PONTIFICIA UNIVERSIDAD CATOLICA DE CHILE
SCHOOL OF ENGINEERING

APPLYING REAL OPTIONS IN THE TV MARKET: A SIMPLE FRAMEWORK FOR VALUING FLEXIBILITY

ENRIQUE T. HEDERRA

Thesis submitted to the Office of Research and Graduate Studies
in partial fulfillment of the requirements for the degree of
Master of Science in Engineering

Advisor:
PATRICIO DEL SOL

Santiago de Chile, November 2013

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Gratefully to my family

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ABSTRACT

This paper develops a simple framework for supporting decisions on whether and when to cancel a television program. The framework uses real options theory to maximize the profits a TV program will earn. A station or network programming director can limit downside losses on an unsuccessful program by being flexible regarding the number of episodes to be shown and reviewing its continuation once it has been launched and uncertainty about viewers' reactions has subsided. The framework was applied to real data from the Chilean television market, generating decisions on the continuation or cancellation of TV series that would have resulted in an increase in profits actually earned on these shows of 12.4%.

Keywords: Decision making/process, Planning and control, Real options, Television audience, Linear mixed model, Case study, Simulation.

RESUMEN

Este artículo desarrolla un método de apoyo simple para decidir si cancelar o no un programa de televisión en un momento dado. El método usa teoría de opciones reales para maximizar las utilidades que un programa de TV puede generar. El director de programación de una cadena o canal limita las pérdidas de un posible fracaso siendo flexible y revizando, luego del lanzamiento de un programa y con más certeza de su recepción por parte del público, el número total de capítulos. El método se aplica a datos reales pertenecientes al mercado de televisión Chileno, generandose decisiones sobre la continuidad o cancelación de series de televisión que habrían resultado en un incremento de las ganancias reales en estos programas de 12.4%.

Palabras Claves: Toma de decisiones/proceso, Planeamiento y control, Opciones reales, Audiencia televisiva, Modelo linear mixto, Caso de estudio, Simulación.

1. GENERAL INTRODUCTION

This thesis contains the text of an article submitted to an academic journal. The abstract is the same for both documents. The article is introduced in Section 2. Section 3 presents a simplified real options framework to illustrate the required elements and the method of calculating expected profits when flexibility is present; Section 4 applies the whole real options framework to the actual Chilean television market; and the last section presents our conclusions on the use of the proposed framework and its advantages.

2. INTRODUCTION

Spending on television advertising in the United States reached $71.8 billion in 2011, 4.5% more than the $68.7 billion spent in 2010 and 12.7% greater than the $63.7 billion paid out in 2009 (Marketing charts, 2012). Spot television rates are a direct function of network ratings (Gensch & Shaman, 1980), an indicator of the performance of TV programs that measures the average percentage of households or viewers in the potential audience who are tuned in to a particular program at a given time.

An important characteristic of the television industry is that it is a *winner-takes-all* business. Having the top few shows can create domino effects all the way down a network's lineup. An analysis of TV ratings in USA reveals that in most weeks, the network with the number one program also has the week's highest average ratings (Anand, 2002). Thus, the advantage of the market leader may be much more fragile than is apparent from market share data. If the leading show changes (and it often does), it is highly probable that the leading channel will too. This is what lies behind the high volatility of program ratings. Network Programming Directors (PDs) must therefore give careful consideration to decisions regarding when a TV program is successful and should be continued and when it is not and should be canceled. This is particularly true of prime-time programs, whose performance has a major impact on a channel's overall revenue.

An increasingly influential tool in management practices for decision making under uncertainty is real options analysis (Brennan & Schwartz, 1985; Cortazar & Schwartz, 1993). Framing real investment decisions as analogous to financial options, real option theory argues that value can be created by breaking one large investment decision into a series of smaller ones. Spreading investments over time allows managers to respond to unfolding contingencies. By investing in flexibility, managers can take advantage of upside (gain) outcomes and avoid downside (loss) outcomes (Miller & Waller, 2003).

The problem of making flexible decisions under uncertainty using real options has been studied for different areas such as natural resource extraction (Paddock, Siegel, & Smith, 1988; Tourinho, 1979; Brennan & Schwartz, 1985; Cortazar, Schwartz, & Casassus, 2001), real estate (Titman, 1985), facilities planning (McDonald & Siegel, 1985), environmental investments (Cortazar, Schwartz, & Salinas, 1998), manufacturing (Kulatilaka, 1988; Kulatilaka & Trigeorgis, 1994; Bengtsson & Olhager, 2002) and the retail trade (Tsai & Hung, 2009). However, in other decision areas closer to marketing, and particularly in television, the literature is sparse.

Applying real option models to a problem in television programming necessarily means incorporating uncertainty. This in turn implies the need for forecasting of both the average and the variance of a program's ratings. The process of television ratings forecasting has received considerable attention, specially in the 1980s, given that any purchase of television advertising time involves buying a predicted audience, yet the recent literature on ratings forecasting remains limited to a few papers (Napoli, 2001; Danaher, Dagger, & Smith, 2011; Gensch & Shaman, 1980).

Though ratings forecasts can be used in deciding whether or not to cancel a show, applying them directly underestimates expected profits because they do not take into account the flexibility management has to cancel a show earlier than planned. The variance of the ratings should also be carefully considered given that the expected profit will be biased if the relationship between earnings and ratings is not linear. There have been efforts to solve this issue through a partially observed Markov decision process (Givon & Grosfeld-Nir, 2008) but no academic contribution, that we are aware of, has attempted to solve the problem using real options.

The purpose of this article is to design a simple and adaptable real options framework (ROF) for calculating the expected profit of a TV program that explicitly considers the value of flexibility. The development of this design is complemented with a case study that uses real data from Chilean television.

The remainder of this paper is organized into three sections. Section 3 presents a simplified real options framework to illustrate the required elements and the method for calculating expected profits when flexibility is present; Section 4 applies the whole real options framework to the actual Chilean television market; and the last section presents our conclusions on the use of the proposed framework and its advantages.

3. A REAL OPTIONS MODEL FOR MAXIMIZING PROFITS OF A TV PROGRAM

Though high ratings are by no means the only objective pursued by network broadcasters, they are obviously a central one and have major financial implications (Horen, 1980). In all previous studies on maximizing a TV program's profit with the exception of Givon and Grosfeld-Nir (2008), the number of episodes of a program is considered fixed and the main objective is to increase profits by gaining audience share over the other networks. To accomplish this, they focus on two widely used approaches: either produce more popular shows than the other networks or design optimal schedules for the whole season by using different competitive scheduling strategies (Danaher & Mawhinney, 2001; Horen, 1980; Henry & Rinne, 1984a). This paper proposes a third approach in which the number of episodes of each program is considered to be variable and is optimized to respond, in an already running schedule, to continual changes in program ratings by extending successful programs and thus increase profits or canceling unprofitable ones to reduce losses.

To formulate our real options model, we assume that a program is initially designed to have 0, 1, 2 or 3 episodes and that its ratings follow a discrete binomial tree as shown in Figure 3.1, where each node represents a possible rating outcome and the probability of increasing or decreasing the rating on the next episode. In this example, the relationship between profits and ratings is $Profit = 4 \times Rating - 60$. We further assume that the Programming Director (PD) is able to cancel the program and replace it at any moment by another one that neither wins nor loses money. Finally, for simplicity we also assume that the discount rate is 0%.

In Trigeorgis (1996) the author notes that the standard net-present-value (NPV) rule cannot capture management's flexibility to subsequently adapt and revise the NPV-based decisions in response to unexpected market developments. To illustrate how the amount of flexibility to review decisions affects a program's expected profit, we define three possible scenarios using the example in Figure 3.1:

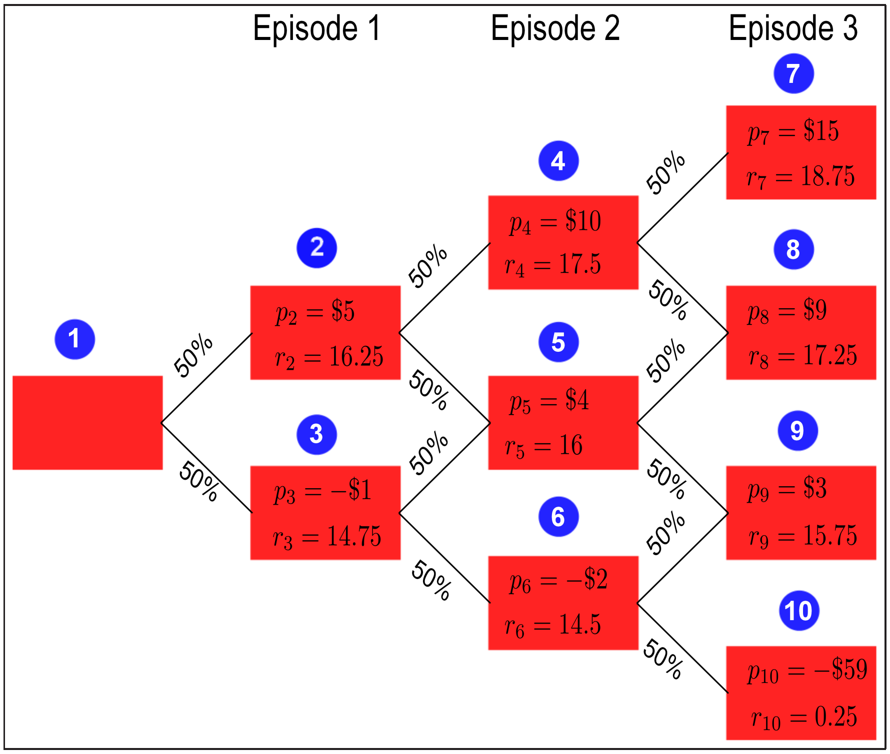

FIGURE 3.1. Three-episode discrete binomial tree. Each node represents a possible outcome of the corresponding episode with its respective rating r and profit p. In this example, the impact ratings have on profit is $Profit = 4 \times Rating - 60$

(i) Number of episodes is fixed: The decision in this case is whether to air all three episodes of the program or none at all. The expected profit is calculated using the NPV under uncertainty technique according to which the program should be shown if its NPV is positive. In our example, the program's NPV is \$5 so the PD should air it.

