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Accounting for Uncertainty in Value Judgements when Applying Multi-Attribute Value Theory

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Abstract In environmental decisions, analysts commonly face substantial uncertainties around stakeholders' values judgments. Multi-Attribute Value Theory (MAVT), a family of multi-criteria decision analysis techniques, is applied in participative settings to articulate stakeholders' values in decision-making. In MAVT, value judgments represent the intensity of individuals' preferences in a set of objectives, which are operationalized as scaling factors or weights. Different sets of weights may express variation in people's preferences or value judgments. Unfortunately, there are still important methodological gaps regarding how to incorporate uncertainty and the substantial variation commonly encountered in stakeholders' preferences. This article presents a model of uncertainty that encompasses the dispersion of value judgments in MAVT. To achieve this goal, we draw on info-gap theory, which provides a mathematically grounded method for exploring sensitivity to preference weights when there are relatively high levels of uncertainties. We experimentally tested the uncertainty model in an environmental decision problem. We found that MAVT can use info-gap analysis to deal with multiple value judgments, avoiding exclusive reliance on nominal expected values to inform decisions. We explored a mechanism to explicitly consider the trade-offs between the performance of alternatives and the level of uncertainty that in any specified context a decision maker is willing to accept. Findings emphasize the potential of MAVT to support environmental management decisions, particularly in situations where multiple stakeholders and their contested value judgments have to be considered simultaneously to explore uncertainties around value trade-offs.

Keywords Multi-criteria decision analysis (MCDA) · Sensitivity analysis · Info-gap · Trade-offs · Expected values

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1 Introduction

Dealing with multiple stakeholders' values and preferences is a critical stage in environmental decision-making [1]. Nevertheless, integrating value judgments in decision-making is not straightforward [2]. Important environmental decisions typically involve adverse impacts for a subset of stakeholders who may disagree about what is relevant for a particular decision [3]. Articulating a multiplicity of perspectives and formally integrating value judgments in decision models have emerged as a methodological challenge for environmental managers and analysts [4, 5].

Multi-criteria decision analysis (MCDA) provides methods to systematically analyze multi-objective problems, evaluating and prioritizing alternatives under a set of parameters and criteria. These methods have been largely applied in different contexts to support environmental decision problems [6–8]. In

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addition, new developments consider formal protocols to explicitly include people's values and participation in decision-making [4, 9, 10].

Multi-Attribute Value Theory (MAVT) is a family of MCDA techniques, which is characterized by estimating expected values for a multi-attribute rank of alternatives. MAVT is based on explicit trade-offs among fundamental objectives [11, 12]. In this sense, MAVT is considered a compensatory approach, whereby poor performance on one objective can be offset by good performance in another objective [13]. In MAVT, value judgments represent the intensity of individuals' preferences in a set of objectives. Formally, value judgments are operationalized as scaling factors or weights, where the relative importance of one objective is related to others [14]. Weights express a subjective judgment: individuals express an equivalent valuation of the increment from the worst to the best performance on one fundamental objective (measured by an attribute) compared with the increment from the worst to best performance on another fundamental objective (measured by a different attribute) [15]. Decision analysis and cognitive psychology theory have developed formal protocols and procedures to elicit these weights from respondents, reducing arbitrary sources of uncertainty arising from measurement error [16].

Some participative environmental decisions are based on multi-sector management committees, which can hold systematic and deep value differences among their members. These differences will be expressed in the final set of attribute weights elicited from each individual. Though different sets of weights may express real discrepancies in people's preferences or value judgments, it may be necessary to aggregate preferences to progress consensus solutions. This is a particularly problematic task, which requires integrating numerous perspectives and potentially conflicting interests in one measure [17]. In these situations, a different kind of uncertainty emerges, associated with the set of weights that best represent a group's interests. In environmental settings, there is little guidance on methods or illustrative applications for incorporating variation in stakeholders' preferences on objectives in decision models. For example, Estévez and Gelcich [5] found that in 14 articles that actually have elicited individual weights from a sample of stakeholders in fisheries management and marine conservation, only four reported results of sensitivity analysis of weights in alternative performances. Therefore, in a multi-stakeholder decision setting, it is worth developing sensitivity analyses to explore a plausible range of weights that might reasonably reflect the individual and collective judgments of stakeholders [18–21].

One-dimensional and three-dimensional sensitivity analyses have been commonly applied to weights in MCDA [22]. One-dimensional analysis estimates the effect of a single weight in the output of the decision model, keeping the original ratios between weights of other criteria constant [23, 24]. This has limited practical application because interpretation obfuscates dissenting opinions. On the other hand, three-dimensional analyses can explore potential interactions among weights, evaluating different performance scores without the restriction of keeping original ratios of weights [15]. Nevertheless, in scenarios where group consensus is difficult to achieve, three-dimensional analysis does not consider measures of dispersion associated with each attribute's average weight.

