PONTIFICIA UNIVERSIDAD CATÓLICA DE CHILE
ESCUELA DE INGENIERÍA

AN ADAPTIVE ROBUST OPTIMIZATION MODEL FOR POWER SYSTEMS PLANNING WITH OPERATIONAL UNCERTAINTY

FELIPE VERÁSTEGUI GRÜNEWALD

Thesis submitted to the Office of Research and Graduate Studies
in partial fulfillment of the requirements for the degree of
Master of Science in Engineering

Advisor:
ÁLVARO LORCA GÁLVEZ
PEDRO GAZMURI SCHLEYER

Santiago de Chile, October 2018

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Gratefully to all

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ABSTRACT

The need for sustainable power systems is driving the adoption of large shares of variable renewable energy. Due to this, there is an increasing necessity for new long-term planning models that can correctly assess the reserve capacity and flexibility requirements to manage significant levels of short-term operational uncertainty. Motivated by this key challenge, this work proposes an adaptive robust optimization model for the Generation and Transmission Expansion Planning Problem. The proposed model has a two-stage structure that separates investment and operational decisions, over a planning horizon with multiple periods. The key attribute of this model is the representation of daily operational uncertainty through the concept of representative days and an uncertainty set for demand and the availability of wind and solar power built over such days. Also, the model employs a DC-power flow representation for the transmission network. This modelling setup allows an effective representation of the reserve capacity and flexibility requirements of a system with large shares of renewable energy. To efficiently solve the problem, the column and constraint generation method is employed. Extensive computational experiments on a 20-bus representation of the Chilean power system over a 20-year horizon show the advantages of the proposed robust expansion planning model, compared to an approach based on deterministic representative days, due to an effective spatial placement of both variable resources and flexible resources.

Keywords: Generation Expansion Planning, Renewable Energy, Robust Optimization, Transmission Expansion Planning.

RESUMEN

La necesidad por sistemas sustentables de energía está impulsando la adopción de grandes cuotas de energía renovable variable en todo el mundo. Debido a esto, existe una creciente necesidad de nuevos modelos de planificación a largo plazo que puedan evaluar correctamente la capacidad de reserva y los requisitos de flexibilidad para gestionar niveles significativos de incertidumbre operativa a corto plazo. Motivado por este desafío clave, este trabajo propone un modelo de optimización adaptable robusto para el Problema de Planificación de la Expansión de la Generación y la Transmisión. El modelo propuesto tiene una estructura de dos etapas que separa las decisiones de inversión y operación, sobre un horizonte de planificación con múltiples períodos. El atributo clave de este modelo es la representación de la incertidumbre operativa diaria a través del concepto de días representativos y un conjunto de incertidumbre para la demanda y la disponibilidad de la energía eólica y solar, construido sobre esos días. Además, el modelo emplea una representación lineal del flujo de potencia para la red de transmisión. Este modelamiento permite una representación efectiva de la capacidad de reserva y de la flexibilidad operacional requeridas para la operación de un sistema de potencia con grandes cuotas de energía renovable. Para resolver eficientemente el problema de optimización, se emplea un método de generación de columnas. Extensos experimentos computacionales en una representación de 20 nodos del sistema eléctrico chileno en un horizonte de 20 años muestran las ventajas del modelo de planificación de expansión robusto propuesto, en comparación con un enfoque basado en días representativos deterministas, debido a una colocación espacial efectiva tanto de recursos variables como de recursos flexibles.

Palabras Clave: Energía Renovable, Planificación de la Expansión de la Generación, Planificación de la Expansión de la Transmisión, Optimización Robusta.

1. INTRODUCTION

1.1. Context

In the recent years, there has been a global need for transitioning to more sustainable systems. In this context, power systems play a major role due to their large impact in greenhouse gas emissions and the fast growth in energy demand due to population growth, fast developing countries and novel technologies advances, such as the block chain technology or the electric vehicles industry.

Due to this, several countries have endorsed the adoption of large shares of variable renewable energy such as wind and solar generation. Also, the investment cost of these generation technologies has greatly decreased in the last few years. This has further increased their expected penetration in power systems, due to larger private investments.

In spite of the benefits of variable renewable energy, this imposes several challenges for traditional power systems. First, there is an increased need for *flexibility* due to increased ramp requirements, which are a consequence of the variable nature of these energy sources in addition to the variations in the systems load. Also, the uncertain nature of wind and solar power, may impose the need for *reserve capacity*, which can become critical in days with complicated realizations of the uncertain parameters related to these resources availability.

Because of the above mentioned reasons, planning power systems in the long term has become a critical task for ensuring a reliable, sustainable, and cost-effective electricity supply. This task has become particularly challenging in recent years, due to the significant levels of uncertainty introduced by large shares of variable renewable energy.

1.2. Generation and Transmission Expansion Planning

In this context, the Generation and Transmission Expansion Planning (EP) problem provides a key support for this process. This problem consists on the joint optimization of investments on generation and transmission infrastructure of a power system over a given planning horizon to ensure a reliable and cost effective energy supply (Conejo et al., 2016). The objective function typically consists on minimizing the total costs of the system, considering both investment and operational costs for the investment decisions. Thus, in order to correctly asses such decisions, an effective representation of the system's operation is required to properly account for the short-term uncertainty and variability introduced by renewable resources.

The rest of this section presents a review on the scientific literature regarding existing methodologies to represent the power systems operation and uncertainty within the EP problem.

1.2.1. Representation of the Operation

The representation of the power system's operation within the Generation and Transmission EP problem has been extensively studied and several methodologies have been proposed to address this issue. This task is complex due to the fact that planning horizons usually span over several years (typically 10-20 years). Thus, the representation of short-term operation in a highly detailed way may cause the EP problem to become computationally untractable.

In this context, early EP models used the concept of the *load duration curve* and *load blocks*, which overlooked the chronology of demand and variable generation profiles in order to reduce the complexity of the planning problem (Oree et al., 2017).

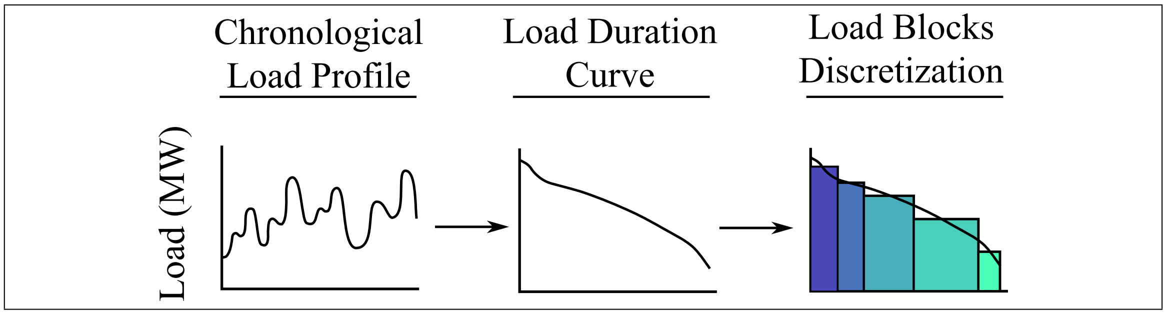


Figure 1.1. Load blocks methodology

The idea of the load blocks concept is to simplify the operational conditions through a discrete set of blocks. This is done by reordering the chronological profile into a decreasing function, namely, the duration curve. Then, this duration curve is approximated by a discrete number of load blocks. Figure 1.1 shows a graphical representation of this methodology.

As previously mentioned, the load blocks concept disregards the chronology of the operation. It has been showned that this simplification becomes less useful as the penetration of variable renewable energy increases, where a low level of temporal detail may have a great impact in the resulting expansion plan (Poncelet et al., 2016). Also, the use of load blocks is not compatible with generator ramps or commitment constraints within the EP problem, which may result in the underestimation of the flexibility requirements of a given power system.

Due to this challenge, recent models have developed new methods for an appropriate representation of the system's flexibility requirements, such as *system states*, *representative blocks* or *representative days*. In specific, the use of a representative set of chronologically ordered time points has allowed the inclusion of ramp constraints and in some cases the inclusion of commitment constraints and reserve requirements in long term planning problems (Belderbos & Delarue 2015, van Stiphout et al. 2017, Jin et al. 2014).

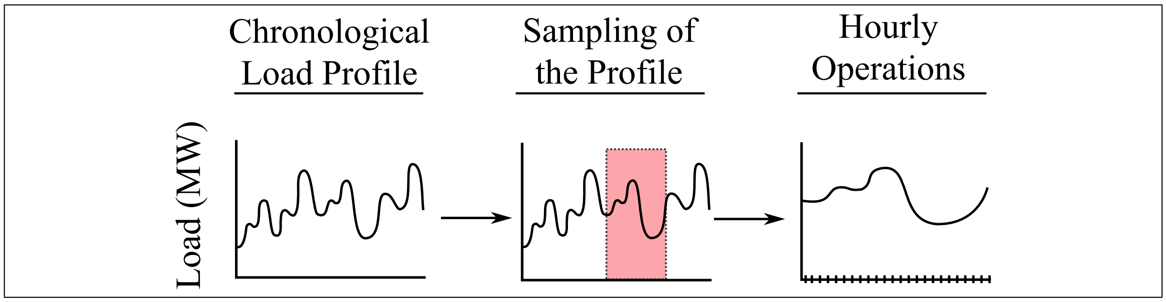


Figure 1.2. Representative intervals methodology

The idea behind the concept of representative intervals or representative days is to simplify a large number of chronological profiles into a smaller set of data, selected in such a way that it provides an effective representation of the different operating conditions within an EP problem. Figure 1.2 provides a graphical representation of this methodology. The selection of the representative intervals is typically done through a *clustering technique*. Note that this approach allows to preserve the operation's chronology.

1.2.2. Representation of the Uncertainty

Another complex task in EP models is the representation of uncertainty. Due to the recent concern regarding the operational challenges imposed by large shares of variable renewable energy, this task has become specially critical for the EP process. Also, due to the fact that the EP problem usually spans over large planning horizons, forecast errors in load growth or other relevant parameters are very likely to occur. Thus, the decision process must take into account the related uncertainty to these parameters.

In the scientific literature regarding the EP problem, the treatment of uncertainty has been extensively studied. It is possible to classify uncertainties by means of their time horizon into short-term and long-term uncertainties (Zhang & Conejo, 2018). Short-term uncertainties may include operational parameters such as renewable resource availability, hourly load variations, and operational contingencies, among others, while long term uncertain parameters may include investment costs for different technologies, fuel prices, load growth, or even hydro inflows. In particular, large shares of variable renewable energy greatly increase short-term or operational uncertainty, and neglecting this uncertainty may lead to significant estimation errors in the system's costs and unprepared expansion plans (Nagl et al. 2013; Seljom & Tomasgard 2015).

There are two main approaches to tackle uncertainty in expansion planning problems, namely, Stochastic Programming (SP) and Robust Optimization (RO). On the one hand, most SP models use a given set of scenarios to represent the probability distribution of the uncertain parameters in the EP problem (Gorenstin et al. 1993, Maluenda et al. 2018, Ahmed et al. 2003). SP is applied mainly to long term uncertainties, due to the fact that including the required number of operational scenarios to represent short-term uncertainty may cause the problem to become computationally intractable, specially under long planning horizons.

On the other hand, RO methods rely on the construction of uncertainty sets to explicitly represent potential uncertainty realizations, while preparing the system for every realization in such set by considering complicated scenarios within the uncertainty set. The combination of load blocks with RO concepts has been used to model uncertainty in Generation EP (Dehghan et al., 2014), and also in Transmission EP (Jabr 2013, Chen et al. 2014, Ruiz & Conejo 2015,Chen & Wang 2016). Further, other authors have employed a set of operational time points in RO models for the Transmission EP (Dehghan et al. 2017, Chatthaworn & Chaitusaney 2017, Zhang & Conejo 2018), and for the Generation and Transmission EP (Baringo & Baringo 2018, Roldán et al. 2018). Moreover, some works consider contingency and reliability constraints through the combination of system states and RO concepts in Transmission EP (Moreira et al. 2015, Wu et al. 2018), and in Generation and Transmission EP (Moreira et al. 2017, Dehghan et al. 2016).

As mentioned above, the adoption of large shares of variable renewable energy greatly increase the short-term or operational uncertainty. In this context, modern RO approaches are very promising, due to the fact that they allow an explicit consideration of the operational uncertainty over long planning horizons while maintaining computational tractability.

1.2.3. Literature Gap

Despite recent progress, some important gaps remain in the EP literature. In fact, EP models usually consider various operational simplifications, such as the consideration of a single future target year as opposed to multi-period models, or the separate consideration of generation and transmission investment decisions, as opposed to their joint consideration. Further, none of the mentioned RO works in this area consider a daily chronologically linked operation. This precludes the inclusion of various key operational aspects, including generator ramp constraints, which are an essential element of power system flexibility.

Due to this, the main focus of this work is to fill the presented gap through the development of an adaptive robust optimization model for the Generation and Transmission EP problem, considering the use of *uncertain representative days* to represent short-term uncertainty and variability, allowing a chronological representation of the system's operation, and thus enabling the inclusion of key operational aspects.

The developed model allows an effective balance between the flexibility requirements motivated by variability, and the reserve capacity requirements motivated by uncertainty of variable renewable energy sources, thus providing an expansion plan that is able to effectively accommodate large shares of renewable energy without comprising reliability standards of the power systems operation. Extensive computational experiments on a 20-bus representation of the Chilean power system show the advantages of the proposed model as compared to a deterministic representative days approach, due to an effective spatial placement of variable and flexible resources, and the consideration of key statistical elements through the constructed uncertainty sets.

1.3. Contributions

Finally, the main contributions of this work can be summarized as follows:

(i) We propose a novel two-stage adaptive robust optimization model for the Generation and Transmission Expansion Planning problem considering an explicit representation of short-term operational uncertainty and capturing the chronology in load and renewable profiles, resulting in a valuable support for the strategic placement of flexible and variable resources.