(ii) Number of episodes is variable but precommitted (*passive PD*): The decision here is whether to air 1, 2 or 3 episodes or not show the program at all. The choice is made by comparing the NPVs of the four alternatives (0, 1, 2 or 3 episodes) and choosing the one whose NPV is highest. In our example the NPV is \$2 for 1 episode, \$6 for 2 episodes and \$5 for 3 episodes, suggesting the number of precommitted episodes should be only 2.

(iii) Number of episodes is variable but flexible in an already running schedule (*active PD*): In this situation the PD must decide whether or not to cancel the subsequent episodes each time an episode has been aired and its rating and profit information comes available. Although at any given moment management needs only to commit to the current decision (whether or not to release the next episode), there is an intrinsic dependence between the current choice and the later possibilities. Because of this relationship, the extra value must be calculated by solving the uncertainty tree starting from the end (the right side) and working backwards, computing the optimal decision in each state in order to find the best strategy. In our example, this would mean that after node 6 the PD would cancel the program. Conditional on this, and rolling back at node 3, the expected profit for it $4. The upper branch of the tree does not change and its expected profit conditional on being at node 2 is $16. Adding the possible profits of episode 1 and the expected profits of episodes 2 and 3, we obtain a value for the program of $12.

The third scenario has greater flexibility and thus higher expected program profits, and is the one we will apply here. The flexibility to cancel a show in a running schedule could imply new costs (e.g., writing a second script, higher filming production costs, loss of audience loyalty) that must be compared with the value added by the option to cancel. In our example the option's value is $6, which is the difference between having a *passive PD* and an *active PD*.

The real options framework we develop in what follows involves the same three steps as the simplified model just presented, namely:

(i) Define the possible cancellation times (Constraints): A channel may be unwilling or unable to cancel a program at certain times because of contractual obligations to the cast, reluctance to interrupt a plot line involving a well-known actor, etc.

(ii) Model the rating's stochastic process (Definition of the uncertainty): As in the above example, ratings are reported at discrete points in time (episodes); however, in real life they are not described by a binomial tree but rather a continuous random variable with interdependencies within a program.

(iii) Estimate the impact ratings have on profits (Definition of the financial impacts of possible outcomes).

4. CASE STUDY AND DISCUSSION

In this section we apply the Real Options Framework (ROF) to determine whether or not to continue an already running program. The same real options framework, which was illustrated in Section 3, is developed in detail and applied using data from the Chilean television industry. We also discuss, and quantify with real data, how the decision policy generated by the framework increases channel revenue for our case.

4.1. Chilean television industry

The free-to-air TV market in Chile is dominated by four channels (Canal 13, Televisión Nacional (TVN), Megavisión (Mega) and Chilevisión (CHV)), all of which exhibit the industry characteristics described in Section 2: domino effects, fragile competitive advantages and volatile program ratings (del Sol, 2009).

In Chile, soap operas (known locally as *Teleseries*) are high-risk bet prime-time programs whose performance greatly affects overall channel revenue. The channel broadcasting the leading soap is very likely to have the highest market share for the entire season. Thus, each channel tries to come up with innovative soaps, which means large investments in publicity campaigns, scripts, directors and casting. The programs usually turn out either to be great successes or colossal failures (Chávez, 2011). In what follows, we apply the real options framework to Chilean soaps.

4.2. Data

The database constructed for this study contains unique and previously unexploited information on the revenues and viewers of all TV programs broadcast during the three-year period 2006-2008 by the four major Chilean channels and the costs of every program shown by one of the channels, Canal 13. Raw viewer data was obtained on a minute-by-minute basis and includes information on the number of viewers during each channel's scheduled periods, which totaled 4,996,159 minutes for the 4 channels combined.[1] A program's rating is the ratio of the number of its viewers to its potential audience (Time Ibope, 2012). Potential audience information was not available in the raw data but can be calculated using the public *Daily top 10 programs* database published on the Time Ibope (2012) website. The method of computation is briefly explained in the headnote to Table 4.1. With the number of viewers and potential audience figures for each program we were able to obtain the minute-by-minute ratings, and taking the average of these we arrived at the program ratings. Values were derived for the three-year period for 91,559 episodes of 2,050 different programs.

TABLE 4.1. The top ten programs of February 6, 2006. The daily potential audience of viewers is computed as the average of the ratios of viewers to program rating for the various programs Potential Audience$_{2006/02/06} = \frac{100}{10} \times (\frac{Viewers_1}{Rating_1} + \frac{Viewers_2}{Rating_2} + \ldots + \frac{Viewers_{10}}{Rating_{10}})$, or 6,097,048 persons. The potential audience of the year is calculated as the average of the daily potential audiences.

Nº	**Channel**	**Program start**	**Program End**	**Program**	**Viewers**	**Rating**
1	Canal 13	20:22	20:59	GATAS Y TUERCAS	437.920	**7,2**
2	Mega	21:58	00:29	HUMORANDE	437.240	**7,2**
3	Canal 13	21:00	21:56	TELETRECE	371.480	**6,1**
4	TVN	21:00	21:56	24 HORAS CENTRAL	324.830	**5,3**
5	TVN	22:03	23:30	MEA CULPA	316.860	**5,2**
6	Canal 13	19:57	20:21	GATAS Y TUERCAS (RESUMEN)	302.280	**5,0**
7	Mega	00:30	00:44	CERO HORAS	294.150	**4,8**
8	CHV	22:00	23:48	LA ESCLAVA ISAURA	293.580	**4,8**
9	Canal 13	21:59	23:36	LA CASA	286.980	**4,7**
10	TVN	20:23	20:58	AMOR EN TIEMPO RECORD	256.510	**4,2**

The raw cost data for Canal 13 were broken down into license fees (e.g., for screenwriters), marketing and other expenses. The cost data table (not shown here) has 7,065 rows for each program episode and 535 summarizing the costs of multiple episodes of a single program (soap operas are in this second group). The revenue generated by every program was calculated using the advertising rates of 1,184,093 television spots.

[1]Scheduled periods usually exclude the early morning hours when no regular programming is broadcast. Of the 4,996,159 minutes, Canal 13 accounted for 1,335,023, TVN for 1,190,377, Mega for 1,206,739 and CHV for 1,264,020.

4.3. Estimation of a profit function (Definition of the financial impact of possible outcomes)

The profit earned on a program episode is the difference between its revenue and its costs. In line with the literature we model profit assuming revenue and cost are independent of each other (Givon & Grosfeld-Nir, 2008). From the data just described (Section 4.2) the average episode cost of Canal 13's soaps can be estimated at $65 thousand, figure which we will use in our model. As Canal 13 and TVN are the historical leaders in Chile's soap opera wars, we may reasonably assume that they have similar costs. On the other hand, Mega and CHV participate less systematically in these battles and their soaps usually have lower ratings (see Figure 4.1), suggesting their operating costs are also lower. Thus, for these two channels an assumed cost similar to the cheapest Canal 13 soap ($50 thousand) is also reasonable.

An advertiser's willingness to pay for time on a given show depends on the program's target audience and *reach*. For estimating the revenue function, the complications posed by the existence of different rates for different target audiences can be controlled for by using only soap opera data for the analysis (i.e., relying on their similarity (Frank, Becknell, & Clokey, 1971)). The *reach effect* refers to the fact that a program with a rating of, say, 40 points brings in more revenue than two programs each with ratings of 20 points. The reason is that some viewers of the two 20-point programs may be the same persons, meaning that an ad placed on both shows will be watched by some persons twice and their combined reach will therefore include some double-counting. This obviously does not occur with an ad placed on a single 40-point program.

To incorporate the reach effect, we modeled the relationship between a program's average revenue and its rating by a quadratic function, as shown in Figure 4.1. The result of the quadratic regression is the expression $Revenue = 1.84 \times Rating + 2.26 \times Rating^2$.

