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QCD sum rules and thermal properties of Charmonium in the vector channel

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Abstract

The thermal evolution of the hadronic parameters of charmonium in the vector channel, i.e. the J/ψ resonance mass, coupling (leptonic decay constant), total width, and continuum threshold is analyzed in the framework of thermal Hilbert moment QCD sum rules. The continuum threshold s_0, as in other hadronic channels, decreases with increasing temperature until the PQCD threshold $s_0 = 4\, m_Q^2$ is reached at $T \simeq 1.22\, T_c$ (m_Q is the charm quark mass) and the J/ψ mass is essentially constant in a wide range of temperatures. The other hadronic parameters behave in a very different way from those of light-light and heavy-light quark systems. The total width grows with temperature up to $T \simeq 1.04\, T_c$ beyond which it decreases sharply with increasing T. The resonance coupling is also initially constant beginning to increase monotonically around $T \simeq T_c$. This behavior strongly suggests that the J/ψ resonance might survive beyond the critical temperature for deconfinement, in agreement with lattice QCD results.

Keywords: Finite temperature field theory, hadron physics.

1. Introduction

We discuss here the thermal evolution of the hadronic parameters of J/ψ in the vector channel, using thermal QCD Sum Rules [1]. We refer the reader to the original article [2] for details. This technique has been used previously in the light-light and in the heavy-light quark sector [3]- [5], with the following emerging picture: (i) For increasing temperature, hadronically stable particles develop a non-zero width, and resonances become broader, diverging at a critical temperature interpreted as the deconfinement temperature (T_c). The thermal resonance broadening was first proposed in [6]. ii) Above the resonance region, the continuum threshold in hadronic spectral functions, i.e. the onset of perturbative QCD (PQCD), decreases monotonically with increasing temperature. When $T \to T_c$ hadrons disappear from the spectrum. (iii) This scenario is also supported by the behavior of hadronic couplings, or leptonic decay constants, which approach zero as $T \to T_c$. Masses, on the other hand, do not to provide information about deconfinement.

The thermal behavior of the heavy-heavy quark correlator should be different from that involving at least one light quark since: a) In the light-light and heavy-light quark sector, the PQCD contribution is dominated by the time-like spectral function (annihilation term), which is relatively unimportant in relation to the light quark condensate contribution, being the scattering PQCD spectral function highly suppressed. Instead, for heavy-heavy quark systems this term becomes increasingly important with increasing temperature while the annihilation term only contributes near threshold; b) The non-perturbative QCD sector in the operator product expansion (OPE) of light-light and heavy-light quark correlators is driven by the light quark condensate, responsible for the behavior of the continuum threshold since $s_0(T)/s_0(0) \simeq \langle\langle \bar{q}q \rangle\rangle / \langle \bar{q}q \rangle$ [4]-[7]. The light quark condensate is the order parameter for chiral symmetry restoration. In contrast, for heavy-heavy quark correlators the leading power correction in the OPE is that of the gluon condensate, which has a very different thermal behavior. In this approach, the criti-

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cal temperature for deconfinement is a phenomenological parameter which does not need to coincide with e.g. the critical temperature obtained in lattice QCD [8]. In fact, results from QCD sum rules lead to values of T_c somewhat lower than those from lattice QCD. In order to compare with other approaches, we express our results in terms of the ratio T/T_c.

We find for charmonium in the vector channel that the continuum threshold, $s_0(T)$, decreases with increasing T, being driven by the gluon condensate and the PQCD spectral function in the space-like region, until it reaches the PQCD threshold $s_0 = 4\, m_Q^2$ at $T \simeq 1.22\, T_c$ (m_Q is the charm quark mass). Below this value of s_0 the sum rules cease to be valid. The J/ψ mass remains basically constant as in the light-light or heavy-light systems. We have, however, a very different thermal evolution of the width and the coupling. Both are almost independent of T up to $T \simeq 0.8\, T_c$ where the width begins to increase substantially, but then above $T \simeq 1.04\, T_c$ it starts to decrease sharply, and the coupling increases also sharply. This suggests the survival of the J/ψ resonance above the deconfinement temperature.