(ii) We propose a new approach for building uncertainty sets in planning models based on the concept of representative days, as an effective representation of load and renewable uncertainty. This is a key element of the proposed planning model and it can also be employed separately to provide useful insight about the future operation of days with complicated realizations in load and renewable resources.

(iii) We provide experimental evidence of the advantages of the proposed model as compared to an approach based on a given set of deterministic representative days by studying the performance of these models under 365 days of operation over multiple planning periods, and under a "complicated day" in the final planning period. In particular, the results show the effectiveness of the resulting expansion plan in handling operational uncertainty reliably and in a cost-effective way.

The remainder of the work is organized as follows. Chapter 2 presents the nomenclature used in the present document. The proposed and studied mathematical models are presented in Chapter 3. Chapter 4 describes the solution methods and algorithms for the proposed model. Chapter 5 presents computational experiments on a case study. Final conclusions and future research topics are presented in Chapter 6.

2. NOMENCLATURE

This Chapter presents the nomenclature used for the EP models presented in this work.

2.1. Sets and Indexes

$e \in \mathcal{E}$**:** Index and set of planning periods.

$d \in \mathcal{D}$**:** Index and set of representative days.

$t \in \mathcal{T}$**:** Index and set of time points.

$b \in \mathcal{B}$**:** Index and set of buses.

$i \in \mathcal{G}$**:** Index and set of generators.

$j \in \mathcal{L}$**:** Index and set of transmission lines.

$\xi \in \Xi$**:** Index and set of uncertainty realizations.

$\mathcal{G}^E \subset \mathcal{G}$**:** Set of existing generators.

$\mathcal{G}^C \subset \mathcal{G}$**:** Set of candidate generators.

$\mathcal{G}^V \subset \mathcal{G}$**:** Set of variable generators.

$\mathcal{G}^{\textbf{Wind}} \subset \mathcal{G}$**:** Set of wind generators.

$\mathcal{G}^{\textbf{Solar}} \subset \mathcal{G}$**:** Set of solar generators.

$\mathcal{L}^E \subset \mathcal{L}$**:** Set of existing transmission lines.

$\mathcal{L}^C \subset \mathcal{L}$**:** Set of candidate transmission lines.

$\mathcal{G}(b) \subset \mathcal{G}$**:** Set of generators at bus b.

$\mathcal{U} \subset \Xi$**:** Uncertainty set for power demand.

$\mathcal{A} \subset \Xi$**:** Uncertainty set renewable capacity factors

2.2. Parameters

C^I_{ei}, C^I_{ej}**:** Investment cost for installing candidate generator i and candidate transmission line j in planning period e, respectively (US$/MW).

C^E_{ei}, C^E_{ej}**:** Investment cost for expanding existing generator i and existing transmission line j in planning period e, respectively (US$/MW).

w_d: Relative weight of representative day d. It represents the number of days which are represented by d in any planning period.

C_{ei}^{g}: Variable generation cost for generator i in planning period e (US$/MWh).

C_{eb}^{LS}: Variable load shedding cost in bus b in planning period e (US$/MWh).

$\overline{x}_i^g$,$\overline{x}_j^l$: Maximum expansion capacity for generator i and existing line j over the planning horizon (MW).

$\overline{I_e^G}$,$\overline{I_e^L}$: Maximum investment budgets for investment in generation and transmission for planning period e, respectively (US$).

$\overline{p_{ei}^g}$: Base maximum generation capacity for generator i in planning period e (MW).

α_{dti}: Renewable capacity factor in time point t of representative day d for variable generator i.

r_i^u, r_i^d: Maximum ramp up and ramp down for two consecutive timepoints, respectively, for generator i (MW).

$\overline{f_{ej}}$: Base power flow limit for transmission line j in planning period e (MW).

M: Big-M parameter.

b_j: Susceptance of transmission line j (UNIT).

p_{edtb}^D: Load at bus b, for planning period e, representative day d, and time point t.

F_i: Factor between 0 and 1, indicating the firm capacity fraction of generator i.

PRM: Planning Reserve Margin.

$\tilde{p}_{edtb}^D, \hat{p}_{edtb}^D$: Nominal and variability parameters for power demand in bus b in time point t of representative day d for planning period e

$\tilde{\alpha}_{dti}, \hat{\alpha}_{dti}$: Nominal and variability parameters for renewable capacity factor of generator i in time point t of representative day d for planning period e.

$\Gamma_D, \Gamma_{\mathbf{v}}$: Uncertainty budget for power demand and capacity factors of variable generation technology v.

2.3. Decision Variables

x^g_{ei}**:** Investment in generation capacity of generator i in planning period e (MW)

x^l_{ej}**:** Investment in transmission capacity of existiing transmission line j in planning period e (MW)

z^l_{ej}**:** Binary decision for installing a new transmission line j in planning period e.

p^g_{edti}**:** Power output of generator i in time point t of representative day d in planning period e (MWh).

p^{LS}_{edtb}**:** Load shedding in bus b in time point t of representative day d in planning period e (MWh).

f^l_{edtj}**:** Power flow through transmission line j in time point t of representative day d in planning period e (MWh).

θ_{edtb}**:** Voltage angle of bus b in time point t of representative day d in planning period e.

3. MATHEMATICAL MODELS

This Chapter presents the studied EP models and their mathematical formulation.

3.1. Modeling Approach

EP models consist of selecting generation and transmission investments over a given planning horizon in order to minimize total system costs and satisfy various requirements. Several approaches have been developed for this purpose. The rest of this Chapter presents a deterministic expansion planning (DEP) model based on the representative days approach, which then is further extended to consider explicit operational uncertainty in the robust expansion planning (REP) model. Finally, a novel way of building an uncertainty set based on the representative days approach is presented. This uncertainty set, considered on its own, may provide valuable support to the EP process by explicitly finding complicated realizations of the daily uncertainty.

3.2. Deterministic Expansion Planning

In order to compare the proposed model, we will begin presenting a DEP model, that employs the concept of representative days. For this, a planning horizon composed of several planning periods is considered, where the systems operation in each period is represented through the operation under a representative set of days (namely, representative days), each of which has a set of chronological time points in it.

3.2.1. Objective Function

$$\min_{\boldsymbol{x}^g, \boldsymbol{x}^l, \boldsymbol{z}^l, \boldsymbol{p}^g, \boldsymbol{p}^{LS}, \boldsymbol{\theta}^b, \boldsymbol{f}^l}, \quad \sum_{e \in \mathcal{E}} \Big(\sum_{i \in \mathcal{G}^C} C^I_{ei}\, x^g_{eg} + \sum_{j \in \mathcal{L}^C} C^I_{ej}\, z^l_{ej} +$$

$$\sum_{i\in\mathcal{G}^E} C^E_{ei}\, x^g_{eg} + \sum_{j\in\mathcal{L}^E} C^E_{ej}\, x^l_{ej} +$$
$$\sum_{d\in\mathcal{D}} w_d \sum_{t\in\mathcal{T}} \big(\sum_{i\in\mathcal{G}} C^g_{ei}\, p^g_{edti} + \sum_{b\in\mathcal{B}} C^{LS}_{eb}\, p^{LS}_{edtb}) \big) \tag{3.1a}$$

The objective function (3.1a) considers investment costs for candidate units and lines, expansion costs for existing units and lines, and generation and load shedding costs for every time point in every representative day. Parameter w_d represents the weight of representative day d, and it corresponds to the number of days in a planning period that such day represents.

Recall that $\mathcal{E}, \mathcal{D}, \mathcal{T}, \mathcal{B}, \mathcal{G}, \mathcal{L}$ are the sets of planning periods, representative days, time points, buses, generators and transmission lines respectively. Sets $\mathcal{G}^E$, $\mathcal{G}^C$, $\mathcal{L}^E$ and $\mathcal{L}^C$ correspond to existing generators, candidate generators, existing lines and candidate lines, hence $\mathcal{G}^E \cup \mathcal{G}^C = \mathcal{G}$ and $\mathcal{L}^E \cup \mathcal{L}^C = \mathcal{L}$.

Also, recall that decisions x^g_{ei}, x^l_{ej} and z^l_{es} represent investments in the generation capacity of generator i, the transmission capacity of transmission line j, and the installation of a new candidate line s, for planning period e, respectively. The other decisions correspond to operational decisions for every time point t within every representative day d, in every planning period e, namely generation at each unit (p^g_{edti}), power flow through each line ($f_{edt,l}$), voltage at each bus (θ_{edtb}), and load shedding (p^{LS}_{edtb}).

3.2.2. Investment Constraints

Constraints can be divided into investment-related or operational. This Section presents the investment constraints of the DEP model. First, (3.1b)-(3.1e) bound the total capacity over the planning horizon for new investments.

$$0 \leq \sum_{e\in\mathcal{E}} x^g_{ei} \leq \overline{x}^g_i \quad \forall\, i \in \mathcal{G} \tag{3.1b}$$

$$0 \leq \sum_{e\in\mathcal{E}} x^l_{ej} \leq \overline{x}^l_j \quad \forall\, j \in \mathcal{L}^E \tag{3.1c}$$

$$0 \leq \sum_{e \in \mathcal{E}} z^l_{ej} \leq 1 \quad \forall j \in \mathcal{L}^C \tag{3.1d}$$

$$z^l_{ej} \in [0, 1] \quad \forall j \in \mathcal{L}^C, \forall e \in \mathcal{E}. \tag{3.1e}$$

$$\tag{3.1f}$$

Here, (3.1b) limits new capacity for both existing and candidate generation units, (3.1c) limits new capacity for existing lines, (3.1d) ensures that a candidate line is only installed once during the planning horizon, and (3.1e) restricts possible values for binary variable z^l. Note that, in contrast with investment in generation units, investment decisions for transmission lines define different variables for existing and candidate lines (x^l and z^l, respectively). This is due to the fact that the voltage angle at both ends of a candidate line must only be related when such line is built, hence a constraint with a binary behaviour is needed. For the sake of simplicity no further variables are defined, but expanding an installed candidate line is a simple extension of the presented model.

Constraints (3.1g)-(3.1h) set investment budgets for generation units and transmission lines in each planning period, as follows:

$$0 \leq \sum_{j \in \mathcal{L}^E} C^E_{ej}\, x^l_{ej} + \sum_{j \in \mathcal{L}^C} C^I_{ej}\, z^l_{ej} \leq \overline{I^L_e} \quad \forall e \in \mathcal{E} \tag{3.1g}$$

$$0 \leq \sum_{i \in \mathcal{G}^E} C^E_{ei}\, x^g_{eg} + \sum_{i \in \mathcal{G}^C} C^I_{ei}\, x^g_{eg} \leq \overline{I^G_e} \quad \forall e \in \mathcal{E}. \tag{3.1h}$$

Here (3.1g) sets the investment budget for transmission lines and (3.1h) set the investment budgets for generation units.

3.2.3. Operational Constraints

This section presents the operational constraints considered for the DEP model ((3.1i)-(3.1p)). Note that this are formulated for every time point t of every representative day d in every planning period e.

First, (3.1i) limits the power output of conventional generators to the sum of their initial capacity and new capacity. For candidate units, the initial capacity is set to zero. Also, (3.1j) limits the power output of variable renewable generators to their resource availability, which is given by the product of the maximum capacity and the *capacity factor* α_{dti}. Note that α_{dti} is indexed by time point t in representative day d, and represents the renewable resource profile. Also, recall that $\mathcal{G}^V \subset \mathcal{G}$ is the set of variable generators such as solar or wind generators.

$$\left\{\begin{aligned} &0 \leq p^g_{edti} \leq \overline{p^g_{ei}} + \sum_{\bar{e}\leq e} x^g_{\bar{e},i} \quad \forall\, i \in \mathcal{G}\backslash\mathcal{G}^V, && (3.1i)\\ &0 \leq p^g_{edti} \leq \ (\overline{p^g_{ei}} + \sum_{\bar{e}\leq e} x^g_{\bar{e}i})\, \alpha_{dti} \quad \forall\, i \in \mathcal{G}^V, && (3.1j) \end{aligned}\right\},$$

$$\forall e \in \mathcal{E},\, \forall d \in \mathcal{D},\, \forall t \in \mathcal{T}$$

Constraint (3.1k) limits the power output changes between two consecutive time points due to technical ramp limits. This is a key constraint for the consideration of flexibility, and it is allowed by the consideration of a chronologically ordered operation through the concept of representative days,

$$r^d_i \leq p^g_{edti} - p^g_{ed,t-1,i} \leq r^u_i \quad \forall\, i \in \mathcal{G},\, t > 0, \tag{3.1k}$$

$$\forall e \in \mathcal{E},\, \forall d \in \mathcal{D},\, \forall t \in \mathcal{T}$$

Constraint (3.1l) limits the power flow through candidate lines, and (3.1m) limits the power flow through previously existing lines. Eq. (3.1n) relates voltage angle variables to power flow variables through a DC power flow model for candidate lines, when such lines have been installed, using a Big-M model. As mentioned above, this particular constraint motivates the definition of binary investment variable z^l, due to the fact that the voltage angle at both ends of a candidate line must only be related when such line is installed. In comparison, (3.1o) determines a DC power flow model for existing lines through a much more simple equation.

$$\Big\{ -\sum_{\bar{e}\leq e} z^l_{ej}\overline{f_{ej}} \leq f_{edtj} \leq \sum_{\bar{e}\leq e} z^l_{ej}\overline{f_{ej}} \quad \forall\, j \in \mathcal{L}^C \tag{3.1l}$$

$$-\overline{f_{ej}} - \sum_{\bar{e}\leq e} x^l_{ej} \leq f_{edtj} \leq \overline{f_{ej}} + \sum_{\bar{e}\leq e} x^l_{ej} \quad \forall\, j \in \mathcal{L}^E \tag{3.1m}$$

$$-M\,(1 - \sum_{\bar{e}\leq e} z^l_{ej}) \leq\; b_j\left(\theta_{edt,j(s)} - \theta_{edt,j(r)}\right) - f_{edtj}$$

$$\leq M\,(1 - \sum_{\bar{e}\leq e} z^l_{ej}) \quad \forall\, j \in \mathcal{L}^C \tag{3.1n}$$

$$b_j\left(\theta_{edt,j(s)} - \theta_{edt,j(r)}\right) = f_{edtj} \quad \forall\, j \in \mathcal{L}^E \Big\}, \tag{3.1o}$$

$$\forall e \in \mathcal{E},\, \forall d \in \mathcal{D},\, \forall t \in \mathcal{T}$$

Finally, (3.1p) ensures the load balance in every time point.

$$\Big\{ \sum_{i\in\mathcal{G}(b)} p^g_{edti} + \sum_{j|j(s)=b} f_{edt,j} - \sum_{j|j(r)=b} f_{edt,j}$$

$$= p^D_{edtb} - p^{LS}_{edtb} \quad \forall\, b \in \mathcal{B} \Big\}, \tag{3.1p}$$

$$\forall e \in \mathcal{E},\, \forall d \in \mathcal{D},\, \forall t \in \mathcal{T}$$

As mentioned above, the operational constraints are formulated in every planning period, for every time point of every representative day. The goal of using representative days is to provide a chronological, more detailed representation of the operation than other early EP approaches (such as those based on a load duration curve and load blocks), while allowing the consideration of different operating conditions, including load or renewable profiles, throughout a specific planning period. It has been shown that for power systems with large shares of renewable energy, the representative days approach is more adequate for an accurate representation of the system's requirements and operating conditions, hence providing more efficient expansion plans in comparison with load-block models (Maluenda et al. 2018, Poncelet et al. 2016, Poncelet et al. 2017).