If this expression is combined with the cost estimates for the 4 channels we have the following channel-specific profit models:

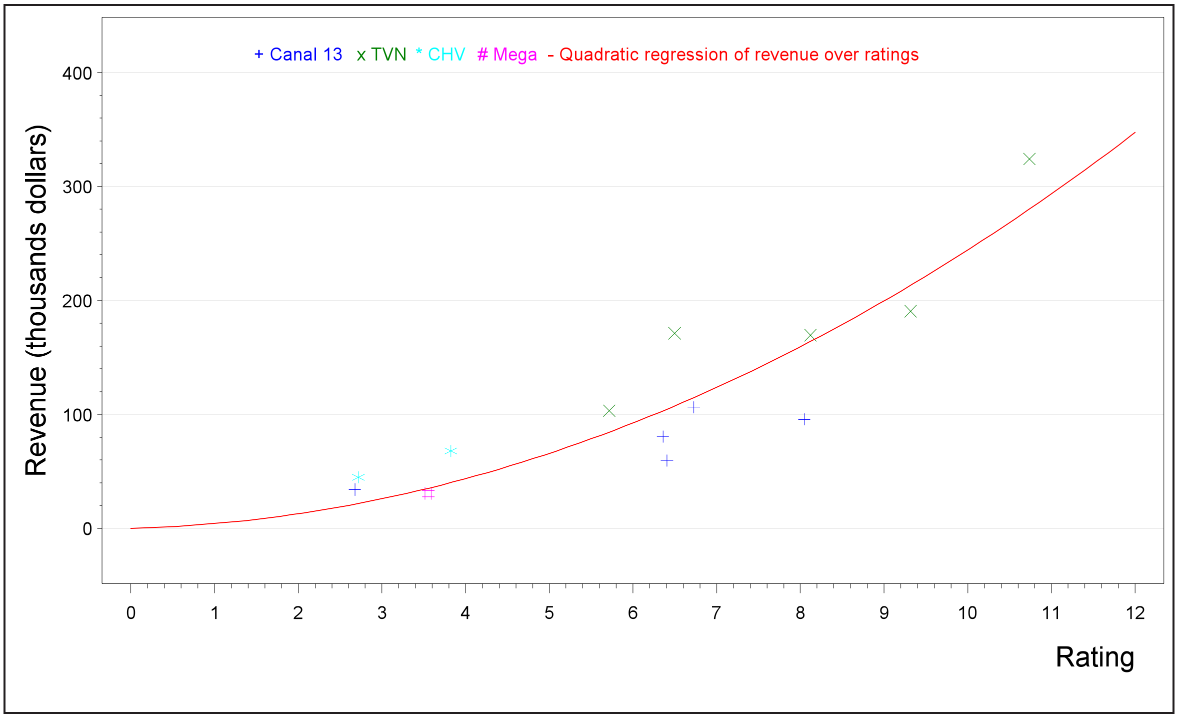

FIGURE 4.1. Quadratic regression of soaps revenue on ratings: $Revenue = 1.84 \times Rating + 2.26 \times Rating^2$.

$$Profit = 1.84 \times Rating + 2.26 \times Rating^2 - 65\} \textbf{ For Canal 13 and TVN} \tag{4.1}$$

$$Profit = 1.84 \times Rating + 2.26 \times Rating^2 - 50\} \textbf{ For Mega and CHV}$$

4.4. Modeling the rating's stochastic process (Definition of the uncertainty)

In the example of Section 3 we assumed a discrete binomial tree with 50% of probabilities of going up or down to model ratings. In this section we will construct an improved model by considering ratings as a continuous random variable with interdependencies within a program.

A number of TV forecasting models have been proposed, of which several of the more significant ones are compared in Danaher et al. (2011). The authors found that the linear mixed effects model (LMEM) was the most accurate at predicting ratings and is therefore the one chosen for our application.

The distinctive feature of an LMEM is that the mean response is modeled as a combination of: (i) fixed effects, which are population characteristics assumed to be shared by all individuals and are described below in Section 4.4.1; and (ii) random effects, which are subject-specific effects unique to a particular individual and are described in Section 4.4.2. The inclusion of the two effects is what prompted the use of the term ”mixed” (Fitzmaurice, Laird, & Ware, 2004). The actual LMEM ratings model summarizing all the effects and defining the stochastic process is presented in Section 4.4.3.

4.4.1. Fixed effects

The fixed effects can be divided into two groups of covariates (Danaher et al., 2011), time-based and program-specific. In what follows we discuss each one in turn.

4.4.1.1. Time-based fixed effects

Gensch and Shaman (1980) show that in the United states, total network audience is strongly influenced by the availability of non-television viewing activities and is thus related to the season, day of the week and hour of the day. Figures 4.2 and 4.3 suggest these factors influence Chilean ratings as well. We model seasonality with a linear combination of trigonometric functions that represent annual, semi-annual, quarterly and three other cycles of shorter length as in Gensch and Shaman (1980). With respect to the day of the week we use dummy variables for each one. Time in the evening is represented by 10 dummy time slot variables of half an hour each.[2] The data we use are restricted to the period 6 pm-11 pm, in line with the most recent studies, and only cover weekdays, when all Chilean soap operas are aired. With these constraints, a total of 25,994 program episodes are included in the analysis. By also incorporating a linear trend we control for a possible long-term decline in television as the primary media device (Helm, 2007; Gensch & Shaman, 1980).

[2]Note that each program is coded into a single slot variable. If the show stretches across two slots, the one containing the larger part of the show is used, and if each contains an equal part, the first one is chosen.

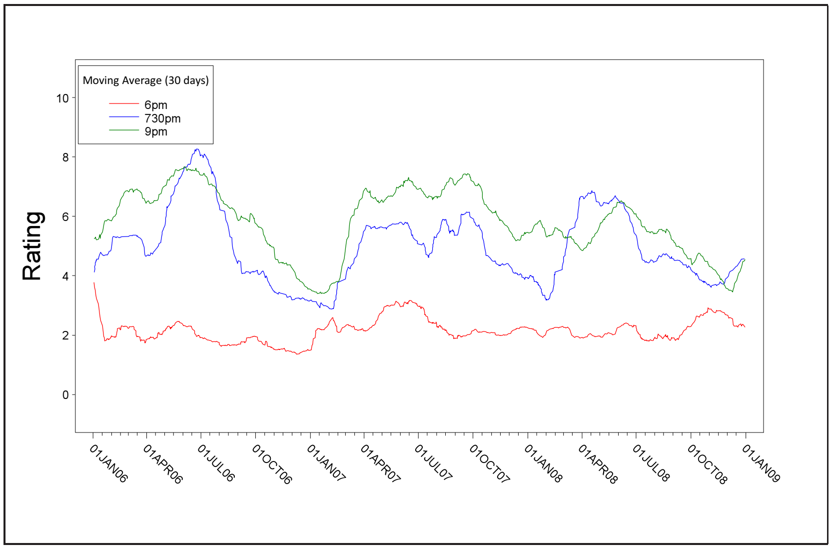

FIGURE 4.2. Ratings of Canal 13 programs broadcast at 6 pm, 7:30 pm and 9:00 pm as 30-day centered moving averages. Note that later shows generally have higher ratings, and winter ratings (June to September in the southern hemisphere) are higher than summer ones.

4.4.1.2. Program-based fixed effects

Although the time-based analysis does suggest some sources of variability, the ratings furnish evidence of much greater variation. For example, viewing behavior shows that audiences prefer some content styles over others (Danaher et al., 2011; Henry & Rinne, 1984b; Frank et al., 1971; Rust & Alpert, 1984). Genres were coded into six categories: *light content* (comedy, variety, game shows, music, children's programming, educational programming), *serious content* (current affairs, magazine, documentaries, drama, news, science, travel, religion, culture), *sports*, *movies*, *soap operas* and *reality TV*. Length of the program (in minutes) can also be an important explanatory factor. A program episode's status as a first run, rerun, or trailer[3] can also influence ratings and is captured by a dummy variable. A channel's identity (Anand, 2002), which builds loyalty and a viewer baseline, is represented by a separate dummy variable for each channel. Finally, dummy variables were also added for the first and last episodes of every soap opera because they usually have greater variability and higher ratings.

[3]Though strictly speaking TV program trailers are not episodes, for present purposes they are included under this heading.

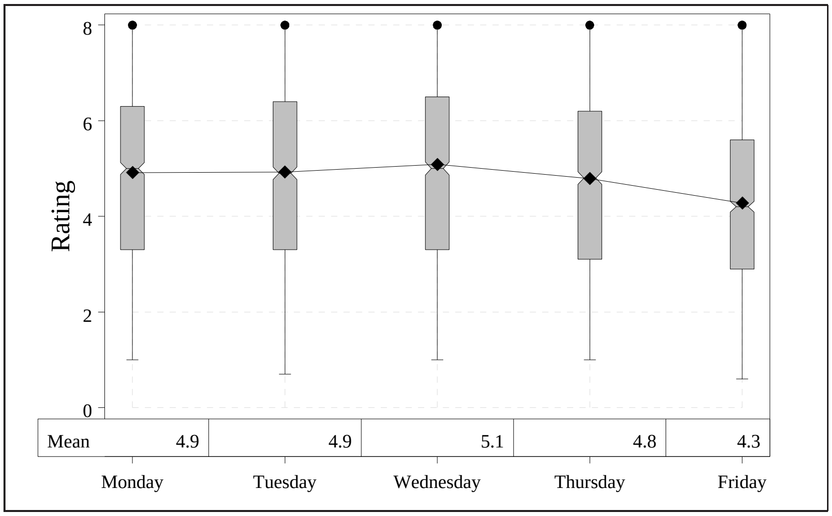

FIGURE 4.3. Canal 13 ratings, 2006-2008. Note that ratings at the end of each working week tend to be lower. Symbols used in figure: Box = 1st - 3rd (Q1 - Q3) Quartiles. ⧫ = Mean. Line inside box = Median. Whisker length = Highest value ● = Figure is clipped at 8 rating points.

4.4.2. Random effects

Random effects usually represent random deviations from the relationships described by fixed effects (West, Welch, Ga, & Crc, 2007). The forecasting model tries to capture the unobserved unique program effects by adding two program specific variables: a random intercept (Danaher et al., 2011) and an episode-wise random trend that controls for variations in a program's level of popularity. For example, the motivation for watching a specific program increases with the number of persons following it (Leibenstein, 1950; Katz & Shapiro, 1985). Given that the number of episodes vary greatly between shows we use the logarithm of the episode number for modeling the trend.

The programs were divided into three clusters, with separate variance parameters for the random effects and errors for each one (see Appendix A). One cluster contains the Canal 13 and TVN soaps, another the Mega and CHV soaps and the third one the remaining programs of all channels.