In this article, we developed two experimental decision problems, involving the hypothetical culling of feral horses in Southern Australia, as a way to operationalize and exemplify our methodological approach. We propose a model of uncertainty that encompasses the dispersion of value judgments around the average of attributes' weights in MAVT. To achieve this goal, we draw on info-gap theory, which provides a mathematically grounded method for exploring alternative performances when there are relatively high levels of uncertainties [25]. We focus on finding robust alternatives relative to the uncertainty in the weights, which increases the prospects for acceptable outcomes for a broad range of stakeholders, as opposed to optimal outcomes for a small subset [26]. We complemented the info-gap model with additional sensitivity analysis to discriminate alternatives that do not perform satisfactorily. Finally, we explored a mechanism to explicitly consider a trade-off between the selection of the best alternative using the expected values of the uncertain weights and the selection of the best alternative that is the most "robust" to uncertainty.

1.1 The Additive Value Model

In MAVT, marginal value functions represent people's judgments about the desirability of individual objectives (measured by attributes), associated with a set of alternatives [13]. The attributes' value scores are generally normalized with a linear or polynomial function [12]. Then, the *additive value function* is commonly used to aggregate independent marginal value functions, according to a set of weights (Eq. (1)).

$$V\left(a_j\right) = \sum_{i=1}^{n} w_i v_i\left(a_j\right), \quad j = 1, \ldots, m \tag{1}$$

Where, $V(a_j)$ is the overall multi-attribute value of alternative a_j, $v_i(a_j)$ is the marginal value function of a_j on attribute i, w_i is the weight of attribute i, n is the number of attributes, and m is the number of alternatives [14]. In the additive value function, the values of w_i indicate the relative importance of the objectives in the specific context provided by the range of performance of alternatives. Each weight is non-negative and the sum of the weights is normalized to 1 (Eq. (2)), where, $w_i \in [0,1]$.

$$\sum_{i=1}^{n} w_i = 1, \quad w_i \geq 0, i = 1, \ldots, n \tag{2}$$

1.2 The Info-gap Model of Uncertainty

In decision science, estimating the uncertainty by deriving for each alternative the minimal value in the set of value functions have been previously considered [27, 28]. In this article, we relied on info-gap analysis to priotitize alternatives in the robustness function, which plots performance robustness against uncertainty, given a nominal model [29]. The info-gap robustness function is based on three elements: a particular decision-making model, the info-gap model of uncertainty, and a set of performance requirements [25]. In the info-gap framework, a more robust decision increases satisfactory results [26]. Therefore, more robustness is preferred to less robustness: the function maximizes the robustness of pre-defined adequate (satisfactory) outcomes [26]. In this context, "satisfactory" refers to a satisfactory aggregate performance for the additive value function.

In this article, $v_i(a_j)$ is a marginal value function with no uncertainty. The true weight (w_i) is an unknown value representing the relative importance of the attribute; the best estimate of w_i is w_i, with a dispersion around the value (w_i) represented by its error estimate (σw_i). In this article, w_i is the nominal value of w_i, which is calculated as the mean of respondents' weights.

In problems based on random samples or probability distribution, we can use the standard deviation or the coefficient of variation as an error estimate in the info-gap model. Nevertheless, the non-probabilistic approach of info-gap allows us to incorporate a subjective estimation of the weights' dispersion in the model, if the standard deviation is not available. We express the absolute fractional error of each w_i, calibrated by its own error estimate (Eq. (3)). We refer to h as the horizon of uncertainty. We adopted a non-probabilistic fractional-error info-gap model of uncertainty, defining $U(h)$ as the set of all possible vectors of weights that satisfy a pre-defined horizon of uncertainty (Eq. (4)) [26].

$$\left| \frac{w_i - w_i}{\sigma w_i} \right| \leq h \tag{3}$$

$$U(h) = \left\{ w : \left| \frac{w_i - w_i}{\sigma w_i} \right| \leq h, w_i \in [0, 1], \sum w_i = 1, i = 1, 2, \ldots, n \right\}, h \geq 0 \tag{4}$$

In this info-gap model, when σw_i is equal to 0, w_i is equal to w_i for all values of h; thus, the model contains only the nominal estimate values of w_i.

The performance requirement establishes a minimum acceptable value for the outcome; it represents what the decision makers aspire to achieve [26]. The robustness function is based on satisfying this performance requirement. In this paper, V_c pre-defines a performance requirement, where the value of $V(a_j)$ is satisfactory if $V(a_j) \geq V_c$ (Eq. (5)).

$$\min_{w \in U(h)} V(a_j) \geq V_c \tag{5}$$

The robustness (h) to uncertainty of the weights for alternative a_j is the greatest horizon of uncertainty, h, which satisfies the performance requirement (V_c). The result cannot be negative (Eq. (3)). Higher values of robustness suggest that the alternative will be more immune to uncertainty in the weights. On the other hand, lower values of robustness suggest that the alternative is more vulnerable to uncertainty in the weights (Eq. (6)).

$$h(a_j, V_c) = \max \left\{ h : \left(\min_{w \in U(h)} V(a_j) \right) \geq V_c \right\} \tag{6}$$

When defining V_c, we must be careful in setting a value such that the set (Eq. (7)) is not empty. To assure this, we need to satisfy (Eq. (8)):

$$\left\{ h : \left(\min_{w \in U(h)} V(a_j) \right) \geq V_c \right\} \tag{7}$$

$$\sum_{i=1}^{n} w_i v_i(a_j) \geq V_c \tag{8}$$

Considering Eq. (6) and Eq. (7), the satisfactory condition of one alternative depends on the results of the optimization problem (Eq. (9)),

$$\min_{w \epsilon U(h)} V(a_j) \tag{9}$$

under different horizons of uncertainty ($h \geq 0$). The evaluation of alternatives has to consider the satisfactory condition (Eq. (5)).