3.2.4. Planning Reserve Margin

The formulation in (3.1), as presented, considers no explicit uncertainty. It is expected that the use of representative days, typically selected by a clustering algorithm, will provide enough information for the model to consider different possible operational situations. However, for large shares of variable renewable energy, this deterministic approach may result insufficient.

Due to this reason, it is a common industry practice to consider a deterministic planning reserve margin (PRM) to prepare the system for errors in peak load forecasting or other unexpected events (*Seventh Northwest Power Plan*, 2016). PRM consists on a percentage capacity requirement above the system's peak load ("Reliability Metrics Specifications Sheet" 2009, *Estimating the Economically Optimal Planning Reserve Margin* 2015, Long range Energy Alternatives Planning System LEAP n.d.). In this work we will consider the following equation for the inclusion of such margin:

$$\sum_{i \in \mathcal{G}} F_i \, (\overline{p^g_{ei}} + \sum_{\bar{e} \leq e} x^g_{\bar{e},i}) \geq \overline{p^D_e} \, (1 + PRM) \tag{3.1q}$$

In (3.1q), $\overline{p^D_e}$ corresponds to the system's net peak load in planning period e. Net load is calculated as total load minus variable or non-dispatchable generation. The factor F_i is included to indicate whether the generation unit i may or may not contribute with reserve capacity ($0 \leq F_i \leq 1$). A usual approach is to consider that non-dispatchable generators may not provide reserve capacity ($F_i = 0$).

3.3. Robust Expansion Planning

As previously stated, an important challenge in EP models is the consideration of operational uncertainty and different operating conditions throughout every planning period. The presented DEP model considers a planning reserve capacity margin to prepare against unexpected realizations of short-term uncertainty, and a set of deterministic representative days to prepare for the different conditions of load and renewable availability. When finding an optimal expansion plan in DEP, it becomes ideal to consider a large number of representative days, however, this may cause the problem to lose computational tractability. On the other hand, the PRM presented in (3.1q) may be insufficient to capture key geographical and statistical aspects when planning for excess load or renewable scarcity. An alternative approach to face these issues is the use of RO techniques.

Motivated by this, the REP model proposed in this work is presented as follows,

$$\min_{\boldsymbol{x} \in X} \left(\boldsymbol{c}^\top \boldsymbol{x} + \max_{\boldsymbol{\xi} \in \Xi} \min_{\boldsymbol{y} \in Y(\boldsymbol{x}, \boldsymbol{\xi})} \boldsymbol{b}^\top \boldsymbol{y} \right). \tag{3.2}$$

In this problem, $\boldsymbol{x} = (\boldsymbol{x}^g, \boldsymbol{x}^l, \boldsymbol{z}^l)$ and X represents DEP investment constraints (3.1b)-(3.1h). Vector $\boldsymbol{\xi}$ represents a realization of the short-term uncertainty in the operational constraints of the problem, and Ξ is the *uncertainty set* for such realizations. In this work, we consider uncertainty in load and renewable capacity factors, hence $\boldsymbol{\xi} = (\boldsymbol{p}^D, \boldsymbol{\alpha})$. Also, we take $\Xi = \mathcal{U} \times \mathcal{A}$, where $\mathcal{U}$ is an uncertainty set for load ($\boldsymbol{p}^D$) and $\mathcal{A}$ is an uncertainty set for the capacity factors of the variable renewable units ($\boldsymbol{\alpha}$). Finally, $\boldsymbol{y} = (\boldsymbol{p}^g, \boldsymbol{p}^{LS}, \boldsymbol{f}^l, \boldsymbol{\theta})$ and $Y(\boldsymbol{x}, \boldsymbol{\xi})$ represents DEP operational constraints (3.1i)-(3.1p) under given investment decisions ($\boldsymbol{x}$) as well as given loads and capacity factors of renewable units ($\boldsymbol{\xi}$).

The formulation in (3.2) consists of a two-stage robust optimization problem (Zhang & Conejo 2018, Zeng & Zhao 2013, Lorca et al. 2016, Bertsimas et al. 2013), where the first stage considers investment decisions and the second stage is an operational problem where uncertainty is realized in a way that maximizes operational costs. The goal of this formulation is to immunize the system towards any possible realization within the uncertainty set Ξ, in other words, it explicitly protects the system against the worst-case realization of uncertain parameters in such set.

3.4. Uncertainty Sets

Uncertainty sets are a key concept in RO. The idea is that the decisions selected by the model will be prepared to any realization of uncertainty within such set. In this work, we consider the following uncertainty set for power demands:

$$\mathcal{U} = \Big\{ \boldsymbol{p}^D : \sum_{b \in \mathcal{B}} \frac{\left| p^D_{edtb} - \tilde{p}^D_{edtb} \right|}{\hat{p}^D_{edtb}} \leq \Gamma_D \sqrt{|\mathcal{B}|} \tag{3.3a}$$

$$p^D_{edtb} \in [\tilde{p}^D_{edtb} - \Gamma_D \, \hat{p}^D_{edtb}, \, \tilde{p}^D_{edtb} + \Gamma_D \, \hat{p}^D_{edtb}] \tag{3.3b}$$

$$\forall e \in \mathcal{E}, d \in \mathcal{D}, t \in \mathcal{T} \Big\},$$

where Γ_D is the uncertainty budget. This parameter allows the control of the level of conservatism of the solution. Also, $\tilde{p}^D_{edtb}$ and $\hat{p}^D_{edtb}$ are, respectively, a nominal value and a variability parameter for power load in period e, day d, time t and bus b. As an example, these parameters can be selected using the historical mean and historical standard deviation.

On the other hand, the uncertainty set for variable generation technologies is defined as $\mathcal{A}$. In this work, we consider both wind and solar generation as uncertain generation technologies, and $\mathcal{A} = \mathcal{A}^{\text{Wind}} \times \mathcal{A}^{\text{Solar}}$, where each individual uncertainty set is described as follows:

$$\mathcal{A}^{\mathrm{v}} = \Big\{ \boldsymbol{\alpha} : \sum_{i \in \mathcal{G}^{\mathrm{v}}} \frac{|\alpha_{dti} - \tilde{\alpha}_{dti}|}{\hat{\alpha}_{dti}} \leq \Gamma_{\mathrm{v}} \sqrt{|\mathcal{G}^{\mathrm{v}}|} \tag{3.4a}$$

$$\alpha_{dti} \in [\tilde{\alpha}_{dti} - \Gamma_{\mathrm{v}} \;\; \hat{\alpha}_{dti}, \tilde{\alpha}_{dti} + \Gamma_{\mathrm{v}}\, \hat{\alpha}_{dti}] \tag{3.4b}$$

$$\forall e \in \mathcal{E}, d \in \mathcal{D}, t \in \mathcal{T} \Big\}.$$

Here, $i \in \{\text{Wind}, \text{Solar}\}$, and $\mathcal{G}^{\mathrm{i}} \subset \mathcal{G}$ is the set of generators corresponding to technology i. The structure of the uncertainty set mirrors that of (3.3), with Γ_{i} as the uncertainty budget that determines the size of each set.

Note that these equations consider every time point in every representative day during the planning period, hence, a realization of the uncertain vector $\boldsymbol{\xi}$ consists of $|\mathcal{E}| \times |\mathcal{D}|$ daily load profiles for each bus, and daily profiles for the capacity factors of each variable generator.

This may provide valuable insights on critical conditions for power systems with a large share of variable renewable energy, by identifying troublesome days and their specific characteristics in terms of spatial resource availability and the dynamics that cause the maximization of operational costs.

4. SOLUTION ALGORITHMS

This Chapter presents an approach for solving an EP problem using the proposed REP model via a Column and Constraint Generation method and an Alternating Direction Method.

4.1. Overall Solution Approach

Due to the structure of the REP model, a convenient reformulation is introduced using the auxiliary variable η:

$$\min_{\boldsymbol{x}\in X,\eta} \left(\boldsymbol{c}^\top \boldsymbol{x} + \eta\right) \tag{4.1a}$$

$$\text{s.t. } \eta \geq \min_{\boldsymbol{y}\in Y(\boldsymbol{x},\boldsymbol{\xi})} \boldsymbol{b}^\top \boldsymbol{y} \quad \forall \boldsymbol{\xi} \in \Xi. \tag{4.1b}$$

Here, η represents the worst-case operational cost in the objective function (4.1a). The inclusion of constraint (4.1b) ensures that this cost is that of the worst possible uncertainty realization within the set Ξ.

Note that given the continuous uncertainty set presented in Section 3.4, the reformulation in (4.1) considers an infinite number of constraints, since there are infinite possible uncertainty realizations. Fortunately, an effective solution method has been developed for problems with this structure, namely, the column constraint and generation (CCG) method (Bertsimas et al. 2013, Zeng & Zhao 2013).

4.2. Column and Constraint Generation

Let $\mathcal{S} \subset \Xi$, represent a subset containing a finite number of uncertainty realizations. Now, consider the following relaxation of the reformulation of REP (4.1):

$$\min_{\boldsymbol{x}\in X,\eta} \left(\boldsymbol{c}^\top \boldsymbol{x} + \eta\right) \tag{4.2a}$$

$$\text{s.t. } \eta \geq \min_{\boldsymbol{y}\in Y(\boldsymbol{x},\boldsymbol{\xi})} \boldsymbol{b}^\top \boldsymbol{y} \quad \forall \boldsymbol{\xi} \in \mathcal{S}. \tag{4.2b}$$

Here, (4.2b) consists of a finite subset of the infinite number of constraints in (4.1b). This problem yields a first stage solution, $\boldsymbol{x}^*$, which consists of an expansion plan prepared to deal with the worst realization of uncertainty within the subset $\mathcal{S}$.

The CCG method consists of the sequential addition of uncertainty realizations, $\boldsymbol{\xi}^*$, to the subset $\mathcal{S}$, until the optimal solution of the relaxation presented in (4.2) remains invariant. Here, $\boldsymbol{\xi}^*$ corresponds to the uncertainty realization within Ξ which maximizes operational costs under a given first stage solution, $\boldsymbol{x}^*$, and it can be found solving the following optimization subproblem:

$$\max_{\boldsymbol{\xi}\in\Xi} \min_{\boldsymbol{y}\in Y(\boldsymbol{x},\boldsymbol{\xi})} \boldsymbol{b}^\top \boldsymbol{y}. \tag{4.3}$$

where $\boldsymbol{y}$ represents operational decisions for every time point of every day during the expansion horizon.

Let $f(\boldsymbol{x})$ be the objective value of problem (4.3), or in other words, the operational cost for the worst realization of uncertainty for fixed first stage investment decisions $\boldsymbol{x}$). With this, the CCG method is formally presented in Algorithm 1.

Algorithm 1 CCG solution method for REP

1: $k \leftarrow 0, S \leftarrow \emptyset$
2: **repeat**
3: $\quad(\boldsymbol{x}, \eta) \leftarrow$ optimal solution of the master problem (4.2)
4: $\quad$Evaluate $f(\boldsymbol{x})$: $\boldsymbol{\xi}^*_{k+1} \leftarrow$ optimal solution of (4.3)
5: $\quad S \leftarrow S \cup \{\boldsymbol{\xi}^*_{k+1}\}$
6: $\quad k \leftarrow k + 1$
7: **until** $f(\boldsymbol{x}) \leq \eta$

It has been shown that the CCG algorithm has finite convergence for polyhedral Ξ and $Y(\boldsymbol{x}, \boldsymbol{\xi})$ (Zeng & Zhao, 2013), which is the case in this work.

It remains to discuss how to solve the operational subproblem (4.3). Since $Y(\boldsymbol{x}, \boldsymbol{\xi})$ is polyhedral, we can take the dual in the inner min problem to obtain an overall bilinear problem, given that Ξ is also polyhedral. This bilinear problem can be approximately solved using an *alternating direction method*.