4.4.3. Stochastic process of program ratings

The various elements described in Sections 4.4.1.1, 4.4.1.2 and 4.4.2 for modeling the rating of an episode (e) of a program (p) are all incorporated into a simple general formula, given below as Equation 4.2.[4] β is used for fixed effects and s and ϵ for random effects.

$$
\begin{aligned}
R_{(p,e)} = Rating_{(p,e)}{}^{5} = \;& \beta_1 + \\
& \left.\begin{aligned}
&\textstyle\sum_{j=1}^{6} \beta_{j+1} cos(\frac{2\pi jt}{365}) + \beta_{j+7} sin(\frac{2\pi jt}{365}) + \\
&\beta_{14} Mon_{(p,e)} + \beta_{15} Tue_{(p,e)} + \beta_{16} Wed_{(p,e)} + \beta_{17} Thu_{(p,e)} + \\
&\beta_{18} 630PM_{(p,e)} + \beta_{19} 700PM_{(p,e)} + \beta_{20} 730PM_{(p,e)} + \\
&\beta_{21} 800PM_{(p,e)} + \beta_{22} 830PM_{(p,e)} + \beta_{23} 900PM_{(p,e)} + \\
&\beta_{24} 930PM_{(p,e)} + \beta_{25} 1000PM_{(p,e)} + \beta_{26} 1030PM_{(p,e)} + \\
&\beta_{27} t +
\end{aligned}\right\} \text{Time-based fixed effects} \\
& \left.\begin{aligned}
&\beta_{28} Light_p + \beta_{29} Movie_p + \beta_{30} Sport_p + \beta_{31} Reality_p + \beta_{32} Soap_p + \\
&\beta_{33} ProgramLength_{(p,e)} + \\
&\beta_{34} Original_{(p,e)} + \beta_{35} Repetition_{(p,e)} + \\
&\beta_{36} Ucat_p + \beta_{37} Tnac_p + \beta_{38} CHV_p + \\
&\beta_{39} SoapStart_{(p,e)} + \beta_{40} SoapEnd_{(p,e)} +
\end{aligned}\right\} \text{Program-based fixed effects} \\
& s_{(1,p)} + s_{(2,p)} episode_{(p,e)} + \;\Big\}\; \text{Program-specific random effects} \\
& \epsilon_{(p,e)} \;\Big\}\; \text{Error term}
\end{aligned}
\qquad (4.2)
$$

The terms on the right-hand side of the equation start with the intercept, followed in blue by the time-based fixed effects (seasonality, day of the working week, hour of the day, trend), and then in red by the program-based fixed effects (content style, program length, episode status, channel, first/last episodes status), followed in green by the program-specific random effects (intercept and trend), ending with the error term in cyan.

Equation 4.2 can be written as Eq. 4.3, which is obtained by stacking the former over the episodes of a specific program. In this abbreviated form, the linear relationship between the covariates and the ratings in the LMEM is clearly revealed. $\mathbf{X_{p_1}}$ and $\mathbf{Z_{p_1}}$ are the design matrices of the fixed and random effects, respectively, representing the known values of the covariates described in Equation 4.2. The index 1 represents the data from the actual sample (already aired episodes). Vector $\vec{\beta}$ contains the fixed effects parameters and $\vec{s}_p$ the

[4]The number 365 in Equation 4.2 is used for common years; for leap years, 366 would be used. t is the number of days between the date of the episode release and January 1st, 2006.

[5]A logit transformation was applied to the rating data to permit their use with the linear model.

program-specific random effects parameters. Finally, $\vec{\varepsilon}_{p_1}$ is the disturbance vector (West et al., 2007).

$$\vec{R_{p_1}} = \{R_{(p,e)} : e \in \text{already-aired episodes}\} = \mathbf{X_{p_1}}\vec{\beta} + \mathbf{Z_{p_1}}\vec{s}_p + \vec{\varepsilon}_{p_1} \tag{4.3}$$

Upon fitting the model to the available data we obtain a calibrated multivariate normal distribution that models the behavior of a program's ratings. The result is shown in Equation 4.4.

$$\vec{R_{p_1}} \sim N(\vec{\hat{\mu}_1}, \hat{\mathbf{\Sigma}}_{\mathbf{11}}) \tag{4.4}$$

In our context of forecasting TV ratings, the idea is that the model predict the ratings of episodes that have not yet been broadcast as accurately as possible. Letting $\vec{R_{p_2}}$ represent as-yet unaired episodes of the same program p, the joint distribution of $\vec{R_{p_1}}$ and $\vec{R_{p_2}}$ (Harville, 1985) (see Appendix A) is given by

$$\begin{pmatrix} \vec{R_{p_1}} \\ \vec{R_{p_2}} \end{pmatrix} \sim N\left(\begin{pmatrix} \vec{\hat{\mu}_1} \\ \vec{\hat{\mu}_2} \end{pmatrix}, \begin{pmatrix} \hat{\mathbf{\Sigma}}_{\mathbf{11}} & \hat{\mathbf{\Sigma}}_{\mathbf{12}} \\ \hat{\mathbf{\Sigma}}_{\mathbf{12}}^{\top} & \hat{\mathbf{\Sigma}}_{\mathbf{22}} \end{pmatrix}\right) \tag{4.5}$$

According to Searle (1971), the conditional distribution of $\vec{R_{p_2}}/\vec{R_{p_1}}$ can then be written as

$$\vec{R_{p_2}}|\vec{R_{p_1}} \sim N(\vec{\hat{\mu}_2} + \hat{\mathbf{\Sigma}}_{\mathbf{12}}^{\top}\hat{\mathbf{\Sigma}}_{\mathbf{11}}^{\mathbf{-1}}(\vec{R_{p_1}} - \vec{\hat{\mu}_1}), \hat{\mathbf{\Sigma}}_{\mathbf{22}} - \hat{\mathbf{\Sigma}}_{\mathbf{12}}^{\top}\hat{\mathbf{\Sigma}}_{\mathbf{11}}^{\mathbf{-1}}\hat{\mathbf{\Sigma}}_{\mathbf{12}}) \tag{4.6}$$

4.5. Results

Having already defined the stochastic process of the ratings and their financial impact on profits, the only element remaining to be defined is the possible cancellation times. Two cases are worked out to illustrate how the model functions when the number of episodes is variable but flexible in an already running schedule. In the first case, developed in Section 4.5.1, the PD has the option to cancel the soap opera following a single fixed episode number, while in the second case, analyzed in Section 4.5.2, the PD has the option to cancel following either of two fixed episodes.

There are many ways of calibrating the model to validate it, starting from the simplest method which is simply to have the PD use his experience to estimate the co-variance parameter values. Various statistical approaches could be used, but given the small number of prime-time soap operas aired in the three-year period we chose to calibrate the model using different in-sample (IS) data for every soap on which data was available. More specifically, to estimate the stochastic process followed by all the episodes of a given soap after the n^{th} one, we leave out-of-sample (OOS) all the data between the day of the n^{th} episode and the last one. The calibration is performed using the data for all of the programs so that the covariates are statistically significant (see Appendix A).

4.5.1. Option to cancel after a single fixed episode number

Assume that after $n = 25$ episodes the PD has the option to cancel the soap or let it continue until episode 100.[6] In this case, the key measure for making a decision is simply the expected profit on the remaining 75 OOS episodes (given the short time horizon, a 0% discount rate is assumed for simplicity). The expected profit is computed using a Monte Carlo simulation with Equation 4.6 and the 25 episodes as $\vec{R_{p_1}}$, applying the profit function 4.1 to each rating's trajectory. The parameters for all the simulations carried out were such that the expected profits had a percentage of error of no more than 0.5% with a probability of approximately 95% (see Appendix B). Table 4.2 presents the results of the simulation along with the observed profits obtained by applying the profit function to the observed ratings (Columns 2, 3 and 4) as well as the interpretation and analysis of the results. The interpretation of the simulated OOS profit (Column 4) is the decision of cancel the soap or let it continue after episode 25 and is shown in Column 5. The analysis of the ex-post

[6] We will assume that all soaps were initially designed to have 100 episodes.

accuracy of the decision (i.e. if signs of Columns 2 and 3 are equal) is shown in Column 6. Finally the impact that using ROF has in the total profit is shown in Column 8, comparing the percentage difference between Column 2 and 7.

TABLE 4.2. Summary of simulation results for the 13 soap operas with only 25 in-sample episodes. Whether or not to continue broadcasting is decided using the OOS expected profit showed in Column 4 (soaps not continued are highlighted in bold). Only 15% (out of 13 soaps) of the recommendations were not optimal ex-post (Column 6). Profits are presented in thousands dollars.