The uncertainty model presented in Eq. (4) can also be expressed as Eq. (10),

$$U(h) = \left\{ w : \sum_{i=1}^{n} w_i = 1, w_i \in [0, 1] \cap \left[w_i - h\sigma_{w_i}, w_i + h\sigma_{w_i} \right], i = 1, \ldots, n \right\}, h \geq 0 \tag{10}$$

Here, it is important to note that,

$$[0, 1] \cap \left[w_i - h\sigma_{w_i}, w_i + h\sigma_{w_i} \right] = [0, 1] \tag{11}$$

if Eq. (12) and Eq. (13),

$$w_i - h\sigma_{w_i} \leq 0 \Rightarrow \frac{w_i}{\sigma_{w_i}} \leq h \tag{12}$$

$$w_i + h\sigma_{w_i} \geq 1 \Rightarrow \frac{1 - w_i}{\sigma_{w_i}} \leq h, i = 1, \ldots, n \tag{13}$$

then, Eq. (14) and Eq. (15),

$$h \geq \max_{i=1,\dots,n} \left[\max \left\{ \frac{w_i}{\sigma_{w_i}}, \frac{1-w_i}{\sigma_{w_i}} \right\} \right] \tag{14}$$

$$U(h) = \left\{ w : \sum_{i=1}^{n} w_i = 1, \quad w_i \epsilon \, [0,1], \quad i = 1, \dots, n \right\}, \quad h \geq 0 \tag{15}$$

and the value of Eq. (9) does not change for those horizons of uncertainty. In doing so, in this range of values for h, and considering Eq. (5), even for a constant increase of the horizon of uncertainty ($h \rightarrow +\infty$), the alternative under evaluation will be satisfactory. In this scenario, the robustness function is defined as Eq. (16), expressed in this case as Eq. (17).

$$h\left(a_j, V_c\right) = \sup \left\{ h : \left(\min_{w \in U(h)} V\left(a_j\right) \right) \geq V_c \right\} \tag{16}$$

$$h\left(a_j, V_c\right) = +\infty \tag{17}$$

In this article, the performance under uncertainty ($m_j(h)$) corresponds to the minimum value of the function $V(a_j)$ for all w belonging to $U(h)$ (Eq. (18)).

$$m_j(h) = \min_{w \epsilon U(h)} V\left(a_j\right) \tag{18}$$

Cost of robustness (C) refers to the quality of the outcome that is sacrificed for an increase in the level of robustness [26]. This can be expressed as the derivative of the function $m(h)$ with respect to the horizon of uncertainty (h), in $h = 0$ (Eq. (19)).

$$C = {}^{d\left(m_j(h)\right)}/_{d}(h) \tag{19}$$

The value 0 on the robustness axis represents the point where the performance requirement is equal to the highest nominal multi-attribute performance score. Such nominal performance scores do not consider the sensitivity of decisions to uncertainty in the value judgments. In this sense, there is no guarantee that alternatives will achieve these values. This property is known as zeroing in the info-gap framework [26]. The negative slope indicates that as the value of h increases, the performance requirement must become more modest.

In this article, we tested the info-gap model of uncertainty for weights in the additive value function in two experimental decision problems about a hypothetical feral horse management problem in Southern Australia.

2 Methods

Environmental management decisions are typical examples of multi-objective decisions where individuals require to perform trade-offs among fundamental objectives. In Southern Australia, decisions involving the management of feral horses are controversial, because decision makers are required to make value judgments on the lives of charismatic and culturally relevant animals such as brumbies (*Equus caballus*). In our experimental setup, we focused on eliciting weights from participants that had to analyze trade-offs in a consequence matrix, including five alternatives and three objectives: *total management cost* (COST), *number of plant species threatened with extinction by horses* (PLANTS), and *number of horses killed* (HORSES) (Table 1).

We generated two decision problems, what only varied between them was the range of consequence for each objective. All participants were introduced to the same hypothetical management decision in the Kosciuszko National Park:

> "Australia has more than 400,000 wild horses or brumbies, the largest population in the world. Memorials, museums, films and books recognize their economic and cultural role in the history of Australia. However, ecologists highlight their impact on native plants. Feral horse management is controversial, attracting much scrutiny and adverse publicity. Kosciuszko National Park was established in 1944 and is one of the largest conservation reserves in Australia. It contains continental Australia's highest mountains and a great variety of floristic

Table 1 Consequence matrices for the hypothetical decision problems. Attribute—specific performance of alternatives in a) decision problem 1 and b) decision problem 2.

