Squared Prediction Error was calculated considering the no change forecast as benchmark. Table 6-3 summarizes the results for each year and for the whole 7-year period.

Table 6-3: Model's Relative Mean Squared Prediction Error

Horizon (months)	General Accuracy	Yearly Accuracy						
		2014	2015	2016	2017	2018	2019	2020
1	**0.978**	**0.810**	1.200	**0.869**	**0.810**	1.184	1.018	**0.972**
3	1.001	**0.839**	1.630	**0.652**	**0.705**	1.343	1.271	**0.863**
6	1.028	**0.911**	2.247	**0.428**	**0.539**	1.451	2.014	**0.650**
9	1.003	**0.902**	2.356	**0.120**	**0.669**	1.730	1.715	**0.645**
12	**0.885**	**0.866**	2.137	**0.151**	**0.749**	1.678	**0.895**	**0.708**
15	**0.780**	**0.846**	1.708	**0.170**	**0.876**	1.561	**0.769**	-
18	**0.688**	**0.815**	1.520	**0.170**	1.332	1.440	**0.473**	-
21	**0.634**	**0.791**	**0.796**	**0.175**	1.168	1.296	**0.580**	-
24	**0.576**	**0.737**	**0.393**	**0.146**	1.301	**0.969**	-	-
Horizons up to 12 months	**0.981**	**0.885**	2.181	**0.240**	**0.657**	1.557	1.380	**0.701**
Horizons between 13 and 24 months	**0.684**	**0.803**	1.206	**0.167**	1.072	1.309	**0.677**	-
Horizons up to 24 months	**0.770**	**0.817**	1.580	**0.180**	**0.838**	1.372	**0.933**	-

In the table above, **boldface** indicates improvements on the no-change forecast. The general accuracy indicates Relative MSPE from 2014 to 2020. The yearly accuracy refers to the year in which the forecast is made. "Horizons up to 12 months" indicates the Relative MSPE for horizons from 1 to 12 months. "Horizons between 13 and 24 months" indicates the Relative MSPE for horizons from 13 to 24 months. "Horizons up to 24 months" indicates the Relative MSPE for horizons from 1 to 24 months. Overall, in the 7 years the model is better than the no change forecast in almost all horizons with exception of 3-, 6- and 9-months horizons. Also, the performance of the model between the 13- and 24-months horizons is better than for shorter horizons, this could mean that our model predicts better longer horizons.

The performance of the model is better than the no change forecast in the horizons of 1-, 12-, 15-, 18-, 21- and 24-months; this is reflected in the General Accuracy column. If we aggregate the horizons between 1 to 12 months, 13 to 24 months, and 1 to 24 months we obtain that the model has a more accurate prediction. Besides, as the horizon increases the metric decreases.

Table 6-4 summarizes the RMSE of the model for each year and for the whole 7-year period.

Table 6-4: Model's RMSE

Horizon (months)	General Accuracy	Yearly Accuracy						
		2014	2015	2016	2017	2018	2019	2020
1	0.147	0.128	0.158	0.139	0.114	0.147	0.112	0.212
3	0.265	0.225	0.325	0.195	0.207	0.247	0.235	0.398
6	0.362	0.298	0.526	0.221	0.254	0.329	0.322	0.558
9	0.415	0.429	0.631	0.152	0.356	0.342	0.331	0.676
12	0.472	0.604	0.659	0.244	0.371	0.381	0.431	0.750
15	0.494	0.719	0.609	0.310	0.293	0.437	0.423	-
18	0.509	0.817	0.493	0.359	0.272	0.512	0.294	-
21	0.516	0.845	0.366	0.367	0.339	0.511	0.348	-
24	0.507	0.804	0.339	0.308	0.348	0.546	-	-
Horizons up to 12 months	0.356	0.370	0.519	0.192	0.283	0.310	0.308	0.473
Horizons between 13 and 24 months	0.505	0.788	0.489	0.334	0.319	0.489	0.411	-
Horizons up to 24 months	0.431	0.615	0.504	0.272	0.301	0.409	0.345	-

In the table above, the yearly accuracy refers to the year in which the forecast is made. "Horizons up to 12 months" indicates the RMSE for horizons from 1 to 12 months. "Horizons between 13 and 24 months" indicates the RMSE for horizons from 13 to 24 months. "Horizons up to 24 months" indicates the RMSE for horizons from 1 to 24 months.

In the table above, we observe that when the horizon increases, the error of the prediction increases too.

Table 6-5 summarizes the Dstat of the model for each year and for the whole 7-year period.

Table 6-5: Model's Dstat

Horizon (months)	General Accuracy	Yearly Accuracy						
		2014	2015	2016	2017	2018	2019	2020
1	**0.579**	**0.755**	0.365	**0.654**	**0.596**	0.423	**0.596**	**0.667**
3	**0.542**	**0.755**	0.308	**0.673**	**0.692**	0.346	0.346	**0.707**
6	**0.529**	**0.736**	0.077	**0.731**	**0.904**	0.269	0.231	**0.963**
9	**0.523**	**0.698**	0.135	**1.000**	**0.750**	0.115	0.308	**1.000**
12	**0.513**	**0.736**	0.154	**1.000**	**0.558**	0.038	**0.577**	**1.000**
15	**0.567**	**0.774**	0.404	**1.000**	**0.558**	0.096	**0.564**	-
18	**0.620**	**0.811**	0.500	**1.000**	**0.519**	0.077	**1.000**	-
21	**0.675**	**0.849**	**0.596**	**1.000**	**0.577**	0.296	**1.000**	-
24	**0.751**	**0.868**	**0.692**	**1.000**	**0.558**	**0.635**	-	-
Horizons up to 12 months	**0.537**	**0.733**	0.191	**0.845**	**0.710**	0.236	0.372	**0.813**
Horizons between 13 and 24 months	**0.629**	**0.813**	**0.510**	**1.000**	**0.550**	0.218	**0.740**	-
Horizons up to 24 months	**0.579**	**0.773**	0.350	**0.922**	**0.630**	0.227	0.491	-

In the table above, the **boldface** indicates improvements on the no-change forecast. The general accuracy indicates Dstat from 2014 to 2020. The yearly accuracy refers to the year in which the forecast is made. "Horizons up to 12 months" indicates the Dstat for horizons from 1 to 12 months. "Horizons between 13 and 24 months" indicates the Dstat for horizons from 13 to 24 months. "Horizons up to 24 months" indicates the Dstat for horizons from 1 to 24 months.

As is shown in the general accuracy, the model is better than the no change forecast for each horizon. Besides the accuracy between the 13- and 24- horizons is better than the shorter horizons, meaning that not only the error is less in those horizons but also the directional accuracy is better.

6.2 Nearest future contract

As a second alternative to predict the copper spot price, we used a proxy of the nearest future contract to its maturity. Like the consensus, given a date the available futures for the different horizons were used to predict the copper price. This forecast is calculated as the weighted average of the two futures closest to the horizon. In this case we used the futures contracts traded on LME exchange and on COMEX exchange.

6.2.1 LME

First, we analyzed the LME contracts. Table 6-6 summarizes the Relative MSPE of the LME's futures contracts for each year and for the whole 7-year period.

Table 6-6: LME's Relative Mean Squared Prediction Error

Horizon (months)	General Accuracy	Yearly Accuracy						
		2014	2015	2016	2017	2018	2019	2020
1	**0.987**	**0.953**	1.015	**0.986**	**0.965**	1.010	**0.972**	**0.993**
3	**0.978**	**0.901**	1.005	**0.967**	**0.938**	1.051	**0.995**	**0.991**
6	**0.972**	**0.907**	**0.992**	**0.964**	**0.911**	1.129	1.066	**0.956**
9	**0.965**	**0.877**	**0.979**	**0.978**	**0.951**	1.205	1.016	**0.949**
12	**0.968**	**0.908**	**0.979**	**0.974**	**0.981**	1.229	**0.955**	**0.933**
15	**0.971**	**0.916**	**0.995**	**0.972**	1.017	1.224	**0.931**	-
18	**0.971**	**0.919**	**0.957**	**0.979**	1.090	1.190	**0.884**	-
21	**0.983**	**0.918**	**0.982**	**0.981**	1.103	1.232	**0.966**	-
24	**0.988**	**0.914**	1.004	**0.981**	1.161	1.154	-	-
Horizons up to 12 months	**0.970**	**0.896**	**0.985**	**0.973**	**0.946**	1.155	1.000	**0.960**
Horizons between 13 and 24 months	**0.976**	**0.917**	**0.986**	**0.978**	1.068	1.202	**0.926**	-
Horizons up to 24 months	**0.974**	**0.913**	**0.986**	**0.977**	1.000	1.190	**0.953**	-

The table above follows the same format as the Table 6-3.

