



PONTIFICIA UNIVERSIDAD CATOLICA DE CHILE  
SCHOOL OF ENGINEERING

# **AGE OF INFORMATION IN IOT-BASED WIRELESS NETWORKED CONTROL SYSTEMS**

**JUAN PABLO MENA**

Thesis submitted to the Office of Research and Graduate Studies  
in partial fulfillment of the requirements for the degree of  
Master of Science in Engineering

Advisor:

FELIPE NÚÑEZ RETAMAL

Santiago de Chile, March, 2021

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*To all my teachers*

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## ABSTRACT

Timeliness of information is critical for ensuring performance of networked control systems (NCSs). Despite timeliness is intrinsically linked to the underlying communication network, up to date design of NCSs considers the control and communication layers independently. As a step towards changing this perspective, this work analyses the interplay between both layers using Age of Information (AoI) and Peak Age in IoT-based networked control systems, where a set of independent feedback loops operate over a shared network. In particular, the analysis focuses on how the metrics based on the concept of status age behave in typical MAC schemes used in IoT deployments, and how this behaviour can be used to explain the Quality of Control (QoC) of the loops inserted in a specific MAC scheme network in the context of two classical NCSs indicators: maximum allowable transmit interval (MATI) and maximum allowable delay (MAD).

Different concepts relevant for studying NCS are reviewed. Then, based on a classic NCS model commonly used in the literature, expressions for AoI, peak age and the probability of the peak age surpassing a certain threshold related to the loop stability in two types of CSMA/CA and TDMA-based wireless NCSs are derived. Alongside, some insights about the usefulness of certain MAC protocols depending on specific design requirements are stated, and effective comparisons are made in the context of design problems.

The principal conclusions involve the obtention of different optimal sampling rates for the AoI in the different protocols analysed for different packet lengths. Also, the relationship between the peak age metric and the NCS stability is established by determining the optimal sampling rate needed in order to avoid a certain stability-related threshold.

## RESUMEN

Una correcta ocurrencia en el tiempo de transmisión de la información es fundamental para garantizar el rendimiento de los sistemas de control sobre redes (NCS). A pesar de que esta ocurrencia está intrínsecamente vinculada a la red de comunicación, hoy en día el diseño de los sistemas de control en red considera las capas de control y comunicación de forma separada. Como un paso hacia el cambio de esta perspectiva, este trabajo analiza la interacción entre ambas capas utilizando los conceptos de la edad de la información (AoI) y la edad pico en sistemas de control en red basados en el internet de las cosas (IoT), donde un conjunto de lazos de control independientes operan a través de una red compartida. En particular, el análisis se centra en cómo se comportan las métricas basadas en el concepto de la edad del estado en los esquemas inalámbricos de control de acceso al medio (MAC) típicamente utilizados en las implementaciones de IoT, y cómo este comportamiento se puede utilizar para explicar la calidad de control (QoC) de los lazos insertados en una red específica, la cual está vinculada con dos indicadores clásicos de NCS: el intervalo de transmisión máximo permitido (MATI) y máximo retardo permitido (MAD).

Se revisan diferentes conceptos relevantes para el estudio de sistemas de control en redes. Luego, sobre la base de modelos NCS comúnmente utilizados en la literatura, se derivan expresiones para AoI, la edad pico y la probabilidad de que la edad pico supere un cierto umbral relacionado con la estabilidad del lazo para NCS inalámbricos que utilizan dos tipos distintos de protocolos CSMA/CA y un tipo de TDMA. Además, se establecen algunas ideas sobre la utilidad de ciertos protocolos MAC dependiendo de los requisitos de diseño específicos, y se realizan comparaciones efectivas en el contexto de ejemplos de problemas de diseño.

Las principales conclusiones involucran la obtención de diferentes tasas de muestreo óptimas para la edad de la información en los diferentes protocolos analizados para diferentes largos de paquetes. Además, la relación entre la edad pico y la estabilidad de los lazos

se logra a través de la determinación de tasas de muestreo optimas para evitar el exceder un cierto umbral relacionado a la estabilidad del lazo.

# 1. INTRODUCTION

## 1.1. Motivation

Among the new technological trends, the Internet of Things (IoT) stands out with the promise to revolutionize our daily lives through its pervasive sensing, information integration, and automated decision making (Atzori, Iera, & Morabito, 2017). The IoT was conceived as a data-providing large-scale network of heterogeneous objects with Internet connectivity (Al-Fuqaha, Guizani, Mohammadi, Aledhari, & Ayyash, 2015); however, as communication technologies have enlarged the set of devices with networking capabilities, the original conception of the IoT has mutated to a distributed, sparsely coupled, system of interacting smart objects, or things (Atzori et al., 2017).

The change in the conceptual vision of the IoT has resulted in an extension of its original purpose (Atzori et al., 2017; J. Lin et al., 2017). The incorporation to the network of devices with advanced processing and actuating capabilities has enabled using the resources of the IoT for closed-loop automatic control tasks (Oróstica & Núñez, 2019; Mendoza & Núñez, 2019), closer to the control point of view of a cyber-physical system (CPS) (Langarica, Rüffelmacher, & Núñez, 2020; Núñez, Langarica, Díaz, Torres, & Salas, 2020). From this point of view, the IoT constitutes a powerful backbone to control external processes using its sensing, actuating, and computational power (Oróstica & Núñez, 2019; Mendoza & Núñez, 2019).

In this context, the paradigm of networked control systems (NCSs) (Baillieul & Antsaklis, 2007), which explicitly includes communication channels in the control loop, is the natural framework to study the integration of feedback control theory and the IoT. In particular, given the wireless nature of the IoT at the field level, the study of wireless networked control systems (WNCSs), conceived as a set of feedback control loops involving communications over an unreliable multipurpose wireless medium (Park, Coleri Ergen, Fischione, Lu, & Johansson, 2018), is relevant for a proper integration of both concepts.

In WNCS, “sensor nodes attached to the physical plant sample and transmit their measurements to the controller over a wireless channel. Controllers then compute control commands based on these sensor data, which are then forwarded to the actuators in order to influence the dynamics of the physical plant” (Park et al., 2018).

The main idea of using WNCSs is to implement fully functional control systems without the need of wires, thus breaking the limits of distance and space in the connection between sensors, actuators and controllers, adding flexibility to the whole system design. In this scenario, each element can be freely designed, thinking specifically in the requirements of the application and not worrying about the physical place where it needs to be. Today, different WNCS applications have been backed up by several international organizations such as Wireless Avionics Intra-Communications Alliance, ZigBee Alliance, Z-wave Alliance, International Society of Automation, among others (Park et al., 2018). WNCSs will play a fundamental role in Industry 4.0, the fourth stage of the industrial revolution, where communication between machines is assured and computers will make decisions without human involvement.

Industry 4.0 and the latest technological advances have brought substantial improvements in different aspects of communication networks, in order to expand their use to more critical and real time applications. Figure 1.1 shows the different needs in terms of latency (the delay in the communications between two nodes) and bandwidth (data rate per second) of different applications, from home sensors and domestic video application to advanced Virtual Reality (VR) systems and industrial control automation.

In the last decade, wireless networks developers have been particularly concerned on increasing transfer rate. This is specially true for WiFi and cellular networks development in the recent years. Table 1.1 shows how the transfer speed has increased with new protocols.

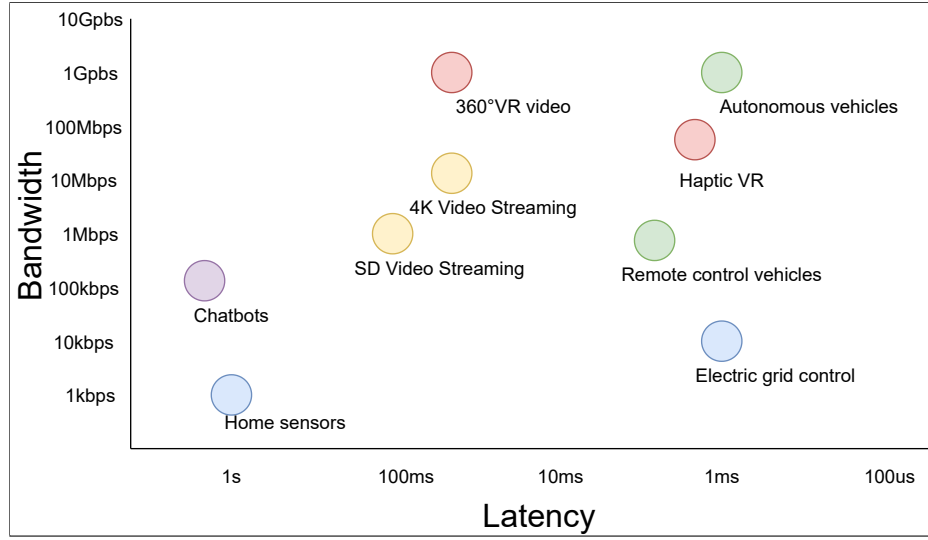


FIGURE 1.1. Illustration of the different domains of new value underpinned by new requirements in bandwidth and latency (Weldon, 2016).

TABLE 1.1. Increase in transfer speed of different protocols throughout the years (Malik et al., 2015) (Ezhilarasan & Dinakaran, 2017).

| Year/Standard | Mobile network          | WiFi                            | Bluetooth    |
|---------------|-------------------------|---------------------------------|--------------|
| 2003          | 100Kbps (2G)            | 54Mbps (802.11g)                | 3Mbps(2.0)   |
| 2007          | 2Mbps (3G)              | 450Mbps (802.11n)               | 3.5Mbps(2.1) |
| 2013          | 100-1000Mbps (4G)       | 1.3Gbps (802.11ac)              | 24Mbps(4.1)  |
| 2020          | 3Gbps (5G), theoretical | 4.8Gbps (802.11ax, one antenna) | 48Mbps(5.1)  |

These improvements have enabled transmitting enormous amounts of data between different devices, allowing the existence of data clouds, cryptocurrencies and stream services, among others. However, Industry 4.0 and some types of WNCSSs not only require more deterministic latency results and better reliability in dense-traffic zones, but also new metrics that can truly relate the communication and the control aspect. From a NCS perspective, the communication network impacts the performance of the system (Baillieul & Antsaklis, 2007) in terms of “control quality”, hence, its presence must be taken into account from the earliest design stage. This motivates a co-design strategy where the control and communication layers are designed concurrently, considering the strong interplay between them

(Park et al., 2018). However, this is not an easy task. Typical communication indicators fail in measuring the effects of the communication aspect on control performance, making this interplay non-evident (Ayan, Gürsu, Hirche, & Kellerer, 2020).

Focusing on maintaining closed-loop stability, the key performance indicator of a control system, achieved by a nominal non-networked control law, several efforts have been made by the control community to establish requirements on the network “quality of service” (QoS) (D. Borgers, Geiselhart, & Heemels, 2017). The introduction of the Maximum Allowable Transmit Interval (MATI) and Maximum Allowable Delay (MAD) (Liu, Selivanov, & Fridman, 2019) of a NCS enables linking communications and control. The MATI and MAD refer to timeliness of information. The MATI can be viewed as an upper bound on the sampling period (Hespanha, Naghshtabrizi, & Xu, 2007), while the MAD is an upper bound on the transmission delay the system tolerates (Heemels, Teel, van de Wouw, & Nešić, 2010). MATI and MAD are metrics derived purely from a control perspective. How these relate from a communications point of view in a networked environment is yet to be studied.

Timeliness has been recognized as a key aspect in sample-data control systems since the early days of digital control, and the sampling period the direct indicator of it. At first sight, based on classical theory from digital control systems, to improve performance of the control loop, the sampling period should be reduced as much as possible (Moyne & Tilbury, 2007), well below the MATI. However, as stated in (Moyne & Tilbury, 2007), in contrast to digital control systems where performance increases monotonically with the sampling frequency, a WNCS has an optimal sampling frequency, above which performance deteriorates due to the effect of congestion in the network, which produces a delay that might be well above the MAD. This observation suggests that, from a communications perspective, fulfilling given values for MATI and MAD involves choosing precisely the nominal sampling period of the control loop.

Recently, the concept of Age of Information (AoI) (S. Kaul, Yates, & Gruteser, 2012) has appeared in the communications community as an application-independent metric that

allows evaluating network performance. AoI is a purely timeliness metric, which works as a basis for designing networked systems (Yates & Kaul, 2019). Given its definition, and the observation in (Yates & Kaul, 2019) that sources can minimize their AoI by optimizing their updating rates, AoI looks like an appealing metric for linking the control and communications layers in a WNCS, hence enabling an effective co-design.

A step in this direction was taken in (Ayan et al., 2020) and (Ayan, Vilgelm, Klügel, Hirche, & Kellerer, 2019), where AoI is used as a cross-layer metric to schedule multiple NCSs that compete for network resources. While AoI alone does not capture all the requirements of the NCSs, it complements other application-related metrics (Ayan et al., 2019). This observation motivates to seek a better understanding of AoI in WNCSs and its interplay with application-related metrics at the design stage.

## **1.2. Objectives**

The principal objective of this thesis is to analyse the interplay between the communication aspect of WNCSs, in certain specific wireless communication protocols, and the control loop aspect, with its stability constraints, using the status age as a cross-layer metric. In this sense, the specific objectives includes analysing theoretical models of different wireless communication protocols with the AoI concept. Another objective is to find relationships between the stability constraints of different control loops sharing their information across a common wireless channel and the peak age metric of the specific channel. The third objective is to compare the performance of the system under the different protocols analysed using the status age.

## **1.3. Assumptions and results generalization**

In this work several assumptions are made in order to obtain the results presented. They are enumerated and ordered in a way that can be useful to the reader, before the derivation of each outcome. This means that all the results, unless tell otherwise, are only valid within the context of each assumption and must not be generalized. In general, the assumptions

made are common in the literature reviewed, and include some aspects as assuming deterministic sampling rates, ideal wireless channel conditions, negligible time-quantization effects, systems with finite amount of devices using networks and ideal hardware functioning, among others specifically pointed out.

#### **1.4. Organization**

In order to accomplish the objectives, in Chapter 2 we first study preliminary topics that will help us to understand WNCSSs. Also, we introduce and explain the wireless protocols, and define the model used in the analysis and the principal two problems under study. In Chapter 3 we study the first problem, related to the purely timeliness aspect of the WNCSSs, while in Chapter 4 we study the second problem related to the effect of the communication network on the stability of WNCSSs. Each problem is studied separately with the specific wireless communication protocols previously introduced, with comparison between the results on the different protocols made afterwards. In Chapter 5 we give insights on how the topics analysed can help in designing WNCSSs. Finally, in Chapter 6 we give the conclusions on this work and directions for future research.

## 2. PRELIMINARIES

This chapter introduces different topics on which the research and the results of this thesis are based. Firstly we introduce the notation. Next, we review Markov chains and a specific type of stochastic hybrid system, which is useful to model a communication protocol. Then we introduce NCS, defining how control loops interact with a communication network. We deepen in the communication aspect defining the basic units and the different layers involved. Then we describe the different protocols that rule how the communication system interacts with other devices and the medium. Finally, we introduce two different metrics related to the timeliness of the information, which are vital for the results of this study.

### 2.1. Notation

In this work,  $\mathbb{R}$  denotes the real numbers,  $\mathbb{R}_{>a}$  the real numbers larger than  $a$ ,  $\mathbb{Z}_{\geq a}$  the integers larger or equal than  $a$ ,  $\mathbb{R}^n$  the Euclidean space of dimension  $n$ , and  $\mathbb{R}^{n \times m}$  the set of  $n \times m$  matrices with real coefficients. Given an  $n$ -dimensional vector  $v \in \mathbb{R}^n$ ,  $v^T$  denotes its transpose. For a real-valued sequence  $\alpha : \mathbb{Z}_{\geq 0} \rightarrow \mathbb{R}$ ,  $\alpha_t$  denotes its  $t$ th element. Random variables are generally denoted in boldface. For a random variable  $\mathbf{y}$ ,  $E[\mathbf{y}]$  denotes the expectation of  $\mathbf{y}$ . The space of functions  $\phi : [-\tau_M, 0] \rightarrow \mathbb{R}^n$ , which are absolutely continuous on  $[-\tau_M, 0]$ , and have square integrable first-order derivatives is denoted by  $W[-\tau_M, 0]$  with the norm  $\|\phi\|_W := \max_{\theta \in [-\tau_M, 0]} |\phi(\theta)| + \left[ \int_{-\tau_M}^0 |\dot{\phi}(s)|^2 ds \right]^{\frac{1}{2}}$ . Given a collection of vectors  $x_i$ , the operator  $\text{col}\{\cdot\}$  generates a column vector as  $\text{col}\{x_1, \dots, x_n\} = [x_1^T, \dots, x_n^T]^T$ .

### 2.2. Markov chains

A Markov chain is a stochastic process in which the future value only depends on its present value. It can represent discrete or continuous-time processes.

DEFINITION 1 ((Billingsley, 1961)). *The sequence  $\{X_t, t \geq 0\}$  is said to be a Markov chain if for all state values  $i_0, i_1, i_2, \dots, i_t$ ,*

$$P[X_{t+1} = j \mid X_0 = i_0, \dots, X_{t-1} = i_{t-1}, X_t = i_t] = P\{X_{t+1} = j \mid X_t = i\}, \quad (2.1)$$

where  $i_0, i_1, i_2, \dots, i_n$  are the states drawn from a state space  $\mathcal{I}$ .

In this work, we focus on Markov chains representing a continuous-time process. If the states are continuous,  $X(t) \in \mathbb{R}$ , let

$$P_{ij}(t) = P\{X(t+s) = j \mid X(s) = i\} \quad (2.2)$$

denote the probability that a process presently in state  $i$  will be in state  $j$  after  $t$  units of time. In a continuous-time Markov chain, the time the process spends in each state is exponentially distributed and independent from the state visited (Ross, 2006). Depending on the structure, Markov chains can have different properties. We are interested in ergodicity.

DEFINITION 2. *A Markov chain is called ergodic if the following hold (Ross, 2006): (i) all states of the Markov chain communicate, in the sense that starting in state  $i$  there is a positive probability of ever being in state  $j$ , for all  $i, j$ ; and (ii) the Markov chain is positive recurrent, in the sense that, starting in any state, the mean time to return to that state is finite.*

If a Markov chain is ergodic, the probability of the process being in state  $j$  at time  $t$  converges to a limiting value, which is independent of the initial state and defined as

$$P_j = \lim_{t \rightarrow \infty} P_{ij}(t).$$

$P_j$  is often called stationary probability and normally denoted in literature as  $\pi_j$ .

Stationary probabilities are calculated using a set of equations balancing the arrivals and departures of each state in the long run (Ross, 2006), as follows,

$$\begin{aligned}
v_j P_j &= \sum_{k \neq j} q_{kj} P_k \quad \text{for all states } j \\
\sum_j P_j &= 1,
\end{aligned} \tag{2.3}$$

where  $v_j$  is the rate at which the process leaves state  $j$  and  $q_{kj}$  is the rate at which the process enters state  $j$  from state  $k$ .

In this thesis, Markov chains will be used to model certain wireless communication protocols, with the form of hybrid stochastic systems for which a continuous component has a dynamic related to a specific discrete state.

### 2.3. Piecewise Linear Stochastic Hybrid Systems (PWL-SHSs) with Linear Reset Maps

In this work we use the Stochastic Hybrid Systems (SHS) framework given in (Hespanha, 2006). A SHS is defined as a dynamical system with state  $(\mathbf{q}, \mathbf{x})$ , where  $\mathbf{q}(t) \in \mathcal{Q} := \{0, 1, \dots, m\}$  is the discrete component and  $\mathbf{x}(t) \in \mathbb{R}^n$  is the continuous component. Given a  $k$ -dimensional vector of independent Brownian motion processes  $\mathbf{z}$ , and smooth mappings  $f : \mathcal{Q} \times \mathbb{R}^n \times [0, \infty) \rightarrow \mathbb{R}^n$  and  $g : \mathcal{Q} \times \mathbb{R}^n \times [0, \infty) \rightarrow \mathbb{R}^{n \times k}$ , the evolution of the SHS is governed by: i) the stochastic differential equation

$$\dot{\mathbf{x}} = f(q, \mathbf{x}, t) + g(q, \mathbf{x}, t)\dot{\mathbf{z}}, \tag{2.4}$$

ii) a set of transitions  $\mathcal{L} = \{1, \dots, \ell_0\}$  such that each  $l \in \mathcal{L}$  defines a discrete transition map  $(\mathbf{q}^+, \mathbf{x}^+) = \phi_l(\mathbf{q}, \mathbf{x}, t)$ ,  $\phi_l : \mathcal{Q} \times \mathbb{R}^n \times [0, \infty) \rightarrow \mathcal{Q} \times \mathbb{R}^n$ , and iii) a set of transition intensities  $\lambda^{(l)}(q, \mathbf{x}, t)$ ,  $\lambda^{(l)} : \mathcal{Q} \times \mathbb{R}^n \times [0, \infty) \rightarrow [0, \infty)$ .

In this work, we restrict ourselves to the sub-class of piecewise linear SHSs (PWL-SHSs) with linear reset maps introduced in (Yates & Kaul, 2019), which are useful to model status update system with queues. Considering that the discrete component  $\mathbf{q}$  represents a continuous-time finite-state Markov chain, we restrict (Yates & Kaul, 2019):  $f(\mathbf{q}, \mathbf{x}, t) = b_q$ ,  $g(\mathbf{q}, \mathbf{x}, t) = 0$ ,  $\lambda^{(l)}(\mathbf{q}, \mathbf{x}, t) = \lambda^{(l)} \delta_{\mathbf{q}_l, \mathbf{q}}$ , and  $\phi_l(\mathbf{q}, \mathbf{x}, t) = (\mathbf{q}_l^+, A_l \mathbf{x})$ , where  $\delta_{\mathbf{q}_l, \mathbf{q}(t)}$  is the Kronecker delta function,  $b_q \in \mathbb{R}^n$ , and  $A_l \in \mathbb{R}^{n \times n}$ .

## 2.4. Networked control systems

NCSs can be formulated using different modeling approaches, such as discrete-time, hybrid/impulsive, and delayed continuous-time (Liu et al., 2019). In this work, we build on the formulation presented in (Liu, Fridman, & Johansson, 2015), where a continuous-time, linear plant

$$\dot{x}(t) = Ax(t) + Bu(t) \quad (2.5)$$

and a continuous-time dynamic output feedback controller

$$\begin{aligned} \dot{x}_c(t) &= A_c x_c(t) + B_c \hat{y}(\tau_k) \\ u(t) &= C_c x_c(t) + D_c \hat{y}(\tau_k), \quad t \in [t_k, t_{k+1}), k \in \mathbb{Z}_{\geq 0} \end{aligned} \quad (2.6)$$

interact over a communication network, forming a closed-loop. In this setup,  $x(t) \in \mathbb{R}^n$  is the state vector,  $u(t) \in \mathbb{R}^m$  is the control input and  $A, B$  are system matrices of appropriate dimensions. The initial condition is given by  $x(0) = x_0$ . where  $x_c(t) \in \mathbb{R}^{n_c}$  is the state of the controller.  $A_c, B_c, C_c$  and  $D_c$  are matrices of appropriate dimensions. We consider that the loop has  $N$  distributed sensors, a controller and an actuator, all connected via wireless network. Their measurements are given by  $y_j(t) = C_j x(t), j = 1, \dots, N$ . Let  $C = [C_1^T \dots C_N^T]^T$  and the output is  $y(t) = [y_1^T(t) \dots y_N^T(t)]^T \in \mathbb{R}^{n_y}$ . Given a sequence of strictly increasing sampling/transmission times  $\tau$ , at each sampling instant  $\tau_k$ , at most one of the outputs  $y_j(\tau_k) \in \mathbb{R}^{n_j}, \sum_{j=1}^N n_j = n_y$ , is sampled and transmitted over the network, processed in the controller and transmitted to the actuator, arriving at destination after a random delay  $\mathbf{d}_k$ . Then,  $t_k$  is defined as the arrival time,  $t_k = \tau_k + \mathbf{d}_k$ . The variable delay is decomposed in  $\mathbf{d}_k = \mathbf{d}_k^{sc} + \mathbf{d}_k^{ca} + \mathbf{d}_k^c$ , where  $\mathbf{d}_k^{sc}$  and  $\mathbf{d}_k^{ca}$  are the network-induced delays (from the sensor to the controller and from the controller to the actuator, respectively), and where  $\mathbf{d}_k^c$  is the computational delay in the controller node. We suppose that the controller and the actuator are event-driven (in the sense that the controller and the ZOH device update their outputs as soon as they receive a new sample).

Let  $\hat{y}(\tau_k) = [\hat{y}_1^T(\tau_k) \cdots \hat{y}_N^T(\tau_k)]^T \in \mathbb{R}^{n_y}$  denote the most recently received output information on the controller side. We consider the error between the system output  $y(\tau_k)$  and the last available information  $\hat{y}(\tau_{k-1})$

$$\begin{aligned} e(t) &= \text{col}\{e_1(t), \dots, e_N(t)\} := \hat{y}(\tau_{k-1}) - y(\tau_k) \\ t &\in [t_k, t_{k+1}), k \in \mathbb{Z}_{\geq 0}, \hat{y}(\tau_{-1}) = 0, e(t) \in \mathbb{R}^{n_y} \end{aligned} \quad (2.7)$$

The sensor  $j$  transmitting its output is called the active node, which forms a wireless link to transmit. Let  $\sigma_k \in \mathcal{I} = \{0, 1, \dots, N\}$  denote the active output node at each  $\tau_k$ , which will be chosen based on a scheduling policy, previously established in the loop design process. Here  $\sigma_k = 0$  means that no node will transmit the samples from sampling instant  $k$ , this can happen when either a link from another loop is active for that sampling period (if the network is shared), or that a packet dropout has occurred.

By using a two-step design approach, where controller (2.6) is designed for plant (2.5) ignoring the communication constraints, values  $\tau_{\text{MATI}}$  and  $\tau_{\text{MAD}}$  can be obtained such that if  $\tau_{k+1} - \tau_k \leq \tau_{\text{MATI}}$  and  $\mathbf{d}_k \leq \tau_{\text{MAD}}$  hold  $\forall k$ , then the closed-loop remain stable after the addition of the communication network. Therefore, the scheduling policy that takes the decision of which output  $y_j(\tau_k)$  sample and transmit at each instant  $\tau_k$  is critical for obtaining the  $\tau_{\text{MATI}}$  and  $\tau_{\text{MAD}}$ .

Based on this paradigm, numerous works have focused on characterizing  $\tau_{\text{MATI}}$  and  $\tau_{\text{MAD}}$  in different scenarios and for different scheduling policies. In (Heemels et al., 2010), using hybrid systems theory,  $\tau_{\text{MATI}}$  and  $\tau_{\text{MAD}}$  are characterized via Lyapunov-based conditions to ensure UGAS and  $\mathcal{L}_p$  stability under two specific scheduling policies, and an explicit tradeoff, which is almost linear for certain systems, is shown to exist. Following a similar approach, (Heijmans, Postoyan, Nešić, & Heemels, 2017) introduces a lower bound (MIATI) on the transmission interval, which allows obtaining a less conservative  $\tau_{\text{MATI}}$ , therefore relaxing constraints. Alongside this, (Liu et al., 2015) analyses mean-square stability for a scheduling policy based on a Markov chain, where each link has assigned a

probability to be transmitted. The reader is referred to the excellent surveys ([Hespanha et al., 2007](#)) and ([Liu et al., 2019](#)) for details on this line of research.

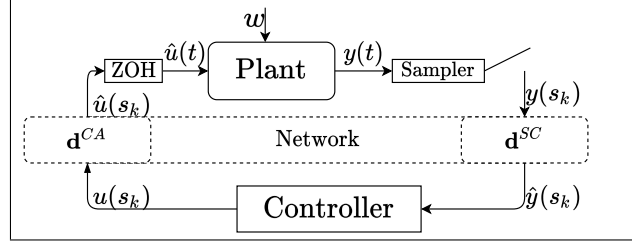


FIGURE 2.1. Networked Control System.

### 2.4.1. Stability problem in relationship with scheduling policy

One of the objectives of analysing the scheduling policy of a loop inserted in a NCS is to obtain the maximum  $\tau_{\text{MATI}}$  and  $\tau_{\text{MAD}}$  value for which the plant achieves stability. In this case, we study the stability in the mean square sense. To get a formal definition we base on ([Liu et al., 2015](#)):

**DEFINITION 3.** *The NCS (2.5)-(2.6) is said to be exponentially mean-square stable with respect to  $x$  if there exist constants  $b > 0$ ,  $\alpha > 0$  such that the following bound holds*

$$\mathbb{E} \{ |x(t)|^2 \} \leq b e^{-2\alpha(t-0)} \mathbb{E} \{ \|x_0\|_W^2 + |e(0)|^2 \}, t \geq 0$$

*for the solutions of the stochastic impulsive system (2.5) and (2.6) initialized with  $e(0) \in \mathbb{R}^{n_v}$  and  $x(t) = \phi(t), t \in [0 - \tau_M, 0]$ . Moreover, the NCS (2.5)-(2.6) is exponentially mean-square stable if additionally the following bound is valid*

$$\mathbb{E} \{ |e(t)|^2 \} \leq b e^{-2\alpha(t-0)} \mathbb{E} \{ \|x_0\|_W^2 + |e(0)|^2 \}, t \geq 0$$

As explained before, there are many possible scheduling policies that accomplish different  $\tau_{\text{MATI}}$  and  $\tau_{\text{MAD}}$  values. In the literature it is common to obtain the result as a sum of  $\tau_{\text{MATI}} + \tau_{\text{MAD}}$  ([Liu et al., 2015](#)) ([Heemels et al., 2010](#)), leaving the design option to distribute both delay and transmission interval according to the available resources, while respecting the sum of those values. In this sense, we study three specific scheduling policies

to obtain a  $\tau_{\text{MATI}} + \tau_{\text{MAD}}$  value that achieves stability in the mean-square sense: Try-Once-Discard, Round Robin and Stochastic. The calculation of the  $\tau_{\text{MATI}} + \tau_{\text{MAD}}$  bound for the different policies is made in (Liu et al., 2015) and (Zhang & Fridman, 2019) by solving the feasibility of a set of Linear Matrix Inequalities (LMI). In all cases, another parameter included among the calculations, in addition to  $\tau_{\text{MATI}} + \tau_{\text{MAD}}$ , is the minimum delay  $0 < \eta_m < \tau_{\text{MATI}} + \tau_{\text{MAD}}$ , which is useful to obtain less conservative results when a bound is predetermined in that sense.

#### 2.4.1.1. Try-Once-Discard (TOD) scheduling

In TOD scheduling, a weighted error method is used to decide which node from the loop will be sampled and transmitted. Each node has a weight matrix  $W_j, j = 1, \dots, N$ , defined during the loop controller design, that indicates which nodes are more critical to the loop. The node chosen is determined as

$$\sigma_k = \arg(\max_{j=1, \dots, N} \{e_j^T W_j e_j\}) \quad (2.8)$$

#### 2.4.1.2. Round Robin scheduling

In the Round Robin schema, the nodes are sampled and transmitted periodically, always in the same order, we can determine the chosen node as

$$\sigma_k = \text{mod}(k, N) + 1 \quad (2.9)$$

#### 2.4.1.3. Stochastic scheduling

In stochastic scheduling we assume that at each instant  $k$  the protocol determines  $\sigma_k$  through a Markov Chain. The conditional probability that node  $j$  gets access to the network for the sample at time  $\tau_k$ , given the values of  $\sigma_{k-1} \in \mathcal{I}$ , is defined by

$$\text{Prob}\{\sigma_k = j \mid \sigma_{k-1} = r\} = \pi_{rj} \quad (2.10)$$

where  $0 \leq \pi_{rj} \leq 1$  for all  $r, j \in \mathcal{I}$ ,  $\sum_{j=0}^N \pi_{rj} = 1$ .  $\sigma_0 \in \mathcal{I}$  is assumed to be given. The transition probability matrix is denoted by  $\Pi = \{\pi_{rj}\} \in \mathbb{R}^{(\mathcal{N}_i+1) \times (\mathcal{N}_i+1)}$ .

## 2.5. Communication network layers

Communication networks are commonly described hierarchically in different layers, each one having different components and functions, which together form a full communications stack. With the purpose of standardizing the layered networking model, in 1977 the Open Systems Interconnection project, or OSI, was founded. The OSI model is a 7 layer networking model that goes from the top application layer, which abstractly specifies the methods and information to send via the networking system, to the physical layer, which refers to the physical hardware where the protocol that allows the transmission to be made is implemented (Dordal, 2014).

However, while OSI represented an attempt at the creation of networking standards independent of any individual government, their protocols failed in the marketplace because its unpractical bureaucracy (Dordal, 2014). Commonly, instead of the 7 layer model, a more concise 5 layer approach is used, based on standard protocols, called the TCP/IP model. This model is the one that will be used to describe this problem statement, as can be seen in Figure 2.1. In this representation, the control system is in the application layer, above the whole communication network on a high level deciding how and which information use and send via the other layers. The transport layer refers to the high level management of connection abstraction to get packages from one node to another, controlling the transport to ensure security and quality of service. The network layer provides a global mechanism for addressing and routing, so that packets can be delivered from any host to any other global host from other networks, by devices known as routers, regardless of whether the host is local or global. The link layer refers to all the digital operations on packets, and is in charge of providing local routing mechanisms. Finally, the PHY layer deals with the analog electrical, optical or radio signaling mechanisms involved physically in the transmission.

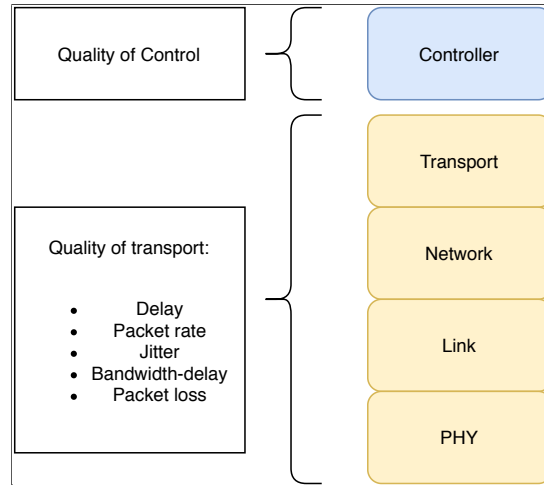


FIGURE 2.2. Quality categories according to system layers.

Figure 2.2 separates the quality of service (QoS), or transport, and the quality of control (QoC) of a NCS in the context of the communications layers description. QoC is exclusive to the application layer, and the communications aspects relate to it in the QoS, which depends on the four layers under the application. QoS is determined by different performance indicators, being the most classical ones the delay, the packet rate, the jitter, the bandwidth and the packet loss. In this section we describe each layer of the simplified model and the performance indicators that describe the QoS.

### 2.5.1. Physical layer

The lowest layer of the model provides the mechanical, electrical, functional, and procedural standards to access the physical medium (Day & Zimmermann, 1984). This layer delivers service for all the above layers, translating logical requests from the link layer into hardware operations for transmission, alongside physically receiving information bits in reception. It mainly consists in the physical chip that encodes the characters to bits, sending them by radio with a specific frequency or by wires, depending on the protocol.

### 2.5.2. Link layer

The link layer in the OSI model divides into two sublayers, the logical link control (LLC) and the medium access control (MAC) layers (Dordal, 2014). LLC is above MAC, and serves the network layer on addressing the destination of host using MAC address, which is a predetermined address unique for each device, independently of the network. LLC also tracks acknowledges and controls the data between the transmission at MAC level. For that reason, LLC is sufficient to provide a mechanism for devices inside the same local network to transmit between themselves using their respective MAC address. MAC layer most important functions are to provide an abstraction of the physical layer to the LLC, resolve collision issues, detect and possibly correct errors which may occur in the physical layer and select the mechanism to access the medium, frequency channel and frame encapsulation. This mechanism can be selected from a variety of options that are usually classified in two groups: (i) contention-based access, in which each sender has a mechanism to sense the medium and a back-off algorithm to dispute the channel access with other nodes; and (ii) a schedule-based access, in which normally the whole network is coordinated and each sender knows previously when it will be the time to transmit (Park et al., 2018).

### 2.5.3. Network layer

The principal function of the network layer is to support and provide mechanism for global connectivity between local networks in a way that allows practical scaling for all sorts of use, being a local network connecting to other local networks, or, as nowadays, form one large virtual network that can connect every device via internet. The key difference between transport information contained at link layer when compared to transport information contained at the network layer is that the information can move beyond the local network to reach hosts in remote network locations. This mechanism is a best effort system; there are no acknowledgments or re-transmissions, and will be used by devices operating at this layer to select appropriate network routes to ensure packet forwarding. The routing information contained within a packet includes the source of the sending host

and the eventual destination of the remote host. This information is contained within the network layer header that encapsulates network frames at the link layer (Dordal, 2014).

#### **2.5.4. Transport layer**

The transport layer is responsible for success and control transmission in the network layer, creating an abstraction of virtual connection that manages the best effort approach of the inferior layer. The most popular mechanism, but not the only one, used in this layer is the Transmission Control Protocol (TCP). In practice, TCP works with IP mechanism present at the network layer to provide different features as reliability, to keep track of lost packets to control re-transmission and acknowledgements, port numbers, to specify the receiving application data, and throughput management, to control congestion (Dordal, 2014).

#### **2.5.5. Application layer**

The application layer as the highest layer of OSI does not provide services to any other layer. The primary concern of this layer is with the semantics of the application (Day & Zimmermann, 1984), so it does not have to take any concern with the communication system itself, but use it to achieve its own goal. For this study, the application layer in the high level concept of communications consists in the control system that rules the relationship sensor-controller-actuator, relying on the transport layer to transfer the data between the different nodes in the network.