4.3. Alternating Direction Method

To solve the operational subproblem in (4.3), it is useful to consider the following reformulation using duality:

$$\max_{\boldsymbol{\xi}\in\Xi} \min_{\boldsymbol{y}} \left\{ \boldsymbol{b}^\top \boldsymbol{y} : \boldsymbol{A}\,\boldsymbol{y} \geq \boldsymbol{f} + \boldsymbol{G}\,\boldsymbol{x} + \boldsymbol{H}\boldsymbol{\xi} \right\} \tag{4.4a}$$

$$= \max_{\boldsymbol{\xi}\in\Xi} \max_{\boldsymbol{\pi}} \left\{ \boldsymbol{\pi}^\top (\boldsymbol{f} + \boldsymbol{G}\boldsymbol{x} + \boldsymbol{H}\boldsymbol{\xi}) : \boldsymbol{\pi}^\top \boldsymbol{A} = \boldsymbol{b}^\top, \boldsymbol{\pi} \geq 0 \right\} \tag{4.4b}$$

$$= \max_{\boldsymbol{\xi},\boldsymbol{\pi}} \left\{ \boldsymbol{\pi}^\top (\boldsymbol{f} + \boldsymbol{G}\boldsymbol{x}) + \boldsymbol{\pi}^\top \boldsymbol{H}\boldsymbol{\xi} : \boldsymbol{\xi} \in \Xi, \boldsymbol{\pi}^\top \boldsymbol{A} = \boldsymbol{b}^\top, \boldsymbol{\pi} \geq 0 \right\}, \tag{4.4c}$$

Here, $\boldsymbol{\pi}$ is the vector of dual variables from the inner minimzation problem in (4.3). Strong duality of the the inner problem holds if $Y(\boldsymbol{x}, \boldsymbol{\xi})$ is bounded and non-empty (Ben-Tal & Nemirovski, 2001). Given the polyhedral uncertainty set and the consideration of DC power flow, the resulting problem is bilinear, and can be solved approximately using the alternating direction method.

The alternating direction method consists of an iterative procedure, where two linear problems are solved consecutively, fixing either the realization of uncertainty $\boldsymbol{\xi}$ or the dual variables $\boldsymbol{\pi}$. For a fixed realization of uncertainty, problem (4.4) becomes the following linear problem:

$$\max_{\boldsymbol{\pi}} \left\{ \boldsymbol{\pi}^\top (\boldsymbol{f} + \boldsymbol{G}\boldsymbol{x}) + \boldsymbol{\pi}^\top \boldsymbol{H}\boldsymbol{\xi} : \boldsymbol{\pi}^\top \boldsymbol{A} = \boldsymbol{b}^\top, \boldsymbol{\pi} \geq 0 \right\}, \tag{4.5}$$

and for fixed dual variables, problem (4.4) becomes the following linear problem:

$$\max_{\boldsymbol{\xi}} \left\{ \boldsymbol{\pi}^\top (\boldsymbol{f} + \boldsymbol{G}\boldsymbol{x}) + \boldsymbol{\pi}^\top \boldsymbol{H}\boldsymbol{\xi} : \boldsymbol{\xi} \in \Xi \right\} \tag{4.6}$$

A formal description for the alternating direction method is presented in Algorithm 2.

Algorithm 2 Alternating direction method for (4.4)

1: Choose an initial $\boldsymbol{\xi} \in \Xi$
2: **repeat**
3: $\quad\boldsymbol{\pi} \leftarrow$ optimal solution of (4.5) with objective θ
4: $\quad\boldsymbol{\xi} \leftarrow$ optimal solution of (4.6) with objective θ'
5: **until** $\theta' = \theta$
6: Output: θ as estimate of $f(\boldsymbol{x})$ with solution $\boldsymbol{\xi}^*$

Note that this problem yields, for a given expansion plan $\boldsymbol{x}$, a realization of the uncertain operational parameters which maximizes the system's operational cost.

Due to the employed modelling setup, this realization of the operational uncertainty over the planning period is composed by daily profiles of load and renewable capacity factors for each representative day in each planning period.

For power systems with large shares of renewable energy, this operational subproblem may represent a valuable tool for assessing complicated and non-trivial realizations of uncertainty. As mentioned above, the only necessary condition over the uncertainty set for the presented solution methodology is that Ξ must be polyhedral. This allows to include, if needed, various linear constraints in the uncertainty sets definition to model temporal or spatial correlations, thus allowing an accurate representation of the operational uncertainty. Then, if the representation is effective, the optimal solution of the operational subproblem will provide valuable information on the loads and renewable sources which are more critical for the operation of a given power system. This could allow a better preparation for facing and understanding such scenarios, thus providing a more reliable and cost-effective energy supply.

5. COMPUTATIONAL EXPERIMENTS

This Chapter presents extensive computational experiments comparing the DEP and REP models with the purpose of understanding how the proposed model can provide support within the EP process, by finding expansion plans that are effectively prepared for significant operational uncertainty.

5.1. Case Study

In all experiments we employ a test case that consists of a 20-bus representation of the Chilean Power System, as presented in Figure 5.1. This test case has 24 transmission corridors, 136 existing generation units and 178 candidate generation units, and is based on *The New Energy Platform: Analysis of Energy Policy and Technology Scenarios for Chile* (2018). Load and renewable capacity factor profiles for the 365 days of the base year (2018), load growth for each planning period, and generation and investment costs are also based on *The New Energy Platform: Analysis of Energy Policy and Technology Scenarios for Chile* (2018). Hydraulic generation was considered to be deterministic.

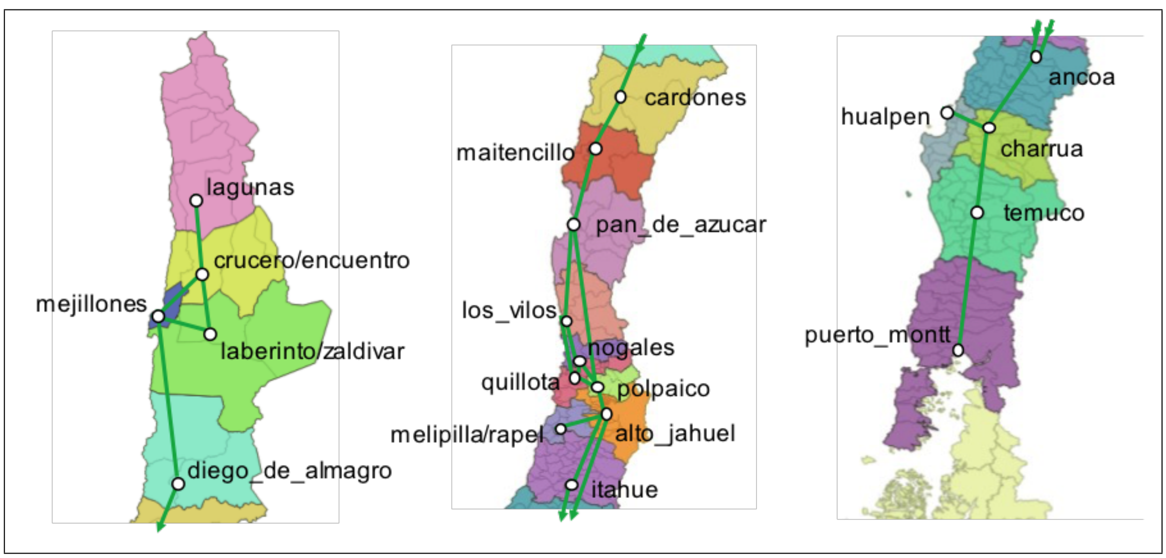


Figure 5.1. 20-bus representation of the Chilean Power System

5.2. Experimental Setup

A 20-year planning horizon was considered, divided into five planning periods of 4 years each, considering a yearly 6% discount rate in the objective function of DEP and REP. In order to determine a set of representative days for both models, three clusters and their respective weights for the 365 days of data for the base year were selected using a *hierarchical clustering* technique (Ward Jr, 1963).

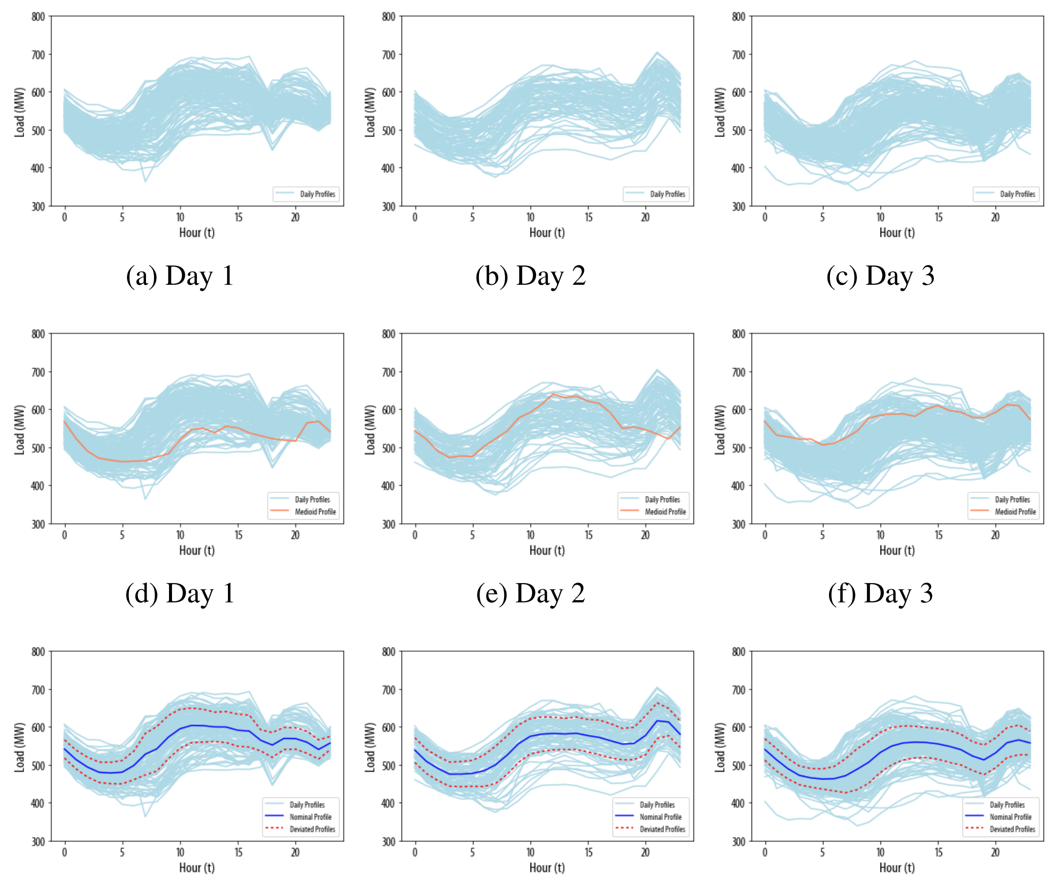
Hierarchical clustering consists on choosing a number of subsets within an initial set, based on a distance measure and a linkage criterion. For this specific case study, the *canberra distance* and the *complete linkage* criterion were considered. Then, the load and renewable capacity factor profiles for 365 days of the base year were divided into three separate clusters considering a different number of days in each cluster.

The goal of this process is to build representative days to serve as input for the DEP and REP models, as an effective way of representing the operating conditions of the system, thus providing an computationally efficient way of estimating the operational costs over the planning horizon. The clusters provide a notion over the number of days over the base year which have similar conditions in terms of load and renewable profiles.

For the DEP model, the medioid of each cluster was selected to to provide the load and capacity factors for each representative day. This approach has the advantage of considering real data based days, which better represents the need for flexible resources. For the REP model, on the other hand, uncertainty sets for each representative day were built from each cluster considering the hourly average and standard deviation as nominal and variation parameters for load and capacity factor profiles. This approach has the disadvantage of smoothing load and specially renewable resource nominal profiles. However, through the consideration of a variation parameter, the impact of this simplification is reduced, due to the fact that the operational subproblem is able to generate highly variable profiles within the defined variation parameters.

Finally, it is important to note that a different number of clusters could be selected, this will ultimately depend on the focus of the problem and the available computational resources.

Regarding the generated clusters, Figure 5.2 presents the load profiles, mediod and uncertainty set considered for each representative day of an example bus (Hualpen).



