

The Range of Values of λ_2/λ_1 and λ_3/λ_1 for the Fixed Membrane Problem

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Abstract

We investigate the region of the plane in which the point $(\lambda_2/\lambda_1, \lambda_3/\lambda_1)$ can lie, where λ_1, λ_2 , and λ_3 are the first three eigenvalues of the Dirichlet Laplacian on an arbitrary bounded domain $\Omega \subset \mathbb{R}^2$. In particular, by making use of a technique introduced by de Vries we obtain the best bounds to date for the quantities λ_3/λ_1 and $(\lambda_2 + \lambda_3)/\lambda_1$. These bounds are $\lambda_3/\lambda_1 \leq 3.90514^+$ and $(\lambda_2 + \lambda_3)/\lambda_1 \leq 5.52485^+$ and give small improvements over previous bounds of Marcellini. Where Marcellini used a bound due to Brands in his argument we use a better version of this bound which we obtain by incorporating deVries' idea. The other bounds that yield the greatest information about the region where points $(\lambda_2/\lambda_1, \lambda_3/\lambda_1)$ can (possibly) lie are those due to Marcellini, Hile and Protter, and us (of which there are several, with two of them being new with this paper).