In the internet model, the application level is based on high level protocols like the file transfer protocol (FTP), Hypertext Transfer Protocol (most commonly known HTTP), which enables easy access and transfer of information in the web (Oracle, 2010) and Secure Shell (SSH), commonly used to securely controlling devices over the command line in different networks (Ylonen, Lonvick, et al., 2006).

### 2.5.6. Packet

The principal unit of transmission in the ISO/OSI model of layers is the packet. More specifically, a packet is the data unit at the network layer. In the transport layer there is not a common name for the data unit as it depends on the implementation (TCP or UDP), mainly because the transport layer function is to control the packet transmission. However, some literature call data at this point segments ([Dordal, 2014](#)), because the segments are parts of the message divided from the application layer. The packets encapsulate the data segments with routing information. At the link layer, on the lower side, the data transmitted is called a frame, which also is the packet encapsulated with the data link layer information from the source and the destination, depending on the protocol.

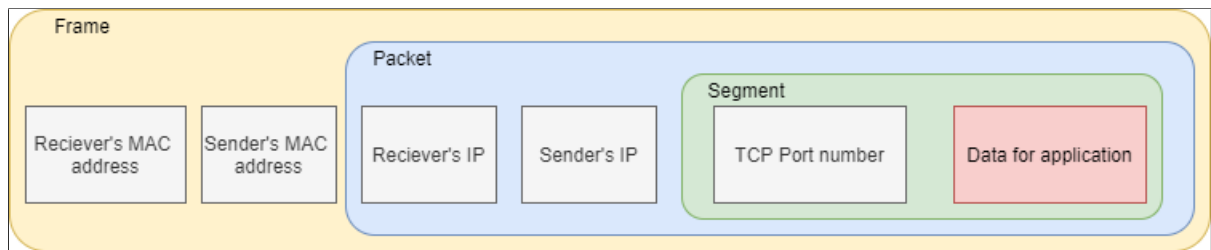


FIGURE 2.3. Packet encapsulation.

Figure 2.3 shows a very simple example of how segments, packets and frames work encapsulating the information of the layer above each one in the TCP/IP protocol. For more realistic and complex applications the packets and frame have a lot more meta-data for different purposes, mainly for protocol configuration, security and tracking acknowledgements, among others.

#### 2.5.6.1. Packet delay

In a WNCS communication service, the delay  $d$  is the time elapsed between the application layer of the sender requesting the lower layers to send the packet and its arrival and processing at the destination device's application layer. It consists in different uncertainties,

some of them more non-deterministic than others, that have to be analyzed and compensated to achieve a correct time synchronization between the network agents (Maróti, Kusy, Simon, & Ledeczi, 2004). The different delays both from the transmitter and the receiver side are:

- (i) Pre-processing time ( $d^{\text{pre}}$ ): is the time used to assemble the packet and issue the send request to the MAC layer on the transmitter side. It highly depends on the processor load and overhead and is considered to be non-deterministic.
- (ii) Queue time ( $d^{\text{queue}}$ ): the packet waits in the queue for other packets to be transmitted, if it is the case, which depends on the queue system used by the sender which itself depends on the routing protocol. On this delay, data must wait in the buffers of the routers or switches before being forwarded. This queue time will depend on the queue discipline implemented by the model, which normally can be categorized in First-Come-First-Serve (FCFS), Last-Come-First-Serve with no preemption (LCFS-NP) and Last-Come-First-Serve with preemption (LCFS-P). The difference between these last two is that when a queue has preemption the packet being transmitted can be dropped out during transmission to be replaced by a most updated sample.
- (iii) Access time ( $d^{\text{MAC}}$ ): this is the delay from waiting to access the transmit channel on the head of the queue, to where the transmission actually begins, depends on the protocol used. According to (Maróti et al., 2004), both queue and access time are the least deterministic part of the message delivery in wireless sensor networks, because they depend mostly on the network traffic.
- (iv) Transmission time ( $d^{\text{TX}}$ ): this is the time it takes to the sender to actually transmit the message. It can be separated in the data transmission time and the propagation time. The data transmission time depends on the radio speed and the length of the message, so it is considered to be deterministic and can be calculated as  $d^{\text{data}} = N/R$ , where  $N$  is the number of bits and  $R$  is the bits per second radio speed rate of transmission. The propagation time ( $d^{\text{prop}}$ ) refers to the time that

it takes to arrive at the receiver. It is deterministic because it depends on the distance between nodes and on the physical medium, which is normally air.

- (v) Receive time ( $d^{\text{post}}$ ): time to process the incoming message and to notify the receiver application. Its characteristics are similar to the pre-processing time.

Therefore, the delay  $d$  is given by

$$d = d^{\text{pre}} + d^{\text{queue}} + d^{\text{MAC}} + d^{\text{TX}} + d^{\text{post}} \quad (2.11)$$

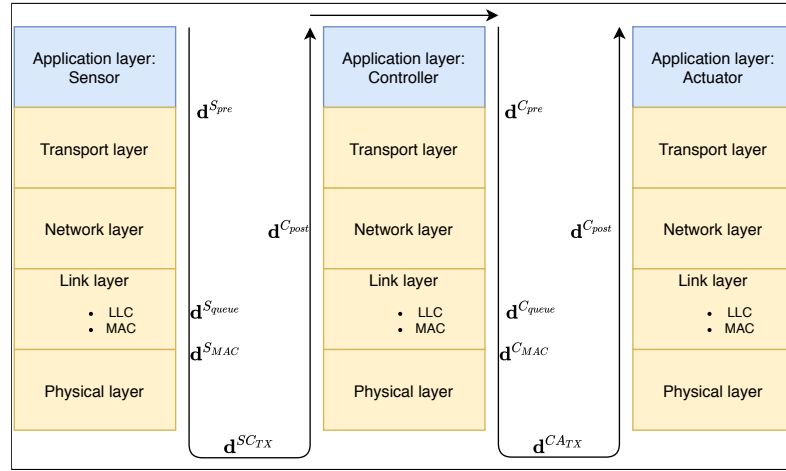


FIGURE 2.4. Time elapsed on a wireless message transmission.

Figure 2.4 shows the time schema of a packet delivery over a network transmission for a NCS, with packet time delay  $d$ , separating the delays for the sensor-controller and the controller-actuator part. On this definition, the most critical and large delay for a wireless transmission are the access and queue time (R. D. Yates, 2018). The time for the packet to wait until being able to actually use the transmission channel can largely surpass the encoding and decoding times and also make the propagation time negligible in comparison. This is the main reason why wireless standards have special protocols for the packet to access the medium. Considering this, in the next discussions we will neglect on the pre and post-processing time.

$$\mathbf{d} \approx \mathbf{d}^{\text{queue}} + \mathbf{d}^{\text{MAC}} + \mathbf{d}^{\text{TX}} \quad (2.12)$$

Alongside this, the terms can be regrouped in a service time  $\mathbf{s} = \mathbf{d}^{\text{MAC}} + \mathbf{d}^{\text{TX}}$  and a waiting time  $\mathbf{w} = \mathbf{d}^{\text{queue}}$ . Therefore we finally define the delay used in this thesis as

$$\mathbf{d} \approx \mathbf{w} + \mathbf{s} \quad (2.13)$$

### 2.5.6.2. Packet rate

Defines the number of packets being transmitted or received per unit of time. It is considered more important than the actual data rate, because packet rate is limited in different ways such as processing time and media access. Data rate, also, depends on the size of the packet ([Gallenmüller et al., 2018](#)). For NCSs aiming towards low latency and high reliability, a high packet rate is not as critical as delay or packet loss.

### 2.5.6.3. Packet loss

In communications, data packets may be loss during transmissions for various reasons, depending on the susceptibility of the channel to blockage, multipath, doppler shift, interference and other unstable channel characteristics ([Park et al., 2018](#)). The loss rate describes the fraction of packets transmitted but no received. One of the most common reasons for packet dropouts is congestion, which is produced when packets arrive at the transmitter faster than they can be sent out ([Dordal, 2014](#)). When this situation occurs, a queue will form for that interface to access the channel. Once that queue is full, packets will be dropped. The most common strategy (though not the only one) is to drop any packets that arrive when the queue is full, or to drop the old packets and put at the queue's head the most fresh ones. Another common cause of packet loss are collisions, which happen when the MAC mechanism allows two nodes to transmit their packets at the same time, causing interference between them and making impossible for the receivers to decode the message ([Dordal, 2014](#)). This is why one of the most important aspects of the MAC protocol is how they avoid collisions. In practice, the loss rate can most easily be determined by the

receiver using sequence numbers in each transmitted packet. In case of unreliable physical links such as wireless networks, those sequence numbers are part of the MAC header and are inserted by the transmitter. When a receiver detects a gap in the sequence number of subsequent packets, it can determine the exact number of missed packets in between (Gallenmüller et al., 2018).

#### **2.5.6.4. Bandwidth-delay product**

For NCS application, bandwidth-delay product can refer to the number of packets currently in flight between source and destination. It can be defined in number of packets or in bits. In NCSs, the packet size is expected to remain constant.

#### **2.5.6.5. Jitter**

Jitter is measured as the mean deviation in actual arrival times, versus theoretical arrival times, and is measured in units of the timestamp clock from the implemented hardware. (Dahmouni, Girard, & Sansò, 2012) uses the definition from IETF, the Jitter is the expected absolute value of the difference between the delays of two consecutive transmitted packets. It allows to consider hardware non-idealities in a network transmission and is critical for mission-critical applications. For most of applications, the variation in the arrival time of packets at the terminal must be compensated by using a playback buffer in order to provide a regular packet stream to the application.

### **2.6. Medium Access Control Protocols**

In this work we consider two classical specific methods for each MAC configuration. The difference in the methods will affect, among other things, the time a packet at the head of the queue of each sensor spends waiting for the transmission to actually start,  $d^{\text{MAC}}$ . For contention-based we describe the carrier sense multiple access protocol with collision avoidance (CSMA/CA), while for schedule-based a time-division multiple access (TDMA) method is selected. These two MAC mechanisms were selected for being among the most popular ones in the communication protocols used by WNCS, specially in time-triggered

systems (Park et al., 2018). For contention-based systems CSMA/CA is the most traditional scheme used in scalable wireless systems, while TDMA is preferred for industrial IoT standards such as WirelessHART and ISA-100.11a over other non contention protocols, like frequency-division multiple access (FDMA). The main difference between the contention and schedule approaches is that in contention-based schemes unrelated devices compete to transmit over the network using a distributed algorithm, which provides high simplicity and easy scalability (Wang et al., 2016; Park et al., 2018), while in scheduled-based schemes a network coordinator determines which link transmits by defining time slots that are assigned to each link according to a scheduling criterion. This last mechanism is less flexible since the coordinator must be reprogrammed when a new link is integrated (T. S. Lin, Rivano, & Le Mouél, 2014).

#### **2.6.1. Contention-based channel access: CSMA/CA**

The carrier sense multiple access protocol was originally created in the times of 10-Mbps classic Ethernet with half duplex connectivity (Dordal, 2014), in which data could flow from and to a device but not simultaneously. In this context, it was necessary to formulate an algorithm that sensed the connection in a transmission to detect collisions and provide a way to organize the network in order to repeat the transmission without causing a new collision. With new Ethernet technology, full duplex communication was achieved, so the algorithm, called carrier sense multiple access with collision detection (CSMA/CD) was not necessary anymore. In a wireless network, however, most transmitting stations can not detect collisions (Dordal, 2014), which brought the problem back and therefore a necessity to implement an algorithm to manage it. The solution for most wireless protocols, specially IEEE802.11 (WiFi) and some applications of IEEE802.15.4 (ZigBee, Thread) is the carrier sense multiple access protocol with collision avoidance (CSMA/CA), which, as the name says, seeks to avoid collisions in transmission before they happen. In this work we consider two different sub-mechanisms or techniques of CSMA/CA, one basic model that is normally used in high-energy devices based on the IEEE802.11 protocol called Distributed Coordinator Function (DCF) and the other one used in IEEE802.15.4, low-energy

IoT oriented, called non beacon-enabled (nBE) CSMA/CA. The implementation of the protocol is based on the idea that when the packets arrive at the head of the queue in a link, to initiate transmission the sender has to compete with all the other links that also have packets to transmit in order to capture the medium. This competition is what motivates the algorithm to avoid collisions. When a link captures the medium, the other links will sense that the channel is not idle, and therefore will not attempt a transmission until they sense otherwise. In both techniques, we do not consider the hidden nodes problem, as each node can sense all the other nodes in the network.

#### 2.6.1.1. DCF CSMA/CA

For this scenario consider the simple DCF CSMA/CA approach used in (Maatouk, Assaad, & Ephremides, 2020) and (S. Kaul, Gruteser, Rai, & Kenney, 2011), as there are many others with optional features and other non-simplifications according to the specific application. The detailed algorithm consist in the following steps:

- (i) Each link, before sending a packet, sets its own back-off timer  $T_B = 0$  to zero and determines whether an interfering transmission is in progress using a carrier-sense mechanism.
- (ii) If the channel is idle, the transmitter will wait for a period called inter-frame space (IFS) to wait for if another link has just started to transmit and the signal has not yet arrived at the waiting link.
- (iii) After IFS, the transmitter back-off timer will start ticking for a period of time randomly chosen over a multiple of mini-slots each of duration  $T_{slot}$ . More specifically, the link picks a random back-off window uniformly distributed from the range  $[0; W - 1]$  where  $W$  is referred to as the Contention Window (CW). Defining the average back-off time for each link as  $\frac{1}{R}$ , we define this time as a function of the CW and  $T_{slot}$ , which is normally an specification for the PHY layer of the transmission protocol, e.g.,  $9\mu s$  for IEEE802.11 and  $16\mu s$  for IEEE802.15.4.

$$T_{slot} \frac{W - 1}{2} = \frac{1}{R} \quad (2.14)$$

- (iv) While the back-off timer is running, the carrier-sense mechanism is still in function. If it senses another transmission, the timer will stop, and resume only when the mechanism sense the channel idle again. Then, only when the timer reaches its end, the sender will start transmitting the packet.
- (v) After transmission and propagation time, which are considered to be exponentially distributed with parameter  $1/H$ , the algorithm optionally requires an ACK, which if not received will trigger a re-send of the packet increasing the back-off timer until a threshold is reached, in which case the transmission is aborted.

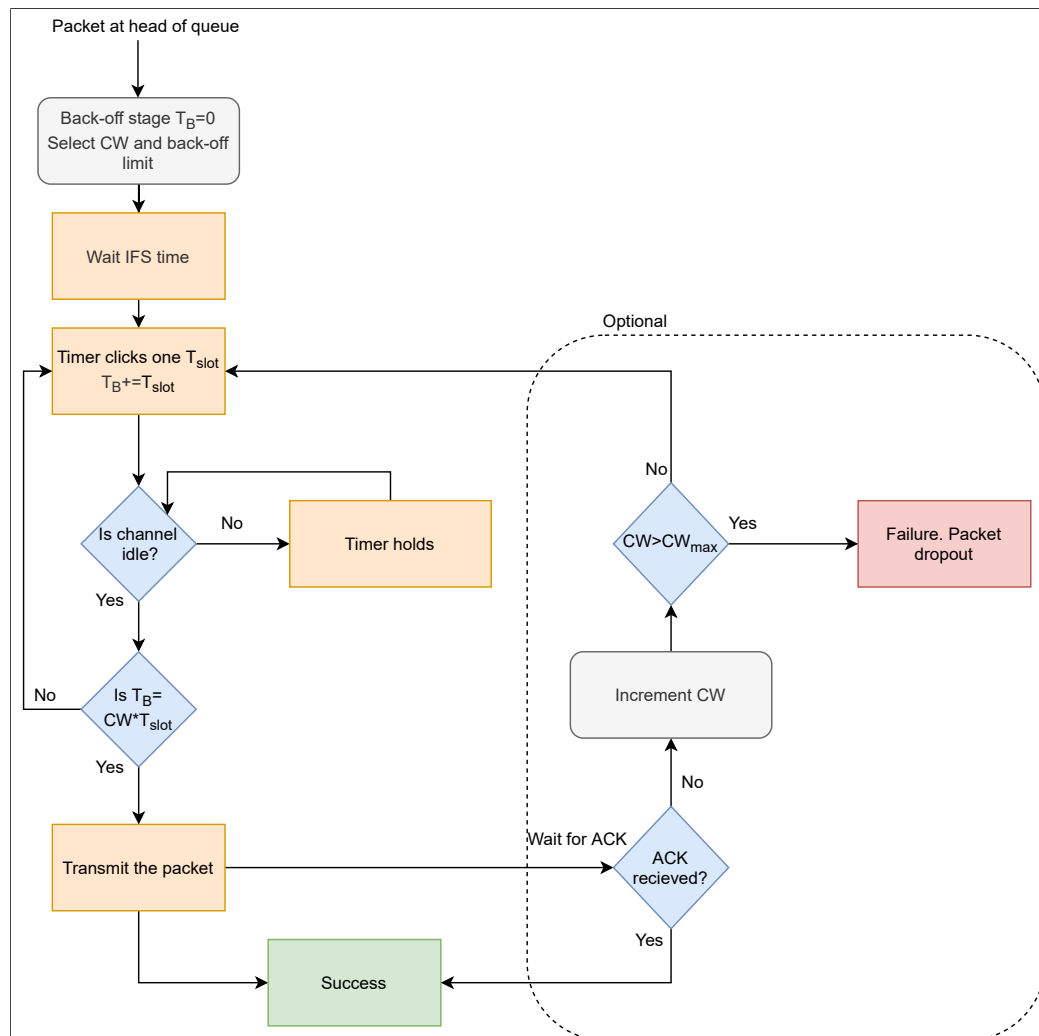


FIGURE 2.5. Simple DCF CSMA/CA flowchart.

Figure 2.5 describes the steps enumerated. This protocol separates the random delay to access the medium and the transmission and propagation of the packet in two parameters,  $1/R$  and  $1/H$  respectively. The value of  $H$  is directly related to the length of the packet and the data transmission rate of the technology used. It can be expressed as  $H = L/Dr$ , where  $L$  is the packet length and  $Dr$  is the data rate. In IEEE802.11, the data rate has increased throughout the years along with the evolution of the protocol amendments (Malik et al., 2015). The respective equations that relate these parameters to a specific service time  $s$  will be treated later in accordance to the model studied and in combination with the SHS formulation.

#### 2.6.1.2. Non beacon-enabled CSMA/CA

In the context of the IoT, we study the non beacon-enabled approach used in IEEE802.15.4 networks from (Buratti & Verdone, 2009). This mode considers the unslotted CSMA/CA, meaning that there is not a WPAN coordinator that continuously sends beacon messages to the nodes in the network, which allows to analyse a scenario in which different nodes share a network in a distributed-algorithm fashion, which implies easy scalability.

The detailed algorithm is based on mainly two parameters: the number of back-off retries  $NB$  and the back-off exponent  $BE$ . Both parameters start in 0 and 3 and have a maximum value of 4 and 5, respectively. The algorithm consist in the following steps:

- (i) Each link, before sending a packet, sets its own back-off timer to zero and determines a random delay uniformly selected between  $[0, 2^{BE} - 1]$  back-off units.
- (ii) When the link finishes waiting for the delay, it senses the channel. If it is IDLE, it attempts transmission, which will last for a specific number of back-off units depending on the length of the packet.
- (iii) If the channel is not IDLE when sensing, the back-off timer is reset to zero and the link determines another random delay uniformly selected between  $[0, 2^{\min(BE+1, BE_{max})} - 1]$  back-off units. The back-off try counter  $NB$  also increases in 1.

- (iv) The steps repeat until a last try with  $NB = 4$ . If the channel has not been sensed IDLE and the transmission attempted by that stage, the packet is dropped-out and the transmission is considered failed.

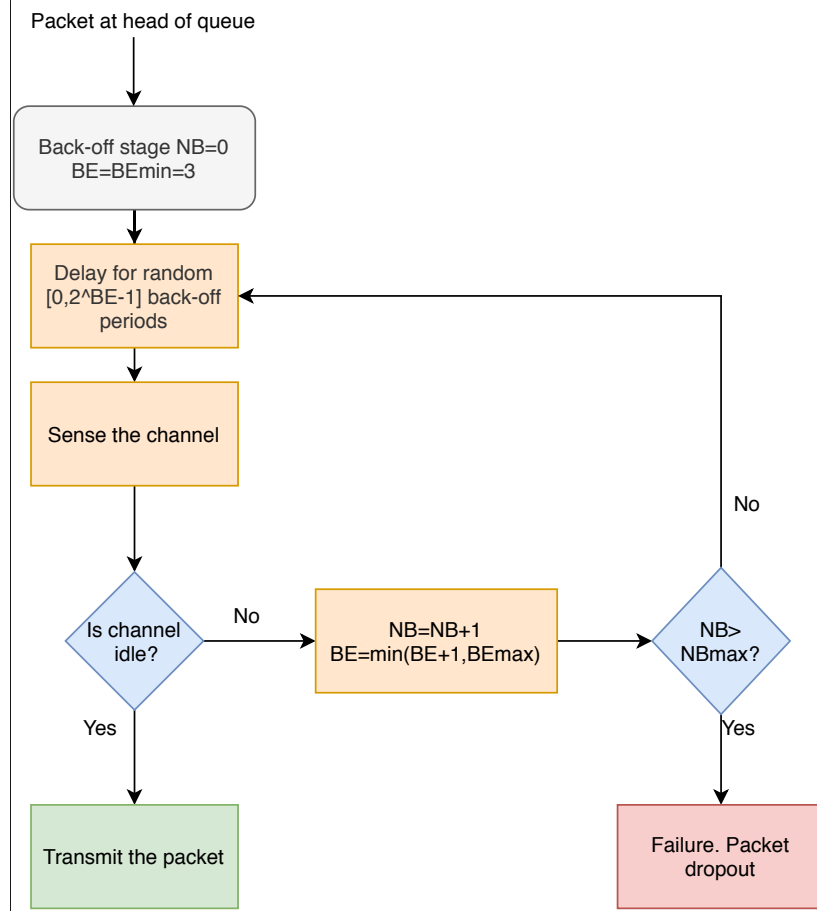


FIGURE 2.6. Simple nBE CSMA/CA flowchart.

The nBE CSMA/CA established is also explained in Figure 2.6. This protocol will determine a  $d^{\text{MAC}}$  probability distribution depending on the number of nodes competing for the channel. To this end we add the delay  $d^{\text{TX}}$  that takes to actually perform the transmission. In this sense, the protocol discussed in (Buratti & Verdone, 2009) considers that each back-off period used for transmission transmits at most 10 bytes. If the back-off period is set to  $db = 20T_s$ , with  $T_s = 16\mu s$  being the symbol time for discretization (the time slot normally used in IEEE802.15.4), the data transfer rate of the protocol would be

of approximately  $\frac{10 \cdot 8}{20 \cdot 16 \cdot 10^{-6}} = 250 kbps$ , which is the real data rate reported for practical applications using the IEEE802.15.4 standard (Petrova, Riihijarvi, Mahonen, & Labella, 2006). In this way, the sum of both delays results in a service time delay  $s = d^{MAC} + d^{TX}$  which has a predetermined distribution based on three parameters: the packet length, the number of links contending for the channel and the symbol time  $T_s$ .

Alongside, the nBE CSMA/CA used in IEEE802.15.4 differs with the DCF normally used in IEEE802.11, mainly on the issue that in the latter nodes continuously sense the channel at every step of the back-off timer, and not only when it ends. (Maatouk et al., 2020) (Buratti & Verdone, 2009). The main reason of this difference is that in IEEE802.15.4 the focus is in low energy usage, which does not allow radios to sense the channel all the time, while in IEEE802.11 energy consumption it is not a big constraint.

### 2.6.2. Schedule-based channel access: TDMA

Time-division multiple access method consists in dividing time into a sequence of slots in order to allow multiple senders to use the same channel frequency without interfering each other. The principal difference with CSMA/CA is that in this case transmitters do not compete for channel access with a method that avoids collisions, but rather there is a network coordinator that determines which node transmits at which time slot (He, Yuan, & Ephremides, 2018). As a consequence, it can be considered that there is not a MAC time to access the channel,  $d^{MAC} \approx 0$ . In multi-hop networks, the coordinator is also in charge of routing the packets in the network, optimizing a communication-aspect objective parameter, such as delay, throughput, etc.

With TDMA, transmission times can be considered deterministic inside the time slot (Ding, Zhang, Yin, & Ding, 2013), making it the principal reason why several real-time applications prefer to use a scheduler coordinator in their network, specially if the number of devices is relatively high (Park et al., 2018). The scheduler ensures that all the transmitters that require service can accomplish their transmission without any interference of the other nodes transmitting in the same frequency. In that way every sender starts transmission at the beginning of its respective time slot. In WNCSs that use this method, the main objective

of the coordinator in charge of scheduling is to optimize an aspect of the application layer, normally a control constraint, depending on the system design.

Schedulers algorithms that use TDMA normally use superframes: a series of time slots that define the communication schedule, a superframe can include the transmission of the packets waiting at the head of the queue from all the senders, or can include the transmission from all the packets in the queue of a single sender, depending on the application. Normally the superframe is used to refer to a single execution of the algorithm, in which the coordinator has to evaluate the whole system state and decide a schedule of transmission for that state. Figure 2.7 is an example of how the scheduler can decide, with each application of the algorithm, the schedule of transmission of the nodes, constructing superframes that define the transmission schedules according to the requirements and constraints of the control layer.

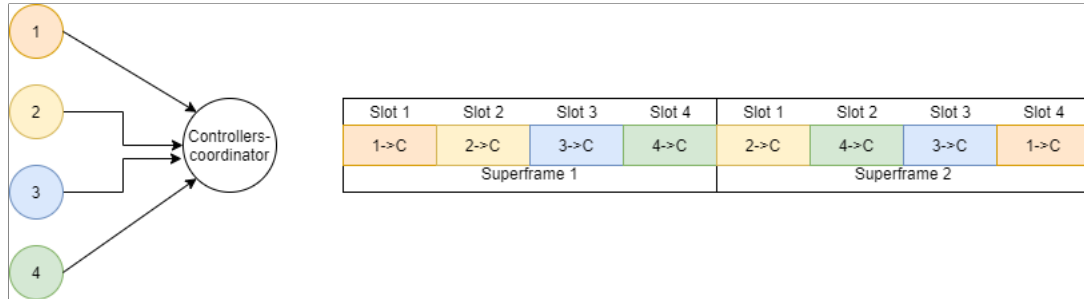


FIGURE 2.7. Example of a superframe.

## 2.7. Age of Information (AoI)

AoI is a timeliness metric defined for each link  $j$  in a status update system. AoI is defined as the average value of the status age  $\Delta_j(t) = t - u_j(t)$ , which refers to the time elapsed between the actual time  $t$  and the timestamp  $u_j(t)$  indicating the sampling (generation) time of the last message arrived at destination in communication link  $j$ . Status age increases linearly with time until a new message arrives, when is updated based on the new timestamp. This form a saw-tooth pattern (Figure 2.8) that is geometrically analysed

in (S. Kaul et al., 2012) and (Yates & Kaul, 2019) to obtain the following result, which will be instrumental for this work.

LEMMA 1 (Theorem 3 in (Yates & Kaul, 2019)). *For a stationary ergodic status updating system, the AoI of link  $j$  is given by*

$$\bar{\Delta}_j = \frac{E[\mathbf{y}\mathbf{d}] + E[\mathbf{y}^2]/2}{E[\mathbf{y}]} \quad (2.15)$$

where  $\mathbf{y}$  and  $\mathbf{d}$  are random variables representing the inter-generation time of the transmitted messages at link  $j$ , and the total delay the message spends in link  $j$  from its generation to the end of its transmission, respectively.

Based on Lemma 1 expressions characterizing AoI for different queue disciplines have been derived, including first-come first-served, last-come first-served, and preemptive schemes. The reader is referred to (S. Kaul et al., 2012) for an introduction on the topic and to (Yates & Kaul, 2019) for an extensive treatment.

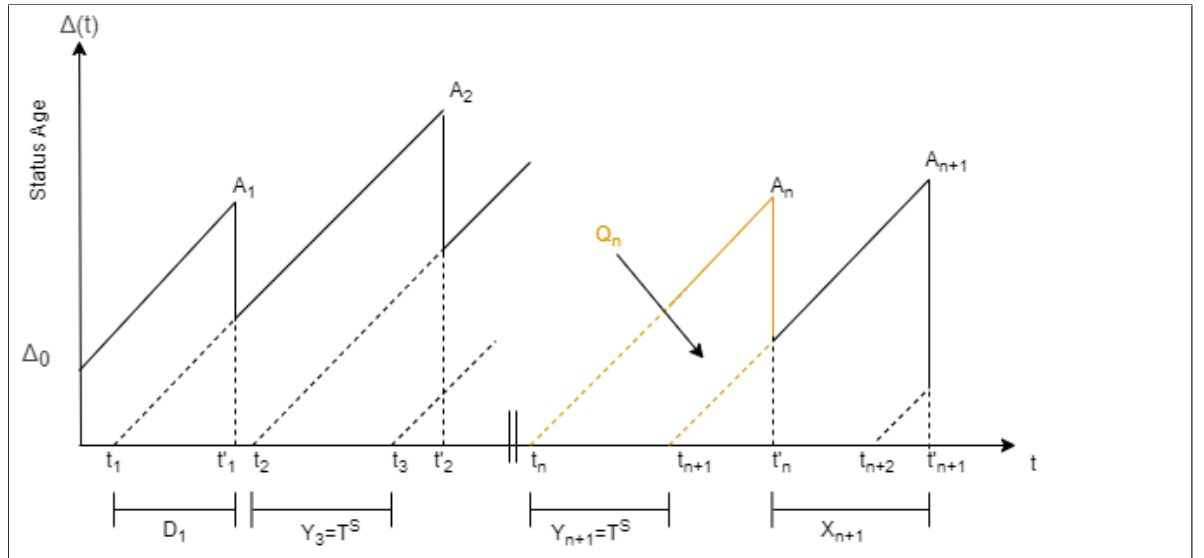


FIGURE 2.8. Example of status age along time from a relative time observer (S. Kaul et al., 2012).

Figure 2.8 shows how status age forms a saw-tooth pattern when each new sensor update message arrives at the controller. Thus, the definition of AoI can be interpreted as the area under the curve of status age normalized by a period. For each  $k$  message transmitted, and to use the simpler notation, we redefine the arrival time of messages as  $t'_k$  and the departure time as just  $t_k$ .

$$\mathbf{d}_k := t'_k - t_k \quad (2.16)$$

It is important to notice that the delay definition of (2.16) based on arrival time analyses the behaviour of the transmission using data that is in practice only available after the message reception, instead of (2.11), which explained how non-deterministic variables prevented an accurate calculation.  $\mathbf{d}_k$  is therefore considered to be a random variable. Another definition from the figure is the inter-generation time, which is defined as

$$\mathbf{y}_k := t_k - t_{k-1} \quad (2.17)$$

Notice that when the samplings of the transmitted packets are constant, the inter-generation time corresponds to the sampling period, which is commonly described as  $\lambda$ . AoI describes the timeliness of one's knowledge of a process (R. Yates, 2018), and so it becomes necessary to analyze it more deeply. It is important to notice here that, while AoI is related to message delay, it does not refer to the same thing, and that minimising status age is not the same than minimising the delay or diminishing the sampling period.

The idea of NCSs aiming to achieve low latency and high reliability for specific applications is to always have the system being updated with fresh information as quickly as possible. One simple solution could be to set the sampling frequency as high as the sensor can sample, but unfortunately that would not solve the problem. When the source generates messages, they have to receive service in the communication channel before arriving to destination, and the channel can only serve one message at a time and at a service rate that is different (and normally much slower) than the message generation capability (sampling period). This is the reason why queue disciplines are important. This characteristic

makes it seem convenient to model the transmission with simple queue theory in order to analyse how AoI deals with fresh updates that cannot arrive to their destination because older updates are still waiting to be transmitted, and which changes can be applied to the queue in order to prevent this.

## 2.8. Peak Age of Information and Outage Probability

Another metric based on status Age is called the Peak Age, which characterizes the maximum value of the status age immediately before an update is received.

DEFINITION 4 (Definition 3 of (Costa, Codreanu, & Ephremides, 2016)). *The Peak Age is the value of the status age obtained immediately before receiving the  $k$ th update, expressed*

$$\hat{\Delta}_k = \mathbf{d}_{k-1} + t'_k - t'_{k-1} \quad (2.18)$$

The definition allows us to manipulate the terms to obtain the following result

LEMMA 2. *The Peak Age is given by*

$$\hat{\Delta}_k = \mathbf{d}_k + \mathbf{y}_k, \quad (2.19)$$

consequently, the expected Peak Age or Peak Age of Information (PAoI), is given by

$$\bar{\Delta} = E[\mathbf{d}] + E[\mathbf{y}] \quad (2.20)$$

PROOF. From (Costa et al., 2016) we have

$$\begin{aligned} \hat{\Delta}_k &= t'_{k-1} - t_{k-1} + t'_k - t'_{k-1} \\ \hat{\Delta}_k &= t'_k - t_{k-1} + (t_k - t_k) \\ \hat{\Delta}_k &= t'_k - t_k + t_k - t_{k-1} \end{aligned} \quad (2.21)$$

which are directly defined in (2.16) and (2.17) as the delay and the inter-sampling period, respectively.  $\square$

It is interesting to note that the average Peak Age, or PAoI, is considerably simpler to calculate than AoI, as there is no need to calculate the expected value of the multiplication of both  $d$  and  $y$  variables.

The peak age can be seen as the worst case status age and is useful to analyse when and in which conditions the status Age exceeds a certain bound, which will be a central topic in this work. We will refer to the probability of this excess occurring when the queue has reached stability as the Outage Probability of Peak Age (OPPA). In simple terms, and without having to compromise for a distribution of the service and arrival processes, we define OPPA as  $P[\hat{\Delta}_\infty > x]$ , where  $x$  is the bound established and  $\hat{\Delta}_\infty$  is the Peak Age when the queue is stable.

### 2.8.1. Outage probability of Peak Age in deterministic arrivals

In (Seo & Choi, 2019), a numerical result for the outage probability  $P[\hat{\Delta}_k > x]$  on systems using D/G/1 queues is found. In (Seo & Choi, 2019), the waiting time of the  $k$ th message in the queue with deterministic arrivals is redefined based on the waiting and the service time of the previous packet as

$$\mathbf{w}_k = (\mathbf{w}_{k-1} + \mathbf{s}_{k-1} - \frac{1}{\lambda})^+ \quad (2.22)$$

where  $(x)^+ = \max(x, 0)$ . Defining  $\mathbf{w}_0 = 0$  and  $\mathbf{s}_0 = 0$ , in (Seo & Choi, 2019) it is proven that we can generalize  $\hat{\Delta}_k$  to

$$\hat{\Delta}_k = \max_{1 \leq s \leq k} \left( \sum_{i=s}^k \mathbf{s}_i - \frac{k-1-s}{\lambda} \right) \quad (2.23)$$

and

$$\mathbf{w}_k = \max_{1 \leq s \leq k-1} \left( \sum_{i=1}^s \mathbf{s}_i - \frac{s}{\lambda} \right) \text{ for } k \geq 2 \quad (2.24)$$

with  $\mathbf{w}_1 = 0$ . From 2.23 we can formally define  $\hat{\Delta}_\infty$  as  $\hat{\Delta}_k$  when  $k \rightarrow \infty$ . With these expressions, (Seo & Choi, 2019) uses the union and Chernoff bounds in order to obtain an

upper-bound on the outage probability of Peak age, which is summarized in the following Lemma:

LEMMA 3. *[from (Seo & Choi, 2019)] An upper-bound on the outage probability of peak Age (OPPA) is given by*

$$\mathbb{P} \left[ \hat{\Delta}_{\infty} \geq x \right] \leq e^{-\theta^*(x-b)} \cdot m(\theta^*), \quad (2.25)$$

where  $b = 1/\lambda$  and  $m(\theta^*)$  is the moment-generating function (MGF) of the service time distribution evaluated at  $\theta^*$ , which in turn is obtained for the D/G/1 queue as

$$\theta^* = \sup_{\theta > 0} \{ \theta : \Lambda(\theta) < \theta b \}, \quad (2.26)$$

with  $\Lambda(\theta)$  the natural logarithm applied on the moment generating function of the service time distribution  $s$ , with its argument being  $\theta$ .

PROOF. The proofs of Lemmas 1 and 2 in (Seo & Choi, 2019) give the result, which is obtained from equation 19 in (Seo & Choi, 2019).  $\square$

For D/G/1 systems, (Seo & Choi, 2019) specifies that  $\theta^* > 0$  can be found by numerically solving the following equation

$$\frac{\Lambda(\theta)}{\theta} + \frac{-\theta}{\lambda\theta} = 0 \quad (2.27)$$

## 2.9. Base model and problem statement

In this work, we focus on the WNCS represented in Figure 2.9 where a set of  $M \in \mathbb{Z}_{\geq 1}$  plants and a set of  $M$  controllers interact over a shared wireless network. Consider that a single controller  $C_i, i \in \{1, \dots, M\} = \mathcal{S}$ , of the form (2.6), is assigned to a single plant  $P_i$  of the form (2.5) to create a control loop  $\mathcal{P}_i$ , yielding a system with  $M$  control loops. To continue the formulation, the following assumption is made use of.