Alternatives	Total management cost (AUD)	Number of plant species threatened with extinction by horses	Number of horses killed
a)			
A	250,000	2	800
B	100,000	3	500
C	200,000	5	400
D	90,000	6	200
E	50,000	10	50
b)			
A	260,000	2	1100
B	170,000	3	700
C	120,000	4	400
D	90,000	6	270
E	80,000	7	100

AUD Australian dollars

Table 2 Mean, standard deviation (*Stdev*) and coefficient of variation (*CV*) for objectives' weights in decision problems 1 and 2

Objectives	Decision problem 1 ($N = 36$)			Decision problem 2 ($N = 34$)		
	Mean	Stdev	CV	Mean	Stdev	CV
Minimize management cost (COST)	0.22	0.10	45.45	0.22	0.11	50.00
Maximize plant species protection (PLANT)	0.48	0.13	27.08	0.55	0.11	20.00
Minimize horses killed (HORSE)	0.30	0.10	33.33	0.23	0.14	63.64

Weights were normalized to sum to 1

communities. Today, 1,700–3,000 feral horses live in the park, increasingly having an impact on the native flora. Decision-makers are developing a Horse Management Plan. In considering the merit of alternative management options, they have identified three relevant objectives: minimize the financial cost of implementing the plan, minimize loss of native plant species and minimize the number of horses killed. Table 1 shows the objectives, their attributes, and the full range of impacts (best and worst outcomes) under the set of candidate management options. Note that the Number of species threatened with extinction refers only to the number of species affected by horses (through grazing or trampling). The number of horses killed refers mainly to aerial shooting, although candidate management plans may include on-ground capture and euthanizing of horses. Costs vary according to different cull methods and the accessibility of horse population."

After reading the problem statement, participants were asked to articulate trade-offs among objectives, using the swing weights method[1] [14].

Responses for both decision problems were collected during the months of July and August 2012. In decision problem 1, participants were 36 post-graduate environmental science students at the University of Melbourne. The questionnaire was offered during coursework and participation was voluntary and not related to students' grades. Students responded to the questionnaire with pen and paper, taking around 20 min. In decision problem 2, participants were recruited from a list of 26 post-graduate students and 54 academics/researchers in the School of Botany, University of Melbourne. A total of 34 individuals (42.5% response rate) participated in the decision problem, consisting of 15 students and 19 academics/researchers. The sample is not representative of researchers and students of School of Botany at University of Melbourne. This research was approved by the Human Ethics Committee of the University of Melbourne (HREC 1135693.2).

2.1 Expected Values, Sensitivity Analysis, and Info-gap

We calculated the mean and standard deviation of respondents' weights, searching for statistical differences among nominal weights (ANOVA). We checked normality of the weights' distributions with the Shapiro-Wilk normality test. Then, we calculated the coefficient of variation (*CV*) (the ratio of the standard deviation to the mean) of the weights to normalize the measure of dispersion. We normalized each objective v_i in the consequence matrix with a linear value function, with 0 representing the worst value and 100 the best, which is an often-made assumption in MAVT [12].[2]

We calculated the nominal expected value ($V(a_j)$) for each alternative by the additive value function (Eq. (1)), using the arithmetic mean of respondents' weights (w_i) (Eq. (2)). These results can identify the best alternative based on the available information for this particular decision problem, assuming no uncertainty in the consequences and the weight average as the best estimation of respondents' opinions. Nevertheless, these nominal expected values do not consider the dispersion of value judgments around the arithmetic mean of respondents' weights.

We developed a three-dimensional sensitivity analysis using *Eclipse* and plotted with *R ggplot2* library. We compared alternative performances for all possible combinations of weights in objectives, regardless of respondent's opinions, and observed structurally dominated alternatives and conditions for the highest nominal values for each alternative. The same exercise was developed considering ranking in weights between objectives.

We resolved the info-gap model of uncertainty (Eq. (4)), resolving the optimization problem (Eq. (9)). We used the function '*linpro*' (toolbox '*Quapro*') (codes were written in

[1] Questionnaires for decision problems 1 and 2 are included in Online Resource.

[2] Marginal value functions for decision problems 1 and 2, respectively: COST ($Y = -0.0005X + 125$; $-0.0005556X + 144.4$), PLANTS ($Y = 12.4X + 125$; $Y = -20X + 140$), HORSES ($Y = -0.1333X + 106.6667$; $-0.1X + 110$).

Table 3 Expected values and cost of robustness for decision problems 1 and 2

Plans	Decision problem 1 (N = 36)			Decision problem 2 (N = 34)		
	Nominal expected values	Respondents (%)	Cost of robustness	Nominal expected values	Respondents (%)	Cost of robustness
A	48.0	16.7	−13.0	55.0	26.5	−11.0
B	64.4	61.1	−5.1	57.4	50.0	−4.7
C	42.2	8.3	−4.0	57.8	23.5	−2.0
D	56.8	13.9	−3.9	46.6	0	−8.2
E	52.0	0	−13.0	45.0	0	−11.0

