PONTIFICIA UNIVERSIDAD CATÓLICA DE CHILE
ESCUELA DE INGENIERÍA

LONG-TERM POWER SYSTEMS PLANNING WITH OPERATIONAL FLEXIBILITY

ALAN BASTIÁN VALENZUELA MEZA

Thesis submitted to the Office of Research and Graduate Studies
in partial fulfillment of the requirements for the degree of
Master of Science in Engineering

Advisors:
MATÍAS NEGRETE PINCETIC
DANIEL OLIVARES QUERO

Santiago de Chile, May 2017

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You have no responsibility to live up to what other people think you ought to accomplish. I have no responsibility to be like they expect me to be. It's their mistake, not my failing.

Richard Feynman

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ABSTRACT

In recent years, there have been unprecedented levels of renewable energy penetration in power systems. As several countries have set ambitious future goals for renewable energy generation, the inclusion of these sources is expected to increase. This have drawn great attention to the operational challenges associated with their volatile nature. In this regard, the concept of flexibility –a power systems's ability to react to sudden changes in demand and supply– becomes key. While significant advances have been made to improve the modelling of flexibility of power systems in operational stages, this issue is generally neglected in expansion planning models, due to computational capabilities. To address this issues, this work presents a tractable power system generation and transmission expansion planning model that enables to obtain a near-optimal capacity mix, considering renewable penetration with detailed operation. This is achieved by considering a relaxation of unit commitment (UC) constraints to account for operational flexibility requirements. The proposed formulation is compared in terms of optimality and computation time with two benchmark models: a planning model with exact UC representation, and another in which UC constraints are not considered. The results show that the proposed formulation is able to represent the operational flexibility in planning decisions, with reduced computation times.

Keywords: Expansion planning, transmission planning, unit commitment, flexibility.

RESUMEN

En los últimos años, han habido niveles sin precedentes de penetración de energía renovable en los sistemas de potencia. Dado que various países se han propuesto ambiciosas metas futuras, se espera que la inclusión de estas fuentes aumente. Esta inclusión ha llamado la atención sobre los desafíos operacionales relacionados con su carácter volátil. En este ámbito, el concepto de flexibilidad -la capacidad de los sistemas de energía para reaccionar a cambios repentinos en la demanda y la suministro- pasa a ser clave. Aunque se han hecho avances significativos para mejorar el modelamiento de la flexibilidad de los sistemas de potencia en las fases operacionales, este problema generalmente se descuida en los modelos de planificación de la expansión, debido a problemas computacionales. Para abordar estos problemas, este trabajo presenta un modelo de planificación de expansión de generación y transmisión de sistemas de potencia manejable que permite obtener un mix de capacidad casi óptimo, considerando penetración renovable con operación detallada. Esto se logra al considerar una relajación de las restricciones de pre-despacho (UC) para tener en cuenta los requisitos de flexibilidad operacional. La formulación propuesta se compara en términos de optimalidad y tiempo de resolución con dos modelos de referencia: un modelo de planificación con representación exacta de UC y otro en el que no se consideran las restricciones de UC. Los resultados muestran que la formulación propuesta es capaz de representar estrechamente la flexibilidad operativa en las decisiones de planificación, con tiempos de resolución reducidos.

Palabras claves: Planificación de la expansión, planificación de transmisión, pre-despacho, flexibilidad.

1. INTRODUCTION

Power systems are currently facing deep changes, as they move to an energy mix less dependent on fossil fuels and with higher penetration of renewable energy sources. These sources bring environmental advantages over conventional generation, mainly due to the reduction of greenhouse gas emissions. As several countries are concerned about these emissions, renewable goals have been a popular scheme to adress this issue, as can be observed in Table 1.1. This table also shows that the inclusion of these sources will increase. Considering high renewable penetration levels and harnessing all the benefits of integrating these sources brings several challenges to the planning, operation and associated markets of electricity systems (Després et al., 2015). The source of many of those challenges are intrinsically related to the requirement of achieving a continuous balance between demand and supply for a reliable operation. In this setting, flexibility which is the ability of a power system to react to sudden changes in demand and supply becomes a key issue. In particular in the case of solar and wind energy, these sources have uncertainty in their forecast, for which achieving a continuous balance of demand and supply is a current challenge in operation (Cochran et al., 2014). A higher inclusion of these renewable sources also needs a higher capacity of flexible generators to account for the uncertainty of their forecast.

Table 1.1. Renewable energy portfolios in the world (REN21, 2015)

Country	Share (2014)	Target
Austria	68.1% (2013)	70.6% by 2020
Belgium	12.3% (2013)	20.9% by 2020
France	20.0%	27.0% by 2020
Germany	28.0%	40-45% by 2025
New Zealand	80.0%	90.0% by 2025
Spain	36.4% (2013)	38.1% by 2020

Historically, power system planning was somehow decoupled from more detailed operational issues, as a simplified operation is considered. However, in systems with large penetration of renewable energy sources in which flexibility of the generation fleet is critical, the inclusion of some features of such operational constraints is required (Palmintier & Webster, 2016). Considering those characteristics, which are usually in hourly timescale, creates several challenges on the planning models in terms of time resolution, complexity and computational capabilities (Pfenninger et al., 2014), for which they are usually are omitted.

In this thesis we explore the planning of future power systems with focus on detailed operational modelling. For this, we review models for planning in both generation and transmission, models for operation of a power system with detailed thermal constraints and literature review of planning models. As most of these models do not consider renewable generation in an accurate way, this work proposes a tractable planning expansion model that enables to obtain a near-optimal capacity mix, considering renewable penetration with detailed operation. To achieve this, we formulate a relaxation of the detailed operation of th system, simplifying the problem of considering binary variables in a detailed operation. The proposed formulation is compared with other two benchmark models, one consisting of a full detailed operational model and other consisting in a simplified operational model. The obtained results show advantages in our model in terms of optimality and computation time, enabling us to represent the operational flexibility in planning decisions.

In what follows of this chapter we will present a brief review of expansion planning tools, which are the main tools for used by academics, researchers and policy makers to plan the future power system. We will explore expansion in both generation and transmission. We will also present an introduction to the Unit Commitment (UC) problem, which is a detailed formulation of the operation of a power system. Current models are also presented and discussed, to give an overview of the modelling features included to overcome the integration of renewable energy.

1.1. Expansion Planning Review

Traditionally, the expansion of the system is analyzed through two problems: Generation expansion planning (GEP) and Transmission expansion planning (TEP). Both problem can be considered as a highly constrained large-scale mixed-integer non-linear program, which generally contain an objective function consisting of investment and operational costs, and several constraints that detail the feasible structure and operation of the system. In the following sections we will describe each one of these problem.

1.1.1. Generation Expansion Planning

GEP is related to the investment in generation projects, and determines what generating units should be constructed and when generating units should come on line over a long-term planning horizon (20 to 30 years), based in the criteria of minimizing total cost and/or enhancing the reliability with different types of constraints. The total costs depend on the investment and the operation costs. Several constraints can be used, which can vary depending on the operational detail to be considered, as can be satisfying expected demand, limiting generation to installed capacity, security constraints (reserves), generation limits for each unit, installation schedule limitations, considering single-bus or multi-bus, etc. In the case of generation mix with high share of hydro capacity, stochastic formulations have been proposed (Kelman, 1998), as the operation is coupled in time, that is, a decision today affects operating costs in the future.

The GEP problem has been widely discussed in the literature (Kagiannas et al., 2004; Zhu & Chow, 1997). The problem is usually analyzed in various versions, among which an important distinction is the consideration of the transmission network, classifying as single bus or multi-bus. The characteristics of the transmission system can be relevant in generation expansion planning, and can even change dramatically due to a proactive transmission expansion and impacts of congestion in the grid (Sauma & Oren, 2006).

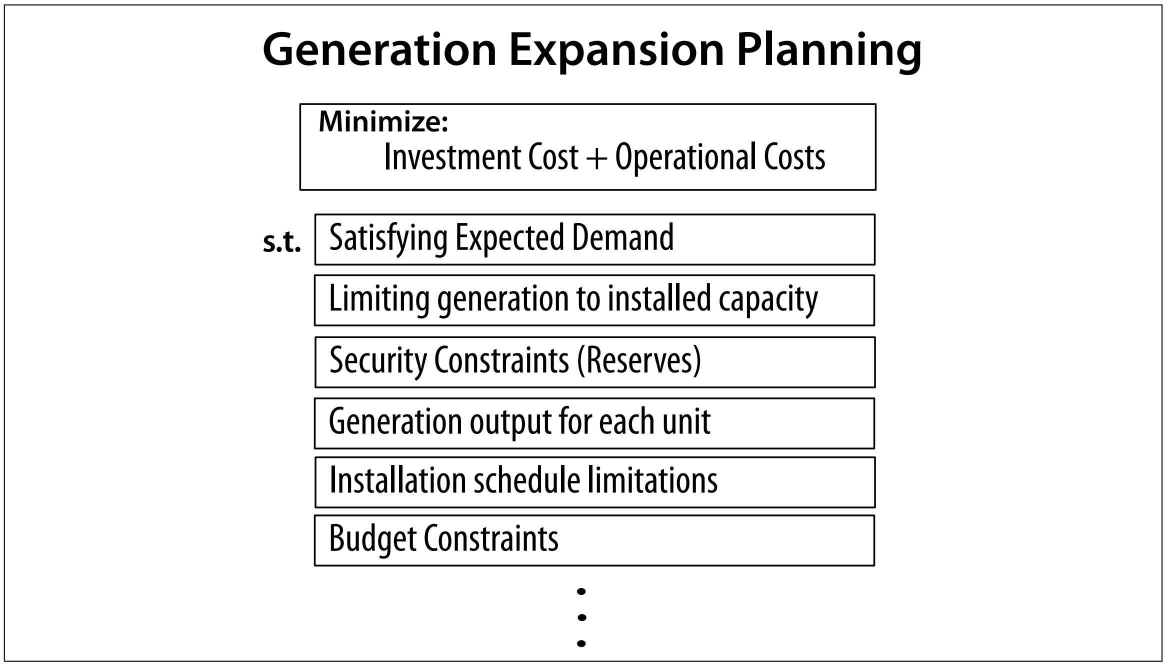

Figure 1.1. Generation Expansion Planning

Historically, formal methods for optimizing least cost investment plans in electric power started in the 1940s and 1950s. Various methodologies, from graphical methods (Kirchmayer et al., 1955) to mathematical optimization models (Masse & Gibrat, 1957), aim to solve the same core problem of minimizing the sum of investment and operation costs. The first linear programs of the GEP problem were developed (Masse & Gibrat, 1957; Beglari & Laughton, 1975; Anderson, 1972), using early advances in computers. These formulations started the trend of using advanced algorithms and computers to solve complex electricity investment planning problems.

One of the early graphical methods, which is still a widely used tool for illustrating concepts today, is the load duration curve. This tool provide a valuable intuition into production costing, as it has been used to explain the economic value of having peaking units (Kirchmayer et al., 1955). The method has been expanded, including more plant types (Galloway et al., 1960), probabilistic production functions cost (Baleriaux et al., 1967) and hydro-storage (Ramos et al., 1989). The improvements of the mathematical methods have widely replaced the use of graphical methods, by considering complex optimization problems.

Modern methods started to appear by the 1970s, among which evolved forms of some of these tools are used today, such as MARKAL (Fishbone & Abilock, 1981) or EGEAS (Caramanis et al., 1982). The improvements in computers enable various mathematical tools to be implemented, being able to consider greater temporal resolution, distinguishing non-sequential variations across diferents scales (daily, weekly, seasonal). The methods enable to balance investment decisions and operations by optimizing.

Advanced techniques were developed in the upcoming years to overcome the different problems, such as dimensionality by using Benders' decomposition (Bloom, 1983) or Dantzig-Wolfe's decomposition (Singh et al., 2009), Dynamic Programming used in EGEAS (Caramanis et al., 1982) and heuristic techniques (Fukuyama & Chiang, 1996).

In the last decades, several extensions have been developed. Demand side management resources have been analyzed (Hill, 1991), consideration of markets and competitive electricity industry (Chuang et al., 2001) and co-optimization of transmission expansion planning (Wenyuan & Billinton, 1993). Uncertainty handling is a particularly relevant topic, as the expansion problem is highly dependent on the considered data, including demand profiles, fuel cost and water availability in hydro-thermal systems. Mathematical optimization models have been developed to overcome these problems, such as stochastic optimization (Gandulfo et al., 2014), robust optimization (Dehghan et al., 2014) and progressive hedging (Munoz & Watson, 2015).

1.1.2. Transmission Expansion Planning

In a similar way to the previous case, TEP decides when, where and how many new transmission lines (and substations) should be constructed, over a long-term planning horizon, also considering criteria of minimizing costs and/or enhancing reliability. Different constraints are considered, as can be the modelling of the network (AC/DC power flow, transportation network model), power limits on transmission lines, demand constraints on each node, security constraints (N-1 or others). A widely used model is the DC model, which is a simplified power flow analysis model, given that it provides similar results compared to the exact model (Padiyar & Shanbhag, 1988). Traditionally, GEP is solved as a first problem, and the obtained solution is considered as an input to the TEP problem, which is solved as a second problem.

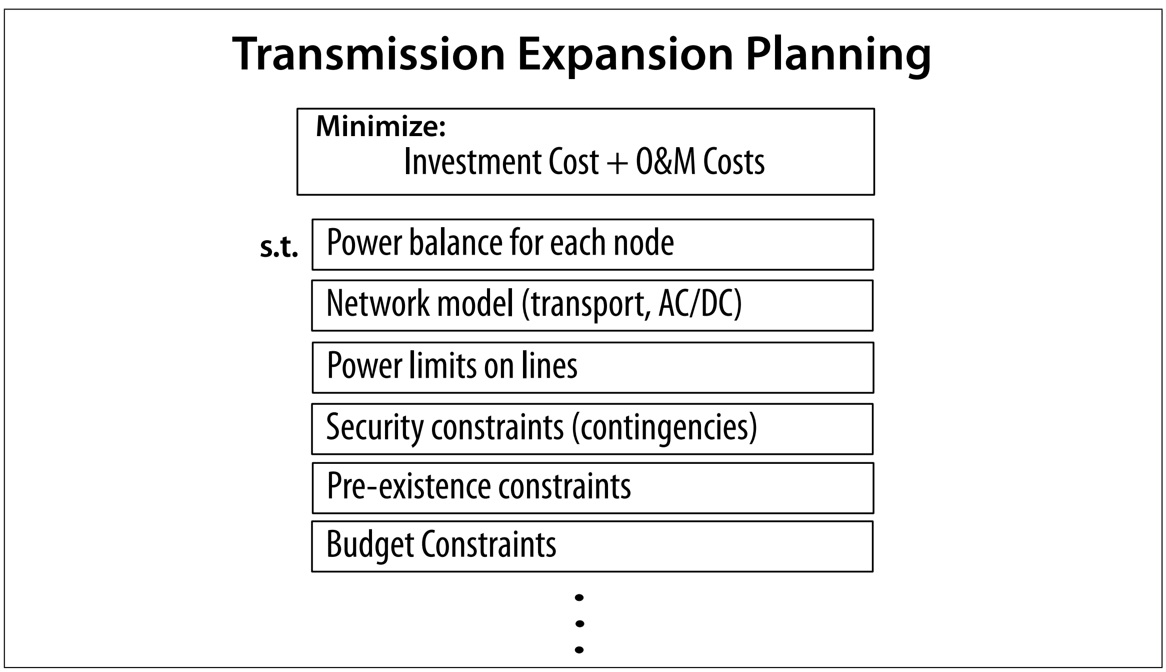

Figure 1.2. Transmission Expansion Planning

TEP has also being widely discussed in the literature (Latorre et al., 2003; Hemmati et al., 2013b). Historically, transportation model was first proposed in (Garver, 1970), using linear programming, and has been later improved (Romero et al., 2003). Other models have considered decomposition approaches (Levi & Calovic, 1991; Romero & Monticelli, 1994) and branch and bound models (Haffner et al., 2000; Rider et al., 2008).

