PONTIFICIA UNIVERSIDAD CATOLICA DE CHILE

SCHOOL OF ENGINEERING

VEHICLE ROUTING PROBLEMS WITH PRODUCT MIXING AND EXTENSIONS

GERMÁN ENRIQUE PAREDES BELMAR

Thesis submitted to the Office of Graduate Studies in partial fulfillment of the requirements for the Degree of Doctor in Engineering Sciences.

Advisor:

VLADIMIR MARIANOV KLUGE

Santiago de Chile, October, 2016

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Members of the Commitee:

VLADIMIR MARIANOV

JORGE VERA

CRISTIÁN CORTÉS

CARLOS OBREQUE

JUAN ANTONIO MESA

CRISTIÁN VIAL

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To my parents Elvira and Luis

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ESCUELA DE INGENIERÍA

PROBLEMAS DE RUTEO DE VEHÍCULOS CON MEZCLA DE PRODUCTOS Y EXTENSIONES

Tesis enviada a la Dirección de Investigación y Postgrado en cumplimiento parcial de los requisitos para el grado de Doctor en Ciencias de la Ingeniería

GERMÁN ENRIQUE PAREDES BELMAR

RESUMEN

Esta tesis se proponen modelos de programación matemática y métodos eficientes para resolver distintos problemas de ruteo de vehículos con mezcla de productos. Esta investigación aborda tres problemas que consideran mezcla de productos: Un Problema de Recolección de Leche con Mezclas; El Problema de Recolección de Leche con Mezclas y Puntos de Recolección; y El Problema de Recolección de HAZMAT con carga de múltiples productos.

El problema de ruteo con mezcla de productos considera transporte productos de distinta calidad/tipo ofrecidos o demandados por un conjunto de clientes utilizando una flota de vehículos. A diferencia de los problemas tradicionales de ruteo de vehículos multi-producto, la mezcla de dos o más productos puede producir cambios en el estado de cada vehículo. Se requiere realizar un seguimiento de cada vehículo a lo largo de una ruta.

En el problema de recolección de leche con mezcla, donde una compañía recolecta leche utilizando una flota de vehículos no-homogénea, desde un conjunto de predios organizados como cooperativa. Existen tres calidades de leche, donde cada predio produce sólo una calidad. Los ingresos aumentan con la calidad de la leche. Además, se requieren cantidades mínimas de cada calidad en la planta. Los predios están distribuidos en una extensa zona geográfica, haciendo que el costo de transporte sea relevante. La novedad del problema es permitir la mezcla de distintas calidades de leche en los camiones, donde la calidad de la mezcla equivale a la leche de más baja calidad cargada. Utilizamos una heurística para resolver el problema, que busca maximizar el beneficio. Se propone un modelo de programación entera y se resuelve con branch-and-cut. El método se prueba en instancias test y en un caso real en el sur de Chile.

En el problema de recolección de leche con mezclas y puntos de recolección, la leche también se mezcla en los camiones. La recolección desde predios lejanos puede generar altos costos de transporte. Se propone ubicar puntos de recolección para que algunos predios lejanos puedan trasladar su producción a esos puntos. Luego, la leche acumulada en esos puntos es recolectada por un camión, evitando así largos recorridos. Se presenta un modelo de programación entera y se resuelve con branch-and-cut en instancias pequeñas. También presentamos una heurística para resolver instancias rápidamente.

En el problema de recolección de materiales peligrosos, un grupo de materiales peligrosos de distinto tipo son recolectados con una flota de camiones homogénea. Cada residuo posee expone a la población en distinta medida ante un accidente. Los residuos peligrosos pueden ser transportados en un mismo vehículo, a menos que la legislación actual lo prohíba. El riesgo generado por un vehículo hacia las personas y el ambiente, cambia cuando se agregan a éste nuevos residuos con otros tipos de riesgo. Se propone un modelo de programación entera que minimiza la población expuesta, y los costos de transporte. Presentamos un caso de estudio en Santiago de Chile.

Miembros de la Comisión de Tesis Doctoral
VLADIMIR MARIANOV
JORGE VERA
CRISTIÁN CORTÉS
CARLOS OBREQUE
JUAN ANTONIO MESA
CRISTIÁN VIAL
Santiago, octubre, 2016

PONTIFICIA UNIVERSIDAD CATÓLICA DE CHILE
ESCUELA DE INGENIERÍA

VEHICLE ROUTING PROBLEMS WITH PRODUCT MIXING AND EXTENSIONS

Thesis submitted to the Office of Research and Graduate Studies in partial fulfillment of the requirements for the Degree of Doctor in Engineering Sciences by

GERMÁN ENRIQUE PAREDES BELMAR

ABSTRACT

In this thesis, new mathematical programming models and efficient methods to solve different routing problems with product mixing are proposed. This research addresses three problems dealing with product mixing: A Milk Collection Problem with Blending; The Blended Milk Collection Problem using Collection Points; and The Hazardous Materials Collection problem with multiproduct loading.

The routing problem with product mixing studies the transportation of different quality/type of products generated or required by a set of clients using a fleet. Unlike the known multi-product routing problems, in the particular cases addressed in this Thesis, the combination of two of more commodities can produce changes in the status of the product or the load of each vehicle, which requires following this status along the route.

In the Milk Collection Problem with Blending, a firm collects milk using a nonhomogeneous truck fleet, from a number of geographically distributed farms organized as a cooperative. There are three qualities of milk, and each farm produces only one quality. Although the firm revenue increases with the quality of the collected milk, minimum amounts are required at the plant of each quality. The farms are spread over a large rural area, which makes transportation cost very relevant. The problem's novelty lies in the fact that different qualities of milk can be blended in some of the trucks en route, and the resulting blend is classified as its lowest quality component. A heuristic is used to solve the problem, which maximizes the profit while fulfilling the minimum requirements for the different milk qualities at the plant. The integer programming model is solved using branch and cut. The model is then run on several test instances and a real case in the south of Chile.

In the blended milk collection problem using milk collection points, the milk is also blended in the trucks. Collecting milk from the farthest farms could have high costs, so collection points are located so that the farthest farms can bring their milk to these points, where it is collected by a plant truck, avoiding a longer trip. A model is presented and solved using Branch and Cut for small instances. A heuristic is presented to solve larger instances quickly.

In the HAZMAT collection problem, a set of different hazardous wastes are transported using a homogeneous truck fleet. Each waste poses a level of risk to the population and environment exposed to an accident. The wastes can be transported in a same truck, unless considered incompatible by the regulations. The risk posed by a truckload to the exposed people and environment, changes when new waste with different risk level is added to the truck. Using an integer programming model, we minimize the total exposed population, as well as the total transportation costs. We present a case study in the city of Santiago of Chile to show the practical application of our proposed approach.

Members of the Doctoral Thesis Committee:
VLADIMIR MARIANOV
JORGE VERA
CRISTÁN CORTÉS
CARLOS OBREQUE
JUAN ANTONIO MESA
CRISTIÁN VIAL
Santiago, october, 2016

1. INTRODUCTION

Mathematical models can be used to help optimizing transportation costs and improving the use of available resources in many real problems dealing with distribution and collection of goods. The broad range of problems known as the Vehicle Routing Problem (VRP), appear in this context. The VRP aims at finding a set of vehicle routes and defining the most efficient sequence in which each vehicle must collect from, or deliver to a set of customers, not exceeding the capacity constraints, at the minimum cost (Irnich, Toth, & Vigo, 2014).

An important variant of VRP is the Multi-Product VRP (MPVRP). The MPVRP addresses transportation of a set of supplies or products of different types (size, class, quality, grade, classification, risk, among other features). When different products cannot be carried in the same truck, a VRP must be solved for each type of product (Abkowitz & Cheng, 1988; Ahluwalia & Nema, 2006; Dooley, Parker, & Blair, 2005; Hu, Sheu, & Huang, 2002; A. Nema & Gupta, 2003; A. K. Nema & Gupta, 1999; Sheu, 2007; Verter & Kara, 2008). In addition, trucks with compartments can be used, so that each compartment carries a different product. This approach is known as the Multi-Compartment Vehicle Routing Problem (MC-VRP). It usually appears in the transportation of products such as livestock, fuel, olive oil and perishable products, among others (Caramia & Guerriero, 2010; Fallahi, Prins, & Wolfler Calvo, 2008; Henke, Speranza, & Wäscher, 2015; Lahyani, Coelho, Khemakhem, Laporte, & Semet, 2015; Masson, Lahrichi, & Rousseau, 2015; Mendoza, Castanier, Guéret, Medaglia, & Velasco, 2010; Reed, Yiannakou, & Evering, 2014; Sethanan & Pitakaso, 2016).

If different products can be carried in a same vehicle or vehicle compartment, and they do not interact in any way with each other, the transportation costs can be decreased, taking advantage of the economies of scope. This problem has been studied in different contexts. Some researchers study the multiple product vehicle routing focusing on inventory routing over a time horizon, satisfying supply and demand for different goods

(Coelho & Laporte, 2013; Moin, Salhi, & Aziz, 2011). Other authors address dimension or weight constraints for products with different weight and volume, e.g. boxes, furniture, etc. (Junqueira, Oliveira, Carravilla, & Morabito, 2013; Russell, Chiang, & Zepeda, 2008; Viswanathan & Mathur, 1997). In all these cases, the products are combined in a same vehicle or compartment, but there is no particular need to study the interactions between the products being transported. Whenever a new product is loaded, the only relevant change in the vehicle status is its available capacity.

In this thesis, we propose, formulate and solve a novel variant of the MPVRP, which arises when different products are carried in the same truck or truck compartment, and these products interact with each other. In this problem, loading two products jointly results in a third, single product, whose features can be either equal to one of the loaded products, or different to both products originally loaded. Solving the problem requires studying the interactions between products and finding the best vehicle routes according to mixing requirements and operational constraints.

A mix of different products can be seen as a change in the status or qualification of the vehicles. For example, in the milk collection context, there may be different qualities of milk. If different qualities of milk are loaded in a same truck, the resulting mix is classified as the lesser quality milk among all those loaded in the truck. In hazardous material transportation, there is the need to account (and minimize) the exposed population of the vehicles when they transport different hazardous materials. The risk to which the vehicles expose the population, depends on the mix of the loaded products.

Note that the problem we address in this thesis is NP-Hard, as its versions are generalizations of the Vehicle Routing Problem or the Location Routing Problem.

To the best of our knowledge, this multi-product problem with product mixing or blending has not been dealt with in the specialized literature. We optimize this approach for three real applications.

The objective of this thesis is to determine the best transportation routes considering the particular complexities of each situation, using mathematical models and ad-hoc solution methods.

This research addresses three real problems including product blending or mixing: A Milk Collection Problem with Blending; The Blended Milk Collection Problem using Collection Points; and The Hazardous Materials Collection Problem with Multiple-Product Loading.

1.1. Milk Collection Problem with Blending

Chapter 2 presents the Milk Collection Problem with Blending (MB), which is the first vehicle routing problem with product mixing, analyzing the interactions between the different products loaded in a same truck.

In the MB problem, a firm collects raw milk to produce its final milk products. The firm has a heterogeneous truck fleet to collect the milk from a set of farms. The milk is collected directly from the farms, which are organized as a cooperative (Palsule-Desai, 2015). This organization requires all the produced milk being collected daily from all farms, regardless the distance, volume or quality.

There are three qualities of milk, and each farm produces a single quality. Is important to note that there are minimum volume requirements for each milk quality at the plant. The farms are spread over a large rural area, which makes the transportation cost an important component of the total cost.

Currently, blending is used in practice, but the milk is collected using an intuitive, empiric method by a dispatcher. Therefore, a mathematical approach is needed to improve the milk collection process.

The main contribution of this Chapter lies in the fact that different qualities of milk can be blended in some of the trucks en route, and the resulting blend is classified as its lowest quality component. It is also possible to blend at the plant, so that some amounts of higher quality can be used to satisfy the requirements of lower quality milk, whenever it is required to meet the quotas of milk.