(a) Day 1 (b) Day 2 (c) Day 3

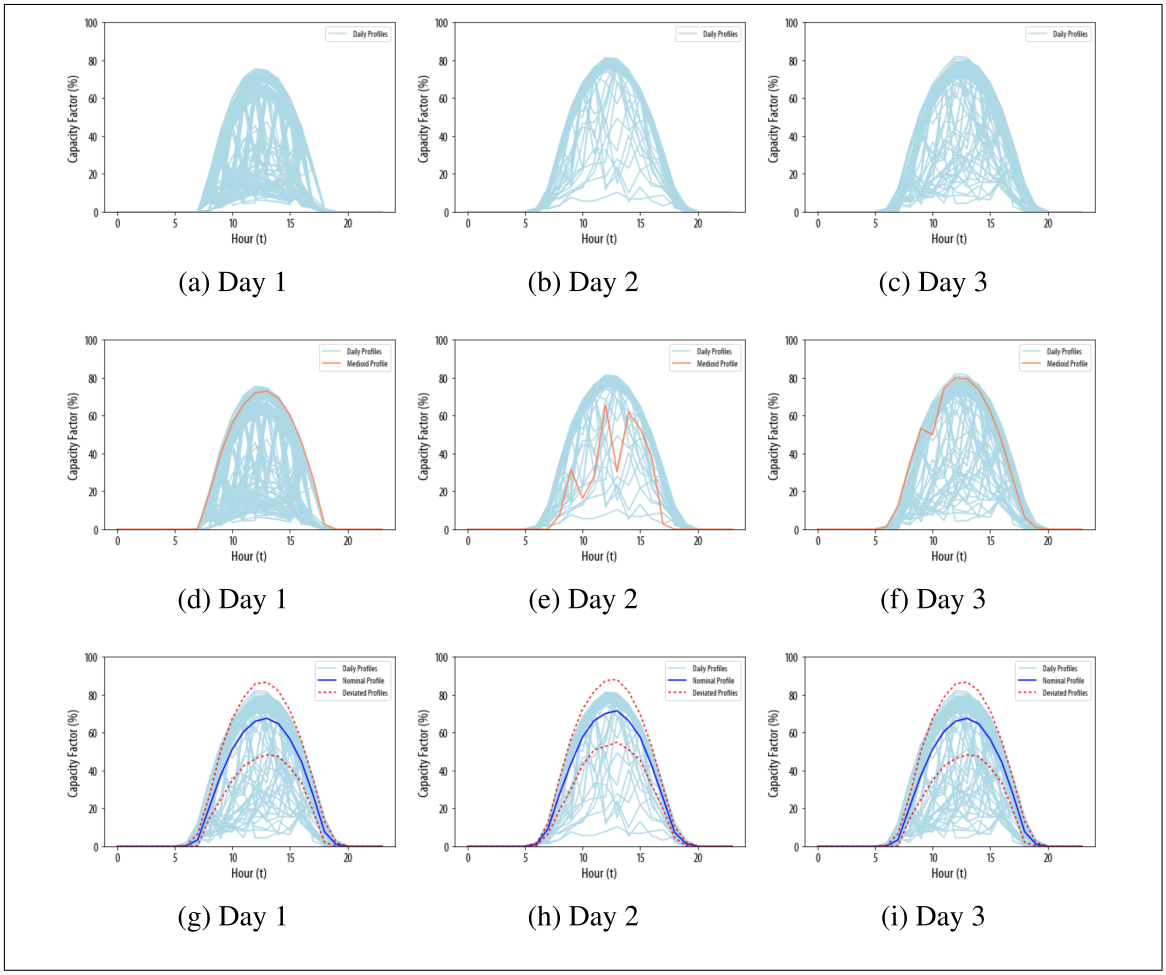
(d) Day 1 (e) Day 2 (f) Day 3

(g) Day 1 (h) Day 2 (i) Day 3

Figure 5.2. Clustering, medioids and uncertainty sets for load (Hualpen Bus)

Figures 5.2a-5.2c show the clusters and representative days profiles selected from the 365 days of the base year. Figures 5.2d-5.2f show the medioid for each cluster, which corresponds to the base load for each representative day in the DEP model. Finally, Figures

5.2g-5.2i show the nominal and variation profiles for the uncertainty set of each representative day in the REP model. Recall that the size of the uncertainty set may be enlargened or tightened through the Γ parameter. In this figure, a value of $\Gamma = 1$ was considered, this is, one standard deviation above and below the nominal value.



(a) Day 1 (b) Day 2 (c) Day 3

(d) Day 1 (e) Day 2 (f) Day 3

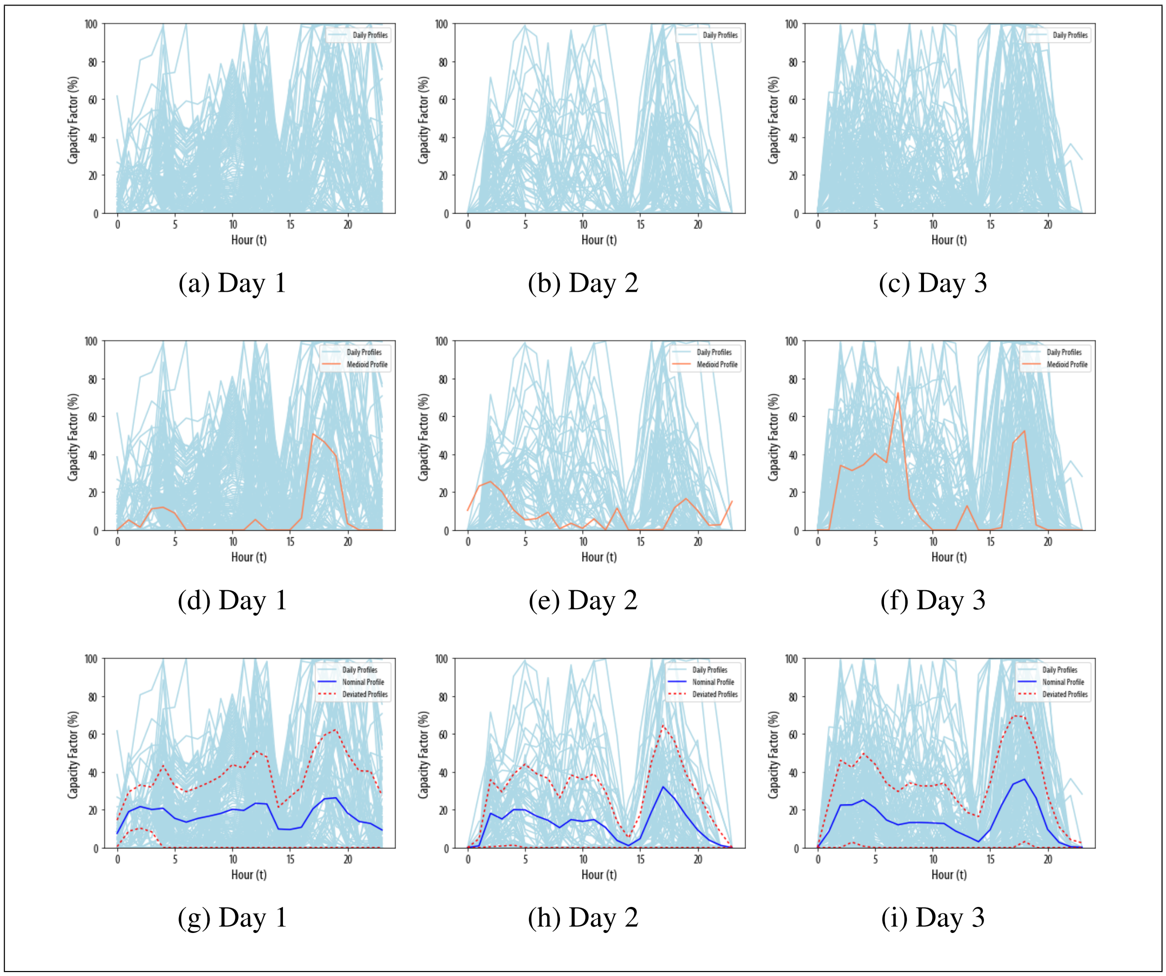
(g) Day 1 (h) Day 2 (i) Day 3

Figure 5.3. Clustering, medioids and uncertainty sets for PV (Alto Jahuel)

Figure 5.3 presents the clusters and selected representative days profiles for the capacity factor of an example solar generator (PV Alto Jahuel). Figures 5.3a-5.3c present the clustered profiles. Figures 5.3d-5.3f present the medioid of each cluster. Finally, Figures

5.3g-5.3i present the uncertainty set built based on each cluster. Note that the nominal renewable profiles are smoother curves than the selected medioids. This becomes specially observable in 5.3h as compared to 5.3e.

Figure 5.4 presents the clusters and selected representative day profiles for the capacity factor of an example wind generator (Wind Alto Jahuel).



(a) Day 1 (b) Day 2 (c) Day 3

(d) Day 1 (e) Day 2 (f) Day 3

(g) Day 1 (h) Day 2 (i) Day 3

Figure 5.4. Clustering, medioids and uncertainty sets for wind (Alto Jahuel)

Figures 5.4a-5.4c present the clustered profiles. Figures 5.4d-5.4f present the medioid of each cluster and, Figures 5.4g-5.4i present the uncertainty set built based on each cluster. Note the higher variability within each cluster of wind capacity factor as compared to solar capacity factors in Figure 5.3. This has a special impact on the variation profile shown in figures 5.4g-5.4i, where considering a value of $\Gamma = 1$ allows the operational subproblem to generate days with very scarse wind resource availability. Also, note that the nominal curves are smoothened when compared to the selected medioids. As mentioned above, this is usually an undesired effect, causing the underestimation of flexibility requirements. However, the inclusion of the variability parameter may greatly reduce the impact of this simplification.

It is important to note that it is possible to consider other approach for selecting the nominal and variability parameters of the uncertainty sets regarding each representative day. For instance, it is possible to use the medioid profile as the nominal profile, and build the variability parameter around such curve. In this work, for sake of simplicity regarding the construction of uncertainty sets, the hourly average and standard deviations were selected.

Regarding the reserve capacity requirements for the DEP model, the presented PRM approach was employed. Mini-hydro technology and variable generators such as solar and wind were not credited with firm capacity for reserve requirements, however, their expected generation was considered as renewable generation in the calculation of net load.

All models were programmed on Pyomo, a Python-based, open-source optimization modelling language (Hart et al., 2017), using Gurobi as solver (Gurobi Optimization, 2016). For all cases, we considered a convergence tolerance of 0.1%. All experiments were implemented in a Dell PowerEdge R360 server with an Intel Xeon CPU E5-2630 v4 processor running at 2.20GHz, and 64 GB of RAM.

In what follows, Section 5.3.1 presents the optimal expansion plans for the proposed models and their computational efficiency, Section 5.3.2 studies the operational performance of the obtained expansion plans using 365 days , and finally Section 5.3.3 presents a comparative analysis of the different expansion plans when facing a worst-case realization of load and renewable sources.

5.3. Experiment Results

This Section presents and discuss the obtained results using several instances of the DEP and REP models for the presented Case Study.

5.3.1. Expansion Plans and Computational Performance

This part studies the solution of the proposed REP model, and its comparison to DEP. To gain insight on each model's characteristics, the EP problem was solved for various levels of PRM in DEP, and various levels of Γ in REP.

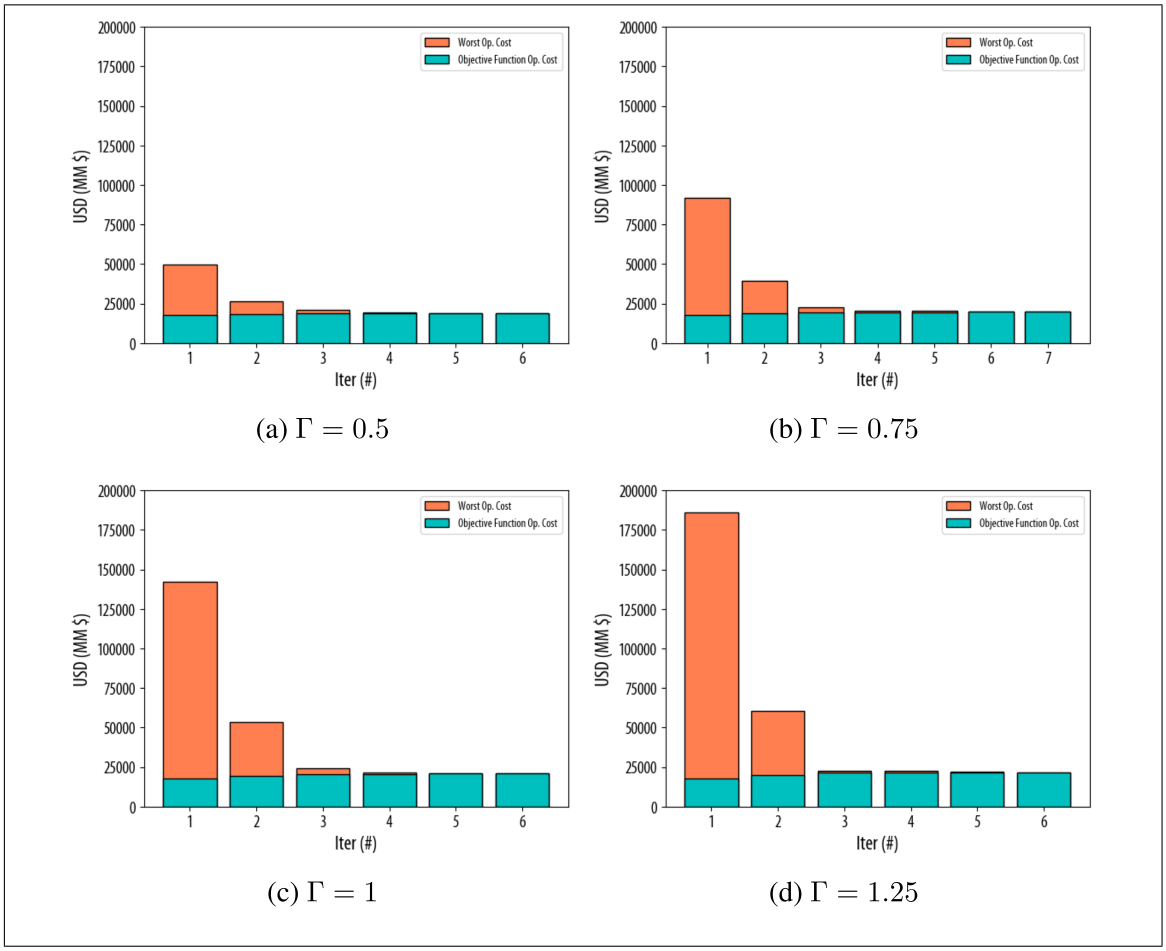
First, Table 5.1 presents the running time and number of master iterations needed until convergence of Algorithm 1 for the proposed REP model, under different values for the uncertainty budget.

Table 5.1. Computational performance of the proposed model

REP	$\Gamma = 0.5$	$\Gamma = 0.75$	$\Gamma = 1.0$	$\Gamma = 1.25$
Running time (h)	7.95	10.9	11.2	9.92
Master iterations	6	7	6	6

From this Table, it can be observed that solving time does not vary significantly with the uncertainty budget, in spite of the fact that some values of Γ require larger computational efforts and a higher number of master iterations until convergence. Note that solving time for the DEP model is not presented due to scale, given the fact that it is solved in minutes for all tested values of PRM. While maintaining tractability, the proposed REP model requires a significant computational effort in comparison to the tested DEP model. Possible approaches to reduce the computational time, such as relaxing the convergence tolerance or decomposition techniques, may be implemented to improve this aspect, but this is out of the focus of this work and will be subject of future work.