The PQCD spectral function in the space-like region plays here a very important role. Non-relativistic approaches to charmonium at finite temperature would normally miss this contribution. In fact, the complex energy plane in the non-relativistic case would only have one cut along the positive real axis, which would correspond to the time-like (annihilation) region of PQCD. The space-like contribution ($q^2 = (\omega^2 - |\mathbf{q}|^2) \leq \mathbf{0}$) in the form of a cut in the energy plane centered at the origin for $-|\mathbf{q}| \leq \omega \leq |\mathbf{q}|$, would not be present in the non-relativistic case.

2. Hilbert Moment QCD Sum Rules

We consider the correlator of the heavy-heavy quark vector current at finite temperature

$$\Pi_{\mu\nu}(q^2,T) = -(g_{\mu\nu}q^2 - q_\mu q_\nu)\, \Pi(q^2,T)$$
$$= i \int d^4x\, e^{iqx}\;\; \theta(x_0)\;\; << |[V_\mu(x)\;,\; V_\nu^\dagger(0)]| >>, \quad (1)$$

where $V_\mu(x) =: \bar{Q}(x)\gamma_\mu Q(x)$:, and $Q(x)$ is the heavy (charm) quark field. The matrix element above is understood to be the Gibbs average in the quark-gluon basis. The imaginary part of the vector correlator in PQCD at finite temperature involves two pieces, one in the time-like region ($q^2 \geq 4m_Q^2$), $Im\ \Pi_a(q^2,T)$, which survives at T=0, and one in the space-like region ($q^2 \leq 0$), $Im\ \Pi_s(q^2,T)$, which vanishes at T=0. To leading order in PQCD we find

$$\frac{1}{\pi}\, Im\, \Pi_a(q^2,T) = \frac{3}{16\pi^2} \int_{-v}^{v} dx\, (1-x^2)$$
$$\left[1 - n_F\left(\frac{|\mathbf{q}|x+\omega}{2T}\right) - n_F\left(\frac{|\mathbf{q}|x-\omega}{2T}\right)\right], \quad (2)$$

where $v^2 = 1 - 4m_Q^2/q^2$, m_Q is the heavy quark mass, $q^2 = \omega^2 - \mathbf{q}^2 \geq 4m_Q^2$, and $n_F(z) = (1+e^z)^{-1}$ is the Fermi thermal function. In the rest frame of the thermal bath, $|\mathbf{q}| \to 0$, the above result reduces to

$$\frac{1}{\pi}\, Im\, \Pi_a(\omega,T) = \frac{v(3-v^2)}{8\pi^2}\;\; [1 - 2n_F(\omega/2T)]$$
$$\times\, \theta(\omega - 2m_Q)\,. \quad (3)$$

The quark mass is assumed independent of T, a good approximation for $T < 200$ MeV [9]. Only the leading order in the strong coupling will be considered here.

The PQCD piece in the space-like region demands a careful analysis. In the complex energy plane, and in the space-like region, the correlator $\Pi(q^2)$, Eq.(1), has a cut centered at the origin and extending between $\omega = -|\mathbf{q}|$ and $\omega = |\mathbf{q}|$. In the rest frame this cut produces a delta function $\delta(\omega^2)$ in the imaginary part of $\Pi(q^2)$. The result is

$$\frac{1}{\pi}\, Im\, \Pi_s(\omega,T) = \frac{2}{\pi^2}\, m_Q^2\, \delta(\omega^2) \times$$
$$\left[n_F\left(\frac{m_Q}{T}\right) + \frac{2\,T^2}{m_Q^2} \int_{m_Q/T}^{\infty} y\, n_F(y)\, dy\right]. \quad (4)$$

The corresponding hadronic representation is parametrized in terms of the ground state resonance,the J/ψ, followed by a continuum given by PQCD after a threshold $s_0 > M_V^2$. In the zero width approximation, the hadronic spectral function is

$$\frac{1}{\pi}\, Im\, \Pi(s,T)|_{HAD} = \frac{1}{\pi} Im\, \Pi(s,T)|_{RES}\;\; \theta(s_0 - s)$$
$$+\frac{1}{\pi} Im\, \Pi(s,T)|_{PQCD}\;\; \theta(s - s_0)$$
$$= 2\, f_V^2(T)\, \delta(s - M_V^2(T)) + \frac{1}{\pi} Im\Pi(s,T)_a\;\; \theta(s - s_0)\,, \quad (5)$$

where $s \equiv q^2 = \omega^2 - \mathbf{q}^2$. The leptonic decay constant is defined as $< 0|V_\mu(0)|V(k) >= \sqrt{2}\, M_V\, f_V\, \epsilon_\mu$.