Blending reduces the revenue, but the transportation cost decreases, resulting in a higher profit. The trucks may start the trip at any node (e.g. house of truck drivers, dispatching point, truck depot, plant, etc.) and end the trip at the plant.

It is important to note that the milk blending is a usual practice worldwide, and it does not violate any regulation when correctly done (CDIC, 2005; Chite, 1991; INIFAP, 2011; MHFW, 2015; Pinzón, 2015; U.S. DHHS, 2009; UNIA, 2011). GHL Incorporated (2014), in a report for the Food and Drug Administration (FDA), declares that "*It is a common practice in some states (United States), that bulk milk pickup tankers collect milk from two grades, Grade A and non-Grade A. Then these loads are delivered as non-Grade A milk at processing facilities*".

Several approaches for the milk collection problem have been presented in the literature, but none them includes blending. Some articles study real milk collection problems considering one quality of milk (Basnet, Foulds, & Wilson, 1999; Butler, Herlihy, & Keenan, 2005; Butler, Williams, & Yarrow, 1997; Claassen & Hendriks, 2007; Igbaria, Ralph H. Sprague, Basnet, & Foulds, 1996; Masson et al., 2015; Prasertsri & Kilmer, 2004; Sankaran & Ubgade, 1994).

If there is more than one quality of milk, there are different approaches: separate trucks (Dooley et al., 2005) or trucks with compartments (Caramia & Guerriero, 2010; Hoff & Løkketangen, 2007; Lahrichi, Gabriel Crainic, Gendreau, Rei, & Rousseau, 2015; Sethanan & Pitakaso, 2016).

We propose a mixed integer programming model for the milk blending problem.

The model allows blending in the trucks and at the plant. The model determines the farms visited by each truck and the route that the trucks must follow so that all the milk produced is delivered to the plant. It also shows where the blending should be done: in the trucks or at the plant. We also test our approach in trucks with compartments. The objective function of the model is to maximize the profit, computed as the revenues less the transportation costs (Paredes-Belmar, Marianov, Bronfman, Obreque, & Lüer-Villagra, 2016).

We solve instances of the problem using a branch-and-cut algorithm with known cuts and a new cut. This exact approach can solve medium size instances up to 100 nodes optimally. However, real cases can have many more producers, like our 500-node real case study. We develop the following three-stage heuristic for these instances:

- The first stage separates the instance into clusters using a suitable clustering method. The geographical accidents (rivers, main roads, lakes, mountains) can be used appropriately to make an efficient clustering. The *k*-means method is also a good and easy standard clustering method.

- As minimum amounts of each quality of milk must be met at the plant, and the there is an available truck fleet with different capacities, both milk quotas and trucks have to be allocated to each cluster. In the second stage, a mixed integer programming model is proposed to allocate milk requirements and correctly share the truck fleet among cluster.

- Finally, each cluster is solved to optimality using the branch-and-cut algorithm.

The models involved in this three-stage heuristic are coded in AMPL and solved with standard options of CPLEX.

The main contributions in this chapter are the introduction of the new Milk Collection Problem with Blending and an efficient solution method.

1.2. The Blended Milk Collection Problem using Collection Points

The simplest way to collect the raw milk from farms is the door-to-door method. However, when the farms are scattered over a large area, the milk collection truck routes could be long and they could contain many farms, resulting in high transportation costs and collection times. This method is inefficient when there are small farms grouped far away from the plant (Anquez & Tiersonnier, 1962), especially when there are several qualities of milk. The high transportation cost of this system has motivated the search for better collection options.

In Chapter 3, we generalize the problem introduced in the previous chapter, proposing an extension called the Milk Blending Collection with Collection Points (MBCP). In this extension, we consider the location of some collection points, to facilitate and expedite the milk collection process.

The aim of MBCP is finding the number and the location of collection (accumulation) points, to which a set of distant and small farms can deliver their production, decreasing the distances traveled by the firm trucks. Thereby, a vehicle can collect the total production of one or more farms in a single stop, saving cost and time and increasing the profit for the company. Those unallocated farms are visited directly by the trucks.

Note that there is an access cost of transporting the milk from the small and distant farms to the collection points. However, this cost is lesser than the cost of operating big tankers visiting the small and distant farms.

The objective is to maximize the profit, considering the milk revenues obtained from the milk received at the plant, the transportation costs, the costs of setting collection points and the access cost from farms to collection points.

The specialized literature presents a set of related problems to the MBCP. For example, the Median Tour Problem, consists in design a main tour for a vehicle, and the nodes not belonging to this main tour are assigned to the nearest node in the tour (Current & Schilling, 1994; Labbé, Laporte, Martín, & González, 2004; Labbé, Laporte, Rodríguez Martín, & González, 2005). Moreover, the MBCP is similar to the Vehicle Routing-Allocation Problem (VRAP), which consists in constructing a main route set to visit a subset of clients, where the non-visited clients can be assigned to a client in the route or be discarded with an associated penalization cost (Beasley & Nascimento, 1996; Ghoniem, Scherrer, & Solak, 2013; Vogt, Poojari, & Beasley, 2007). Other extensions of VRAP appears in the context of school bus routing problems (Bowerman, Hall, & Calamai, 1995; Riera-Ledesma & Salazar-González, 2012; P Schittekat, Sevaux, & Sorensen, 2006; Patrick Schittekat et al., 2013). None of these papers consider the mixing of different products in the same vehicle.

To the best of our knowledge, the MBCP has not been studied or solved in the literature of VRP nor milk collection, despite to be a practical and efficient approach.

To solve the MBCP we design the following approximate methodology. First, we propose a mixed linear integer programming model for the MBCP. Each truck can load milk in the farms and in the location points. Milk blending is allowed if it is profitable. As an output of the proposed model, we obtain the optimal route for each truck considering blending and collection points simultaneously; the set of collection points to be located, the farms allocated to an open collection point, and the unallocated farms which are loaded directly by trucks. We solve small size instances to optimality using a branch-and-cut procedure.

Second, we design a three-stage heuristic to solve large size instances, including exact models and heuristics:

- The first stage solves optimally a covering model that allocates the small and distant farms to collection points. The output of this stage indicates which collection points needs to be open, and the total milk volume in each one.

- The collection points have accumulated milk from a subset of farms. So, the collection points are now “virtual farms” producing potentially the three qualities of milk. The second stage generates feasible routes for the routing problem (considering all unallocated farms and collection points) using the ant colony meta-heuristic (Bell & McMullen, 2004; Bullnheimer, Hartl, & Strauss, 1999; Dorigo & Stützle, 2004; Mazzeo & Loiseau, 2004; Montemanni, Gambardella, Rizzoli, & Donati, 2005; Yu, Yang, & Yao, 2009).

- The third stage selects the best routes (between the routes generated in the previous stage) for a capacitated available truck fleet using a mixed-integer programming model.

The first and the third stage of the heuristic (mathematical models) are coded in AMPL and solved with CPLEX. The second stage (ant colony meta-heuristic) is coded in C++.

We describe computational results for a rural area in Chile. The real instance has 500 farms and 112 feasible location nodes (for collection points) and we compare those results with alternative approaches in the literature.

1.3. Hazardous materials collection with multi-product loading

Industrial hazardous wastes or materials (HAZMAT) are generated daily by industries located in urban sites and must be transported to a disposal sites or to final treatment.

The HAZMAT transportation involves risks of possible accidents with important consequences for human and natural environment. Particularly, in an urban zone, the accident probability of HAZMAT incident is low, but the consequences can be catastrophic (Zografos & Androutsopoulos, 2004). Hence, there is a general concern to develop mathematical models to find efficient and low-risk routes.

In Chapter 4, we address a new HAZMAT multiple-product collection problem, which is another application for the routing problem with product mixing.

Most of literature study the HAZMAT transportation problem with a single product (Androutsopoulos & Zografos, 2012; Bronfman, Marianov, Paredes-Belmar, & Lüer-Villagra, 2015; Giannikos, 1998; Jacobs & Warmerdam, 1994; Leonelli, Bonvicini, & Spadoni, 2000; Pradhananga, Taniguchi, & Yamada, 2010; Shih & Lin, 2003; Siddiqui & Verma, 2014; Tarantilis & Kiranoudis, 2001; Zografos & Androutsopoulos, 2002, 2004) presenting different variants as objectives, risk models, etc.

Moreover, there are authors considering the transport of more than one type/class of product (Abkowitz & Cheng, 1988; Ahluwalia & Nema, 2006; Hu et al., 2002; A. Nema & Gupta, 2003; A. K. Nema & Gupta, 1999; Samanlioglu, 2013; Sheu, 2007; Verter & Kara, 2008). These authors study the risk estimation as the main concern.

In our HAZMAT problem, a homogeneous truck fleet must collect a set of different hazardous materials from waste generators which are scattered over a large urban area. There are compatible wastes/materials (defined by regulation) that can be transported together in the same truck.

Each waste or material has a known population exposure risk. So, each waste exposes a different population size. We define the risk of the truck as the riskiest material being transported by the truck. The risk of a truck varies along his route and it is related to the products transported by the truck on each arc of the route.

Note that the physical mix is different to that of the MB or MBCP, since in this case, there is no blending, but the concept is similar for truck status effects. In the HAZMAT waste collection problem, different risks are combined in a same truck and we study how the risk to which each truck exposes the population changes when new waste is added to the truck.

The objectives of the HAZMAT collection problem are minimizing the routing costs and minimizing the total exposed population. Note that both objectives are in conflict: low-cost routes can be riskier in terms of total population exposure, and low-risk routes tend to be expensive. The literature on HAZMAT transportation has studied these objectives, offering a set of efficient solutions to decision makers.

We propose a bi-objective integer-programming model and we solve it optimally using the solver CPLEX. The model is coded in AMPL. We solve a hypothetical case in the urban area of Santiago, Chile. To the best of our knowledge, the proposed work has not been dealt with in the literature on HAZMAT nor vehicle routing.

Unlike the previous literature, we propose this new HAZMAT collection and we optimize it. We compare our results with the VRP approach (exclusive vehicles for each material) obtaining good results, in terms of number of vehicles, exposed population and transportation costs.

1.4. Thesis Contributions

In synthesis, the main scientific contributions of this thesis are the following:

First, we present a new class of routing problems with product mixing. This approach is new and different from other multi-product transportation problems presented in the vehicle routing literature. We study the interaction between the different materials loaded in a same vehicle and quantify the related effect. Three real applications are presented, modeled and solved in this thesis.

Second, we present a milk collection problem with blending, inspired in a real problem of a milk company. We formulate a mathematical model for the problem and solve small instances with a branch-and-cut scheme. We propose a three-stage heuristic to solve large instances. The methodology achieves good results and improves the current empiric method used by the company.

Third, we generalize the milk collection problem with blending, adding collection points to improve the solutions of the problem. We formulate the problem using a mixed-integer programming model and solve it with a three-stage heuristic obtaining good results in terms of solution quality (when they are compared with other approaches) and solution time.

Fourth and finally, we model and solve a HAZMAT collection problem with multiple-products, in which loading different products in a truck is allowed, provided that the compatibility constraints given by current regulations are met. The methodology provides a set of non-dominated solutions for the HAZMAT collection problem. A case in the urban area of Santiago-Chile is presented and solved.

The remainder of the thesis is organized as follows. Chapter 2 contains the paper "*The Milk Collection Problem with Blending"*, published in Transportation

Research Part E: Logistics and Transportation Review*;* Chapter 3 presents the paper "*The Milk Blending Problem using Collection Points*", submitted to Computers & Electronics in Agriculture, Chapter 4 presents the paper "*Hazardous Materials Collection with Multiple-Product loading*" published in Journal of Cleaner Production.