In this case we observed that the general accuracy of the futures is clearly better than the no change forecast. On both short and long horizons, using the nearest future contract can provide a better prediction than no change.

Table 6-7 summarizes the RMSE of the LME's futures contracts for each year and for the whole 7-year period.

Table 6-7: LME's RMSE

Horizon (months)	General Accuracy	Yearly Accuracy						
		2014	2015	2016	2017	2018	2019	2020
1	0.147	0.139	0.146	0.148	0.124	0.135	0.110	0.214
3	0.262	0.233	0.254	0.237	0.239	0.218	0.208	0.426
6	0.352	0.298	0.350	0.331	0.331	0.290	0.234	0.677
9	0.407	0.423	0.407	0.435	0.424	0.285	0.255	0.820
12	0.494	0.618	0.446	0.620	0.425	0.326	0.446	0.861
15	0.551	0.748	0.465	0.743	0.316	0.387	0.465	-
18	0.605	0.868	0.391	0.861	0.246	0.466	0.402	-
21	0.642	0.910	0.407	0.869	0.330	0.498	0.449	-
24	0.664	0.895	0.543	0.797	0.328	0.596	-	-
Horizons up to 12 months	0.354	0.373	0.349	0.386	0.340	0.267	0.262	0.553
Horizons between 13 and 24 months	0.604	0.842	0.442	0.809	0.318	0.468	0.480	-
Horizons up to 24 months	0.485	0.651	0.398	0.634	0.329	0.381	0.348	-

The table above follows the same format as the Table 6-4.

As the other cases, the error increases as the horizon increases.

Table 6-8 summarizes the Dstat of the LME's futures contracts for each year and for the whole 7-year period.

Table 6-8: LME's Dstat

Horizon (months)	General Accuracy	Yearly Accuracy						
		2014	2015	2016	2017	2018	2019	2020
1	**0.587**	**0.736**	0.288	**0.692**	**0.596**	0.481	**0.654**	**0.667**
3	**0.571**	**0.755**	0.442	**0.788**	**0.692**	0.327	0.462	**0.512**
6	**0.644**	**0.830**	**0.615**	**0.712**	**0.904**	0.308	0.346	**0.926**
9	**0.606**	**0.925**	**0.577**	**0.577**	**0.750**	0.228	0.404	**1.000**
12	**0.589**	**0.962**	**0.538**	**0.577**	**0.558**	0.212	**0.673**	**1.000**
15	**0.570**	**0.962**	0.385	**0.596**	**0.558**	0.250	**0.692**	-
18	**0.564**	**0.943**	0.481	**0.538**	**0.519**	0.212	**0.808**	-
21	**0.533**	**0.925**	**0.519**	**0.538**	**0.577**	0.077	**0.615**	-
24	**0.544**	**0.925**	0.385	**0.519**	**0.558**	0.327	-	-
Horizons up to 12 months	**0.599**	**0.841**	**0.521**	**0.665**	**0.710**	0.317	0.470	**0.734**
Horizons between 13 and 24 months	**0.558**	**0.942**	0.457	**0.553**	**0.550**	0.199	**0.727**	-
Horizons up to 24 months	**0.580**	**0.892**	0.489	**0.609**	**0.630**	0.258	**0.553**	-

The table above follows the same format as the Table 6-5.

In the horizons analyzed, we observed that using the nearest futures contract is a good approach to predict the direction in which the copper price will move, but we see that the performance is better in shorter horizons (less than 13 months).

6.2.2 COMEX

Second, we analyzed the COMEX contracts. Table 6-9 summarizes the Relative MSPE of the COMEX's futures contracts for each year and for the whole 7-year period.

Table 6-9: COMEX's Relative Mean Squared Prediction Error

Horizon (months)	General Accuracy	Yearly Accuracy						
		2014	2015	2016	2017	2018	2019	2020
1	1.018	**0.961**	1.078	1.026	**0.957**	1.008	1.054	1.030
3	**0.997**	**0.915**	1.063	1.012	**0.912**	1.007	1.037	1.018
6	**0.984**	**0.970**	1.053	**0.979**	**0.846**	1.157	1.069	**0.955**
9	**0.975**	**0.944**	1.045	**0.951**	**0.889**	1.263	1.007	**0.947**
12	**0.985**	**0.967**	1.035	**0.947**	**0.936**	1.329	**0.955**	**0.899**
15	**0.992**	**0.974**	1.031	**0.943**	**0.980**	1.349	**0.924**	-
18	**0.997**	**0.979**	**0.986**	**0.946**	1.139	1.323	**0.862**	-
21	1.009	**0.980**	**0.954**	**0.949**	1.163	1.365	**0.908**	-
24	1.010	**0.979**	**0.951**	**0.945**	1.230	1.223	-	-
Horizons up to 12 months	**0.983**	**0.954**	1.047	**0.960**	**0.891**	1.193	1.005	**0.965**
Horizons between 13 and 24 months	1.000	**0.978**	**0.988**	**0.946**	1.086	1.319	**0.910**	-
Horizons up to 24 months	**0.995**	**0.974**	1.010	**0.949**	**0.977**	1.287	**0.945**	-

The table above follows the same format as the Table 6-3.

In this case, we observed that the general accuracy of the futures indicates that these are better in horizons from 3 to 18 months, but they are better in horizons shorter than 13 months.

Table 6-10 summarizes the RMSE of the COMEX's futures contracts for each year and for the whole 7-year period.

Table 6-10: COMEX's RMSE

Horizon (months)	General Accuracy	Yearly Accuracy						
		2014	2015	2016	2017	2018	2019	2020
1	0.150	0.140	0.150	0.151	0.124	0.135	0.114	0.218
3	0.265	0.235	0.262	0.243	0.236	0.214	0.212	0.432
6	0.354	0.308	0.360	0.334	0.319	0.293	0.235	0.677
9	0.410	0.439	0.420	0.429	0.410	0.292	0.254	0.819
12	0.498	0.638	0.459	0.611	0.415	0.339	0.446	0.846
15	0.557	0.772	0.474	0.731	0.310	0.407	0.463	-
18	0.613	0.896	0.397	0.846	0.251	0.491	0.397	-
21	0.651	0.941	0.401	0.855	0.338	0.524	0.436	-
24	0.671	0.926	0.528	0.782	0.338	0.614	-	-
Horizons up to 12 months	0.3577	0.385	0.359	0.383	0.330	0.271	0.263	0.555
Horizons between 13 and 24 months	0.611	0.870	0.443	0.796	0.321	0.490	0.476	-
Horizons up to 24 months	0.490	0.672	0.403	0.625	0.325	0.396	0.347	-

The table above follows the same format as the Table 6-4.

As the other cases mentioned before, the error increases as the horizon increases but, in this case, the error is greater.

Table 6-11 summarizes the Dstat of the COMEX's futures contracts for each year and for the whole 7-year period.

Table 6-11: COMEX's Dstat

Horizon (months)	General Accuracy	Yearly Accuracy						
		2014	2015	2016	2017	2018	2019	2020
1	**0.507**	**0.623**	0.442	0.442	**0.558**	**0.558**	0.404	**0.521**
3	**0.503**	**0.736**	0.269	**0.538**	**0.692**	0.442	0.423	0.390
6	**0.559**	**0.660**	0.365	**0.692**	**0.904**	0.231	0.288	**0.963**
9	**0.554**	**0.642**	0.269	**0.769**	**0.750**	0.365	0.404	**1.000**
12	**0.510**	**0.623**	0.327	**0.788**	**0.558**	0.173	**0.577**	**1.000**
15	**0.563**	**0.642**	**0.519**	**0.865**	**0.558**	0.212	**0.590**	-
18	**0.610**	**0.642**	**0.596**	**0.962**	**0.519**	0.135	**1.000**	-
21	**0.628**	**0.642**	**0.692**	**0.942**	**0.577**	0.192	**1.000**	-
24	**0.659**	**0.660**	**0.692**	**0.942**	**0.558**	0.442	-	-
Horizons up to 12 months	**0.526**	**0.664**	0.311	**0.657**	**0.705**	0.338	0.409	**0.664**
Horizons between 13 and 24 months	**0.600**	**0.643**	**0.599**	**0.917**	**0.550**	0.212	**0.763**	-
Horizons up to 24 months	**0.560**	**0.653**	0.455	**0.787**	**0.627**	0.271	**0.524**	-

The table above follows the same format as the Table 6-5.

As the case of LME futures contracts, the nearest futures contract is a good approach to predict the direction in which the copper price will move.

6.3 Analysts' consensus

The analysts' consensus was proposed as an alternative method to predict copper spot price. Given a date the available analysts' consensus for the different horizons were used to predict the price. This forecast is calculated as the weighted average of the two analysts' consensus closest to the horizon.

Table 6-12 summarizes the Relative MSPE of the consensus for each year and for the whole 7-year period.