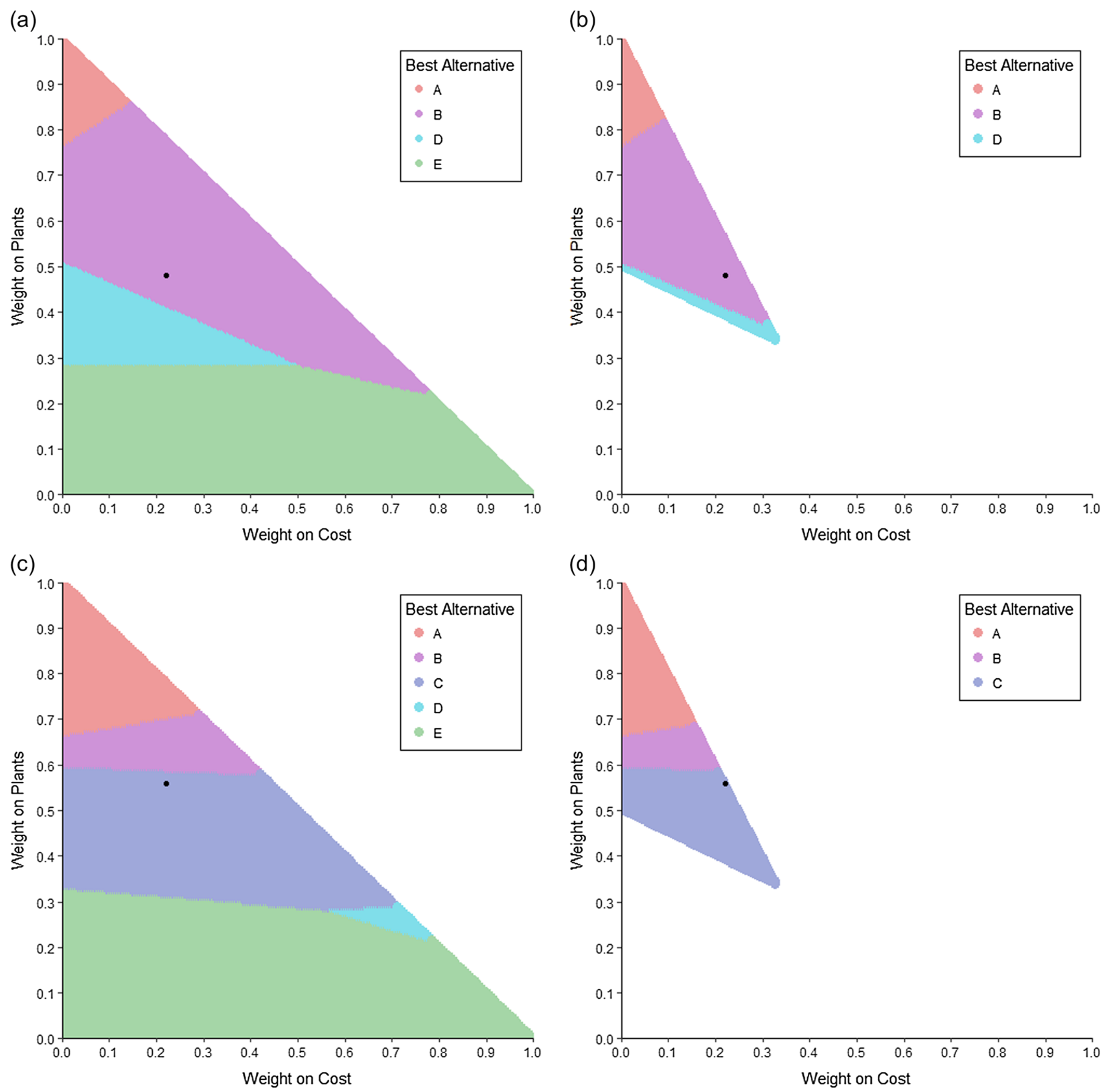

Fig. 1 Alternative with the highest nominal expected value at all weight combinations: **a** decision problem 1, **c** decision problem 2. Alternative with the highest nominal expected value at all weight combinations with ranking constraint: **b** decision problem 1, **d** decision problem 2. *Black dots* correspond to the nominal weights

Scilab). We finally developed an uncertainty analysis to explore how far it is worth to sacrifice the performance requirement for robustness. We hypothesized that, in some cases, the more robust alternative may show its benefits (higher expected value under uncertainty) only under rare situations (very big deviations from nominal weights).

3 Results

Table 2 presents the mean, standard deviation and coefficient of variation for weights in decision problems 1 and 2. In general, both decision problems have similar results, despite independent samples. PLANTS was the most important objective for respondents in decision problem 1 (ANOVA-F test = 41.4, $p < 0.001$) and decision problem 2 (ANOVA-F test = 26.3, $p < 0.001$). Then, COST and HORSES have a relatively similar degree of importance, with slightly higher mean weights for HORSES. In general, weights were normal or near-normally distributed, except for HORSES in decision problems 1 and 2 and PLANTS in decision problem 2 (Appendix). In both decision problems, PLANTS presents a relatively small dispersion of weights ($CV_{\text{decision problem 1}}$ = 27.1%, $CV_{\text{decision problem 2}}$ = 20.0%), which is interpreted as a higher level of consensus about its degree of importance in this particular decision context. In contrast, COST and HORSES had higher dispersion of weights, representing a lower degree of consensus about their relative importance.

Table 3 presents the nominal expected values for each alternative. In decision problem 1, alternatives B and D have the highest expected values (64.4 and 56.8, respectively); alternatives A, C, and E have less than 53 points of performance. Interestingly, most respondents selected alternative B (61.7%), concordant with the alternative with the highest nominal expected value. In decision problem 2, alternatives B and C have the highest nominal expected values (more than 57), being selected by 73.5% of the respondents. It is important to note that despite differences described between respondents' choices and nominal expected values, in both decision problems, the two alternatives with the highest nominal expected value were selected by more than the 70% of respondents (75.0% in decision problem 1).