TEP can be traditionally classified as static (single-stage) or as dynamic (multi-stage) planning. Static planning means that only where and how many elements have to be added, not considering when, in contrast to dynamic planning. As dynamic planning is highly complex, static problems have been more developed, enabling considering more characteristics, while dynamic planning has been solved considering a sequential planning problems (Binato & Oliveira, 1995).

Another classification of the problem can be considered as the power system structure. In regulated systems, the main objective of TEP is to meet the demand of loads while maintaining reliability and service quality. Based on known information of loads, generating units, load and dispatch patterns, a solution can be designed by considering the least cost transmission plan, while achieving certain reliability criteria. In deregulated environment, TEP include maximization of the investor's profit as its constraints, resulting in a different decision. Furthermore, power system uncertainties have increased by deregulation (De la Torre et al., 1999), for which a more detailed study must be considered. The planification in a deregulated power system can be considered as centralized or decentralized, which differ if a central entity, e.g. system operator, takes all planning decisions.

As a last differentiation, which takes more importance recently, is the consideration of uncertainty. A deterministic approach consider a known realization, i.e. all information is known, and is usually designed to be worst case of the system. A non-deterministic approach consider several possible realizations each with own probabilities, preparing for all the possible cases.

1.2. Unit Commitment

Operational costs take a very important role in expansion planning problems, as they are integral part of the objective function. Generally, the operation is considered as an economic dispatch, which is the problem of obtaining the minimum generating costs subject to demand balance and reserve constraints. Economic dispatch considers a very simplified operation for the system, as a generating unit can cycle indefinitely and other operational constraints are omitted. Under a large penetration of renewable energy, the economic dispatch can result in an unfeasible state for the generating units, as several of these units are not designed to have large variations in their power outputs. Therefore, detailed operation needs to be included in order to account for the flexibility and hourly behavior. In order to do so, we can consider the UC problem.

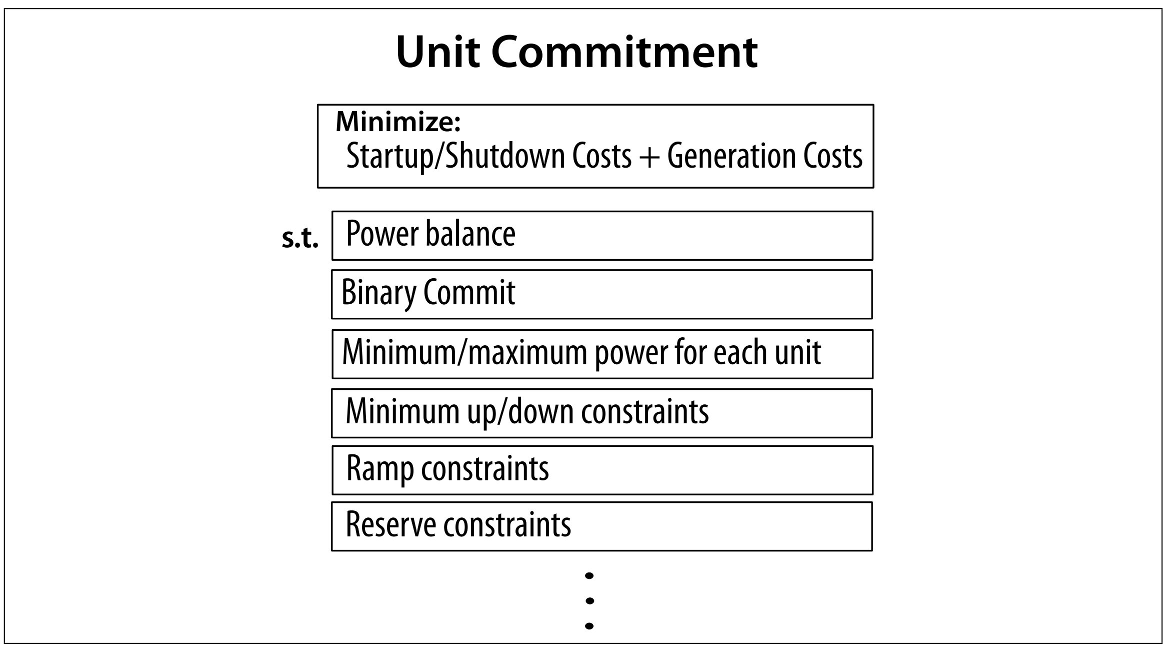

Figure 1.3. Unit Commitment

The UC problem is a milestone problem in power systems operations. It aims to obtain the commitment of each unit, i.e. the state of each unit in each time period (on or off), and the power generated by each unit, while balancing demand and supply of electricity and satisfying operational constraints, such as minimum power, maximum power, ramps constraints, minimum up and down constraints, reserve and transmission constraints, etc, over a restricted time horizon.

This problem has also been widely discussed in literature (Padhy, 2004; Saravanan et al., 2013) and is actively researched, as better solutions mean less costly dispatch. Minor differences, e.g. 0.5% reduction of fuel use, can result in savings of million dolars per years to large utilities (C.-A. Li et al., 1997). UC is a mixed integer linear program (MILP) by nature hard to solve, due to its high dimensionality and the use of binary variables, as the solution states which unit is either on or off in each time period. This increases the possible combination of feasible solutions of the UC making hard to find optimal solutions, and added to the different constraints considered to model the systems, such as thermal generator constraints or reserve constraints, increases its complexity even more. Various approaches have been developed, which ranges from complex mathematical optimization models to simple "rule-of-thumb" methods, e.g. priority lists.

In a regulated environment, the UC problem is solved by the independent system operator to minimize overall cost in a centralized system and plan the next 24 to 72 hours of operation. In a market system, generating companies submit bids, in order to maximize their revenue, hence the UC schedule is also essential, resulting in prices to use as a signal by the operator to enable a feasible operation of the system.

Various versions of UC can be distinguished. First, we can consider price based UC, in which there is no restriction of satisfying hourly loads, as the objective is to maximize the payoff. The prices enables to decide is a unit goes on/off (T. Li & Shahidehpour, 2005). Profit based UC represents deregulated environments, which is a more complex and competitive approach. Finally, a security based UC involves determining efficient operation of the units, also considering power flow constraints and generator maintenance.

1.3. State of the art

Recently, there are approaches that combines GEP and TEP in a unified model, generation and transmission expansion planning, and are broadly used in environmental policy and planning studies (Nelson et al., 2012; Short et al., 2011). The combination of both problems have been discussed in literature (Hemmati et al., 2013a), however is more recent and restricted than the discussion of the separate problems. Despite having a higher dimensionality with its corresponding difficulties, integrating GEP and TEP would yield better results overall (Sharan & Balasubramanian, 2012). In the case of renewable energy, the performance of these energy sources is highly dependant on the location, and the integration of TEP also enables to account for flexibility in transmission (Cochran et al., 2014), as extensions in the transmission system can benefit power plants and reduce unwanted effects (Schaber et al., 2012), such as reduced revenues.

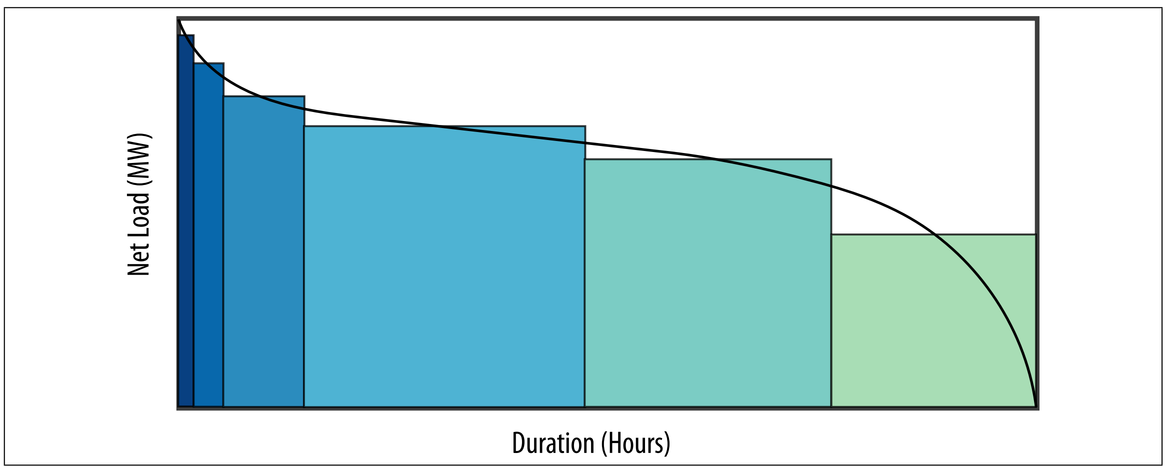

Figure 1.4. Load Duration Curve: There is no flexibility considered classically

Classically, expansion models consider load duration curves to represent the demand. A load duration curve is a curve constructed by considering hourly demand in a year. That is, all the demand of a year is ordered decreasingly with respect to the hours, obtaining a single curve as in Figure 1.4. This load duration curve is modeled consisting of six blocks, as each block is an approximation of a segment of the load duration curve. As the load

Model	Citation	Time horizon (year)	Temporal Granularity	Transmission model	TEP
E2M2	(Spiecker et al., 2011), (Keppler & Cometto, 2012)	40	144 time slices (12 representative days)	No	No
EGEAS	(Rastler et al., 2011)	40	12 time slices	No	No
PERSEUS	(Rosen, 2008), (Rosen et al., 2007)	20 (5 year step)	36 time slices (8 days)	No	No
PLEXOS	(Johnson, 2014)	30	Chronological or load duration curve (6 blocks)	NTC or DC	Yes
ReEDS	(Short et al., 2011)	44 (2 year step)	17 time slices (4 representative days)	NTC or DC	No
SWITCH	(Nelson et al., 2012)	40 (4 years step)	144 time slices (6 representative days)	NTC	Yes

Table 1.2. Review of Expansion Models

duration curve considers a year restricted to a few states, representative blocks, which are dispatched independently, the chronological information of the demand is omitted, therefore no consideration of flexibility is included.

The growth and inclusion of new technologies defies the classical approach, as hourly behavior becomes more relevant each day, which is not considered in these models as they aggregate similar hours, not considering temporal resolution (Shortt et al., 2013). Flexibility is omitted in classical planning models and even current accurate long-term energy models do not consider a proper temporal representation to account for inter-temporal constraints (Després et al., 2015). Current models, which are summarized in Table 1.2, have started to approach the time resolution which higher detail, instead of considering load duration curves. However, the considered time slices are usually not chronological, for which they may not be suitable for high renewable penetration.

We can also analyze the operational capabilities of the different models. Current power system expansion models (Nelson et al., 2012; Short et al., 2011; Cometto et al., 2012; Capros, 2013) do not account for UC constraints, omitting several thermal generators characteristics. For low penetration of renewable energy sources these omissions have not been an issue. However, for large penetrations of renewable energy sources the lack of consideration of those thermal unit constraints could increase the costs once the system is operated. Indeed, renewable energy might have even to be curtailed which may be or not compensated (Rogers et al., 2010; C. Li et al., 2015) due to the infeasibility of the system to cope with the renewable energy flexibility requirements.

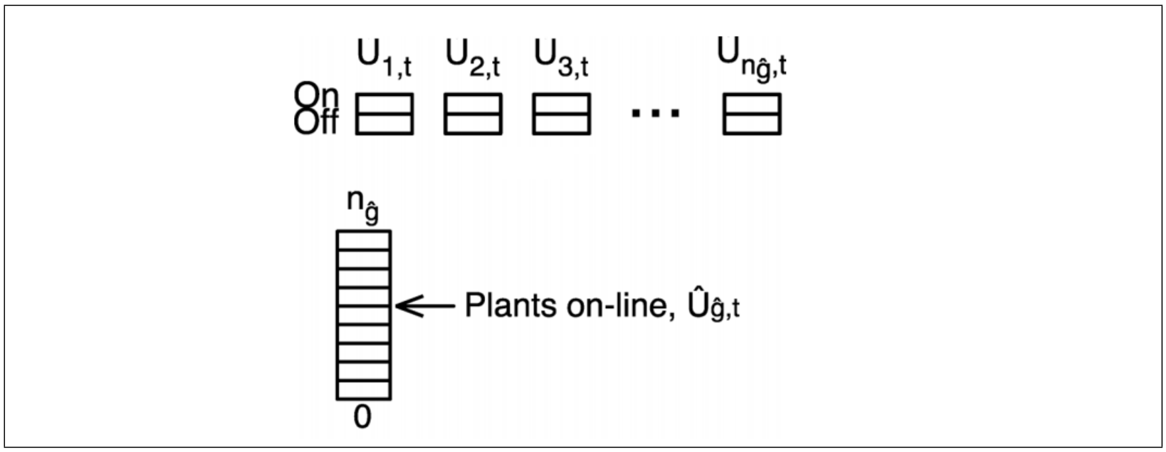

Figure 1.5. Integer Clustering Method (Palmintier & Webster, 2016)

In the literature, efforts have been developed recently to overcome the different challenges, while trying to model the different detail of operation to effectively account for renewable integration. Clustering techniques have been adopted recently (Palmintier & Webster, 2016; Zhang et al., 2016) to overcome the problems of using binary variables in UC. These techniques are based on replacing a set of different generators by a reduced set of similar generators, replacing binary variables by a lower number of integer variables, as can be observed in Figure 1.5, or even relaxing to continuous variables. This method obtains fast solutions, however, the generator units are modified and the transmission system is omitted, which is relevant due to the strong dependence of the location of renewable projects. Heuristic approaches, which have been widely used for solving UC, have also been proposed in planning models in (Bent et al., 2011; Rajesh et al., 2016). These formulations allows fast solutions without guarantees of global optimality. In addition, these methods are dependent on tuning parameters and omit real UC operations. Other methods, as screening curve (Batlle & Rodilla, 2013), have similar disadvantages.

1.4. Main Contributions

In this work, we present an expansion planning model with an embedded relaxation of the UC problem which allows capturing flexibility attributes of the generation fleet and the transmission system in an efficient way. These flexibility attributes enables us to represent a system with renewable penetration with more detail, which in practice means a near-optimal mix capable of supporting higher levels of renewable energy. The UC relaxation maintains a tractable form of the UC, considering similar restrictions to the original problem but bypassing the problem of the binary variables in operation, as the desired result is investment in an optimum operation, but not the operation scheduling itself. In specific our contributions include:

- Formulation of a combined generation and transmission expansion planning model, considering renewable portfolio constraints for high levels of renewable energy.
- Formulation of a relaxation of the UC problem, simplifying the use of binary variables in the operation and achieving better representation of thermal units.
- Presentation of extensive numerical simulations using the proposed model for systems with different levels of flexibility and time resolution which reinforce the impact of considering operational constraints in planning decisions.