ASSUMPTION 1. *Each control loop  $\mathcal{P}_i$  operates at a constant sampling frequency  $\lambda_i$ .*

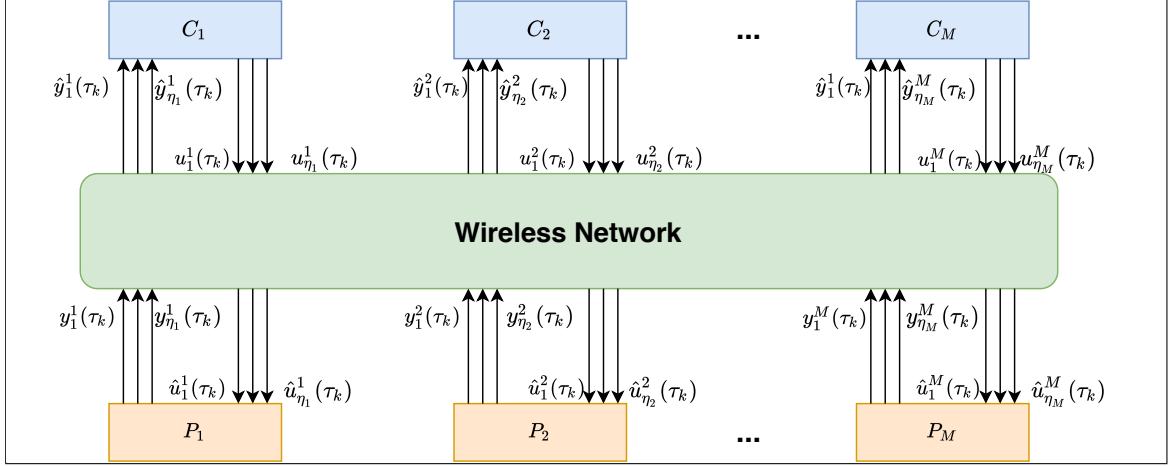


FIGURE 2.9.  $M$ -loop WNCs considered in this work.

REMARK 1. *Assumption 1 is reasonable for IoT systems where a time-driven transmission strategy is more common than an event-driven one, as the time-driven strategy looks for obtaining samples in regular, and commonly previously defined, periods of time.*

Based on the previous discussion, we refer to the system in Figure 2.9 as an  $M$ -loop WNCs and we model each loop  $\mathcal{P}_i$  as a tuple  $\mathcal{P}_i = (P_i, C_i, \mathcal{N}_i, \lambda_i, \tau_{\text{MATI}}^i, \tau_{\text{MAD}}^i)$ , where  $\mathcal{N}_i \in \mathbb{Z}_{\geq 1}$  is the number of wireless nodes required by  $\mathcal{P}_i$  to operate,  $\lambda_i \in \mathbb{R}_{>0}$  is the sampling rate ruling  $\mathcal{P}_i$ , and  $\tau_{\text{MATI}}^i, \tau_{\text{MAD}}^i$  are values for MATI and MAD obtained during the two-step design approach. As determined before, at each sampling time  $\tau_k$ , at most one node per loop samples its outputs  $y_j^i(\tau_k) \in \mathbb{R}^{n_j^i}, \sum_{j=1}^{\mathcal{N}_i} n_j^i = n_y^i$ , forming therefore at most one link per loop at each sampling time. We make the following assumptions on the loops, and on the network, respectively.

ASSUMPTION 2. *Each loop chooses which node will be sampled at sampling time  $\tau_k$  according to an internal, previously determined, scheduling policy, which will determine the loop own  $\tau_{\text{MATI}}^i + \tau_{\text{MAD}}^i$  values to ensure mean-square stability. The active output node is denoted by  $\sigma_k^i$ . This means that at each sampling instant no more than  $M$  links are trying to transmit over the network. The scheduling policy can be any of the previously described, i.e., TOD, RR, or stochastic.*

ASSUMPTION 3. *For each link of all  $M$  loops, the wireless network is assumed to be error-free, with no dropouts and negligible quantization effects.*

REMARK 2. *Assumption 3 may appear strong at first. However, the conditions assumed are common in the literature (Maatouk et al., 2020) (Heemels et al., 2010) (Liu et al., 2015). From the control perspective, dropouts affects the MATI of the loop, and can be easily included if we define an integer  $\delta$  of maximum successive dropouts, obtaining a  $\tau_{MATI}^i = \tau_{MATI}^i / (\delta + 1)$  (Heemels et al., 2010). From the communications aspect, assuming an ideal channel is common for obtaining results in the scope of investigation, therefore not having to consider other stochastic parameters. All the protocols analysed in this work take precautions to include retries and parameters to avoid dropouts in a way that its occurrence is only marginally probable.*

At this point, we formally introduce the two concrete problems under analysis. These problems are formulated aiming to resolve the objective of this thesis through the principal hypothesis that the status age can be used as a cross-layer metric to formally link the communication and control aspect in WNCSS design under specific MAC protocols.

PROBLEM 1. *For an  $M$ -loop WNCSS for which Assumptions 1 and 3 hold, derive expressions for the AoI of the links of loop  $\mathcal{P}_i$ ,  $\bar{\Delta}_i$ , under concrete MAC schemes.*

PROBLEM 2. *For an  $M$ -loop WNCSS for which Assumptions 1 and 3 hold, derive expressions involving the  $\tau_{MATI}^i + \tau_{MAD}^i$  stability constraints and the Peak Age of the links of loop  $\mathcal{P}_i$ ,  $\hat{\Delta}_i$ , under concrete MAC schemes.*

REMARK 3. *Since for a given loop  $i$  all its nodes  $j$  are constrained by the same  $\tau_{MATI}^i + \tau_{MAD}^i$  values, and, moreover, at each sampling instant at most one link per loop transmits, the notation of the metrics defined over status age can avoid specifying which node  $j$  of loop  $i$  is referring to, as it is irrelevant for the communication and control analysis. Therefore, the average Age  $\bar{\Delta}_i$ , the average Peak Age  $\bar{\hat{\Delta}}_i$  and the Peak Age of the  $k$ th message  $\hat{\Delta}_k^i$  are valid for the link of loop  $i$ , which is used by the sampling node defined by the loop scheduling policy.*

The following results work as starting point for the analysis. From the AoI concept, and referring to Problem 1, we can state:

LEMMA 4. *Consider an  $M$ -loop WNCS for which Assumptions 1 and 3 hold, the AoI of the link of loop  $\mathcal{P}_i$  is*

$$\bar{\Delta}_i = \left( E[\mathbf{d}_i] + \frac{1}{2\lambda_i} \right), \quad (2.28)$$

where  $E[\mathbf{d}_i] = E[\mathbf{w}_i] + E[\mathbf{s}_i]$  is the expected delay of a message in the link, decomposed in a queue waiting time  $\mathbf{w}_i$  and a service time  $\mathbf{s}_i$ , both depending on the MAC scheme.

PROOF. From Assumption 1 we know that every link from  $\mathcal{P}_i$  is governed by a constant sampling rate  $\lambda_i$ , since from Assumption 3 no dropouts take place, we can define the inter-generation time in Lemma 1 as  $\mathbf{y}_i = 1/\lambda_i$ . Furthermore, since  $\mathbf{y}_i$  is constant, then  $\mathbf{y}_i$  and  $\mathbf{d}_i$  are independent  $\forall i$ , yielding  $E[\mathbf{y}_i \mathbf{d}_i] = E[\mathbf{y}_i]E[\mathbf{d}_i] = 1/\lambda_i E[\mathbf{d}_i]$ . Replacing in (2.15) gives the result.  $\square$

Similarly, from the Peak Age notion, and concerning Problem 2, we state:

LEMMA 5. *Consider an  $M$ -loop WNCS for which Assumptions 1 and 3 hold, each loop will be stable in the mean-square sense if the Peak Age of every loop satisfies*

$$\hat{\Delta}_k^i \leq \tau_{MATI}^i + \tau_{MAD}^i \quad \forall k \quad (2.29)$$

PROOF. From 2.19, we can analyse separately the restriction on the delay  $\mathbf{d}_{k-1} \leq \tau_{MAD}^i$  and on the transmission interval (which is constant)  $\mathbf{y}_k = \mathbf{y} \leq \tau_{MATI}^i$ . Summing gives the result.  $\square$

REMARK 4. *It is important to emphasize from the previous lemma that for every transmission  $k$  the relationship must be respected in order to ensure stability. With this, we have established an explicit relation between the Peak Age and the stability constraints of a loop inserted in a shared network that uses a specific MAC protocol. This result represents a fundamental key in the analysis of the interplay between loop stability and communication protocol constraints.*

In the following chapters we address both Problems 1 and 2 for two examples of a contention-based and one example of a scheduled-based MAC scheme. From now on, we will analyse the WNCS in Figure 2.9 considering only the link sensor-controller of each loop, assuming that the controller-actuator link is instantaneous, which is a valid consideration if we note that the majority of the literature assume that controller and actuator are wired connected or attached between each other (Liu, Fridman, & Hetel, 2012) (Liu et al., 2015) (D. P. Borgers & Heemels, 2014).

### 3. AGE OF INFORMATION (AOI) IN WNCSS

In this chapter we derive expressions for the Age of Information (AoI)  $\bar{\Delta}_i$  of the links from different loops inserted in a WNCS as the one described in the previous section. In particular, we analyze two cases of Carrier Sense Multiple Access with Collision Avoidance (CSMA/CA) and Time-division Multiple Access (TDMA).

#### 3.1. AoI in CSMA-based WNCSSs

In this section we analyse the previously discussed mode with the two different CSMA/CA-based sub-mechanisms previously described: the DCF CSMA/CA algorithm and the un-slotted nBE CSMA/CA. In terms of notation, we will refer to the models as “CSMA-A” and “CSMA-B”, respectively. Before going into detail on each one, we have to make the following assumption

*ASSUMPTION 4. Each link of loop  $\mathcal{P}_i$  transmitting over a CSMA-A network with constant sampling rate  $\lambda_i$  operates in a D/M/1 First-Come First-Serve (FCFS) queue discipline, while the loops transmitting over CSMA-B operate in a D/G/1 FCFS policy. Furthermore, the use of FCFS defines a utilization rate  $\rho_i = \lambda_i E[\mathbf{s}_i^{CSMA}]$ , for which  $\rho_i \leq 1$  is assumed to hold  $\forall i$ .*

*REMARK 5. Although other disciplines like Last-Come First-Serve (LCFS), with and without preemption, have been proven theoretically better in terms of AoI (S. K. Kaul, Yates, & Gruteser, 2012), the most common IoT-oriented MAC protocols that use CSMA/CA, as IEEE802.15.4 and IEEE802.11, handles packets with a FCFS policy (Mishra, Na, & Rosenburgh, 2007) (Moo-Yeong Jeong, Bum-Gon Choi, Ju Yong Lee, & Chung, 2010).*

##### 3.1.1. AoI in DCF CSMA/CA based WNCSSs

In this section we follow the approach for the idealized DCF CSMA/CA described in (Maatouk et al., 2020). CSMA-A is modeled as a Markov chain that defines a PWL-SHS and considers a back-off time for each link to capture the medium and a holding

(transmission) time after which the channel is freed and becomes ready to be captured again, with no collisions considered. In the following, this formulation is used to analyse the expected service time  $E[s_i^{\text{CSMA-A}}]$  for links of  $\mathcal{P}_i$ , considering only the point of view of a link from  $\mathcal{P}_i$  when transmitting packets over a CSMA-A network in the presence of  $M - 1$  other loops.

To characterize  $E[s_i^{\text{CSMA-A}}]$ , the PWL-SHS model is formulated by first defining a state  $(\mathbf{q}, \mathbf{x})$ . The discrete component  $\mathbf{q} \in \mathbb{Q} = \{0, 1..M\}$  represents the Markov chain in Figure 3.1, also used in (Maatouk et al., 2020), where  $\mathbf{q}(t) = 0$  implies that the channel is idle and ready to be captured by a node from any loop, while  $\mathbf{q}(t) = i, i \neq 0$ , implies that a node from loop  $i \in \mathcal{S}$  has the channel under its control and has already started a transmission through a wireless link. This chain makes noticeable the notion of increasing congestion when the number of competing links increases. The transition from  $\mathbf{q} = i \neq 0$  to  $\mathbf{q} = 0$  is ruled by the exponentially-distributed holding time, with expected value  $\frac{1}{H_j}$ , which measures the time the active link from loop  $i$  uses to transmit a packet. Similarly, the transition from  $\mathbf{q} = 0$  to  $\mathbf{q} = i \neq 0$  is ruled by the exponentially-distributed back-off time, with expected value  $\frac{1}{R_i}$ , which measures the waiting time the link from loop  $i$  experiences when trying to capture the medium.

The continuous component  $\mathbf{x} \in \mathbb{R}$  measures the time a packet from the active node of loop  $i \in \mathcal{S}$  spends in the system, from its arrival to the head of the queue until the end of its transmission, hence,  $b_q = 1$ . To keep the chain static, it is assumed that whenever a link from loop  $i$  captures the channel it already has a packet to transmit (all loops have permanently at least one packet in their respective queues), as done in (Yates & Kaul, 2019) and (Maatouk et al., 2020). This means that we calculate the bound case for when  $\rho_i \geq 1$ , which implies queue instability in the long term.

Regarding transitions, the Markov chain defines a set  $\mathcal{L}$  of  $2M$  transitions, where each element  $l$  of  $\mathcal{L}$  represents a change in the loop transmitting  $q_l \rightarrow q_l^+$ , meaning a transition from state  $\mathbf{q}_l$  to  $\mathbf{q}_l^+$ , with associated intensity  $\lambda^{(l)} = R_i$  for transitions of the form  $0 \rightarrow k$  and  $\lambda^{(l)} = H_i$  for transitions of the form  $i \rightarrow 0$ , and linear reset map  $\phi_l = (\mathbf{q}_l^+, z_l \mathbf{x})$  with

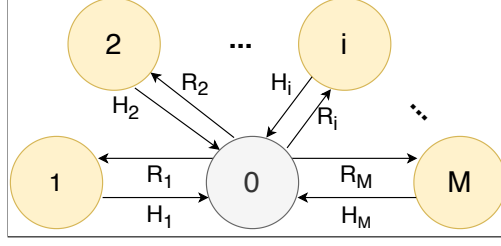


FIGURE 3.1. Markov chain of the CSMA-A transmission scheme.

TABLE 3.1. SHS transitions of packet delay in CSMA-A

| $l$   | $q_l \rightarrow q_l^+$ | $\lambda^{(l)}$ | $z_l \mathbf{x}$ | $z_l$    | $z_l \bar{v}_{q_l}$ |
|-------|-------------------------|-----------------|------------------|----------|---------------------|
| 1     | $0 \rightarrow 1$       | $R_1$           | $[x]$            | $[1]$    | $[\bar{v}_0]$       |
|       | $\vdots$                | $\vdots$        | $\vdots$         | $\vdots$ | $\vdots$            |
| $M$   | $0 \rightarrow M$       | $R_M$           | $[x]$            | $[1]$    | $[\bar{v}_0]$       |
| $M+1$ | $1 \rightarrow 0$       | $H_1$           | $[x]$            | $[1]$    | $[\bar{v}_1]$       |
|       | $\vdots$                | $\vdots$        | $\vdots$         | $\vdots$ | $\vdots$            |
| $M+i$ | $i \rightarrow 0$       | $H_i$           | $[0]$            | $[0]$    | $[0]$               |
|       | $\vdots$                | $\vdots$        | $\vdots$         | $\vdots$ | $\vdots$            |
| $2M$  | $M \rightarrow 0$       | $H_M$           | $[x]$            | $[1]$    | $[\bar{v}_M]$       |

$z_l \in \{0, 1\}$  a binary map such that  $z_l = 0$  only when  $l$  is associated to transition  $i \rightarrow 0$  indicating that transmission at the link of interest is completed, hence the link service time is reset to 0.

Additionally, following (Yates & Kaul, 2019) we define the discrete component probability  $\pi_q(t) = \mathbb{E} [\delta_{q,q(t)}] = P(q(t) = q)$ , the vector function  $v_q(t) = [v_{q0}(t), \dots, v_{qN}(t)] = \mathbb{E} [\mathbf{x}(t) \delta_{q,q(t)}]$ , which measures the correlation between the continuous component  $\mathbf{x}(t)$  and the discrete state  $\mathbf{q}(t)$ , and for each discrete state  $\mathbf{q} \in \mathbb{Q}$  we define the incoming transition set as  $\mathbb{L}_q^+ = \{l \in \mathcal{L} : q_l^+ = q\}$ , and the outgoing transition set as  $\mathbb{L}_q = \{l \in \mathcal{L} : q_l = q\}$ .

Now we can introduce a bound for  $E[\mathbf{s}_i^{\text{CSMA-A}}]$  considering this PWL-SHS

**THEOREM 1.** *Consider an  $M$ -loop WNCS for which Assumptions 1, 3 and 4 hold. If the WNCS operates over a CSMA-A network, then for every packet originated at a node of loop  $\mathcal{P}_i$  the expected service time  $E[\mathbf{s}_i^{\text{CSMA-A}}]$  satisfies*

$$\frac{\bar{\pi}_0}{R_i} + \frac{\pi_i}{H_i} \leq E[\mathbf{s}_i^{\text{CSMA-A}}] \leq E[\mathbf{s}_i^{\text{MCSMA-A}}], \quad (3.1)$$

and

$$E[\mathbf{s}_i^{\text{MCSMA-A}}] = E[\mathbf{x}(t)] = \sum_{q \in \mathcal{Q}} \bar{v}_q = \sum_{j=0, j \neq i}^M \bar{v}_j + \frac{\bar{\pi}_i}{H_i}, \quad (3.2)$$

where

$$\begin{aligned} \bar{v}_0 &= \frac{\bar{\pi}_0}{\sum_{j=1}^M R_j} + \sum_{j=1}^M \frac{\bar{\pi}_j}{R_j} \\ \bar{v}_j &= \frac{\bar{\pi}_j + R_j \bar{v}_0}{H_j}. \end{aligned} \quad (3.3)$$

And the stationary distributions are given by

$$\begin{cases} \bar{\pi}_0 = \frac{1}{C(R)} \\ \bar{\pi}_j = \frac{\frac{R_j}{H_j}}{C(R)} \quad j = 1, \dots, M \end{cases} \quad (3.4)$$

with  $C(R) = 1 + \sum_{j=1}^M \frac{R_j}{H_j}$ .

PROOF. Since the Markov chain in Figure 3.1 is irreducible and positively recurrent, there exists a unique positive stationary distribution (Serfozo, 2009), which can be obtained using the balancing equations from Lemma 2 in (Yates & Kaul, 2019) as

$$\bar{\pi}_q \sum_{l \in \mathcal{L}_q} \lambda^{(l)} = \sum_{l \in \mathcal{L}'_q} \lambda^{(l)} \bar{\pi}_{q_l}, \quad \sum_{q \in \mathcal{Q}} \bar{\pi}_q = 1, \quad q \in \mathcal{Q} \quad (3.5)$$

using the information in Table 3.1 results in (3.4). From Theorem 4 in (Yates & Kaul, 2019), a non-negative solution of the correlation vector limit  $\bar{v}_q \in \mathbb{R}^{N+1}$  is obtained as

$$\bar{v}_q \sum_{l \in \mathcal{L}_q} \lambda^{(l)} = b_q \bar{\pi}_q + \sum_{l \in \mathcal{L}'_q} \lambda^{(l)} \bar{v}_{q_l} z_l, \quad q \in \mathcal{Q} \quad (3.6)$$

replacing the results in (3.4) gives (3.3). Finally, from (Yates & Kaul, 2019),  $E[\mathbf{x}(t)] = \sum_{q \in \mathcal{Q}} \bar{v}_q(t)$ , which leads to (3.2). The right inequality in (3.1) follows by noting that, by construction,  $E[\mathbf{x}(t)]$  is the expected service time for a packet when  $M$  loops constantly compete for the channel. In case a link presents an empty queue, the expected service time is strictly smaller, hence (3.1) holds. The left inequality in (3.1) follows by noting that

the lowest expected service corresponds to the case when  $M = 1$ , where only the loop of interest is competing for the channel.  $\square$

From Lemma 4, to obtain the AoI of the link of loop  $\mathcal{P}_i$ ,  $\bar{\Delta}_i^{\text{CSMA-A}}$ , the queue waiting time  $E[\mathbf{w}_i]$  needs to be characterized. Note that since the link of  $\mathcal{P}_i$  transmitting over a CSMA-A network is inserted in a D/M/1 FCFS scenario, the expected waiting time can be written as (S. Kaul et al., 2012):  $E[\mathbf{w}_i^{\text{CSMA-A}}] = E[\mathbf{s}_i^{\text{CSMA-A}}] \frac{\beta_i}{(1-\beta_i)}$ , where  $\beta_i$  is given in terms of the utilization rate  $\rho_i$  and the Lambert  $\mathcal{W}$  function (Corless, Gonnet, Hare, Jeffrey, & Knuth, 1996) as:  $\beta_i = -\rho_i \mathcal{W}(-(\rho_i)^{-1} e^{(-1/\rho_i)})$ .

Combining this expression with Lemma 4 and Theorem 1 gives the following result for  $\bar{\Delta}_i^{\text{CSMA-A}}$ .

**THEOREM 2.** *Consider an  $M$ -loop WNCS for which Assumptions 1, 3 and 4 hold. If the WNCS operates over a CSMA-A network, then the AoI,  $\bar{\Delta}_i^{\text{CSMA-A}}$ , of the link of loop  $\mathcal{P}_i$  satisfies*

$$\bar{\Delta}_i^{\text{CSMA-A}} = \frac{1}{2\lambda_i} + \frac{E[\mathbf{s}_i^{\text{CSMA-A}}]}{1 - \beta_i} \leq \bar{\Delta}_i^{\text{MCSMA-A}} = \frac{1}{2\lambda_i} + \frac{E[\mathbf{s}_i^{\text{MCSMA-A}}]}{1 - \beta_i} \quad (3.7)$$

Theorem 2 gives an expression for  $\bar{\Delta}_i^{\text{CSMA-A}}$  that, as stated in (Yates & Kaul, 2019), can be optimized by properly choosing the sampling rate. Figure 3.2 shows how the sampling rate that minimizes  $\bar{\Delta}_i^{\text{MCSMA-A}}$  lowers as the number of loops grows, while the minimum upper bound itself increases. A concrete result in this regard can be derived when the links of the WNCS are homogeneous.

**PROPOSITION 1.** *Consider an  $M$ -loop WNCS operating over a CSMA-A network for which Assumptions 1, 3 and 4 hold. If  $H_i = H$  and  $R_i = R \forall i \in \mathcal{S}$ , the least upper-bound on the AoI of the link of loop  $\mathcal{P}_i$  is given by*

$$\bar{\Delta}_i^{*\text{MCSMA-A}} = 2.2526 E[\mathbf{s}_i^{*\text{MCSMA-A}}], \quad (3.8)$$

where

$$E[s_i^{*MCMA-A}] = \frac{1}{M^2 R^2 / H + MR} + \frac{M}{MR + H} + \frac{R}{H(MR + H)} + \frac{R(\frac{1}{M^2 R^2 / H + MR} + \frac{M+1}{MR+H})(M-1)}{H}. \quad (3.9)$$

Moreover,  $\bar{\Delta}_i^{*MCMA-A}$  is achieved when  $\mathcal{P}_i$  operates at the optimal sampling rate  $\lambda_i^{*MCMA-A} = \frac{0.515}{E[s_i^{*MCMA-A}]}$ .

PROOF. From (S. Kaul et al., 2012), the AoI of a link transmitting over a CSMA-A network under a D/M/1 FCFS scenario is optimized at  $\rho^* = 0.515$ , which implies that the optimal sampling rate is given by  $\lambda_i^{*MCMA-A} = \frac{0.515}{E[s_i^{*MCMA-A}]}$ . Evaluating  $\beta_i$  for  $\rho_i = 0.515$  yields  $\beta_i = 0.2198 \forall i$ . Evaluating in (3.7) results in (1), while (3.9) follows by evaluating the right hand side of (3.2) with  $H_i = H$  and  $R_i = R \forall i$ .  $\square$

REMARK 6. Proposition 1 considers the scenario where  $H_i = H$  and  $R_i = R \forall i$ , which applies when every set of sensors/actuators sends packets of the same length, with the network not giving priority to any link, as stated in (Maatouk et al., 2020). This is reasonable for IoT systems with similar devices. Furthermore, Proposition 1 considers the same sampling rate for every loop. It should be noted that this does not imply synchronized sampling, as sampling instants can be lagged between loops. In this context,  $\bar{\Delta}_i^{CSMA-A}$  is lower-bounded by the case when the link finds the channel with no other links contending for it, where (3.9) is evaluated for  $M = 1$ .

### 3.1.1.1. Simulations

We simulate the CSMA-A system in MATLAB in order to confirm the results. We consider a WNCS with 5 loops in which we calculate the average age of each link, simulated five times and 100 seconds for each time, for each value of  $\lambda$ , with a channel holding time of  $1/H = 4.256ms$ . These parameters allow us to simulate from 100 (for  $\lambda = 1$ ) to 9000 (for  $\lambda = 90$ ) updates for each link in each instance, which, once averaged, gives us a fair approximation to the steady-state information.

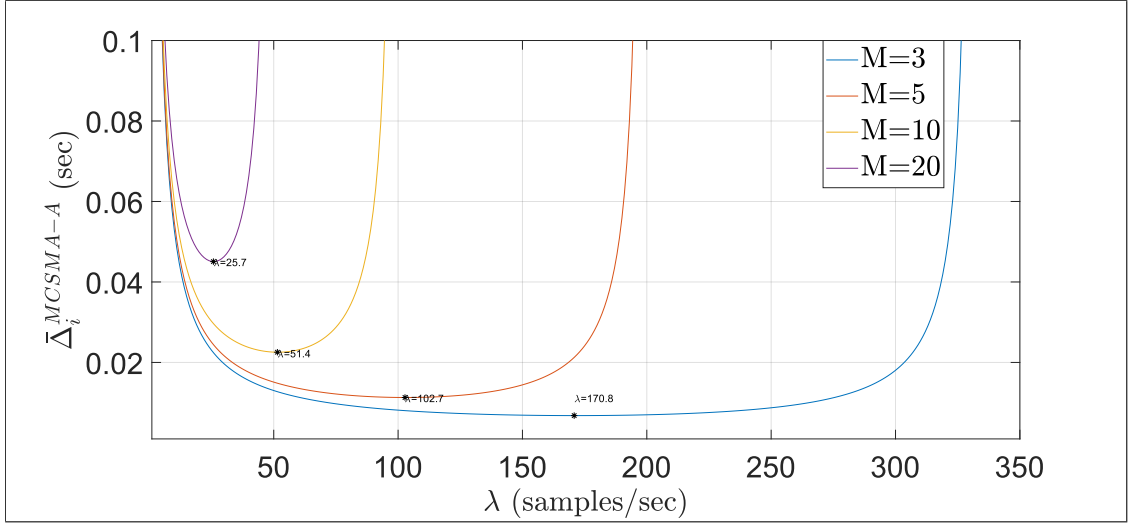


FIGURE 3.2.  $\bar{\Delta}_i^{MCSMA-A}$  as a function of the sampling rate for different number of links in the system with optimal points.  $\lambda_i = \lambda \forall i$ .  $1/H = 1ms$  ( $L=2304$  bytes) and  $R = 14.8H \forall i$ .

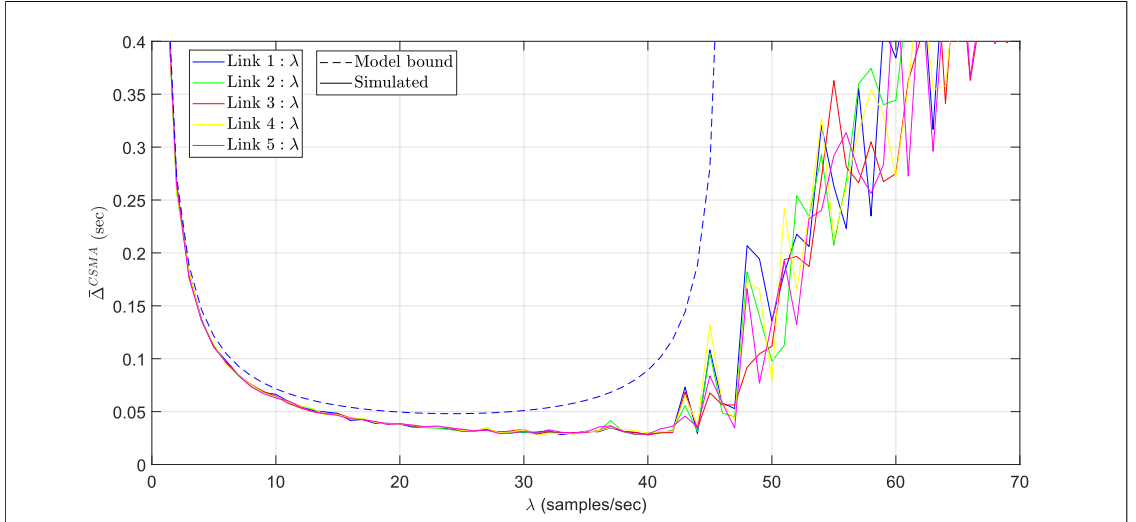


FIGURE 3.3. Simulation of  $\bar{\Delta}^{CSMA-A}$  with  $M = 5$  and  $1/H = 4.256ms$ .  $\lambda_i = \lambda$ .

In Figure 3.3, we can appreciate that the bound is quite tight for all  $\lambda$  in the case where all links sample at the same rate, being closer for the smaller values. When we consider loops with different sampling rates, it can be noticed in Figure 3.4 that a faster loop benefits in the sense of AoI when the other loops sample at a slower rate, because from its perspective it does not have to compete with the other loops all the time. Both type

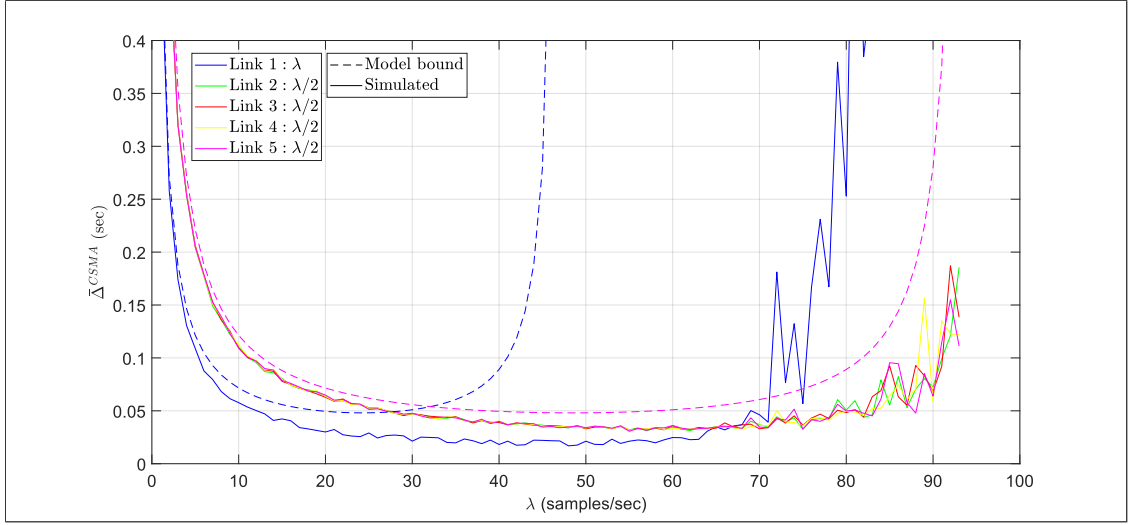


FIGURE 3.4. Simulation of  $\bar{\Delta}^{CSMA-A}$  with  $M = 5$  and  $1/H = 4.256ms$ . One link sampling with  $\lambda$  and four with  $\lambda/2$ .

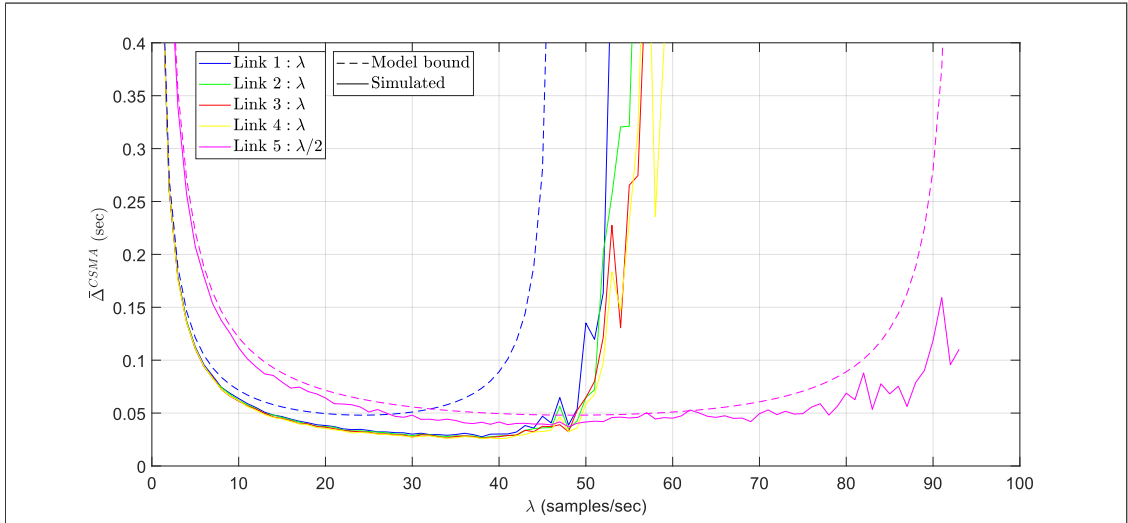


FIGURE 3.5. Simulation of  $\bar{\Delta}^{CSMA-A}$  with  $M = 5$  and  $1/H = 4.256ms$ . Four links sampling with  $\lambda$  and one with  $\lambda/2$ .

of loops, fast and slow, are below the respective bound, with the bound on slower loops being tighter. Finally, in Figure 3.5 is appreciated that if we consider 4 fast and one slow loop, all the links are below the bound, but the slower one is hampered and very close to the bound when compared to the other cases, as from its perspective it has to compete almost all the time with all the other loops.

### 3.1.2. AoI in nBE CSMA/CA based WNCSSs

To model CSMA-B, we follow the approach for the idealized unslotted CSMA/CA described in (Buratti & Verdone, 2009). CSMA-B is modeled by calculating the probability that a certain packet from a certain link finishes its transmission after a certain number of back-off periods since its arrival to the head of the queue. In the following, this formulation is used to analyse the expected service time  $E[s_i^{\text{CSMA-B}}]$  for the link of  $\mathcal{P}_i$ , considering only the point of view of the link from  $\mathcal{P}_i$  when transmitting packets over a CSMA-B network in the presence of  $M - 1$  other loops.

As discussed previously, the unslotted non beacon-enabled CSMA/CA protocol gives all the links a distributed algorithm designed to avoid collisions. The algorithm in Figure 2.6 is applied to all the devices that intend to share the network. From Assumption 2, we can determine that a maximum of  $M$  nodes will contend for the channel at the same time. For each node forming a link accessing the channel a state is assigned modeled as a bi-dimensional process  $Q_i(t) = \{B_c(t), B_s(t)\}$ , where  $t$  is the integer multiple of the back-off period  $db$  that indicates the time slot at which the node is,  $B_c$  is the back-off counter and  $B_s$  is the back-off stage. For this model we consider the parameters of the IEEE802.15.4 standard (Buratti & Verdone, 2009) (IEEE 802.15 - Wireless Specialty Networks (WSN) Working Group, 2007), which indicates that the back-off period  $db$  is equal to 20 times the time symbol, which is  $16\mu s$ . Therefore,  $db = 320\mu s$ . It is considered that every packet from every link attempting to capture the channel has a length of  $L$  bytes, with a minimum of  $L = 10$ . In the first back-off stage,  $B_s = 0$ , the back-off counter  $B_c$  starts counting down from a uniformly chosen number of  $[0, \dots, W_0]$  slots. For  $B_s = 1$  the back-off counter goes backwards from a number chosen between  $[0, \dots, W_1]$  and so on for all the stages.  $W$  is called the contention window, and its value is given by the back off exponent  $BE$  previously discussed. The values that relate the stage, the counter and the contention window are given in table 3.2.

TABLE 3.2. Relationship between back-off stages and back-off exponents in CSMA-B protocol.

| $B_s$ | BE | $W_s$ |
|-------|----|-------|
| 0     | 3  | 8     |
| 1     | 4  | 16    |
| 2     | 5  | 32    |
| 3     | 5  | 32    |
| 4     | 5  | 32    |

The model is used mainly in the calculation of two different probabilities: the probability of the node being in a sensing state in a certain slot  $t$ , which is denoted by  $P\{C^t\}$ , and the probability that in a certain slot  $t$  the channel is found to be busy after sensing, which is denoted by  $p_b^t$ .