When considering a finite (total) width the following replacement will be understood

$$\pi\, \delta(s - M_V^2(T)) \to \frac{M_V(T)\Gamma_V(T)}{(s - M_V^2(T))^2 + M_V^2(T)\Gamma_V^2(T)}\,, \quad (6)$$

The hadronic scattering term, due to current scattering off D-mesons, is negligible [2]. The correlation function $\Pi(q^2, T)$, Eq.(1), satisfies a once subtracted dispersion relation. To eliminate the subtraction one can use Hilbert moments, i.e.

$$\varphi_N(Q^2, T) \equiv \frac{(-)^N}{(N)!}\Big(\frac{d}{dQ^2}\Big)^N \Pi(Q^2, T) = \frac{1}{\pi}\int_0^\infty \frac{ds}{(s+Q^2)^{N+1}}\, Im\,\Pi(s,T)\,, \tag{7}$$

where $N = 1, 2, ...,$ and $Q^2 \geq 0$ is an external four-momentum squared, to be considered as a free parameter. Using Cauchy's theorem in the complex s-plane, the Hilbert moments become Finite Energy QCD sum rules (FESR), i.e.

$$\varphi_N(Q^2, T)|_{RES} = \varphi_N(Q^2, T)|_{QCD}\,, \tag{8}$$

where

$$\varphi_N(Q^2, T)|_{RES} \equiv \frac{1}{\pi}\int_0^{s_0(T)} \frac{ds}{(s+Q^2)^{N+1}}\, Im\,\Pi(s,T)|_{RES}\,, \tag{9}$$

$$\varphi_N(Q^2, T)|_{QCD} \equiv \frac{1}{\pi}\int_{4m_Q^2}^{s_0(T)} \frac{ds}{(s+Q^2)^{N+1}}\, Im\,\Pi_a(s,T) + \frac{1}{\pi}\int_0^\infty \frac{ds}{(s+Q^2)^{N+1}}\, Im\,\Pi_s(s,T) + \varphi_N(Q^2,T)|_{NP}\,, \tag{10}$$

and $Im\,\Pi(s,T)|_{RES}$ is given by the first term in Eq.(5) modified in finite-width according to Eq.(6), and the PQCD spectral functions are given by Eqs.(3) and (4).

The dimension d=4 non perturbative term in the OPE is well known in the literature, see [2] for details. The dependence on N is quite cumbersome and it is proportional to the gluon condensate $\left\langle\left\langle\frac{\alpha_s}{\pi}G^2\right\rangle\right\rangle$. At low temperatures, this condensate has been calculated in chiral perturbation theory [10]. In this framework the condensate remains essentially constant up $T \sim T_c \simeq 100$ MeV, after which it decreases sharply. In order to go beyond the low temperature regime of chiral perturbation theory, lattice QCD provides the right tool. A good approximation [11] is given by the expression

$$\left\langle\left\langle\frac{\alpha_s}{\pi}G^2\right\rangle\right\rangle = \left\langle\frac{\alpha_s}{\pi}G^2\right\rangle\left[\theta(T^*-T) + \frac{1-\frac{T}{T_C^*}}{1-\frac{T^*}{T_C^*}}\theta(T-T^*)\right] \tag{11}$$

where $T^* \approx 150$ MeV is the breakpoint temperature where the condensate begins to decrease appreciably, and $T_C^* \approx 250$ MeV is the temperature at which $\left\langle\left\langle\frac{\alpha_s}{\pi}G^2\right\rangle\right\rangle_{T_C} = 0$.

Returning to the Q^2 dependence of the Hilbert moments, Eq.(7), we shall fix Q^2 and $s_0(0)$ from the experimental values of the mass, the coupling, and the width at T=0. At finite temperature there are non-diagonal (Lorentz non-invariant) condensates that might contribute to the OPE. Non-gluonic operators are highly suppressed [5], [12] so that they can be safely ignored. We have considered also a gluonic twist-two term in the OPE introduced in [13], and computed on the lattice in [14]. Its impact is small,(2-6)%, and plays no appreciable role in the results.