Regarding the convergence of the CCG algorithm for the proposed REP model, Figure 5.5 presents the operational cost in the objective function against the operational cost considering the worst realization of uncertainty for every representative day over the planning horizon.



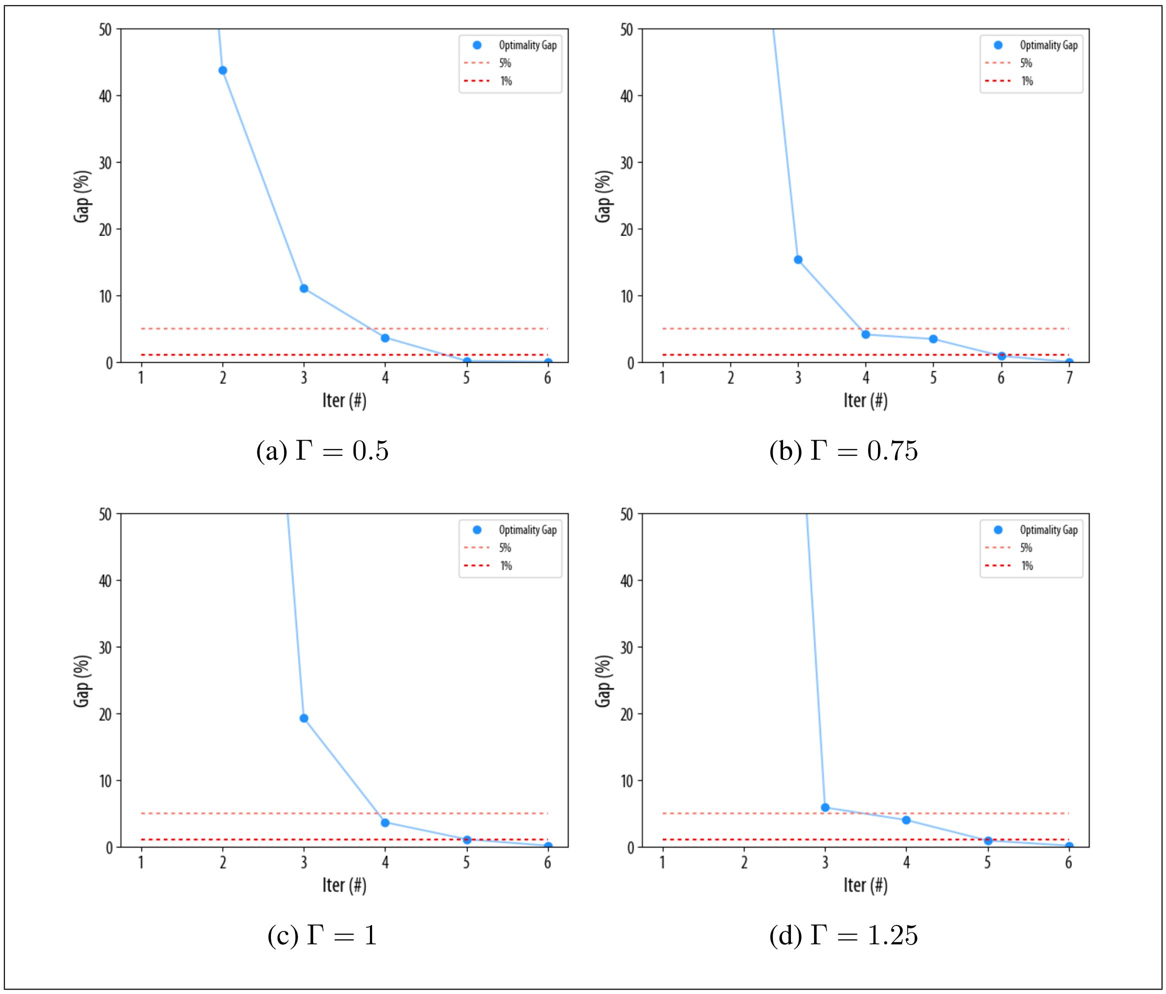
(a) $\Gamma = 0.5$ (b) $\Gamma = 0.75$

(c) $\Gamma = 1$ (d) $\Gamma = 1.25$

Figure 5.5. Worst operational cost and operational cost in the objective function for each master iteration of the CCG algorithm

In this Figure, note that the total cost under the worst scenario significantly decreases for the first 3 iterations for every solved instance. Afterwards, the difference between the objective value and the total cost under the worst scenario becomes smaller and it requires 3 or 4 additional iterations to converge for the predefined convergence tolerances of 0.1%.

This shows how critical the first scenarios become as they are included in the planning problem for REP. It also shows that for a more relaxed convergence criteria, running times could significantly diminish, considering the fact that as the number of iteration increases, more computational effort is required since the model grows in number of variables and constraints.



(a) $\Gamma = 0.5$ (b) $\Gamma = 0.75$

(c) $\Gamma = 1$ (d) $\Gamma = 1.25$

Figure 5.6. Optimality gap for each master iteration of the CCG algorithm

This insight is further supported by Figure 5.6. This figure shows the optimality gap through the master iterations for various levels of Γ. The optimality gap is estimated as the percentual difference between the lower and upper bounds of the operational cost.

These bounds are, respectively, the operational cost in the objective function of the master problem and the worst operational cost for the current expansion plan estimated through the operational subproblem. Note that these values were explicitly presented in Figure 5.5.

From Figure 5.6, it can be observed that after 4 iterations all instances of the EP problem converge below a 5% optimality gap threshold. After 5 iterations, almost all instances converged below a 1% optimality gap threshold, except for the instance considering $\Gamma = 0.75$, which needed one additional iteration until total convergence in comparison to every other REP solution (7 vs. 6). These results show that relaxing the convergence tolerance (set to 0.1% in this work) could greatly reduce the computational times presented in 5.1, due to the previously mentioned fact that the last iterations are the most expensive from a computational perspective. Specifically, the first 4 iterations presented total running times between 1.65 and 2.65 hours for all solved instances.

Regarding the objective function of the tested models, Table 5.2 presents the cost structure for the solutions obtained by the DEP and REP models.

From this table, it can be observed that investment cost and investment on generation capacity increases for more conservative solutions (i.e. as PRM or Γ increase). However, investment on transmission capacity does not follow this trend in the REP model, decreasing when Γ shifts from 1.0 to 1.25. This is due to the fact that transmission investments get postponed due to increased generation reserve capacities. Also, note that the operational costs in the objective function are not comparable between DEP and REP models, since this value is an estimation of real operation in DEP, whereas in REP it represents the worst operational cost as determined by the uncertainty set. Thus, the operational costs increase with Γ in the REP model, but decrease with PRM in the DEP model.

Regarding the obtained expansion plans, Figure 5.7 presents the final generation capacity mix for a conservative and non-conservative solution for both the DEP model (15% and 45% PRM) and for the REP model ($\Gamma = 0.50$ and $\Gamma = 1.25$).

Table 5.2. Cost decomposition for DEP and REP

DEP	15% PRM	25% PRM	35% PRM	45% PRM
Gen Inv. (MM $)	11,107	11,272	11,616	12,178
Line Inv. (MM $)	5.6443	5.8749	6.1317	6.4246
Inv. Cost (MM $)	11,112	11,278	11,622	12,184
Op. Cost (MM $)	17,132	17,073	16,852	16,517
Total Cost (MM $)	28,360	28,467	28,591	28,818
REP	$\Gamma = 0.5$	$\Gamma = 0.75$	$\Gamma = 1.0$	$\Gamma = 1.25$
Gen Inv. (MM $)	11,546	11,970	12,343	12,927
Line Inv. (MM $)	5.2057	5.9421	6.0166	5.9851
Inv. Cost (MM $)	11,551	11,976	12,349	12,933
Op. Cost (WC) (MM $)	18,907	19,860	20,910	21,671
Total Cost (WC) (MM $)	30,487	31,860	33,286	34,629

Observe from this Figure that for the DEP model, a low PRM results in a highly renewable mix. When shifting towards a higher PRM, the investment in variable renewable capacity increases (10.4 GW vs 11.5 GW in wind, 5.7 GW vs 6.0 GW in solar), together with an increased investment in firm generation capacity, particularly in gas generation technology (1.0 GW vs 3.3 GW). This is due to the fact that renewable generation reduces net load, thus lowering reserve requirements, and also gas generation units present the cheapest capacity investment, thus these units are the most cost-effective way of satisfying the PRM requirement. It is important to recall that flexibility and energy requirements are equivalent for both DEP instances, so all the changes in investment between DEP with PRM 15% and DEP with PRM 45% are motivated by reserve capacity requirements. Regarding the REP model, $\Gamma = 0.5$ provides a highly renewable mix as well, but considering

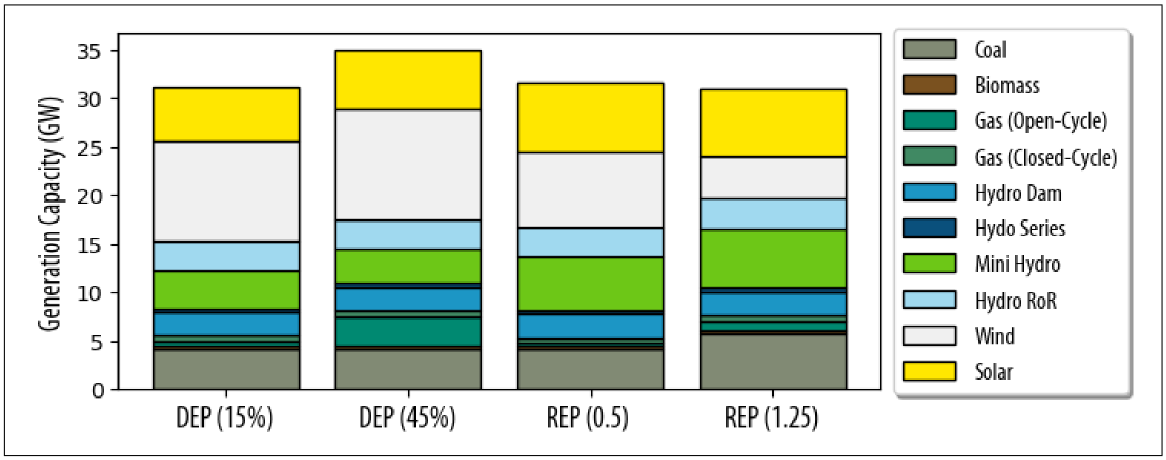
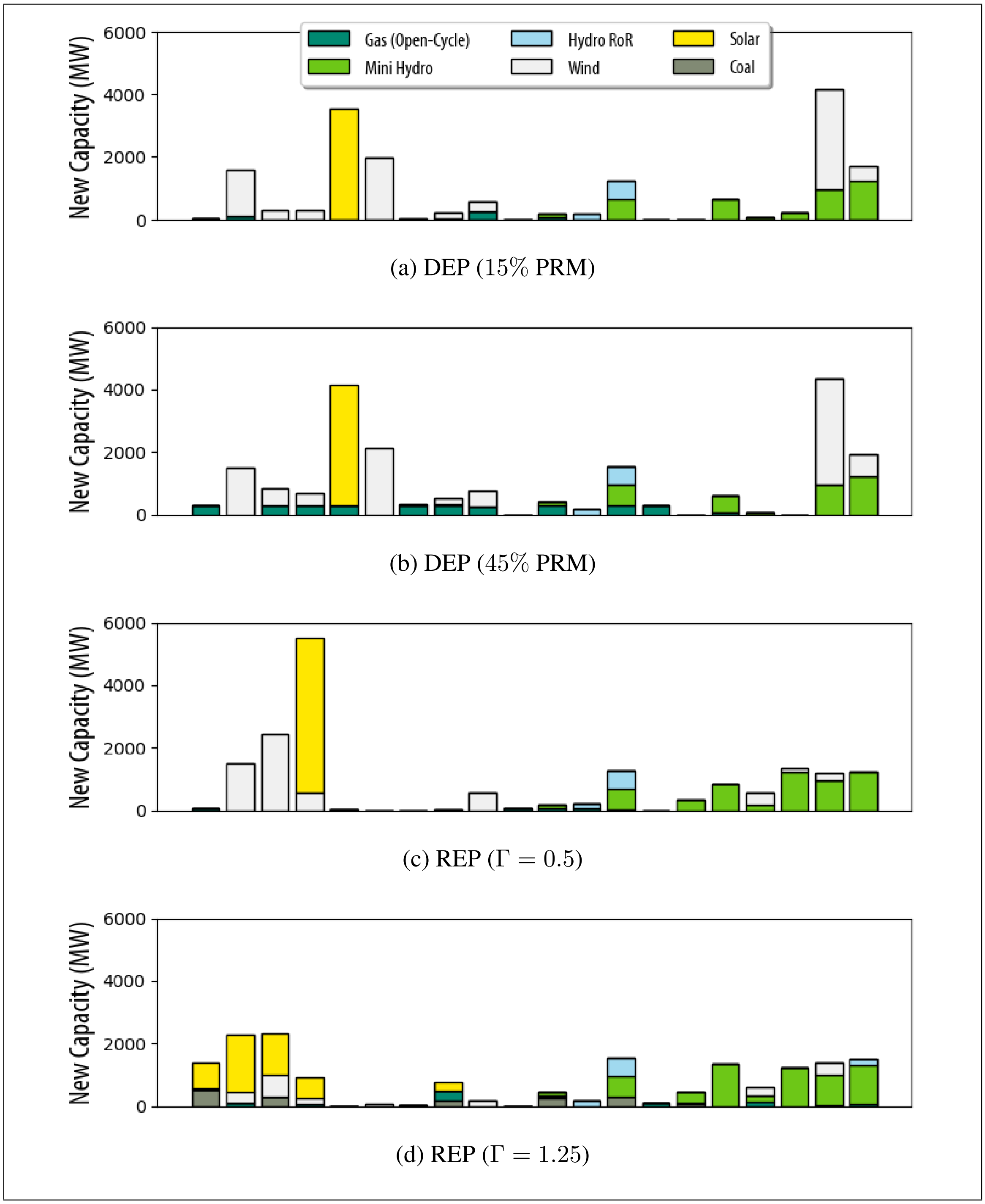


Figure 5.7. Generation capacity mix under DEP and REP in the last planning period

more investments on solar (7.1 GW) and less investments on wind (7.9 GW) than both DEP models. This is due to the fact that wind generation gets penalized, even in the non-conservative solution of the REP model, because it presents a higher variability through the year than solar generation. This is also observed when comparing the conservative REP solution ($\Gamma = 1.25$) with the non conservative REP solution ($\Gamma = 0.5$), where wind is further penalized (4.3 GW vs 7.8 GW), in addition to increased investments in reserve capacity through coal (5.7 GW vs 4.1 GW) and total gas (1.5 GW vs 0.8 GW) generation capacity.