2. A MILK COLLECTION PROBLEM WITH BLENDING

A milk collection problem with blending is introduced. A firm collects milk from farmers, and each farm produces one out of three possible qualities of milk. The revenue increases with quality, and there is a minimum requirement at the plant for each quality. Different qualities of milk can be blended in the trucks, reducing revenues, but also transportation costs, resulting in higher profit. A mixed integer-programming model, a new cut, and a branch-and-cut algorithm are proposed to solve medium-sized instances. A three-stage heuristic is designed for large instances. Computational experience for test instances and a large-sized real case is presented.

This chapter was formatted as a manuscript and submitted to Transportation Research Part E: Logistics and Transportation Review in December, 2015. It was accepted in July 21, 2016 and published (Paredes-Belmar et al., 2016). This chapter contains the modifications done to the manuscript.

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2.1. Introduction

The cost of collecting milk from producers in the milk production supply chain has a significant impact on profit (Lahrichi et al., 2015; Rojas & Lusa, 2005). Milk producers are frequently scattered over extended rural areas, sometimes far from processing plants, making transportation cost a relevant component of total cost. It is also common for small producers to organize themselves into cooperatives, able to obtain better commercial terms with, usually, a single buyer (FAO, 2012). Each cooperative sells the milk produced by its members to the buyer, or firm, who performs the collection process (Palsule-Desai, 2015). This arrangement is convenient for the cooperative members, but poses some challenges to the buyer, who must collect milk from all farms in the cooperative, although some may be located far from the plant. Moreover, milk produced by different farms can have different qualities or grades, used for different final products. Currently, firms address the differences in quality by either using separate trucks for collecting different qualities, or using tanks with separate compartments for different qualities. Both solutions are expensive, and particularly if some farms produce small quantities of milk, as in the real case in this study.

A different approach, consisting of mixing or blending different qualities of milk in some of the trucks, is also possible. Blending degrades the quality of part of the collected milk, as the blended product is classified as its lowest quality component, which reduces the firm's revenue. However, the savings in transportation cost exceed the reduction in revenue and ultimately increase profit.

Blending is a common practice. GLH Incorporated (2014), in a report for the United States' Food and Drug Administration (FDA), explicitly declares that "It is a common practice in some states, including two large milk production states, that bulk milk pickup tankers pick up milk from both Grade 'A' and non-Grade

'A' milk producers on the same tanker. Then, these loads are delivered to non-Grade 'A' processing facilities." New York regulations allow commingled milk in trucks (New York SDAM, 2003). A plant in the south of Chile, which is this paper's case study, also uses blending. The common use of blending makes this a relevant practice that, to the best of the authors' knowledge, has never been analyzed in the literature.

Note that blending does not violate any regulations, as long as the resulting milk is correctly classified at the processing facility. Therefore, this procedure could be extended to wherever the grading of milk or dairy products is performed by bacterial limits and somatic cell count, or by fat content. Examples of countries with such regulations, among others, are the United States (Chite, 1991; U.S. DHHS, 2009), Canada (CDIC, 2005), Bolivia (UNIA, 2011), Mexico (INIFAP, 2011), Panama (Pinzón, 2015), and India (MHFW, 2015). This procedure also applies when the industry imposes such a classification, as in the case of the Murray Goulburn Co-operative in New South Wales and Sydney, Australia (Devondale Murray Goulburn, 2014).

The blending of different products or product qualities in the same vehicle also applies to other industries. Bing et al. (2014) solve a waste collection problem, in which collection is either performed after separating some recyclables at collection points, or by loading different classes of waste in the same truck and classifying them at the processing site. These are also possible applications for this study's approach.

Finally, yet importantly, Sethanan & Pitakaso (2016) state that the mixing of raw milk from different collection centers in the same compartment would be a valuable extension of their research, as they do not use blending, and this would "add to the ability of (their) technique to model real world problems."

An optimization of the blending procedure is proposed, as this is of practical relevance, and has not been previously addressed. This study's contributions are several. First, the Milk Collection Problem with Blending (MB) is introduced which, rather than an algorithmic contribution, describes and solves an innovative milk collection method. For each truck in a heterogeneous fleet, the MB solution indicates what farms each truck must visit, and the route it must follow, to deliver all produced milk to the plant. This also specifies whether it is more convenient to perform blending in the trucks or at the plant. The objective is to maximize the firm's profits. Second, a mixed integer formulation for the problem is proposed, as well as a branch-and-cut algorithm, using a new cut and known cuts to solve medium-sized instances optimally. Third, a heuristic procedure is designed to solve large instances, which partitions the set of farms into clusters or areas, each with a fewer number of farms. Solving the problem by clustering is non-trivial, as there are milk quotas to be fulfilled and a given truck fleet; therefore, trucks and milk quotas must be efficiently assigned to clusters. Finally, the problem is solved for each cluster using the branch-and-cut algorithm.

Test instances of up to 100 nodes are solved, and a real instance is solved that includes 500 farms. The solutions obtained using this new approach are then compared to the solutions currently implemented by the firm, and with the solutions obtained by collecting each quality of milk separately, using the vehicle routing problem (VRP) for each. Managerial insight is provided. Finally, solutions for trucks with and without compartments, and with and without blending are compared, which demonstrates that blending dominates all solutions.

Note that the problem is NP-Hard, as for one quality of milk, it reduces the VRP, which is NP-Hard (Irnich et al., 2014; P Toth & Vigo, 2002).

The remainder of the chapter is organized as follows: subsection 2.2 presents the literature review. Subsection 2.3 describes the milk collection problem with blending. Subsection 2.4 details the development of the mixed integer programming (MIP) model, the valid cuts, and their separation algorithms. Subsection 2.5 illustrates a procedure for solving large instances. Subsection 2.6 is devoted to numerical experience, with the test instances, the actual case, and the full heuristic. Different alternative approaches are compared in this section, including the use of compartments. Section 2.7 concludes.

2.2. Literature review

The literature provides a large number of articles studying the VRP, considering different variants, applications and solution methods (Golden, Raghavan, & Wasil, 2008; P Toth & Vigo, 2001; Paolo Toth & Vigo, 2014). A relevant variant of the VRP is the Multi-Product Vehicle Routing Problem (MPVRP). The MPVRP allows to reduce costs, by consolidating different products in a same vehicle (Liu, Lei, & Park, 2008). This variant has been addressed in different ways. Some authors consider as the main concern the inventory management over a time horizon (L C Coelho, Cordeau, & Laporte, 2012; Huang & Lin, 2010; Moin et al., 2011; Zhalechian, Tavakkoli-Moghaddam, Zahiri, & Mohammadi, 2016; Zhang, Qi, Miao, & Liu, 2014) or dimension constraints for products with different weight and volume, e.g. boxes, furniture, etc. (Junqueira et al., 2013; Russell et al., 2008; Viswanathan & Mathur, 1997). Other studies solve the MPVRP using trucks with compartments for liquids, avoiding product blending in a same compartment (Caramia & Guerriero, 2010; Fallahi et al., 2008; Henke et al., 2015; Lahyani et al., 2015; Lai, Crainic, Di Francesco, & Zuddas, 2013; Mendoza et al., 2010; Reed et al., 2014; Sethanan & Pitakaso, 2016). In all those cases, the products are loaded on a same vehicle but remain separable, so they can also be unloaded separately.

Product collection, specifically, has also been profusely addressed in a broad range of fields, going from blood collection from donors (Gunpinar & Centeno, 2016) to waste collection from islands (Miranda, Blazquez, Vergara, & Weitzler, 2015), to cite extreme examples. None has used different product blending. In terms of milk collection, there are different variants, as well as a number of solution methods, none of which include blending. Sankaran & Ubgade (1994) were the first to address milk collection as a special case of the VRP. They designed collection routes to minimize transportation costs, using a Decision Support System (DSS) to solve a 70-farm case in Etah, India, obtaining yearly savings of USD $15,000. Igbaria et al. (1996), Prasertsri & Kilmer (2004) and Butler et al. (2005) solved similar real problems using a DSS tool.

Butler et al. (1997) solve a milk collection problem in which some producers must be visited twice, while the milk from the rest of the farms is collected only once a day. Basnet et al. (1999) propose an exact model and a heuristic procedure for assigning trucks to predefined routes in New Zealand, the goal of which is to minimize the time at which the last truck delivers its load to the plant. Hoff & Løkketangen (2007) solve a real case in Norway, using the model defined by Chao (2002) for a truck and trailer routing problem. Claassen & Hendriks (2007) address a goat milk collection problem in the Netherlands. The producers are visited according to individual frequencies, rather than daily. The authors minimize the deviations, whether surplus or deficit, between collection and production. Dayarian, Crainic, Gendreau & Rei (2013) and Dayarian, Crainic, Gendreau & Rei (2015b) solve a milk collection and distribution problem over multiple periods, considering significant seasonal production variations over a tactical time horizon. Dayarian, Crainic, Gendreau & Rei (2015a) introduce a routing problem with multiple depots, a heterogeneous fleet, and time windows, motivated by a real milk collection problem in Canada. They solve an integer programming formulation using column generation and a branch-and-price algorithm.

Other authors examine the collection of different qualities of milk. Dooley et al. (2005), in a New Zealand application, classify the milk into two qualities. Each quality of milk is collected separately, and the transportation cost is minimized. Caramia & Guerriero (2010) address a problem with four qualities of milk, which are not allowed to be blended. They use trucks with compartments and apply a local search heuristic, which first allocates producers to truck compartments to minimize the number of trucks. The routing problem is solved in the second stage, with the goal of minimizing travel distance. Lahrichi et al. (2015) addressed a case in Canada, in which trucks with two compartments start their route at a single depot in multiple periods and collect three qualities of milk, which are not allowed to be blended. The milk is transported to a set of plants, and the trucks travel back to the depots, at a minimum travel cost. The authors solve the problem using a tabu search heuristic. Sethanan & Pitakaso (2016) determine the milk collection routes for a set of milk collection centers. They consider the use of different qualities of milk without blending, as they use trucks with compartments. The article suggests, as a future work, the blending of milk from different collection points in the same compartment. Masson et al. (2015) study an annual dairy transportation problem, inspired by a Canadian milk collection problem. They generalize the problem proposed by Lahrichi et al. (2015), which considers variations on a basis of daily demand.

Literature also handles the collection of fresh agricultural products, in terms of related problems. A review of this literature is provided in the work of Shukla & Jharkharia (2013). The trucks in that case are not usually equipped to maintain a product’s freshness; therefore, delivery time constraints are required.

2.3. Milk collection with blending

Farms in this study's version of the problem produce three qualities of milk with decreasing revenues; quality A is better than B, which is better than C. Quotas for each quality must be satisfied at the plant. Blending of the different qualities of milk is allowed in the trucks along the route to save transportation costs, and at the plant to satisfy its minimum requirements. Blending makes the problem non-separable by quality of milk. In the numerical tests, all routes end at the plant, but they can begin anywhere, and the trucks are heterogeneous. For example, Figure 2-1 illustrates a five-node network, in which the node marked "0" is the plant. In this example, the routes start and end at the plant, and the number beside each arc is the transportation cost. The quota for each quality of milk is 200 liters, and the unit revenues per liter of milk are 1.0, 0.7, and 0.3 monetary units (MU) for milk A, B, and C, respectively. The capacity of each truck is 220 liters.

If no blending occurs, the problem becomes a separable VRP for each milk quality. The solution requires three trucks. The transportation cost is 280, following the routes 0-1-2-4-2-0 (milk A), 0-2-0 (milk B), and 0-3-0 (milk C), while the revenue is 420, with a total profit of 140. If blending is allowed, the MB requires the same three trucks. The optimal routes are 0-1-0 (milk A), 0-2-4-2-0 (milk B, resulting from the blending of milks B and A), and 0-3-0 (milk C) at a cost of 220 and a revenue of 414. The total profit is 194, or 39% higher than the profit obtained without blending. The revenue in the latter case is reduced by 6 units, while costs are reduced by 60 units.

Note that at the beginning of each season, or with a frequency that depends on the regulations (e.g., FDA, 2015), each farm's production line is inspected and a grade or quality, and consequently, a unit price, is assigned to that producer's milk for that season. The farmer is paid for the volume of milk he delivers, at the price set during the last inspection. Both volume and price are known *a priori*. Consequently, the total payment is a fixed amount, not possible to optimize, which does not depend on whether or not it is blended. Hence, payment is not included in the objective. The plant bears the cost of the reduction in revenue because of blending.