We developed a tridimensional sensitivity analysis for both decision problems (Fig. 1). In decision problem 1, alternative C is structurally dominated by other alternatives, that is, for no weight combination, this alternative has the highest nominal expected value (Fig. 1a). Nevertheless, it was chosen by 8.3% of respondents. Additionally, it can be observed that alternative E has the highest nominal value only when the weight of PLANTS is less than 0.3, a condition which explains why no respondent chose this alternative (Table 3). Figure 1b shows alternatives with the highest nominal expected value based on the constraint that the order of importance is PLANT, HORSE, and COST. In this scenario, there are only three alternatives available. Alternative B is dominant in the majority of weight combinations. In decision problem 2, Fig. 1c shows that alternative C is dominant in almost all scenarios when the weight of PLANTS is between 0.35 and 0.60. Alternative B is almost always dominant in scenarios when the weight of PLANTS fluctuates between 0.6 and 0.7.

Based on the three-dimensional analysis, the nominal expected values, and respondents' choices, we could narrow the competing alternatives to two for each decision problem (B and D in decision problem 1, B and C in decision problem 2). Figure 2 presents the robustness curves for the alternatives in both decisions problems. In decision problem 1 (Fig. 2a), alternative B has the highest nominal expected value (70.5). Nevertheless, it has a higher cost of robustness than alternative D, with the second highest nominal expected value (65.6) (Table 3). The curves of alternatives D and B intersect at the critical point of 50.0, under a level of uncertainty (h) of 4.1. If we accept a performance requirement lower than the critical value of 50.0, alternative D could be recommended

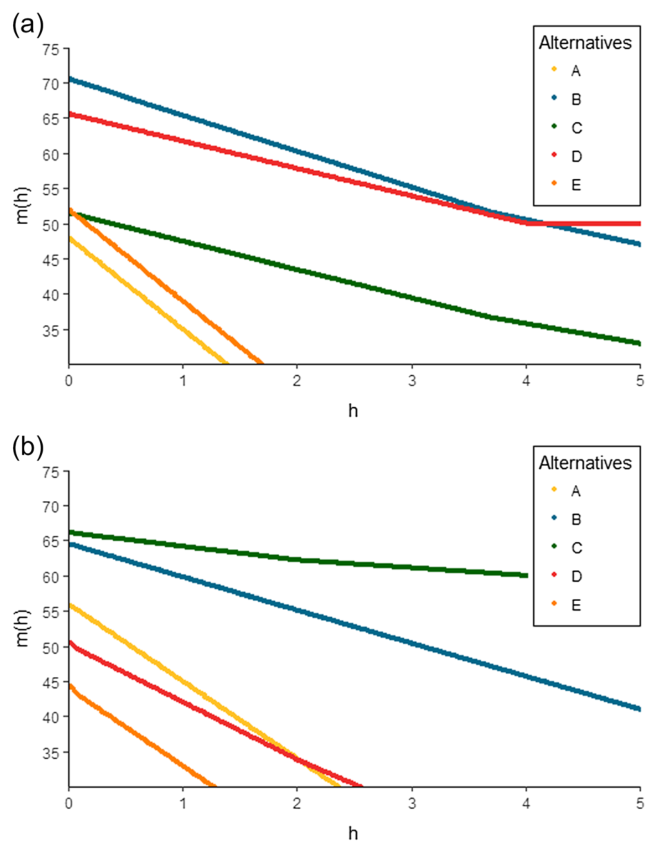

Fig. 2 Robustness curves for the two alternatives with the highest nominal expected values: **a** decision problem 1, **b** decision problem 2. h = horizon of uncertainty, $m(h)$ = performance under uncertainty

over alternative B. In decision problem 2, we find that alternative C held the highest performance score and the lowest cost of robustness; consequently, alternative C would be preferred over alternatives A, B, D, and E at any performance requirement (Fig. 2b).

4 Discussion

We found that MAVT can use info-gap analysis to explore sensitivity of multiple value judgments, avoiding exclusive reliance on nominal expected values to inform decisions. We developed an info-gap analysis to identify acceptable alternatives, based not only on nominal performances scores but also on relative immunity to dispersion on weights. Under such circumstances, an alternative with a lower nominal performance score may be preferred over an alternative with a higher nominal value, if its behavior against sensitivity on weights is more desirable. This is referred to as preference reversal in info-gap literature [26].

Robustness about the best set of weight that represents a group position is definitely a desirable characteristic of the alternative to be chosen. Nevertheless, it might be reasonable to consider for which value of uncertainty this robustness is actually a reference to make a better choice.