On its side,  $P\{C^t\}$  is calculated from the probability of the node being in a sensing state in a certain slot  $t$  when being at a certain back-off stage. This term is denoted as  $P\{S_s^t\}$ , using the notation from (Buratti & Verdone, 2009), where  $s$  references the back-off stage of the node,  $B_s(t)$ . In order to calculate  $P\{S_s^t\}$ , the model establishes the transition probabilities between each back-off stage following the algorithm described in 2.6.1.2. Each probability  $P\{S_s^t\}$  is calculated for each slot  $t$  recursively from the calculations of the previous slot  $P\{S_s^{t-1}\}$  starting with

$$P\{S_0^t\} = \begin{cases} \frac{1}{W_0}, & \text{for } t \in [0, W_0 - 1] \\ 0, & \text{for } t > W_0 - 1 \end{cases} \quad (3.10)$$

The probability of sensing the channel in upper stages includes passing from one stage to the other, which happens when one node senses the channel busy, with probability denoted by  $p_b^t$ . This term is also calculated recursively, starting from  $p_b^0 = 0$ . As  $p_b^t = 1 - p_f^t$ , with  $p_f^t$  being the probability that the channel is found to be free after sensing, which is given by the following equations (Buratti & Verdone, 2009), based on defining a number  $D = \lceil \frac{L}{10} \rceil$ .  $D$  is based on the notion that, considering the IEEE802.15.4 standard (IEEE 802.15 - Wireless

Specialty Networks (WSN) Working Group, 2007), in one back-off period  $db$ , which is the resolution time used for the protocol analysis, at most 10 bytes of information can be transmitted. Therefore, each packet holds the channel for  $D$  back-off periods. Then, if  $D = 1$

$$p_f^t = (1 - p_b^{t-1}) \prod_{i=0}^{NB_{\max}} (1 - P\{S_i^{t-1}\})^{M-1} + p_b^{t-1} \quad (3.11)$$

Otherwise, if  $D > 1$

$$p_f^t = (1 - p_b^{t-1}) \prod_{i=0}^{NB_{\max}} (1 - P\{S_i^{t-1}\})^{M-1} + (1 - p_b^{t-D-1}) \cdot \left[ 1 - \prod_{i=0}^{NB_{\max}} (1 - P\{S_i^{t-D-1}\})^{M-1} \right] \quad (3.12)$$

with  $NB_{\max} = 4$ , according to the standard protocol discussed. As both definitions,  $p_b^t$  and  $P\{S_s^t\}$ , depend on each other to be calculated, the calculation of the probability at each step is done in an algorithmic style.  $P\{S_s^t\}$  depends on  $p_b^t$ , which on itself depends on  $P\{S_s^{t-1}\}$  and so on. Then, the probability of the node sensing the channel in a back-off stage that is not zero includes both the probability of finding the channel busy and the previous probabilities of sensing the channel in a previous stage.

$$P\{S_1^j\} = \begin{cases} 0, & \text{for } j < 2 \\ \sum_{v=1}^{j-1} P\{S_0^v\} \cdot \frac{p_b^v}{W_1}, & \text{for } j \in [2, W_0] \\ P\{S_1^{W_0}\}, & \text{for } j \in [W_0 + 1, W_1 + 1] \\ P\{S_1^{W_0}\} - \sum_{v=1}^{j-W_1-1} P\{S_0^v\} \cdot \frac{p_b^v}{W_1}, & \text{for } j \in [W_1 + 2, W_{0,1} - 1] \\ 0, & \text{for } j > W_{0,1} - 1 \end{cases} \quad (3.13)$$

$$P \{S_2^j\} = \begin{cases} 0, & \text{for } j < 3 \\ \sum_{v=2}^{j-1} P \{S_1^v\} \cdot \frac{p_b^v}{W_2}, & \text{for } j \in [3, W_{0,1}] \\ P \{S_2^{W_{0,1}}\}, & \text{for } j \in [W_{0,1} + 1, W_2 + 2] \\ P \{S_2^{W_{0,1}}\} - \\ - \sum_{v=2}^{j-W_2-1} P \{S_1^v\} \cdot \frac{p_b^v}{W_2}, & \text{for } j \in [W_2 + 3, W_{0,1,2} - 1] \\ 0, & \text{for } j > W_{0,1,2} - 1 \end{cases} \quad (3.14)$$

$$P \{S_3^j\} = \begin{cases} 0, & \text{for } j < 4 \\ \sum_{v=3}^{j-1} P \{S_2^v\} \cdot \frac{p_b^v}{W_2}, & \text{for } j \in [4, W_2 + 3] \\ \sum_{v=3}^{j-1} P \{S_2^v\} \cdot \frac{p_b^v}{W_2} - \sum_{v=3}^{j-W_2-1} P \{S_2^v\} \cdot \frac{p_b^v}{W_2}, & \text{for } j \in [W_2 + 4, W_{0,1,2}] \\ P \{S_3^{W_{0,1,2}}\} - \sum_{v=W_{0,1}}^{j-W_2-1} P \{S_2^v\} \cdot \frac{p_b^v}{W_2}, & \text{for } j \in [W_{0,1,2} + 1, W_{0,1,2,3} - 1] \\ 0, & \text{for } j > W_{0,1,2,3} - 1 \end{cases} \quad (3.15)$$

$$P \{S_4^j\} = \begin{cases} 0, & \text{for } j < 5 \\ \sum_{v=4}^{j-1} P \{S_3^v\} \cdot \frac{p_b^v}{W_2}, & \text{for } j \in [5, W_2 + 4] \\ \sum_{v=4}^{j-1} P \{S_3^v\} \cdot \frac{p_b^v}{W_2} - \sum_{v=4}^{j-W_2-1} P \{S_3^v\} \cdot \frac{p_b^v}{W_2}, & \text{for } j \in [W_2 + 5, W_{0,1,2,3}] \\ P \{S_4^{W_{0,1,2,3}}\} - \sum_{v=W_{0,1,2}}^{j-W_2-1} P \{S_3^v\} \cdot \frac{p_b^v}{W_2}, & \text{for } j \in [W_{0,1,2,3} + 1, W_{0,1,2,3,4} - 1] \\ 0, & \text{for } j > W_{0,1,2,3,4} - 1 \end{cases} \quad (3.16)$$

$W_{x,y,z}$  is defined as the sum of  $W_x + W_y + W_z$ . All the definitions and the  $P\{S_s^t\}$  probabilities derivations can be found in (Buratti & Verdone, 2009). Then,  $P\{C^t\}$  is calculated as

$$P\{C^t\} = \begin{cases} P\{S_0^0\}, & \text{for } t = 0 \\ P\{S_0^1\}, & \text{for } t = 1 \\ \sum_{i=0}^{t-1} P\{S_i^t\}, & \text{for } t \in [2, \dots, 4] \\ \sum_{i=0}^{NB_{\max}} P\{S_i^t\}, & \text{for } t \in [5, \dots, W_0 - 1] \\ \sum_{i=1}^{NB_{\max}} P\{S_i^t\}, & \text{for } t \in [W_0, \dots, W_{0,1} - 1] \\ \sum_{i=2}^{NB_{\max}} P\{S_i^t\}, & \text{for } t \in [W_{0,1}, \dots, W_{0,1,2} - 1] \\ \sum_{i=3}^{NB_{\max}} P\{S_i^t\}, & \text{for } t \in [W_{0,1,2}, \dots, W_{0,1,2,3} - 1] \\ P\{S_4^t\}, & \text{for } t \in [W_{0,1,2,3}, \dots, W_{0,1,2,3,4} - 1] \end{cases} \quad (3.17)$$

Finally, (Buratti & Verdone, 2009) establishes the probability that a generic node finishes its packet transmission successfully on a certain slot  $t \in [0, t_{\max} + D - 1]$ , in the context of  $M$  links competing for the channel access. As the time finishing the transmission since its arrival to the queue is a multiple of the back-off period  $db$ , we describe the probability of the packet having a certain service time as

$$P\{Z^t\} = P\{C^{t-D}\} \cdot (1 - p_b^{t-D}) \cdot \prod_{i=0}^{NB_{\max}} (1 - P\{S_i^{t-D}\})^{M-1} \quad t \in [0 \dots t_{\max} + D - 1] \quad (3.18)$$

which refers that the probability of a node correctly ending its transmission on a given slot  $t$  is given by the chance that only one node senses the channel free in slot  $t - D$ .

We notice that  $\sum_{t=0}^{t_{\max}+D-1} P\{Z^t\} < 1$  for the majority of the combinations of packet lengths and number of loops. The main reason of this are the collisions produced when two or more links are assigned the same back-off counter, and therefore sense the channel free in the same slot. Both loops then transmit and the packet is lost due to the collision. Extending the model from (Buratti & Verdone, 2009), we assume that there exist an acknowledge mechanism that allows the loop to know if the packet was received at destination or not.

If the packet was not received, the link from the loop will attempt a retry on capturing the channel to send the same packet that was not received. The probability of a link not receiving an acknowledge is the probability that the link does not transmit successfully  $(1 - \sum_{t=0}^{t_{max}+D-1} P\{Z^t\})$ . Therefore, if the link attempts a retry, in the next  $t_{max}+D-1$  slots the probability of a successful transmission is given by  $P\{Z^t\} = (1 - \sum_{k=0}^{t_{max}+D-1} P\{Z^k\}) \cdot P\{Z^{t-(t_{max}+D)}\}$   $t \in [t_{max} + D \dots 2(t_{max} + D - 1)]$ . With this, setting the retries as  $r \in [0 \dots R]$ , with the maximum number of retries being  $R$ , we extend the model and present the distribution for the service time of a CSMA-B successful transmission

$$P[s_i^{\text{MCSMA-B}} = t \cdot db] = \left(1 - \sum_{k=0}^{t_{max}+D-1} P\{Z^k\}\right)^r P\{Z^{t-r(t_{max}+D)}\} \quad (3.19)$$

for  $t \in [r(t_{max} + D), \dots, (r+1)(t_{max} + D - 1)]$  and  $r \in [0, \dots, R]$ . If we consider a sufficiently large  $R$ , the sum of the probabilities  $\sum_{t=0}^{R(t_{max}+D-1)} P[s_i^{\text{MCSMA-B}} = t \cdot db] \approx 1$ , which allows us to assume that no are no dropouts with this protocol, therefore maintaining Assumption 3.

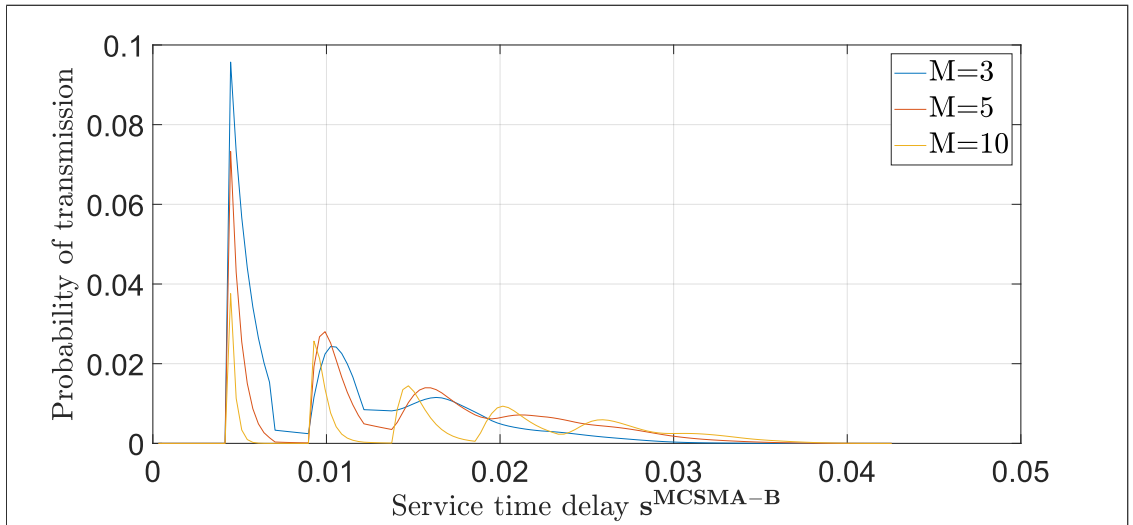


FIGURE 3.6. Service time delay distribution on CSMA-B network with  $M$  links contending for the channel.  $L = 133$  bytes,  $db = 320\mu s$  and  $R = 0$ .

In order to analyse the principal characteristics of the service time distribution, we calculate its moment-generating function (MGF) as

$$m^{\text{MCSMA-B}}(\theta) = \sum_{t=0}^{(R+1)(t_{\max}+D-1)} e^{\theta \cdot t \cdot db} P[s^{\text{MCSMA-B}} = t \cdot db] \quad (3.20)$$

REMARK 7. *As the upper limit of the sum in (3.20) includes the parameter  $D$ , and the number of loops  $M$  competing for the channel is included in the exponent of the parameters to solve, it becomes very complex, computationally speaking, to solve a symbolic expression including  $D$  or  $M$ . Therefore, the only option to obtain results is to fix both parameters before evaluating. Figure 3.6 shows an example of how the distribution behaves when no retries are attempted.*

REMARK 8. *Notice that the service time distribution calculated considers  $M$  links constantly competing for the channel access, with all the loops starting the mechanism at the same time and with the same sampling rate. This means that it does not consider if one link accomplishes transmission in an early slot, and then only the  $M - 1$  links left compete with each other in the next slots. However, according to (Buratti & Verdone, 2009), this issue does not affect significantly the results, as can be seen in Figure 3.7, where the red bar symbolizes including a Bernoulli variable that estimates when each link finishes transmitting, defining a variable  $M$  ( $N$  in (Buratti & Verdone, 2009)); and the white bar symbolizes the case where  $M$  is constant.*

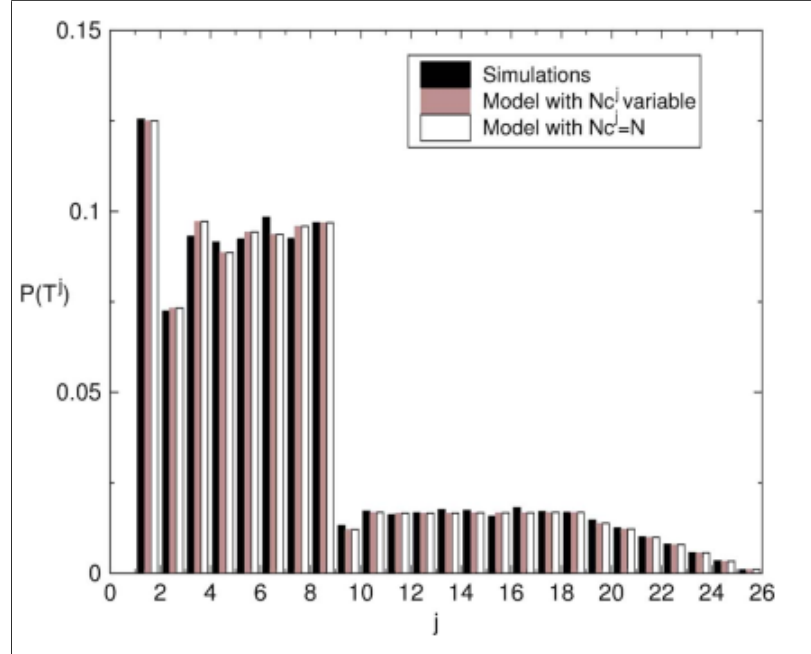


FIGURE 3.7. Figure from (Buratti & Verdone, 2009) showing the distribution in the sensing slots for the CSMA/CA 802.15.4 algorithm.  $N=5$ .  $L=10$  bytes.

REMARK 9. As the model does not consider loops with different sampling rates, we will separate the cases and make an additional analysis. Three scenarios are considered: (i) analysing a loop of interest  $i$  in the context of all the loops in the network sampling at the same rate; (ii) analysing a loop of interest  $i$  for the case where its sample rate is the fastest of all the other loops in comparison; and (iii), analysing a loop of interest  $i$  in the context of having at least one loop in the network that samples faster than the loop of interest.

LEMMA 6. Consider an  $M$ -loop WNCS for which Assumptions 1, 2, 3 and 4 hold. If the WNCS operates over a CSMA-B network with every loop sampling at the same rate, then for every packet originated at the of loop  $\mathcal{P}_i$  the expected service time  $E[s_i^{\text{CSMA-B}}]$  satisfies

$$E[s_i^{\text{CSMA-B}}] \leq E[s^{\text{MCSMA-B}}] \quad (3.21)$$

Before stating a result for the AoI of the the packets in CSMA-B, the queue waiting time  $E[w_i^{\text{MCSMA-B}}]$  needs to be characterized.

LEMMA 7. Consider an  $M$ -loop WNCS for which Assumptions 1, 2, 3 and 4 hold. If the WNCS operates over a CSMA-B network with every loop sampling with the same rate, then the expected waiting time  $E[\mathbf{w}_i^{\text{CSMA-B}}]$ , of the link from loop  $\mathcal{P}_i$  satisfies

$$E[\mathbf{w}_i^{\text{CSMA-B}}] \leq E[\mathbf{w}^{\text{MCSMA-B}}] \approx \frac{\lambda_i \sigma^2}{2(1 - \rho_i)} \exp\left(\frac{-2(1 - \rho_i)}{3\lambda_i \sigma^2 \mu_i}\right) \quad (3.22)$$

with  $\mu_i = 1/E[\mathbf{s}^{\text{MCSMA-B}}] = m'(0)$  and  $\sigma^2 = m''(0) - \mu_i^2$ .

PROOF. From Assumption 4, the link of  $\mathcal{P}_i$  transmitting over a CSMA-B network is inserted in a D/G/1 FCFS scenario, a distribution for which there is not an exact characterization of the average waiting time (Janssen & Van Leeuwen, 2005). Therefore, we make an approximation based on the literature. Specifically, we take the Kraemer and Langenbach-Belz approximation (Kraemer & Langenbach-Belz, 1976), which defines the waiting time in a queue with any  $\rho$  as

$$L_q \approx \frac{\rho^2 (C_a^2 + C_s^2)}{2(1 - \rho)} g \quad (3.23)$$

with

$$g = \exp\left(\frac{-2(1 - \rho)(1 - C_a^2)^2}{3\rho(C_a^2 + C_s^2)}\right) \text{ when } C_a^2 \leq 1 \quad (3.24)$$

$$g = \exp\left(\frac{(1 - \rho)(1 - C_a^2)}{C_a^2 + 4C_s^2}\right) \text{ when } C_a^2 > 1$$

where  $C_a^2 = \frac{\sigma_a^2}{(1/\lambda)^2}$ ,  $C_s^2 = \frac{\sigma_s^2}{(1/\mu)^2}$  refer to the arrival and service coefficient of variation, respectively. As the arrivals are constant,  $C_a = 0$ . Then, applying the mean and variance of the service time distribution obtained from the MGF (3.20), and the Little's formula  $L_q = \lambda W_q$  ((Little, 1961)), (3.22) is obtained.  $\square$

Based on the previous results, at this point we can introduce a result for the AoI.

THEOREM 3. Consider an  $M$ -loop WNCS for which Assumptions 1, 2, 3 and 4 hold. If the WNCS operates over a CSMA-B network with every loop sampling at the same rate, then the AoI,  $\bar{\Delta}_i^{\text{CSMA-B}}$ , satisfies

$$\bar{\Delta}_i^{CSMA-B} \leq \bar{\Delta}_i^{MCSMA-B} = \frac{1}{2\lambda_i} + E[\mathbf{s}_i^{MCSMA-B}] + E[\mathbf{w}_i^{MCSMA-B}] \quad (3.25)$$

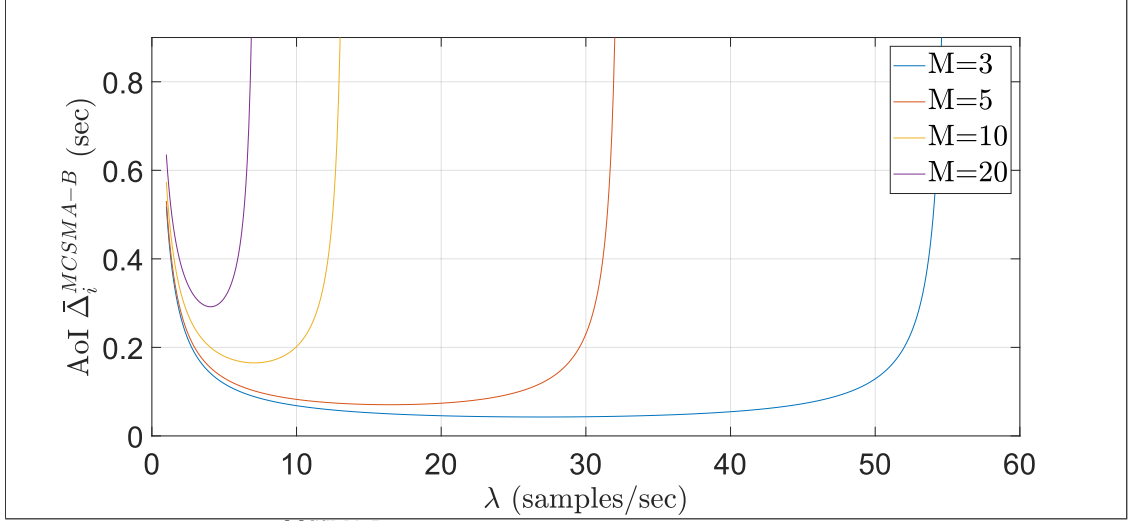


FIGURE 3.8.  $\bar{\Delta}_i^{MCSMA-B}$  as a function of the sampling rate for different number of links in the system.  $\lambda_i = \lambda \forall i$ .  $L = 133$  bytes,  $db = 320\mu s$  and  $R = 25$ .

Now we will analyse the cases when the loops in the system have different sampling rates. Firstly, if the loop of interest samples faster than the other loops in the network, from its point of view it will not compete constantly against all the other loops, but only when the other ones have packets to transmit, which happens with a lower frequency than the transmissions of the loop of interest. In this sense, to consider that the loop is always competing with a number of  $M - 1$  other loops is in fact an upper bound of performance, which is intuitively less tight than for the case when all loops sample at the same rate. This notion allows us to extend the results with the following corollary.

**COROLLARY 1.** *Consider an  $M$ -loop WNCS for which Assumptions 1, 2, 3 and 4 hold. If the WNCS operates over a CSMA-B network with every loop sampling at the same rate, or the loop of interest  $i$  has the largest sampling rate, then the AoI,  $\bar{\Delta}_i^{CSMA-B}$ , satisfies*

$$\bar{\Delta}_i^{CSMA-B} \leq \bar{\Delta}_i^{MCSMA-B} = \frac{1}{2\lambda_i} + E[\mathbf{s}_i^{MCSMA-B}] + E[\mathbf{w}_i^{MCSMA-B}] \quad (3.26)$$

REMARK 10. From the results in Lemma 6, Corollary 1 and the statements of Remark 9, it can be established that the optimal AoI for a specific loop is achieved when it samples faster than all the other loops. Because of this, the optimal AoI for all the loops will naturally occur when all the loops sample at the same rate,  $\lambda_i = \lambda$ . Figure 3.8 shows how  $\bar{\Delta}_i^{MCSMA-B}$  behaves with different number of loops sharing the network. Notice that, similar to CSMA-A, the optimal  $\lambda$  that minimizes AoI decreases as the number of loops grows. In Figure 3.9 we can appreciate how the sampling rate at which AoI is minimized varies with the number of loops competing to access the medium. For each packet size, the curve approximates like an exponential.

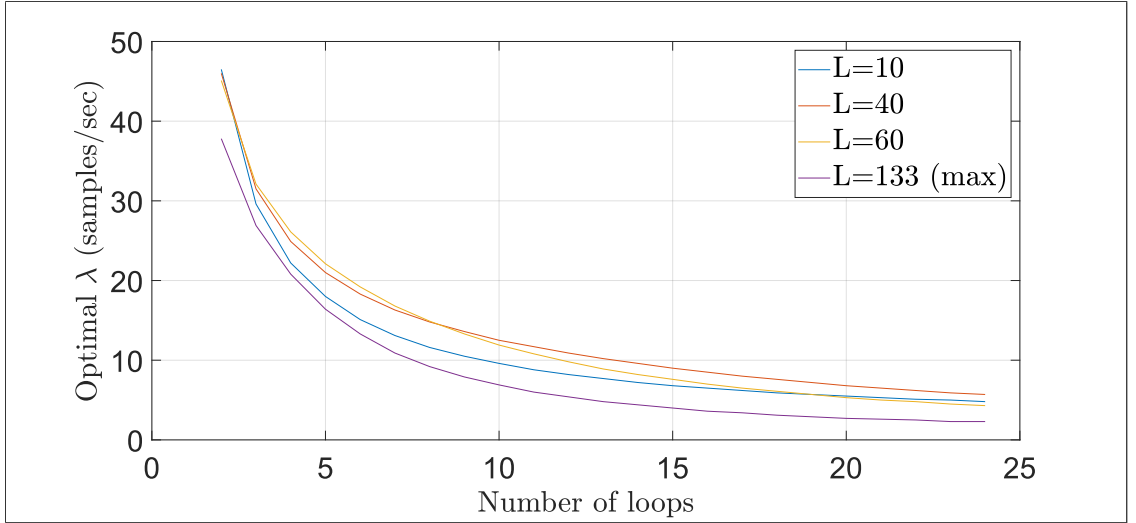


FIGURE 3.9. Optimal  $\lambda_i = \lambda$  for minimizing  $\bar{\Delta}_i^{CSMA-B}$  with different packet sizes.

$db = 320\mu s$ .

On the other hand, when a loop  $i$  is sampling at a lower rate than others in the same system, from its point of view it has to compete with all the loops at a frequency relatively much higher than the loops that sample faster, because the probability that those loops will have packets at the head of the queue when the slow loop starts transmission is much higher. Moreover, as they sample faster, it is more likely that the links from the other loops found themselves in time slots where the probability of sensing the channel free is higher. This is confirmed by analyzing Figure 3.6 and Figure 3.7 from (Buratti & Verdone, 2009),

where it is shown that a successful transmission is more probable in the first back-off slots. This is because, as explained, in the nBE CSMA/CA algorithm the back-off time assigned in the first stage is smaller, and therefore the links have more chances to sense the medium in a short time period. Then, as the links competing with the link of the loop of interest sample faster, the loop of interest will see the other loops transmitting their packets with a short service time with more frequency, and therefore, from its perspective, will see much more packets transmitted from the other loops than it would if those other loops sampled at the same rate than it, thus emulating the situation if the slower loop were competing with a number of loops larger than the actual number.

Therefore, we consider that this case is equivalent as if the slower link were competing with even more links than the actual ones. In this sense, we define a specific number of virtual links that will be added to the “real” ones, having in total  $X \geq M$  links, which are based on the difference between the sampling rate of the loops. To determine  $X$ , the sampling rates of all the other loops are compared to the one of interest, if a sampling rate is slower than the one from the loop of interest, it is considered as one link. On the contrary, the number of links considered is the division between the faster link and the link of interest, rounded to the greatest integer.

$$X_i = \sum_{j=1}^M \left\lceil \frac{\lambda_j}{\lambda_i} \right\rceil \quad (3.27)$$

REMARK 11. *In a WNCS with Assumptions 1,2, 3 and 4, if a loop  $i$  operates with a sampling rate that is not the fastest in the network, the service time  $s_i^{CSMA-B}$  can be bounded by  $E[s_i^{CSMA-B}] \leq E[s^{X_i CSMA-B}]$ . Following this, its waiting time and AoI are considered to be  $E[w_i^{CSMA-B}] \leq E[w^{X_i CSMA-B}]$  and  $\bar{\Delta}_i^{CSMA-B} \leq \bar{\Delta}_i^{X_i CSMA-B}$ , respectively.*

The results of this section and the values expressed in Remark 11 will be confirmed with the following simulations.

### 3.1.2.1. Simulations

In order to confirm the calculated model and the notions about the virtual links, we simulate the CSMA-B system in MATLAB. We consider a WNCS with 5 loops in which we calculate the average age of each link, simulated 3 times and 300 seconds for each time, for each value of  $\lambda$ , for a packet of 133 bytes. This means that in each simulation we generate between 300 (for  $\lambda = 1$ ) and 30000 (for  $\lambda = 100$ ) independent updates in each instance, 3 different times, which, once averaged, give us a statistically fair comparison. In Figure 3.10, we first consider the case where all loops sample at the same rate, therefore simulating the results of Theorem 3.

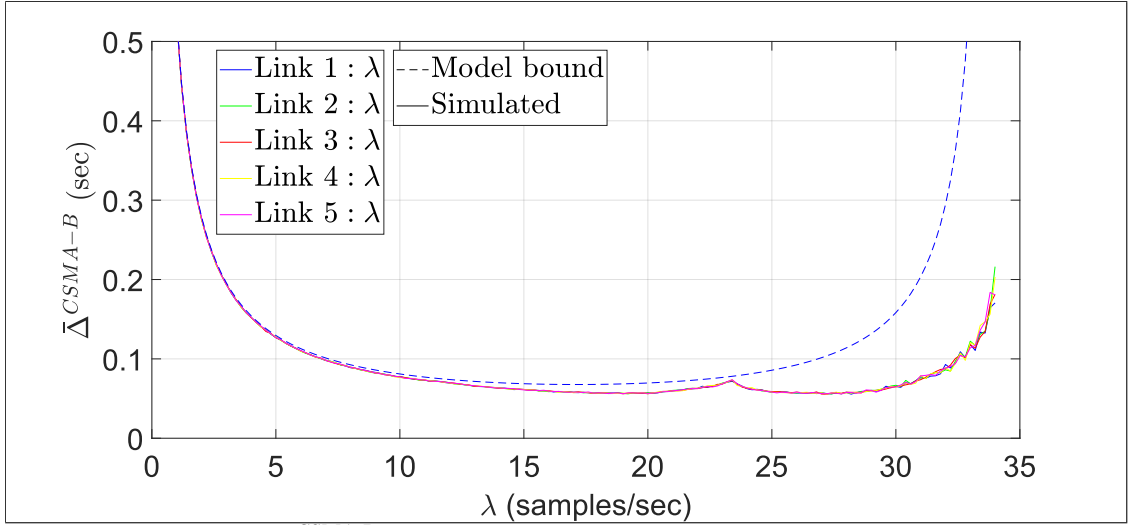


FIGURE 3.10.  $\bar{\Delta}_i^{\text{CSMA-B}}$  simulation and theoretical bound as a function of the sampling rate for all loops sampling at the same rate.  $L = 133$  bytes,  $db = 320\mu s$  and  $R = 15$ .

If we now consider the scenario with 4 loops sampling at  $\lambda$  and one slower loop sampling at  $\lambda/2$ , the loop sampling at the slower rate will be simulated with the equations of Remark 11, considering  $X = 9$  links in the network. Figure 3.11 show this scenario.

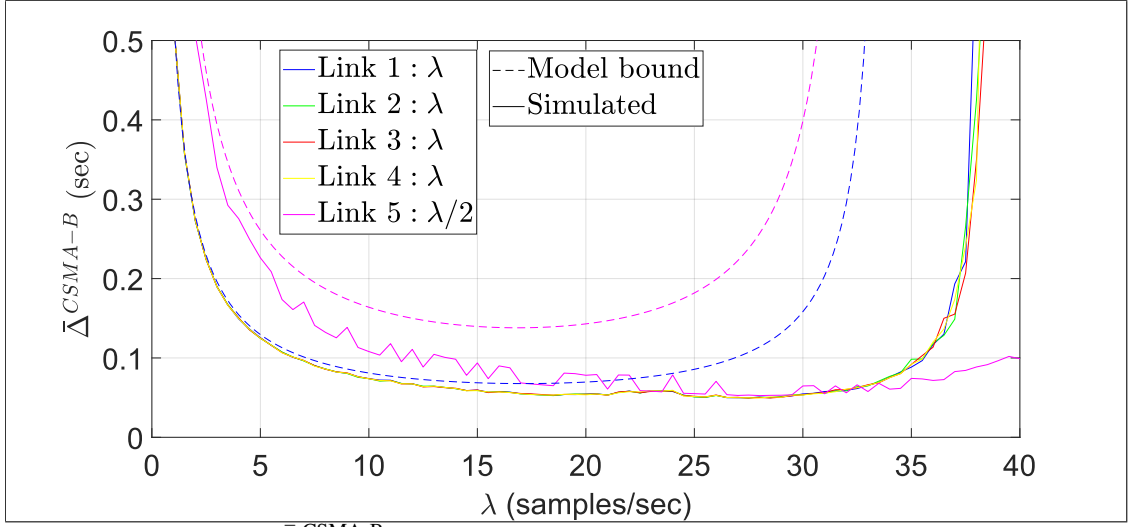


FIGURE 3.11.  $\bar{\Delta}_i^{CSMA-B}$  simulation and theoretical bound as a function of the sampling rate for different loops sampling at different rates.  $L = 133$  bytes,  $db = 320\mu s$  and  $R = 15$ .

Then, in Figure 3.12 we simulate the scenario where there are 4 loops sampling at  $\lambda/2$  and only one loop faster sampling at  $\lambda$ . In this case the slower loops consider that they are competing in a network of  $X = 6$  links in total.

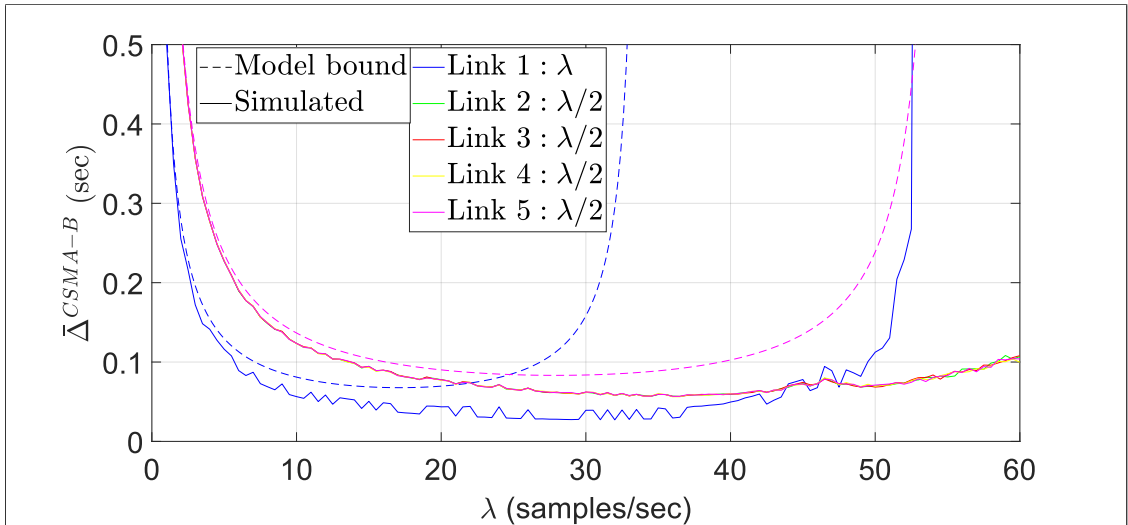


FIGURE 3.12.  $\bar{\Delta}_i^{CSMA-B}$  simulation and theoretical bound as a function of the sampling rate for different loops sampling at different rates.  $L = 133$  bytes,  $db = 320\mu s$  and  $R = 15$ .

The simulations confirm the results given, with the bound being tighter in the situations where the majority of the loops have the same sampling rate. This is specially important for analysing the optimal  $\lambda$  in terms of AoI, as a tight bound means that the optimal rate from the model is closer to the simulated one. In Figure 3.13 we consider the case with different loops having even more differentiated sampling rates.

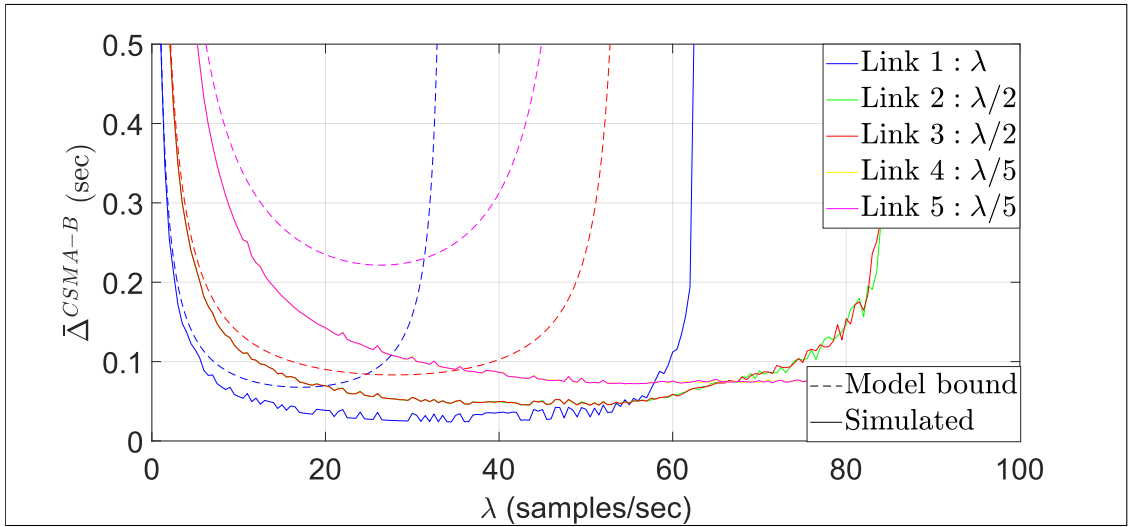


FIGURE 3.13.  $\bar{\Delta}_i^{\text{CSMA-B}}$  simulation and theoretical bound as a function of the sampling rate for different loops sampling at different rates.  $L = 133$  bytes,  $db = 320\mu s$  and  $R = 15$ .

Finally, in Figure 3.14 we simulate if every single loop had a different sample rate.

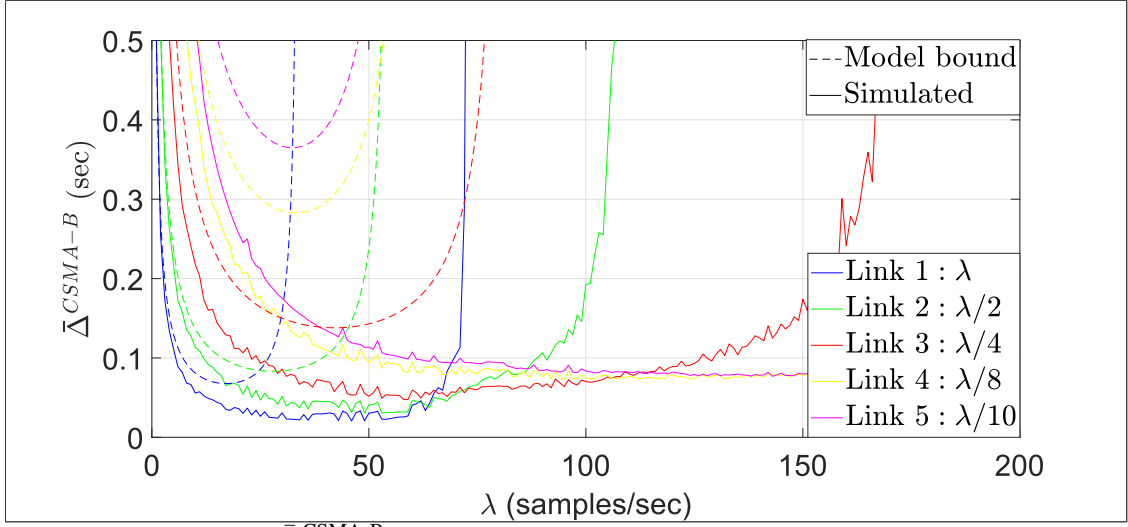


FIGURE 3.14.  $\bar{\Delta}_i^{CSMA-B}$  simulation and theoretical bound as a function of the sampling rate for different loops sampling at different rates.  $L = 133$  bytes,  $db = 320\mu s$  and  $R = 15$ .