Further analyzing the obtained expansion plans, Figure 5.8 presents the spatial placements of generation capacity investments, for the most conservative and non-conservative solutions of DEP and REP. In this Figure, the x-axis spans the different buses of the Chilean power system from north to south.

From Figs. 5.8a and 5.8b it can be observed that both the conservative and non conservative DEP solutions present an extremely similar placement of renewable resources. The main difference between these expansion plans lies in an increased investment in gas generation throughout the power system. As mentioned above, this investment is induced by the fact that, for this case study, gas technology is the cheapest capacity investment



(a) DEP (15% PRM)

(b) DEP (45% PRM)

(c) REP ($\Gamma = 0.5$)

(d) REP ($\Gamma = 1.25$)

Figure 5.8. Spatial distribution of generation capacity investments under DEP and REP (from north to south)

and thus it is the most cost-efficient way of meeting PRM requirements. In contrast, from Figs. 5.8c and 5.8d it can be observed that for REP: i) There is fewer wind investments under the conservative solution, due to its higher variability (as discussed above), and ii) Under the conservative solution, the associated risk of uncertain generation is diversified by spreading solar investments across several buses in the system. Also, the additional firm generation capacity of the conserative REP solution is installed in a different form as compared to the conservative DEP solution, in terms of both spatial location and type of generation.

From the above results, we can observe a different approach towards understanding flexibility and reserve requirements from the DEP and REP models. This is explained by the fundamental difference in their representation of operational uncertainty. In particular, we observed a diversified spatial distribution of both variable renewable and firm generation capacities under the conservative REP model. In what follows, we provide experimental evidence of the impact of this modelling approach in the system's future operation.

5.3.2. Operational Performance over the Planning Horizon

This part studies the operational performance of the expansion plans obtained by DEP and REP over the planning horizon under 365 days for each planning period. To perform this analysis, real data for 365 days in the base year, in combination with a load growth factor, were used to generate daily operational profiles for both load and renewable capacity factors for a total of 365 future days in each planning period. Using this data, an economic dispatch problem was solved for every day over the whole planning horizon, for a total of 1825 days of operation (365 days multiplied by 5 planning periods). The costs are extrapolated to represent the fact that each planning period has a duration of 4 years. Note that this experimental setup allows the evaluation of the operational performance and the total costs of the expansion plans obtained by DEP and REP over the planning horizon, for a particular realization of the uncertain parameters. In fact, it shows the operational outcomes of the planned systems if the future uncertain parameters were realized following the true data from the base year (which was used to define the representative days in DEP and REP).

Table 5.3 presents the total cost, the investment cost, the operational cost, the frequency of load shedding (LS), the total LS, and the cost associated with LS, for DEP and REP under various values of PRM and the uncertainty budget Γ, respectively. The total cost is calculated as the sum of the investment cost and the operational cost. The investment cost is obtained directly from the solutions of DEP and REP, respectively, and the operational cost and other operational metrics are based on the evaluation of the obtained expansion plans over 365 days for every planning period, as explained above.

Note from this Table that the lowest total cost is achieved by the REP model considering $\Gamma = 0.75$. The DEP model achieves its lower total cost considering a 35% PRM, but it incurs in higher LS frequency and total LS. Also, notice the decrease in operational cost, LS frequency and total LS, and the increase in the investment cost, as the solutions become more conservative in both DEP and REP models (as PRM and Γ increase, respectively). Recall that these results are for the whole planning horizon. The rest of this Section focus on the final planning period, which is of interest since it is where the power system presents the largest shares of variable renewable energy.

Figure 5.9 presents a visual comparison of the daily cost histograms for the 365 days of operation in the last planning period under DEP and REP, under different levels of conservatism. The heights and colors of the bars represent the frequency for each bin in the histogram. Note that the last bin includes outlier values for scale.

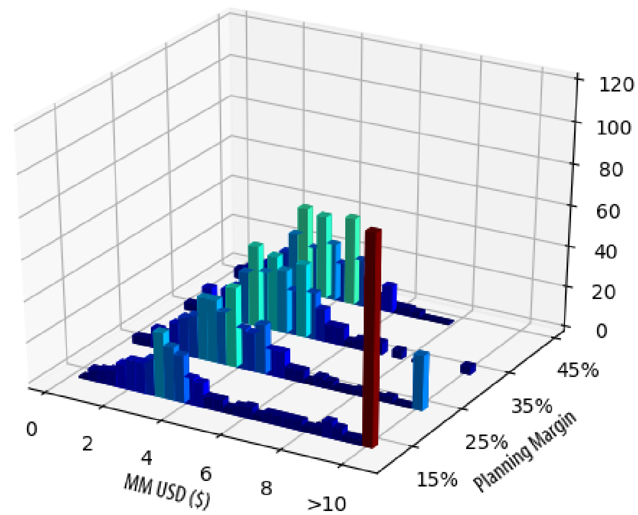
From Fig. 5.9a, it can be observed that as PRM increases, the DEP model manages to eliminate the days with operational costs above 8 MM \$ due to the increased investments in additional generation capacity. However, the daily operational cost remains highly variable throughout the year, specially in comparison to the REP model, as shown in Fig.

Table 5.3. DEP and REP performance over the planning horizon under 365 days of operation in each planning period

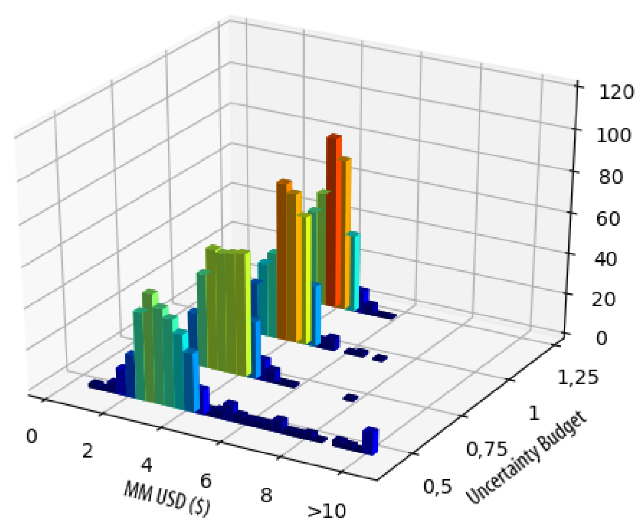
DEP	15% PRM	25% PRM	35% PRM	45% PRM
Total Cost (MM$)	31,364	29,132	28,708	28,819
Inv. Cost (MM $)	11,112	11,278	11,622	12,184
Op. Cost (MM$)	20,251	17,854	17,086	16,634
LS Frequency (Days %)	12.1	4.77	1.59	0.05
Total LS (GWh)	1,532	321.1	44.54	0.467
LS Cost (MM$)	6,067	1,272	176.4	1.849
REP	$\Gamma = 0.5$	$\Gamma = 0.75$	$\Gamma = 1.0$	$\Gamma = 1.25$
Total Cost (MM$)	28,621	28,571	28,822	28,920
Inv. Cost (MM $)	11,551	11,976	12,349	12,933
Op. Cost (MM$)	17,070	16,596	16,473	15,988
LS Frequency (Days %)	5.10	0.60	0.38	0.05
Total LS (GWh)	176.3	6.064	5.438	0.558
LS Cost (MM$)	698.1	24.02	21.54	2.209

5.9b. In fact, the proposed REP model presents a considerably more stable operational cost throughout the year.

This insight is further supported by the results presented in Figure 5.10. This figure presents the LS histograms for the 365 analyzed days in the final planning period of DEP and REP models for different levels of conservatism. The value of 0% LS has been removed from these histograms. Note that the last bin includes outlier values for scale.



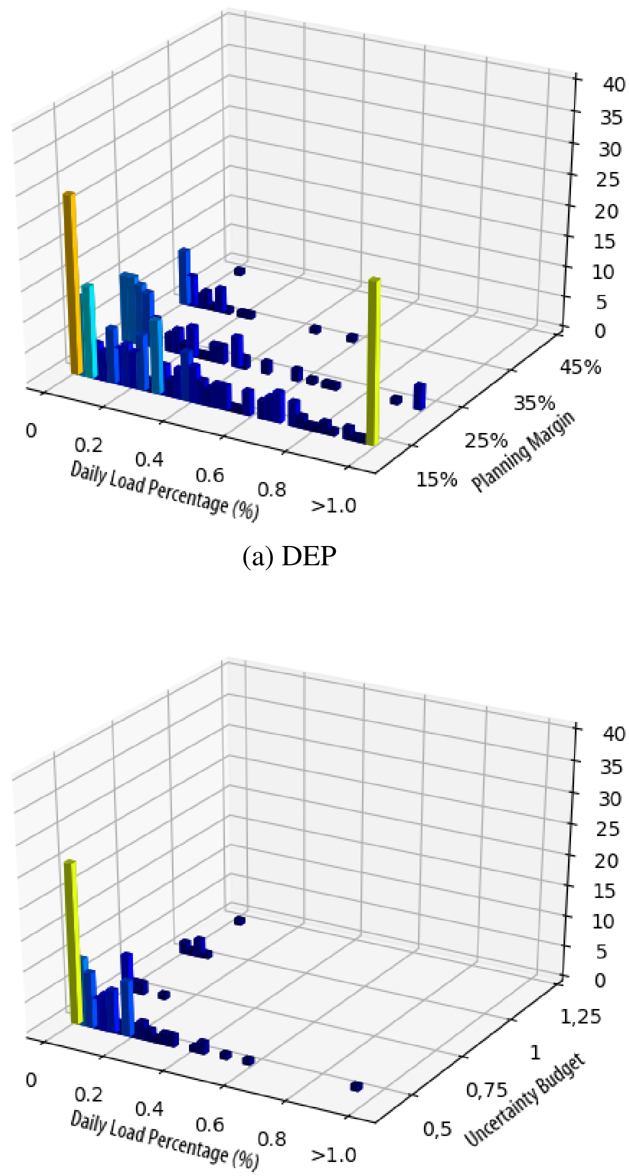
(a) DEP



(b) REP

Figure 5.9. Histogram for the daily operational cost of DEP and REP under 365 days of operation in the last planning period

In this figure, it can be observed that for most of the tested DEP model solutions there is a large number of days with LS. In contrast, most of the REP model solutions are able to greatly reduce the number of days with LS. Notably, the comparison between the solutions



(a) DEP

(b) REP

Figure 5.10. Histogram for the daily load shedding percentage of DEP and REP under 365 days of operation in the last planning period (the value of 0% has been removed from these histograms)

with lower total cost for the whole planning period, (DEP considering 35% PRM and REP considering 0.75Γ) shows the effectiveness of the proposed model to prepare for various realizations of operational uncertainty without comprising operational standards.

To further understand the variability of the operational costs achieved by DEP and REP, Table 5.4 presents the maximum daily operational cost and the average of the 36 days with the highest operational cost, over the 365 days of operation in the final planning periods.

Table 5.4. DEP and REP performance for the days with higher costs in the final planning period

DEP	15% PRM	25% PRM	35% PRM	45% PRM
Max Daily Cost (MM$)	85.39	41.08	17.75	7.687
Max 36 Avg. Cost (MM $)	35.04	15.01	7.421	6.011
REP	$\Gamma = 0.5$	$\Gamma = 0.75$	$\Gamma = 1.0$	$\Gamma = 1.25$
Max Daily Cost (MM$)	22.11	7.644	7.122	5.864
Max 36 Avg. Cost (MM $)	9.199	4.816	5.131	4.717

It can be observed that all the expansion plans obtained under DEP incur in very high operational costs under such worst 36 days. Only the most conservative solution, with a PRM of 45%, manages to reduce the max daily cost value below the 8 MM \$ threshold, but still maintaining a high average cost under the worst 36 days. When compared to the proposed REP model, only the least conservative solution ($\Gamma = 0.5$) incurs in higher costs than the most conservative DEP solution; in fact, every other REP solution performs better under both metrics. These results further support the notion that the proposed REP model is significantly better prepared for high levels of operational uncertainty.

Finally, if we compare the best DEP and REP solutions in terms of total cost (considering a 35% PRM and $\Gamma = 0.75$, respectively), we can observe that REP achieves a 0.86%

lower total cost. Moreover, regarding metrics related with reliability (LS frequency, total LS) and cost stability in the final planning period, a significant difference can be observed in favour of the REP model, providing experimental evidence of its potential impact to support the EP process.

As previously mentioned, the experimental setup considered in the present subsection allows to gain insights on the tested models performance for an expected realization of the operational uncertainty, this is, considering an accurate prediction of load growth factor and facing the same uncertainty realizations as the data used for building the representative days. The following Section provides experimental evidence of the advantages of using the proposed REP as compared to a deterministic representative days approach, under a complicated and unexpected realization of the operational uncertainty.