1.5. Document Structure

This work is structured as follows. In **Chapter 2** we describe in detail our proposed mathematical model, presenting the convex relaxation along with the characteristics of the optimization model. The constraints used in expansion models and UC problems are discussed.

In **Chapter 3** we illustrate two alternative formulations to compare our proposed model: one with full UC which is a overdetailed mode, and other with no UC constraints, resembling a traditional formulation. We also present two test systems used, one corresponding to ISO New England and other corresponding to Chile (CDEC-SIC and CDEC-SING).

In **Chapter 4** we obtain and analyze results, comparing our formulation to the other models, while varying modeling characteristics such as renewable penetration or time resolution. Finally, concluding remarks and future work are presented in **Chapter 5**.

2. MODEL FORMULATION

2.1. Nomenclature

2.1.1. Indices and sets

$g, \mathcal{G}$	Index, set of generator unit.
$l, \mathcal{L}$	Index, set of transmission lines.
$t, \mathcal{T}$	Index, set of scheduling hours.
$\omega, \mathcal{W}$	Index, set of representative week.
$y, \mathcal{Y}$	Index, set of year.
$a, \mathcal{A}$	Index, set of load zones.
e	Index of generator, transmission or renewable project e.
$\mathcal{G}_c$	Set of candidate generator units to build.
$\mathcal{G}_a$	Set of generator units in load zone a.
$\mathcal{L}_c$	Set of candidate transmission lines to build.
$\mathcal{V}_a$	Set of load zones connected to load zone a.

2.1.2. Variables

$I_{g,y}$	Binary construction status of candidate generator unit g in year y.
$I^{\mathrm{tr}}_{l,y}$	Binary construction status of candidate transmission line l in year y.
$I^{\mathrm{wind}}_{a,y}, I^{\mathrm{solar}}_{a,y}$	Continuous cumulated MW investment in wind, solar power in load zone a and year y.
$P_{g,y,\omega,t}$	Generated power of generator unit g in year y, week ω and time t.
$P^{\mathrm{wind}}_{a,y,\omega,t}$, $P^{\mathrm{solar}}_{a,y,\omega,t}$	Generated power of wind, solar units in load zone a, in year y, week ω and time t.
$P^{\mathrm{cur,wind}}_{a,y,\omega,t}$, $P^{\mathrm{cur,solar}}_{a,y,\omega,t}$	Curtailed load of wind, solar energy in load zone a, in year y, week ω and time t.
$w_{g,y,\omega,t}$	Commitment of generator unit g in year y, week ω and time t.
$u_{g,y,\omega,t}$	Start up of generator unit g in year y, week ω and time t.

$T_{y,\omega,t}(a,a')$	Power supplied by transmission from load zone a to load zone a' in year y, week ω and time t.
$C^{g}_{g,y,\omega,t}$	Generating cost of generator unit g in year y, week ω and time t.

2.1.3. Parameters

$P^{\min}_{g}$	Minimum power output of generator g.
$P^{\max}_{g}$	Maximum power output of generator g.
$P^{D}_{a,y,\omega,t}$	Power demand at load zone a in year y, week ω and time t.
$G^{\text{wind}}_{\omega,t}, G^{\text{solar}}_{\omega,t}$	Hourly generation profile of an unitary wind, solar generation unit in time t.
$f^{\text{wind}}_{a}, f^{\text{solar}}_{a}$	Capacity factor of wind, solar generation in load zone a.
$\alpha_{g,k}$	Variable cost of generator g in segment k.
$\beta_{g,k}$	No-load cost of generator g in segment k.
$T^{\text{cost},e}$	Total cost of project e.
$C^{inv,e}$	Annualized investment cost of project e.
n_e	Lifetime of investment project e.
C^{inv}_{g}	Annualized investment cost of candidate generator g.
$C^{\text{tr,inv}}_{l}$	Annualized investment cost of transmission line l.
$C^{\text{wind}}, C^{\text{solar}}$	Annualized cost of wind, solar investment per MW built.
$E^{\text{wind}}_{a}, E^{\text{solar}}_{a}$	Existing capacity of wind, solar capacity in area a.
SC_g	Start up cost of generator g.
mup_g	Minimum up time of generator unit g.
mdw_g	Minimum down time of generator unit g.
R_g	Ramp up/down capacity of generator unit g.
R^{su}_{g}	Ramp capacity at start up of generator unit g.
r	Reserve capacity parameter of the system.
$T^{\max}_{l}(a,a')$	Maximum transfer capacity of transmission line l between load zone a and a'.
$\eta(a,a')$	Path efficiency in transfer capacity from load zone a to a'.

γ	Annual discount rate.
γ_y	Discount factor of year y.
s	Scaling factor of yearly operation.
w	Weighted average capital cost.
n_y	Number of chronological years in a representative year.
G_y	Renewable generation goal for year y.

2.2. Overview

The proposed model is referred to as the Convex Relaxation, as the main idea is to solve a planning model that includes a relaxation of the UC problem. Basically, this relaxation comes from considering a convex approximation of the UC, in which the commitment and start-up variables, which are binary variables, become continuous. Despite that the formulation is not entirely linear, given that the investment variables are still binary, the operation is entirely linear, reducing the computational burden of including a full UC, while maintaining the flexibility insights provided in generation and transmission. In this section we provide specific details of the objective function, planning constraint, dispatch constraints, the UC relaxation used and UC constraints.

2.3. Objective Function

The objective function in our model depends on the total investment cost and the total operation costs. The investment costs are calculated as the sum of the inversion in each different project in each different representative year, considering a discount factor γ_y

$$\begin{aligned} C^{\text{total,inv}} = \sum_{y\in\mathcal{Y}} \gamma_y \Bigg(& \sum_{g\in\mathcal{G}_c} C^{\text{inv}}_g I_{g,y} + \sum_{l\in\mathcal{L}_c} C^{\text{tr,inv}}_l I^{\text{tr}}_{l,y} + \\ & \sum_{a\in\mathcal{A}} (C^{\text{wind}} I^{\text{wind}}_{a,y} + C^{\text{solar}} I^{\text{solar}}_{a,y}) \Bigg) \end{aligned} \tag{1a}$$

The investment decision variables in generation and transmission projects are considered as binary variables, as they are either build entirely or not. In the case of renewable projects, the investments are considered as continuous as the renewable sources are aggregated in large scale in the respective load zone.

All the costs are calculated at the first year, for which we consider the respective discount factor γ_y:

$$\gamma_y = \frac{1}{(1+\gamma)^{n_y(y-1)}} \sum_{j=1}^{n_y} \frac{1}{(1+\gamma)^{(j-1)}} \tag{1b}$$

where γ is the annual discount rate and n_y is the number of chronological years in a representative year.

For each project e, either generation, transmission or renewable, the investment cost is calculated respectively as the annualized cost $C^{\text{inv},e}$ based on the weighted average capital cost w using a capital recovery factor:

$$C^{\text{inv},e} = T^{\text{cost},e} \frac{w}{(1-(1+w)^{-n_e})} \tag{1c}$$

where $T^{\text{cost},e}$ is the total cost and n_e is the lifetime.

The total operation costs depends on the sum of operational cost of all generators along with their respective start-up cost, which are the terms usually considered in the objective function of an UC problem:

$$C^{\text{total,op}} = \sum_{\substack{y\in\mathcal{Y}\\ \omega\in\mathcal{W}}} \sum_{\substack{t\in\mathcal{T}\\ g\in\mathcal{G}}} \gamma_y s \left(C^{\text{gen}}_{g,y,\omega,t} + SC_g u_{g,y,\omega,t} \right) \tag{1d}$$

The operation costs are scaled with a factor s to account for yearly generation. Shutdown costs are not considered for simplification, but can be added to the formulation.

2.4. Planning Constraints

Planning constraints restrict the investment variables between different representative years. We have a first set of logic constraints, which apply for each $y \in Y - \{y_0\}$, where y_0 is the initial year:

$$I_{g,y-1} \leq I_{g,y} \qquad \forall g \in \mathcal{G}_c \tag{2a}$$

$$I^{\text{tr}}_{l,y-1} \leq I^{\text{tr}}_{l,y} \qquad \forall l \in \mathcal{L}_c \tag{2b}$$

$$I^{\text{wind}}_{a,y-1} \leq I^{\text{wind}}_{a,y} \qquad \forall a \in \mathcal{A} \tag{2c}$$

$$I^{\text{solar}}_{a,y-1} \leq I^{\text{solar}}_{a,y} \qquad \forall a \in \mathcal{A} \tag{2d}$$

To keep track of the existence of already constructed elements across the different generation, transmission and renewable projects, constraints (2a) - (2d) force such elements to appear built in future years.

To consider renewable integration, we formulate a renewable portfolio constraint. This constraint requires that a percentage of the total generation is obtained from renewable sources each year and it is applied for each $y \in Y$:

$$\begin{aligned} &\sum_{\substack{a \in \mathcal{A} \\ \omega \in \mathcal{W},\, t \in \mathcal{T}}} (P^{\text{wind}}_{a,y,\omega,t} + P^{\text{solar}}_{a,y,\omega,t}) \geq \\ G_y &\sum_{\substack{\omega \in \mathcal{W} \\ t \in \mathcal{T}}} \left(\sum_{a \in \mathcal{A}} (P^{\text{wind}}_{a,y,\omega,t} + P^{\text{solar}}_{a,y,\omega,t}) + \sum_{g \in \mathcal{G}} P_{g,y,\omega,t} \right) \end{aligned} \tag{2e}$$

Depending on the goal G_y, we can achieve different goals of penetration of renewable sources. The convergence problems associated with the UC may be hindered with this constraint in a typical UC, as this constraint couple all the dispatch decision in each time period.

2.5. Dispatch constraint

Dispatch constraints apply for each $y \in Y, \omega \in W$. The generation cost is formulated as a variable which is bounded by a piecewise linear function, and applies for each $g \in G, t \in T, k \in [1, K]$:

$$C^{\text{gen}}_{g,y,\omega,t} \geq \alpha_{g,k} P^{t}_{g,y,\omega,t} + \beta_{g,k} w^{t}_{g,y,\omega,t} \tag{3a}$$

where $\alpha_{g,k}$ and $\beta_{g,k}$ are constants dependent of the cost of a generator unit and K is the number of cuts.

The maximum/minimum constraints limits the output of an unit, depending on the commitment state. The following constraints apply for each $g \in \mathcal{G}, t \in \mathcal{T}$:

$$P^{\min}_{g} w_{g,y,\omega,t} \leq P_{g,y,\omega,t} \tag{3b}$$

$$P^{\max}_{g} w_{g,y,\omega,t} \geq P_{g,y,\omega,t} \tag{3c}$$

In the case of NCRE sources, the availability of such energy depends on the renewable profile considered, the capacity factor of the allocated resources, and the existing capacity. We also consider that renewable energy may be curtailed, which can already be seen in practice (C. Li et al., 2015). We propose the following constraints which apply for each $a \in \mathcal{A}, t \in \mathcal{T}$:

$$P^{\text{wind}}_{a,y,\omega,t} + P^{\text{cur,wind}}_{a,y,\omega,t} = G^{\text{wind}}_{\omega,t} f^{\text{wind}}_{a} (E^{\text{wind}}_{a} + I^{\text{wind}}_{a,y}) \tag{3d}$$

$$P^{\text{solar}}_{a,y,\omega,t} + P^{\text{cur,solar}}_{a,y,\omega,t} = G^{\text{solar}}_{\omega,t} f^{\text{solar}}_{a} (E^{\text{solar}}_{a} + I^{\text{solar}}_{a,y}) \tag{3e}$$

The power balance constraint enables to balance hourly demand in the different load zones, and considers the transmission between the zones through a network transport model. This constraint apply for each $a \in \mathcal{A}, t \in \mathcal{T}$:

$$\begin{aligned} &\sum_{g \in \mathcal{G}_a} P_{g,y,\omega,t} + P^{\text{wind}}_{a,y,\omega,t} + P^{\text{solar}}_{a,y,\omega,t} + \\ &\sum_{a' \in \mathcal{V}_a} \Big(\eta(a,a') T_{y,\omega,t}(a',a) - T_{y,\omega,t}(a,a') \Big) = P^{D}_{y,\omega,t,a} \end{aligned} \tag{3f}$$

The first terms represents the generated power from local generators and renewable sources, while the terms in parenthesis represents the imported power from other load zones, multiplied by the path efficiency, and the exported power to other load zones. The sum of generation must match the demand at all time periods.

The transfer capacity between load zones is constrained within the limits of the existing lines, which is expressed in the following constraint and applies for each $a \in \mathcal{A}$, $t \in \mathcal{T}$, $a' \in \mathcal{V}_a$:

$$T_{y,\omega,t}(a,a') \leq \sum_{l \in \mathcal{L}_a} T^{\max}_{l}(a,a') I^{\text{tr}}_{l,y} \tag{3g}$$

To achieve a reliable system, as projected demand might not match real demand, the system must consider a reserve policy to cope with contingencies. The following constraint represents the reserve requirements in all load zones by maintaining a maximum capacity of the generators, in each load zone, larger that the net demand. This is represented with the following constraint, which applies for each $a \in \mathcal{A}, t \in \mathcal{T}$:

$$\begin{aligned} \sum_{g \in \mathcal{G}_a} P^{\max}_{g} w_{g,y,\omega,t} + \sum_{a' \in \mathcal{V}_a} \Big(\eta(a,a') T_{y,\omega,t}(a,a') - T_{y,\omega,t}(a',a) \Big) \\ \geq P^{D}_{y,\omega,t,a}(1+r) - (P^{\text{wind}}_{a,y,\omega,t} + P^{\text{solar}}_{a,y,\omega,t})(1-r) \end{aligned} \tag{3h}$$

where r is the reserve parameter. The constraint considers that the different load zones can also provide power, as in constraint (3f). The terms of renewable generation are subtracted to obtain a net power and are reduced by the factor $(1-r)$ due to the uncertainty associated to their generation.