Table 6-12: Relative Mean Squared Prediction Error of the Consensus

Horizon (months)	General Accuracy	Yearly Accuracy						
		2014	2015	2016	2017	2018	2019	2020
1	2.392	1.113	3.695	1.595	3.766	2.526	1.492	2.476
3	1.466	**0.883**	2.455	**0.954**	1.937	1.644	1.008	1.363
6	1.317	**0.916**	2.815	**0.671**	1.726	1.271	1.801	**0.749**
9	1.322	1.072	2.628	**0.818**	1.255	2.100	2.090	**0.484**
12	1.215	1.089	2.531	**0.706**	**0.944**	2.017	1.065	1.071
15	1.151	1.139	2.273	**0.654**	1.001	1.951	**0.927**	-
18	1.031	1.163	2.165	**0.559**	**0.502**	1.809	**0.518**	-
21	**0.981**	1.242	1.493	**0.495**	**0.504**	1.643	**0.243**	-
24	**0.839**	1.221	**0.618**	**0.422**	**0.494**	**0.923**	-	-
Horizons up to 12 months	1.340	1.041	2.661	**0.776**	1.414	1.793	1.460	**0.861**
Horizons between 13 and 24 months	1.019	1.191	1.744	**0.546**	**0.697**	1.575	**0.747**	-
Horizons up to 24 months	1.113	1.166	2.095	**0.589**	1.101	1.630	1.007	-

The table above follows the same format as the Table 6-3.

In this case, we observed that the general accuracy of its prediction is worse than the no change forecast for almost every horizon, except the longer ones. This could mean than the analysts' forecast available in Bloomberg can be useful for forecasting copper prices in longer horizons.

Table 6-13 summarizes the RMSE of the consensus for each year and for the whole 7-year period.

Table 6-13: RMSE of the Consensus

Horizon (months)	General Accuracy	Yearly Accuracy						
		2014	2015	2016	2017	2018	2019	2020
1	0.230	0.150	0.278	0.188	0.246	0.214	0.136	0.338
3	0.321	0.230	0.397	0.236	0.343	0.273	0.209	0.500
6	0.409	0.299	0.589	0.277	0.455	0.307	0.305	0.599
9	0.477	0.468	0.666	0.398	0.487	0.376	0.365	0.585
12	0.553	0.677	0.718	0.528	0.417	0.418	0.471	0.923
15	0.600	0.834	0.703	0.609	0.313	0.489	0.464	-
18	0.623	0.977	0.588	0.650	0.167	0.574	0.308	-
21	0.642	1.059	0.501	0.617	0.223	0.575	0.225	-
24	0.611	1.035	0.426	0.523	0.214	0.533	-	-
Horizons up to 12 months	0.416	0.402	0.573	0.344	0.415	0.332	0.317	0.524
Horizons between 13 and 24 months	0.617	0.959	0.588	0.605	0.257	0.536	0.431	-
Horizons up to 24 months	0.518	0.735	0.581	0.492	0.345	0.446	0.358	-

The table above follows the same format as the Table 6-4.

In this case, we observe a similar behavior to the model, as the horizon becomes longer the error also increases. Meaning that the accuracy of the forecast worsens, and the expectations of each analyst reflected in the consensus is more inaccurate. Table 6-14 summarizes the Dstat of the consensus for each year and for the whole 7-year period.

Table 6-14: Dstat of the Consensus

Horizon (months)	General Accuracy	Yearly Accuracy						
		2014	2015	2016	2017	2018	2019	2020
1	**0.565**	**0.717**	0.442	**0.635**	**0.538**	**0.577**	**0.673**	0.354
3	**0.545**	**0.755**	0.346	**0.731**	0.500	**0.558**	**0.538**	0.341
6	0.462	**0.642**	0.077	**0.788**	0.269	0.500	0.327	**0.778**
9	0.450	**0.566**	0.135	**0.846**	0.462	0.327	0.250	**0.857**
12	0.494	0.453	0.154	**0.827**	**0.654**	0.327	**0.558**	0.000
15	**0.533**	0.340	0.404	**0.846**	**0.731**	0.288	**0.615**	-
18	**0.592**	0.340	0.500	**0.846**	**0.846**	0.269	**0.923**	-
21	**0.620**	0.264	**0.596**	**0.865**	**0.808**	0.481	**1.000**	-
24	**0.663**	0.189	**0.692**	**0.865**	**0.846**	**0.731**	-	-
Horizons up to 12 months	0.493	**0.607**	0.204	**0.768**	0.490	0.450	0.413	**0.542**
Horizons between 13 and 24 months	**0.586**	0.297	**0.510**	**0.854**	**0.787**	0.412	**0.743**	-
Horizons up to 24 months	**0.536**	0.452	0.357	**0.811**	**0.639**	0.431	**0.521**	-

The table above follows the same format as the Table 6-5.

If we look at the general accuracy column, we see that the consensus is better to predict the direction of copper prices than the no change forecast, with the exception of the horizon 6-, 9- and 12-months. Also, we observed that the general accuracy between the 13- and 24- horizons is better than the shorter horizons.

7. ANALYSIS AND RECOMMENDATIONS

In this section, we try to give an advice of which forecast to use for different horizons and the reason of this suggestion. First, we compare the performance of the LME future contracts and the COMEX futures contracts. In Tables 7-1, 7-2 and 7-3 we compared the general accuracy of each metric and selected in each horizon which of them had a better metric. This way each row indicates which prediction is better: the LME prediction or the COMEX prediction.

Table 7-1: LME's and COMEX's Relative MSPE general accuracy

Horizon (months)	LME	COMEX
1	**0.987**	1.018
3	**0.978**	0.997
6	**0.972**	0.984
9	**0.965**	0.975
12	**0.968**	0.985
15	**0.971**	0.992
18	**0.971**	0.997
21	**0.983**	1.009
24	**0.988**	1.010
Horizons up to 12 months	**0.970**	0.983
Horizons between 13 and 24 months	**0.976**	1.050
Horizons up to 24 months	**0.974**	1.031

In the table above, the **boldface** indicates that for that horizon the performance is better based on the selected metric.

Table 7-2: LME's and COMEX's RMSE general accuracy

Horizon (months)	LME	COMEX
1	**0.147**	0.150
3	**0.262**	0.265
6	**0.352**	0.354
9	**0.407**	0.410
12	**0.494**	0.498
15	**0.551**	0.557
18	**0.605**	0.613
21	**0.642**	0.651
24	**0.664**	0.671
Horizons up to 12 months	**0.354**	0.357
Horizons between 13 and 24 months	**0.604**	0.611
Horizons up to 24 months	**0.485**	0.490

In the table above, the **boldface** indicates that for that horizon the performance is better based on the selected metric.

Table 7-3: LME's and COMEX's Dstat general accuracy

Horizon (months)	LME	COMEX
1	**0.587**	0.507
3	**0.571**	0.503
6	**0.644**	0.559
9	**0.606**	0.554
12	**0.589**	0.510
15	**0.570**	0.563
18	0.564	**0.610**
21	0.533	**0.628**
24	0.544	**0.659**
Horizons up to 12 months	**0.599**	0.526
Horizons between 13 and 24 months	0.558	**0.600**
Horizons up to 24 months	**0.580**	0.560

In the table above, the **boldface** indicates that for that horizon the performance is better based on the selected metric.

As observed on the tables the LME predictions are better than the COMEX predictions. This result was expected since the copper spot price that were forecasted was selected from the LME exchange.

Once determined that LME prediction was more accurate, we compare its performance to the model's forecast and the consensus' forecast. In Tables 7-4, 7-5 and 7-6 we compared the general accuracy of each metric and selected in each horizon which of them had a better metric.

Table 7-4: Model's, LME's, and Consensus' Relative MSPE general accuracy

Horizon (months)	Model	LME	Consensus
1	**0.978**	0.987	2.392
3	1.001	**0.978**	1.466
6	1.028	**0.972**	1.317
9	1.003	**0.965**	1.322
12	**0.885**	0.968	1.215
15	**0.780**	0.971	1.151
18	**0.688**	0.971	1.031
21	**0.634**	0.983	0.981
24	**0.576**	0.988	0.839
Horizons up to 12 months	0.981	**0.970**	1.340
Horizons between 13 and 24 months	**0.684**	0.976	1.019
Horizons up to 24 months	**0.770**	0.974	1.113

In the table above, the **boldface** indicates that for that horizon the performance is better based on the selected metric.

Table 7-5: Model’s, LME’s, and Consensus’ RMSE general accuracy

Horizon (months)	Model	LME	Consensus
1	**0.147**	0.147	0.230
3	0.265	**0.262**	0.321
6	0.362	**0.352**	0.409
9	0.415	**0.407**	0.477
12	**0.472**	0.494	0.553
15	**0.494**	0.551	0.600
18	**0.509**	0.605	0.623
21	**0.516**	0.642	0.642
24	**0.507**	0.664	0.611
Horizons up to 12 months	0.356	**0.354**	0.416
Horizons between 13 and 24 months	**0.505**	0.604	0.617
Horizons up to 24 months	**0.431**	0.485	0.518

In the table above, the **boldface** indicates that for that horizon the performance is better based on the selected metric.