Soap's name (1)	Observed total soap's profit (2)	OOS observed profit (3)	OOS expected profit (4)	Continue broad-casting? (5)	Ex-post accuracy of the decision (6)	Observed total soap's profit using ROF (7)	Total soap's profit percent change (8)
DESCARADO	4188	2810	1242	Yes	Correct	4188	0%
COMPLICES	18862	12689	18434	Yes	Correct	18862	0%
CHARLY TANGO	**-2332**	**-1500**	**-1382**	**No**	**Correct**	**-831**	**64%**
FLORIBELLA	7779	4587	6176	Yes	Correct	7779	0%
VIVIR CON 10	**-2971**	**-2216**	**-1820**	**No**	**Correct**	**-755**	**75%**
CORAZON DE MARIA	12292	8592	14453	Yes	Correct	12292	0%
PAPI RICKY	6074	4074	7306	Yes	Correct	6074	0%
FORTUNATO	**-1449**	**-1694**	**-382**	**No**	**Correct**	**245**	**117%**
AMOR POR ACCIDENTE	**912**	**80**	**-354**	**No**	**Incorrect**	**832**	**-9%**
LOLA	5975	2694	16	Yes	Correct	5975	0%
MALA CONDUCTA	-826	-676	43	Yes	Incorrect	-826	0%
DON AMOR	2432	2107	938	Yes	Correct	2432	0%
VIUDA ALEGRE	2742	1708	3934	Yes	Correct	2742	0%
Total	**53679**				**85%**	**59010**	**10%**

Using ROF instead of just continuing the soaps for the pre-committed total number of episodes results in a profit increase of almost 10%.[7] Note, however, that the PD can also make ex-post erroneous decisions using the proposed framework. Two types of errors can be defined. A Type I error occurs when the PD continues running a soap that should have been canceled (*Mala Conducta*) and a Type II occurs when the PD should have continued running a soap but cancels it instead (*Amor por accidente*). A Type I error does not reduce total profits compared with non-model-based decision policies that continue the program until the end, which in effect present the same error. Whenever a tool is used to decide

[7]This measure incorporates the profits of the 25 first episodes, which in both cases are equal. Not including them does not change the decision, but does obscure the effects of taking more time to decide.

whether or not to cancel a program, there is a potential for Type II errors. A simple way of reducing them would be to check whether the probability of a positive OOS profit is high enough before canceling. This is easily calculated with ROF.

4.5.1.1. Sensitivity analysis: One optimal review point

One of the assumptions of the example given in Section 3 was that after each episode, the PD was able to review the continuation of the soap. If this is not feasible because, say, a certain number of episodes must initially be filmed, a review point must be defined ex-ante, for example, we assumed 25 episodes in Table 4.2. An approximation to the optimal reviewing point can be found using Figure 4.4, which compares the profit increase by using ROF instead of the non-model-based procedures at different review points (i.e., different numbers of IS episodes n). The figure shows a lot of noise in the first 25 episodes with a few local maxima, mainly because 13 programs are not enough data to get a smooth curve. Using the smoothing technique presented in (Garcia, 2010), we find that the maximum of the smoothed values occurs after the 6th episode where we obtain a 8.5% profit increase (non-smoothed, actually earned profits). In this case the ROF recommends canceling *Charly Tango*, *Vivir con 10* and *Amor por accidente*. With only 13 programs, our best estimate of the optimal review point is after 6 episodes. The right vertical axis in the figure shows the progression in the percentage of ex-post erroneous decisions. As expected, it decreases to 0 as more information about the soap is captured.

4.5.2. Option to cancel after either of two fixed episode numbers

This section develops the multi-stage example of Section 3 assuming that the PD will review the continuation of the soap after each of two fixed episodes n_1 and n_2. The extra cancellation time n_2 means that a decision to continue at n_1 is less dependent on high ratings compared with Section 4.5.1, the reason being the possibility of reviewing the PD's decision later when there is less uncertainty regarding the soap's success. The convexity introduced by the quadratic profit function also limits the requirements on n_1. This is so because the potential downside losses resulted from bad ratings are less than the upside potential gains resulting from good ratings.

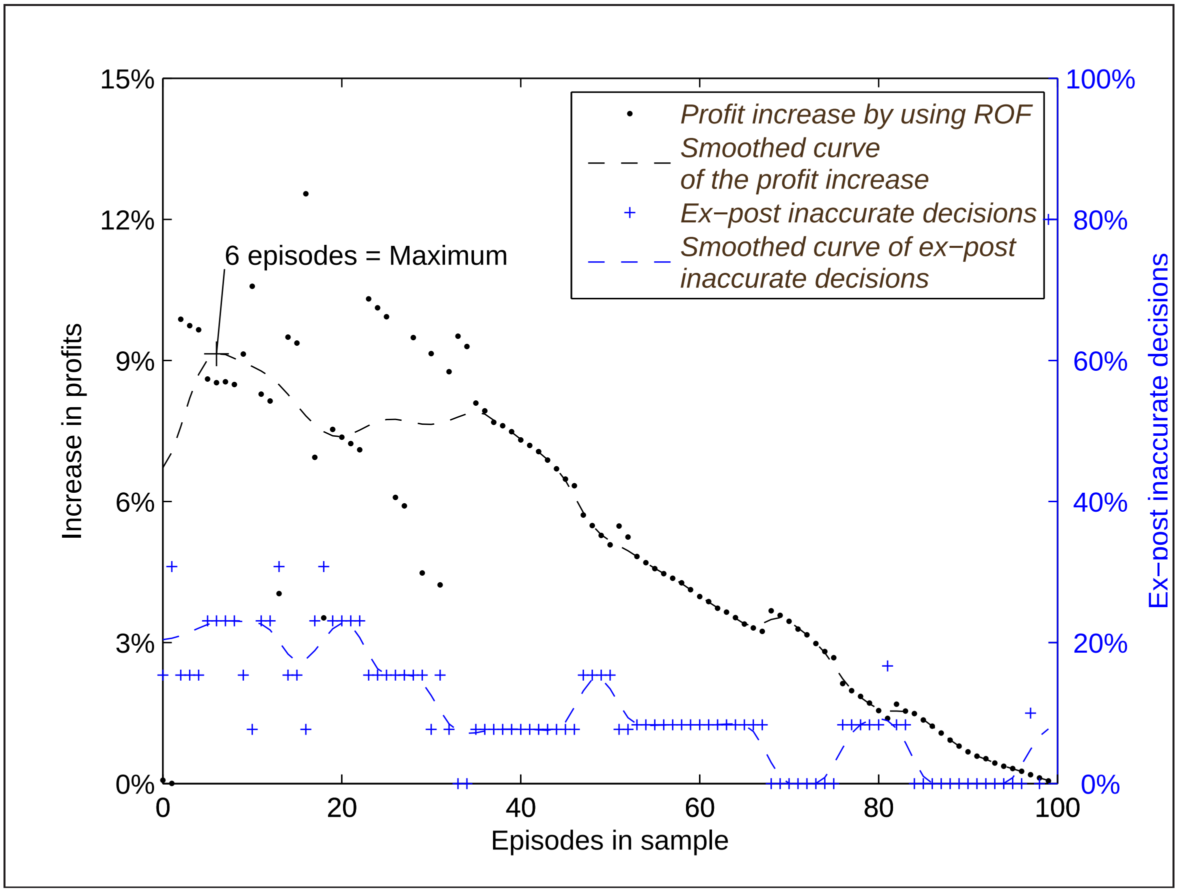

FIGURE 4.4. The left vertical axis shows the profit increase using ROF instead of letting soaps continue. The black dots represent the real increase (e.g., 10% with 25 in-sample episodes, as shown in Table 4.2) while the black dashed line is the smoothed curve. The right vertical axis shows the ex-post percentage of inaccurate decisions if the ROF had been used (e.g., 15% with 25 in-sample episodes, as shown in Table 4.2).

We begin by arbitrarily assuming that $n_1 = 6$ and $n_2 = 25$. As in Section 4.5.1, the expected profit will be used to decide whether or not to cancel the soap after 6 episodes. This is an American real options problem where we want to know if it is optimal to exercise at n_1 having the option of postponing the decision until n_2. This type of problem is complex and several approaches for approximating the decision have been proposed, including the popular Longstaff and Schwartz (2001) method applied to a real options problem by Cortazar, Gravet, and Urzua (2008). In this case it is particularly complex given the multi-dimensionality of the ratings vector $\vec{R}_p$ and implementations of the models just cited will be explored in future articles. As an approximation, we use a standard simulation method in which the first step is to simulate the trajectories in the ratings for episodes 6 to 100. The second step is a backward induction in which the profitability of each trajectory is calculated by first evaluating the profit between episodes 25 to 100. Only if this is positive will it be added to the profit between episodes 6 to 25, and the maximum between this and \$0 is then the corresponding profit for each trajectory. The approximated value of the option at episode 6 is the average over all paths, indicating that the soap should continue if the value is positive. A rigorous simulation solution would first simulate ratings between episode 6 and 25 and then conditionally simulate episodes 25 to 100 for each of the trajectories, thus increasing quadratically the number of trajectories needed. This, however, would be too demanding for personal computers, even if variance reduction techniques were used.

In results not shown, for the 13 soaps the ROF indicates that only *Charly Tango* and *Vivir con 10* should be canceled at the 6th episode while *Fortunato* and *Amor por Accidente* should be canceled at the 25th episode. The two to be canceled at the 6th episode are different from those so indicated with a single 6th episode cancellation option (see Section 4.5.1.1) illustrating the relatively lower dependency on ratings referred to above. These cancellations generate a 12.5% profit increase compared to continuing them until the end, as opposed to the 8.5% increase with the single option (non-smoothed values). This result highlights the importance of a having a second review point.

4.5.2.1. Sensitivity analysis: Two optimal review points

Section 4.5.1.1 presented the case in which a single episode had to be chosen before the soap started as the point for reviewing its continuation. The same situation is analyzed in this section, except that this time two reviewing points are chosen. One approximate way of finding the optimal points is to calculate, for each combination of n_1 and n_2, how much of the soap's profit using ROF would have been obtained compared to the observed soap's profit without using ROF. The contour lines of the smoothed curve (Garcia, 2010) are presented in figure 4.5. As can be seen, the maximum of the smoothed values occurs when the first decision is taken after 5 episodes and the second after 42 where we obtain a

12.4% profit increase (non-smoothed, actually earned profits). This is our best estimate of the optimal reviewing points for the above case.