With all the examples given, it can be seen then that the bound is always tighter, and therefore more useful for optimization purposes, when analysing the faster loops in the network. For the slower sampling loops, the bound is still always valid, but a precise evaluation of the optimal sampling rate cannot be done with the sufficient accuracy. In this sense, for design purposes, it is established that the faster the loop, the closer the actual AoI is to its model bound. Another aspect to take into account is that, when analysing the faster loop, while more loops also sample at the same faster rate, the tighter the bound will be. Therefore, it comes the conclusion that the case where the simulated AoI comes closer to the bound model is when all loops sample at the same rate, as seen in Figure 3.10.

### 3.2. AoI in TDMA-based WNCSs

In contrast to CSMA/CA, in TDMA a network coordinator allocates to each link a time slot based on a scheduling algorithm. This configuration allow us to make the following assumption, based on an idealized TDMA setup.

ASSUMPTION 5. *Every link in the WNCS operating over a TDMA network transmits its updates in a single packet in one time slot. Therefore, service times are deterministic and given by the duration of the time slot, i.e.,  $s_i^{TDMA} = T^{slot} \forall i$ . The waiting time  $w_i^{TDMA}$  measures the time elapsed between sampling and the start of the time slot allocated for transmitting the corresponding sample.*

REMARK 12. *Assumption 5 is reasonable for an ideal TDMA scheme with negligible synchronization time (Wang et al., 2016) and constant packet length for all the links  $\forall i$ . In practice, synchronization time is not negligible and the time slot incorporates offsets and acknowledges in order to ensure reliability (Khader, Willig, & Wolisz, 2011).*

In this work we base our scheduling strategy in the greedy policy from (Kadota, Sinha, Uysal-Biyikoglu, Singh, & Modiano, 2018), which seeks to minimize AoI. To continue the analysis, the following assumption is in order.

ASSUMPTION 6. *Every loop  $\mathcal{P}_i$  operating over a TDMA network is ruled by the same sampling rate  $\lambda \leq \frac{1}{T^{slot}}$ , with every link of every loop  $\mathcal{P}_i$  sampling at the same instant  $\tau_q$ . Moreover, the TDMA network is governed by a coordinator using a greedy scheduling policy, which schedules each time slot to the link with the highest age at the moment of the allocation, i.e., choosing link  $p = \operatorname{argmax}_h \Delta_h(t)$ , where  $\Delta(t) \in \mathbb{R}^N$  is the status age vector of the system at time  $t$ , to transmit its most updated sample. If there is no packet to transmit, the time passes and the slot is lost. When two or more links share the highest age at the moment of allocation, the scheduler randomly chooses one with equal probability. It is assumed that the coordinator is synchronized with the sampling process, meaning that the first slot is allocated at a sampling instant. Service time starts at the moment of the slot allocation, and when sampling and allocation occur at the same time, it is assumed that sampling occurs first.*

An important issue in the operation of the proposed TDMA network occurs when links have a queue of length larger than 1 when granted access to the channel. The algorithm in

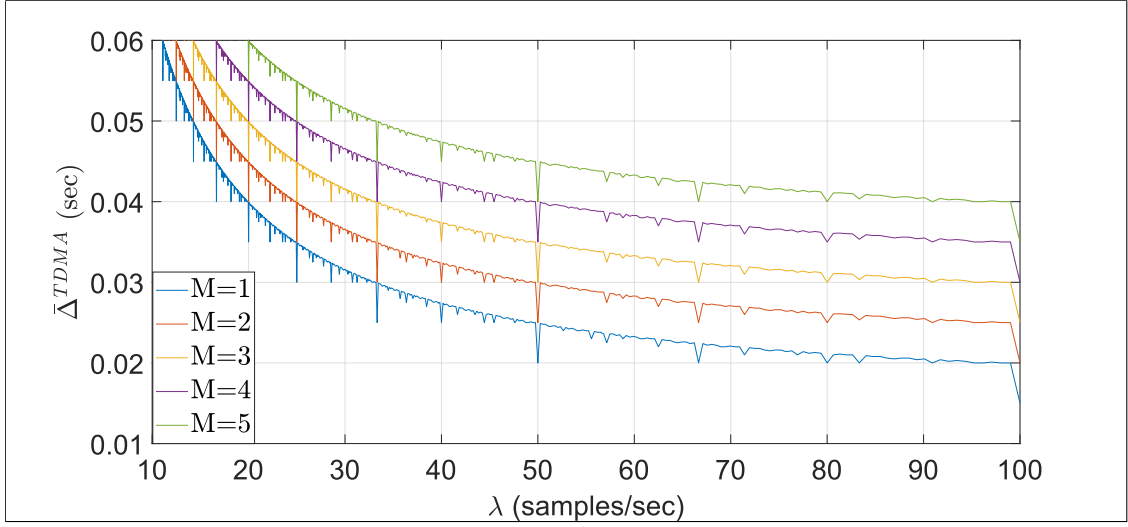


FIGURE 3.15.  $\bar{\Delta}^{TDMA}$  as a function of the sampling rate with for different number of links.  $T^{\text{slot}} = 10\text{ms}$ .

Assumption 6 establishes that only the most updated sample is transmitted. This motivates the definition of an effective sampling rate  $\lambda_{\text{eff}}$  for the TDMA network.

LEMMA 8. Consider an  $M$ -loop WNCS operating over a TDMA network for which Assumptions 1, 3, 5, and 6 hold. For every loop of the WNCS,  $\lambda_{\text{eff}}$  is the rate at which effectively transmitted messages are generated with average  $\bar{\lambda}_{\text{eff}}$  given by

$$\bar{\lambda}_{\text{eff}} = \begin{cases} \frac{1}{MT^{\text{slot}}} & \text{if } T^{\text{slot}} \leq \frac{1}{\lambda} < MT^{\text{slot}} \\ \lambda & \text{if } \frac{1}{\lambda} \geq MT^{\text{slot}}. \end{cases} \quad (3.28)$$

Moreover, the number of links  $J_q$  that transmit messages sampled at time instant  $\tau_q$  is given by

$$J_q = \begin{cases} \lceil \frac{q+1}{\lambda T^{\text{slot}}} \rceil - \lceil \frac{q}{\lambda T^{\text{slot}}} \rceil & \text{if } T^{\text{slot}} \leq \frac{1}{\lambda} < MT^{\text{slot}} \\ M & \text{if } \frac{1}{\lambda} \geq MT^{\text{slot}} \end{cases}, \quad (3.29)$$

which is periodic with period  $Q$ , where  $Q$  is the least integer such that  $\frac{Q}{\lambda}$  is a multiple of  $T^{\text{slot}}$

PROOF. From Assumption 6, all loops, hence all links, operate with sampling rate  $\lambda$  and sample at the same time  $\tau_q$ . For each  $\tau_q$ ,  $M$  samples are generated, yet  $J_q \leq M$  links transmit samples in the time slots allocated between  $\tau_q$  and  $\tau_{q+1}$ . Note that the total number of slots allocated until  $\tau_q$  can be obtained using the ceiling function as  $\lceil \frac{q}{\lambda T^{\text{slot}}} \rceil$ , therefore the slots available for transmission in the time period from  $\tau_q$  to  $\tau_{q+1}$  can be obtained by taking the total number of slots until  $\tau_{q+1}$ ,  $\lceil \frac{q+1}{\lambda T^{\text{slot}}} \rceil$ , and subtracting the number until  $\tau_q$ , hence the first case in (3.29) holds. Note then that when  $\frac{1}{\lambda} \geq MT^{\text{slot}}$ , the slots allocated between  $\tau_q$  and  $\tau_{q+1}$  are always equal or greater than  $M$ , then all the samples from  $\tau_q$  are transmitted, therefore  $J_q = M \forall q$  and  $\lambda_{\text{eff}} = \lambda$ , hence (3.29) holds. On the other hand, when  $T^{\text{slot}} < \frac{1}{\lambda} < MT^{\text{slot}}$ ,  $J_q$  is variable with  $q$ . Note that, since  $\frac{Q}{\lambda}$  is a multiple of  $T^{\text{slot}}$ , the values of  $J_q$  repeat from  $q = Q + 1$ , hence  $J_q$  is periodic with period  $Q$ . Due to the randomness incorporated when several links have the same age, each link  $p$  has its own  $\lambda_{p\text{eff}}$ , which is variable for each  $\tau_q$ . However, periodicity of  $J_q$  allows us to claim that on average each link transmits the same amount of samples,  $\frac{J_q}{M}$ , in a time period of length  $J_q T^{\text{slot}}$ . Then, the average effective sampling rate for every link is  $\bar{\lambda}_{\text{eff}} = \frac{1}{MT^{\text{slot}}}$ . Finally, when  $\frac{1}{\lambda} = T^{\text{slot}}$ , from (3.29)  $J_q = 1 \forall q$ , and the effective sampling interval is constant and equal to  $MT^{\text{slot}}$ , since in this case only one message sampled at  $\tau_q$  is transmitted.  $\square$

REMARK 13. Note that, although  $\lambda_{i\text{eff}}$  is not constant for  $T^{\text{slot}} < \frac{1}{\lambda} < MT^{\text{slot}}$ , given  $\lambda$  and  $T^{\text{slot}}$ , the sampling interval is still a value independent from  $\mathbf{d}_i$ . Therefore, Lemma 4 holds.

At this point, the main result of this section is stated.

THEOREM 4. Consider an  $M$ -loop WNCS operating over a TDMA network for which Assumptions 1, 3, 5, and 6 hold. The AoI of every link of a loop  $i$  is given by

$$\bar{\Delta}^{\text{TDMA}} = \frac{1}{2\bar{\lambda}_{\text{eff}}} + T^{\text{slot}} + \bar{w}^{\text{TDMA}}, \quad (3.30)$$

where  $\bar{w}^{TDMA}$  is the average waiting time, given by

$$\bar{w}^{TDMA} = \begin{cases} \frac{\sum_{q=1}^Q \sum_{j=1}^{J_q} [(j-1)T^{\text{slot}} + w_q]}{Q/\lambda T^{\text{slot}}} & \text{if } T^{\text{slot}} \leq \frac{1}{\lambda} < MT^{\text{slot}} \\ \frac{\sum_{q=1}^Q \sum_{j=1}^{J_q} [(j-1)T^{\text{slot}} + w_q]}{MQ} & \text{if } \frac{1}{\lambda} \geq MT^{\text{slot}}, \end{cases} \quad (3.31)$$

with  $w_q = rT^{\text{slot}} - \frac{q}{\lambda}$ , where, given  $q \in \mathbb{Z}_{\geq 1}$ ,  $r$  is the least integer such that  $rT^{\text{slot}} \geq \frac{q}{\lambda}$  holds.

PROOF. Assume, without loss of generality, that both the sampling and transmission process begin at the same time, then, because of Assumption 6, the first link allocated after  $\tau_0$  has waiting time equal to 0. For sampling time  $\tau_1$ ,  $J_1$  links, as defined in Lemma 8, are allocated and the waiting time for the first link is given by  $w_1$ . For the  $l$ th link allocated, with  $l \in \{2, \dots, J_1\}$ , the waiting time is given by  $w_1 + (l-1)T^{\text{slot}}$ . For subsequent sampling times  $\tau_q$ , with  $q \in \{2, \dots, Q\}$ , a similar argument can be applied, noting that the waiting time for the first link allocated is  $w_q$ . By definition of  $Q$  from Lemma 8,  $w_q$  is periodic with period  $Q$ , hence only  $Q$  possible values for  $w_q$  exist, which without loss of generality can be taken from the interval  $q \in \{1, \dots, Q\}$ . Therefore, for each sampling instant  $\tau_q$  there are  $J_q$  possible waiting times values that repeat periodically and in the long term are assigned to each link with equal probability. Periodicity of both  $J_q$  and  $w_q$ , and the fact that the order of the links allocated changes randomly at every instant where  $J_q > 1$ , which when  $M > 1$  happens at least once in the interval  $q = \{1, \dots, Q\}$ , allows us to define a long term average for the waiting time. To this end, note that the total waiting time for  $\tau_q$  is  $\sum_{j=1}^{J_q} [(j-1)T^{\text{slot}} + w_q]$ . Adding over  $q \in \{1, \dots, Q\}$  is dividing by the total number of transmitted messages,  $\frac{Q}{\lambda T^{\text{slot}}}$  for  $T^{\text{slot}} \leq \frac{1}{\lambda} < MT^{\text{slot}}$  and  $MQ$  for  $\frac{1}{\lambda} \geq MT^{\text{slot}}$ , results in (3.31). Invoking Lemma 4, considering Assumption 5 and Lemma 8, yields (3.30).  $\square$

As with the CSMA-A case, the AoI can be optimized by properly selecting the transmission rate  $\lambda$ . A concrete result in this regard is the following.

**PROPOSITION 2.** *Consider an  $M$ -loop WNCS operating over a TDMA network for which Assumptions 1, 3, 5, and 6 hold. The minimum AoI of a link of the WNCS is achieved*

at the optimal sampling rate  $\lambda^{*TDMA} = \frac{1}{T^{slot}}$ , and given by

$$\bar{\Delta}^{*TDMA} = \frac{MT^{slot}}{2} + T^{slot} = \frac{(M+2)T^{slot}}{2} \quad (3.32)$$

PROOF. When  $\frac{1}{\lambda} = T^{slot}$ , we have that  $J_q = 1 \forall q$ . Furthermore,  $Q = 1$  and hence  $w_q = 0 \forall q$ . Therefore,  $\bar{w}^{TDMA} = 0$  for all transmitted packets. Using (3.30) considering that from Lemma 8,  $\bar{\lambda}_{eff} = \frac{1}{MT^{slot}}$  results in (3.32).  $\square$

REMARK 14. Lemma 2 and Corollary 3 in (Kadota et al., 2018) state that in a system where Assumptions 3 and 6 hold, when sampling at the optimal rate, the greedy scheduling policy described drives the system into a steady-state, where slot allocations follow a Round Robin scheme, and the number of time slots elapsed since the last update of the respective link from loop  $i$  is constant and minimal  $\forall i$ , thus AoI is optimal for every link in the system. This is consistent with the result in Proposition 2.

REMARK 15. Due to hardware constraints, it is not possible to ensure that sensors sample with a period exactly equal to the time slot. Hence, in practice (3.32) is an approximation. Figure 3.15 shows (3.30) for different sampling rates.

### 3.2.1. Simulations

We simulate the TDMA system in MATLAB in order to confirm the results. We consider a WNCS with 5 loops in which we calculate the average age of each link, simulated two times and 10 seconds for each time, for each value of  $\lambda$ . These parameters allow us to simulate from 10 to 1000 updates for each link in each instance, which, once averaged, gives us a fair approximation to the steady-state information. We consider the transfer rates of the IEEE802.15.14 protocol, defining a  $T^{slot} = 10ms$  for TDMA, similar to the Wireless HART protocol (Wang et al., 2016).

For the TDMA network, in Figure 3.16 it can be noticed that, as expected, all the links closely follow the model for every possible value of  $\lambda$ . For some specific values, some links may have some peaks or be out of sync with the others because of numerical approximations and the non-infinite time horizon for which the average age is calculated.

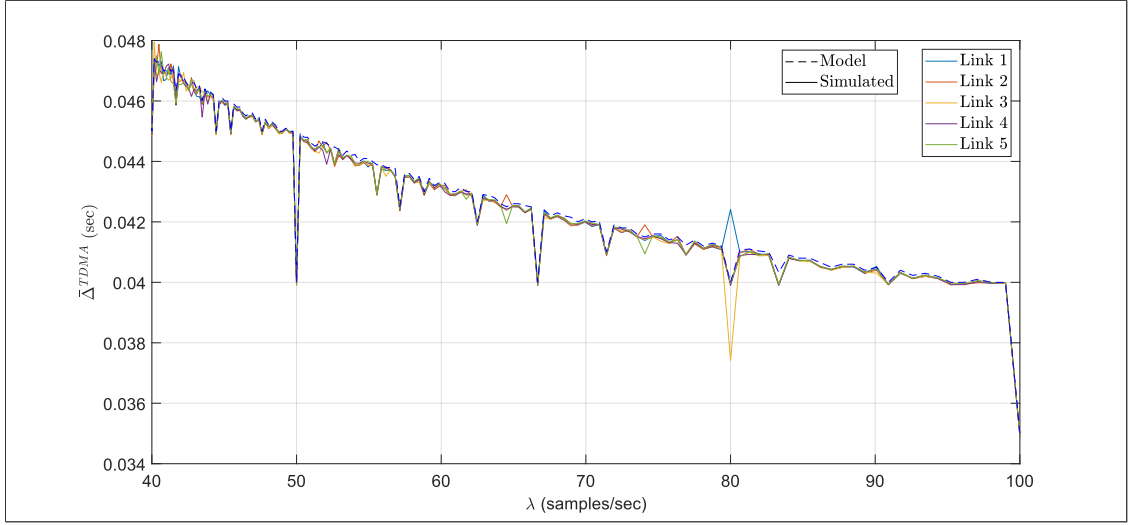


FIGURE 3.16. Simulation of  $\bar{\Delta}^{TDMA}$  with  $M = 5$  and  $T^{\text{slot}} = 10ms$  following greedy algorithm ( $\lambda_i = \lambda$ ).

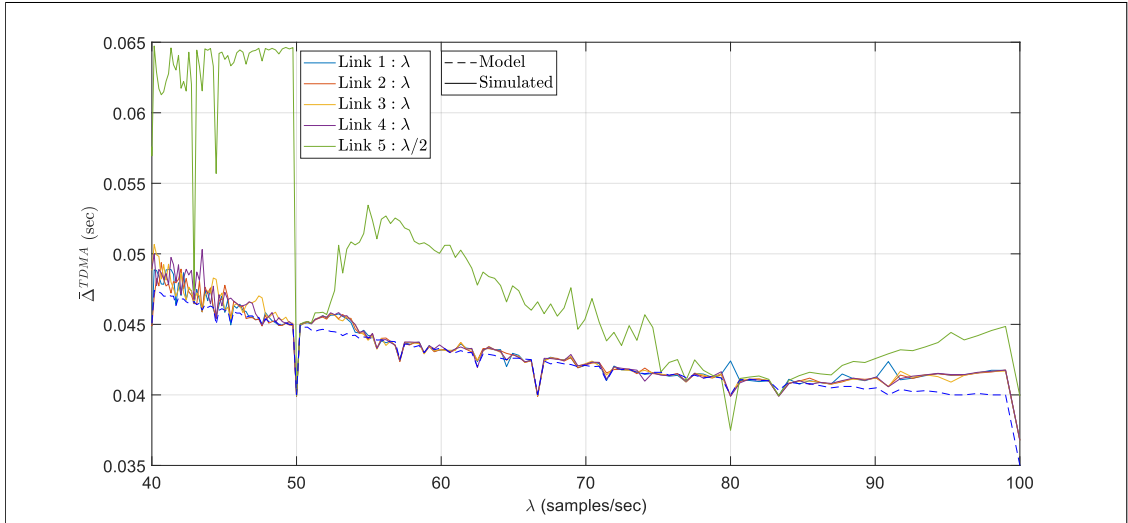


FIGURE 3.17. Simulation of  $\bar{\Delta}^{TDMA}$  with  $M = 5$  and  $T^{\text{slot}} = 10ms$  following greedy algorithm but with four links sampling with  $\lambda$  and one with  $\lambda/2$ .

This is related to the problems stated in Remark 15. In order to confirm that the algorithm described in Assumption 6 optimizes the AoI, in Figure 3.17 we break the rule that forces all the links to sample at the same rate, maintaining the greedy scheduling and  $T^{\text{slot}} = 10ms$ , but with one link sampling at  $\lambda/2$ . We see that not only the slower link raises its Age, but it also affects all the other links in comparison with the model calculated with

Assumption 6 at some specific values of  $\lambda$ , because of the slots that are assigned to the slower link (due to having the biggest AoI) and lost because there are not new packets, and then assigned again, causing the transmission of the slower link sometimes to take 2 slots at certain values of  $\lambda$ . It can also be seen that the links that sample at  $\lambda$  do not achieve the same optimal age when sampling at  $\lambda = \frac{1}{T^{\text{slot}}}$ .

## 4. PEAK AGE AND WNCS STABILITY

In this chapter we derive expressions for the Peak Age  $\hat{\Delta}_i$  of the links from different loops inserted in a WNCS as the one described in the problem statement section under the same three MAC schemes analysed for the AoI. Also, we establish the relationship between the Peak Age related to the communication protocol used and the stability constraints determined for each control loop in order to ensure mean-square stability.

### 4.1. Peak Age and OPPA on stability in CSMA/CA-based WNCSs

From the results in the previous sections, we can define a bound on the Peak Age of the links transmitting in a CSMA/CA network as follows

LEMMA 9. *Consider an  $M$ -loop WNCS operating over a CSMA/CA network for which Assumptions 1, 2, 3 and 4 hold. The network can be either type “A” for loops sampling with any rate or “B” for loops sampling with the same rate. The Peak Age of the  $k$ -th transmitted message of loop  $i$  is bounded by the Peak Age of a message transmitted in the context of the same network with a constant number of links transmitting.*

$$\hat{\Delta}_{i_k}^{CSMA} \leq \hat{\Delta}_{i_k}^{MCSMA} = \mathbf{s}_{\mathbf{k}-1}^{MCSMA} + \mathbf{w}_{\mathbf{k}-1}^{MCSMA} + \frac{1}{\lambda_i} \quad (4.1)$$

Moreover, the same holds for the average Peak Age, or Peak Age of Information (PAoI).

$$\bar{\Delta}_i^{CSMA} \leq \bar{\Delta}_i^{MCSMA} = E[\mathbf{s}^{MCSMA}] + E[\mathbf{w}^{MCSMA}] + \frac{1}{\lambda_i} \quad (4.2)$$

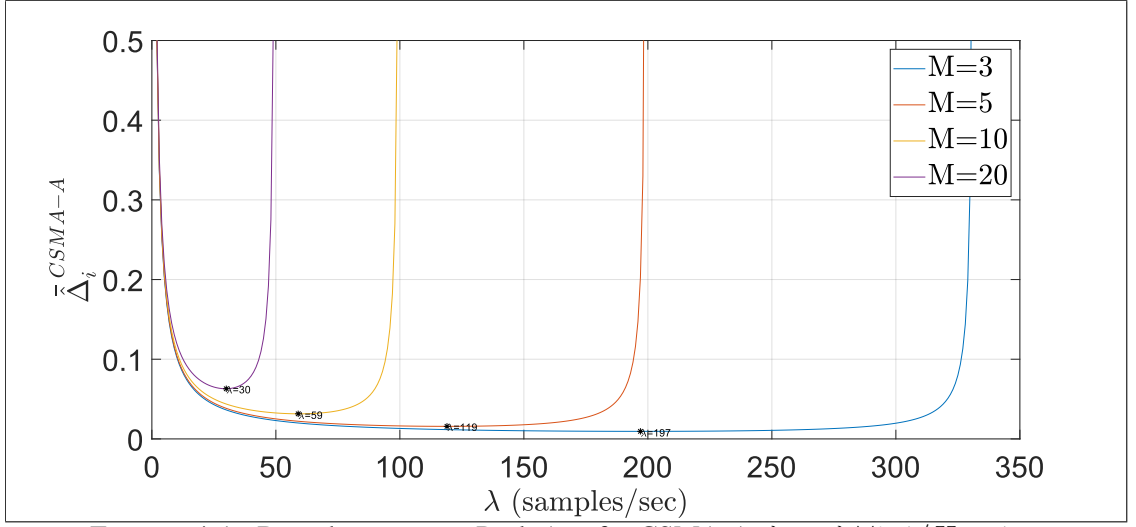


FIGURE 4.1. Bound on average Peak Age for CSMA-A.  $\lambda_i = \lambda \forall i$ .  $1/H = 1ms$  ( $L=2304$  bytes) and  $R = 14H$ . Black points denote the minimum  $\bar{\Delta}_i^{CSMA}$  at the optimal  $\lambda$ .

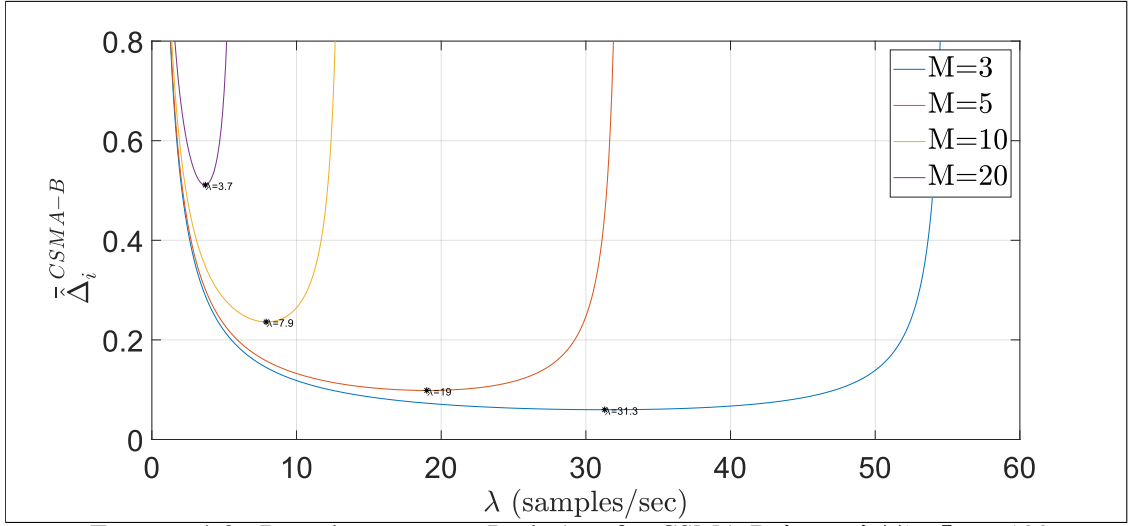


FIGURE 4.2. Bound on average Peak Age for CSMA-B  $\lambda_i = \lambda \forall i$ .  $L = 133$  bytes,  $db = 320\mu s$  and  $R = 20$ . Black points denote the minimum  $\bar{\Delta}_i^{CSMA}$  at the optimal  $\lambda$ .

Figures 4.1 and 4.2 show how the average Peak Age  $\bar{\Delta}_i$  behaves in each CSMA/CA protocol when  $\lambda_i = \lambda$ . It is interesting to notice that the values of  $\lambda$  that optimize the

average Peak Age are not the same than the ones that optimize the AoI. This occurs because, although the equations of average Peak Age and AoI only differ in  $\frac{1}{\lambda}$ ,  $\lambda$  is also included in the waiting time, inside the delay term.

However, the interest of studying the Peak Age is its relation with the stability constraints of the loops inserted in the WNCS. As CSMA/CA is based on a probability distribution of the service time, stability in the sense of (Heemels et al., 2010) or (Liu et al., 2015) can not be guaranteed, since with positive probability a transmission may violate the MAD constraint, specially in the case when the MAD value is inferior to the maximum possible service time of the CSMA/CA transmission.

Now, we are interested in calculating the outage probability of the peak age surpassing  $\tau_{MATI}^i + \tau_{MAD}^i$ , and analysing in which situations this probability is minimised, therefore improving the probabilities of the system being stable. For this, we propose the following

**PROPOSITION 3.** *Consider an  $M$ -loop WNCS operating over a CSMA network for which Assumptions 1,2, 3 and 4 hold. The network can be either type “A” for loops sampling with any rate or “B” for loops sampling at the same rate. The probability of the WNCS exceeding the stability constraints  $\tau_{MATI}^i + \tau_{MAD}^i$  given for each loop is bounded by*

$$P \left[ \hat{\Delta}_{i\infty}^{CSMA} \geq \tau_{MATI}^i + \tau_{MAD}^i \right] \leq e^{-\theta^*(\tau_{MATI}^i + \tau_{MAD}^i - \frac{1}{\lambda_i})} \cdot m(\theta^*) \quad (4.3)$$

where  $\theta^*$  is given by 2.26 and  $m(\theta^*)$  is the MGF of the service time distribution evaluated in  $\theta^*$ , 3.20 in CSMA-B.

**PROOF.** Combining Lemma 3 and 5, it can be established that  $P[\hat{\Delta}_{i\infty}^{MCSMA} \geq \tau_{MATI}^i + \tau_{MAD}^i] \leq e^{-\theta^*(\tau_{MATI}^i + \tau_{MAD}^i - \frac{1}{\lambda_i})}$ , where  $\hat{\Delta}_{i\infty}^{MCSMA}$  refers to the case of a CSMA network with  $M$  loops constantly contending for the channel and a  $\theta^*$  calculated with the MGF from 3.20. Then, from Lemma 9,  $\hat{\Delta}_{i\infty}^{CSMA} \leq \hat{\Delta}_{i\infty}^{MCSMA}$ , which results in  $P[\hat{\Delta}_{i\infty}^{CSMA} \geq \tau_{MATI}^i + \tau_{MAD}^i] \leq P[\hat{\Delta}_{i\infty}^{MCSMA} \geq \tau_{MATI}^i + \tau_{MAD}^i]$ , giving the result.  $\square$

#### 4.1.1. OPPA in DCF CSMA/CA

In systems operating over CSMA-A, the bound from Proposition 3 can be tightened if we extend the result from Lemma 3 using Lemma 2 from (Seo & Choi, 2019), which exploits the continuous service time distribution to stretch the bound.

**PROPOSITION 4.** *Consider an  $M$ -loop WNCS operating over a CSMA-A network for which Assumptions 1, 2, 3 and 4 hold. The probability of the WNCS exceeding the stability constraints  $\tau_{MATI}^i + \tau_{MAD}^i$  given for each loop is bounded by*

$$P \left[ \hat{\Delta}_{i\infty}^{CSMA-A} \geq \tau_{MATI}^i + \tau_{MAD}^i \right] \leq e^{-\theta^* (\tau_{MATI}^i + \tau_{MAD}^i - \frac{1}{\lambda_i})} \quad (4.4)$$

where  $\theta^*$  is given by 2.26 for the MGF of the service time distribution evaluated in  $\theta^*$ ,  $m(\theta^*)^{MCSMA-A} = \frac{\mu}{\mu - \theta^*}$ , where  $\mu$  is the service rate  $\mu = 1/E[s^{MCSMA-A}]$ .

**PROOF.** From the result of Proposition 3, we use the complete result of Lemma 2 from (Seo & Choi, 2019), in whose proof it is stated that, if  $m(\theta^*)$  is independent of  $x - b$ , we have that  $P \left[ \hat{\Delta}_{i\infty} \geq x \right] \approx ce^{\theta^*(x-b)}$ , where  $c$  is a constant.  $c = 1$  is set due to  $P \left[ \hat{\Delta}_{i\infty} \geq b \right] \leq 1$ . Considering the threshold  $x$  to be  $\tau_{MATI}^i + \tau_{MAD}^i$  gives the result.  $\square$

Therefore, we now need to find a  $\theta^* > 0$  from the following equation

$$\frac{\ln(\frac{\mu}{\mu - \theta})}{\theta} + \frac{-\theta}{\lambda\theta} = 0 \quad (4.5)$$

As stated in (Seo & Choi, 2019), there is not a closed expression that solves  $\theta^* > 0$  for this distribution, so it must again be numerically solved for each different sampling rate and number of loops in the system. Figure 4.3 shows how the same threshold has different OPPA according to the number of loops in the system.

Figure 4.4 shows the form of the curve for the probability of trespassing the stability constraints with different number of loops in the system is different depending on the  $\tau_{MATI}^i + \tau_{MAD}^i$  value.

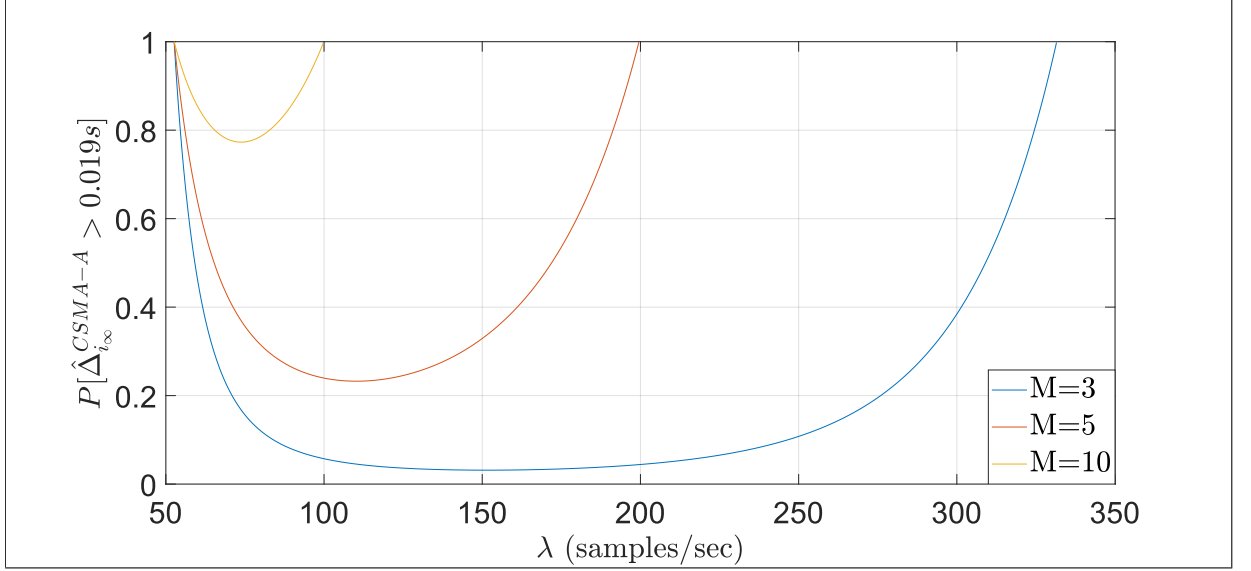


FIGURE 4.3. Bound on  $P \left[ \hat{\Delta}_{i\infty}^{CSMA-A} \geq \tau_{MATI}^i + \tau_{MAD}^i \right]$  as a function of the sampling rate for different number of loops in the system.  $\lambda_i = \lambda \forall i$ .  $1/H = 1ms$  ( $L=2304$  bytes) and  $R = 14H$ .  $\tau_{MATI}^i + \tau_{MAD}^i = 0.019s$ .

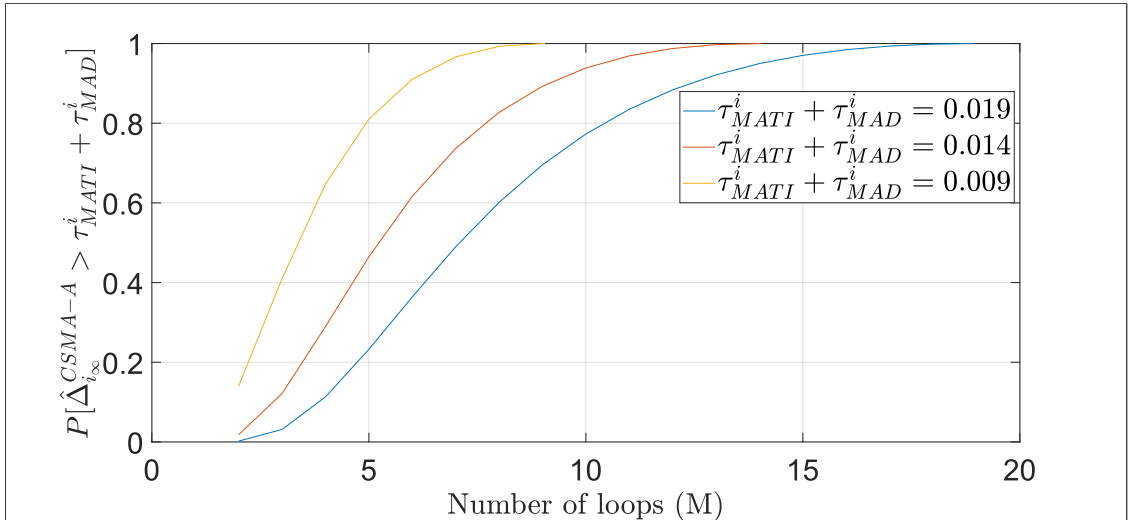


FIGURE 4.4. Bound on  $P \left[ \hat{\Delta}_{i\infty}^{CSMA-A} \geq \tau_{MATI}^i + \tau_{MAD}^i \right]$  on the optimal sampling rate for OPMA minimization with CSMA-A  $\lambda_i = \lambda \forall i$ .  $1/H = 1ms$  ( $L=2304$  bytes) and  $R = 14H$ .

#### 4.1.1.1. Simulations

In order to confirm the calculated model, we simulate the both Peak Age and OPPA for the DCF CSMA/CA protocol on MATLAB. We consider a WNCS with 5 loops in which we calculate the average age of each link, simulated in the same conditions than the CSMA-A AoI, with a channel holding time of  $1/H = 4.256ms$ . We consider different scenarios of links sampling at different rates.