Table 7-6: Model’s, LME’s, and Consensus’ Dstat general accuracy

Horizon (months)	Model	LME	Consensus
1	0.579	**0.587**	0.565
3	0.542	**0.571**	0.545
6	0.529	**0.644**	0.462
9	0.523	**0.606**	0.450
12	0.513	**0.589**	0.494
15	0.567	**0.570**	0.533
18	**0.620**	0.564	0.592
21	**0.675**	0.533	0.620
24	**0.751**	0.544	0.663
Horizons up to 12 months	0.537	**0.599**	0.493
Horizons between 13 and 24 months	**0.629**	0.558	0.586
Horizons up to 24 months	0.579	**0.580**	0.536

In the table above, the **boldface** indicates that for that horizon the performance is better based on the selected metric.

As observed on the tables, model’s predictions tend to be better when the horizon increases compared to the LME’s predictions. Also, the tables show that the consensus’ predictions are the worst in each metric, however as the horizon increases the consensus’ metrics improves meaning that the information provided in it can be useful for longer horizons. Before making a recommendation for when to use the model’s predictions, LME’s predictions, or the consensus’ predictions, we analyzed the model’s performance *a priori* and *a posteriori*, dividing the results based on the behavior of the data.

7.1 Model's results analysis

In order to understand in which scenario is more accurate to use the model we recalculate the metrics Relative MSPE and RMSE dividing the data set using a spot filter, a futures filter and an expected filter. The first was *a posteriori* and the last two *a priori*, meaning that for the first analysis we used the spot that we tried to predict to divide the sample and in the last two we used the curve and the current spot.

7.1.1 Spot filter

We divided the sample based on the spot price: we set a filter equal to one if the spot went up and zero if it went down. The purpose of this was to test if the model performed better if the spot went up, as the analysts' expectations tended to be on the upside. The following table compute the general accuracy of each metric:

Table 7-7: Model's performance based on the spot filter

Horizon (months)	Relative MSPE		RMSE		Dstat	
	Spot Filter 1	Spot Filter 0	Spot Filter 1	Spot Filter 0	Spot Filter 1	Spot Filter 0
1	**0.703**	1.269	0.123	0.169	**0.900**	0.222
3	**0.600**	1.488	0.220	0.301	**0.953**	0.168
6	**0.514**	1.768	0.298	0.404	**1.000**	0.167
9	**0.423**	1.640	0.304	0.478	**1.000**	0.188
12	**0.261**	1.436	0.282	0.561	**1.000**	0.203
15	**0.226**	1.254	0.275	0.609	**1.000**	0.240
18	**0.207**	1.077	0.274	0.649	**1.000**	0.288
21	**0.218**	**0.980**	0.287	0.673	**1.000**	0.346
24	**0.212**	**0.890**	0.284	0.683	**1.000**	0.454
Horizons up to 12 months	**0.437**	1.590	0.259	0.419	**0.975**	0.183
Horizons between 13 and 24 months	**0.224**	1.076	0.285	0.641	**1.000**	0.298
Horizons up to 24 months	**0.292**	1.211	0.272	0.530	**0.987**	0.235

In the table above, the **boldface** indicates improvements on the no-change forecast.

Based on the table above we concluded that the model performs better relative to the no change forecast if the copper price in the futures goes up and worse if it goes down. This happens for every horizon tested, meaning that the model's forecast is more optimistic. This tendency is also reflected when we see the RMSE. This last metric reflects that in the case that the spot goes up the model's performances are significantly better than in the case that the spot goes down, notwithstanding we cannot know in advance if the spot price will go up or down. So, this analysis showed us that the model performs better in one case, but we were not able to set a rule for when to use the model from this analysis.

7.1.2 Futures filter

We split the sample based on the ratio between the price given by the futures curve of the model and the current spot price. We calculated that ratio each week for every horizon in the in sample set then for each horizon we computed the median of this ratio and use it as a threshold to split the out-of-sample results. So, each week on the out-of-sample set we choose one horizon and calculate the ratio, then we compared it with the median ratio of that horizon in the in-sample set, if the ratio for that week and horizon was above the median, the filter was set equal to one, in other case it was set equal to zero. In other words, if the price of the expectative implied by the future contracts price, reflected in the future curve, was upwards compared to the historical the filter was one and zero in the opposite case.

The following table compute the general accuracy of each metric:

Table 7-8: Model's performance based on the futures filter

Horizon (months)	Relative MSPE		RMSE		Dstat	
	Future Filter 1	Future Filter 0	Future Filter 1	Future Filter 0	Future Filter 1	Future Filter 0
1	**0.983**	**0.972**	0.153	0.140	**0.576**	**0.582**
3	**0.881**	1.187	0.264	0.267	**0.552**	**0.531**
6	**0.862**	1.428	0.364	0.358	**0.593**	0.440
9	**0.880**	1.185	0.393	0.444	**0.534**	**0.507**
12	**0.789**	**0.989**	0.420	0.573	0.495	**0.538**
15	**0.716**	**0.848**	0.429	0.586	**0.521**	**0.643**
18	**0.565**	**0.808**	0.396	0.689	**0.582**	**0.699**
21	**0.555**	**0.732**	0.425	0.691	**0.605**	**0.848**
24	**0.506**	**0.680**	0.429	0.674	**0.696**	**0.900**
Horizons up to 12 months	**0.858**	1.170	0.345	0.370	**0.549**	**0.522**
Horizons between 13 and 24 months	**0.599**	**0.786**	0.424	0.642	**0.585**	**0.723**
Horizons up to 24 months	**0.680**	**0.886**	0.387	0.494	**0.567**	**0.600**

In the table above, the **boldface** indicates improvements on the no-change forecast.

Based on the table above we can say that the model performs better relative to the no change forecast if the ratio is one and worse if it is zero. This tendency is also reflected when we see the RMSE. In the case of the Dstat, the metric does not vary significantly. But using this filter we can know in advance if the forecast will be more reliable. So, if we compute the ratio and it is one, the forecast will tend to be more accurate than if it were zero.

7.1.3 Expected filter

In this case, we separate the sample with a criterion like the futures filter. We split the sample based on the ratio between the price given by the expected curve of the model and the current spot price. We calculate that ratio each week for every horizon in the in sample set then for each horizon we compute the median of this ratio and use it as a threshold to split the out-of-sample results. So, each week on the out-of-sample set we choose one horizon and calculate the ratio, then we compared it with the median ratio of that horizon in the in-sample set. If the ratio for that week and horizon was above the median, the filter was set equal to one, in other case it was set equal to zero. In other words, if the price of the expectative implied by the model was upwards, compared to the historical, the filter was one and zero in the opposite case.

The following table compute the general accuracy of each metric:

Table 7-9: Model's performance based on the expected filter

Horizon (months)	Relative MSPE		RMSE		Dstat	
	Expected Filter 1	Expected Filter 0	Expected Filter 1	Expected Filter 0	Expected Filter 1	Expected Filter 0
1	**0.997**	**0.948**	0.144	0.152	**0.562**	**0.609**
3	1.004	**0.995**	0.267	0.261	**0.526**	**0.583**
6	**0.994**	1.141	0.374	0.331	**0.547**	0.490
9	**0.960**	1.118	0.418	0.410	**0.587**	0.382
12	**0.802**	1.023	0.434	0.541	**0.581**	0.375
15	**0.662**	**0.947**	0.422	0.619	**0.632**	0.427
18	**0.523**	**0.879**	0.392	0.703	**0.662**	**0.528**
21	**0.472**	**0.826**	0.389	0.737	**0.686**	**0.650**
24	**0.417**	**0.774**	0.377	0.751	**0.758**	**0.732**
Horizons up to 12 months	**0.943**	1.072	0.355	0.359	**0.563**	0.481
Horizons between 13 and 24 months	**0.539**	**0.868**	0.402	0.685	**0.670**	**0.537**
Horizons up to 24 months	**0.677**	**0.912**	0.378	0.531	**0.612**	**0.506**

In the table above, the **boldface** indicates improvements on the no-change forecast.

Based on the table above we can say that the model performs better relative to the no change forecast if the ratio is one and worse if it is zero. This tendency is also reflected when we see the RMSE. In the case of the Dstat we saw a little improvement on this metric when the filter was equal to ones. So, when using this filter, we can know in advance when the forecast will be more reliable. If we compute the ratio and it is one, the forecast will tend to be more accurate than if it were zero.