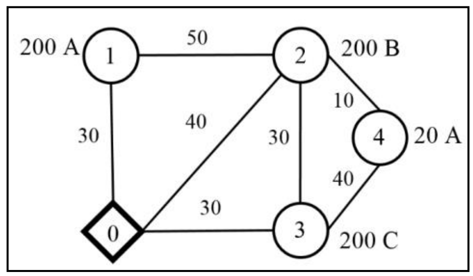


Figure 2-1. Network for comparing MB and VRP solutions

The model allows for the blending of different qualities of milk at the plant, as this is sometimes profitable, as noted in the following example. Producers 1, 2, and 3 in Figure 2-2 each produce 100 liters of milk B, and producers 4 and 5 each produce 200 liters of milk A. The quotas at the plant are 350 liters of milk B and 200 liters of milk A. The capacity of each truck is 500 liters, and two trucks are required for collecting all the milk. In Figure 2-2a), blending at the plant is not allowed. Truck 1 follows the route shown as a continuous line, and truck 2 follows the dotted route; truck 1 collects 500 liters of milk B, while truck 2 collects 200 liters of milk A. The revenue is 550 units, the cost is 330 units, and the profit is 220 units. Figure 2-2b) displays the optimal solution when blending at the plant is allowed, with truck 1 collecting 300 liters of milk B, and truck 2 collecting 400 liters of milk A. At the plant, 50 liters of milk A are blended with milk B to satisfy the milk B quota. The revenue is 595 units, and the cost is 320 units. The final profit is 275 units, which is 55 units higher than the solution illustrated in Figure 2-2a).

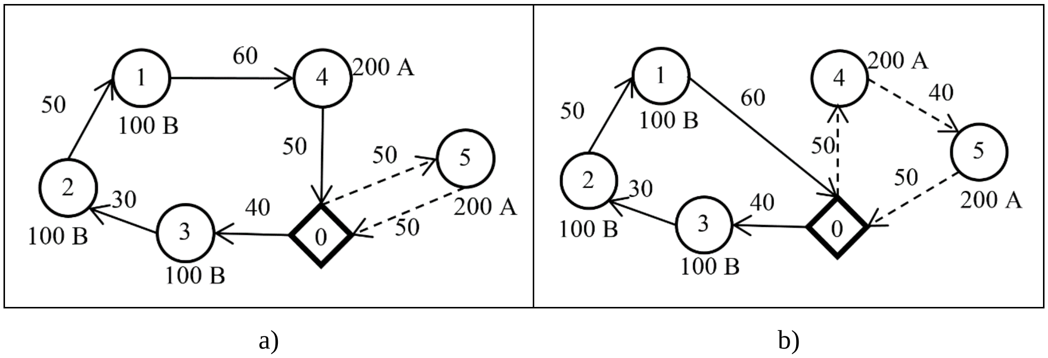


Figure 2-2. Effects of blending at the plant

Note that to satisfy the quotas for the three qualities of milk, the following relations must hold:

$$q_A \geq P_A \tag{2.1}$$

$$\left(q_A - P_A\right) + q_B \geq P_B \tag{2.2}$$

$$\left(q_A - P_A\right) + \left(q_B - P_B\right) + q_C \geq P_C \tag{2.3}$$

where q_A, q_B, and q_C are the total production of milk A, B, and C, respectively, and P_A, P_B, and P_C are the minimum quotas at the plant for milk A, B, and C, respectively.

It can be observed that blending can also be applied to trucks with compartments; that is, each compartment in such a truck can carry blended milk.

2.4. Mixed integer model

Let $G(N, A)$ be a complete graph, with N as the set of nodes representing producers, and A as the set of arcs, or roads. N_0 is defined as $N \cup \{0\}$, where 0 is the node that identifies the plant location. A^0 defines the set of arcs connecting the plant with the producers. K is the set of trucks, and T is the set of qualities of milk. N^t identifies the producers of milk quality $t \in T$; D^t = {milk quality r| blend of r and t results in r. Includes $r = t$}. IT is the set of ordered pairs (i, t) of producer i and milk quality t, as each client produces only one quality of milk. However, it is easy to generalize the model by making one copy of each farm for each quality of milk it produces, with all copies sited in the same location. Q^k is the capacity of truck k; q_i^t is the amount of milk t produced by farm i; c_{ij}^k the travel cost of truck k over the arc $(i, j) \in A \cup A^0$; α^t is the revenue per unit of milk quality t; and P^t is the quota for milk quality t at the plant.

The notation from the work of Yaman (2006) is used for the heterogeneous truck fleet. Let $K_i = \{k \in K: t \in T, q_i^t \leq Q^k \ \ \forall t\}$ be the set of trucks that can visit producer i and, for each arc $(i, j) \in A \cup A^0$, let the truck set $K_{ij} = \{k \in K: q_i^t + q_j^t \leq Q^k \ \ \forall t\}$ be those trucks that can travel from producer i to producer j, collecting all milk from both producers without exceeding truck capacity. Node 0_k is the node at which truck k starts its trip, which ends at the plant. Finally, the set $AK = \{(i, j, k): (i, j) \in A \cup A^0, k \in K_{ij}\}$ is defined.

Decision Variables

$$x_{ij}^k = \begin{cases} 1 & \text{If truck } k \text{ travels directly from node } i \text{ to node } j \\ 0 & \text{otherwise} \end{cases}$$

$$y_i^{kt} = \begin{cases} 1 & \text{If truck } k \text{ loads milk type } t \text{ from producer } i \\ 0 & \text{otherwise} \end{cases}$$

$$z^{kt} = \begin{cases} 1 & \text{If truck } k \text{ delivers milk type } t \text{ to the plant} \\ 0 & \text{otherwise} \end{cases}$$

$w^{kt} =$ Volume of milk quality t that truck k delivers to the plant.

v^{tr} = Volume of milk of quality t delivered to the plant, blended for its use as milk of quality r.

The formulation of the MB problem is as follows:

$$Z = Max \sum_{t \in T} \sum_{r \in T} \alpha^r v^{tr} - \sum_{(i,j,k) \in AK} c_{ij}^k x_{ij}^k \tag{2.4}$$

Subject to

$$\sum_{t \in T} \sum_{i \in N:(i,t) \in IT} q_i^t y_i^{kt} \le Q^k \qquad \forall k \in K \tag{2.5}$$

$$\sum_{k \in K_i} y_i^{kt} = 1 \qquad \forall i \in N, t \in T:(i,t) \in IT \tag{2.6}$$

$$\sum_{j:(0_k, j,k) \in AK} x_{0_k j}^k \le 1 \qquad \forall k \in K \tag{2.7}$$

$$\sum_{i:(i,j,k) \in AK} x_{ij}^k = \sum_{h:(j,h,k) \in AK} x_{jh}^k \qquad \forall k \in K_j, j \in N_0 \tag{2.8}$$

$$\sum_{p:(p,i,k) \in AK} x_{pi}^k = y_i^{kt} \qquad \forall k \in K_i, i \in N, t \in T:(i,t) \in IT \tag{2.9}$$

$$z^{kt} \le 1 - \sum_{\substack{r \in D^t: r \ne t, \\ (i,r) \in IT}} y_i^{kr} \qquad \forall k \in K_i, i \in N, t \in T \tag{2.10}$$

$$\sum_{t \in T} z^{kt} \le 1 \qquad \forall k \in K \tag{2.11}$$

$$w^{kt} \le z^{kt} Q^k \qquad \forall k \in K, t \in T \tag{2.12}$$

$$w^{kt} \le \sum_{r:t \in D^r} \sum_{h \in N^r} q_h^r y_h^{kr} \qquad \forall k \in K, t \in T \tag{2.13}$$

$$\sum_{k \in K} \sum_{t \in T} w^{kt} = \sum_{(i,t) \in IT} q_i^t \tag{2.14}$$

$$\sum_{r \in D^t} v^{tr} = \sum_{k \in K} w^{kt} \qquad \forall t \in T \tag{2.15}$$

$$\sum_{t \in T} v^{tr} \ge P^r \qquad \forall r \in D^t \tag{2.16}$$

$$y_i^{kt} + y_i^{kr} \le 1 \qquad \forall (t,r) \in PM; (i,t),(j,t) \in IT \tag{2.17}$$

$$\sum_{i \in S} \sum_{j \in S} x_{ij}^k \le |S| - 1 \qquad \forall S \subseteq N, k \in K \tag{2.18}$$

$$y_i^{kt}, z^{kt} \in \{0,1\} \qquad \forall i \in N, k \in K_i, t \in T:(i,t) \in IT \tag{2.19}$$

$$x_{ij}^k \in \{0,1\} \qquad \forall (i,j,k) \in AK \tag{2.20}$$

$$w^{kt}, v^{tr} \ge 0 \qquad \forall k \in K; t, r \in T, r \in D^t \tag{2.21}$$

Objective (2.4) maximizes profit, which is the revenue from the milk received at the plant, minus the transportation cost. Recall that the price paid to the farmers is not included in the objective because it is constant. Constraints (2.5) limit the capacity of each truck. Constraints (2.6) require collection of the milk of each farm by exactly one truck. Constraint (2.7) imposes one route at most for each truck, which must start at some node 0_k for each truck k. Constraints (2.8) are flow balance equations for each node and each truck. The route of truck k, by virtue of constraints (2.9), must stop at node i, if it collects the milk from that node. Constraints (2.10) avoid a truck loading milk of a quality lesser than t if it is delivering milk quality t to the plant. Constraints (2.11) require a truck k to contain only one quality of milk, possibly blended. The relationship between continuous variables w^{kt}, or volume of milk t, and the binary variables z^{kt}, or the assignment of milk quality t to truck k, is set by constraints (2.12). Constraints (2.13) measure the volume of milk A, B or C arriving at the plant in truck k.

Note that constraints (2.10) - (2.13) set the blending rules, and different rules could be included by changing these constraints. Constraint (2.14) forces the transportation of all the produced milk to the plant. Constraints (2.15) balance the amount of each quality of milk arriving at the plant, and the amount of remaining milk of each quality after blending at the plant. Constraints (2.16) enforce the plant’s quotas. Constraints (2.17) avoid prohibited blends. Constraints (2.18) prevent unwanted sub-tours for each truck. Finally, constraints (2.19) - (2.21) set the variables’ domain.

The agreement between the firm and the cooperative stipulates collection of all produced milk, for this study’s real case. However, visiting some low-production, distant farms could be more expensive than the profit obtained from collecting their milk. In this case, if the agreements between the members of the cooperative allow for this practice, these farms could be paid for their milk, but not visited. This case is a variant of the Price Collecting Routing Problem (Balas, 1989; Tang & Wang, 2006). This study's model can be easily modified to solve this problem, replacing constraints (2.6) and (2.14) with the following constraints:

$$\sum_{k \in K_i} y_i^{kt} \leq 1 \qquad \forall i \in N, t \in T : (i,t) \in IT \qquad (2.22)$$

$$\sum_{k \in K} \sum_{t \in T} w^{kt} = \sum_{k \in K} \sum_{(i,t) \in IT} q_i^t y_i^{kt} \qquad (2.23)$$

It is also easy to modify the model to represent a situation in which trucks have compartments, each of which can carry blended milk. Each compartment in this modified model is treated as if it was a single truck. However, all the "trucks," representing compartments of the same actual truck, are forced to travel together. The following constraints replace the original constraints in the model:

$$\sum_{j:(0_k, j, k) \in AK} x_{0_k j}^k \leq |C| \qquad \forall k \in K \qquad (2.24)$$

$$\sum_{p:(p,i,k) \in AK} x_{pi}^k \geq y_i^{kt} \qquad \forall k \in K_i, i \in N, t \in T : (i,t) \in IT \qquad (2.25)$$

Constraint (2.24) now states that the number of compartments that depart from node 0_k is at most the highest number |C| of compartments in any truck, and constraint (2.25), as opposed to requiring exactly one truck visiting farm *i*, allows all compartments belonging to the same truck visiting that node, if one or more of its compartments collects milk there. Finally, the following constraint is added:

$$x_{ij}^k = x_{ij}^s \qquad \forall (i, j) \in A, (k, s) \in CM \qquad (2.26)$$

where *CM* is the set of compartments *k* and *s* in the same truck. The constraint (2.26) forces all compartments in the set *CM* to follow the same route.