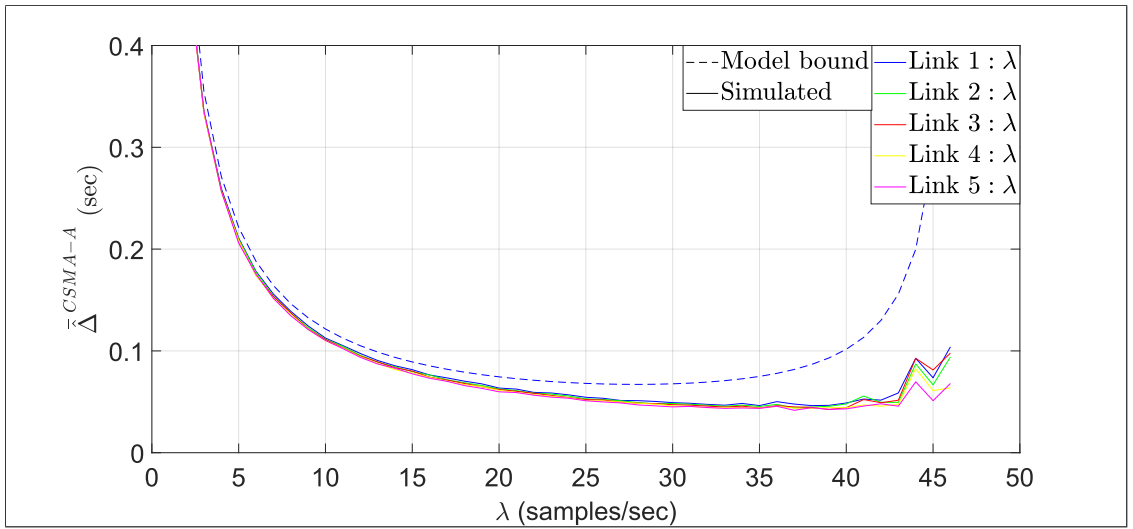


FIGURE 4.5. Simulation of  $\bar{\Delta}^{CSMA-A}$  with  $M = 5$  and  $1/H = 4.256ms$ .  $\lambda_i = \lambda$ .

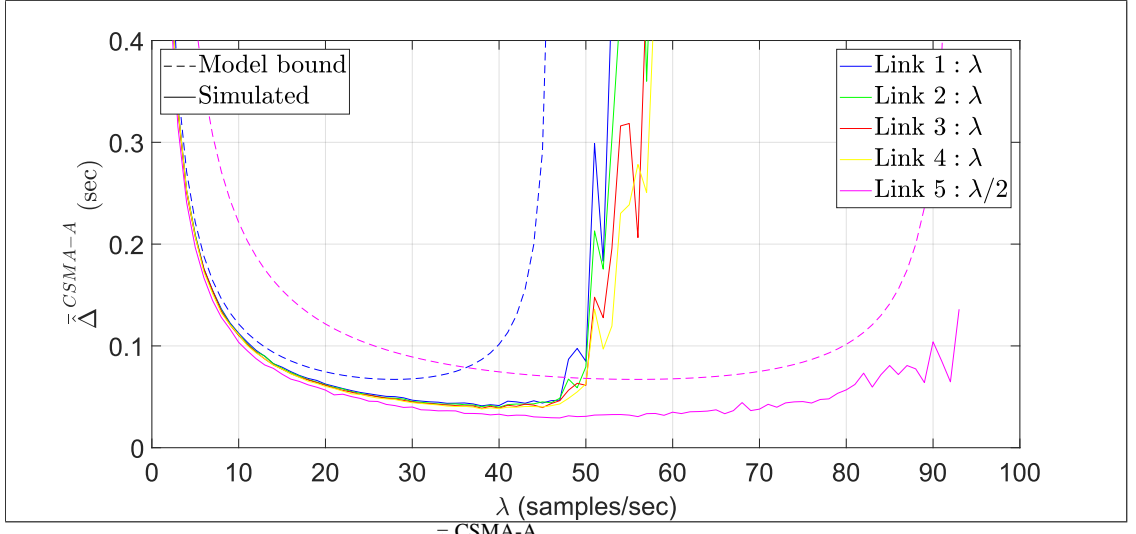


FIGURE 4.6. Simulation of  $\bar{\Delta}^{CSMA-A}$  with  $M = 5$  and  $1/H = 4.256ms$ . Four links sampling with  $\lambda$  and one with  $\lambda/2$ .

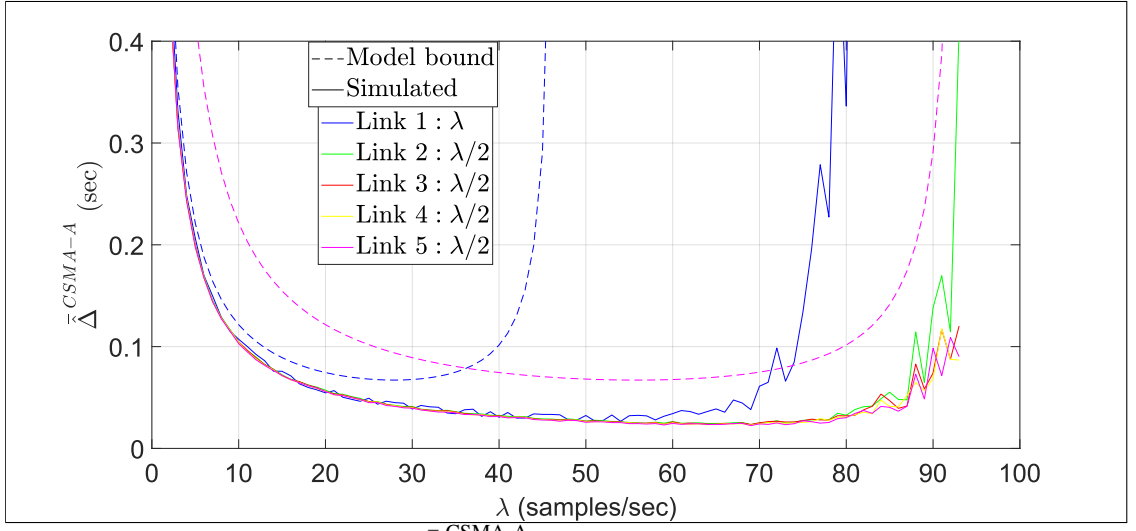


FIGURE 4.7. Simulation of  $\bar{\Delta}^{CSMA-A}$  with  $M = 5$  and  $1/H = 4.256ms$ . One link sampling with  $\lambda$  and four with  $\lambda/2$ .

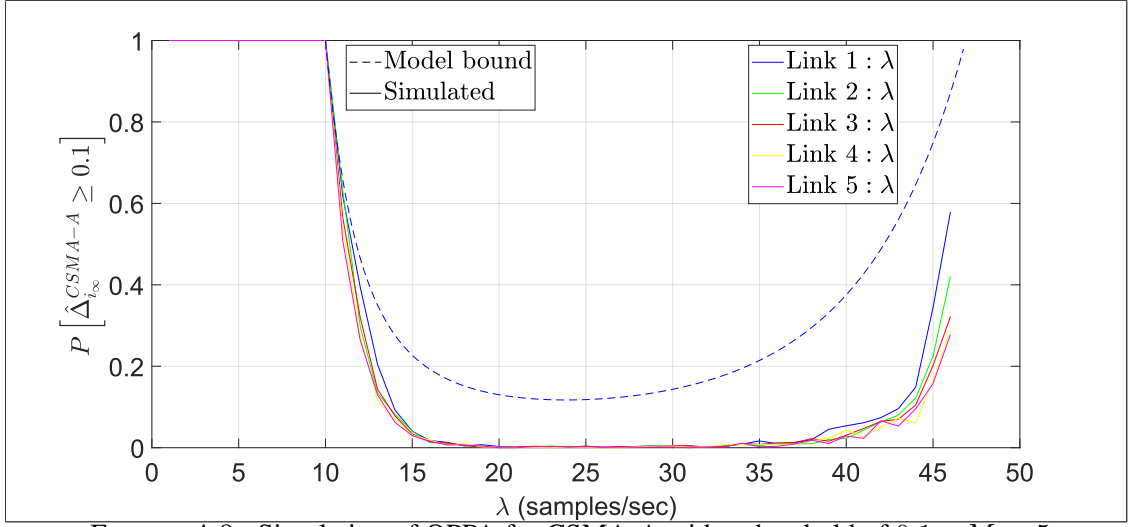


FIGURE 4.8. Simulation of OPPA for CSMA-A with a threshold of  $0.1s$ .  $M = 5$  and  $1/H = 4.256ms$ .  $\lambda_i = \lambda$ .

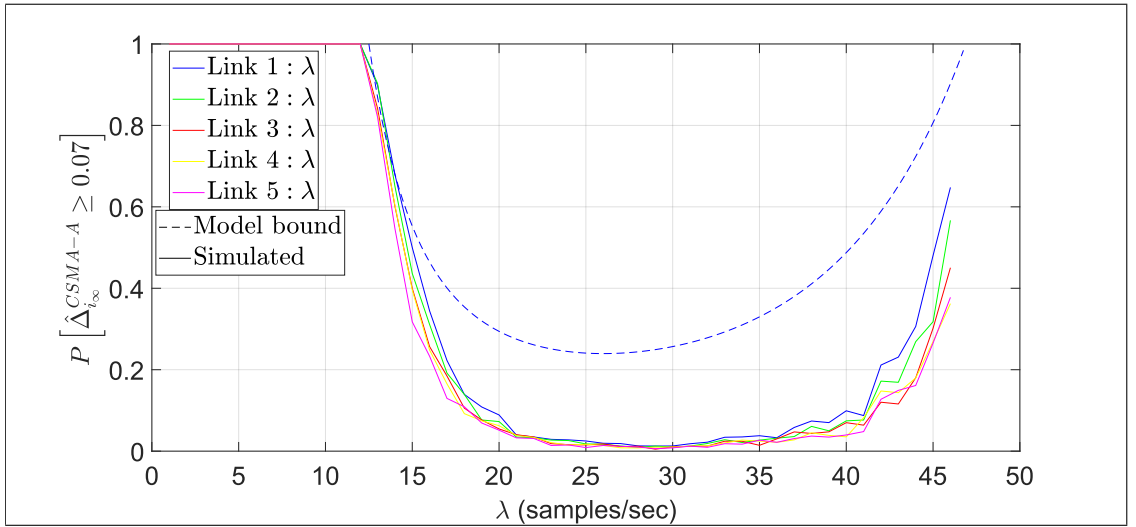


FIGURE 4.9. Simulation of OPPA for CSMA-A with a threshold of  $0.07s$ .  $M = 5$  and  $1/H = 4.256ms$ .  $\lambda_i = \lambda$ .

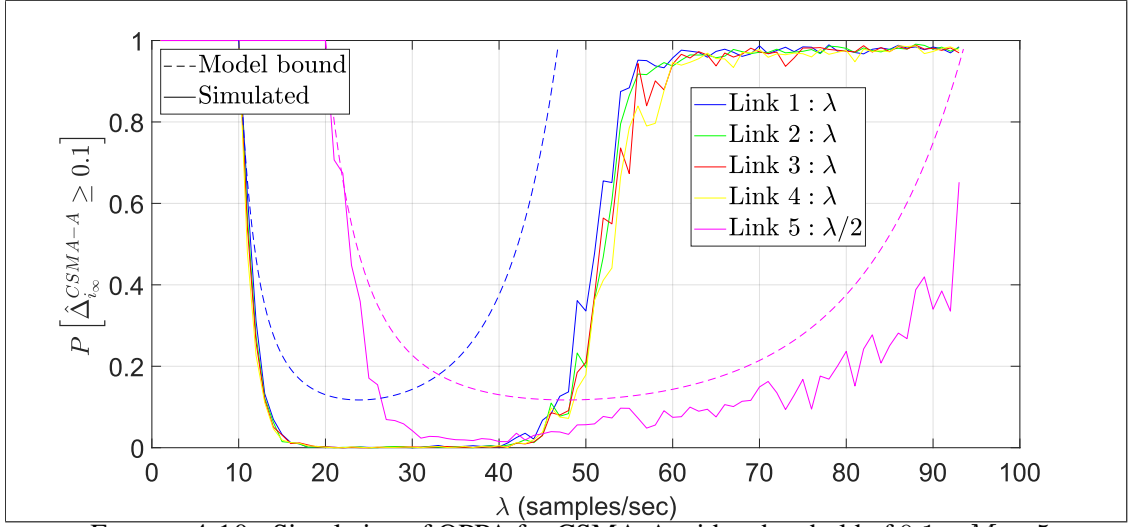


FIGURE 4.10. Simulation of OPPA for CSMA-A with a threshold of  $0.1s$ .  $M = 5$  and  $1/H = 4.256ms$ . Four links sampling with  $\lambda$  and one with  $\lambda/2$ .

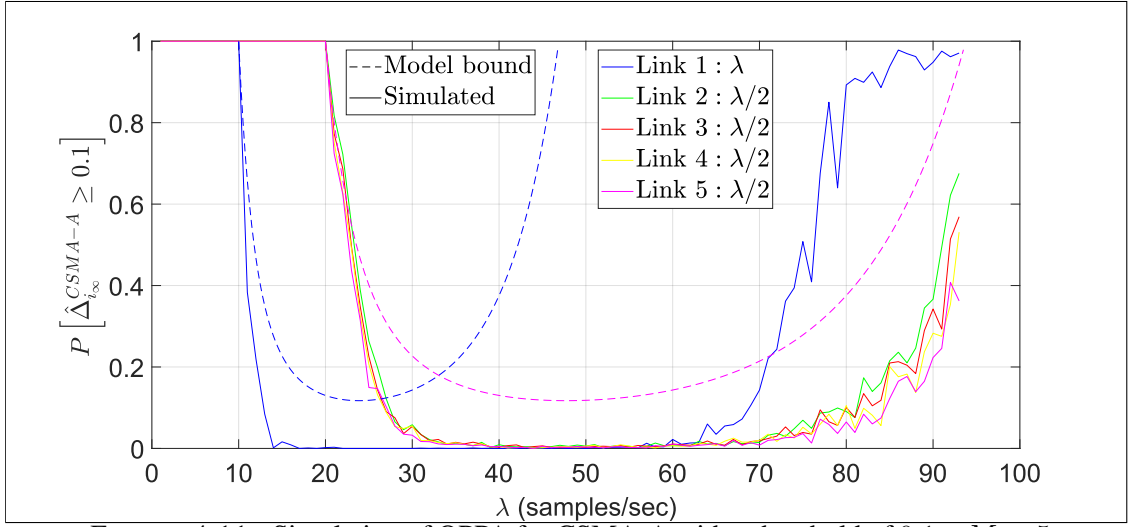


FIGURE 4.11. Simulation of OPPA for CSMA-A with a threshold of  $0.1s$ .  $M = 5$  and  $1/H = 4.256ms$ . One link sampling with  $\lambda$  and four with  $\lambda/2$ .

We can appreciate that the bound is respected for every case, confirming the results. For the PAoI, it can be seen that the bound is tighter when all the loops sample at the same rate (Figure 4.5). When four loops sample at  $\lambda$  and one at  $\lambda/2$  (Figure 4.6), the bound is still tight for the first four loops but looser for the loop sampling at a slower rate. The

opposite happens in Figure 4.7, where the bound for the faster loop is less tight than for the loops sampling at  $\lambda/2$ . This is similar to what happened when simulating AoI, the more loops sampling at the same rate, the tighter the bound is.

When simulating OPPA, another parameter is to be considered, which is the OPPA threshold. In Figures 4.8 and 4.9 it can be seen that, while both cases have all the loops sampling at the same rate, the bound is slightly tighter when the threshold is higher. However, it is important to remark that the bound is respected for all the scenarios analysed. In Figures 4.10 and 4.11 it can be seen that, when loops sample at different sampling rates, the bound is tighter for the case when most loops sample at a relatively fast rate.

#### 4.1.2. OPPA in nBE CSMA/CA

In the CSMA-B-based systems, we are not able to use Proposition 4, because the service time distribution is calculated from a discrete model, making the last approximation of Lemma 2 in (Seo & Choi, 2019) inaccurate, as the probability  $P\left[\hat{\Delta}_{i\infty} \geq x\right] = e^{\theta^*(x-b)} < 1$  when  $x - b > 0$ , which is not true due to the existence of a minimum strictly positive transmission time. Therefore, we apply Proposition 3. We calculate  $\theta^*$  from 2.26, where it is required to use the moment generating function of the protocol, which is given in 3.20. Therefore, we need to find a  $\theta^* > 0$  from the following equation

$$\frac{\ln(m^{\text{MCSMA-B}}(\theta))}{\theta} + \frac{-1}{\lambda} = 0 \quad (4.6)$$

As stated in (Seo & Choi, 2019), there is not a closed expression that solves  $\theta^* > 0$ , so it must be numerically solved for each different sampling period and number of loops in the system. We present the following figures to illustrate the results.

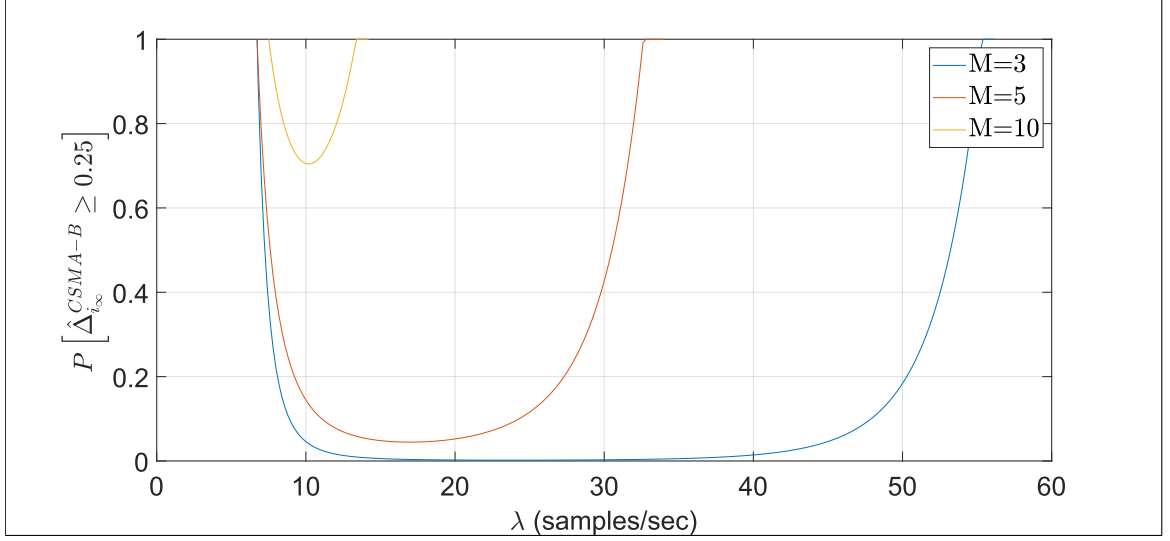


FIGURE 4.12. Bound on  $P \left[ \hat{\Delta}_{i_\infty}^{CSMA-B} \geq \tau_{MATI}^i + \tau_{MAD}^i \right]$  as a function of the sampling rate for different number of links in the system.  $\lambda_i = \lambda \forall i$ .  $L = 133$  bytes,  $db = 320\mu s$  and  $R = 20$ .  $\tau_{MATI}^i + \tau_{MAD}^i = 0.25s$ .

Figure 4.12 shows the OPPA bound for WNCSS with different number of loops. In order to make the CSMA-B bound tighter, we propose the following based on the arguments of Lemma 2 in (Seo & Choi, 2019)

**PROPOSITION 5.** *Consider an  $M$ -loop WNCSS operating over a CSMA-B network for which Assumptions 1, 2, 3 and 4 hold. The probability of the WNCSS exceeding the stability constraints  $\tau_{MATI}^i + \tau_{MAD}^i$  given for each loop is bounded by*

$$P \left[ \hat{\Delta}_{i_\infty}^{CSMA-B} \geq \tau_{MATI}^i + \tau_{MAD}^i \right] \leq e^{-\theta^* (\tau_{MATI}^i + \tau_{MAD}^i - \frac{1}{\lambda_i} - (D+1)0.00032)} \quad (4.7)$$

where  $\theta^*$  is given by 2.26 for the MGF of the service time distribution evaluated in  $\theta^*$ , given by 3.20.

**PROOF.** From the result of Proposition 3, and from the proof of Lemma 2 in (Seo & Choi, 2019), it is stated that, if  $m(\theta^*)$  is independent of  $x - b$ , we have that  $P \left[ \hat{\Delta}_{i_\infty} \geq x \right] \approx ce^{\theta^*(x-b)}$ , where  $c$  is a constant. Considering that the service time distribution of CSMA-B

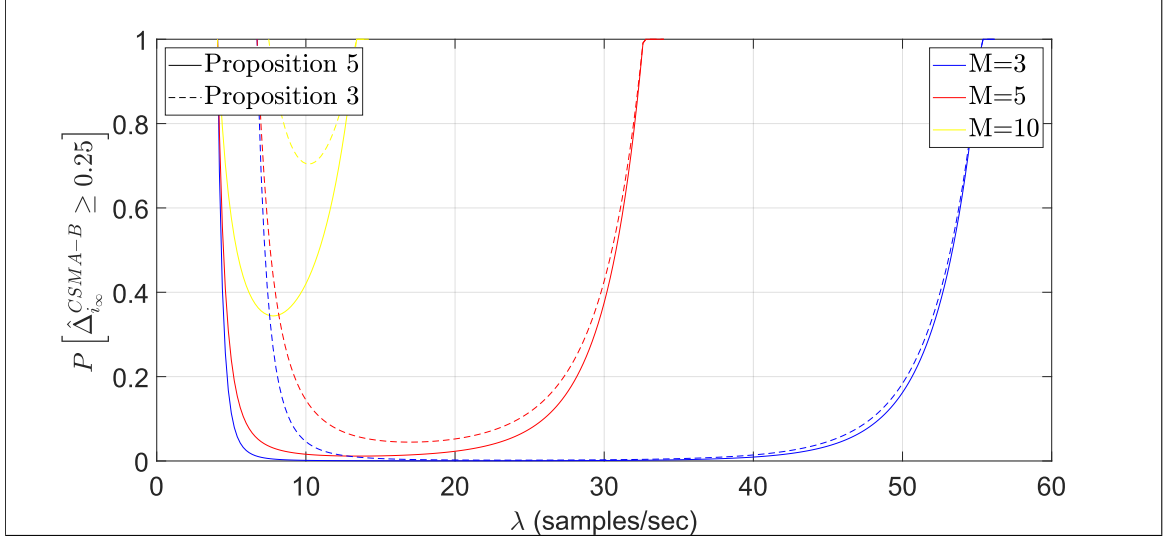


FIGURE 4.13. Bound on  $P \left[ \hat{\Delta}_{i\infty}^{\text{CSMA-B}} \geq \tau_{\text{MATI}}^i + \tau_{\text{MAD}}^i \right]$  as a function of the sampling rate for different number of links in the system.  $\lambda_i = \lambda \forall i$ .  $L = 133$  bytes,  $db = 320\mu s$  and  $R = 20$ .  $\tau_{\text{MATI}}^i + \tau_{\text{MAD}}^i = 0.25s$ .

consists in discrete slots, the minimum possible transmission occurs at least  $(D + 1) \cdot t$  slots after the protocol is initiated, because the transmission lasts for  $D$  slots and at least one slot must be the node sensing the channel. As each slot consists of  $320\mu s$ , the constant  $c = e^{\theta^*((D+1)0.00032)}$  is set due to  $P \left[ \hat{\Delta}_{i\infty} \geq x \right] \geq 1 \quad \forall b \geq x - (D + 1)0.00032$ . Taking the threshold  $x$  as  $\tau_{\text{MATI}}^i + \tau_{\text{MAD}}^i$  gives the result.  $\square$

Figure 4.13 compares how the bound tightens with the new proposition. It can be seen that the bound from Proposition 5 differentiates from the one of Proposition 3 more as the number of loops in the system grows, showing that the new proposition gives new information and therefore is useful.

Figure 4.14 shows the form of the curve for the probability of trespassing the stability constraints with different number of loops in the system is different depending on the  $\tau_{\text{MATI}}^i + \tau_{\text{MAD}}^i$  value. As the OPPA is calculated with a numerical method, there is not a direct formula that can be used.

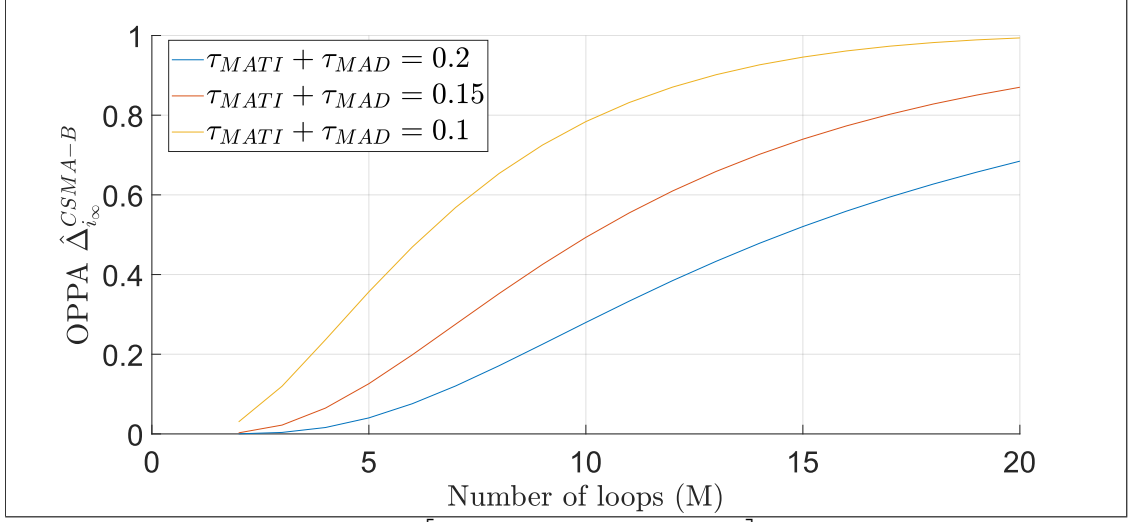


FIGURE 4.14. Bound on  $P \left[ \hat{\Delta}_{i_\infty}^{CSMA-B} \geq \tau_{MATI}^i + \tau_{MAD}^i \right]$  on the optimal sampling rate for OPPA minimization with  $\lambda_i = \lambda \forall i$  using Proposition 5.  $L = 10$  bytes,  $db = 320\mu s$  and  $R = 20$ .

REMARK 16. Both Lemma 9 and Proposition 5 can be extended when considering CSMA-B for all loops for the cases where loops have different sampling rates considering the virtual links insight from the previous section. In that case, the Peak Age can be generalized with the following equation

$$\hat{\Delta}_{i_k}^{CSMA-B} \leq \hat{\Delta}_{i_k}^{X_i CSMA-B} = \mathbf{s}_{k-1}^{X_i CSMA-B} + \mathbf{w}_{k-1}^{X_i CSMA-B} + \frac{1}{\lambda_i} \quad (4.8)$$

where  $X_i$  is given in (3.27) for CSMA-B.

#### 4.1.2.1. Simulations

In order to confirm the calculated model, we simulate the Peak Age and OPPA of a CSMA-B system on MATLAB using Propositions 3 and 5. We consider a WNCS with 5 loops in which we calculate the average age of each link, simulated with the same conditions than the CSMA-B in AoI, for each value of  $\lambda$ , for a packet of 133 bytes, with  $R = 15$  retries maximum. We first consider the case where all links sample at the same rate, and then scenarios of links sampling at different rates, for these scenarios we consider the virtual links notions indicated in Remark 11.

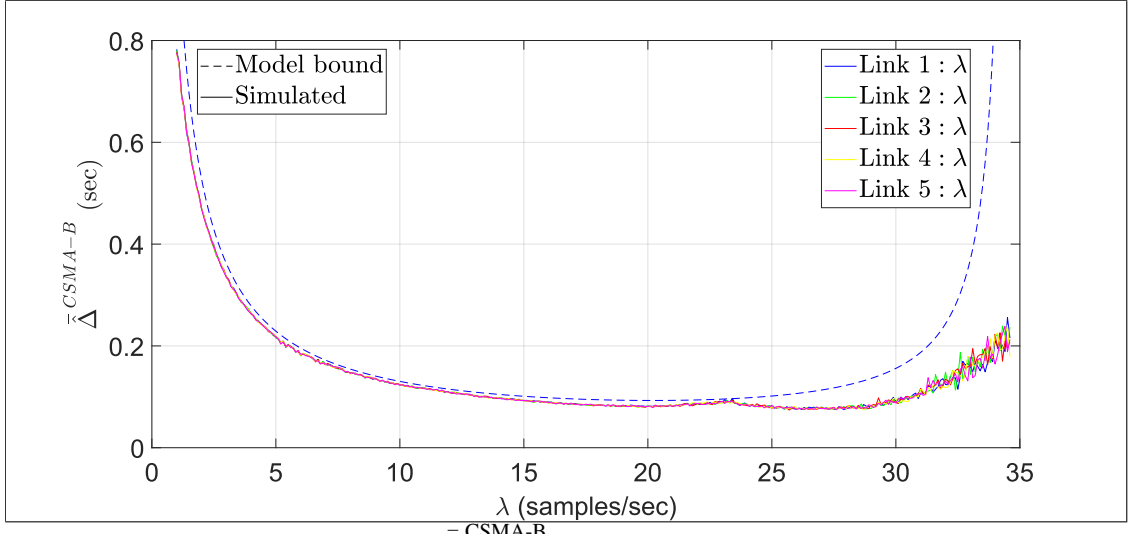


FIGURE 4.15. Simulation of  $\bar{\Delta}^{CSMA-B}$  with  $M = 5$ ,  $L = 133$  bytes,  $db = 320\mu s$  and  $R = 15$ .  $\lambda_i = \lambda$ .

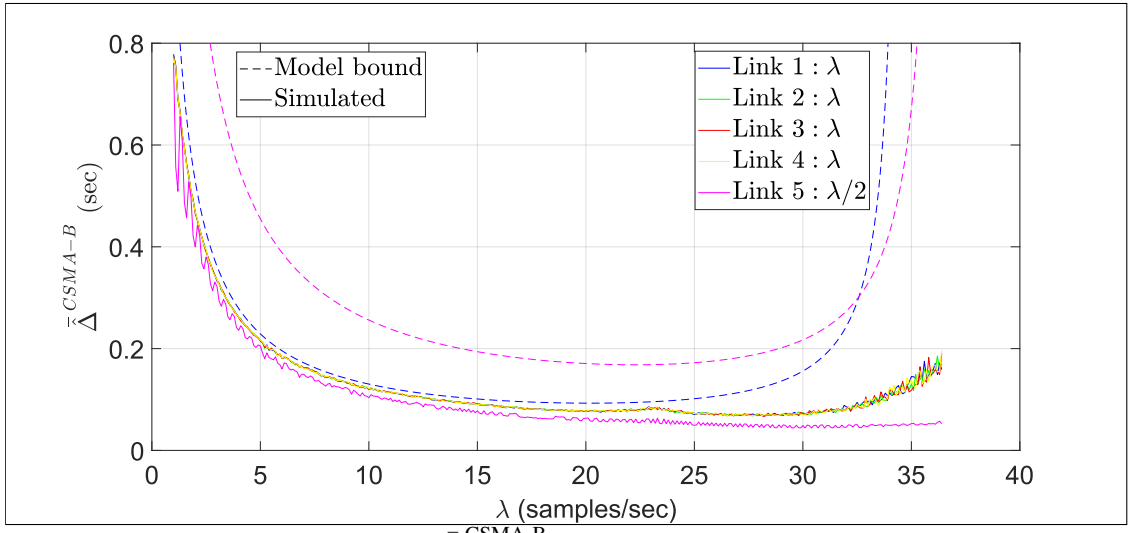


FIGURE 4.16. Simulation of  $\bar{\Delta}^{CSMA-B}$  with  $M = 5$ ,  $L = 133$  bytes,  $db = 320\mu s$  and  $R = 15$ . Four links sampling with  $\lambda$  and one with  $\lambda/2$ .

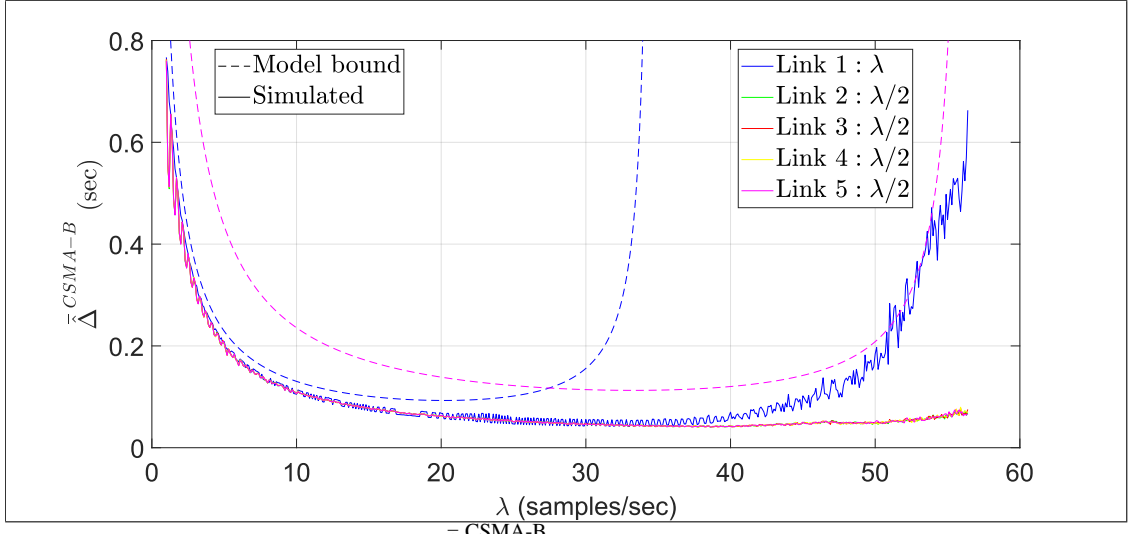


FIGURE 4.17. Simulation of  $\bar{\Delta}^{CSMA-B}$  with  $M = 5$ .  $L = 133$  bytes,  $db = 320\mu s$  and  $R = 15$ . One link sampling with  $\lambda$  and four with  $\lambda/2$ .

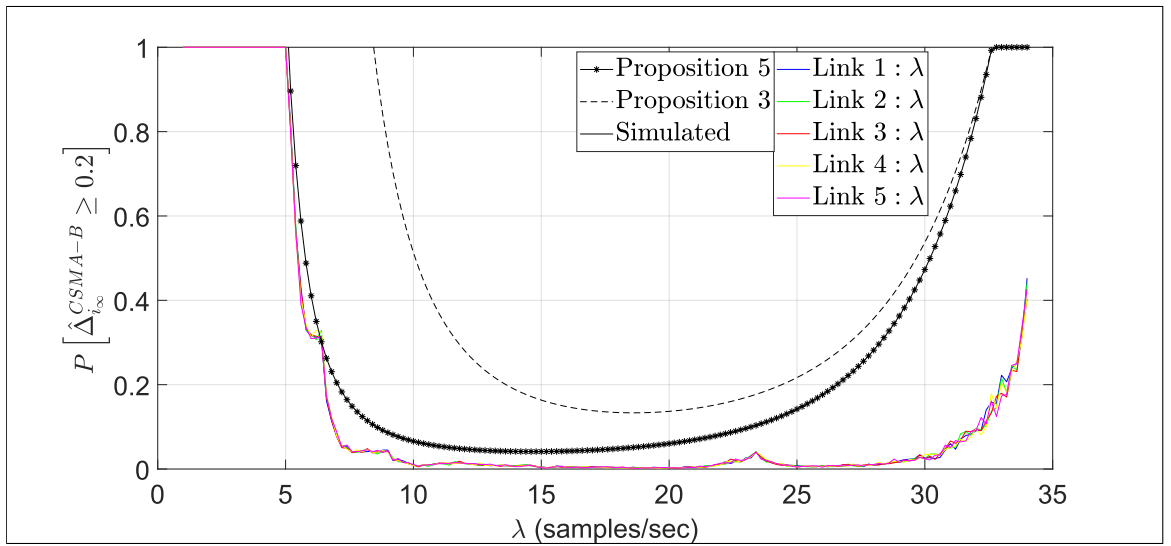


FIGURE 4.18. Simulation of OPPA for CSMA-B with a threshold of  $0.2s$ .  $M = 5$ .  $L = 133$  bytes,  $db = 320\mu s$  and  $R = 15$ .  $\lambda_i = \lambda$ .

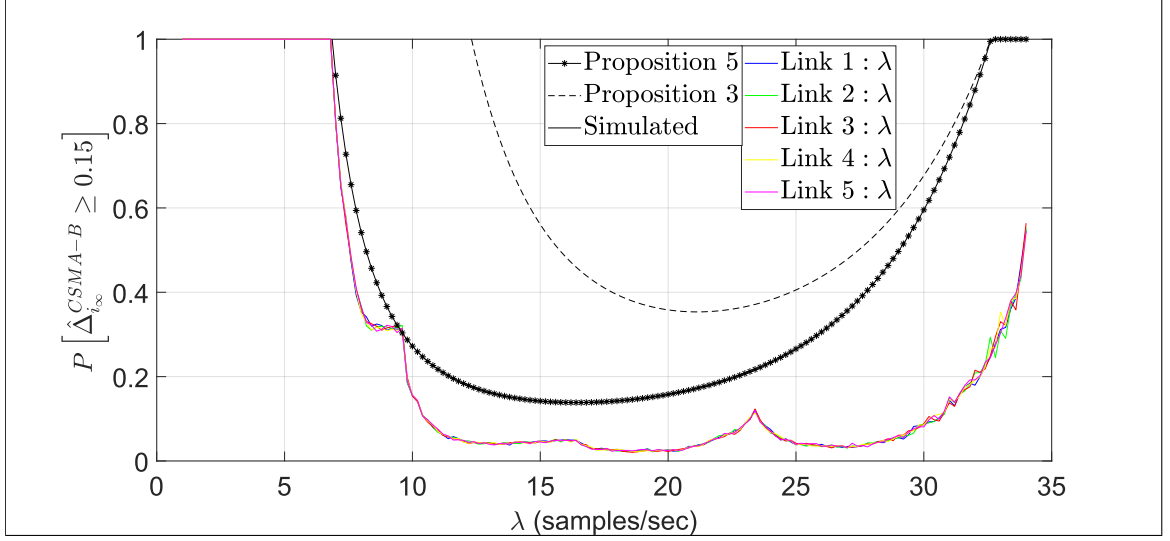


FIGURE 4.19. Simulation of OPPA for CSMA-B with a threshold of  $0.15s$ .  $M = 5$ .  $L = 133$  bytes,  $db = 320\mu s$  and  $R = 15$ .  $\lambda_i = \lambda$ .