7.2 Prediction's comparison using different filters

As we saw a changes in the performance of the model based on the different filters. We decided to compute the same metrics on the LME's and Consensus' predictions using the last two filters since they can be used *a priori*.

7.2.1 Future filter

In Tables 7-10, 7-11 and 7-12 we compared the general accuracy of each metric and selected in each horizon which of them had a better metric.

Table 7-10: Relative MSPE general accuracy comparison (futures filter)

Horizon (months)	Futures Filter 1			Futures Filter 0		
	Model	LME	Consensus	Model	LME	Consensus
1	**0.983**	0.989	2.237	**0.972**	0.984	2.583
3	**0.881**	0.984	1.263	1.187	**0.968**	1.781
6	**0.862**	0.982	1.191	1.428	**0.948**	1.622
9	**0.880**	0.989	1.208	1.185	**0.930**	1.490
12	**0.789**	0.997	1.148	0.989	**0.937**	1.287
15	**0.716**	1.005	1.050	**0.848**	0.935	1.258
18	**0.565**	1.011	0.831	**0.808**	0.931	1.227
21	**0.555**	1.029	0.821	**0.732**	0.926	1.178
24	**0.506**	1.032	0.650	**0.680**	0.923	1.120
Horizons up to 12 months	**0.858**	0.988	1.218	1.170	**0.941**	1.531
Horizons between 13 and 24 months	**0.599**	1.016	0.859	**0.786**	0.929	1.210
Horizons up to 24 months	**0.680**	1.007	0.972	**0.886**	0.932	1.293

In the table above, the **boldface** in the columns below Futures Filter 1 and Futures Filter 0 indicates the best performance of the three predictions.

Table 7-11: RMSE general accuracy comparison (futures filter)

Horizon (months)	Future Filter 1			Future Filter 0		
	Model	LME	Consensus	Model	LME	Consensus
1	**0.153**	0.154	0.231	**0.140**	0.141	0.228
3	**0.264**	0.279	0.316	0.267	**0.241**	0.327
6	**0.364**	0.388	0.428	0.358	**0.292**	0.382
9	**0.393**	0.417	0.461	0.444	**0.394**	0.499
12	**0.420**	0.472	0.507	0.537	**0.523**	0.613
15	**0.429**	0.508	0.520	**0.586**	0.615	0.714
18	**0.396**	0.529	0.479	**0.689**	0.739	0.849
21	**0.425**	0.578	0.516	**0.691**	0.778	0.877
24	**0.429**	0.613	0.486	**0.674**	0.786	0.866
Horizons up to 12 months	**0.345**	0.370	0.411	0.370	**0.332**	0.424
Horizons between 13 and 24 months	**0.424**	0.552	0.508	**0.642**	0.698	0.797
Horizons up to 24 months	**0.387**	0.470	0.462	**0.494**	0.507	0.597

In the table above, the **boldface** in the columns below Futures Filter 1 and Futures Filter 0 indicates the best performance of the three predictions.

Table 7-12: Dstat general accuracy comparison (futures filter)

Horizon (months)	Future Filter 1			Future Filter 0		
	Model	LME	Consensus	Model	LME	Consensus
1	0.576	**0.582**	0.582	0.582	**0.593**	0.548
3	0.552	**0.557**	0.547	0.531	**0.586**	0.543
6	**0.593**	**0.593**	0.472	0.440	**0.716**	0.447
9	**0.534**	**0.534**	0.435	0.507	**0.706**	0.471
12	0.495	0.489	**0.533**	0.538	**0.731**	0.438
15	0.521	0.473	**0.596**	0.643	**0.732**	0.429
18	0.582	0.464	**0.670**	0.699	**0.774**	0.430
21	0.605	0.462	**0.682**	**0.848**	0.709	0.468
24	0.696	0.476	**0.754**	**0.900**	0.729	0.414
Horizons up to 12 months	0.549	**0.550**	0.510	0.522	**0.663**	0.472
Horizons between 13 and 24 months	0.585	0.470	**0.657**	0.723	**0.740**	0.437
Horizons up to 24 months	0.567	0.510	**0.584**	0.600	**0.693**	0.458

In the table above, the **boldface** in the columns below Futures Filter 1 and Futures Filter 0 indicates the best performance of the three predictions.

After analyzed the three metrics we noted that this filter indicates that when its value is one, we can trust the model's forecast and when the filter is zero we can trust the LME's forecast.

7.2.2 Expected filter

In Tables 7-13, 7-14 and 7-15 we compared the general accuracy of each metric and selected in each horizon which of them had a better metric.

Table 7-13: Relative MSPE general accuracy comparison (expected filter)

Horizon (months)	Expected Filter 1			Expected Filter 0		
	Model	LME	Consensus	Model	LME	Consensus
1	0.997	**0.992**	2.562	**0.948**	0.979	2.128
3	1.004	**0.984**	1.487	0.995	**0.963**	1.413
6	0.994	**0.966**	1.372	1.141	**0.992**	1.136
9	**0.960**	0.965	1.406	1.118	**0.966**	1.098
12	**0.802**	0.967	1.313	1.023	**0.971**	1.051
15	**0.662**	0.975	1.165	**0.947**	0.966	1.130
18	**0.523**	0.976	0.953	**0.879**	0.964	1.121
21	**0.472**	0.998	0.869	**0.826**	0.965	1.113
24	**0.417**	1.005	0.649	**0.774**	0.968	1.077
Horizons up to 12 months	**0.943**	0.968	1.412	1.072	**0.972**	1.167
Horizons between 13 and 24 months	**0.539**	0.985	0.948	**0.868**	0.966	1.110
Horizons up to 24 months	**0.677**	0.979	1.106	**0.912**	0.967	1.122

In the table above, the **boldface** in the columns below Expected Filter 1 and Expected Filter 0 indicates the best performance of the three predictions.

Table 7-14: RMSE general accuracy comparison (expected filter)

Horizon (months)	Expected Filter 1			Expected Filter 0		
	Model	LME	Consensus	Model	LME	Consensus
1	0.144	**0.143**	0.231	**0.152**	0.154	0.228
3	0.267	**0.264**	0.325	0.261	**0.257**	0.311
6	0.374	**0.369**	0.440	0.331	**0.309**	0.330
9	**0.418**	0.419	0.506	0.410	**0.381**	0.406
12	**0.434**	0.477	0.556	0.541	**0.527**	0.548
15	**0.422**	0.512	0.560	**0.619**	0.626	0.677
18	**0.392**	0.535	0.529	**0.703**	0.737	0.794
21	**0.389**	0.566	0.528	**0.737**	0.797	0.856
24	**0.377**	0.584	0.470	**0.751**	0.840	0.886
Horizons up to 12 months	**0.355**	0.360	0.434	0.359	**0.342**	0.374
Horizons between 13 and 24 months	**0.402**	0.544	0.533	**0.685**	0.723	0.775
Horizons up to 24 months	**0.378**	0.454	0.483	**0.531**	0.547	0.589

In the table above, the **boldface** in the columns below Expected Filter 1 and Expected Filter 0 indicates the best performance of the three predictions.

Table 7-15: Dstat general accuracy comparison (expected filter)

Horizon (months)	Expected Filter 1			Expected Filter 0		
	Model	LME	Consensus	Model	LME	Consensus
1	0.562	**0.579**	0.562	**0.609**	0.602	0.570
3	0.526	**0.578**	0.522	0.583	0.553	**0.602**
6	0.547	**0.661**	0.419	0.490	**0.606**	0.558
9	0.587	**0.591**	0.409	0.382	**0.637**	0.539
12	**0.581**	0.571	0.467	0.375	**0.625**	0.548
15	**0.632**	0.539	0.588	0.427	**0.635**	0.417
18	**0.662**	0.551	0.652	0.528	**0.596**	0.461
21	**0.686**	0.515	0.670	**0.650**	0.575	0.500
24	**0.758**	0.526	0.742	**0.732**	0.592	0.451
Horizons up to 12 months	0.563	**0.600**	0.466	0.481	**0.597**	0.552
Horizons between 13 and 24 months	**0.670**	0.537	0.639	0.537	**0.604**	0.465
Horizons up to 24 months	**0.612**	0.571	0.546	0.506	**0.600**	0.513

In the table above, the **boldface** in the columns below Expected Filter 1 and Expected Filter 0 indicates the best performance of the three predictions.

The same conclusion can be drawn from the expected filter: If its value is one we can trust the model's forecast and when the filter is zero we can trust the LME's forecast.

7.3 Recommendation

In order to obtain an accurate prediction of the copper spot price we recommend to use the model by Cifuentes et al. (2020) when the price of the expectative implied by the futures contracts price, reflected in the futures curve, is upwards compared to the historical (futures filter) or when the price of the expectative implied by the model is upwards compared to the historical (expected filter). Otherwise, we recommend to use the LME's futures contracts.