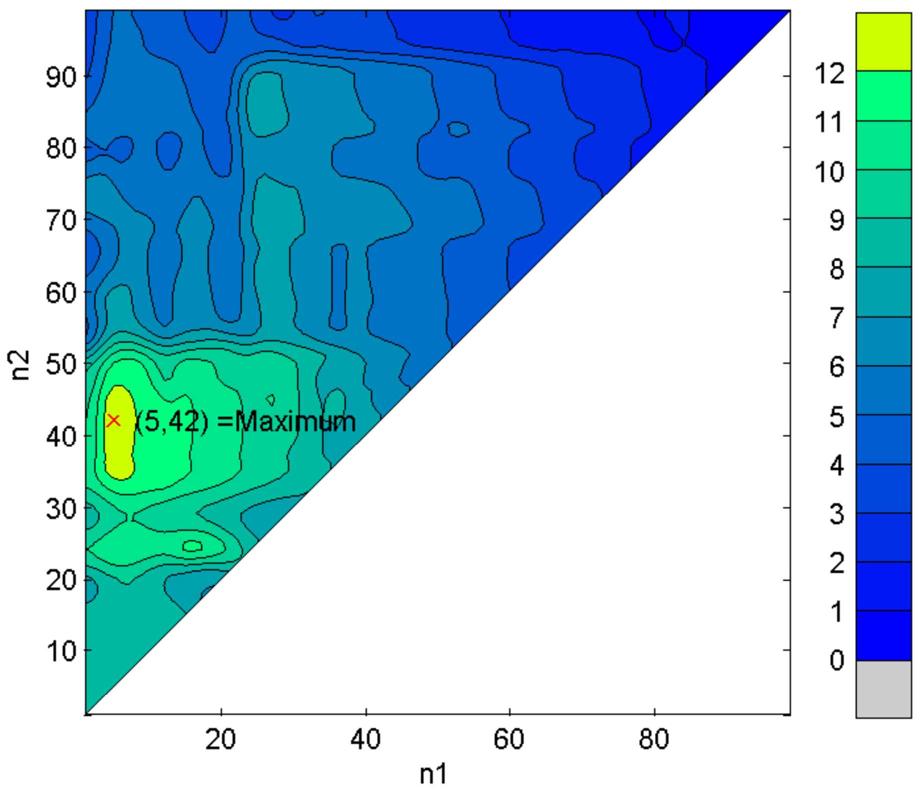

FIGURE 4.5. Contour lines of the percentage increase in total revenue by applying the decision model with different cancellation decision times $n1$ and $n2$.

5. CONCLUSIONS

This article presents a simple framework based on real options theory that supports decision-making under uncertainty by active managers in the television industry. The real options framework (ROF) is used to maximize the profits of a television program by optimizing the number of episodes. No existing studies, that we are aware of, have used real options in this market where value can be created by capitalizing on favorable future opportunities and on canceling programs with low ratings to mitigate losses. To illustrate the functioning of the proposed decision framework we applied it using real rating, cost and revenue data from a Chilean TV channel database never previously exploited. The use of the Monte Carlo method for simulating ratings ensures the framework is highly adaptable and could be readily extended by adding factors such as the initial investment, a cancellation price, different profit functions for different programs, more possible cancellation times, a revenue discount rate or different stopping criteria.

The results of our real application are encouraging even for a small TV market such as Chile's, demonstrating that networks could make significant savings by adopting a well-designed program cancellation decision policy. Applying the ROF to the data with the single option of canceling a program after the 6th episode increased the total profit of all channels by 8.5% while the option to decide if canceling or not a program after 5 episodes, and then again after 42, increased profit by 12.4%. These outcomes are interesting because they highlight the applicability of real options analysis to industries where it has not previously been employed such as media and other markets with data not yet exploited.

The modular structure of the proposed framework facilitates parallel efforts to develop better models of the ratings' stochastic process, improve the ratings profit function and enhance the implementation of real options valuation using state-of-the-art advances in these areas.

The framework could also be used to help networks sell television commercial time. Advertisers typically aim to achieve a target number of gross rating points (GRPs) over the duration of an advertising campaign. The literature on the subject has focused on accurate prediction of future ratings given that over- or underachievement of the target GRPs will result in financial losses. Predicting not only the expected value of the GRPs but also their variance would enable spots where forecasting errors are less costly to be scheduled in programs with greater variance.

The availability of additional data would facilitate significant improvements to the framework as presented here. A more detailed breakdown of a channel's expenditures into categories such as initial investment, actors' fees, script development and so on would allow for a formulation that more closely specified the cost structure. Also, a larger sample of soap operas than was included here would result in smoother curves for determining the points at which cancellation decisions should be reviewed without the need for additional techniques.

Overall, the ROF results represent a promising step towards the effort to increase profits by investing in flexibility. The empirical findings provide a solid baseline against which future models for supporting decisions on whether and when to cancel a television program may be compared.

References

Anand, B. (2002). Competing over the airwaves. *Smart Manager*, *v. Q102*, 22–36.

Bengtsson, J., & Olhager, J. (2002). The impact of the product mix on the value of flexibility. *Omega*, *v. 30*(n. 4), 265–273.

Brennan, M. J., & Schwartz, E. S. (1985). Evaluating natural resource investments. *Journal of Business*, *v. 58*(n. 2), 135–157.

Brockwell, P. J., & Davis, R. A. (1991). *Time series: Theory and methods* (2nd ed.). Springer-Verlag.

Brockwell, P. J., & Davis, R. A. (2002). *Introduction to time series and forecasting* (2nd ed.). Springer-Verlag.

Chávez, F. (2011). Los intereses económicos detrás de la guerra de las teleseries. *Comunicación y Medios*, *v. 0*(n. 16). Retrieved from http://www.comunicacionymedios.uchile.cl/index.php/RCM/article/viewArticle/11548/11905

Cortazar, G., Gravet, M., & Urzua, J. (2008). The valuation of multidimensional american real options using the lsm simulation method. *Computers & Operations Research*, *v. 35*(n. 1), 113–129.

Cortazar, G., Schwartz, E., & Salinas, M. (1998). Evaluating environmental investments: A real options approach. *Management Science*, *v. 44*(n. 8), 1059–1070.

Cortazar, G., & Schwartz, E. S. (1993). A compound option model of production and intermediate inventories. *The Journal of Business*, *v. 66*(n. 4), 517-540.

Cortazar, G., Schwartz, E. S., & Casassus, J. (2001). Optimal exploration investments under price and geological-technical uncertainty: a real options model. *R&D Management, v. 31*(n. 2), 181–189.

Danaher, P. J., Dagger, T. S., & Smith, M. S. (2011). Forecasting television ratings. *International Journal of Forecasting, v. 27*(n. 4), 1215–1240.

Danaher, P. J., & Mawhinney, D. F. (2001). Optimizing television program schedules using choice modeling. *Journal of Marketing Research, v. 38*(n. 3), 298–312.

del Sol, P. (2009). *Ganar sin competir ganar por suerte* (1st ed.). Aguilar/ El Mercurio.

Fitzmaurice, G. M., Laird, N. M., & Ware, J. H. (2004). *Applied longitudinal analysis* (1st ed.). Wiley-Interscience.

Frank, R. E., Becknell, J. C., & Clokey, J. D. (1971). Television program types. *Journal of Marketing Research, v. 8*(n. 2), 204-211.

Garcia, D. (2010). Robust smoothing of gridded data in one and higher dimensions with missing values. *Computational Statistics & Data Analysis, v. 54*(n. 4), 1167–1178.

Gensch, D., & Shaman, P. (1980). Models of competitive television ratings. *Journal Of Marketing Research (JMR), v. 17*(n. 3), 307–315.

Givon, M., & Grosfeld-Nir, A. (2008). Using partially observed markov processes to select optimal termination time of tv shows. *Omega, v. 36*(n. 3), 477 - 485.

Harville, D. A. (1985). Decomposition of prediction error. *Journal of the American Statistical Association, v. 80*(n. 389), 132-138.

Harville, D. A. (1990). Advances in statistical methods for genetic improvement of livestock. In K. Gianola D.;Hammond (Ed.), (chap. BLUP (Best Linear Unbiased Prediction), and Beyond). Springer-Verlag.

Heidelberger, P., & Welch, P. D. (1981). A spectral method for confidence interval generation and run length control in simulations. *Commun. ACM*, *v. 24*(n. 4), 233–245.

Helm, C. (2007, august). *Ibm consumer survey shows decline of tv as primary media device.* Retrieved from http://www-03.ibm.com/press/us/en/pressrelease/22206.wss

Henderson, C. R. (1984). *Applications of linear models in animal breeding* (1st ed.). University of Guelph.

Henry, M. D., & Rinne, H. J. (1984a). Offensive versus defensive tv programming strategies. *Journal of Advertising Research*, *24*(n. 3), 45–56.

Henry, M. D., & Rinne, H. J. (1984b). Predicting program shares in new time slots. *Journal of Advertising Research*, *v. 24*(n. 2), 9–17.

Horen, J. H. (1980). Scheduling of network television programs. *Management Science*, *v. 26*(n. 4), 354–370.

Katz, M. L., & Shapiro, C. (1985). Network externalities , competition , and compatibility. *Network*, *v. 75*(n. 3), 424–440.

Kulatilaka, N. (1988). Valuing the flexibility of flexible manufacturing systems. *IEEE Transactions on Engineering Management*, *v. 35*(n. 4), 250–257.

Kulatilaka, N., & Trigeorgis, L. (1994). The general flexibility to switch: Real options revisited. *International Journal Of Finance*, *v. 6*(n. 2), 778–798.

Law, A. M., & Kelton, W. D. (2000). *Simulation modeling and analysis* (3rd ed.). McGraw-Hill Higher Education.

Leibenstein, H. (1950). Bandwagon, snob, and veblen effects in the theory of consumers' demand. *Quarterly Journal of Economics*, *v. 64*(n. 2), 183–207.