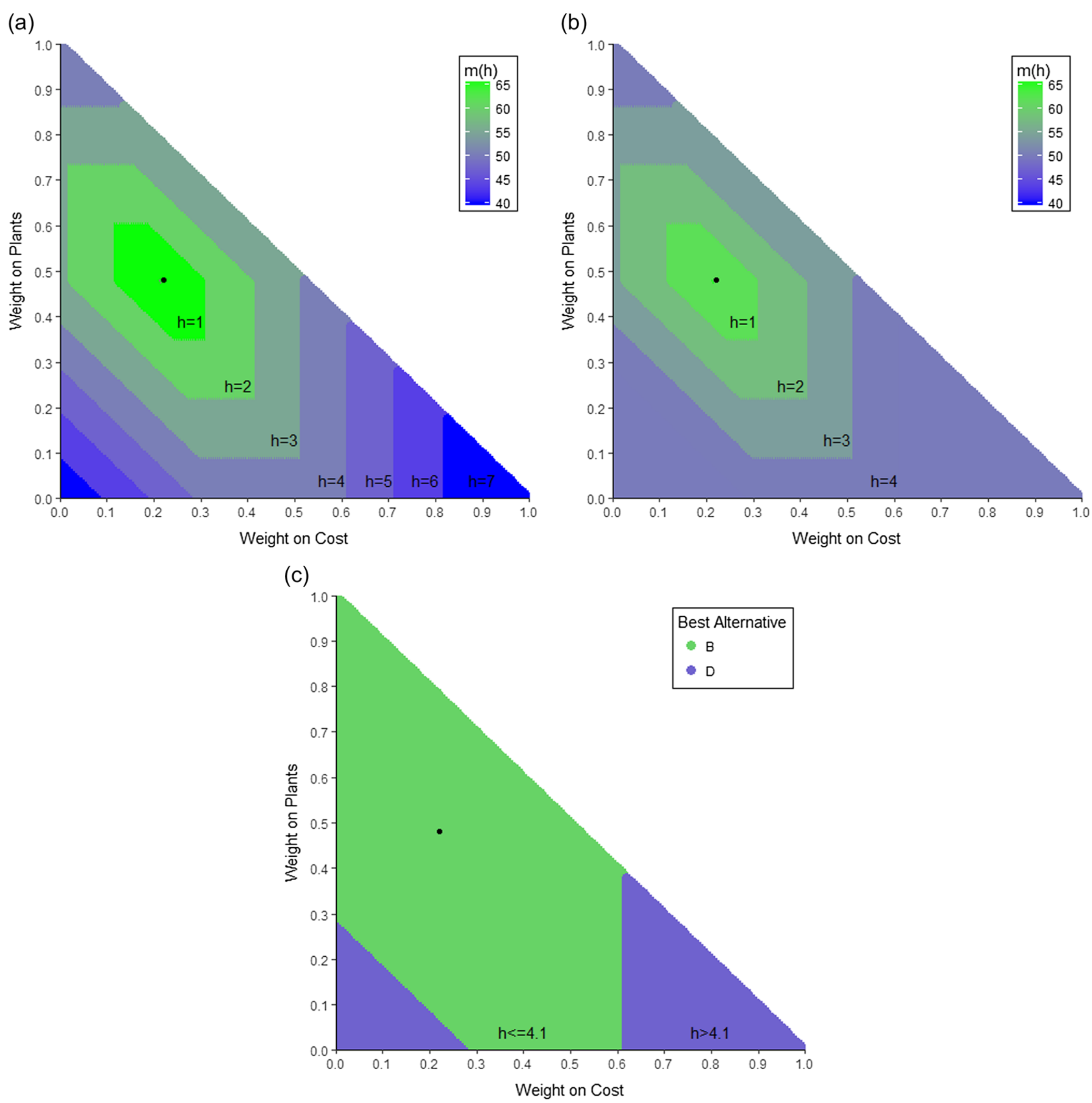

Fig. 3 Performance under uncertainty ($m(h)$) at all weight combinations, indicating levels of uncertainty in decision problem 1: **a** alternative B and **b** alternative D. **c** Comparison of alternatives B and D indicating which has higher $m(h)$ at all weight combinations. *Black dots* correspond to the nominal expected value

In other words, under which circumstances the more robust alternative has a higher expected value than the other less robust option. If the level of uncertainty (h) that makes the robust alternative and the most desirable one is too high, choosing this alternative seems too costly compared to the expected value that is lost for all those situations where the level of uncertainty is smaller. The question can be formulated in terms of the trade-offs between the level of uncertainty considered and the difference in the expected value that implies a change in alternative with the best performance score.

In decision problem 1, for levels of uncertainties higher than $h = 4.1$, we conclude that it would be plausible to choose alternative D over alternative B, if the performance requirement is lower than 50. Nevertheless, we should consider if it is useful to evaluate the expected value of alternatives for values of h equal or bigger than 4.1. In other words, what is the probability that the real weights are at such levels of dispersion from the estimated weights? In Fig. 3a and b, different colors indicate the values of $m(h)$ achieved at all weight combinations, indicating the respective different levels of uncertainty, for alternatives B and D. Figure 3c compares both alternatives, indicating which option has a higher performance under uncertainty ($m(h)$) at each weight combination. We can observe that in the area with $h \leq 4.1$, alternative B has higher $m(h)$ than D, covering most of the potential combinations of weights in objectives, particularly in the area around the nominal weights. In consequence, the benefit of the robust alternative D, is only under circumstances where $h \geq 4.1$, which seems to be too high. If we assume normal distributions of the weights (Appendix) and considering the data presented in Table 2, the probability of real weights at a distance h bigger or equal to 4.1 from the nominal weights is around 0.018%.