8. CONCLUSION

This study analyzed the forecasting power of a model proposed by Cifuentes et al. (2020). The proposed model uses jointly futures contracts and analysts' expectations to estimate two curves: a futures curve and an expected curve. With this last curve, we predict copper spot prices between 2014 and 2020 in different horizons. This prediction was compared against the no change forecast, a consensus proxy build with the analysts' consensus provided by Bloomberg and a futures proxy obtained with the nearest futures contract of LME and COMEX exchange.

Three metrics were applied to obtain the level accuracy and the directional accuracy of the predictions. Analyzing the metrics of each prediction three main conclusion can be drawn: first the LME's futures contracts had a better performance than the COMEX's futures contracts when predicting the spot price, the metrics of COMEX were worse in level and directional accuracy. Second, the model is better than the consensus forecast in both level and directional accuracy, although the consensus did not obtain good metrics, a better fit is observed for periods longer than one year, which may suggest that it provides information that could be useful at these horizons. Third, when comparing the model with the LME futures contracts, mixed results are obtained, for the case of horizons up to twelve months the futures contracts are better than the model and the no change forecast. However, as the horizon increases, the futures contracts have worse results and the model's prediction begins to take on more value since its predictions are more accurate.

A recommendation was made to use for when to trust the model's predictions. A ratio calculated with the price of the futures contract given by the model, or the expected spot price determined by the model with respect to the current spot price. Using this ratio, the rule is as follows: if the ratio is above the historical median, the prediction is possibly more reliable, and it is recommended to use the model over the other options mentioned.

This results provide useful insight for people involved in the copper market, either to evaluate mining projects or to invest in these projects, as a large part of the performance of these projects is impacted by the price of the metal (Dooley & Lenihan, 2005). Obtaining the best prediction is a difficult task, but a good approximation can be useful in these cases, so we propose using our model when predicting copper spot price for horizon above 12 months and LME futures contracts for horizons below.

For future research, we propose using the futures contracts and analysts' expectations in other models since they provide useful information to forecast copper prices. It can also be tested for horizon longer than 24 months.

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A P P E N D I X

APPENDIX A: MODEL'S PERFORMANCE BASED ON DIFFERENT FILTERS

In this appendix, we compute the Relative MSPE, RMSE and Dstat for each filter and for the different horizons forecasted each year.

A.1 Spot filter

Table A-1: Model's Relative MSPE, spot up

Horizon (months)	General Accuracy	Yearly Accuracy						
		2014	**2015**	2016	2017	2018	2019	2020
1	**0.703**	1.147	**0.723**	**0.613**	**0.547**	**0.502**	**0.569**	**0.838**
3	**0.600**	**0.661**	**0.428**	**0.399**	**0.563**	**0.489**	**0.383**	**0.747**
6	**0.514**	**0.135**	38.388	**0.193**	**0.509**	**0.269**	1.143	**0.649**
9	**0.423**	243.660	42.893	**0.120**	**0.451**	**0.768**	**0.266**	**0.645**
12	**0.261**	-	**0.055**	**0.151**	**0.414**	**0.054**	**0.424**	**0.708**
15	**0.226**	-	**0.159**	**0.170**	**0.310**	18.015	**0.435**	-
18	**0.207**	-	**0.227**	**0.170**	**0.284**	18.123	**0.473**	-
21	**0.218**	-	**0.324**	**0.175**	**0.174**	**0.338**	**0.580**	-
24	**0.212**	-	**0.232**	**0.146**	**0.441**	**0.592**	-	-
Horizons up to 12 months	**0.437**	**0.649**	**0.894**	**0.177**	**0.468**	**0.427**	**0.363**	**0.668**
Horizons between 13 and 24 months	**0.224**	-	**0.252**	**0.167**	**0.311**	**0.544**	**0.492**	-
Horizons up to 24 months	**0.292**	**0.649**	**0.289**	**0.169**	**0.421**	**0.517**	**0.466**	**0.668**

The table above follows the same format as the Table 6-3.

Table A-2: Model's Relative MSPE, spot down

Horizon (months)	General Accuracy	Yearly Accuracy						
		2014	2015	2016	2017	2018	2019	2020
1	1.269	**0.745**	1.354	2.256	2.099	1.371	1.471	1.230
3	1.488	**0.858**	1.853	6.318	2.168	1.470	1.561	1.392
6	1.768	**0.920**	2.212	15.417	2.992	1.593	2.043	9.971
9	1.640	**0.901**	2.274	-	1.817	1.746	2.450	-
12	1.436	**0.866**	2.511	-	2.128	1.686	1.585	-
15	1.254	**0.846**	2.329	-	2.294	1.542	1.792	-
18	1.077	**0.815**	2.318	-	2.041	1.434	-	-
21	**0.980**	**0.791**	3.114	-	1.621	1.345	-	-
24	**0.890**	**0.737**	3.992		1.599	1.109	-	-
Horizons up to 12 months	1.590	**0.889**	2.258	7.846	2.079	1.634	1.935	1.399
Horizons between 13 and 24 months	1.076	**0.803**	2.467	-	1.822	1.367	1.672	-
Horizons up to 24 months	1.211	**0.817**	2.346	7.846	1.881	1.435	1.857	1.399

The table above follows the same format as the Table 6-3.

Table A-3: Model's RMSE, spot up

Horizon (months)	General Accuracy	Yearly Accuracy						
		2014	2015	2016	2017	2018	2019	2020
1	0.123	0.102	0.101	0.132	0.111	0.068	0.077	0.901
3	0.220	0.126	0.116	0.182	0.212	0.086	0.109	0.358
6	0.298	0.037	0.241	0.172	0.258	0.089	0.091	0.568
9	0.304	0.096	0.330	0.152	0.309	0.087	0.136	0.676
12	0.282	-	0.105	0.244	0.331	0.024	0.302	0.750
15	0.275	-	0.156	0.310	0.197	0.165	0.368	-
18	0.274	-	0.166	0.359	0.111	0.148	0.294	-
21	0.287	-	0.276	0.367	0.097	0.120	0-348	-
24	0.284	-	0.307	0.308	0.137	0.321	-	-
Horizons up to 12 months	0.259	0.103	0.180	0.178	0.267	0.084	0.154	0.490
Horizons between 13 and 24 months	0.285	-	0.236	0.334	0.163	0.198	0.374	-
Horizons up to 24 months	0.272	0.103	0.223	0.274	0.227	0.145	0.283	0.490

The table above follows the same format as the Table 6-4.

Table A-4: Model's Relative RMSE, spot down

Horizon (months)	General Accuracy	Yearly Accuracy						
		2014	2015	2016	2017	2018	2019	2020
1	0.169	0.141	0.184	0.150	0.119	0.184	0.150	0.258
3	0.301	0.248	0.382	0.219	0.195	0.307	0.280	0.434
6	0.404	0.320	0.543	0.318	0.212	0.380	0.364	0.141
9	0.478	0.433	0.666	-	0.469	0.362	0.387	-
12	0.561	0.604	0.715	-	0.416	0.389	0.562	-
15	0.609	0.719	0.779	-	0.381	0.457	0.485	-
18	0.649	0.817	0.677	-	0.375	0.526	-	-
21	0.673	0.845	0.469	-	0.509	0.579	-	-
24	0.683	0.804	0.403	-	0.499	0.693	-	-
Horizons up to 12 months	0.419	0.397	0.507	0.253	0.320	0.351	0.371	0.364
Horizons between 13 and 24 months	0.641	0.788	0.656	-	0.439	0.532	0.501	-
Horizons up to 24 months	0.530	0.638	0.604	0.253	0.397	0.454	0.395	0.364

The table above follows the same format as the Table 6-4.

Table A-5: Model's Dstat, spot up

Horizon (months)	General Accuracy	Yearly Accuracy						
		2014	2015	2016	2017	2018	2019	2020
1	**0.900**	0.421	**0.947**	**1.000**	**1.000**	**0.909**	**1.000**	**0.853**
3	**0.953**	**0.692**	**1.000**	**1.000**	**1.000**	**0.900**	**1.000**	**0.935**
6	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**
9	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**
12	**1.000**	-	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**
15	**1.000**	-	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	-
18	**1.000**	-	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	-
21	**1.000**	-	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	-
24	**1.000**	-	**1.000**	**1.000**	**1.000**	**1.000**	-	-
Horizons up to 12 months	**0.975**	**0.708**	**0.992**	**1.000**	**1.000**	**0.973**	**1.000**	**0.946**
Horizons between 13 and 24 months	**1.000**	-	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	-
Horizons up to 24 months	**0.987**	**0.708**	**0.998**	**1.000**	**1.000**	**0.985**	**1.000**	**0.946**

The table above follows the same format as the Table 6-5.