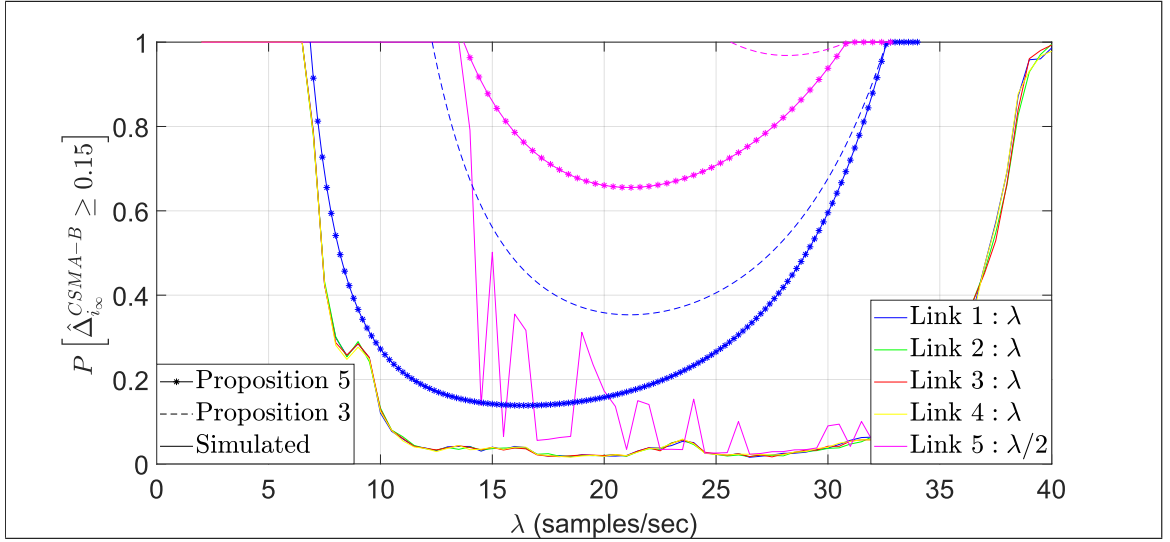


FIGURE 4.20. Simulation of OPPA for CSMA-B with a threshold of  $0.2s$ .  $M = 5$ .  $L = 133$  bytes,  $db = 320\mu s$  and  $R = 15$ . Four links sampling with  $\lambda$  and one with  $\lambda/2$ .

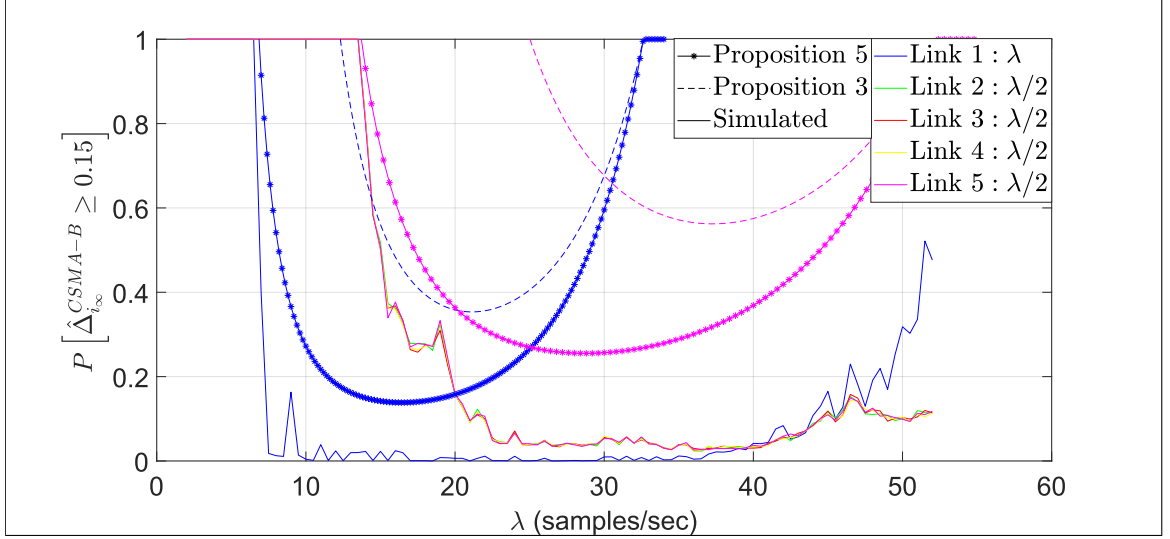


FIGURE 4.21. Simulation of OPPA for CSMA-B with a threshold of  $0.2s$ .  $M = 5$ .  $L = 133$  bytes,  $db = 320\mu s$  and  $R = 15$ . One link sampling with  $\lambda$  and four with  $\lambda/2$ .

For the PAoI, it can be seen that the bound is very tight when all the loops sample at the same rate (Figure 4.15), even more than the CSMA-A case. When four loops sample at  $\lambda$  and one at  $\lambda/2$  (Figure 4.16), the bound is still very tight for the first four loops but less for the loop sampling slower. In Figure 4.17, the bound for the faster loop is less tight than for the loops sampling at  $\lambda/2$ , particularly at higher sampling rates.

When simulating OPPA, the Figures confirm that Proposition 5 offers a tighter bound than 3. Specifically, when comparing Figures 4.18 and 4.19 with Figures 4.20 and 4.21, it can be seen that the bound is tighter when all loops sample at the same rate. When different loops sample at different rates, the loops that share sampling rates will have a tighter bound than the loops that have a differentiated sampling rate.

#### 4.2. Peak Age and OPPA on stability in TDMA-based WNCSSs

As established in Lemma 5, the peak age and the  $\tau_{\text{MATI}}^i + \tau_{\text{MAD}}^i$  loop restrictions are extremely related. In the idealized TDMA with greedy scheduling used for this work, the

central constraint is the Time slot, which can be used to tune an optimal sampling period for the loops present in the network in order to avoid unnecessary queue waiting time for the packets.

With this, given a certain set of loops with known  $\tau_{MATI}^i + \tau_{MAD}^i$  values, we can establish the maximum number of loops that can share the TDMA network proposed ensuring mean-square stability for all the loops, depending on the time slot value of the network. To continue, first we need to establish the following result.

**LEMMA 10.** *Consider an  $M$ -loop WNCS operating over a TDMA network for which Assumptions 1, 3, 5, and 6 hold. The minimum Peak Age is achieved in the case where the waiting time  $\bar{w}^{TDMA} = 0$  for all transmitted packets, which is achieved at the optimal sampling rate  $\lambda^{*TDMA} = \frac{1}{T^{slot}}$ .*

**PROOF.** The proof is straight forward: as the Peak Age also depends on the waiting time of the packets, and as the service time is deterministic, having the minimum possible waiting time would also achieve the minimum possible Peak Age. The minimum waiting time was established in the optimal AoI scenario in Proposition 2.  $\square$

**PROPOSITION 6.** *Consider an  $M$ -loop WNCS operating over a TDMA network for which Assumptions 1, 3, 5, and 6 hold and all the loops sample at the optimal sampling rate  $\frac{1}{\lambda_i} = T^{slot}$ . The optimal Peak Age  $\hat{\Delta}^{*TDMA} = (M + 1)T^{slot}$  is constant  $\forall i, k$ , which therefore implicates that closed-loop stability of loop  $\mathcal{P}_i$  is ensured if the WNCS satisfies*

$$MT^{slot} + T^{slot} = (M + 1)T^{slot} \leq (\tau_{MATI}^i + \tau_{MAD}^i) \forall i \quad (4.9)$$

which from a set  $\mathcal{T} = \{\tau_{MATI}^1 + \tau_{MAD}^1, \tau_{MATI}^2 + \tau_{MAD}^2, \dots, \tau_{MATI}^M + \tau_{MAD}^M\}$  is equivalent to

$$MT^{slot} + T^{slot} = (M + 1)T^{slot} \leq \min(\mathcal{T}) \quad (4.10)$$

**PROOF.** Consider  $\hat{\Delta}_{i_k}^{TDMA} = \frac{1}{\lambda_{i_{eff}}} + T^{slot} + w_{i_k}^{TDMA} \forall k$ . If  $\frac{1}{\lambda_i} = T^{slot} \forall i$ , the Peak Age is considered to be constant and optimal (Lemma 10) in the context of the assumptions given. Then from 2.19, and considering the effective sampling period from 3.28 used to consider the drop-outs from the greedy scheduling,  $\bar{\Delta}^{*TDMA} = \bar{\Delta}_i^{TDMA} = (M + 1)T^{slot}$ .

Then, from Lemma 5, if  $\bar{\bar{\Delta}}^{*TDM A} \leq \tau_{\text{MATI}}^i + \tau_{\text{MAD}}^i$ , then  $Pr[\hat{\Delta}_{\infty} \geq \tau_{\text{MATI}}^i + \tau_{\text{MAD}}^i] = 0$ . On the contrary, if  $\bar{\bar{\Delta}}^{*TDM A} > \tau_{\text{MATI}}^i + \tau_{\text{MAD}}^i$ , then  $Pr[\hat{\Delta}_{\infty} \geq \tau_{\text{MATI}}^i + \tau_{\text{MAD}}^i] = 1$ . To analyze the whole WNCS, in order to ensure stability  $\bar{\bar{\Delta}}^{*TDM A} \leq \tau_{\text{MATI}}^i + \tau_{\text{MAD}}^i \forall i$ , which gives 4.9. □

## 5. COMPARISONS AND NUMERICAL EXAMPLES

After analysing the AoI and the OPPA of the WNCS described for different MAC schemes, we will now compare both CSMA and TDMA protocols under different contexts and scenarios, considering the usefulness of the results obtained for the design of WNCSs. In order to make a fair comparison, the CSMA-A model will be considered with a transfer rate of 250kbps, as it were based on the IEEE802.15.4 PHY layer, therefore having the same constraints in a number of physical issues. Besides this, IEEE802.15.4 is used in a number of industrial IoT applications, being then of a special interest for this thesis.

Notice that this work has focused in analysing different MAC mechanisms, some of whose parameters depend on the PHY layer on which they are built. In this sense, it is important to mention that the PHY layer affects the communication performance in different aspects such as the transfer rate, the range of communication between the devices, the possible interference between devices because of crowded frequency bands, which may increment the need for re-transmissions, among other considerations. Different PHY layers would affect the absolute values of the status metrics analysed, for example, IEEE802.11 offers transfer rates far higher than IEEE802.15.4, which is the reason why a comparison between protocols should consider using the same PHY layer.

### 5.1. AoI in DCF CSMA/CA vs nBE CSMA/CA

We first make a comparison between the two CSMA protocols seen (considering both relying on the transfer rates of the IEEE802.15.4 PHY layer).

It can be seen in Figures 5.1 and 5.2 that the AoI in networks using CSMA-A outperforms in every case the AoI with CSMA-B. This is expected, as even if we compare both protocols with the same transfer speed of 250 kbps from IEEE802.15.4, the loops using the CSMA-A protocol continuously sense the medium at all times, and not only when the back-off counter reaches zero as in CSMA-B. This is because CSMA-A is not designed with any energetic efficiency focus, as the algorithm is mainly expected to be used in the IEEE802.11 protocol, where the energetic consumption is not one of the main issues, and

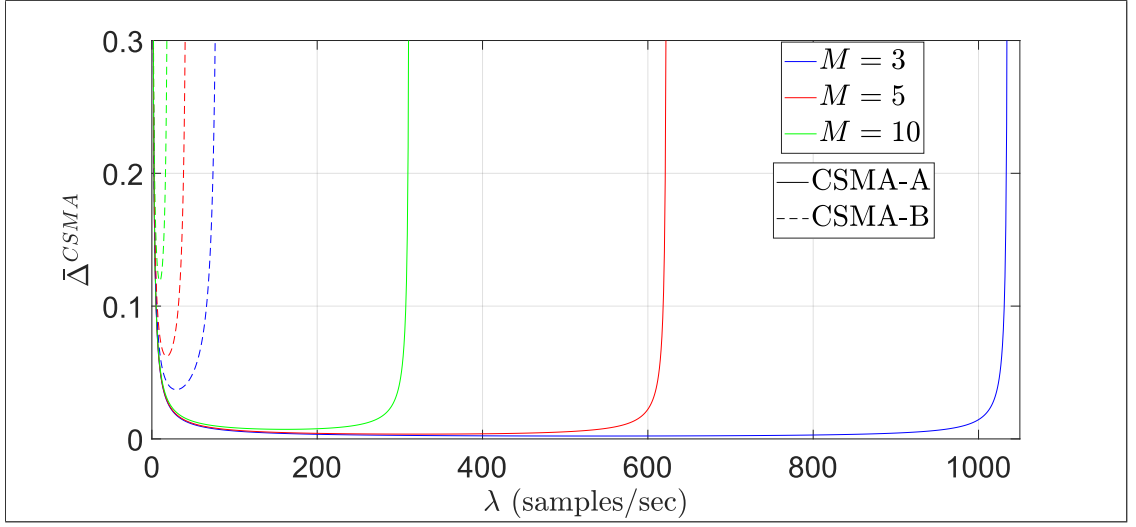


FIGURE 5.1. AoI comparison between CSMA-A and CSMA-B for different number of loops and packet length  $L = 10$  bytes.

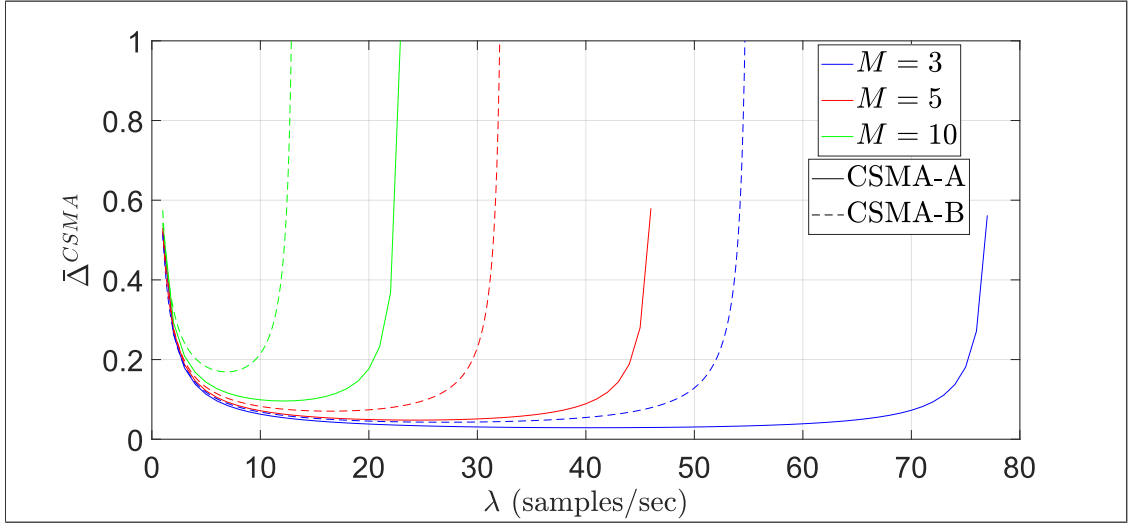


FIGURE 5.2. AoI comparison between CSMA-A and CSMA-B for different number of loops and packet length  $L = 133$  bytes.

it also makes it easy to assume that there will be no dropouts. The loops using CSMA-B sense with much less frequency, therefore having greater delays, and therefore making the assumption of no dropouts realistic only after adding the retries, which add even more delay.

Therefore, even if a comparison between the two CSMA protocol may seem interesting at first, and that CSMA-A may appear to be better, in the practicality of contexts in which both algorithms are used the comparison does not have much sense, as there are other parameters and characteristics that are not considered in the purely timeliness performance aspect that have to be taken account of.

## 5.2. Theoretical AoI comparison between DCF CSMA/CA and TDMA

A fair, theoretical comparison can be made between CSMA-A and TDMA considering all design variables, because, unlike CSMA-B, both systems do not need numerical solving. To this end, the case when the least upper-bound of AoI in CSMA-A,  $\bar{\Delta}^{*MCSMA-A}$  is equal to the optimal AoI of TDMA,  $\bar{\Delta}^{*TDMA}$  can be characterized.

**PROPOSITION 7.** *Consider an  $M$ -loop WNCS for which Assumptions 1 and 3 hold, operating either over a CSMA-A network, in case Assumption 4 holds, or over a TDMA network, in case Assumptions 5, and 6 hold. The least upper-bound of AoI in CSMA-A,  $\bar{\Delta}^{*MCSMA-A}$  is equal to the optimal AoI of TDMA,  $\bar{\Delta}^{*TDMA}$ , if  $T^{slot}$  satisfies*

$$T^{slot} = \frac{(406 - 200\beta^*)(M^3R^2 + H(H - R + MR + M^2R))}{103HMR(1 - \beta^*)(MR + H)(M + 2)}, \quad (5.1)$$

where  $\beta^* = 0.2198$ .

**PROOF.** We invoke Theorems 2 and 4 and the results in Propositions 1 and 2 to solve for  $T^{slot}$  in (3.8)=(3.32). After some algebra we obtain (5.1).  $\square$

We illustrate the impact of this result in the context of an example. Consider an open-loop unstable plant, which is controlled using a closed loop  $\mathcal{P}_i$  with  $\mathcal{N}_i = 2$  links. In this example, we address the issue of deciding the number of loops,  $M$ , yielding an  $M$ -loop WNCS, that is reasonable to allocate in the same network, and which MAC mechanisms should be preferred.

Consider that each loop schedules which of their nodes transmits using a Round Robin strategy, and assume that each loop can be stabilized only if its average age is below a

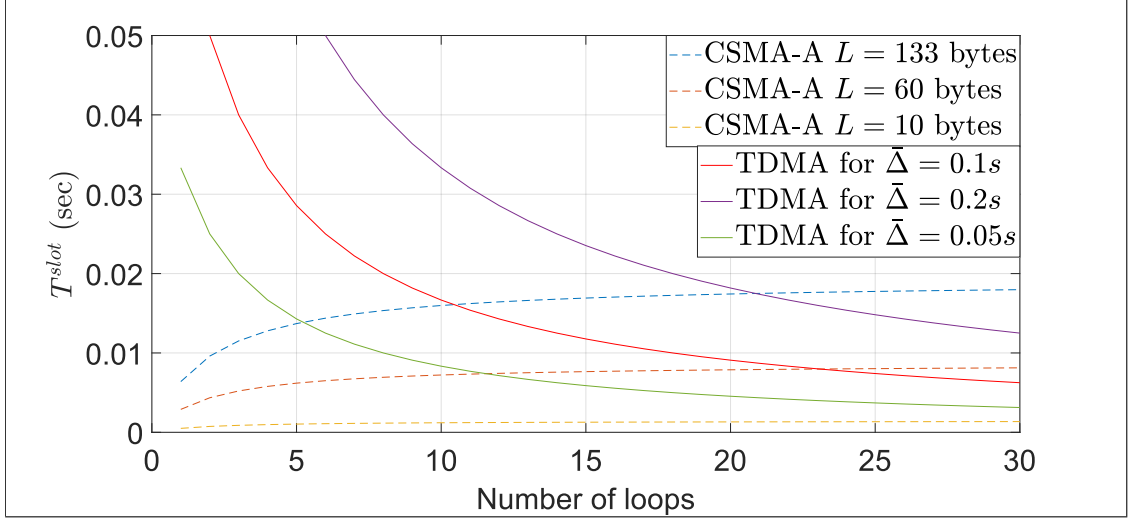


FIGURE 5.3.  $T^{\text{slot}}$  for  $\bar{\Delta}^{\text{TDMA}} = \bar{\Delta}_i^{\text{CSMA-A}}$  for different number of links.

certain value,  $\bar{\Delta} \leq 0.1s$ . We can establish which is the optimal sampling rate for different network conditions. Figure 5.3 illustrates how Proposition 7 determines the value for  $T^{\text{slot}}$  such that the system under TDMA achieves the same AoI than the bound on CSMA-A, considering the constraint on optimal sampling rate. It can be seen that the TDMA (solid) lines, which represent the trade-off between number of loops and the  $T^{\text{slot}}$  needed to achieve a given AoI value, intersect with different (dashed) lines at a value of time slot needed for achieving the same AoI under a CSMA-A scheme, for a given packet length. Using Figure 5.3, we can compare the CSMA-A system with the TDMA system for the AoI of interest. If the design needs 10 loops in the network, to fulfill the AoI constraint, a time slot below  $15ms$  must be chosen for TDMA. Equivalently, the same AoI performance can be obtained in a CSMA-A network with a packet length of 133 bytes. In this relative comparison, if the design supports a shorter packet, the CSMA-A network becomes more convenient, as TDMA would need a far more stricter time slot to have the same performance in terms of AoI.

Intending to have a more practical approach, consider a CSMA-A network where every loop  $\mathcal{P}_i$  transmits with  $1/H_i = 1/H = 4.3ms$  and  $R_i = 14H$ . The value of  $R$  is taken from (Maatouk et al., 2020) characterizing a network with extremely low probability of

collisions, while  $1/H$  refers to a frame of 133 bytes at 250kbps. Consider also a TDMA network with a fixed  $T^{\text{slot}} = 10ms$ , which is used in one of the most relevant TDMA protocols in the IoT context based on IEEE802.15.4 and for process automation, Wireless HART (Wang et al., 2016) (Khader, Willig, & Wolisz, 2011). In this latter case, Proposition 2 indicates that the maximum number of loops than can be allocated in the network is  $M = 18$ . On the other hand, a CSMA-A network can hold (considering its bound), under the optimal sampling scenario described in Proposition 1, a maximum of  $M = 10$  loops for a packet length of 133 bytes, while if reducing the packet size to 10 bytes is supported, the network could hold as far as  $M = 138$  loops.

This example illustrates how the analytical results given in this work can serve as a tool for designing a WNCS that requires to fulfill constraints in terms of AoI. An important insight is that TDMA networks offer more reliability on transmission times, but lack flexibility for adding more links. Furthermore, note that in TDMA the optimal sampling rate is higher than in CSMA-A for achieving the same AoI, which may imply higher energy requirements. On the other hand, as TDMA networks are scheduled, a more complex energy management scheme is available, as links can go into sleep-mode when not transmitting (Khader et al., 2011).

### 5.3. Optimal sampling rates in AoI, Peak and OPPA

An interesting topic is to see how the optimal sampling for AoI, PAoI and OPPA are compared. One value ensures that, at a certain  $\lambda^*$ , the status age of the updates of every link will be the lowest possible on average, the other one intends to obtain the value for which the maximum status age before updates is minimized and the last one minimizes the probability of the maximum time difference in the system between the generation of the sample and the monitor receiving the update exceeding a certain threshold.

In the results of this work, the optimal  $\lambda$  in AoI and PAoI depends only on the protocol chosen and on the length of the packet, while for the OPPA it will also depend on which threshold is trying to be avoided. In Figures 5.4 and 5.5 we can see how the 3 concepts

behave. Remember that the only values that can be analytically calculated is  $\lambda^*$  for AoI and PAoI in CSMA-A and TDMA; all the other optimal sampling rates can only be numerically obtained.

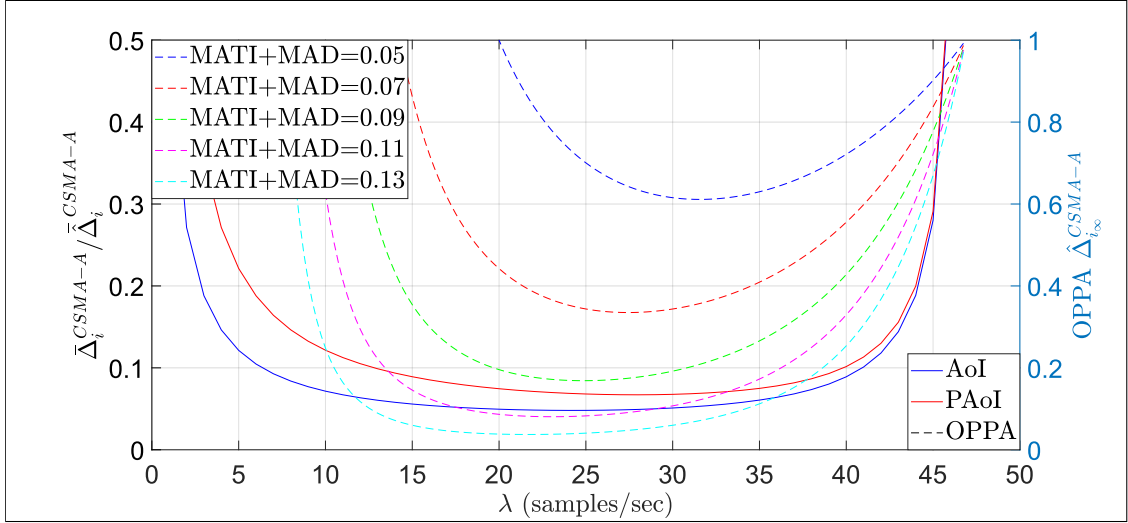


FIGURE 5.4. AoI, PAoI and OPPA for CSMA-A with  $M = 5$ .  $L = 133$  bytes.

$1/H = 4.256ms$   $R = 14.8H$ .  $\lambda_i = \lambda$ .

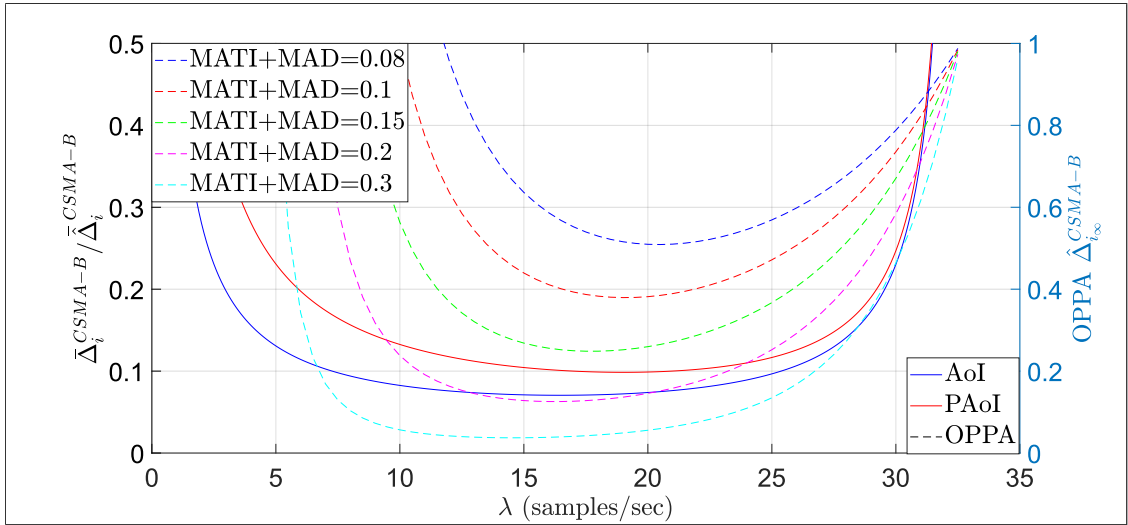


FIGURE 5.5. AoI, PAoI and OPPA for CSMA-B with  $M = 5$ .  $L = 133$  bytes. 15

retries.  $db = 320\mu s$ ,  $\lambda_i = \lambda$ .

It can be seen that, for both CSMA-A and CSMA-B protocols, the optimal sampling rate for the OPPA with different thresholds, considering the threshold to be a specific  $\tau_{\text{MATI}} + \tau_{\text{MAD}}$  value, can be considered near the optimal sampling value that minimizes AoI. Moreover, a similar situation occurs with the sampling value that optimizes PAoI. However, it is not the same value. This means that, if what the system looks is to minimize the frequency of the Peak Age surpassing a specific threshold, the optimal  $\lambda$  for AoI or PAoI can be an acceptable guidance, but will not indicate the specific optimal value, which will depend on the threshold intending to avoid.

With this, we can also compare the AoI and average Peak Age behaviour for the three protocols analysed for different number of loops in the network when sampling at the optimal rate. In comparison, considering the same transfer rates, Figure 5.6 and 5.7 show that TDMA is in general better for AoI at an optimal sampling rate than CSMA-B, but considering that TDMA must sample at a higher sampling rate (at least 3 times higher in the case of Figure 3.8 with  $L = 133$  bytes) to achieve the optimal AoI. This must be considered in the WNCS design, as the sensors involved may have some physical constraints that make impossible to sense as fast as TDMA requires. CSMA-A, on its side, shows a better AoI performance than the other two protocols only when the packet size is small, which shows the importance of also considering the packet size on the design process. However, CSMA-A also incurs in far higher energetic requirements on the devices involved.

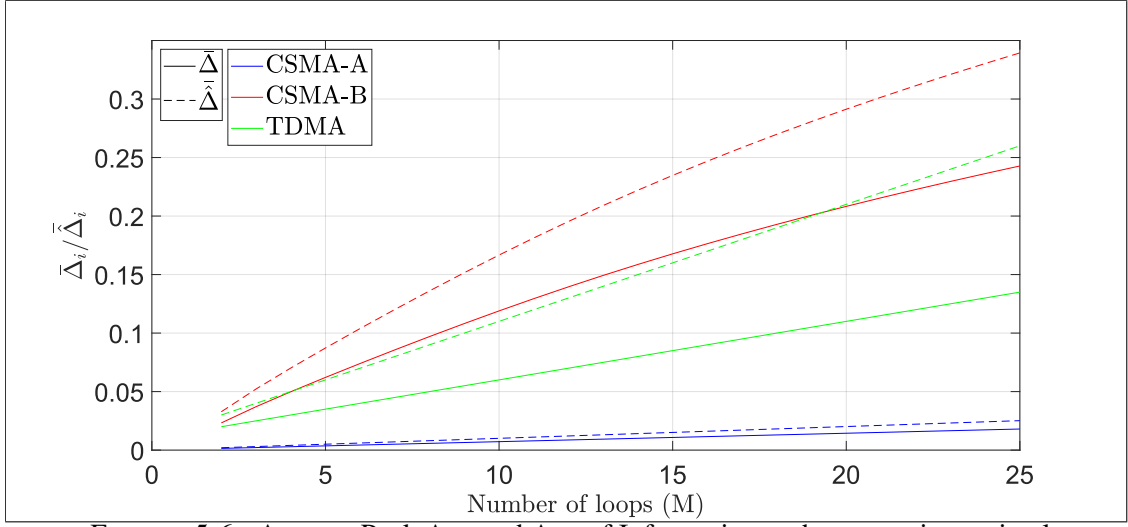


FIGURE 5.6. Average Peak Age and Age of Information at the respective optimal sampling rate for the different protocols analysed based on PHY 802.15.4 transfer rates.  $L = 10$  bytes.  $R = 20$  retries.  $T^{\text{slot}} = 10ms$ .

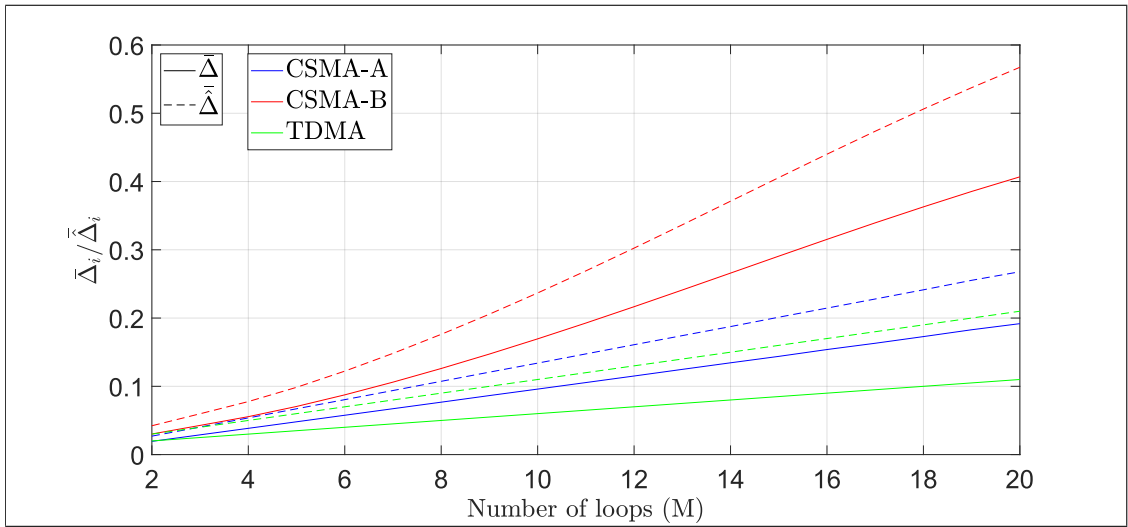


FIGURE 5.7. Average Peak Age and Age of Information at the respective optimal sampling rate for the different protocols analysed based on PHY 802.15.4 transfer rates.  $L = 133$  bytes.  $R = 20$  retries.  $T^{\text{slot}} = 10ms$ .

#### 5.4. An OPPA comparison between DCF CSMA/CA, nBE CSMA/CA and TDMA

We are interested in numerically studying the differences between the performance of the protocols analyzed in this thesis, in the sense of loop stability in the WNCS, and how the protocols offer different attributes depending on the design objectives. When comparing TDMA and CSMA in the outage probability of the Peak Age surpassing the established stability constraint, and considering that both protocols are different in the sense that TDMA can ensure stability for a number of loops depending on the  $T^{\text{slot}}$  value but CSMA is based on OPPA values, the importance is then centered in the tolerance that the designer can hold in the OPPA in the CSMA scenarios (A or B).

Therefore, the objective of comparing CSMA-A, -B, and TDMA is to provide an insight on how many loops can share the network while maintaining the stability, providing a tool for the WNCS design. Consider a WNCS with  $M$  loops where the  $\tau_{\text{MATI}}^i + \tau_{\text{MAD}}^i$  values form a set  $\mathcal{T} = \{\tau_{\text{MATI}}^1 + \tau_{\text{MAD}}^1, \tau_{\text{MATI}}^2 + \tau_{\text{MAD}}^2, \dots, \tau_{\text{MATI}}^M + \tau_{\text{MAD}}^M\}$ . Therefore, when we analyse the stability of the WNCS as a whole, it is expected that the Peak Age of each loop does not surpass or surpass with the least possible probability its respective  $\tau_{\text{MATI}}^i + \tau_{\text{MAD}}^i$  value. We now compare both CSMA/CA and the TDMA protocol with a fixed  $T^{\text{slot}} = 10ms$  considering it as similar as possible to the WirelessHART. For the first example, consider a system with  $\tau_{\text{MATI}}^i + \tau_{\text{MAD}}^i = 0.15s, \forall i$ .

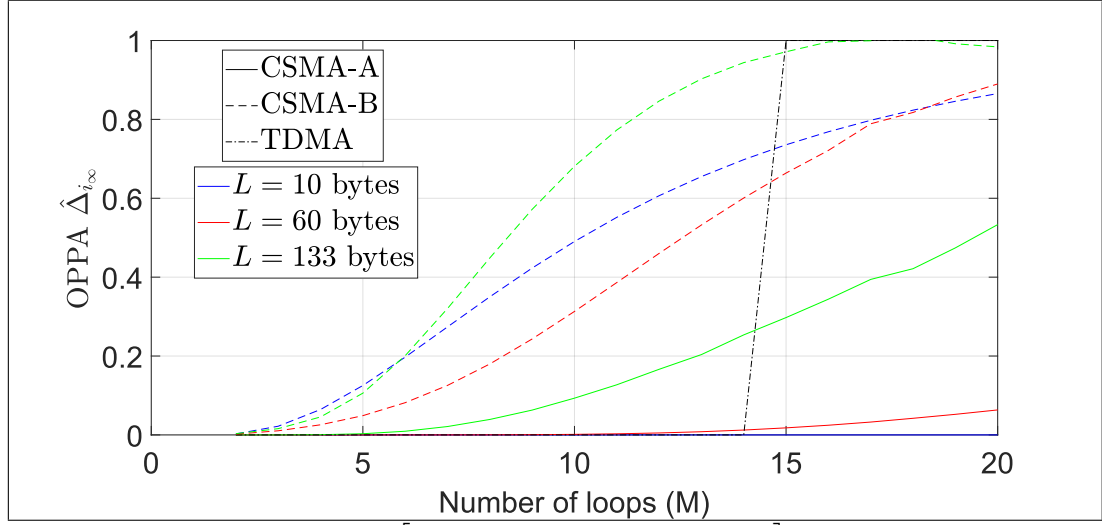


FIGURE 5.8. Bound on  $P \left[ \hat{\Delta}_{i\infty} \geq \tau_{\text{MATI}} + \tau_{\text{MAD}} = 0.15s \right]$  for the different protocols analysed with different packet lengths.  $db = 320\mu s$ ,  $\lambda_i = \lambda$ .