Before making a decision between the two alternatives, where one is more robust than the other but has a smaller nominal expected value, it is advisable to consider the range of dominance of each alternative under the different combinations of weights that the different values of h allow. As an example, the scenario presented in decision problem 1 shows that the less robust alternative has a higher performance under uncertainty, $m(h)$, for most of the weight combinations, and that assuming a situation of $h > 4.1$, where the robust alternative dominates, seems quite unlikely. Robustness curves make plain the trade-off between expected performance and insulation to the adverse implications of uncertainty. Decision makers need to consider their level of risk aversion in arriving at a preferred alternative.

In environmental decision problems, variations in weights may represent irreducible differences in value judgments. In these cases, estimating a nominal expected value based on a single underlying true weight may be inappropriate. In this article, we present a sensitivity analysis to explore alternative performances that could be satisfactory to different stakeholders, despite different combinations of weights. The findings presented here emphasize the potential of multi-criteria decision analysis to support resource management decisions, particularly in situations when multiple value judgments have to be considered simultaneously to explore uncertainties around value trade-offs. In these scenarios, decision analysis can articulate the many objectives of those stakeholders that stand to win or lose in management decisions. These methods, supported by advances in mathematical tools such as info-gap analysis, can be used to negotiate and satisfy stakeholders' expectations.

5 Conclusion

Decision makers commonly have to make difficult environmental decisions, characterized by substantial uncertainties around the things that people value. Sometimes, these decisions trigger clashes among social groups holding different value judgments, making it difficult to find consensus solutions. Despite such difficulties, participatory approaches require decision makers to evaluate trade-offs and the implications of uncertainty explicitly. In this context, we have provided a tool for MAVT that will assist decision makers to consider the uncertainties of value judgments in the decision analysis process, thereby promoting effective democratic and participative decision-making. We showed the potential of using the info-gap model of uncertainty to consider uncertainties around value judgments, when the decision is potentially affected by disagreements about the weights. We explicitly consider the trade-offs between the performance of alternatives and the level of uncertainty that in any specified context a decision maker is willing to accept. We found that MAVT can use info-gap analysis to deal with multiple value judgments, improving on the reliance of the average weights (nominal performance score) to inform decisions.

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Appendix

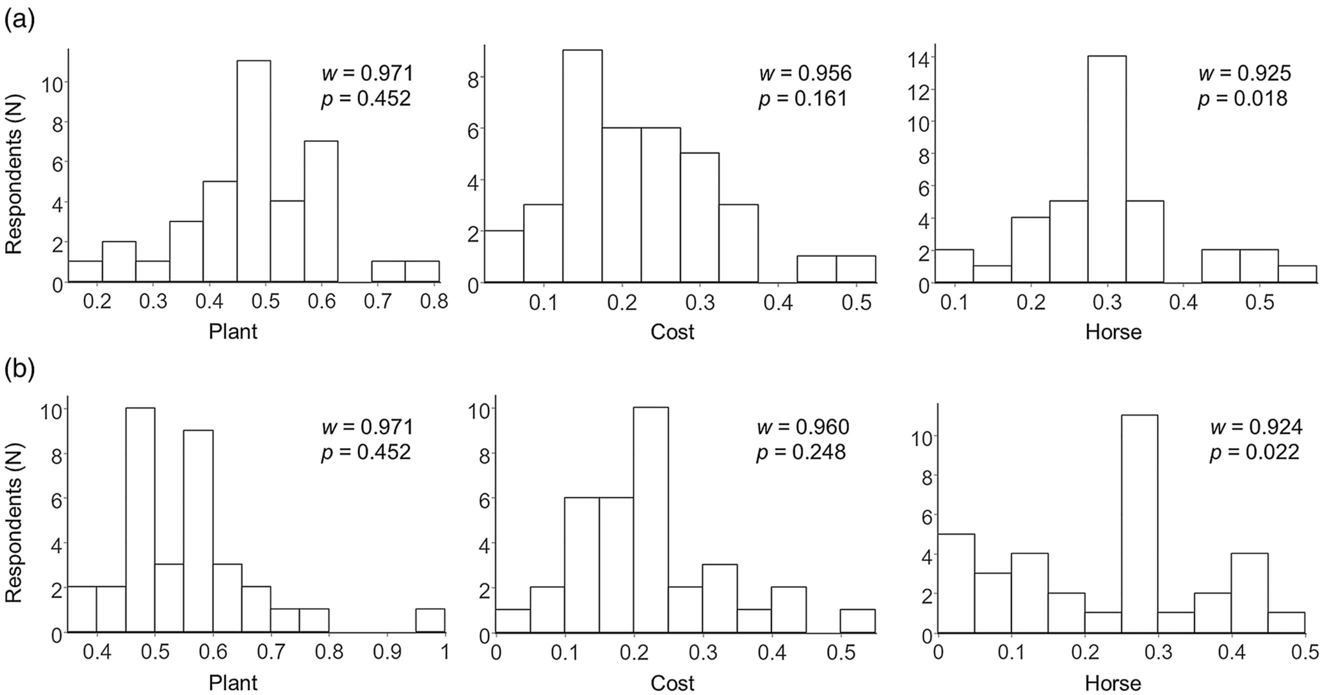

Fig. 4 Normality test and distributions of weights for decision problem 1 (**a**) and decision problem 2 (**b**). *w* = Shapiro-wilk normality test, *p* = *p* value

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