5.3.3. Operational Performance under a Worst-Case Day

In Section 5.3.2 we studied the operational performance of the expansion plans obtained from the DEP and REP models under 365 days in every planning period, where the load and capacity factor profiles of such days were directly extrapolated from actual data. We are now interested in studying the operational performance of DEP and REP under a "worst-case day". To perform this analysis, an uncertainty set was built using the hourly average and standard deviation through the 365 days of data, to select the nominal and variability parameters in this set. Then, the *min-max* operational subproblem (4.3) was solved under the various DEP and REP expansion plans, considering different levels of conservatism. The uncertainty budget for this operational subproblem was set to $\Gamma = 1$. Then, the operation of each plan under the obtained worst-case day was analyzed. Note that this experimental setup allows to individually find the worst daily realization (in terms of operational cost) of uncertain parameters for each tested expansion plan, and evaluate the operational performance of these models under such realization. This provides a notion of how much operational costs could vary under unexpected and complicated realizations of operational uncertainty.

Table 5.5 presents the cost decomposition and load shedding (LS) for the operation of DEP and REP under the nominal day (based on the nominal parameters of the uncertainty set) and under the worst-case day, in the final planning period.

Table 5.5. Operational performance of DEP and REP under a nominal and a worst-case day in the last planning period

DEP	15% PRM	25% PRM	35% PRM	45% PRM
Nominal Op. Cost (MM \$)	3.212	3.071	2.997	2.935
Worst Op. Cost (MM \$)	114.9	58.95	18.17	8.848
Worst Gen Cost (MM \$)	6.812	8.145	8.768	8.848
Worst LS Cost (MM \$)	108.1	50.80	9.404	0.0
Worst LS (Load %)	5.6	2.6	0.5	0.0
REP	$\Gamma = 0.5$	$\Gamma = 0.75$	$\Gamma = 1.0$	$\Gamma = 1.25$
Nominal Op. Cost (MM \$)	2.938	3.308	3.769	3.560
Worst Op. Cost (MM \$)	27.76	5.498	5.565	5.335
Worst Gen Cost (MM \$)	5.657	5.498	5.565	5.335
Worst LS Cost (MM \$)	22.11	0.0	0.0	0.0
Worst LS (Load %)	1.1	0.0	0.0	0.0

In this Table, it can be observed that under the nominal values of uncertain parameters, the DEP model has a similar performance as compared to the proposed REP model. However, under the worst-case day, the difference between DEP and REP becomes critical. Only the most conservative solution of DEP, considering a 45% PRM, manages to reduce LS to zero, whereas the only expansion plan obtained through the REP model that incurs in LS is the least conservative one ($\Gamma = 0.5$). Every other solution obtained through the proposed REP approach manages to avoid LS under a worst-case day. Further, for this specific uncertainty set, the expansion plans provided by the REP model with $\Gamma \geq 0.75$ are able to maintain operational costs under 6 MM \$, whereas the best DEP model solution, even though managing to avoid LS, still incurs in a considerably higher generation cost, over 8 MM \$. This is due to the conceptual differences in the modelling approach, specifically, the fact that the proposed REP model estimates reserve capacity requirements by optimizing real operation under inconvenient realizations of uncertainty and adjusting investment decisions for a reliable and cost-efficient operation under such scenarios.

Recall from subsection 5.3.2 that under 365 days of operation for each planning period over the whole planning horizon, considering an expected realization of uncertain parameters, the proposed REP model achieved lower total costs and a more stable operation, specially for the last planning period. In addition to this, the proposed REP model has a more reliable and cost-efficient operation under unexpected worst-case daily realizations of the operational uncertainty. Thus, it may be concluded that the proposed REP approach provides expansion plans which are better prepared to deal with various realizations of the operational uncertainty, especially for power systems with large shares of variable renewable energy, at a lower total expected cost as compared to a deterministic representative days approach. As presented in subsection 5.3.1, this is due to a comprehensive understanding of key statistical and spatial aspects regarding the specific flexibility and reserve-capacity requirements for a given power system.

6. CONCLUSIONS AND FINAL REMAKRS

This Chapter presents the main conclusions and discuss possible future research topics related to this work.

6.1. Conclusions

We have developed an adaptive robust optimization model for the multi-period Generation and Transmission EP problem, considering the use of uncertain representative days as an effective representation of the operational uncertainty and the flexibility and reserve capacity requirements needed to provide a reliable energy supply in systems with large shares of variable renewable energy.

This problem may be efficiently solved using the CCG method. Extensive computational experiments on a 20-bus representation of the Chilean Power System show the effectiveness of the proposed REP model under different realizations of the short-term uncertainty.

The modeling framework is innovative, specially regarding the joint consideration of load and renewable profiles chronology and the uncertain nature of operational parameters, which result on a endogenous comprehension of both flexibility and reserve capacity requirements for a given power system.

Finally, through a comparative analysis of the operational performance under 365 days of operation for each planning period over the whole planning horizon, and the operational performance under a worst-case day, we provide experimental evidence of the advantages, both in costs and security of energy supply, of the proposed REP model, as compared to a deterministic representative days approach with a planning reserve margin, due to an effective spatial placement of variable and flexible resources, and explicit penalization of variability between different renewable resources.

6.2. Future Work

Regarding future work, remaining challenges include further operational detail on uncertain representative days, such as commitment constraints (through convex relaxations or other computationally efficient approach) or alternate current power flow, the inclusion of outages and extreme events in additional uncertainty sets, and the extension of the model to consider long term uncertainties. Also, the reduction of computational times may be studied via decomposition techniques or other approaches.

Real applications of interest, given the characteristics of the proposed model, could regard optimal battery placement and the study of the interaction between the REP model and renewable portfolio standards, where the effective consideration of spatial and statistical aspects of the system may become critical. Also, the presented operational sub problem may be a useful tool for real power systems to identify complicated realizations of the operational uncertainty.

REFERENCES

Ahmed, S., King, A. J., & Parija, G. (2003, May 01). A multi-stage stochastic integer programming approach for capacity expansion under uncertainty. *Journal of Global Optimization*, *26*(1), 3-24.

Baringo, L., & Baringo, A. (2018, Jan). A stochastic adaptive robust optimization approach for the generation and transmission expansion planning. *IEEE Transactions on Power Systems*, *33*(1), 792-802.

Belderbos, A., & Delarue, E. (2015). Accounting for flexibility in power system planning with renewables. *International Journal of Electrical Power and Energy Systems*, *71*, 33 - 41.

Ben-Tal, A., & Nemirovski, A. (2001). *Lectures on modern convex optimization: analysis, algorithms, and engineering applications* (Vol. 2). SIAM.

Bertsimas, D., Litvinov, E., Sun, X. A., Zhao, J., & Zheng, T. (2013). Adaptive robust optimization for the security constrained unit commitment problem. *IEEE Transactions on Power Systems*, *28*(1), 52-63.

Chatthaworn, R., & Chaitusaney, S. (2017). Robust transmission expansion planning considering the effect of uncertain generation from renewable energy source. *Journal of Engineering and Applied Science*, *12*, 7805 - 7814.

Chen, B., Wang, J., Wang, L., He, Y., & Wang, Z. (2014, Nov). Robust optimization for transmission expansion planning: Minimax cost vs. minimax regret. *IEEE Transactions on Power Systems*, *29*(6), 3069-3077.

Chen, B., & Wang, L. (2016, Nov). Robust transmission planning under uncertain generation investment and retirement. *IEEE Transactions on Power Systems*, *31*(6), 5144-5152.

Conejo, A. J., Morales, L. B., Jalal, K., & Siddiqui, A. S. (2016). *Investment in electricity generation and transmission* (1st ed.). Springer International Publishing.

Dehghan, S., Amjady, N., & Conejo, A. J. (2016, May). Reliability-constrained robust power system expansion planning. *IEEE Transactions on Power Systems*, *31*(3), 2383-2392.

Dehghan, S., Amjady, N., & Conejo, A. J. (2017, Sept). Adaptive robust transmission expansion planning using linear decision rules. *IEEE Transactions on Power Systems*, *32*(5), 4024-4034.

Dehghan, S., Amjady, N., & Kazemi, A. (2014, March). Two-stage robust generation expansion planning: A mixed integer linear programming model. *IEEE Transactions on Power Systems*, *29*(2), 584-597.

Estimating the economically optimal planning reserve margin (Tech. Rep.). (2015, March). 101 Montgomery Street, Suite 1600 San Francisco, CA 94104: Energy and Environmental Economics, Inc. Retrieved from `https://www.epelectric.com/files/html/PRM_Report.pdf`

Gorenstin, B. G., Campodonico, N. M., Costa, J. P., & Pereira, M. V. F. (1993, Feb). Power system expansion planning under uncertainty. *IEEE Transactions on Power Systems*, *8*(1), 129-136.

Gurobi Optimization, I. (2016). *Gurobi optimizer reference manual.* Retrieved from `http://www.gurobi.com`

Hart, W. E., Laird, C. D., Watson, J.-P., Woodruff, D. L., Hackebeil, G. A., Nicholson, B. L., & Siirola, J. D. (2017). *Pyomo–optimization modeling in python* (Second ed., Vol. 67). Springer Science and Business Media.

Jabr, R. A. (2013, Nov). Robust transmission network expansion planning with uncertain renewable generation and loads. *IEEE Transactions on Power Systems*, *28*(4), 4558-4567.

Jin, S., Botterud, A., & Ryan, S. M. (2014, Sept). Temporal versus stochastic granularity in thermal generation capacity planning with wind power. *IEEE Transactions on Power Systems*, *29*(5), 2033-2041.

Long range Energy Alternatives Planning System LEAP. (n.d.). *Planning reserve margin.* Retrieved 2018-08-30, from `https://www.energycommunity.org/Help/Transformation/Planning_Reserve_Margin.htm`

Lorca, Á., Sun, X. A., Litvinov, E., & Zheng, T. (2016). Multistage adaptive robust optimization for the unit commitment problem. *Operations Research*, *64*(1), 32–51.

Maluenda, B., Negrete-Pincetic, M., Olivares, D. E., & Álvaro Lorca. (2018). Expansion planning under uncertainty for hydrothermal systems with variable resources. *International Journal of Electrical Power and Energy Systems*, *103*, 644 - 651.

Moreira, A., Pozo, D., Street, A., & Sauma, E. (2017, July). Reliable renewable generation and transmission expansion planning: Co-optimizing system's resources for meeting renewable targets. *IEEE Transactions on Power Systems*, *32*(4), 3246-3257.

Moreira, A., Street, A., & Arroyo, J. M. (2015, July). An adjustable robust optimization approach for contingency-constrained transmission expansion planning. *IEEE Transactions on Power Systems*, *30*(4), 2013-2022.

Nagl, S., Fürsch, M., & Lindenberger, D. (2013). The costs of electricity systems with a high share of fluctuating renewables: A stochastic investment and dispatch optimization model for europe. *The Energy Journal*, *34*(4), 151-179.

The new energy platform: Analysis of energy policy and technology scenarios for chile (Tech. Rep.). (2018, 2). Av. Vicuña Mackenna 4860, Macul, Santiago, Chile: OCM Lab.

Oree, V., Hassen, S. Z. S., & Fleming, P. J. (2017). Generation expansion planning optimisation with renewable energy integration: A review. *Renewable and Sustainable Energy Reviews*, *69*, 790 - 803.

Poncelet, K., Delarue, E., Six, D., Duerinck, J., & D'haeseleer, W. (2016). Impact of the level of temporal and operational detail in energy-system planning models. *Applied Energy*, *162*, 631 - 643.

Poncelet, K., Höschle, H., Delarue, E., Virag, A., & D'haeseleer, W. (2017, May). Selecting representative days for capturing the implications of integrating intermittent renewables in generation expansion planning problems. *IEEE Transactions on Power Systems*, *32*(3), 1936-1948.

Reliability metrics specifications sheet (Computer software manual No. ALR-13). (2009, 2). Retrieved from `https://www.nerc.com/comm/PC/Performance%20Analysis%20Subcommittee%20PAS%202013/1-3%20July%209.pdf`

Roldán, C., de la Nieta, A. S., García-Bertrand, R., & Mínguez, R. (2018). Robust dynamic transmission and renewable generation expansion planning: Walking towards sustainable systems. *International Journal of Electrical Power and Energy Systems*, *96*, 52 - 63.

Ruiz, C., & Conejo, A. (2015). Robust transmission expansion planning. *European Journal of Operational Research*, *242*(2), 390 - 401.

Seljom, P., & Tomasgard, A. (2015). Short-term uncertainty in long-term energy system models — a case study of wind power in denmark. *Energy Economics*, *49*, 157 - 167.

Seventh northwest power plan (Tech. Rep.). (2016, March). 851 Sw Sixth Ave, Portland OR: Northwest Power and Conservation Council. Retrieved from `https://www.nwcouncil.org/sites/default/files/7thplanfinal_chap10_opplanningreserves_1.pdf` van Stiphout, A., Vos, K. D., & Deconinck, G. (2017, Jan). The impact of operating reserves on investment planning of renewable power systems. *IEEE Transactions on Power Systems*, *32*(1), 378-388.

Ward Jr, J. H. (1963). Hierarchical grouping to optimize an objective function. *Journal of the American statistical association*, *58*(301), 236–244.

Wu, Z., Liu, Y., Gu, W., Wang, Y., & Chen, C. (2018). Contingency-constrained robust transmission expansion planning under uncertainty. *International Journal of Electrical Power and Energy Systems*, *101*, 331 - 338.

Zeng, B., & Zhao, L. (2013). Solving two-stage robust optimization problems using a column-and-constraint generation method. *Operations Research Letters*, *41*(5), 457-461.

Zhang, X., & Conejo, A. J. (2018, March). Robust transmission expansion planning representing long- and short-term uncertainty. *IEEE Transactions on Power Systems*, *33*(2), 1329-1338.