Figure 5.8 shows the results for the example given. As seen previously, the optimal sampling rate for both TDMA and CSMA/CA in this case is to have all loops sampling at the same rate  $\lambda_i = \lambda$ . For the TDMA case, with Proposition 6 we can directly establish that the maximum number of loops that can share the network in the conditions exposed is  $(M + 1)0.01 \leq 0.15 \rightarrow M = 14$ . In CSMA/CA we use a numerical solver to obtain the optimal  $\lambda$  rate at which the OPPA is minimized, in both CSMA-A and CSMA-B cases for different packet lengths. As expected and explained in 5.2, CSMA-A outperforms CSMA-B and therefore has a lower probability of exceeding the constraint for all cases. It can be seen how for CSMA the OPPA is always very low for a small number of loops, rapidly increasing. CSMA-B reaches the almost certain probability of always exceeding the OPPA at approximately 16 loops for a transmission with a packet of 133 bytes.

The previous example showed the simplistic scenario in where all the loops had the same stability constraint, which solved the problem on determining the sampling rate of the loops, as the optimal OPPA for all the loops is obtained when all loops sample at the same rate, similar to the AoI case. It has been established in the TDMA algorithm that all links sample at the same time. However, in CSMA, we have already confirmed in the

simulations that when the sampling rate are different for the loops, the faster loops benefit in relation to the performance it would have, had all loops sampled at the same rate, while the opposite happens for slower loops. This means that if we have a loop with stricter stability constraints, it would be wise to have the sampling rate of that link faster than the other ones from loops with more flexible constraints.

Consider then a new scenario where only one specific loop  $i$  has  $\tau_{\text{MATI}}^i + \tau_{\text{MAD}}^i = 0.1s$ , while all the other loops  $j \neq i$  have the constraints  $\tau_{\text{MATI}}^j + \tau_{\text{MAD}}^j = 0.5s$ . In this case, from Proposition 6 the maximum number of loops that can hold the TDMA mechanisms is still 14, because of the greedy algorithm that does not establish priority for any node from any loop. In CSMA, however, we can numerically solve the optimal  $\lambda$  that optimizes the OPPA in each loop for different total number of loops, following the next steps. For this new example, we consider only CSMA-B with a packet length of 10 or 133 bytes.

We start by determining which sampling value we solve first, if the stricter one or the most flexible ones. This is because the calculation of the virtual links  $X_i$  depends on knowing if any loop will sample faster than the others, so it is crucial to fix one sampling rate considering  $M$  links and then calculate the optimal sampling rate of the other loops considering the virtual links technique. As  $X_i \geq M$ , we first resolve the optimal  $\lambda$  value for the loop with the stricter stability constraint, which is expected to have a higher sampling rate, and then we resolve for the other 4 loops, obtaining the following figure:

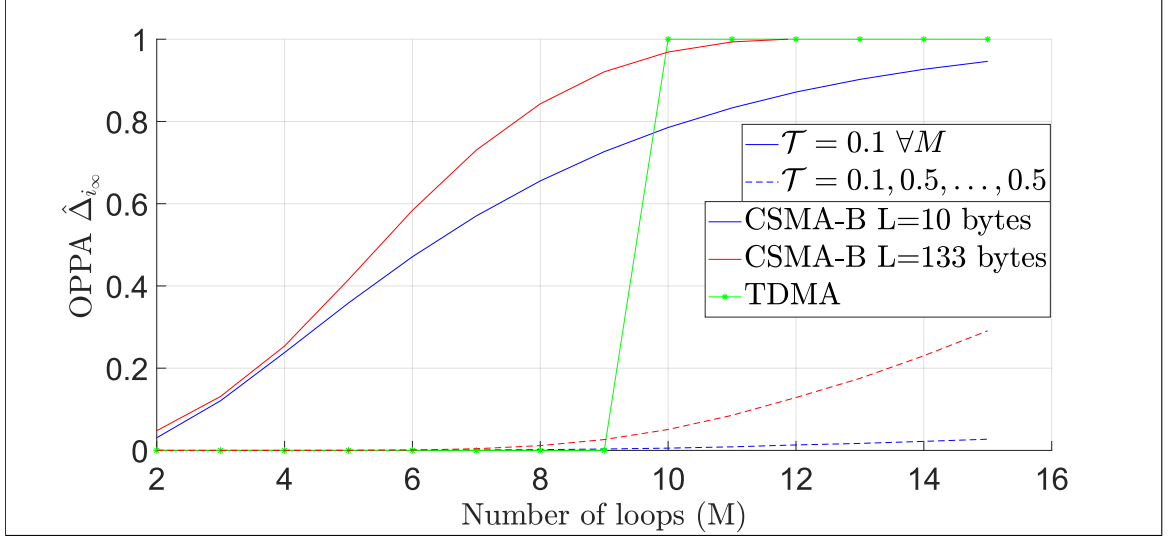


FIGURE 5.9. Bound on  $P \left[ \hat{\Delta}_{i_\infty} \geq \tau_{\text{MATI}} + \tau_{\text{MAD}} \right]$  for different number of loops with different stability constraints.

In Figure 5.9, the straight lines indicate the bound for the loop with  $\tau_{\text{MATI}}^i + \tau_{\text{MAD}}^i = 0.1s$ , where the total number of loops can be other loops with also  $\tau_{\text{MATI}}^i + \tau_{\text{MAD}}^i = 0.1s$  or loops with  $\tau_{\text{MATI}}^i + \tau_{\text{MAD}}^i = 0.5s$ . The dashed lines indicate the OPPA for the loop with the constraint  $\tau_{\text{MATI}}^i + \tau_{\text{MAD}}^i = 0.5s$  inside a system with only one loop with  $\tau_{\text{MATI}}^i + \tau_{\text{MAD}}^i = 0.1s$  and all the other ones with  $\tau_{\text{MATI}}^i + \tau_{\text{MAD}}^i = 0.5s$ . The green line indicates the OPPA using TDMA in system with one or more loops with  $\tau_{\text{MATI}}^i + \tau_{\text{MAD}}^i = 0.1s$ . The figure is useful as it illustrates that the TDMA network for sure will break the stability of the system for an excessive number of loops, regardless if there is only one loop with a strict constraint or many, while in CSMA the analysis can be done separately, having an OPPA for the stricter loop in combination with the OPPA for the more flexible loops. It also shows the difference between having a model with a packet of 10 or 133 bytes, and how a small packet can fit more loops with a lower probability of exceeding the stability constraint.

Another perspective of how this work can be helpful in relation with CSMA might be to obtain the optimal  $\lambda$  for each loop to minimize the OPPA in order to tune the system according to some sampling rate constraints given by the physical limitation of the

sensors of each loop. In the previous example, if the design needs  $M = 6$  loops in CSMA-B, with  $\mathcal{T} = \{0.1, 0.5, 0.5, 0.5, 0.5, 0.5\}$ , the optimal sampling rates would be  $\lambda = \{17.2, 8.6, 8.6, 8.6, 8.6, 8.6\}$  respectively, considering a packet length of 133 bytes. With this, the designer can choose the adequate sensors and parameters that allow the system to have the loops with sampling rates near to the optimal value to have the lowest probability of surpassing the respective loop stability constraints.

In this section, we analyzed how this work provides tools to find the sampling rates that minimizes the probability of surpassing the stability threshold of the different loops that share a network. However, in the next section we will see that sampling at this rate is not always the best option to ensure stability.

### 5.5. A WNCS design example

In order to emphasize the aspects in which this work can be applied to WNCSs we analyse the following application based on the Batch reactor example, a classic NCS example which has been extensively used in the literature since 1995 (Green & Limebeer, 2012) for different models (Walsh, Hong Ye, & Bushnell, 2002) (Nešić & Teel, 2004). In (Heemels et al., 2010) and (Liu et al., 2015), the model is adapted to be used in a wireless environment. As in the preliminaries we used the NCS model from (Liu et al., 2015), we replicate and simulate the example using that model under a Round Robin scheduling, which gives us the constraint  $\tau_{\text{MATI}} + \tau_{\text{MAD}} = 0.043s$  (slightly above the theoretical  $\tau_{\text{MATI}} + \tau_{\text{MAD}} = 0.042s$  value calculated in (Liu et al., 2015)).

The objective of simulating a concrete example is to analyse how the stability of the system behaves when the  $\tau_{\text{MATI}} + \tau_{\text{MAD}}$  constraint is violated only with a relative frequency. It is already established that, for the Batch reactor example, if  $\tau_{\text{MATI}} + \tau_{\text{MAD}}$  is constantly above 0.44, the system will fall in instability. However, up to date there is no known method that specifies what happens if the  $\tau_{\text{MATI}} + \tau_{\text{MAD}}$  values are exceeded only once in ten samples, for example. This idea is critical for this thesis because it directly relates to the concept of OPPA, specially in CSMA networks. As we have developed a model that establishes

the probability of a link surpassing a certain  $\tau_{\text{MATI}} + \tau_{\text{MAD}}$  value when transmitting over a wireless channel, this probability can be interpreted as the relative frequency in which the constraint is violated. For example, when transmitting over a CSMA network, having an OPPA of 0.2 of surpassing the  $\tau_{\text{MATI}} + \tau_{\text{MAD}}$  established value means that in average, in 1 out of 5 consecutively transmitted samples the controller will update the sample after the stability time constraint. Assume that the packet length is 133 bytes, the maximum frame length in PHY 802.15.4 ([Petersen & Carlsen, 2009](#)).

Simulating a concrete example allows us to define a certain degree of tolerance for which when surpassing  $\tau_{\text{MATI}} + \tau_{\text{MAD}}$  the system could remain stable, therefore making the OPPA analysis useful for a specific model in detail. In this case, that tolerance will depend on different aspects of the batch reactor model, like the scheduling protocol, the initial node transmitted and the difference between the  $\tau_{\text{MATI}} + \tau_{\text{MAD}}$  value and the surpassed value, as it is not the same to exceed it once in 10 by  $0.05s$  than to exceed it with the same frequency but by  $1s$ .

Consider then the system with the form described in (2.5) and (2.6) with the following parameters from ([Liu et al., 2015](#))

$$A = \begin{bmatrix} 1.380 & -0.208 & 6.715 & -5.676 \\ -0.581 & -4.2902 & 0 & 0.675 \\ 1.067 & 4.273 & -6.654 & 5.893 \\ 0.048 & 4.273 & 1.343 & -2.104 \end{bmatrix}$$

$$B = \begin{bmatrix} 0 & 0 \\ 5.679 & 0 \\ 1.136 & -3.146 \\ 1.136 & 0 \end{bmatrix}, C = \begin{bmatrix} C_1 \\ C_2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 1 & -1 \\ 0 & 1 & 0 & 0 \end{bmatrix}$$

$$\left[ \begin{array}{c|c} A_c & B_c \\ \hline C_c & D_c \end{array} \right] = \left[ \begin{array}{cc|cc} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ \hline -2 & 0 & 0 & -2 \\ 0 & 8 & 5 & 0 \end{array} \right]$$

As stated before, under a Round Robin scheduling protocol, the  $\tau_{\text{MATI}} + \tau_{\text{MAD}}$  achieved in the simulation is 0.043, near the 0.42 from (Liu et al., 2012) (Liu et al., 2015). Remember from Proposition 3 and 6 that  $\tau_{\text{MATI}} + \tau_{\text{MAD}}$  is related to the Peak Age, as  $\tau_{k+1} - \tau_k + \mathbf{d}_k = \hat{\Delta}_k$ . In Figure 5.10 and 5.11, the outputs of the system are plotted for different, considered constant  $\forall k, \hat{\Delta}_k$  values.

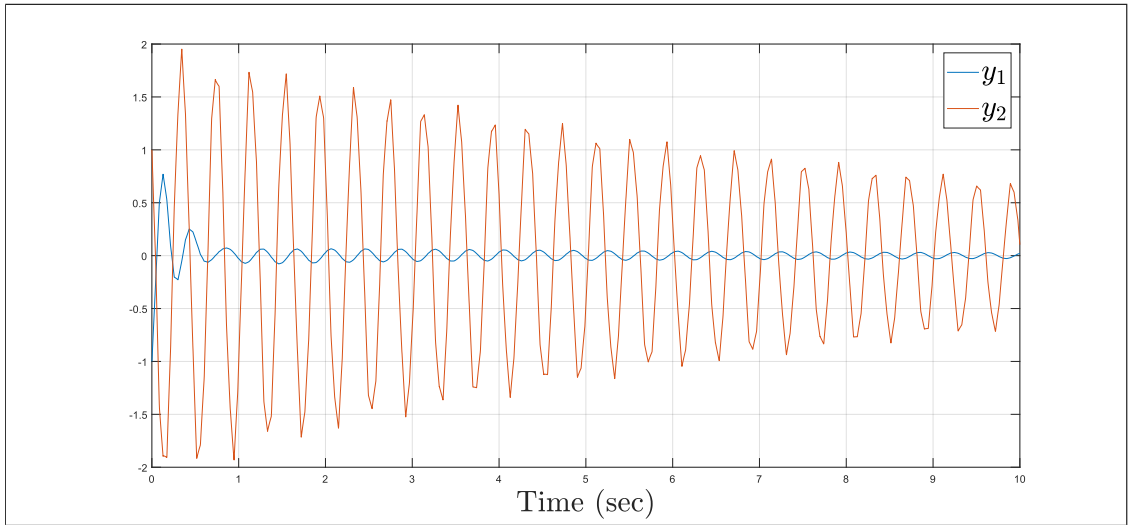


FIGURE 5.10. Batch reactor simulation for  $\hat{\Delta}_k = 0.043 < \tau_{\text{MATI}} + \tau_{\text{MAD}} \forall k$ .

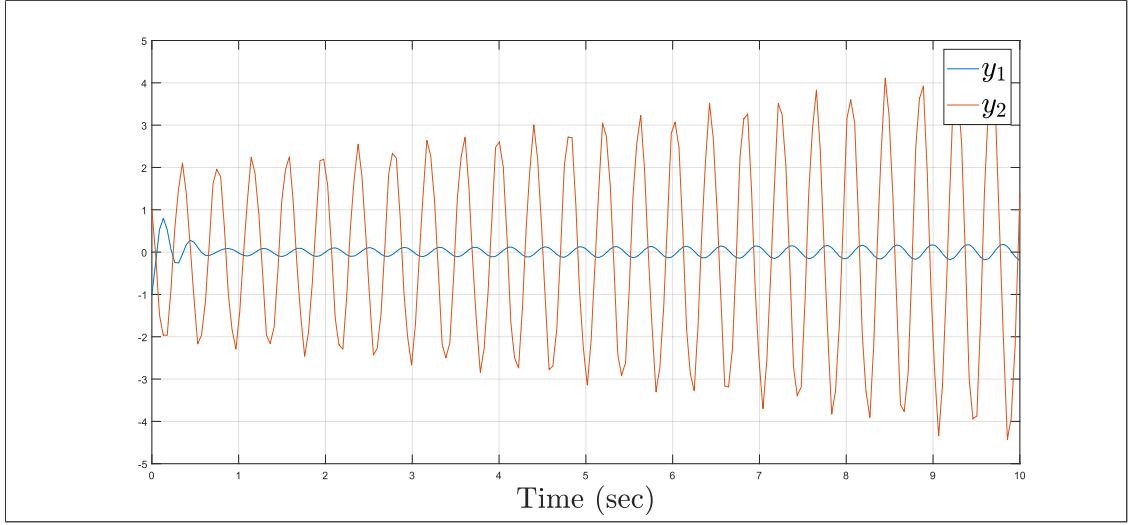


FIGURE 5.11. Batch reactor simulation for  $\hat{\Delta}_k = 0.045 > \tau_{\text{MATI}} + \tau_{\text{MAD}} \forall k$ .

Figures 5.10 and 5.11 show the critical, bound scenarios for when the  $\tau_{\text{MATI}} + \tau_{\text{MAD}}$  is always respected and for when it is always violated, respectively. In this case, the analysis for the probability of the different communication protocols respecting the  $\tau_{\text{MATI}} + \tau_{\text{MAD}}$  at different sampling rates can be done with the OPPA analysis presented in chapter 4, as shown in the examples of the subsections 4.1.1, 4.1.2 and 4.2, establishing a bound on  $P \left[ \hat{\Delta}_{i_\infty} \geq \tau_{\text{MATI}}^i + \tau_{\text{MAD}}^i \right]$  with the expressions stated in Proposition 3 and 6.

However, if what we want is a specific analysis for the example simulated using the Peak Age and OPPA concepts, it is important to establish by how much the  $\tau_{\text{MATI}} + \tau_{\text{MAD}}$  value has to be violated and in which frequency in order to truly destabilize the system. With this, assume a certain, average  $\hat{\Delta}_k = \bar{\bar{\Delta}} < \tau_{\text{MATI}} + \tau_{\text{MAD}}$ , which changes to an extraordinary outage value  $\hat{\Delta}_{k+1} > \tau_{\text{MATI}} + \tau_{\text{MAD}}$  in the next transmission. This change occurs with a certain frequency, and we assume it is deterministic, this means that if the frequency of change is 0.2, 4 transmissions will happen with the average Peak Age  $\bar{\bar{\Delta}}$  and the next one will have an extraordinary Peak Age. After that, another 4 average transmissions will come and the pattern repeats. Therefore, one in a certain number of samples are transmitted with the value that exceeds  $\tau_{\text{MATI}} + \tau_{\text{MAD}}$ . Figure 5.12 shows what this frequency should be, and for which value of  $\hat{\Delta}_{k+1} > \tau_{\text{MATI}} + \tau_{\text{MAD}}$ , in order to destabilize the system.

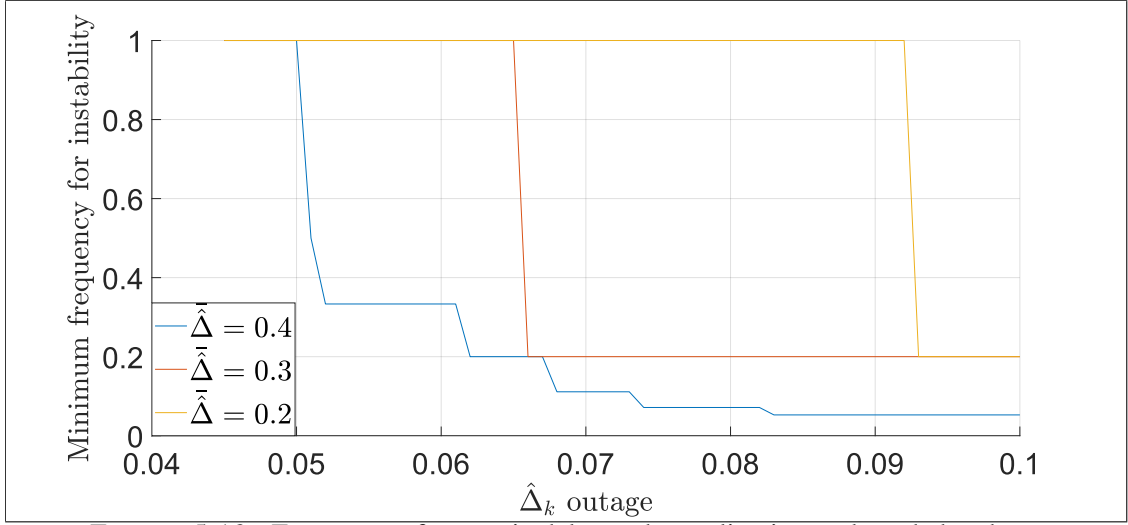


FIGURE 5.12. Frequency of excess in delay and sampling interval needed to destabilize the system for different nominal  $\hat{\Delta}$  values.

Notice that, when  $\hat{\Delta}_k$  is on average  $0.041s$ , if the next transmission occurs with  $\hat{\Delta}_{k+1} = 0.055$ , this would need to happen at least one third of the times to destabilize the system. However, if the next transmission has  $\hat{\Delta}_{k+1} = 0.09$ , this would need to happen 1 out of 19 times and the system would become unstable.

Therefore, for an effective application of this thesis into this example, we are interested in analysing how the information of Figure 5.12 can be used in an OPPA analysis when is assumed that the transmission will occur under one of the three wireless protocols already presented in a WNCS model as the one proposed in 2.9. In this sense, our analysis considers that all the loops in the network will be Batch reactors as the one from (Liu et al., 2015), but it can be easily extended for systems with different dynamics.

The analysis has to be done separately for the contention and the scheduling-based scenarios. In TDMA, from Assumption 5 and 6 we know that the delay  $d_k$  is not random and that at the optimal sampling rate  $\lambda^{*TDMA} = T^{\text{slot}}$  the Peak Age and AoI are minimized. Therefore, the analysis of in which frequency the  $\hat{\Delta}_k$  value surpasses the stability constraint does not have any sense and therefore the course of action is the same than the established

in Proposition 6. Applying it shows us that, considering a  $T^{\text{slot}} = 10\text{ms}$ , a network using TDMA can hold  $M = 3$  Batch reactors.

In CSMA, the exponentially distributed parameters of the channel holding time and the back-off for CSMA-A and the service time distribution based on the retries for CSMA-B do not allow to establish a concrete maximum possible delay ensuring transmission, but allow to establish the exact probability of achieving stability. For this WNCS with multiple batch reactors, we will consider only CSMA-A as an option, as the average Peak Age for CSMA-B is above the  $\tau_{\text{MATI}} + \tau_{\text{MAD}} = 0.044$  constraint for even the minimum possible value at optimal sampling (Figure 4.2).

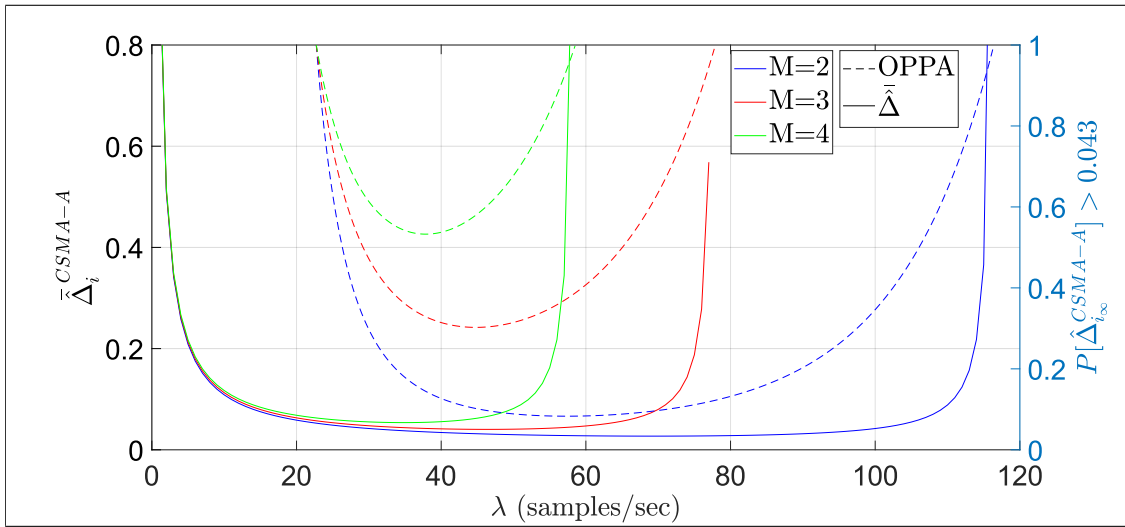


FIGURE 5.13. Average Peak Age and Outage probability of Peak Age in a CSMA-A transmission for  $\tau_{\text{MATI}} + \tau_{\text{MAD}} = 0.043$  ( $L=133$  bytes).

We first execute the same numerical analysis from subsection 4.1.1, establishing the optimal sampling frequency that allow us to minimize the probability that  $\tau_{\text{MATI}} + \tau_{\text{MAD}} = 0.044$  is surpassed and considering also the average peak Age. In Figure 5.13, it can be seen that the only configuration that achieves an average Peak Age lower than 0.044 are the ones with  $M = 2$  and  $M = 3$ . The optimal sampling rates to analyse are, for  $M = 2$ ,  $\lambda = 69.5$  for average Peak Age and  $\lambda = 57.3$  to minimize OPPA; for  $M = 3$  is  $\lambda = 46$  for

average Peak Age and  $\lambda = 44.75$  to minimize OPPA. We will analyse which sampling rate ensures a greater probability of stability for each configuration of number of loops.

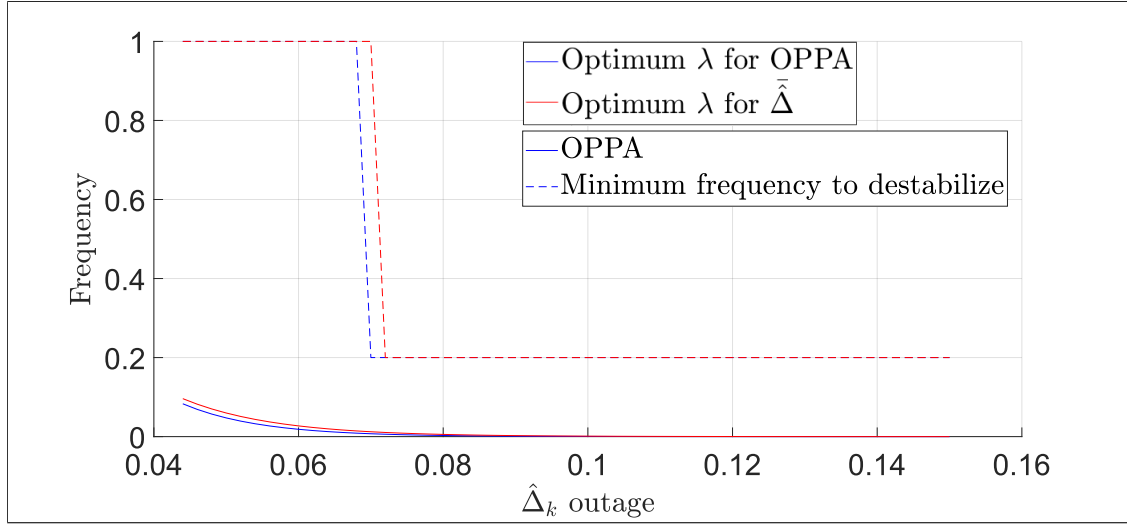


FIGURE 5.14. OPPA and outage frequency needed to destabilize the system for  $M = 2$  in CSMA-A, using two different sampling rates.

Figure 5.14 shows the minimum frequency of the outage needed to destabilize the example system for a short range of possible outage values, considering the system sampling at both the optimal sampling rate that minimizes OPPA and the sampling rate that minimizes the PAoI. It also includes the probability for each possible outage to occur theoretically when using CSMA-A for  $M = -2$ . It can be appreciated that, independent if we choose the value of  $\lambda$  that optimizes Peak Age or the OPPA for  $\tau_{\text{MATI}} + \tau_{\text{MAD}} = 0.044$ , the probability of exceeding the threshold is always below the frequency needed to destabilize the system, which means that we can assure that in the long term a WNCS with 2 batch reactors using CSMA-A with a packet length of 133 bytes will remain stable.

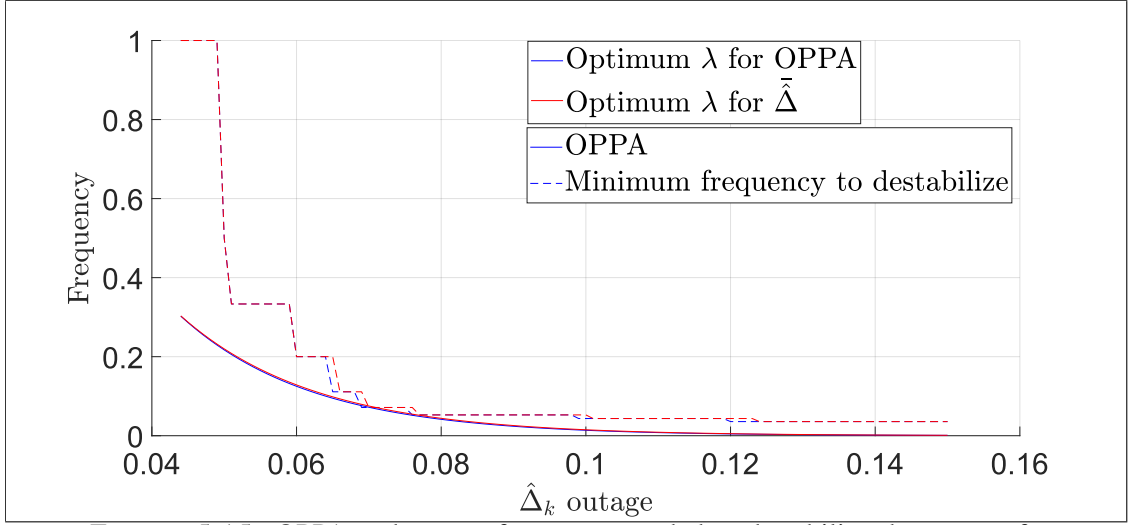


FIGURE 5.15. OPPA and outage frequency needed to destabilize the system for  $M = 3$  in CSMA-A, using two different sampling rates.

On the other side, in Figure 5.15 we have the same scenario but with  $M = 3$  batch reactors. However, here we found that the stability is not ensured. The OPPA curves for both sampling periods analysed slightly surpass the stability bound simulated when  $\hat{\Delta}_k = 0.07$  and  $\hat{\Delta}_k = 0.068$  for the case with the optimal sampling rate for Peak Age and OPPA for  $\tau_{\text{MATI}} + \tau_{\text{MAD}} = 0.044$ , respectively.

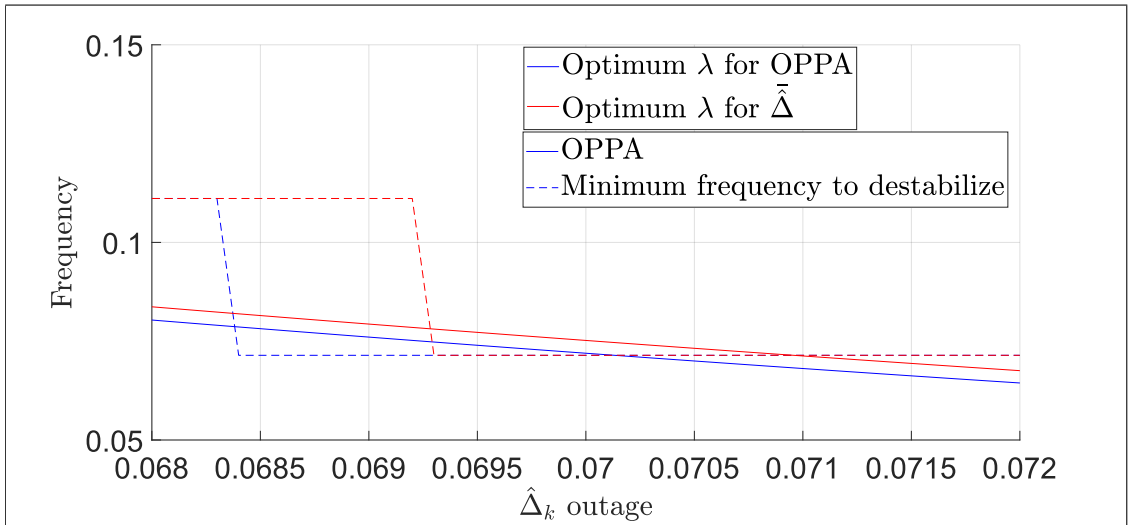


FIGURE 5.16. OPPA and outage frequency needed to destabilize the system for  $M = 3$  in CSMA-A, using two different sampling rates.

Conducting a deeper analysis, shown in Figure 5.16, allow us to establish that, when considering the optimal average Peak Age rate, the system is unstable for a range of delays value that represent a probability of 0.68%, while if the optimal OPPA rate is considered the probability of falling into instability is of 0.74%, which is slightly above, therefore determining that the optimal sampling rate for minimizing OPPA is not necessarily the optimal for obtaining the better chances for ensuring stability.

This discussion leads us to the following insights on this example: between CSMA and TDMA, in this case TDMA is preferable for a design that centers on stability. It can hold  $M = 3$  loops ensuring stability, while using CSMA only CSMA-A can hold the same 3 loops but with a slight probability (0.68%) of falling in instability. In this concrete example, the average Peak Age of CSMA-B with any  $M$  or CSMA-A with  $M > 3$  is above the stability constraint, which makes them not preferable. However, the designer should choose based on the design requirements. For example, if the design could hold a smaller packet length, the results of OPPA and average age for CSMA would be very different, and more flexible in terms of the number of loops the network can hold with a tolerable degree of instability probability, as can be seen in Figure 5.8. On the other side, if the packet length can not be changed, but the stability constraint is more flexible and the number of loops using the network is variable, CSMA can also be preferable, with the expense of a small uncertainty in stability.

Finally, the guidelines for a design based on stability is to choose the MAC protocol that allows more loops to share the network and for which grade of stability, depending on the packet length parameter. If a certain network does not meet the design stability requirements, a small number of loops will have to share the network in order to fulfil them.

## 6. CONCLUSIONS

### 6.1. Concluding remarks

The principal conclusions to this thesis must relate to the two problems stated in section 2.9. The first problem only considered the communication aspect of a WNCS, analysing the behaviour of the AoI on the three MAC protocols presented. In this issue, the results presented show that the protocols form different types of queues and different service time distributions, which cause different AoI curves, where the sampling rate that optimizes the AoI for each case is different. It was resolved that TDMA had a better performance for both AoI and PAoI, but with the trade-off of being more complex and having to sample at a higher rate. CSMA-A was also proven better for AoI and PAoI than CSMA-B because of the algorithm design and fewer energetic constraints.

The first problem served as a preliminary for the second one, as an important number of the results are based on the theoretical analysis realized for the first one. This second problem addressed the issue of expanding the metrics based on the status age to one concept that could be related to the stability of the loops integrating a WNCS using a specific MAC protocol. In this sense, the results presented can be used in different ways according to the modeling of the NCS loops and the design requirements. The OPPA calculations, on its side, serve to avoid surpassing a certain stability threshold frequently used in the NCS literature. This, however, may not be the optimal solution if the design wants to avoid the loops falling in instability, as seen in the example of section 5.5, where it was shown that minimizing the Peak Age may be a better strategy, depending on the specific loop behaviour. The optimal average Peak Age for the 3 protocols analysed have the same conclusions than AoI, as seen in Figures 5.6 and 5.7.

Most of the results in this work were presented in the form of theoretical bounds which involved assuming certain parameters and contexts. These bounds were confirmed by simulations, and were relatively tight in most of the cases, which proves that the assumptions made for the theoretical calculations had support. Also, the results showed that the global

AoI and Peak Age of the WNCS are optimized when all loops sample at the same time. This has implications for the system design when it is intended that different kind of loops share the network, as the aspect sensed may have different nominal sampling rates, that will require some sort of coordination if the objective is to have all the status updated as timely as possible.

The final conclusion is that the objective of studying the interplay between the communication aspect and the control loop performance in WNCS was fulfilled in a satisfactory way, through out the analysis of the metrics based on status Age, specially in the relationship between Peak Age and the MATI+MAD concept. However, in order to achieve a more concrete relationship between the time-related limits to stability and the delay provided by different MAC mechanisms and transmission techniques, different NCS reformulations should be studied. The next section state some ideas on how this work can be extended to accomplish further objectives.

## **6.2. Future research**

The work presented can be extended in many ways and from different perspectives. From the communications aspect, the AoI metric can be studied for new types of queue (Last-come first-serve) in other, specific Medium Access Control protocols such as OFDMA or SDMA, relying on new technologies related to the IoT and the new trends of Industry 4.0. Some technologies like 5G, 802.11ax and the future 6G may offer different solutions that can configure new specific queue types for the links involved in a WNCS whose analysis may be worth doing. However, while those grandiloquent advances can be used in the WNCS context, they also focus on increasing the transfer rate, which is not the most important aspect for our investigation. Other technologies, like the ones grouped as the low-power wide-area network (LPWAN), may offer different solutions as they center in offering long range communications in low power devices, with a slow data rate. Specifically, technologies as LoRa offers a PHY layer transmitting in radio frequency bands between 400MHz and 800 MHz with a data rate between 0.3kbps and 27kbps ([Adelantado et al., 2017](#)), far lower than the 250 kbps of the IEEE802.15.4 PHY layer, and therefore having a

greater AoI and PAoI if used with the MAC protocols analysed in this work, but specifically designed for obtaining greater range and for a greatly optimized power consumption in the devices using it.

In relation with the stability of the loops integrating a WNCS, the work could be extended in developing new ways to relate the relatively new age metrics studied (AoI, Peak and OPPA) with the time-related stability constraint of the loops. This involves exploring new concepts to integrate the importance of the delay and sampling in the modeling of control loops beyond MATI and MAD, to address the problem presented in section 5.5, related to theoretically answering the question of what is the limit for a certain stability constraint to be surpassed, and in which frequency, in order to fall in instability.

Finally, some improvements to the presented models can also be performed. For example, a change in the greedy TDMA algorithm can be proposed, in order to consider the stability constraints of the loops in the network, changing the focus on adapting the algorithm to keep the Peak Age of each transmission from each loop below the respective MATI + MAD value. For CSMA/CA, on the other side, a dynamic optimal sampling algorithm can be studied, based on the amount of loops competing for accessing the network. These new algorithms can be evaluated and compared with the already studied models, with the idea of improving the performance of the network in maintaining the loops stability.

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## **APPENDIXES**

## **APPENDIX A. ACRONYMS**

**AoI:** Age of Information.

**CSMA/CA:** Carrier sense multiple access with collision avoidance.

**DCF:** Distributed coordinator function.

**IFS:** Inter-frame space.

**IoT:** Internet of Things.

**LLC:** Logical link control.

**LPWAN:** Low-power Wide-area Network.

**MAC:** Medium Access Control.

**MAD:** Maximum allowable delay.

**MATI:** Maximum allowable transmission interval.

**MIATI:** Minimum allowable transmission interval.

**nBE:** non beacon-enabled.

**NCS:** Networked Control System.

**OPPA:** Outage Peak Age Probability.

**OSI:** Open Systems Interconnection.

**PAoI:** Peak Age of Information.

**QoS:** Quality of Service.

**SHS:** Stochastic hybrid system.

**TDMA:** Time-division multiple access.

**WNCS:** Wireless Networked Control System.