2.6. UC Relaxation

The UC formulation presented in our proposed model exploits convex hull approximations presented in (Falk, 1969) and used in the context of pricing mechanisms for UC problems in (Hua & Baldick, 2016). Consider vectorial terms $\boldsymbol{P_g}, \boldsymbol{w_g}, \boldsymbol{u_g}, \boldsymbol{P^D}$ over the set of scheduling hours, $\mathcal{T}$, and let X_g be the feasible operational region for unit generator g, defined by its physical and operational constraints. The UC problem aims to minimize the total operation costs, characterized by continuous operation cost functions $C_g^{\mathrm{op}}(.)$, while satisfying the balance between demand and supply and physical operation of the generation and transmission system. A standard UC problem is defined as follows:

$$\begin{aligned} \min_{\boldsymbol{P_g},\boldsymbol{w_g},\boldsymbol{u_g},g\in\mathcal{G}} \quad & \sum_{g\in\mathcal{G}} C_g^{\mathrm{op}}(\boldsymbol{P_g},\boldsymbol{w_g},\boldsymbol{u_g}) \\ \text{s.t.} \quad & \sum_{g\in\mathcal{G}} \boldsymbol{P_g} = \boldsymbol{P^D} \\ & (\boldsymbol{P_g},\boldsymbol{w_g},\boldsymbol{u_g}) \in X_g \qquad\qquad \forall g \in \mathcal{G} \end{aligned} \tag{4a}$$

Instead of solving directly the UC problem, as the problem is hard to solve, we consider to focus on the Lagrangian dual problem, which is convex. By relaxing the demand constraint in (4a) with a dual variable $\boldsymbol{q}$, the Lagrangian dual function of the UC problem is obtained:

$$\begin{aligned} & L(\boldsymbol{q}) = \\ & \min_{(\boldsymbol{P_g},\boldsymbol{w_g},\boldsymbol{u_g})\in X_g} \Big\{ \sum_{g\in\mathcal{G}} C_g^{\mathrm{op}}(\boldsymbol{P_g},\boldsymbol{w_g},\boldsymbol{u_g}) + \boldsymbol{q^T}\big(\boldsymbol{P^D} - \sum_{g\in\mathcal{G}} \boldsymbol{P_g}\big)\Big\} \end{aligned} \tag{4b}$$

Then, the Lagrangian dual problem is given by

$$\max_{\boldsymbol{q}} L(\boldsymbol{q}) \tag{4c}$$

As the set X_g is convex and the cost function is continuous, it can be shown by considering Theorem 3.3 in (Falk, 1969), that the objective function of the Lagrangian dual problem (4c) is equal to the modified primal formulation:

$$\begin{aligned} \min_{\boldsymbol{P_g},\boldsymbol{w_g},\boldsymbol{u_g},g\in\mathcal{G}} \quad & \sum_{g\in\mathcal{G}} C^{\mathrm{op}*}_{g,X_g}(\boldsymbol{P_g},\boldsymbol{w_g},\boldsymbol{u_g}) \\ & \sum_{g\in\mathcal{G}} \boldsymbol{P_g} = \boldsymbol{P^D} \\ (\boldsymbol{P_g},\boldsymbol{w_g},\boldsymbol{u_g}) \in \mathrm{conv}(X_g) & \qquad \forall g\in\mathcal{G} \end{aligned} \tag{4d}$$

where $C^{\mathrm{op}*}_{g,X_g}$ refers to the convex envelope of C^{op}_g taken over X_g and $\mathrm{conv}(\cdot)$ is the convex envelope.

The convex hull of individual units commitment and dispatch decisions along with the convex envelopes of their cost functions are evaluated in (Hua & Baldick, 2016). The resulting problem captures essential UC features and constraints in which the commitment and start up variables becomes continuous variables, using alternative constraints. The advantage of considering this method is that similar overall costs can be obtained from this formulation as compared with the full binary UC, while maintaining the problem as a linear program (Hua & Baldick, 2016). This is due to the fact that the relative gap between the UC and the dual problem approaches 0 as the number of different generators increases to infinity (Bertsekas & Sandell, 1982). Considering this linear formulation enables us represent the UC dispatch to obtain a near optimal solution, as the desired result in our problem is the investment decision and not the real operation of the system.

In the proposed formulation, we also consider renewable generation, renewable portfolio constraints, transmission network and reserve constraints. Adding renewable generation does not change the formulation, as new variables in a convex set are added to the problem. The rest of the constraints are linear constraints, therefore the original set remain convex and the approach remains similar.

2.7. UC Constraints

The UC constraints presented in this section apply for each $y \in \mathcal{Y}, \omega \in \mathcal{W}$. We use $w_{g,y,\omega,t}$ and $u_{g,y,\omega,t}$ as the commitment and start up variables, respectively. In UC problems, these variables are considered as binary variables $(w_{g,y,\omega,t}\ ,\ u_{g,y,\omega,t}) \in \{0,1\}^2$, $\forall g \in \mathcal{G}, \forall t \in \mathcal{T}$. By considering the convex hull approximation discussed in section 2.6, these binary constraints are replaced, for each $g \in \mathcal{G}, t \in \mathcal{T}$, by:

$$u_{g,y,\omega,t} \geq 0 \tag{5a}$$

$$1 \geq w_{g,y,\omega,t} \geq 0 \tag{5b}$$

The commitment and start up variables are related in the following logical constraint, which in a binary formulation forces a start-up variable to be 1 when the commitment goes from 0 to 1, and it is usually considered in UC formulations as follows:

$$u_{g,y,\omega,t} \geq w_{g,y,\omega,t} - w_{g,y,\omega,t-1} \qquad \forall g \in \mathcal{G}, \forall t \in \mathcal{T} - \{t_0\} \tag{5c}$$

where t_0 is the initial hour of the scheduling horizon. Variations of this constraint can be seen in literature when a third binary variable, a shutdown variable, is included.

The consideration of UC features at planning stages does not only require the inclusion of constraints representing operation features of the UC, e.g., constraints of thermal units. In addition, constraints which allow to setup the UC into the planning time-scales are required to define. In our model, the relaxed UC problem is solved for representative time horizons and its operation cost is considered into the planning stages. Generally, UC problems start with an initial commitment of the units, which is taken as a parameter. Due to the nature of the planning problem, this is not possible as we will dispatch in years in which the existence of generator units is a decision variable. Instead, by periodicity of the demand, we will consider that the commitment state of the unit at the beginning of the time period is the same that at the end:

$$w_{g,y,\omega,t_0} = w_{g,y,\omega,t_f} \qquad \forall g \in \mathcal{G} \tag{5d}$$

where t_0 and t_f are the initial and final hours, respectively.

The relationship between the commitment and investment variables, because a generation plant can only operate when it has been built, is considered by the following constraint:

$$w_{g,y,\omega,t} \leq I_{g,y} \qquad \forall g \in \mathcal{G}_c, \forall t \in \mathcal{T} \tag{5e}$$

We also consider the minimum up and down time constraints, which models a more realistic approach to thermal generators, as they need to be turned on/off a minimum time in a set of consecutive hours. The following constraint represents the minimum up time, which is applied for $\{t \in \mathcal{T} \mid t \geq \mathrm{mup}_g + 1\}$:

$$\sum_{\tau=t-\mathrm{mup}_g+1}^{t} u_{g,y,\omega,\tau} \leq w_{g,y,\omega,t} \qquad \forall g \in \mathcal{G} \tag{5f}$$

The minimum down time is captured in constraint (5g), which is applied for $\{t \in \mathcal{T} \mid t \geq \mathrm{mdw}_g + 1\}$

$$\sum_{\tau=t-\mathrm{mdw}_g+1}^{t} u_{g,y,\omega,\tau} \leq 1 - w_{g,y,\omega,t-\mathrm{mdw}_g} \qquad \forall g \in \mathcal{G} \tag{5g}$$

Both constraints restrict the value of $u_{g,y,\omega,t}$ as the sum cannot be greater than the right side, which depends of the commitment state.

Thermal generators are also subject to ramp constraints which bound the up and down differences in power output according to the commitment of an unit. In a binary formulation, the following constraints apply for each $g \in \mathcal{G}, t \in \mathcal{T} - \{t_0\}$ and represents respectively the ramp-up and ramp-down constraints:

$$P_{g,y,\omega,t} - P_{g,y,\omega,t-1} \leq R_i w_{g,y,\omega,t-1} + R_i^{\text{SU}}(1 - w_{g,y,\omega,t-1}) \tag{5h}$$

$$P_{g,y,\omega,t-1} - P_{g,y,\omega,t} \leq R_i w_{g,y,\omega,t} + R_i^{\text{SU}}(1 - w_{g,y,\omega,t}) \tag{5i}$$

These ramp constrains are formulated for binary values of w and u, and are not suitable if these variables take continuous values resulting from the convex relaxation. Therefore, we need to consider another set of constraints for ramp rates if we consider (5a) - (5b). In particular, we will use the 2-time period polytope (Damcı-Kurt et al., 2015). This polytope is the set of linear ramp constraints that are an exact representation of the binary ramp constraints in a 2-time period UC. For the sake of simplicity, we replace $u_{g,y,\omega,t}$ by $u_{g,t}$ and $w_{g,y,\omega,t}$ by $w_{g,t}$, as the omitted terms are constant in each set. Thus, the following constraints apply in $\{g \in \mathcal{G}, t \in \mathcal{T} \mid t \leq t_{\text{end}} - 1\}$:

$$P_{g,t} \leq R_g^{\text{su}} w_{g,t} + (P_g^{\max} - R_g^{\text{su}})(w_{g,t+1} - u_{g,t+1}) \tag{5j}$$

$$P_{g,t+1} \leq P_g^{\max} w_{g,t+1} - (P_g^{\max} - R_g^{\text{su}}) u_{g,t+1} \tag{5k}$$

$$P_{g,t+1} - P_{g,t} \leq (P_g^{\min} + R_g) w_{g,t+1} - P_g^{\min} w_{g,t} - (P_g^{\min} + R_g - R_g^{\text{su}}) u_{g,t+1} \tag{5l}$$

$$P_{g,t} - P_{g,t+1} \leq R_g^{\text{su}} w_{g,t} - (R_g^{\text{su}} - R_g) w_{g,t+1} - (P_g^{\min} + R_g - R_g^{\text{su}}) u_{g,t+1} \tag{5m}$$

Additional constraints can be considered to obtain a more precise representation, such as the 3-time period polytope (Pan & Guan, 2016). However, preliminary results shows that resolution time highly increases, as the number of constraints in the polytopes grow exponentially with the number of time periods considered, while there are no significant changes in investment decisions.

2.7.1. Model Formulation

Considering the proposed constraints explained in previous sections, the Convex Relaxation model is structured as follows:

$$
\begin{aligned}
\min_{X_{\mathrm{CR}}} \quad & C^{\mathrm{total,inv}} + C^{\mathrm{total,op}} \\
\text{s.t.} \quad & (2a) - (2e) \\
& (3a) - (3h) \\
& (5a) - (5g) \\
& (5j) - (5m)
\end{aligned}
$$

where

$$
X_{\mathrm{CR}} = \Big\{ I_{g,y}, I^{\mathrm{tr}}_{l,y}, I^{\mathrm{wind}}_{a,y}, I^{\mathrm{solar}}_{a,y}, P_{g,y,\omega,t}, P^{\mathrm{wind}}_{a,y,\omega,t}, P^{\mathrm{solar}}_{a,y,\omega,t}, P^{\mathrm{cur,wind}}_{a,y,\omega,t}, \\
P^{\mathrm{cur,solar}}_{a,y,\omega,t}, w_{g,y,\omega,t}, u_{g,y,\omega,t}, T_{y,\omega,t}(a,a'), C^{\mathrm{gen}}_{g,y,\omega,t} \Big\}
$$

This model enables us to obtain a investment and operation that considers the flexibility of the UC problem, as similar constraints are used. However, this model is still a relaxation. The relaxation will achieve good results if the system has numerous heterogeneous generators.

3. CASE STUDIES

In this section, we will show two benchmark models to compare our proposed model and two test systems, which will be used to demonstrate the effectiveness of the proposed method.

3.1. Benchmark Models

We use two benchmark to compare the proposed model: one with full detail in the operation which considers a complete characterization of the UC problem, and another one with a simplified dispatch in which chronological features are not considered.

3.1.1. Binary Formulation

As a first benchmark, we use a binary formulation for the expansion planning model in which w and u are binary variables. This formulation includes all the UC constraints, as the idea is to include a full UC in the planning model along with usual planning and dispatch constraints. The Binary Formulation is expressed as:

$$\begin{aligned}
\min_{X_{\mathrm{BF}}} \quad & C^{\text{total,inv}} + C^{\text{total,op}} \\
\text{s.t.} \quad & (2a) - (2e) \\
& (3a) - (3h) \\
& (w_{g,y,\omega,t} \;,\; u_{g,y,\omega,t}) \in \{0,1\}^2 \quad \forall g \in \mathcal{G}, \forall t \in \mathcal{T} \\
& (5c) - (5i)
\end{aligned}$$

where

$$\begin{aligned}
X_{\mathrm{BF}} = \Big\{ & I_{g,y}, I^{\mathrm{tr}}_{l,y}, I^{\mathrm{wind}}_{a,y}, I^{\mathrm{solar}}_{a,y}, P_{g,y,\omega,t}, P^{\mathrm{wind}}_{a,y,\omega,t}, P^{\mathrm{solar}}_{a,y,\omega,t}, P^{\mathrm{cur,wind}}_{a,y,\omega,t}, \\
& P^{\mathrm{cur,solar}}_{a,y,\omega,t}, w_{g,y,\omega,t}, u_{g,y,\omega,t}, T_{y,\omega,t}(a,a'), C^{\mathrm{gen}}_{g,y,\omega,t} \Big\}
\end{aligned}$$

The embedded UC model in this formulation is already hard to solve for large problems due to the use of binary variables, for which is it not practical to have this implementation in a generation and transmission expansion planning problem.

3.1.2. Dispatch-Only Formulation

As another benchmark, we consider a case that resembles the traditional approach. Though w is used, it serves only to handle reserve requirements and investment decisions, taking continuous values as in Eq. (5b). The rest of UC constraints and start-up costs are omitted, obtaining a dispatch-only operation. The Dispatch-Only Formulation is expressed as:

$$\begin{aligned}
\min_{X_{\mathrm{DF}}} \; & C^{\text{total,inv}} + \sum_{\substack{y \in \mathcal{Y} \\ \omega \in \mathcal{W}}} \sum_{\substack{t \in \mathcal{T} \\ g \in \mathcal{G}}} \gamma_y s C^{\text{gen}}_{g,y,\omega,t} \\
\text{s.t.} \quad & (2a) - (2e) \\
& (3a) - (3h) \\
& (5b),\ (5e)
\end{aligned}$$

where

$$\begin{aligned}
X_{\mathrm{DF}} = \Big\{ & I_{g,y}, I^{\text{tr}}_{l,y}, I^{\text{wind}}_{a,y}, I^{\text{solar}}_{a,y}, P_{g,y,\omega,t}, P^{\text{wind}}_{a,y,\omega,t}, P^{\text{solar}}_{a,y,\omega,t}, \\
& P^{\text{cur,wind}}_{a,y,\omega,t}, P^{\text{cur,solar}}_{a,y,\omega,t}, w_{g,y,\omega,t}, T_{y,\omega,t}(a,a'), C^{\text{gen}}_{g,y,\omega,t} \Big\}
\end{aligned}$$

Variations of these constraints are usually considered in most generation and transmission expansion planning models.

3.2. Study Cases

We consider two test systems, one based in ISO New England, and other one based in the Chilean ISO's CDEC-SIC and CDEC-SING. The characteristics of both test systems are summarized in Table 3.1. We also consider several parameters for both study cases: each representative week as a time period of 96 hours, an annual discount rate $\gamma = 5\%$, a WACC $w = 10\%$, 4 representative year of $n_y = 5$ chronological years to model a total of 20 years. All costs are discount to obtain net present value.

Table 3.1. Study Cases Characteristics

Test System	Units			Lines			Load Zones
	Total	Existing	Candidate	Total	Existing	Candidate	
8-Zone Test System	137	76	61	48	12	36	8
Chilean System	167	121	46	50	14	36	7

3.3. ISO New England: 8-Zone Test System

We consider the 8-zone test system (Krishnamurthy et al., 2016) which is based on data from ISO New England. The system consists in 76 existent generator units and 8 load zones. The minimum power of the centrals were obtained from (Papaefthymiou et al., 2014). The value of R_i^{su} is calculated as the minimum power plus half the ramp up/down value. We also consider two cuts ($K = 2$), as the costs are given as a quadratic function. To model the candidate units, we consider 61 new units, with similar characteristics to the existent units. The investment cost for the new units are obtained also from (Papaefthymiou et al., 2014). We consider no initial renewable generation in the system.