Table A-6: Model's Dstat, spot down

Horizon (months)	General Accuracy	Yearly Accuracy						
		2014	2015	2016	2017	2018	2019	2020
1	0.222	**0.941**	0.030	0.000	0.000	0.067	0.000	0.214
3	0.168	**0.775**	0.000	0.000	0.000	0.000	0.000	0.000
6	0.167	**0.696**	0.000	0.000	0.000	0.000	0.000	0.000
9	0.188	**0.692**	0.000	-	0.000	0.000	0.000	-
12	0.203	**0.736**	0.000	-	0.000	0.000	0.000	-
15	0.240	**0.774**	0.000	-	0.000	0.000	0.000	-
18	0.288	**0.811**	0.000	-	0.000	0.020	-	-
21	0.346	**0.849**	0.000	-	0.000	0.050	-	-
24	0.454	**0.868**	0.000	-	0.000	0.296	-	-
Horizons up to 12 months	0.183	**0.737**	0.002	0.000	0.000	0.008	0.000	0.065
Horizons between 13 and 24 months	0.298	**0.813**	0.000	-	0.000	0.047	0.000	-
Horizons up to 24 months	0.235	**0.778**	0.001	0.000	0.000	0.028	0.000	0.065

The table above follows the same format as the Table 6-5.

A.2 Futures filter

Table A-7: Model's Relative MSPE, futures filter = 1

Horizon (months)	General Accuracy	Yearly Accuracy						
		2014	2015	2016	2017	2018	2019	2020
1	**0.983**	-	-	**0.837**	**0.810**	1.204	1.028	**0.992**
3	**0.881**	-	3.825	**0.479**	**0.705**	1.356	1.259	**0.838**
6	**0.862**	-	2.156	**0.164**	**0.539**	1.473	2.234	**0.628**
9	**0.880**	-	3.281	**0.067**	**0.669**	1.715	1.685	**0.645**
12	**0.789**	3.831	2.826	**0.190**	**0.749**	1.650	**0.800**	**0.708**
15	**0.716**	1.573	1.703	**0.187**	**0.876**	1.548	**0.666**	-
18	**0.565**	1.236	1.020	**0.175**	1.332	1.439	**0.476**	-
21	**0.555**	1.082	**0.499**	**0.165**	1.168	1.319	**0.387**	-
24	**0.506**	1.021	**0.349**	**0.144**	1.301	1.006	-	-
Horizons up to 12 months	**0.858**	2.635	2.716	**0.190**	**0.657**	1.551	1.355	**0.677**
Horizons between 13 and 24 months	**0.599**	1.111	**0.891**	**0.169**	1.072	1.323	**0.605**	-
Horizons up to 24 months	**0.680**	1.115	1.199	**0.173**	**0.838**	1.381	**0.873**	**0.677**

The table above follows the same format as the Table 6-3.

Table A-8: Model's Relative MSPE, futures filter = 0

Horizon (months)	General Accuracy	Yearly Accuracy						
		2014	2015	2016	2017	2018	2019	2020
1	**0.972**	**0.810**	1.200	**0.905**	-	**0.952**	**0.877**	**0.933**
3	1.187	**0.839**	1.624	**1.443**	-	1.208	1.325	**0.929**
6	1.428	**0.911**	2.303	**1.562**	-	**0.991**	1.484	**0.820**
9	1.185	**0.902**	2.131	**0.195**	-	1.868	1.997	-
12	**0.989**	**0.863**	1.899	**0.052**	-	2.127	1.372	-
15	**0.848**	**0.839**	1.715	**0.113**	-	1.749	1.500	-
18	**0.808**	**0.795**	2.109	**0.149**	-	1.445	**0.254**	-
21	**0.732**	**0.753**	2.367	**0.227**	-	**0.384**	**0.755**	-
24	**0.680**	**0.702**	1.334	**0.168**	-	**0.708**	-	-
Horizons up to 12 months	1.170	**0.883**	2.020	**0.364**	-	1.645	1.508	0.880
Horizons between 13 and 24 months	**0.786**	**0.779**	1.976	**0.155**	-	1.156	1.151	-
Horizons up to 24 months	**0.886**	**0.797**	2.003	**0.212**	-	1.263	1.299	0.880

The table above follows the same format as the Table 6-3.

Table A-9: Model's RMSE, futures filter = 1

Horizon (months)	General Accuracy	Yearly Accuracy						
		2014	2015	2016	2017	2018	2019	2020
1	0.153	-	-	0.140	0.114	0.160	0.117	0.271
3	0.264	-	0.284	0.206	0.207	0.267	0.228	0.427
6	0.364	-	0.728	0.171	0.254	0.364	0.310	0.537
9	0.393	-	0.659	0.121	0.356	0.365	0.339	0.676
12	0.420	0.280	0.692	0.323	0.371	0.408	0.401	0.750
15	0.429	0.475	0.638	0.369	0.293	0.457	0.407	-
18	0.396	0.689	0.373	0.371	0.272	0.518	0.334	-
21	0.425	0.921	0.307	0.353	0.339	0.553	0.250	-
24	0.429	0.806	0.361	0.305	0.348	0.566	-	-
Horizons up to 12 months	0.345	0.350	0.689	0.198	0.283	0.335	0.303	0.516
Horizons between 13 and 24 months	0.424	0.806	0.447	0.350	0.319	0.513	0.404	-
Horizons up to 24 months	0.387	0.796	0.506	0.297	0.301	0.436	0.337	0.516

The table above follows the same format as the Table 6-4.

Table A-10: Model's RMSE, futures filter = 0

Horizon (months)	General Accuracy	Yearly Accuracy						
		2014	2015	2016	2017	2018	2019	2020
1	0.140	0.128	0.158	0.137	-	0.080	0.073	0.157
3	0.267	0.225	0.325	0.181	-	0.147	0.274	0.348
6	0.358	0.298	0.465	0.264	-	0.126	0.382	0.781
9	0.444	0.429	0.621	0.180	-	0.237	0.281	-
12	0.573	0.608	0.644	0.109	-	0.236	0.593	-
15	0.586	0.727	0.577	0.192	-	0.307	0.490	-
18	0.689	0.830	0.651	0.312	-	0.479	0.053	-
21	0.691	0.833	0.504	0.448	-	0.110	0.464	-
24	0.674	0.803	0.263	0.341	-	0.419	-	-
Horizons up to 12 months	0.370	0.370	0.478	0.185	-	0.183	0.340	0.341
Horizons between 13 and 24 months	0.642	0.786	0.554	0.277	-	0.331	0.436	-
Horizons up to 24 months	0.494	0.605	0.503	0.220	-	0.258	0.381	0.341

The table above follows the same format as the Table 6-4.

Table A-11: Model's Dstat, futures filter = 1

Horizon (months)	General Accuracy	Yearly Accuracy						
		2014	2015	2016	2017	2018	2019	2020
1	**0.576**	-	-	**0.654**	**0.596**	0.415	**0.622**	**0.650**
3	**0.552**	-	0.000	**0.857**	**0.692**	0.317	0.400	**0.600**
6	**0.593**	-	0.000	**0.963**	**0.904**	0.220	0.273	**0.960**
9	**0.534**	-	0.000	**1.000**	**0.750**	0.146	0.364	**1.000**
12	0.495	0.000	0.125	**1.000**	**0.558**	0.048	**0.667**	**1.000**
15	**0.521**	0.000	0.407	**1.000**	**0.558**	0.114	**0.688**	-
18	**0.582**	0.000	**0.667**	**1.000**	**0.519**	0.091	**1.000**	-
21	**0.605**	0.143	**0.718**	**1.000**	**0.577**	0.136	**1.000**	-
24	**0.696**	0.250	**0.744**	**1.000**	**0.558**	**0.568**	-	-
Horizons up to 12 months	**0.549**	0.000	0.039	**0.918**	**0.710**	0.223	0.423	**0.825**
Horizons between 13 and 24 months	**0.585**	0.098	**0.628**	**1.000**	**0.550**	0.177	**0.797**	-
Horizons up to 24 months	**0.567**	0.095	**0.506**	**0.966**	**0.630**	0.199	**0.539**	**0.825**

The table above follows the same format as the Table 6-5.