The production of a farm P can exceed the compartment size C. In that case, we make $\lceil P/C \rceil$ copies of the farm, located at the same point. For example, if P = 3.5C, there will be four copies of the farm, with three of them producing C, and one producing 0.5C.

This model is solved using the same cuts as in the case with no compartments.

2.4.1. Valid Inequalities

The following cuts are used in the branch-and-cut procedure.

Proposition 1

The cut

$$x_{ij}^k \leq \sum_{h \neq i} x_{jh}^k \qquad \forall (i, j) \in A \cup A^0 : i, j \neq \{0\}, k \in K \qquad (2.27)$$

is valid for the MB.

Proof: Dror, Laporte & Trudeau (1994). Note that constraints (2.5)–(2.9) and (2.18) are a model for a capacitated VRP, for each truck. The cut in Proposition 1, proven to be valid for the VRP by Dror et al. (1994), works together with these constraints, uses only the VRP variables, and performs exactly the same function as in that problem. ■

Proposition 2

$$\sum_{i \in S} x_{ij}^k \leq \sum_{h \in S^c, m \in S : h \neq j} x_{hm}^k \qquad \forall S \subseteq N, j \in S^c, k \in K \qquad (2.28)$$

is a valid cut for the MB.

Proof. This inequality follows from the balance constraints (2.8): for a set $S \subseteq N$, if a truck k uses an arc (i, j) with $i \in S$ and $j \in S^c \subseteq N \backslash S$, there must be another arc (h, m) with $m \in S$; otherwise, there is no continuity in the route. It must hold that $h \neq j$, since otherwise, there will be a sub-tour including node j and nodes belonging to S.

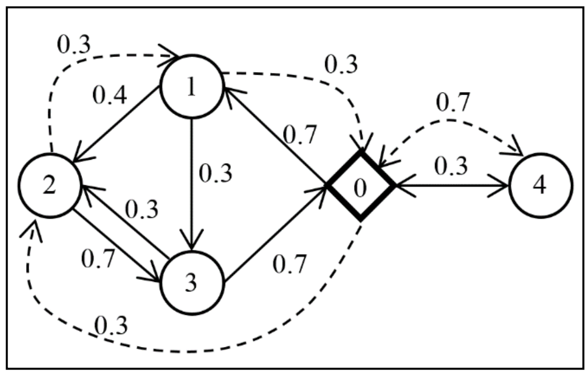


Figure 2-3. Sub-graph of a non-integer solution that shows the use of cut (2.28)

Cut (2.28) is a tighter extension of the connectivity constraint in the work of Drexl (2014). Figure 2-3 displays part of a non-integer solution of the problem with two routes, in which the route for the truck k = 1 is the continuous line, and the route for truck k = 2 is the dashed line.

Truck routes originate at the plant. The reader can be convinced that the connectivity constraint from the work of Drexl (2014) holds for any set S. However, cut (2.28) does not. For example, with S = {1, 2}, i = 1, j = 3, k = 1, $x^1_{1,3} + x^1_{2,3} = 0.3 + 0.7 \not\leq x^1_{0,1} = 0.7$. ■

Proposition 3

The following "multi-star inequality"

$$\sum_{t\in T}\sum_{\substack{i\in S^c\\ j\in S}}\left(Q^k - q_i^t\right)x_{ij}^k \geq \sum_{a\in S}\sum_{t\in T} q_a^t y_a^{kt} + \sum_{t\in T}\sum_{\substack{h\in S\\ m\in S^c}} q_m^t x_{hm}^k \qquad \forall S\subseteq N, k\in K \tag{2.29}$$

is valid for the MB.

Proof: Yaman (2006). Each vehicle k traveling from node $i\in S^c$ to node $j\in S$ must have enough capacity to carry the production of incoming node i, the production of visited nodes in set S, and the production of the subsequent node m.□

Proposition 4

The following inequality

$$\sum_{i\in S^c, j\in S} x_{ij}^k \geq \sum_{t\in T} y_h^{kt} \qquad \forall S\subseteq N, h\in S, k\in K \tag{2.30}$$

is a valid cut for the MB.

Proof: Toth & Vigo (2002). If a farmer $h\in S$ with milk $t\in T$ must be visited by a vehicle k, such that $y_h^{kt} = 1$, then the route of vehicle k must visit the set S, from a node $i\in S^c$ to a node $j\in S$, that is, $x_{ij}^k=1$.□

Constraints (2.18) cannot be directly used in the model, as there is an exponential number of them. These constraints, and the integrality constraints (2.19) - (2.20), are relaxed to solve this relaxed problem. If the solution found by the relaxed formulation is not a feasible integer optimum, a constraint (2.18) is added, as well as cuts (2.27) - (2.30) as required, and solve again. Separation algorithms are used for each cut to find what constraints and cuts to add at every iteration.

Once new cuts are no longer found, the procedure continues with a branch and bound step, after which the separation algorithms are applied again, and so on. The branch and bound used is the standard procedure in the CPLEX software package, using standard options.

2.4.2. Separation algorithms

Once a solution is found, its *supporting graph* $G_S^k(N^k, A_S^k)$ is built for each truck $k \in K$, where $A_S^k = \{(i, j) \in A \cup A^0 \colon x_{ij}^k > 0\}$; that is, the graph includes all arcs for which the associated decision variables are nonzero in the solution. Three independent separation algorithms exist, corresponding to different cuts.

2.4.2.1. Separation algorithm for (2.27)

This algorithm searches for arcs (i, j) of the supporting graph, such that $i \neq 0$ and $j \neq 0$, that is, the arc does not begin or end at the plant and, for each such arc, verifies the violation of cut (2.27). Note that cut (2.27) could have been added to the main model, generating $|K|\cdot|N|^2$ new constraints; however, this would increase the solution time. The order of this algorithm is $O(|K|\cdot|N|^2)$.

2.4.2.2. Separation algorithm for (2.28) and (2.29)

This algorithm analyzes all arcs belonging to the supporting graph and not connected to node 0, or the plant. For each arc (i, j) in the sub-graph corresponding to truck k of the supporting graph, the set S of nodes connected to j, or the head of the arc, is found. After a node is added to the set, the violation of cut (2.28) is checked and the cut is added as required. If a cut is added, the process continues with the next truck k+1. The order of this algorithm is $O(|K|\cdot|N|^4)$.

For cut (2.29), the sets S that are just found are used, as well as the *greedy randomized algorithm* proposed in the works of Augerat et al. (1995) and Baldacci, Hadjiconstantinou & Mingozzi (2004). This procedure starts from a known set S. Let $t \in S^c$ be a node, such that $\sum_{j \in S} x_{tj}^k = \max_{i \in N \setminus S} (\sum_{j \in S} x_{ij}^k)$. If cut (2.29) is violated for $S' = S \cup \{t\}$ in the supporting graph, then a cut is added to the problem. This procedure is repeated while there are remaining nodes to add to S'. The order of this algorithm is $O(|K|\cdot|N|^3)$.

2.4.2.3. Separation algorithm for (2.30) and (2.18)

A set S' is built, starting from node 0. All nodes connected to S' through an arc $(i, j) \in A_S^k$ (i.e., arcs corresponding to truck k) are added one by one until no more connected nodes are found. Then, a set S in S'^c is built using the same procedure, starting with any node $l \notin S'^c$. A cut is added for each node $h \in S$. The process is repeated for each truck k. The order of this algorithm is $O(|K|\cdot|N|^2)$.

Finally, the same sets S are analyzed for the violation of the constraint (2.18).

2.5. A three-stage heuristic for large instances

Solving an instance larger than 100 farms using the branch-and-cut method would be time consuming and, given the available tools, impractical. Some trial runs were conducted to solve the problem, using both VRP, or separate trucks for different qualities of milk, and the formulation presented in subsection 2.3.

Instances of up to 80 – 100 nodes and five to ten vehicles could be solved in both cases.

A three-stage heuristic (TSH) is proposed that divides the large instances into smaller sub-problems, and solves each sub-problem separately.

A set of farms is partitioned into clusters in the first stage. Cluster formation is the subject of an extensive body of literature; for a comprehensive review, see the work of Xu & Tian (2015). Any known method could be used for the problem at hand, provided the amount of milk in each cluster can fill the smallest truck available, and the maximum number of farms in each cluster does not exceed 100, which is the maximum number of farms the blending procedure can manage at this time.

As a clustering procedure, *k*-means (MacQueen, 1967; Žalik, 2008) is a fast, efficient method. *K*-means locates *k* virtual "mean" points and allocates each farm to its closest mean, considering distances over the road network, in such a way as to minimize the sum of the farm-mean distances. Each group of farms around a virtual mean, or cluster, provides shape to an area of the partition. The value of *k* is chosen heuristically, so that each cluster results in a manageable instance for the method in subsection 2.3. Note that as the number of clusters increases, the sub-problems become easier, but the quality of the global solution decreases. Further details on the implementation of *k*-means can be found in the work of Hartigan & Wong (1979).

The *k*-means is a general procedure that can be applied to any large instance. However, this does not explicitly use any information that may be available regarding the geographical region, such as the existence of natural barriers that vehicles cannot cross as highways, rivers, or mountains. As information exists in this case study regarding these natural and fabricated barriers, it is used to perform an *ad-hoc* heuristic ("geographical") clustering, which leads to improved results. The use of such a method must be evaluated on a case-by-case basis. In this study's real case, this geographical partition works more efficiently than *k*-means, as further discussed in the subsection regarding computational results.

The second stage consists of assigning milk quotas to each cluster, which guarantees *a priori* the global quota of each quality of milk required at the plant, and allocating trucks to each cluster. These tasks are performed using a Mixed Integer Programming (MIP) formulation.

Let C be the set of clusters or areas, and $ASIG$ the set of producer-cluster pairs (i, c). Let $\underline{T}$ be the lower bound of the number of trucks per cluster. This lower bound is computed as the minimum number of trucks required to collect all the milk in the cluster, if all the milk had the same quality. Let also $\bar{T}$ be the upper bound of the number of trucks per area. This bound is computed as the number of trucks required to collect all the milk without the use of blending, which is solving a VRP for every milk quality in the cluster. The variables are:

$$\hat{y}_{ck} = \begin{cases} 1 & \text{If truck } k \text{ is allocated to cluster } c \\ 0 & \text{otherwise} \end{cases}$$

$u^{trk} =$ Volume of milk of quality t delivered to the plant by truck k, blended for its use as milk of quality r, used to satisfy the minimum milk volume requirements at the plant.

$u_{+}^{trk} =$ Volume of milk of quality t delivered to the plant by truck k, blended for its use as milk of quality r, not used to satisfy the minimum milk volume requirements at the plant (surplus).