For transmission, we consider 12 existing transmission lines, connecting the zones of the original data, with a capacity of 1200 MVA. The efficiency of the lines is calculated as proportional to the line extension, assuming 1% of power transmitted is lost for every 161 km. We also add three candidate lines for each of these transmission lines, two with a capacity of 500 MVA and another with 1400 MVA. The costs of these lines are calculated respectively as proportional in capacity and length to projects of 220 kV and 500 kV respectively obtained from Chilean ISO CDEC-SIC (CDEC-SIC, 2016).

The demand in the first year is obtained from the scenarios of the original data. The renewable profiles are obtained also from aggregated wind and solar generation of Chilean ISO (CDEC-SIC, 2016). We consider a growth rate of demand of 3.8% each year. The reserve constant r takes the value of 15%.

The demand in the first year is obtained from the scenarios of the original data. The renewable profiles are obtained also from aggregated wind and solar generation of Chilean ISO (CDEC-SIC, 2016). We consider a growth rate of demand of 3.8% each year and three test cases:

- low renewable penetration goal, $G_y = [5\%, 10\%, 15\%, 20\%]$, and one representative week for each each year.
- high renewable penetration goal, $G_y = [10\%, 20\%, 30\%, 40\%]$, and one representative week for each each year.
- high renewable penetration goal, $G_y = [10\%, 20\%, 30\%, 40\%]$, and three representative weeks each year.

In the low renewable penetration, we consider one representative week for each year, modelling each year as a 96 hours period. In the first high renewable penetration, only one representative week is considered and in the second case we consider three representative weeks, modelling each year with an equivalent of 288 hours.

3.4. Chilean System: 7-Zone Test System

The Chilean System is based on the data collected from different sources, corresponding to both Chilean ISO's CDEC-SIC and CDEC-SING. The generator data is obtained from (Comisión Nacional de Energía, 2016), to which we take thermal, hydro, solar and wind units. We treat hydro units as dispatchable units, with capacity reduced to a 35% of their maximum capacity, and not considering water reserve constraints, as they complicate the formulation which is not within our scope. The generators with a capacity lower than 15 MW were aggregated by type in the corresponding load zone, which reduces the total number of existent generators to 121. The fuel prices were obtained from data of the year 2015 in (CDEC-SING, 2016), which were used to obtain the variable cost of centrals. The minimum power were obtained respectively from the ISO's database (CDEC-SING, 2016; CDEC-SIC, 2016), and were also obtained from (Papaefthymiou et al., 2014) in the case of missing data. The parameters from UC such as minimum up/down, start up cost and no load cost were obtained from (Schröder et al., 2013). The ramp up/down parameters were obtained from (Tanaka, 2011) and the start up ramp was also calculated as the minimum power plus half the ramp up/down. We consider one cut ($K = 1$), as the obtained cost are linear. To model the candidate units, we consider 46 new units, with similar characteristics to the previous units. The investment cost for the new units are obtained from (Papaefthymiou et al., 2014).

The transmission lines were obtained from (Comisión Nacional de Energía, 2016) and we consider 7 load zones, two of CDEC-SING and 5 of CDEC-SIC. We add candidate lines for each transmission lines, with a capacity between 350-700 MVAR and with a capacity between 1400-1700 MVAR, depending of the demand requirements of the zone and adding more candidates to the major consumer load zones. The costs of these lines are also calculated respectively as proportional in capacity and length to projects of 220 kV and 500 kV obtained from (CDEC-SIC, 2016).

The demand in the first year is obtained also from (CDEC-SING, 2016; CDEC-SIC, 2016), and for the next years we take a growth rate of 4.51% for CDEC-SING demand and 3.72% for CDEC-SIC demand. The reserve parameter r takes the value of 15% as well. We consider one test case with renewable goals of $G_y = [10\%, 20\%, 30\%, 40\%]$ and one week, motivated by Chile's renewable goals.

3.5. Implementation

For each case we solve all the different formulations, which result in different investment decisions across several years. As we want to compare these investment solutions, all the investment decisions will be tested in a full UC problem for the different time periods. This is solved for all years separately, obtaining a realistic estimate of impact of the investment decisions. These results are referred as the "post investment operation" in our results.

All the models are implemented in FICO Xpress FICO Xpress Optimization Suite (2015). We set the barrier method to solve linear programs, as our preliminary results show that all cases are solved faster. In the cases of one week representation we consider a target MILP tolerance or "MIP gap" of 0.15% and in the case of three week representation a "MIP gap" of 0.5% is considered as the problem is more complex. We also consider a maximum running time of 4 hours. All runs were executed in a Dell PowerEdge R630, with a processor Intel Xeon CPU E5-2620 v3 @ 2.40GHz, running Ubuntu 14.04 (Linux) with 32 GB of RAM.

4. RESULTS

We present our results for the different test cases using the formulations previously described. All cases are analyzed separately, providing table comparison and final generation mix figures to effectively illustrate the results.

4.1. 8-Zone Test System: Low Renewable Penetration

We first analyze the obtained results for the case of low penetration, which can be observed in Table 4.1. The objective values of the Binary Formulation and Convex Relaxation are very similar, and the Dispatch-Only Formulation is lower due to the simplifications. The time resolution is very different across the different formulations, as the Dispatch-Only Formulation is easily solved, the Convex Relaxation takes more time but is solved still in a reasonable time and the binary formulation exhaust the maximum resolution time, not converging with a mipgap of 0.37%. This is due to the difficulty of the binary model to start up or turn off units to achieve an optimal solution.

To analyze operation differences in the approximations of each model, we performed a simple test comparing only 4 days of operation of the first year, considering the obtained solution of the Convex Relaxation and a renewable penetration of 5%. The Convex Relaxation formulation obtains values 0.5% lower than the Binary Formulation, while the

Table 4.1. Low Renewable Penetration

Model	Binary Formulation	Convex Relaxation	Dispatch-Only Formulation
Objective Value (MM USD$)	68,032	67,713	65,828
Time (s)	>14,435	3,059	1,631
Post Inv. Operation (MM USD$)	67,933	67,906	68,505

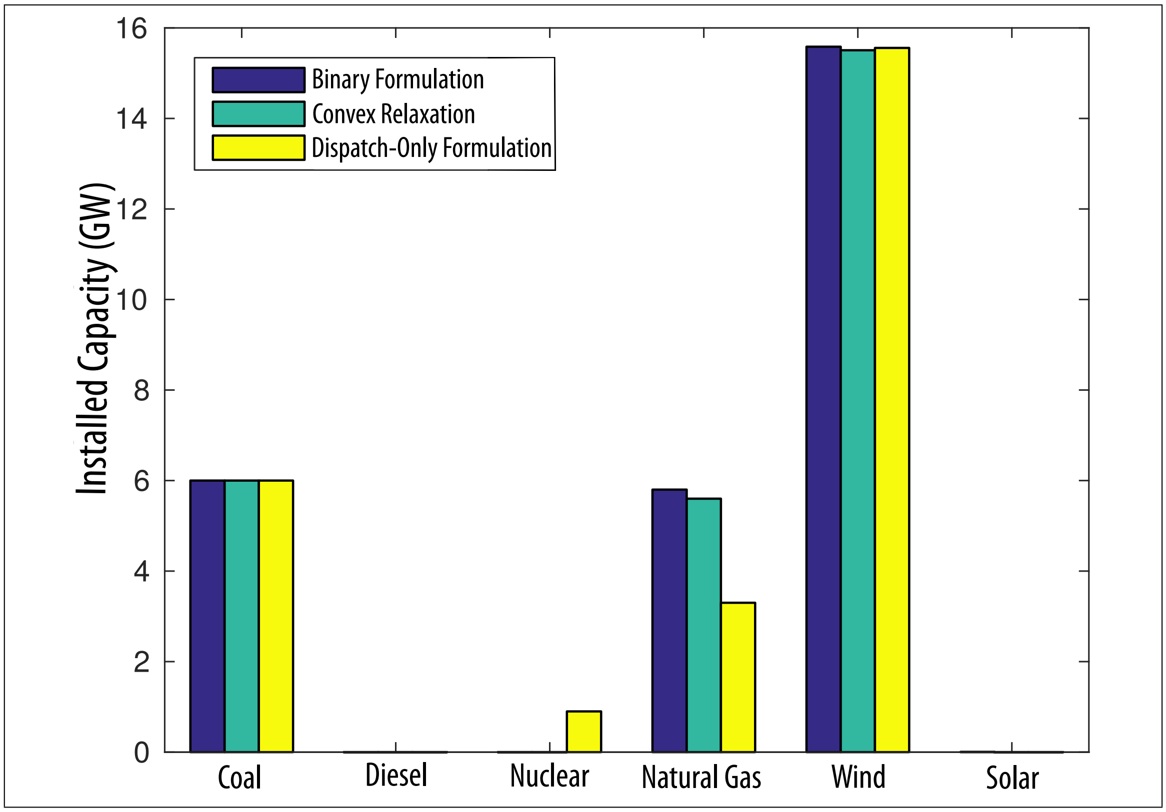

Figure 4.1. Investment in Final Year: Low Renewable Penetration

Dispatch-Only Formulation obtains values 4% lower. Therefore, in this case even as an operational tool only, the Convex Relaxation formulation obtains close results.

In the post investment operation, it can be observed that all models obtain similar solutions, being optimal the Binary Formulation with a practically equal solution to the Convex Relaxation, and with a more expensive, though still close, solution of the Dispatch-Only Formulation.

In a low renewable penetration case, the models act similarly, since there is no high flexibility needs. However, there are differences in each model generation mix, which can be observed in Figure 4.1. The Dispatch-Only Formulation underestimate the operational costs, which result in less investment in overall capacity, such as natural gas, and obtains nuclear investment, as no flexibility is considered. The differences between the

Binary Formulation and the Convex Relaxation are still minor. The allocation of renewable sources are similar in general, however, major differences are observed in the final year, resulting also in different transmission investment decisions.

4.2. 8-Zone Test System: High Renewable Penetration

In this case, the differences of the models are increased, while following the same pattern, as can be observed in Table 4.2. The values of the objective function keep being close in the case of Binary Formulation and Convex Relaxation formulation, while the value of the Dispatch-Only Formulation is lower due to the simplifications. The time resolution is also similar to the previous case across the different formulations, as the lower is the Dispatch-Only Formulation, followed by the Convex Relaxation formulation and the Binary Formulation which still does not converge, with a mipgap of 0.46%.

Table 4.2. High Renewable Penetration

Model	Binary Formulation	Convex Relaxation	Dispatch-Only Formulation
Objective Value (MM USD$)	74,381	73,962	71,058
Time (s)	>14,401	3,061	702
Post Inv. Operation (MM USD$)	74,304	74,204	78,890

We perform another test to analyze operation differences in the approximations of each model, by comparing 4 days of operation of the last year, considering the obtained solution of the Convex Relaxation and a renewable penetration of 40%. The Convex

Relaxation obtains values 3.8% lower than the Binary Formulation, while the Dispatch-Only Formulation obtains values 9.1% lower. Though the differences increases, Convex Relaxation formulation still is much closer than the Dispatch-Only Formulation, which has major differences.

In the post investment operation, the Binary Formulation and Convex Relaxation obtain very similar values too, yielding almost equivalent solutions. As the renewable penetration grows in the system, the Dispatch-Only Formulation, which does not consider the flexibility and underestimate the generation investment, obtains a solution 6.3% higher than the best solution, in which the costs increases as a result of omitting the UC. The difference in the costs become even more pronounced in the final in year, in which the costs obtained by the Dispatch-Only Formulation are 18.3% higher than the obtained by the Convex Relaxation. This result, which is not entirely new (Shortt et al., 2013; Palmintier & Webster, 2016), reinforces the importance of an appropriate planning with renewable penetration.

The investment decision for each formulation can be observed in Figure 4.2. The Dispatch-Only Formulation underestimate the flexibility of the system, as can be observed with the difference of investment in natural gas, while the others formulation obtain similar results, as well as resulting in a lower investment of wind generation, though investing more in coal generators. The system, unlike the previous case, does not invest in nuclear technology, as the size of the units are high and highly inflexible. The allocation of renewable sources vary highly in the third and fourth year along with the transmission, as renewable penetration plays a major role, and may also be the cause of the infeasibility of the Dispatch-Only Formulation.

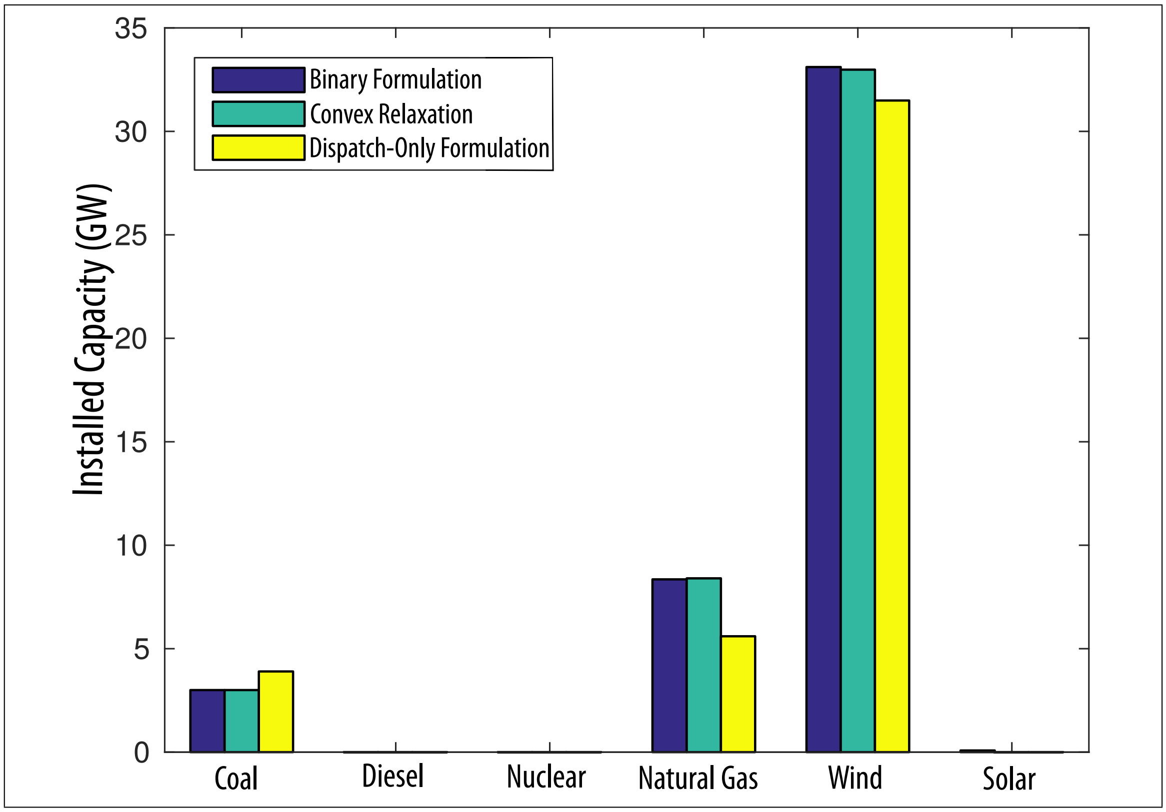

Figure 4.2. Investment in Final Year: High Renewable Penetration

4.3. 8-Zone Test System: High Renewable Penetration with Multiple Weeks

We analyze the results obtained for a high renewable penetration with multiple weeks. In this case, as demand and renewable sources vary in each week, a better decision can be made to account for the uncertainty of these elements, which is similar to an stochastic formulation.