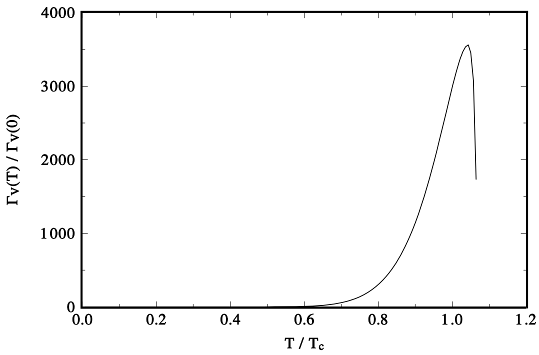

Figure 1: The ratio $\Gamma_V(T)/\Gamma_V(0)$ as a function of T/T_c.

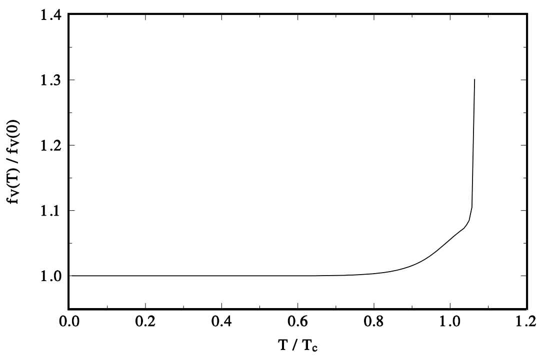

Figure 2: The ratio $f_V(T)/f_V(0)$ as a function of T/T_c.

3. Results

We begin by determining s_0 and Q^2 at T=0 from the moments, Eq.(8), and using as input the experimental values [15] $M_V = 3.097$ GeV, $f_V = 196$ MeV, and $\Gamma_V = 93.2$ keV, as well as $m_Q = 1.3$ GeV, and [16] $\langle 0|\frac{\alpha_s}{12\pi}G^2|0\rangle \simeq 5\times 10^{-3}\mathrm{GeV}^4$. In the zero-width approximation one finds from Eq.(9) that

$$\frac{\varphi_1(Q^2)|_{RES}}{\varphi_2(Q^2)|_{RES}} = \frac{\varphi_2(Q^2)|_{RES}}{\varphi_3(Q^2)|_{RES}} \,. \tag{12}$$

Given the extremely small total width of the J/ψ it turns out that the above relation also holds with extreme accuracy in finite width. Using Eq.(8) this leads to

$$\frac{\varphi_1(Q^2)|_{QCD}}{\varphi_2(Q^2)|_{QCD}} = \frac{\varphi_2(Q^2)|_{QCD}}{\varphi_3(Q^2)|_{QCD}} \,, \tag{13}$$

which depends only on the two unknowns s_0 and Q^2, and provides the first equation to determine this pair of parameters. The second equation can be e.g. Eq.(8) with $N = 1$. In this way we find that $s_0 = 11.64$ GeV^2, and $Q^2 = 10$ GeV^2 reproduce the experimental values of the mass, coupling, and width of J/ψ within less than 1%. This whole set of hadronic parameters will then be used to normalize the corresponding parameters at finite temperature. In this way, see [2] for details, we were able to find the thermal evolution of s_0, the J/ψ mass, its width and its coupling. We show here only the behavior of the width and the coupling (Figs. 1 and 2) since these are the most important results of this analysis.

Both the width and the coupling can only be determined up to $T_f \simeq\ 1.1\,T_c$ beyond which $s_0(T) < M_V^2(T)$ and the FESR integrals have no longer a support. The temperature behavior of the width and the coupling shown in Figs. 1 and 2 strongly suggests the survival of the J/ψ above the critical temperature for deconfinement. This conclusion agrees with results from lattice QCD [8], but disagrees with non-relativistic determinations. As pointed out earlier, the reason for this disagreement might very well be the absence of the central cut (QCD scattering term) in the energy plane in non-relativistic frameworks.

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