The formulation of the model that assigns trucks and allocates milk quotas to each cluster is as follows:

$$Z = Max\sum_{t\in T}\sum_{r\in T}\sum_{k\in K}\alpha^{r}\left(u^{trk} + u_{+}^{trk}\right) \tag{2.31}$$

Subject to

(2.5) - (2.6), (2.10) - (2.15), and (2.18)

$$\sum_{r\in T}\left(u^{trk} + u_{+}^{trk}\right) = w^{kt} \qquad \forall t \in T, k \in K \tag{2.32}$$

$$\sum_{t\in T: r\in D^{t}} u^{trk} = P^{r} \qquad \forall r \in T \tag{2.33}$$

$$\sum_{c\in C}\hat{y}_{ck} = 1 \qquad \forall k \in K \tag{2.34}$$

$$\underline{T} \leq \sum_{k\in K}\hat{y}_{ck} \leq \overline{T} \qquad \forall c \in C \tag{2.35}$$

$$\sum_{\substack{i\in N:\\ (i,t)\in IT \wedge\\ (i,c)\in ASIG}} q_{i}^{t} \leq \sum_{k\in K} Q_{k}\hat{y}_{ck} \qquad \forall c \in C \tag{2.36}$$

$$\sum_{\substack{t\in T:\\ (i,t)\in IT}} y_{i}^{kt} \leq \sum_{\substack{c\in C:\\ (i,c)\in ASIG}} \hat{y}_{ck} \qquad \forall i \in N, k \in K \tag{2.37}$$

$$u^{trk}, u_{+}^{trk} \geq 0 \qquad \forall k \in K; t, r \in T : r \in D^{t} \tag{2.38}$$

$$\hat{y}_{ck} \in \{0,1\} \qquad \forall c \in C, k \in K \tag{2.39}$$

The model maximizes the revenue from the milk received at the plant (31). The constraints (2.5) - (2.6), (2.10) - (2.15), and (2.18) guarantee truck capacity feasibility, and measures the volumes of milk delivered to the plant. Constraints (2.32) and (2.33) require all the milk collected by every truck being delivered to the plant, and used to satisfy the minimum requirements, and possibly some extra amount. Constraints (2.34) force the allocation of every truck to an area. Constraints (2.35) allocate the correct number of trucks to every area, while constraints (2.36) assure that the allocated truck capacity is sufficient. Constraints (2.37) relate the decision variables, forcing that if a truck visits a producer, it is assigned to the area in which the producer is located. Finally, (2.38) - (2.39) state the domain of decision variables.

The third stage of the heuristic involves the branch-and-cut method from subsection 2.4.

2.6. Computational Results

2.6.1. Test instances

The model is first applied to 40 test instances, which range from 23 to 101 nodes. Nine of these instances (eil22–eil101, att48) belong to the TSPLib set (Reinelt, 1991). The instances (a32-a80) are taken from the work of Augerat et al. (1995). The instances (tai75A – tai75D and c50-c75) belong to the work of Taillard (1999). Instances f45 and f72 are taken from the work of Fisher (1994). In all instances, the coordinates and the production (or demand) of the nodes are known.

The branch-and-cut method is used with CPLEX Version 12.5 and AMPL version 20130109. All experiments were run on a PC Intel i7-2600, 3.4 GHz, 16GB RAM, and Ubuntu Server 12.04 LTS.

Three heterogeneous trucks are used in these instances, and the total capacity is sufficient to transport all the milk. The networks are symmetric; that is, $c_{ij} = c_{ji}$. The transportation cost on each arc $(i, j) \in A$ is equal to the Euclidean distance d_{ij} between the end nodes of the arc, that is, $c_{ij} = d_{ij}$. Distances have been approximated to their closest integer value. The income per liter is 1.0, 0.7, and 0.3 units for qualities A, B, and C, respectively. The milk produced at each farm is equal to the demand in each node of the test instances, multiplied by a scale factor *f*, which is used for all test results to have the same order of magnitude. Qualities are assigned to farms using the following rule, which makes a cyclic assignment, that is: node 1 produces milk A, node 2 produces milk B, node 3 produces milk C, node 4 produces milk A, and so on.

The quotas of milk are defined arbitrarily and, in the test instances, are the same for all qualities of milk. Equations (1) – (3) are followed to fix the minimum amounts.

Table 2-1, Table 2-2 and Table 2-3 and display the results. Net indicates the instance. The notation *Q*[*,*;*] indicates the capacity for each truck in thousands of liters. *P*[*,*,*] is the quota of milk A, B, and C, respectively, in thousands of liters. $|N_0|$ is the number of nodes in the network, including the plant. Each truck starts and ends its route at the plant. *Z* is the optimal profit, in monetary units. *VA*, *VB*, and *VC* are the amounts of milk of each quality (liters) after delivery and blending at the plant. *T* is the CPU time in seconds, *TB* indicates if there are blends in the trucks, and *PB* indicates if there are blends at the plant.

Note that blending is used in more than half of the cases, improving profit over the alternative of a separate collection. Running times are reasonable, and these are the summation of the times taken by all four processors of the computer. Therefore, clock time is approximately one quarter of the times shown in the Tables.

Table 2-1. Results for test instances, (Reinelt, 1991)

Net	***Q***	***P***	$\|N_0\|$	***f***	***Z***	***VA***	***VB***	***VC***	***T***	***TB***	***PB***
eil22	[10;15;20]	[6;5;4]	22	1	15,947	9,800	7,200	5,500	12	no	no
eil23	[6;7;8]	[1; 1.5; 2]	23	1	7,207	6,100	2,009	2,080	6	no	no
eil30	[5;5.5;6]	[2.2;2.4;2.6]	30	1	7,117	2,400	6,000	4,350	99	yes	no
eil31	[55;50;40]	[10;5;8]	31	1	60,080	28,300	34,800	27,200	66	no	no
eil33	[20;20;15]	[8;7;6]	33	1	20,409	11,200	11,690	6,480	58	no	no
eil51	[25;30;35]	[22;23;24]	51	1	50,128	22,400	29,600	25,700	154	no	no
eil76	[45;48;51]	[45;43;40]	76	1	91,461	46,800	46,700	42,900	1,700	no	no
eil101	[50;55;60]	[47;48;49]	101	1	96,115	48,800	48,000	49,000	66,843	yes	yes
att48	[50;45;40]	[38;34;30]	48	1	17,452	40,000	37,500	40,000	284	yes	no

Table 2-2. Results for test instances, Augerat et al. (1995)

Net	***Q***	***P***	$\|N_0\|$	***f***	***Z***	***VA***	***VB***	***VC***	***T***	***TB***	***PB***
a32	[10;15;20]	[12;10;8]	32	1	26,660	16,200	10,000	14,800	23	no	yes
a33	[15;20;25]	[15;8;6]	33	100	29,417	17,600	11,400	15,600	62	no	no
a34	[20;20;25]	[10;12;14]	34	100	30,496	15,900	16,000	14,000	40	no	yes
a36	[20;15;15]	[10;12;14]	36	100	29,233	16,000	14,200	14,000	110	no	yes
a37	[20;15;10]	[10;8;6]	37	100	24,837	10,000	16,200	14,500	45	yes	no
a38	[20;20;10]	[10;15;15]	38	100	28,596	10,000	20,000	18,100	570	yes	no
a39	[20;20;20]	[10;12;14]	39	100	30,808	14,600	17,300	16,600	110	no	no
a44	[25;20;15]	[20;16;12]	44	100	38,771	23,400	16,000	17,600	101	yes	yes
a45	[25;20;20]	[20;18;18]	45	100	40,282	23,300	18,000	18,000	136	yes	no
a46	[30;25;20]	[16;17;18]	46	100	40,696	22,400	19,900	18,000	66	no	no
a48	[30;25;20]	[20;20;20]	48	100	39,800	20,100	20,000	22,500	230	yes	yes
a53	[30;30;30]	[20;20;20]	53	30	46,662	23,490	24,870	23,250	183	no	no
a54	[15;15;15]	[5;5;5]	54	50	22,414	12,600	11,650	9,200	304	no	no
a55	[15;15;20]	[5;10;15]	55	50	24,694	11,900	12,250	17,800	270	no	no
a60	[20;10;20]	[8;12;16]	60	50	25,041	11,800	13,650	16,000	3,565	yes	yes
a61	[35;35;35]	[30;20;10]	61	100	60,644	30,400	34,600	23,500	561	no	no
a62	[15;15;15]	[10;11;12]	62	50	22,917	12,500	11,000	13,150	1,022	yes	no
a63	[20;20;20]	[5;10;20]	63	50	24,447	10,050	13,600	20,000	2,930	yes	yes
a64	[20;20;20]	[5;10;20]	64	50	24,100	11,750	10,650	20,000	5,395	yes	yes
a65	[15;15;15]	[10;12;14]	65	50	28,046	14,350	15,000	14,500	478	yes	no
a69	[20;20;20]	[10;15;15]	69	50	25,822	11,750	15,500	15,000	1,552	yes	no
a80	[20;20;20]	[16;10;16]	80	50	29,977	16,250	14,650	16,200	5,626	no	no

Table 2-3. Results for test instances, Taillard (1999) and Fisher (1994)

Net	***Q***	***P***	**|N_0|**	***f***	***Z***	***VA***	***VB***	***VC***	***T***	***TB***	***PB***
c50	[35;30;30]	[15;30;26]	51	100	49,803	21,700	30,000	26,000	84	yes	yes
c75	[40;50;50]	[40;45;50]	76	100	86,677	40,000	46,400	50,000	13,760	yes	yes
tai75A	[25;30;35]	[10;15;20]	76	5	65,477	21,980	15,000	31,800	4,806	yes	yes
tai75B	[30;30;30]	[20;25;25]	76	5	48,238	24,530	25,000	25,000	15,056	yes	yes
tai75C	[15;20;25]	[5;10;15]	76	5	25,906	13,085	10,000	24,515	4,537	yes	yes
tai75D	[20;35;30]	[15;15;20]	76	5	65,477	20,935	19,880	30,060	2,645	no	no
f45	[20;15;10]	[5;10;5]	45	5	23,705	9,760	18,020	8,320	80	yes	no
f71	[50;50;50]	[25;20;10]	72	1	72,864	26,865	49,998	37,977	3,483	yes	no
f72	[60;60;60]	[20;30;40]	72	1	72,072	26,861	47,979	40,000	3,659	yes	yes

The optimal solution was found for all test instances. Although the main objective of Tables 2-1, 2-2 and 2-3 is to show that the problem is solvable for medium size instances, a difference can be noticed between the three sets of instances: the number of instances in each set requiring blending is different. This suggests that the convenience of blending depends on the structure of the network, and the model will choose not blending if such choice is the best.

2.7. Real Case

This study's methodology is applied to a real case, and its performance is compared with the VRP and with the result of the current procedure used by the firm, located in the south of Chile. The firm uses trucks with no compartments. An expert planner designs the routes by hand, and there is occasional heuristic blending of small amounts of milk in the trucks. The firm collects milk from 500 farms spread across a geographical region of approximately 9,600 square kilometers. The average daily production of the farms ranges from 57 to 25,000 liters, as illustrated in Figure 2-4. Note that 53.2% of farmers produce less than 2,326 liters.

Of these 500 farms, 313 produce milk of quality A – in short, milk A; 159 produce milk B; and 28 produce milk C. The volume of milk is 1,435,168 liters of milk A, 268,564 liters of milk B, and 74,475 liters of milk C. The quotas for milk A, B, and C at the plant are 1,250,000, 300,000, and 100,000 liters, respectively. The current criterion is maximizing the amount of milk A. The revenue, in monetary units per liter of milk, is $1.5 \cdot 10^{-2}$, $1.05 \cdot 10^{-2}$, and $4.5 \cdot 10^{-3}$ for milk A, B, and C, respectively.