Table 4.3. High Renewable Penetration - Multiple Weeks

Model	Binary Formulation	Convex Relaxation	Dispatch-Only Formulation
Objective Value (MM USD$)	775,563	82,659	80,188
Time (s)	>14,407	7,623	6,611
Post Inv. Operation (MM USD$)	745,688	82,878	Infeasible[1] (85,175)

The obtained results can be observed in Table 4.3. In this formulation, which is computationally much higher than the previous cases, the formulations obtain even more different results. The Binary Formulation is not suitable at all, given that after the maximum resolution time, achieves only a solution with a mipgap of 89.37% due to the high complexity of the case, as much more variables, specifically binary, are considered. This results in a very high cost, which is only a feasible solution but very far from optimal. The Convex Relaxation obtains a cost which is much more comparable to the order of the previous case and the Dispatch-Only Formulation obtains a lower cost as expected.

[1]No solution is found within the maximum resolution time in the fourth representative year.

All the times resolution are higher across all formulations, as the case is more complex to solve. The Dispatch-Only Formulation is solved in less time and the Convex Relaxation formulation is solved in a still reasonable time. The post investment operation costs clearly manifest the advantages of the Convex Relaxation formulation, as the Binary Formulation obtains a feasible solution, but far from optimal, and the Dispatch-Only Formulation obtains a infeasible solution, though its estimated cost suffer the same disadvantages as the previous case.

The investment decision are very different across the formulations, as can be observed in Figure (4.3). The Binary Formulation is not presented, as there is an over investment due to the difficulty of solving the problem, resulting in high investment across all technologies, which is the reason of the high obtained costs. The Dispatch-Only Formulation

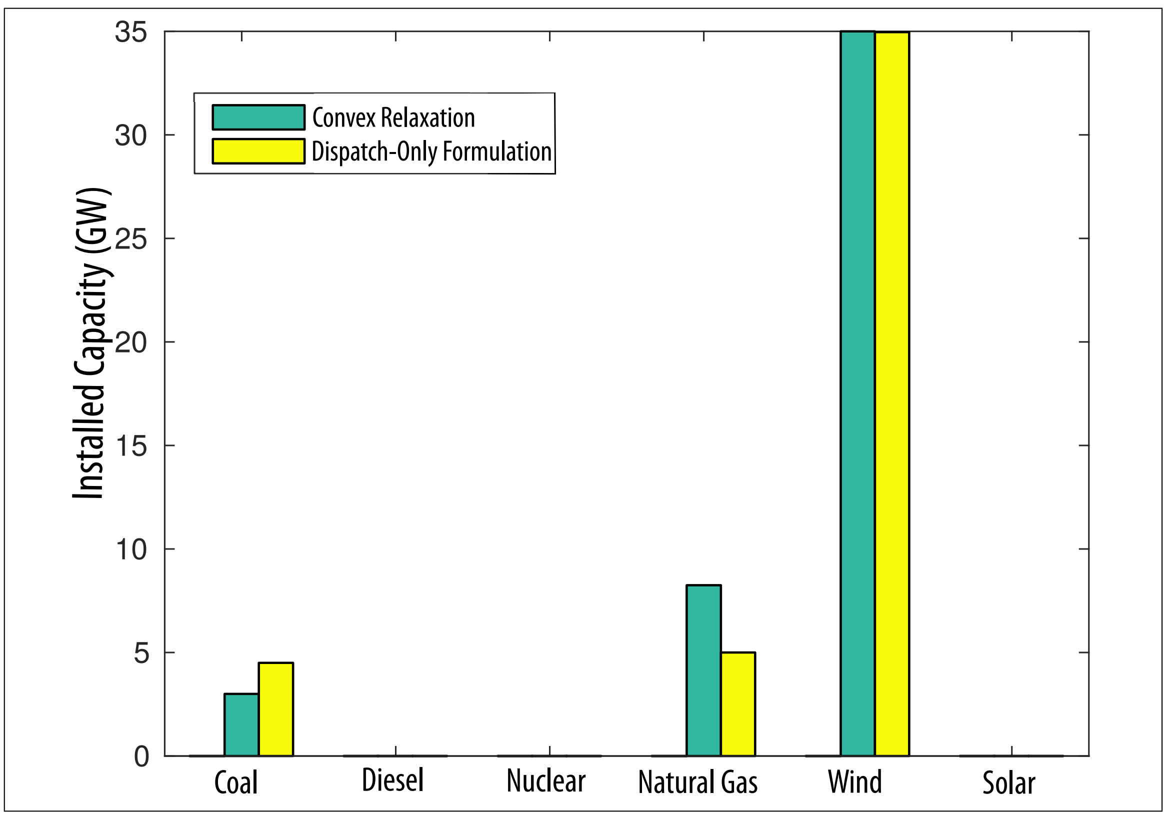

Figure 4.3. Investment in Final Year: High Renewable Penetration With Multiple Weeks

prefers coal over natural gas, once again by preferring only minimizing costs with no flexibility considerations, however, takes a similar decision in wind investment. The Convex Relaxation formulation takes into consideration the flexibility, which is more important in this case as several renewable profiles are considered, and prefers natural gas investments instead of coal.

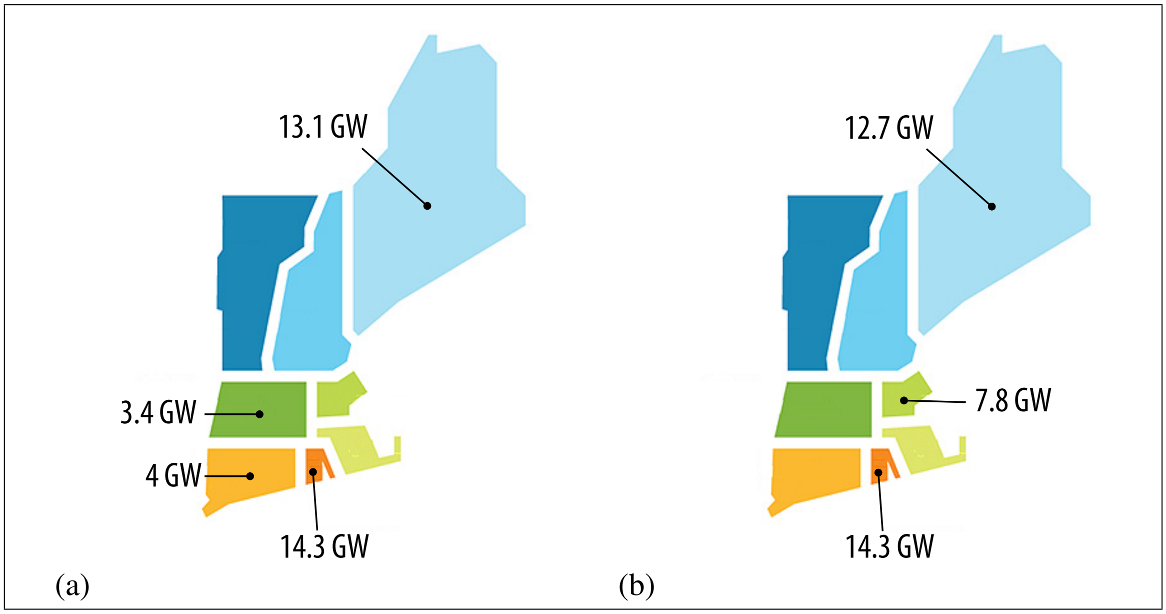

Figure 4.4. Wind investment by zone in the final year. (a) Convex Relaxation. (b) Dispatch-Only Formulation.

The allocation of renewable sources also vary in this case, which can be observed in Figure 4.4. As the Binary Formulation results are not suitable, we only compare the Convex Relaxation and Dispatch-Only Formulation solutions. Though major resources installed in two areas are similar, there are considerable differences in allocating 22% of the capacity, to which the models obtain different allocations. The obtained transmission investment decisions remains similar in both cases, favoring each renewable allocation planning by a small margin.

4.4. Chilean Test System

We show the results of the formulations in the Chilean Test System, which can be seen in Table 4.4. This case shows similar results to the previous cases, as Dispatch-Only Formulation underestimate the overall costs, the time resolution shows similar results in each case, however with higher difficulty in the Convex Relaxation, and the post investment operation is also better in the Convex Relaxation, though in this case the differences are minor.

Table 4.4. Chilean Systems Results

Model	Binary Formulation	Convex Relaxation	Dispatch-Only Formulation
Objective Value (MM USD$)	123,723	123,664	121,383
Time (s)	>14,418	6,288	1,067
Post Inv. Operation (MM USD$)	123,671	123,664	123,910

This case enables us to once again show the advantages of our proposed method in comparison to the Binary Formulation. By comparing to the Dispatch-Only Formulation we can see that the cost do not have big differences, as the flexibility of the system is not a problem. Chilean system already have high flexible capacity, as there is hydro and diesel capacity, the last one motivated by capacity payment. Besides, it should be noted that our focus is the validity of our method, and various values used in this system, mainly UC characteristics, are taken from standard values of known literature. At last, which is highly relevant concerning to the modelation of this system, the dispatch of hydro generators is

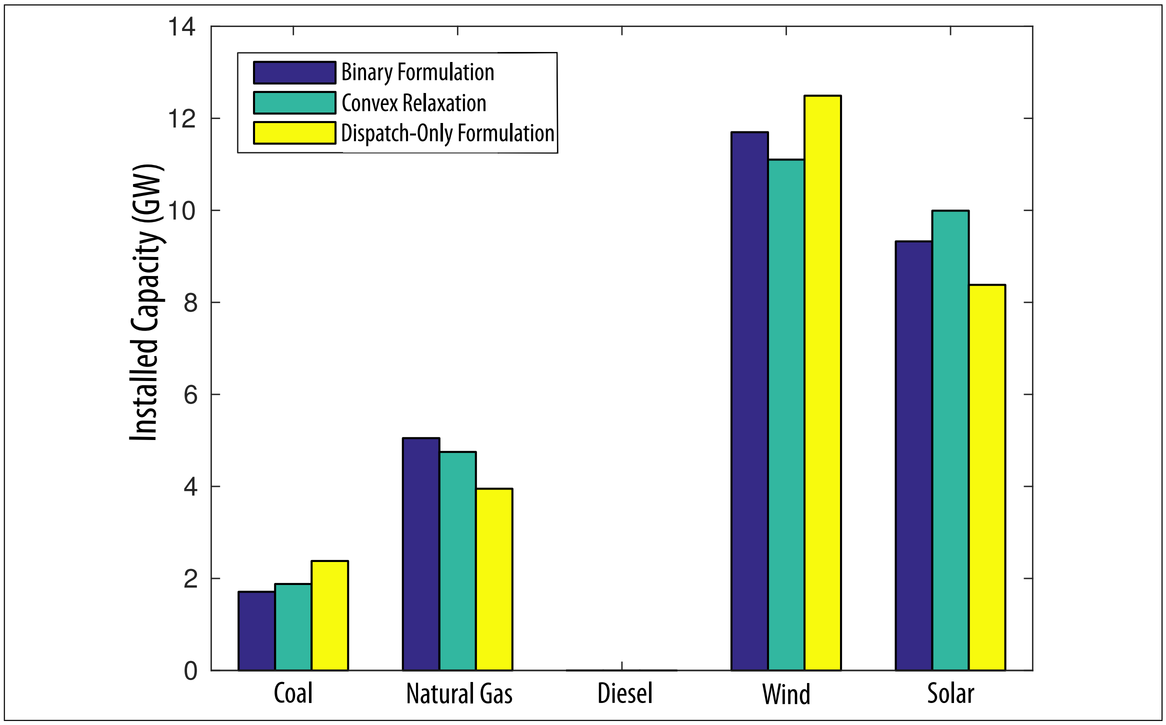

Figure 4.5. Investment in Final Year: Chile

highly simplified as thermal generators, for which the obtained results may not be a valid reference for the real system.

There are still differences in each formulation optimal mix. Though the differences are not big, the Convex Relaxation prefers a more flexible mix by preferring more natural gas and less coal than the Dispatch-Only Formulation. The Binary Formulation goes even further in the differences. The Dispatch-Only Formulation prefers wind resources to solar, as they are available with different power at all hours. The Binary Formulation also has that difference, though with less gap, while the Convex Relaxation obtains minor differences between these resources. Considering flexibility of the model, does not only change the optimal mix in conventional technologies, but also can change renewable technologies. This effect may be increased considering several weeks, as it was showed in the previous case. The allocation of renewable sources vary in the third and fourth year, considering different installed capacity in the same areas. The transmission investment decisions are very similar in all models.

5. CONCLUSIONS

This paper proposes a long term generation and transmission expansion planning model with an embedded approximation of the UC problem using a MILP formulation. The approximation of the UC problem is performed through exploiting convex hull approximations results. The model allows to perform expansion planning considering the flexibility of the system, obtaining effective solutions in the required time. The model has been compared to planning models with a full binary UC problem representation and a dispatch-only planning model without consideration of UC features. In all the cases, the proposed model outperforms the binary formulation, obtaining a better or equivalent solution in less time and outperforms the dispatch-only planning model in terms of costs.

The model provides insights about the need to incorporate flexibility considerations into planning models. The results show how the effectiveness of different formulations is closely related with the level of flexibility of the system. In a system with low flexibility, as renewable penetration grows, the dispatch-only model fails to obtain a solution that meet the requirements of the system. If more time periods are considered, due to the uncertainty of renewable sources or demand, the mix differences can also increase, and a full binary model even fails to obtain a practical solution. Even in a flexible system, the models show that there may be differences in the optimal mix capacity of conventional and renewable technologies.

Future work include the integration of a more complex hydro model representation, the integration of further approximations of operational constraints and the use of the model for energy policy and planning studies.