Longstaff, F. A., & Schwartz, E. S. (2001). Valuing american options by simulation: a simple least-squares approach. *Review of Financial Studies*, *14*(1), 113–147.

Marketing charts. (2012, may). *Tv ad spend rose 4.5% in '11; cable on par with network tv.* Retrieved from http://www.marketingcharts.com/television/tv-ad-spend-rose-45-in-11-cable-on-par-with-network-tv-22051/nielsen-tv-ad-spend-in2011-may2012jpg/

McDonald, R., & Siegel, D. (1985). Investment and the valuation of firms when there is an option to shut down. *International Economic Review*, *v. 26*(n. 2), 331–349.

Miller, K. D., & Waller, H. G. (2003). Scenarios, real options and integrated risk management. *Long Range Planning*, *v. 36*(n. 1), 93–107.

Napoli, P. M. (2001). The unpredictable audience : An exploratory analysis of forecasting error for new prime-time network television programs. *Journal of Advertising*, *v. 30*(n. 2), 53–60.

Paddock, J. L., Siegel, D. R., & Smith, J. L. (1988). Option valuation of claims on real assets: The case of offshore petroleum leases. *Quarterly Journal of Economics*, *v. 103*(n. 3), 479–508.

Ross, S. M. (2006). *Simulation* (4th ed.). Academic Press, Inc.

Rubinstein, R. Y., & Kroese, D. P. (2007). *Simulation and the monte carlo method* (2nd ed.). Wiley Series in Probability and Statistics.

Rust, R. T., & Alpert, M. I. (1984). An audience flow model of television viewing choice. *Marketing Science*, *v. 3*(n. 2), 113–124.

SAS Institute Inc. (2008). Sas/stat® 9.2 user's guide (SAS Institute Inc ed.) [Computer software manual].

Searle, S. R. (1971). *Linear models* (1st ed.). Wiley.

Time Ibope. (2012, november). *Top ten diario.* Retrieved from http://www.timeibope.cl/timeibope/

Titman, S. (1985). Urban land prices under uncertainty. *American Economic Review*, *v. 75*(n. 3), 505–514.

Tourinho, O. (1979). *The option value of reserves of natural resources* (Research Program in Finance Working Papers No. n. 94). University of California at Berkeley.

Trigeorgis, L. (1996). *Real options: Managerial flexibility and strategy in resource allocation* (1st ed.). MIT Press.

Tsai, W.-H., & Hung, S.-J. (2009). Dynamic pricing and revenue management process in internet retailing under uncertainty: An integrated real options approach. *Omega*, *v. 37*(n. 2), 471–481.

West, B. T., Welch, K. B., Ga, A. T., & Crc, H. (2007). *Linear mixed models a practical guide using statistical software* (1st ed.; AnonymousEditors, Ed.). Chapman and Hall/CRC.

APPENDICES

APPENDIX A. LINEAR MIXED RATING MODEL

A.1. Linear mixed effects model and estimation of out-of-sample observations

A linear mixed effects model (LMEM) is a parametric linear model for clustered, longitudinal, or repeated-measures data that quantifies the relationships between a continuous dependent variable and various predictor variables. An LMEM may include both fixed-effect parameters associated with one or more continuous or categorical covariates and random effects associated with one or more random factors. It is this combination of fixed and random effects that the ”mixed” in the model name refers to. Whereas fixed-effect parameters describe the relationships of the covariates to the dependent variable for an entire population, random effects are specific to clusters or subjects within a population. Consequently, random effects are used directly for modeling the random variation in the dependent variable at different data levels. For a detailed discussion of data types, see, for example, West et al. (2007). This appendix does not contain an in-depth explanation of how the LMEM was built, focusing instead on the construction of the specific statistics used here, namely, the conditional expected value and co-variance matrix of the predictions.

Linear mixed models take the following form:

$$\vec{R} = \mathbf{X}\vec{\beta} + \mathbf{Z}\vec{s} + \vec{\varepsilon} \tag{A.1}$$

where $\vec{\beta}$ is a vector of parameters, $\mathbf{X}$ and $\mathbf{Z}$ are known matrices and $\vec{s}$ and $\vec{\varepsilon}$ are unobservable and uncorrelated random vectors.

LMEMs assume the following:

(i) $\vec{R} \sim N(\vec{\mu}, \mathbf{V})$
(ii) $\vec{s} \sim N(\vec{0}, \mathbf{G})$
(iii) $\vec{\varepsilon} \sim N(\vec{0}, \mathbf{K})$

We divide vector $\vec{R}$ into two parts: one, denoted $\vec{R_1}$, for the in-sample episodes and the other, called $\vec{R_2}$, for the out-of-sample episodes. We assume that we do not know $\vec{R_2}$, which are the episode rating to be forecast. The same notation will be used with $\mathbf{X}$, $\mathbf{Z}$ and $\vec{\varepsilon}$.

$$\vec{R_1} = \mathbf{X_1}\vec{\beta} + \mathbf{Z_1}\vec{s} + \vec{\varepsilon_1} \tag{A.2}$$

$$\vec{R_2} = \mathbf{X_2}\vec{\beta} + \mathbf{Z_2}\vec{s} + \vec{\varepsilon_2} \tag{A.3}$$

Using the same notation,

$$\vec{\mu}^{\top} = \begin{pmatrix} \vec{\mu_1}^{\top} & \vec{\mu_2}^{\top} \end{pmatrix} \tag{A.4}$$

$$\mathbf{V} = \begin{pmatrix} \mathbf{V_{11}} & \mathbf{V_{12}} \\ \mathbf{V_{12}}^{\top} & \mathbf{V_{22}} \end{pmatrix} \tag{A.5}$$

The variance of $\vec{R_1}$ is thus

$$\mathrm{Var}(\vec{R_1}) = \mathbf{V_{11}} = \mathbf{Z_1}\mathbf{G}\mathbf{Z_1^{\top}} + \mathbf{K_{11}} \tag{A.6}$$

The variance of $\vec{R_1}$ can be modeled by specifying the structures of $\mathbf{G}$ and $\mathbf{K}$. In the present case, for $\mathbf{G}$ we use a variance components (VC) structure, in effect assuming that the two random effects are uncorrelated, while for $\mathbf{K}$ we use a first order autoregressive moving-average (ARMA(1,1)) structure (see, e.g., Brockwell & Davis, 1991, 2002), reflecting our assumption about the correlation between the errors. The following are examples of a $\mathbf{G}$ variance component matrix with two mixed effects and a $\mathbf{K}$ matrix with only 4 episodes of a program.

$$\mathbf{G} = \begin{pmatrix} g_1 & 0 \\ 0 & g_2 \end{pmatrix}$$

$$ARMA(1,1) = \mathbf{K} = \sigma^2 \begin{pmatrix} 1 & \gamma & \gamma\rho & \gamma\rho^2 \\ \gamma & 1 & \gamma & \gamma\rho \\ \gamma\rho & \gamma & 1 & \gamma \\ \gamma\rho^2 & \gamma\rho & \gamma & 1 \end{pmatrix}$$

Estimates of $\mathbf{G}$ and $\mathbf{K}$ are usually obtained using maximum likelihood (ML) or restricted maximum likelihood (REML); for further details on minimizing these functions, see, e.g., SAS Institute Inc (2008). As was observed in Section 4.5, a specific pair of these matrices exists for each program cluster, and in particular for the soap opera clusters.

Once the algorithm has generated the estimates of $\mathbf{G}$ and $\mathbf{K}$, which are denoted $\hat{\mathbf{G}}$ and $\hat{\mathbf{K}}$, we can derive estimates of $\vec{\beta}$ and $\vec{s}$ by solving the *mixed model equations* (see, e.g., Henderson, 1984).

$$\begin{pmatrix} \mathbf{X_1^\top K_{11}^{-1} X_1} & \mathbf{X_1^\top K_{11}^{-1} Z_1} \\ \mathbf{Z_1^\top K_{11}^{-1} X_1} & \mathbf{Z_1^\top K_{11}^{-1} Z_1} + \mathbf{G}^{-1} \end{pmatrix} \begin{pmatrix} \hat{\vec{\beta}} \\ \hat{\vec{s}} \end{pmatrix} = \begin{pmatrix} \mathbf{X_1^\top K_{11}^{-1}} \vec{R}_1 \\ \mathbf{Z^\top K_{11}^{-1}} \vec{R}_1 \end{pmatrix} \tag{A.7}$$

The solutions can be written as

$$\hat{\mathbf{V}}_{\mathbf{11}} = \mathbf{Z_1} \hat{\mathbf{G}} \mathbf{Z_1^\top} + \hat{\mathbf{K}}_{\mathbf{11}} \tag{A.8}$$

$$\hat{\vec{\beta}} = (\mathbf{X_1^\top} \hat{\mathbf{V}}_{\mathbf{11}}^{-1} \mathbf{X_1})^{-} \mathbf{X_1^\top} \hat{\mathbf{V}}_{\mathbf{11}} \vec{R}_1 \tag{A.9}$$

$$\hat{\vec{s}} = \hat{\mathbf{G}} \mathbf{Z_1^\top} \hat{\mathbf{V}}_{\mathbf{11}}^{-1} (\vec{R}_1 - X_1 \hat{\vec{\beta}}) \tag{A.10}$$