Table A-12: Model's Dstat, futures filter = 0

Horizon (months)	General Accuracy	Yearly Accuracy						
		2014	2015	2016	2017	2018	2019	2020
1	**0.582**	**0.755**	0.365	**0.654**	-	0.455	0.429	**0.679**
3	**0.531**	**0.755**	0.314	0.458	-	0.455	0.000	**0.875**
6	0.440	**0.736**	0.095	0.480	-	0.455	0.000	**1.000**
9	**0.507**	**0.698**	0.179	**1.000**	-	0.000	0.000	-
12	**0.538**	**0.750**	0.167	**1.000**	-	0.000	0.000	-
15	**0.643**	**0.804**	0.400	**1.000**	-	0.000	0.000	-
18	**0.699**	**0.896**	0.211	**1.000**	-	0.000	**1.000**	-
21	**0.848**	**0.957**	0.231	**1.000**	-	**1.000**	**1.000**	-
24	**0.900**	**0.978**	0.538	**1.000**	-	**1.000**	-	-
Horizons up to 12 months	**0.522**	**0.735**	0.220	**0.763**	-	0.285	0.067	**0.782**
Horizons between 13 and 24 months	**0.723**	**0.889**	0.310	**1.000**	-	0.434	**0.508**	-
Horizons up to 24 months	**0.600**	**0.808**	0.248	**0.842**	-	0.349	0.242	**0.782**

The table above follows the same format as the Table 6-5.

A.3 Expected filter

Table A-13: Model's Relative MSPE, expected filter = 1

Horizon (months)	General Accuracy	Yearly Accuracy						
		2014	2015	2016	2017	2018	2019	2020
1	**0.997**	-	1.454	**0.869**	**0.810**	1.220	**0.998**	**0.996**
3	1.004	-	1.636	**0.652**	**0.685**	1.442	1.244	**0.833**
6	**0.994**	-	2.264	**0.428**	**0.506**	1.445	2.220	**0.637**
9	**0.960**	9.237	2.356	**0.120**	**0.508**	2.323	1.703	**0.645**
12	**0.802**	3.831	2.137	**0.151**	**0.551**	3.730	**0.807**	**0.708**
15	**0.662**	1.623	1.708	**0.170**	**0.618**	2.262	**0.717**	-
18	**0.523**	1.304	1.520	**0.170**	1.137	1.834	**0.475**	-
21	**0.472**	1.095	**0.796**	**0.175**	1.052	1.715	**0.499**	-
24	**0.417**	1.036	**0.393**	**0.146**	1.329	**0.894**	-	-
Horizons up to 12 months	**0.943**	2.245	2.196	**0.240**	**0.539**	1.851	1.369	**0.678**
Horizons between 13 and 24 months	**0.539**	1.118	1.206	**0.167**	**0.872**	1.559	**0.625**	-
Horizons up to 24 months	**0.677**	1.126	1.582	**0.180**	**0.661**	1.615	**0.882**	**0.678**

The table above follows the same format as the Table 6-3.

Table A-14: Model's Relative MSPE, expected filter = 0

Horizon (months)	General Accuracy	Yearly Accuracy						
		2014	2015	2016	2017	2018	2019	2020
1	**0.948**	**0.810**	1.067	-	-	1.117	1.116	**0.937**
3	**0.995**	**0.839**	3.649	-	1.552	1.264	1.364	**0.948**
6	1.141	**0.911**	1.532	-	2.958	1.452	1.422	**0.868**
9	1.118	**0.898**	-	-	1.690	1.604	1.833	-
12	1.023	**0.863**	-	-	1.798	1.472	1.321	-
15	**0.947**	**0.839**	-	-	1.860	1.379	1.391	-
18	**0.879**	**0.802**	-	-	1.584	1.251	**0.126**	-
21	**0.826**	**0.752**	-	-	1.326	1.113	**0.766**	-
24	**0.774**	**0.704**	-	-	1.245	1.038	-	-
Horizons up to 12 months	1.072	**0.882**	1.226	-	1.822	1.478	1.438	**0.914**
Horizons between 13 and 24 months	**0.868**	**0.781**	-	-	1.492	1.199	1.197	-
Horizons up to 24 months	**0.912**	**0.798**	1.226	-	1.581	1.276	1.321	**0.914**

The table above follows the same format as the Table 6-3.

Table A-15: Model's RMSE, expected filter = 1

Horizon (months)	General Accuracy	Yearly Accuracy						
		2014	2015	2016	2017	2018	2019	2020
1	0.144	-	0.150	0.139	0.114	0.142	0.111	0.231
3	0.267	-	0.328	0.195	0.206	0.232	0.225	0.414
6	0.374	-	0.527	0.221	0.272	0.235	0.310	0.547
9	0.418	0.206	0.631	0.152	0.324	0.274	0.335	0.676
12	0.434	0.280	0.659	0.244	0.333	0.292	0.397	0.750
15	0.422	0.492	0.609	0.310	0.247	0.368	0.414	-
18	0.392	0.678	0.493	0.359	0.208	0.475	0.312	-
21	0.389	0.920	0.366	0.367	0.266	0.434	0.307	-
24	0.377	0.663	0.339	0.308	0.309	0.479	-	-
Horizons up to 12 months	0.355	0.380	0.532	0.192	0.265	0.239	0.301	0.505
Horizons between 13 and 24 months	0.402	0.775	0.489	0.334	0.261	0.421	0.401	-
Horizons up to 24 months	0.378	0.757	0.511	0.272	0.263	0.348	0.337	0.505

The table above follows the same format as the Table 6-4.

Table A-16: Model's RMSE, expected filter = 0

Horizon (months)	General Accuracy	Yearly Accuracy						
		2014	2015	2016	2017	2018	2019	2020
1	0.152	0.128	0.165	-	-	0.157	0.117	0.190
3	0.261	0.225	0.085	-	0.243	0.263	0.278	0.365
6	0.331	0.298	0.479	-	0.158	0.372	0.405	0.798
9	0.410	0.432	-	-	0.454	0.375	0.298	-
12	0.541	0.608	-	-	0.478	0.421	0.636	-
15	0.619	0.726	-	-	0.424	0.482	0.494	-
18	0.703	0.825	-	-	0.471	0.545	0.034	-
21	0.737	0.833	-	-	0.599	0.593	0.460	-
24	0.751	0.837	-	-	0.532	0.627	-	-
Horizons up to 12 months	0.359	0.370	0.195	-	0.368	0.353	0.351	0.347
Horizons between 13 and 24 months	0.685	0.789	-	-	0.504	0.545	0.473	-
Horizons up to 24 months	0.531	0.607	0.195	-	0.447	0.454	0.392	0.347

The table above follows the same format as the Table 6-4.

Table A-17: Model's Dstat, expected filter = 1

Horizon (months)	General Accuracy	Yearly Accuracy						
		2014	2015	2016	2017	2018	2019	2020
1	**0.562**	-	0.292	**0.654**	**0.596**	0.432	**0.651**	**0.600**
3	**0.526**	-	0.314	**0.673**	**0.700**	0.393	0.419	**0.630**
6	**0.547**	-	0.078	**0.731**	**1.000**	0.421	0.261	**0.962**
9	**0.587**	0.000	0.135	**1.000**	**0.902**	0.316	0.348	**1.000**
12	**0.581**	0.000	0.154	**1.000**	**0.725**	0.111	**0.652**	**1.000**
15	**0.632**	0.000	0.404	**1.000**	**0.707**	0.227	**0.629**	-
18	**0.662**	0.000	0.500	**1.000**	**0.628**	0.120	**1.000**	-
21	**0.686**	0.000	**0.596**	**1.000**	**0.682**	0.345	**1.000**	-
24	**0.758**	0.364	**0.692**	**1.000**	**0.644**	**0.767**	-	-
Horizons up to 12 months	**0.563**	0.000	0.177	**0.845**	**0.804**	0.350	0.420	**0.819**
Horizons between 13 and 24 months	**0.670**	0.133	**0.510**	**1.000**	**0.670**	0.328	**0.784**	-
Horizons up to 24 months	**0.612**	0.125	0.348	**0.922**	**0.738**	0.338	**0.539**	**0.819**

The table above follows the same format as the Table 6-5.

Table A-18: Model's Dstat, expected filter = 0

Horizon (months)	General Accuracy	Yearly Accuracy						
		2014	2015	2016	2017	2018	2019	2020
1	**0.609**	**0.755**	0.429	-	-	0.400	0.333	**0.739**
3	**0.583**	**0.755**	0.000	-	0.500	0.292	0.000	**0.857**
6	0.490	**0.736**	0.000	-	0.500	0.182	0.000	**1.000**
9	0.382	**0.712**	-	-	0.182	0.000	0.000	-
12	0.375	**0.750**	-	-	0.000	0.000	0.000	-
15	0.427	**0.804**	-	-	0.000	0.000	0.000	-
18	**0.528**	**0.860**	-	-	0.000	0.037	**1.000**	-
21	**0.650**	**0.978**	-	-	0.000	0.174	**1.000**	-
24	**0.732**	**1.000**	-	-	0.000	0.455	-	-
Horizons up to 12 months	0.481	**0.737**	0.405	-	0.181	0.151	0.060	**0.792**
Horizons between 13 and 24 months	**0.537**	**0.884**	-	-	0.000	0.113	0.417	-
Horizons up to 24 months	**0.506**	**0.807**	0.405	-	0.083	0.133	0.168	**0.792**

The table above follows the same format as the Table 6-5.