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APPENDICES

A. APPENDICES

A.1. 8-Zone Test System Data

i	α_1	β_1	$P^{\min}$	α_2	β_2	$P_2^{\min}$	$P^{\max}$	SC	R^{su}	R	mup	mdn	Zone	Type
1	884	10	622	710	10	933	1244	950000	682	120	24	49	NH	NUC
2	1118	10	618	921	11	926	1235	850000	678	120	24	48	CT	NUC
3	1456	5	441	1391	5	661	881	900000	501	120	24	48	CT	NUC
4	1213	7	342	1158	7	514	685	875000	402	120	24	48	SEM	NUC
5	1468	11	310	1421	11	465	620	925000	370	120	24	47	VT	NUC
6	3030	18	245	3002	18	428	612	20915	305	120	24	60	RI	BIT
7	1475	21	149	1329	21	261	372	17336	209	120	24	24	CT	SUB
8	1470	21	149	1324	21	261	372	17500	209	120	24	24	CT	SUB
9	1211	18	98	1206	18	171	244	16860	158	120	8	16	RI	BIT
10	1209	18	97	1204	18	170	244	13947	157	120	8	16	RI	BIT
11	744	18	60	742	18	105	150	13872	120	120	16	13	ME	BIT
12	717	18	58	716	18	101	144	13727	118	120	8	9	WCM	BIT
13	407	18	33	407	18	57	82	3486	93	120	16	12	ME	BIT
14	397	18	32	397	18	56	80	2441	92	120	16	12	ME	BIT
15	237	20	19	235	20	34	48	5774	79	120	8	8	NH	BIT
16	235	20	19	233	20	33	48	5774	79	120	8	8	NH	BIT
17	9355	172	120	3212	183	360	600	115287	180	120	10	8	ME	RFO
18	9411	169	112	1635	184	335	559	272400	172	120	24	32	SEM	RFO
19	11557	207	111	3932	223	332	553	191144	171	120	24	32	SEM	RFO
20	6432	168	90	3023	177	269	448	100000	150	120	14	8	CT	RFO
21	9217	204	87	4516	216	261	435	120000	147	120	12	9	RI	RFO
22	7565	244	86	3392	255	259	431	85000	146	120	16	12	ME	RFO
23	7248	235	81	6533	237	244	407	86332	141	120	16	15	CT	RFO
24	3148	55	80	2987	55	240	400	32566	140	120	6	5	NH	RFO
25	10939	207	80	8423	214	240	400	62461	140	120	16	11	CT	RFO
26	4769	199	47	3380	205	142	236	120000	107	120	16	40	CT	RFO
27	3882	156	34	3177	160	101	168	45000	94	120	17	10	CT	RFO
28	3429	156	26	3006	159	78	131	45000	86	120	12	8	CT	RFO

29	2743	195	23	2402	199	70	117	80000	83	120	16	11	CT	RFO
30	2945	157	23	2612	161	69	116	45000	83	120	8	8	ME	RFO
31	2098	54	16	2089	54	49	81	12000	76	120	16	12	CT	RFO
32	3689	25	139	2670	26	416	694	66889	339	400	12	12	ME	NGLN
33	3694	27	137	2707	28	411	685	66889	337	400	12	12	ME	NGLN
34	451	59	135	-508	60	405	676	76560	335	400	12	12	SEM	NGA4
35	505	91	111	-142	93	333	555	25000	311	400	24	24	ME	NGA1
36	-93	29	103	-652	30	309	516	325	303	400	8	6	RI	NGTN
37	-90	24	102	-633	26	305	508	1000	302	400	12	6	NH	NGMN
38	-84	24	98	-590	25	294	490	1000	298	400	14	6	ME	NGMN
39	-70	51	90	-492	52	269	448	2500	290	400	8	6	CT	NGIR
40	-202	33	54	-1412	38	163	271	2352	254	400	6	4	RI	NGTN
41	475	58	53	323	58	159	265	12000	253	400	8	6	RI	NGA4
42	700	22	50	699	22	149	249	11112	250	400	8	5	RI	NGA4
43	700	22	50	699	22	149	248	11112	250	400	8	5	RI	NGA4
44	700	22	50	699	22	149	248	11112	250	400	8	5	RI	NGA4
45	-22	53	49	-151	53	147	245	62500	249	400	12	12	ME	NGPN
46	446	59	49	274	60	147	245	12000	249	400	12	8	SEM	NGA4
47	461	58	48	318	59	143	239	12000	248	400	8	6	RI	NGA4
48	224	26	48	71	26	143	238	7500	248	400	6	6	WCM	NGT2
49	470	57	47	320	58	142	236	12000	247	400	8	6	RI	NGA4
50	238	91	31	169	91	92	154	5000	231	400	17	12	ME	NGA1
51	229	57	30	101	58	89	149	7250	230	400	8	6	RI	NGT4
52	315	58	30	108	60	89	149	4000	230	400	4	6	RI	NGA4
53	278	59	30	-1	61	89	149	3500	230	400	4	6	RI	NGA4
54	316	60	30	-35	62	89	149	4000	230	400	4	6	RI	NGA4
55	284	59	28	69	61	85	141	5000	228	400	6	5	SEM	NGA4
56	192	95	28	142	96	85	141	7250	228	400	3	1	WCM	NGT1
57	293	58	21	254	58	63	105	2500	221	400	8	8	SEM	NGA4
58	240	101	21	182	101	62	104	7750	221	400	6	6	WCM	NGT2
59	608	58	9	578	59	26	44	325	209	400	1	1	CT	NGA4
60	608	58	9	579	59	26	44	325	209	400	1	1	CT	NGA4

61	608	58	9	579	59	26	44	325	209	400	1	1	CT	NGA4
62	608	58	9	580	58	26	43	325	209	400	1	1	CT	NGA4
63	-38	376	55	-267	377	165	275	0	255	400	1	1	WCM	NGTN
64	-38	401	60	-265	402	180	300	0	260	400	1	1	RI	NGTN
65	-46	351	65	-319	352	195	325	0	265	400	1	1	RI	NGTN
66	-208	331	45	-1454	337	135	225	0	245	400	1	1	CT	RFO
67	-22	376	30	-151	377	90	150	0	230	400	1	1	WCM	NGT2
68	-33	351	35	-232	353	105	175	0	235	400	1	1	WCM	NGT2
69	-33	301	35	-232	303	105	175	0	235	400	1	1	ME	NGPN
70	-8	291	17	-55	291	51	85	0	217	400	1	1	ME	NGPN
71	-3	310	11	-23	311	33	55	0	211	400	1	1	ME	NGPN
72	-24	326	30	-170	327	90	150	0	230	400	1	1	ME	NGTN
73	-38	351	40	-269	353	120	200	0	240	400	1	1	ME	NGTN
74	-20	341	30	-142	342	90	150	0	230	400	1	1	ME	NGTN
75	-10	376	20	-71	376	60	100	0	220	400	1	1	ME	NGTN
76	-18	311	25	-125	312	75	125	0	225	400	1	1	ME	NGTN
77	1474	19	120	1435	19	210	300	10080	180	120	15	15	CT	Coal
78	1474	19	120	1435	19	210	300	10080	180	120	15	15	SEM	Coal
79	1474	19	120	1435	19	210	300	10080	180	120	15	15	VT	Coal
80	1474	19	120	1435	19	210	300	10080	180	120	15	15	RI	Coal
81	2913	19	240	2759	20	420	600	20160	300	120	15	15	ME	Coal
82	2913	19	240	2759	20	420	600	20160	300	120	15	15	WCM	Coal
83	2913	19	240	2759	20	420	600	20160	300	120	15	15	NH	Coal
84	2913	19	240	2759	20	420	600	20160	300	120	15	15	CT	Coal
85	2913	19	240	2759	20	420	600	20160	300	120	15	15	SEM	Coal
86	2913	19	240	2759	20	420	600	20160	300	120	15	15	VT	Coal
87	2913	19	240	2759	20	420	600	20160	300	120	15	15	RI	Coal
88	2913	19	240	2759	20	420	600	20160	300	120	15	15	ME	Coal
89	2508	178	24	2228	180	72	120	480	84	120	10	10	WCM	RFO
90	2508	178	24	2228	180	72	120	480	84	120	10	10	NH	RFO
91	2508	178	24	2228	180	72	120	480	84	120	10	10	CT	RFO
92	2508	178	24	2228	180	72	120	480	84	120	10	10	SEM	RFO

93	6095	181	60	4346	188	180	300	1200	120	120	10	10	VT	RFO
94	6095	181	60	4346	188	180	300	1200	120	120	10	10	RI	RFO
95	6095	181	60	4346	188	180	300	1200	120	120	10	10	ME	RFO
96	6095	181	60	4346	188	180	300	1200	120	120	10	10	WCM	RFO
97	11608	188	120	4609	201	360	600	2400	180	120	10	10	NH	RFO
98	11608	188	120	4609	201	360	600	2400	180	120	10	10	CT	RFO
99	11608	188	120	4609	201	360	600	2400	180	120	10	10	SEM	RFO
100	11608	188	120	4609	201	360	600	2400	180	120	10	10	VT	RFO
101	1273	9	300	1233	9	450	600	45600	360	120	24	48	RI	NUC
102	1273	9	300	1233	9	450	600	45600	360	120	24	48	ME	NUC
103	1273	9	300	1233	9	450	600	45600	360	120	24	48	WCM	NUC
104	1239	9	450	1148	9	675	900	68400	510	120	24	48	NH	NUC
105	1239	9	450	1148	9	675	900	68400	510	120	24	48	CT	NUC
106	-43	301	40	-302	303	120	200	0	240	400	1	1	SEM	NGPN
107	-43	301	40	-302	303	120	200	0	240	400	1	1	VT	NGPN
108	-43	301	40	-302	303	120	200	0	240	400	1	1	RI	NGPN
109	-173	303	80	-1210	306	240	400	0	280	400	1	1	ME	NGPN
110	-173	303	80	-1210	306	240	400	0	280	400	1	1	WCM	NGPN
111	-173	303	80	-1210	306	240	400	0	280	400	1	1	NH	NGPN
112	-68	352	50	-473	354	150	250	0	250	400	1	1	CT	NGTN
113	-68	352	50	-473	354	150	250	0	250	400	1	1	SEM	NGTN
114	-68	352	50	-473	354	150	250	0	250	400	1	1	VT	NGTN
115	-68	352	50	-473	354	150	250	0	250	400	1	1	RI	NGTN
116	-270	354	100	-1890	357	300	500	0	300	400	1	1	ME	NGTN
117	-270	354	100	-1890	357	300	500	0	300	400	1	1	WCM	NGTN
118	-270	354	100	-1890	357	300	500	0	300	400	1	1	NH	NGTN
119	-270	354	100	-1890	357	300	500	0	300	400	1	1	CT	NGTN
120	517	63	60	-908	68	180	300	3900	260	400	4	6	SEM	NGA4
121	517	63	60	-908	68	180	300	3900	260	400	4	6	VT	NGA4
122	517	63	60	-908	68	180	300	3900	260	400	4	6	RI	NGA4
123	517	63	60	-908	68	180	300	3900	260	400	4	6	ME	NGA4
124	517	63	60	-908	68	180	300	3900	260	400	4	6	WCM	NGA4

125	517	63	60	-908	68	180	300	3900	260	400	4	6	NH	NGA4
126	517	63	60	-908	68	180	300	3900	260	400	4	6	CT	NGA4
127	517	63	60	-908	68	180	300	3900	260	400	4	6	SEM	NGA4
128	517	63	60	-908	68	180	300	3900	260	400	4	6	VT	NGA4
129	517	63	60	-908	68	180	300	3900	260	400	4	6	RI	NGA4
130	560	68	120	-5143	78	360	600	7800	320	400	4	6	ME	NGA4
131	560	68	120	-5143	78	360	600	7800	320	400	4	6	WCM	NGA4
132	560	68	120	-5143	78	360	600	7800	320	400	4	6	NH	NGA4
133	560	68	120	-5143	78	360	600	7800	320	400	4	6	CT	NGA4
134	560	68	120	-5143	78	360	600	7800	320	400	4	6	SEM	NGA4
135	560	68	120	-5143	78	360	600	7800	320	400	4	6	VT	NGA4
136	560	68	120	-5143	78	360	600	7800	320	400	4	6	RI	NGA4
137	560	68	120	-5143	78	360	600	7800	320	400	4	6	ME	NGA4

Table A.1. 8-Zone Test System Generator Data

l	a	a'	$T_l^{\max}(a,a')$	$\eta(a,a')$	$C_l^{\mathrm{tr,inv}}$	Length (Km)
1	ME	NH	1200	0,98		
2	VT	NH	1200	0,99		
3	VT	WCMASS	1200	0,98		
4	WCMASS	NH	1200	0,99		
5	NEMA/BOST	WCMASS	1200	0,99		
6	NEMA/BOST	NH	1200	0,99		
7	NEMA/BOST	SEMASS	1200	0,99		
8	WCMASS	CT	1200	0,99		
9	WCMASS	RI	1200	0,99		
10	NEMA/BOST	RI	1200	0,99		
11	CT	RI	1200	0,99		
12	SEMASS	RI	1200	0,99		
13	ME	NH	500	0,98	63,478,431	115
14	VT	NH	500	0,99	55,198,635	100
15	VT	WCMASS	500	0,98	82,797,953	150
16	WCMASS	NH	500	0,99	47,470,826	86
17	NEMA/BOST	WCMASS	500	0,99	44,158,908	80
18	NEMA/BOST	NH	500	0,99	34,775,140	63
19	NEMA/BOST	SEMASS	500	0,99	16,559,591	30
20	WCMASS	CT	500	0,99	16,559,591	30
21	WCMASS	RI	500	0,99	35,879,113	65
22	NEMA/BOST	RI	500	0,99	22,079,454	40
23	CT	RI	500	0,99	35,327,127	64
24	SEMASS	RI	500	0,99	11,039,727	20
25	ME	NH	1400	0,98	106,769,580	115
26	VT	NH	1400	0,99	92,843,113	100
27	VT	WCMASS	1400	0,98	139,264,670	150
28	WCMASS	NH	1400	0,99	79,845,077	86
29	NEMA/BOST	WCMASS	1400	0,99	74,274,490	80
30	NEMA/BOST	NH	1400	0,99	58,491,161	63
31	NEMA/BOST	SEMASS	1400	0,99	27,852,934	30

32	WCMASS	CT	1400	0,99	27,852,934	30
33	WCMASS	RI	1400	0,99	60,348,023	65
34	NEMA/BOST	RI	1400	0,99	37,137,245	40
35	CT	RI	1400	0,99	59,419,592	64
36	SEMASS	RI	1400	0,99	18,568,623	20
37	ME	NH	500	0,98	63,478,431	115
38	VT	NH	500	0,99	55,198,635	100
39	VT	WCMASS	500	0,98	82,797,953	150
40	WCMASS	NH	500	0,99	47,470,826	86
41	NEMA/BOST	WCMASS	500	0,99	44,158,908	80
42	NEMA/BOST	NH	500	0,99	34,775,140	63
43	NEMA/BOST	SEMASS	500	0,99	16,559,591	30
44	WCMASS	CT	500	0,99	16,559,591	30
45	WCMASS	RI	500	0,99	35,879,113	65
46	NEMA/BOST	RI	500	0,99	22,079,454	40
47	CT	RI	500	0,99	35,327,127	64
48	SEMASS	RI	500	0,99	11,039,727	20