The $^-$ sign in the equation denotes the generalized inverse. Note that $(\mathbf{X}^\top \widehat{\mathbf{V}}^{-1} \mathbf{X})^-$ is the co-variance matrix of $\hat{\vec{\beta}}$. In the SAS procedure of mixed linear models (PROC MIXED) it is known as the COVB matrix; hereafter it will be denoted $\hat{C}$. The $\hat{\vec{s}}$ estimates are not directly affected by chronologically future ratings but indirectly through the co-variance matrices. The rest of matrix $\hat{K}$ is also known because we have $\hat{\gamma}$, $\hat{\rho}$ and $\hat{\sigma}^2$. Using properties of matrix variance and co-variance we find $\mathbf{V_{22}}$ and $\mathbf{V_{12}}$ as follows:

$$\begin{aligned}\mathbf{V_{22}} &= \mathrm{Var}(\mathbf{X_2}\vec{\beta} + \mathbf{Z_2}\vec{s} + \vec{\varepsilon_2}) \\ &= \mathbf{Z_2}\,\mathrm{Var}(\vec{s})\mathbf{Z_2^\top} + \mathrm{Var}(\varepsilon_2) \\ &= \mathbf{Z_2GZ_2^\top} + \mathbf{K_{22}}\end{aligned} \tag{A.11}$$

$$\mathbf{\hat{V}_{22}} = \mathbf{Z_2\hat{G}Z_2^\top} + \mathbf{\hat{K}_{22}} \tag{A.12}$$

$$\begin{aligned}\mathbf{V_{12}} &= \mathrm{Cov}(\mathbf{X_1}\vec{\beta} + \mathbf{Z_1}\vec{s} + \vec{\varepsilon_1}, \mathbf{X_2}\vec{\beta} + \mathbf{Z_2}\vec{s} + \vec{\varepsilon_2}) \\ &= \mathrm{Cov}(\mathbf{Z_1}\vec{s}, \mathbf{Z_2}\vec{s}) + \mathrm{Cov}(\vec{\varepsilon_1}, \vec{\varepsilon_2}) \\ &= \mathbf{Z_1GZ_2^\top} + \mathbf{K_{12}}\end{aligned} \tag{A.13}$$

$$\mathbf{\hat{V}_{12}} = \mathbf{Z_1\hat{G}Z_2^\top} + \mathbf{\hat{K}_{12}} \tag{A.14}$$

Using elementary properties of the multivariate normal distribution, the conditional distribution of $\vec{R_2}$ given $\vec{R_1}$ (see, e.g., Searle, 1971) is found to be

$$\vec{R_2}|\vec{R_1} \sim N(\vec{\mu_2} + \mathbf{V_{12}^\top V_{11}^{-1}}(\vec{R_1} - \vec{\mu_1}), \mathbf{V_{22}} - \mathbf{V_{12}^\top V_{11}^{-1} V_{12}}) \tag{A.15}$$

This is the distribution of the out-of-sample episodes with full knowledge of the in-sample ones. Using the estimators obtained for the variance matrix and adding a factor for the indeterminacy of $\vec{\beta}$ (see, e.g., Harville, 1990) we obtain the following multivariate normal distribution, which is used to forecast the out-of-sample episodes.

$$\vec{R_2}|\vec{R_1} \sim N(\vec{\hat{\mu_2}} + \mathbf{\hat{V}_{12}^\top \hat{V}_{11}^{-1}}(\vec{R_1} - \vec{\hat{\mu_1}}), \\ \mathbf{\hat{V}_{22}} - \mathbf{\hat{V}_{12}^\top \hat{V}_{11}^{-1} \hat{V}_{12}} + (\mathbf{X_2} - \mathbf{\hat{V}_{12}^\top \hat{V}_{11}^{-1} X_1})\hat{C}(\mathbf{X_2} - \mathbf{\hat{V}_{12}^\top \hat{V}_{11}^{-1} X_1})^\top) \tag{A.16}$$

where $\vec{\hat{\mu_1}} = \mathbf{X_1}\vec{\hat{\beta}}$ and $\vec{\hat{\mu_2}} = \mathbf{X_2}\vec{\hat{\beta}}$.

A.2. Statistical significance of fixed and random effects

The statistical significance of the fixed and random effects is tested using the approximate co-variance matrix of $(\vec{\hat{\beta}} \quad \vec{\hat{s}})$, given by

$$\hat{M} = \begin{pmatrix} \mathbf{X_1^\top \hat{K}_{11}^{-1} X_1} & \mathbf{X_1^\top \hat{K}_{11}^{-1} Z_1} \\ \mathbf{Z_1^\top \hat{K}_{11}^{-1} X_1} & \mathbf{Z_1^\top \hat{K}_{11}^{-1} Z_1} + \mathbf{\hat{G}}^{-1} \end{pmatrix}^{-} \tag{A.17}$$

Following SAS Institute Inc (2008), consider any estimable linear combination of the following form:

$$L \begin{pmatrix} \vec{\beta} \\ \vec{s} \end{pmatrix}$$

If L is a single row, a general t statistic can be constructed as follows:

$$t = L \frac{\begin{pmatrix} \vec{\hat{\beta}} \\ \vec{\hat{s}} \end{pmatrix}}{\sqrt{L \hat{M} L^\top}}$$

The statistic is approximately t-distributed and its degrees of freedom must be approximated. One way of accomplishing this is to use $n - rank(\mathbf{X_1})$, where n is the number of observations.

APPENDIX B. QUANTIFICATION OF THE ERROR IN THE EXPECTED PROFIT

The stopping criterion for the simulation of an estimate is determined by the accuracy requirement, which in turn depends on the estimate's intended application. Stopping the simulation before the criterion is reached would mean foregoing potentially necessary information, while running it longer would simply waste computing time (Heidelberger & Welch, 1981). In our case we want to obtain a point estimate and a confidence interval for the expected profit μ = E[Profit] of a given soap opera. Following Ross (2006), we make n independent replications of the out-of-sample episodes' ratings and then transform them using the profit function described in Section 4.3 into n independent replications (P_1, P_2, ..., P_n) of the soap's profit. An unbiased point estimator ($\bar{P}(n)$) and an approximate 100(1-α) percent ($0 < \alpha < 1$) confidence interval for μ is given by

$$\bar{P}(n) = \sum_{i=1}^{n} P_i \quad \text{sample mean} \tag{B.1}$$

$$\mathrm{P}(\bar{P}(n) - z_{\alpha/2}\frac{S^2(n)}{\sqrt{n}} < \mu < \bar{P}(n) + z_{\alpha/2}\frac{S^2(n)}{\sqrt{n}}) \approx 1 - \alpha$$

where $z_{\alpha/2}$ is the $(1-\alpha/2)$100th percentile of the standard normal distribution and the sample variance is $S^2(n) = \frac{\sum_{i=1}^{n}(P_i - \bar{P}(n))^2}{n-1}$.

Using Equation B.1 we can define the half-width and relative half-width of the confidence interval as follows:

$$w_{\text{half-width}} = z_{\alpha/2}\frac{S^2(n)}{\sqrt{n}} \tag{B.2}$$

$$w_{\text{relative half-width}} = \frac{w_{\text{half-width}}}{|\bar{P}(n)|} \tag{B.3}$$

The stopping criterion we use is to continue simulating until the relative error $|\bar{P}(n) - \mu|/\mu$ falls below a pre-defined γ. At that point, $\bar{P}(n)$ will have a relative error of at most γ or the percentage error in $\bar{P}(n)$ is at most 100γ percent with a probability of $1 - \alpha$.

To reach this criterion we choose initial values for α and γ and continue generating data until $w_{\text{relative half-width}}$ is less than $\gamma/1+\gamma$. Then $\mathrm{P}(\frac{|\bar{P}(n)-\mu|}{|\mu|} < \gamma) \geq 1-\alpha$. Following the demonstration in Law and Kelton (2000), we have

$$
\begin{aligned}
1-\alpha &\approx \mathrm{P}(\bar{P}(n) - z_{\alpha/2}\frac{S^2(n)}{\sqrt{n}} < \mu < \bar{P}(n) + z_{\alpha/2}\frac{S^2(n)}{\sqrt{n}}) && \text{(B.4)}\\
&= \mathrm{P}(\bar{P}(n) - w_{\text{half-width}} < \mu < \bar{P}(n) + w_{\text{half-width}})\\
&= \mathrm{P}(|\bar{P}(n) - \mu| < w_{\text{half-width}})\\
&= \mathrm{P}(\frac{|\bar{P}(n) - \mu|}{|\bar{P}(n)|} < \frac{w_{\text{half-width}}}{|\bar{P}(n)|})\\
&= \mathrm{P}(\frac{|\bar{P}(n) - \mu|}{|\bar{P}(n)|} < w_{\text{relative half-width}})\\
&\leq \mathrm{P}(|\bar{P}(n) - \mu| < \frac{\gamma|\bar{P}(n)|}{1+\gamma})\\
&= \mathrm{P}(|\bar{P}(n) - \mu| < \frac{\gamma|\bar{P}(n) - \mu + \mu|}{1+\gamma})\\
&\leq \mathrm{P}(|\bar{P}(n) - \mu| < \frac{\gamma}{1+\gamma}(|\bar{P}(n) - \mu| + |\mu|))\\
&= \mathrm{P}(|\bar{P}(n) - \mu|(1 - \frac{\gamma}{1+\gamma}) < \frac{\gamma}{1+\gamma}|\mu|)\\
&= \mathrm{P}(|\bar{P}(n) - \mu|\frac{1}{1+\gamma} < \frac{\gamma}{1+\gamma}|\mu|)\\
&= \mathrm{P}(\frac{|\bar{P}(n) - \mu|}{|\mu|} < \gamma)
\end{aligned}
$$

The half-width and relative half-width may also be used as stopping rules (criteria) to control the length of the simulation (Rubinstein & Kroese, 2007). The stopping rule in these cases is simply a matter of choosing an initial α value and a required half-width or relative half-width and continue generating data until w_a or w_r meets the criteria.