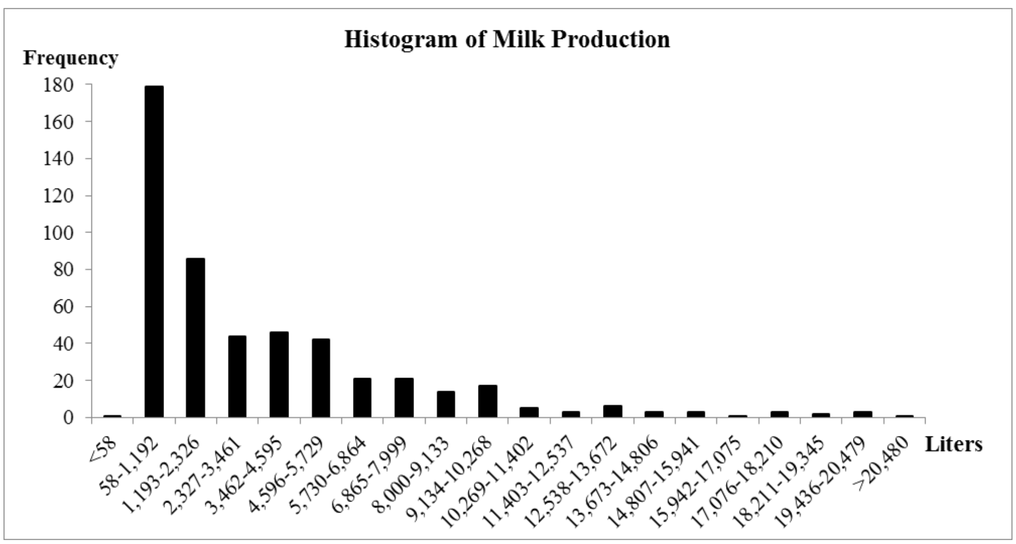
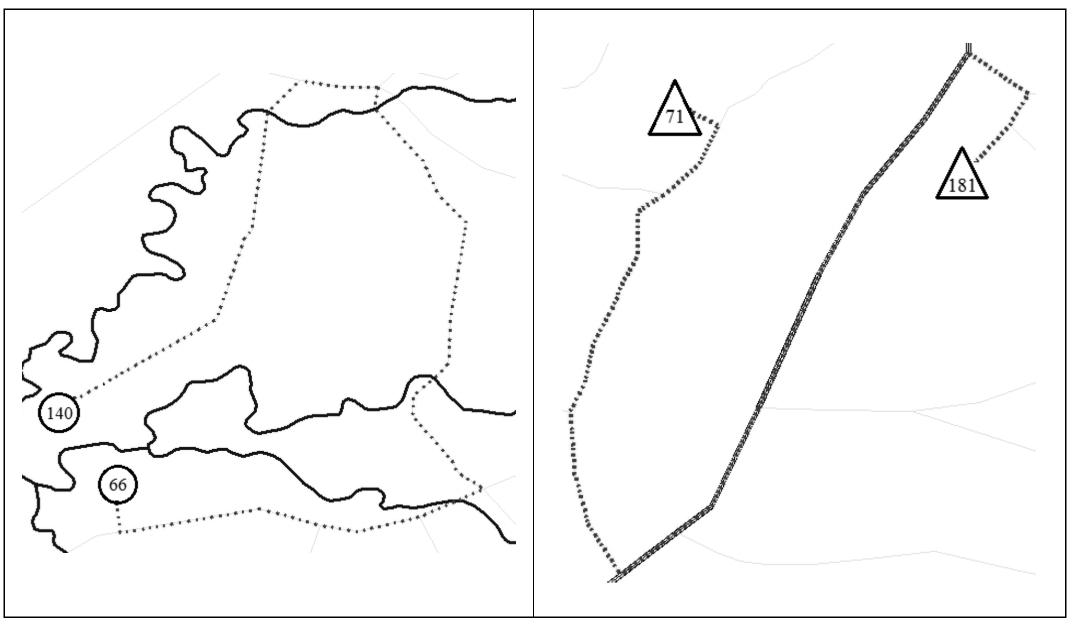


Figure 2-4. Production histogram of actual farms

The fleet is composed of 100 trucks, whose capacities are as follows: 15 15,000-liter trucks; 20 20,000-liter trucks; 15 25,000-liter trucks; and 50 30,000-liter trucks.

As this is too large to use the procedure described in subsection 2.4, the region is partitioned into areas, or clusters. First, this study takes advantage of such geographical barriers as rivers, lakes, mountains, and highways to form the clusters. The road network is tree-like, rather than meshed. Consequently, the network distance between producers can be long, even if the Euclidean distance is short. Figure 2-5 displays an example. The dotted line in Figure 2-5a) notes the 19 km route from farm 66 to farm 140, separated by a Euclidean distance of 2 km. The solid lines are rivers. The dotted line in Figure 2-5b) indicates the shortest route between farms 71 and 181, partially using the solid line, which is a highway. The route distance is 21 km, while the Euclidean distance is 4 km. Different trucks will likely visit these pairs of farms.

Using these barriers as boundaries, the region was divided into 25 areas, containing clusters of between 3 and 39 farms each, as displayed in Figure 2-6 ("geographical" partition).



a) b)

Figure 2-5. Natural barriers in the area

Once the set of farms was partitioned into clusters, the model in subsection 2.5 was used to assign trucks and allocate milk requirements to clusters, or the **second stage of the heuristic**. This stage takes 5,103 seconds of CPU time.

The collection problem was then optimally solved for each cluster, using the branch-and-cut algorithm, or the **third stage of the heuristic**. Table 2-4 notes the results of the three-stage heuristic, and compares it with VRP, or the optimized routes for trucks collecting each quality of milk separately, solved for each cluster independently, and the firm's current procedure.

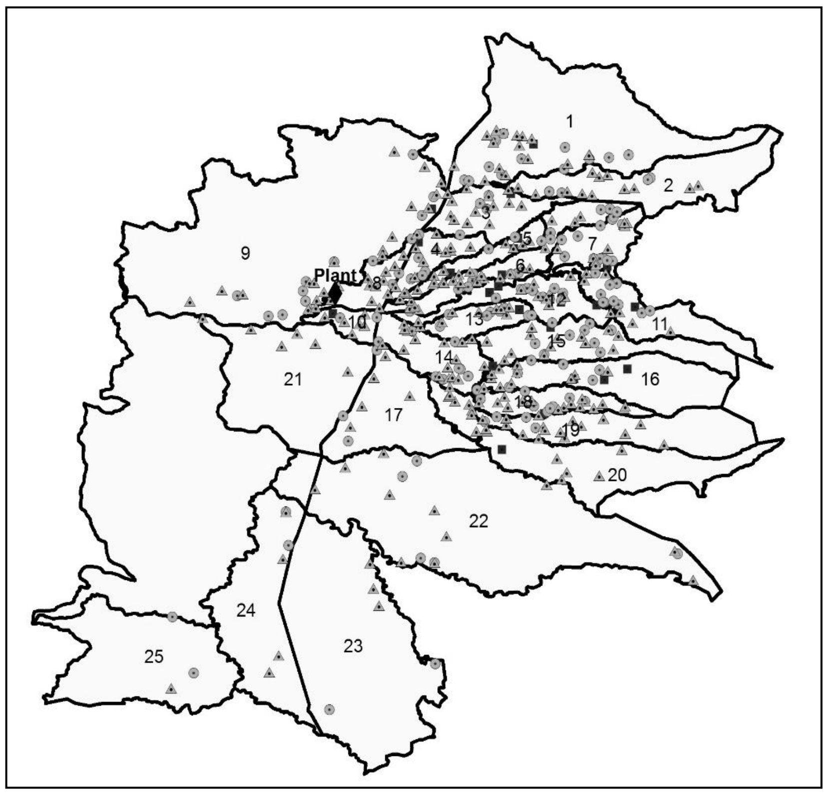


Figure 2-6. Partition of the region into 25 independent areas. A diamond denotes the plant location.

The VRP model presented by Irnich et al. (2014) was employed, using the objective (1) with the same cuts used in MB. The MB solution in this case does not require blending at the plant, while VRP requires large amounts of milk being blended at the plant to satisfy the quotas. Using the current procedure, 627 liters of milk B are used to complete the milk C quota. Note that although the

MB requires more blending in the trucks, which seems counterintuitive, the profit is significantly higher.

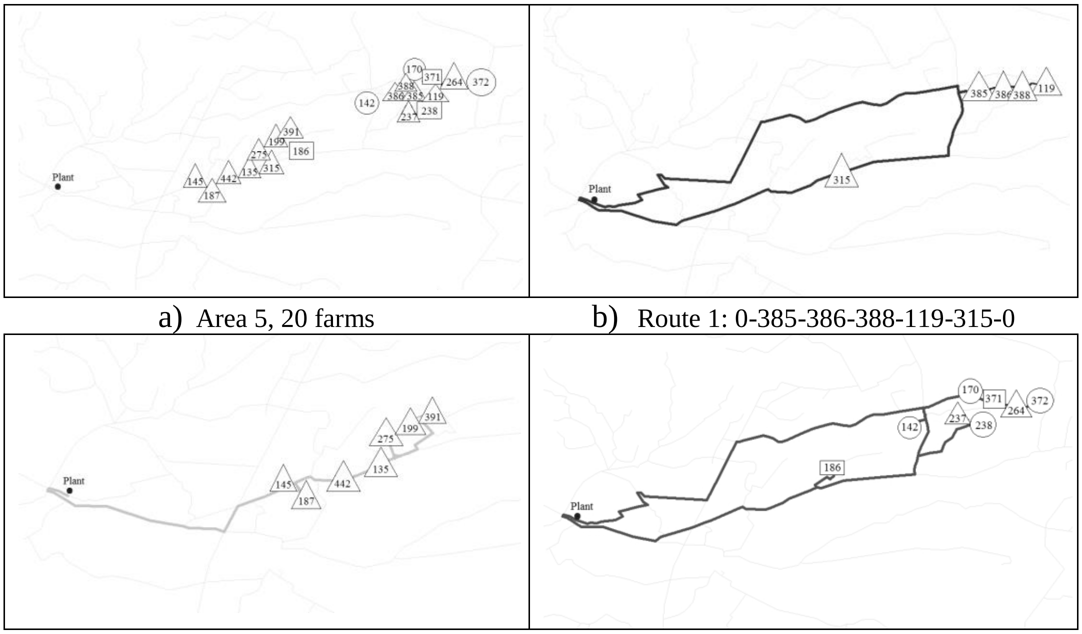
The reported CPU time is the sum of all CPU times required to solve the 25 clusters. As a 4-core computer was used, the clock time was 1.65 hours. Note that all the milk must be collected daily from all farms belonging to the cooperative. However, the routes' programming does not need to be performed on a daily basis, as the production volume in each farm changes slightly; this only occurs in exceptional situations owing to meteorological variations, cattle nutrition changes, or cattle diseases (Dayarian et al., 2015b). Rather, the production changes follow a seasonal cycle.

Table 2-4. Results for the real case for a region partitioned into 25 clusters. Profit denoted in boldface.

	MB	**VRP**	**Current procedure**
Trucks	81	99	100
Revenue [MU]	23,266	24,273	22,804
Costs [MU]	10,093	12,319	18,659
Profit [MU]	**13,173**	**11,954**	**4,145**
Milk A [l]	1,278,815	1,435,168	1,051,791
Milk B [l]	306,060	268,564	627,043
Milk C [l]	193,332	74,475	99,373
A → B [l]	0	31,436	-
A → C [l]	0	25,525	-
B → C [l]	0	-	627
CPU time	23,836	6,241	-

Table 2-4 provides some interesting managerial insight. The VRP solution increases the profit over the firm's current procedure in 2.8 times, and it is obtained in 26 minutes. However, it requires roughly the same number of routes (trucks). When blending is allowed, the transportation costs decrease to roughly one-half of the current transportation costs, and the profit is 3.2 times that of the current procedure and 10% higher than that of VRP. Although the difference between profits using MB and VRP may not seem too significant, it must be noted that MB saves 18 routes over VRP, which could mean significant additional savings and administrative burden because of the reduced number of required trucks. These gains are obtained at the expense of a longer run time (1 hour and 40 minutes, approximately).