Table A.2. 8-Zone Test System Transmission Data

A.2. Chilean System Data

i	α_1	β_1	$P^{\min}$	$P^{\max}$	SC	R^{su}	R	mup	mdn	Zone	Type
1	758	75	84	153	15305	94	46	14	14	SING2	carbon
2	1235	77	122	249	24932	141	75	14	14	SING2	carbon
3	1257	77	122	253	25373	141	76	14	14	SING2	carbon
4	738	80	91	149	14896	99	45	14	14	SING1	carbon
5	396	86	44	80	7981	50	24	14	14	SING2	carbon
6	396	84	44	80	8000	50	24	14	14	SING2	carbon
7	635	81	66	128	12805	75	38	14	14	SING2	carbon
8	617	79	67	124	12443	75	37	14	14	SING2	carbon
9	770	79	79	155	15536	90	46	14	14	SING2	carbon
10	815	77	79	164	16449	92	49	14	14	SING2	carbon
11	765	74	84	154	15436	94	46	14	14	SING2	carbon
12	655	75	56	132	13226	67	40	14	14	SING2	carbon
13	633	76	56	127	12782	67	38	14	14	SING2	carbon
14	237	246	9	24	993	24	24	1	1	SING1	petroleo_diesel
15	246	249	10	25	1033	25	25	1	1	SING2	petroleo_diesel
16	248	249	10	25	1042	25	25	1	1	SING2	petroleo_diesel
17	236	238	10	24	989	24	24	1	1	SING1	petroleo_diesel
18	279	177	11	28	1172	28	28	1	1	SING2	petroleo_diesel
19	112	140	11	37	2330	19	22	6	6	SING2	gas_natural
20	730	86	95	243	15235	140	146	6	6	SING2	gas_natural
21	1179	82	118	393	24617	200	236	6	6	SING2	gas_natural
22	1135	90	113	378	23696	193	227	6	6	SING2	gas_natural
23	1169	90	117	390	24398	199	234	6	6	SING2	gas_natural
24	1173	77	94	236	23670	116	71	14	14	SING2	carbon
25	1173	77	94	236	23670	116	71	14	14	SING2	carbon
26	0	20	20	62	0	62	62	1	1	SIC3	hidro
27	325	216	13	33	1365	33	33	1	1	SIC1	petroleo_diesel
28	0	20	37	48	0	48	48	1	1	SIC4	hidro
29	0	20	37	48	0	48	48	1	1	SIC4	hidro
30	503	174	20	50	2112	50	50	1	1	SIC4	petroleo_diesel

31	510	174	20	51	2141	51	51	1	1	SIC4	petroleo_diesel
32	0	20	5	112	0	112	112	1	1	SIC4	hidro
33	607	74	70	122	12256	78	37	14	14	SIC4	carbon
34	1603	73	129	322	32344	158	97	14	14	SIC4	carbon
35	1237	74	110	249	24973	131	75	14	14	SIC3	carbon
36	374	132	60	125	7809	79	75	6	6	SIC3	gas_natural
37	384	132	60	128	8012	80	77	6	6	SIC3	gas_natural
38	0	20	40	60	0	60	60	1	1	SIC5	hidro
39	1523	179	70	152	6395	152	152	1	1	SIC1	petroleo_diesel
40	0	20	22	39	0	39	39	1	1	SIC3	hidro
41	0	20	0	37	0	37	37	1	1	SIC3	hidro
42	66	57	7	22	1378	11	13	6	6	SIC3	gas_natural
43	0	20	100	165	0	165	165	1	1	SIC3	hidro
44	577	185	25	58	2423	58	58	1	1	SIC3	petroleo_diesel
45	140	117	15	47	2932	25	28	6	6	SIC4	gas_natural
46	0	20	55	32	0	32	32	1	1	SIC3	hidro
47	360	164	14	36	1512	36	36	1	1	SIC5	petroleo_diesel
48	237	248	9	24	994	24	24	1	1	SIC1	petroleo_diesel
49	0	20	5	21	0	21	21	1	1	SIC3	hidro
50	808	165	32	81	3395	81	81	1	1	SIC2	petroleo_diesel
51	237	248	9	24	994	24	24	1	1	SIC1	petroleo_diesel
52	0	20	0	157	0	157	157	1	1	SIC4	hidro
53	330	216	13	33	1386	33	33	1	1	SIC1	petroleo_diesel
54	357	232	14	36	1499	36	36	1	1	SIC1	petroleo_diesel
55	710	75	60	143	11716	72	43	14	14	SIC1	carbon_-_petcoke
56	710	75	60	143	11716	72	43	14	14	SIC1	carbon_-_petcoke
57	681	74	60	137	11242	72	41	14	14	SIC1	carbon_-_petcoke
58	691	74	60	139	11404	72	42	14	14	SIC1	carbon_-_petcoke
59	655	74	60	132	10799	71	40	14	14	SIC1	carbon_-_petcoke
60	243	261	12	24	1020	24	24	1	1	SIC4	petroleo_diesel
61	0	20	1	21	0	21	21	1	1	SIC3	hidro
62	0	20	27	57	0	57	57	1	1	SIC3	hidro

63	0	20	32	54	0	54	54	1	1	SIC3	hidro
64	238	301	10	24	997	24	24	1	1	SIC3	petroleo_diesel
65	214	301	9	21	897	21	21	1	1	SIC3	petroleo_diesel
66	1240	166	50	124	5208	124	124	1	1	SIC2	petroleo_diesel
67	1383	182	65	138	5809	138	138	1	1	SIC4	petroleo_diesel
68	1028	146	41	103	4319	103	103	1	1	SIC4	petroleo_diesel
69	1313	199	60	131	5516	131	131	1	1	SIC3	petroleo_diesel
70	0	20	19	33	0	33	33	1	1	SIC3	hidro
71	1084	87	108	361	22638	184	217	6	6	SIC3	gas_natural
72	1069	130	43	107	4490	107	107	1	1	SIC3	petroleo_diesel
73	1171	81	160	390	24447	229	234	6	6	SIC3	gas_natural
74	1110	133	111	370	23170	189	222	6	6	SIC3	gas_natural
75	1237	74	110	249	24973	131	75	14	14	SIC3	carbon
76	1152	173	46	115	4838	115	115	1	1	SIC2	petroleo_diesel
77	0	20	90	163	0	163	163	1	1	SIC4	hidro
78	0	20	120	199	0	199	199	1	1	SIC3	hidro
79	0	195	56	63	0	63	63	1	1	SIC4	petcoke
80	0	20	14	30	0	30	30	1	1	SIC4	hidro
81	1271	181	70	127	5338	127	127	1	1	SIC3	petroleo_diesel
82	1281	181	70	128	5380	128	128	1	1	SIC3	petroleo_diesel
83	0	20	90	241	0	241	241	1	1	SIC4	hidro
84	0	20	30	132	0	132	132	1	1	SIC3	hidro
85	920	268	37	92	3864	92	92	1	1	SIC3	petroleo_diesel
86	0	20	15	21	0	21	21	1	1	SIC5	hidro
87	0	20	16	62	0	62	62	1	1	SIC4	hidro
88	1103	90	110	368	23028	187	221	6	6	SIC3	gas_natural
89	708	201	130	236	14776	162	142	6	6	SIC3	gas_natural
90	1180	141	130	393	24640	209	236	6	6	SIC3	gas_natural
91	284	251	15	28	1193	28	28	1	1	SIC1	petroleo_diesel
92	259	279	15	26	1088	26	26	1	1	SIC1	petroleo_diesel
93	1376	196	60	138	5779	138	138	1	1	SIC4	petroleo_diesel
94	1700	71	240	342	34301	255	100	14	14	SIC4	carbon

95	0	20	0	27	0	27	27	1	1	SIC3	hidro
96	369	128	37	123	7714	63	74	6	6	SIC1	gas_natural
97	364	128	36	121	7595	62	73	6	6	SIC1	gas_natural
98	589	165	24	59	2472	59	59	1	1	SIC3	petroleo_diesel
99	861	198	34	86	3614	86	86	1	1	SIC1	petroleo_diesel
100	808	165	32	81	3395	81	81	1	1	SIC5	petroleo_diesel
101	564	77	60	113	11374	68	34	14	14	SIC3	carbon
102	1037	75	83	209	20918	102	63	14	14	SIC3	carbon
103	524	208	25	52	2200	52	52	1	1	SIC4	petroleo_diesel
104	521	188	25	52	2187	52	52	1	1	SIC4	petroleo_diesel
105	535	203	25	53	2246	53	53	1	1	SIC4	petroleo_diesel
106	410	220	25	41	1722	41	41	1	1	SIC4	petroleo_diesel
107	458	179	18	46	0	46	46	1	1	SING1	petroleo_diesel
108	1157	202	46	116	0	116	116	1	1	SIC3	petroleo_diesel
109	44	134	4	15	0	8	15	6	6	SIC3	gas_natural
110	10	173	1	2	0	1	2	14	14	SIC3	carbon
111	959	231	38	96	0	96	96	1	1	SIC1	petroleo_diesel
112	507	192	20	51	0	51	51	1	1	SIC4	petroleo_diesel
113	180	219	7	18	0	18	18	1	1	SIC2	petroleo_diesel
114	472	196	19	47	0	47	47	1	1	SIC5	petroleo_diesel
115	129	161	5	13	0	13	13	1	1	SING2	petroleo_diesel
116	895	80	63	180	8346	81	54	8	8	SING2	carbon
117	895	76	63	180	8346	81	54	8	8	SIC1	carbon
118	895	81	63	180	8346	81	54	8	8	SIC2	carbon
119	895	84	63	180	8346	81	54	8	8	SIC3	carbon
120	895	79	63	180	8346	81	54	8	8	SIC4	carbon
121	895	78	63	180	8346	81	54	8	8	SIC5	carbon
122	1740	78	123	350	16229	157	105	8	8	SING1	carbon
123	1740	80	123	350	16229	157	105	8	8	SING2	carbon
124	1740	76	123	350	16229	157	105	8	8	SIC1	carbon
125	1740	84	123	350	16229	157	105	8	8	SIC2	carbon
126	1740	75	123	350	16229	157	105	8	8	SIC3	carbon

127	1740	83	123	350	16229	157	105	8	8	SIC4	carbon
128	2485	75	175	500	23185	224	120	8	8	SIC5	carbon
129	2485	78	175	500	23185	224	120	8	8	SING1	carbon
130	2485	84	175	500	23185	224	120	8	8	SING2	carbon
131	2485	82	175	500	23185	224	120	8	8	SIC1	carbon
132	2485	84	175	500	23185	224	120	8	8	SIC2	carbon
133	2485	78	175	500	23185	224	120	8	8	SIC3	carbon
134	1050	127	88	350	12600	179	245	4	4	SIC4	ccgt
135	1050	116	88	350	12600	179	245	4	4	SIC5	ccgt
136	1050	127	88	350	12600	179	245	4	4	SING1	ccgt
137	1050	126	88	350	12600	179	245	4	4	SING2	ccgt
138	1050	114	88	350	12600	179	245	4	4	SIC1	ccgt
139	1050	105	88	350	12600	179	245	4	4	SIC2	ccgt
140	1050	124	88	350	12600	179	245	4	4	SIC3	ccgt
141	1050	128	88	350	12600	179	245	4	4	SIC4	ccgt
142	1050	117	88	350	12600	179	245	4	4	SIC5	ccgt
143	1050	111	88	350	12600	179	245	4	4	SING1	ccgt
144	1500	124	125	500	18000	256	320	4	4	SING2	ccgt
145	1500	110	125	500	18000	256	320	4	4	SIC1	ccgt
146	1500	118	125	500	18000	256	320	4	4	SIC2	ccgt
147	1500	111	125	500	18000	256	320	4	4	SIC3	ccgt
148	1500	110	125	500	18000	256	320	4	4	SIC4	ccgt
149	1500	117	125	500	18000	256	320	4	4	SIC5	ccgt
150	1500	113	125	500	18000	256	320	4	4	SING1	ccgt
151	1500	125	125	500	18000	256	320	4	4	SING2	ccgt
152	1500	128	125	500	18000	256	320	4	4	SIC1	ccgt
153	1500	116	125	500	18000	256	320	4	4	SIC2	ccgt
154	600	164	50	200	4600	200	200	1	1	SIC3	ocgt
155	600	185	50	200	4600	200	200	1	1	SIC4	ocgt
156	600	167	50	200	4600	200	200	1	1	SIC5	ocgt
157	600	150	50	200	4600	200	200	1	1	SING1	ocgt
158	1200	166	100	400	9200	400	400	1	1	SING2	ocgt

159	1200	158	100	400	9200	400	400	1	1	SIC1	ocgt
160	1200	179	100	400	9200	400	400	1	1	SIC2	ocgt
161	1200	181	100	400	9200	400	400	1	1	SIC3	ocgt
162	2000	201	80	200	4200	200	200	1	1	SIC4	petroleo_diesel
163	2000	201	80	200	4200	200	200	1	1	SIC5	petroleo_diesel
164	2000	201	80	200	4200	200	200	1	1	SING1	petroleo_diesel
165	4000	201	160	400	8400	400	400	1	1	SING2	petroleo_diesel
166	4000	201	160	400	8400	400	400	1	1	SIC1	petroleo_diesel
167	4000	201	160	400	8400	400	400	1	1	SIC2	petroleo_diesel

Table A.3. Chilean System Generator Data

l	a	a'	$T_l^{\max}(a,a')$	$\eta(a,a')$	$C_l^{\mathrm{tr,inv}}$	Length (Km)
1	SING1	SING2	42	0,98		174
2	SING1	SING2	245	0,99		201
3	SING1	SING2	245	0,98		201
4	SIC1	SIC2	197	0,98		92
5	SIC1	SIC2	79	0,99		71
6	SIC2	SIC3	49	0,98		62
7	SIC2	SIC3	224	0,98		102
8	SIC3	SIC4	1766	0,99		192
9	SIC3	SIC4	1786	0,98		206
10	SIC3	SIC4	107	0,98		243
11	SIC3	SIC4	125	0,98		65
12	SIC4	SIC5	18	0,99		37
13	SIC4	SIC5	193	0,99		209
14	SIC4	SIC5	193	0,99		107
15	SING1	SING2	350	0,99	77,664,480	201
16	SING1	SING2	500	0,99	110,949,257	201
17	SING1	SING2	1400	0,98	186,614,657	201
18	SING1	SING2	1400	0,98	186,614,657	201
19	SING1	SING2	1400	0,98	186,614,657	201
20	SING1	SING2	1400	0,98	186,614,657	201
21	SING1	SING2	1400	0,98	186,614,657	201
22	SING2	SIC1	350	0,99	139,100,561	360
23	SING2	SIC1	500	0,99	198,715,088	360
24	SING2	SIC1	1400	0,98	334,235,207	360
25	SING2	SIC1	1400	0,98	334,235,207	360
26	SING2	SIC1	1400	0,98	334,235,207	360
27	SING2	SIC1	1400	0,98	334,235,207	360
28	SING2	SIC1	1400	0,98	334,235,207	360
29	SIC1	SIC2	1400	0,98	85,824,174	102
30	SIC1	SIC2	1400	0,98	85,824,174	102
31	SIC1	SIC2	1400	0,98	85,824,174	102

32	SIC1	SIC2	1400	0,98	85,824,174	102
33	SIC1	SIC2	1400	0,98	85,824,174	102
34	SIC1	SIC2	500	0,99	51,025,619	92
35	SIC1	SIC2	1400	0,98	85,824,174	102
36	SIC1	SIC2	1400	0,98	85,824,174	102
37	SIC2	SIC3	1400	0,98	94,653,554	102
38	SIC2	SIC3	500	0,99	56,275,009	102
39	SIC2	SIC3	1400	0,98	94,653,554	102
40	SIC2	SIC3	1400	0,98	94,653,554	102
41	SIC2	SIC3	1400	0,98	94,653,554	102
42	SIC2	SIC3	1400	0,98	94,653,554	102
43	SIC2	SIC3	1400	0,98	94,653,554	102
44	SIC2	SIC3	1400	0,98	94,653,554	102
45	SIC3	SIC4	1700	0,98	232,642,327	206
46	SIC3	SIC4	700	0,99	159,468,361	206
47	SIC3	SIC4	1700	0,98	232,642,327	206
48	SIC3	SIC4	1700	0,98	232,642,327	206
49	SIC3	SIC4	1700	0,98	232,642,327	206
50	SIC3	SIC4	1700	0,98	232,642,327	206

Table A.4. Chilean System System Transmission Data