Figure 2-7a) illustrates area 5 of the real case and its collection routes (Figures 7b, 7c, 7d) as an example of how the MB uses blending. The two first routes collect 49,762 liters of milk A, while the third route collects a blend of 10,455 liters of milk A, B, and C, resulting in 10,455 liters of C milk. The transportation costs of routes 1, 2, and 3 are 83, 49, and 105 monetary units, respectively. The total cost is 237 monetary units. The profit in this area is 556 monetary units. The VRP requires four routes, with at least two vehicles collecting milk A, one collecting milk B, and one collecting milk C. The total amount of milk collected, before blending, is 52,551 liters of milk A, 2,285 liters of milk B, and 5,381 liters of milk C. The transportation costs for the VRP solution are 361 monetary units, the revenue 836, and the profit 475 monetary units.



a) Area 5, 20 farms

b) Route 1: 0-385-386-388-119-315-0

c) Route 2: 0-135-199-391-275-442-145-187-0

d) Route 3: 0-186-238-237-142-170-264-372-371-0

Figure 2-7. Area 5 and collection routes

The effect of using different partitions is also explored. A second partition was defined, with 13 larger areas, by aggregating some of the 25 areas in the "geographical" partition. Contiguous areas were aggregated for, as the natural barriers were the easiest to cross. Third and fourth partitions were also constructed, with 19 and 25 areas using *k*-means.

Table 2-5 shows the results using different partitions.

As expected, the trucks' utilization is more efficient for larger areas, and fewer vehicles are required to collect the milk. In the limit, using just one area would lead to the best of all solutions and would not require the full heuristic but only the mixed-integer formulation, but it could be intractable in terms of the required run time for large instances. As the number of areas increases, the quality of the solution decreases but the run time does too. A manager should decide what the best quality-time trade-off in the particular instance she needs to solve is and, before applying the method, trials should be run with different numbers of clusters, to arrive at the best choice.

Increasing milk quotas for the highest quality milk also has an effect on revenue and CPU time. Table 2-6 displays the results of two runs for the 25-area geographical partition, in which the required amounts of milk A were 1,200,000 and 1,300,000 liters, while the requirements for milk B and C were the same as previously, or 300,000 and 100,000 liters.

Table 2-5. Different partitions. Profits and CPU time are indicated in boldface.

# of Areas	**25 (geographical)**	**25 (*k*-means)**	**19 (*k*-means)**	**13 (geographical)**
Trucks	81	88	72	72
Revenue [UM]	23,266	23,652	23,205	23,719
Costs [UM]	10,093	10,776	9,256	9,564
Profit [UM]	**13,173**	**12,876**	**13,949**	**14,155**
A [l]	1,278,815	1,315,653	1,262,638	1,330,337
B [l]	306,060	315,702	324,208	295,110
C [l]	193,332	146,852	191,361	152,760
A→B [l]	0	0	0	4,890
A→C [l]	0	0	0	0
B→C [l]	0	0	0	0
CPU Time [s]	**23,836**	**18,491**	**61,142**	**57,962**

Table 2-6. Tests with different milk quotas. Profits are indicated in boldface.

Milk A	1,200,000	1,300,000
Trucks	77	84
Revenue [UM]	21,904	23,572
Costs [UM]	9,946	10,794
Profit [UM]	**13,348**	**12,778**
A [l]	1,235,361	1,397,757
B [l]	217,204	213,699
C [l]	325,642	166,571
A→B [l]	82,796	86,301
A→C [l]	0	0
B→C [l]	0	0
CPU Time [s]	24,537	35,267

The solutions in Table 2-6 indicate that the profit could decrease for increasing requirements of milk A because this necessarily incurs a higher cost of transportation and increases the number of required trucks.

2.8. Prize-collecting version

Some of the clusters in the real case were used to demonstrate that implementing a prize-collecting version of the problem, that is, replacing constraints (2.6) and (2.14) by (2.23) and (2.24), increases the profit. Table 2-7 displays how the optional collection, or the prize-collecting version, dominates the implementation in which all farms must be visited.

Table 2-7. Results for three clusters using the prize-collecting version

		Optional visit			Must visit all		
Net	*Q*, *P* [Ml]	Profit	Revenue	Cost	Profit	Revenue	Cost
Cluster 16	*Q* = [30;15;15]; *P* = [20;5;2]	**319**	625	306	**187**	662	475
	Q = [30;15;15]; *P* = [20;10;10]	**256**	561	305	**180**	650	470
	Q = [30;15;15]; *P* = [43;0;11]	**151**	702	551	**141**	705	564
Cluster 2	*Q* = [30;30;20;15]; *P* = [74;0;0]	**389**	1,115	726	**252**	1,212	960
	Q = [30;30;20;15]; *P* = [74;9;0]	**284**	1,208	924	**252**	1,212	960
	Q = [30;30;20;15]; *P* = [30;30;23]	**105**	875	770	**104**	876	772
Cluster 20	*Q* = [30;30;30]; *P* = [45;0;0]	**553**	868	315	**470**	894	424
	Q = [30;30;30]; *P* = [57;0;6]	**470**	894	424	**470**	894	424
	Q = [30;30;30]; *P* = [30;10;10]	**403**	718	315	**386**	807	421

The results of Table 2-7 indicate that a profit-increasing strategy for the firm would be not to collect the milk from unprofitable farms (in terms of amount and quality), even paying for it. Naturally, the firm could negotiate a lower price in those cases, as the farms keep the milk and can even resell it. A different and complementary strategy for the firm would be to encourage some low-quality-milk farms to enhance their production quality.

2.9. Blending and the use of trucks with compartments

Tests were performed on some of the case study's original clusters (clusters 4, 7, 10, 11, 16, 17, 23, and 24), as shown in Table 2-8, to demonstrate that blending is convenient even with compartmentalized trucks. Different settings are compared in these tests: single-compartment trucks using VRP (VRP), single-compartment trucks with blending (MB), and different compartment-sized multiple-compartment trucks without (TC) and with (MBTC) blending. Truck capacities are noted in the first column. It is considered that all compartments have the same size in each test, and two compartment capacities (CC) were attempted: 10,000 and 15,000 liters. The figures in bold font in the table indicate the best solutions found for the case.

An analysis of the results in Table 2-8 yields the following conclusions:

- **Blending versus compartments**: Generally, the solutions using blending and those using compartments do not dominate one another, although the compartment solutions in the instances tested lead to a higher profit in most cases; an exception is case 2, with cluster 24. The dominance of each of these solutions depends on the size of the compartments and the milk volume from each farm. However, while not illustrated in the table, the use of compartments generally requires larger numbers of trucks, and trucks with compartments are more expensive.

- **Blending versus not blending:** Solutions with blending dominate solutions without blending, whether with or without compartments. Note that the number of trucks with blending is less than the number of trucks without blending.

Table 2-8. Profits for different truck configurations

Truck Capacities [liters]	Solutions	Cluster 4	Cluster 7	Cluster 10	Cluster 11	Cluster 16	Cluster 17	Cluster 23	Cluster 24
30,000 30,000 30,000	VRP	307.56 (3)	-136.22 (3)	-	-95 (3)	-	581.10 (3)	**298.83 (2)**	162.6 (3)
	MB	336.74 (2)	-51.34 (2)	523.56 (3)	-5.83 (2)	206.67 (3)	581.10 (3)	**298.83 (2)**	255.94 (2)
	TC (CC = 10,000)	**365.89 (2)**	**101.22 (1)**	535.98 (3)	56.68 (2)	302.56 (2)	**665.54 (2)**	**298.83 (2)**	286.87 (2)
	TC (CC = 15,000)	360.27 (2)	-6.42 (2)	535.12 (3)	52.19 (2)	249.13 (2)	607.11 (3)	138.69 (3)	**291.05 (2)**
	MBTC (CC = 10,000)	**365.89 (2)**	**101.22 (1)**	**564.95 (2)**	**124.56 (1)**	**364.00 (2)**	**665.54 (2)**	**298.83 (2)**	286.87 (2)
	MBTC (CC = 15,000)	361.88 (2)	47.98 (1)	535.12 (3)	52.19 (2)	342.87 (2)	607.11 (3)	138.69 (3)	**291.05 (2)**
30,000 20,000 10,000	VRP	307.56 (3)	-136.22 (3)	-	-95 (3)	-	-	**159.22 (3)**	162.6 (3)
	MB	336.74 (2)	-51.34 (2)	523.56 (3)	-5.83 (2)	206.67 (3)	490.98 (3)	**159.22 (3)**	**255.94 (2)**
	TC (CC = 10,000)	**363.10 (2)**	**101.22 (1)**	-	56.68 (2)	-	-	**159.22 (3)**	171.6 (3)
	MBTC (CC = 10,000)	**363.10 (2)**	**101.22 (1)**	**528.32 (3)**	**124.56 (1)**	**276.12 (2)**	**494.96 (3)**	**159.22 (3)**	229.86 (2)
30,000 30,000	VRP	-	-	-	-	-	-	**298.83 (2)**	-
	MB	336.74 (2)	-56.99 (2)	371.65 (2)	-5.83 (2)	197.71 (2)	524.93 (2)	**298.83 (2)**	255.94 (2)
	TC (CC = 10,000)	**365.89 (2)**	**101.22 (1)**	-	56.68 (2)	-	-	**298.83 (2)**	286.87 (2)
	TC (CC = 15,000)	360.27 (2)	-6.42 (2)	-	52.19 (2)	-	-	-	**291.05 (2)**
	MBTC (CC = 10,000)	**365.89 (2)**	**101.22 (1)**	**564.95 (2)**	**124.56 (1)**	**364.00 (2)**	**543.67 (2)**	**298.83 (2)**	286.87 (2)
	MBTC (CC = 15,000)	361.88 (2)	47.98 (1)	511.14 (2)	52.19 (2)	342.87 (2)	-	-	**291.05 (2)**

- **Solution feasibility**: Some solutions using the VRP are infeasible, as the truck fleet is limited. The use of compartments improves the situation, but infeasible problems still exist. However, in all tested instances, blending makes solutions fully feasible. Conversely, note that if the size of the compartments is inadequate, the solution can again be infeasible even when blending is used. This is especially important when the farms and amounts of milk produced change over time.

- **Compartment size**: A smaller compartment size in the instances tested results in improved solutions in most clusters, except for cluster 24. Note that in a majority of the clusters analyzed, the farms produce between 1,000 and 3,000 liters; see also Figure 2-4.

- **General conclusion**: Blending improves solutions in all cases in terms of feasibility, profit, and efficiency in the use of trucks.

2.10. Conclusions

This study introduced the MB, motivated by the milk collection procedure currently used in several places. Small amounts of milk of different qualities are blended to reduce transportation costs. The procedure prescribes a set of farms to be visited by each vehicle, the route, and the blending pattern, and defines whether there is a need for blending at the plant to satisfy the quotas for each quality of milk. This model can be useful for other kinds of products, in which the mixing of different qualities of products in the same truck changes the truck's status.

The MIP model is solved using a branch-and-cut method, for which a model is proposed using both known cuts and a new cut. Polynomial separation algorithms are defined, and the procedure is tested on known test instances of up to 101 nodes. A three-stage heuristic procedure is then designed to solve the real case, involving 500 farms scattered over a large region. The region under study is partitioned in the first stage. A mathematical model is used in the second stage designed to set each partition's requirements and select the truck fleet allocated to each partition. The real case is solved in the third stage, obtaining favorable results in comparison with the current collection system. As the problem is new, there are no further efficient ad-hoc heuristic methods. This study's goal, in any case, was to design a heuristic that would provide favorable results within the time limits, given how frequently the problem needs to be solved, rather than aiming for faster times. Time is not critical, as routing does not change daily, but in a 4-core desktop computer, the solution takes 1.65 hours.

Different approaches are also compared as follows: single compartment trucks with and without blending, multiple-compartment trucks with and without blending, and optional and mandatory collection; several instances are solved for these comparisons. The results indicate that, in the instances that were solved, blending always dominates unblended milk collection. Additionally, in most cases, using multiple-compartments trucks is better than using single-compartment trucks with blending. However, if multiple-compartment trucks are available, blending in the compartments always dominates. Furthermore, blending enables feasibility in all cases, and selecting an incorrect compartment size can make the problem infeasible. Finally, using the actual single-compartment truck fleet and blending, significantly improves profit for the actual case, primarily due to the savings in transportation cost.

If the farms in the compartment case exceed the size of a compartment, neither the size of the farm subdivisions nor the truck compartments' size are optimized, although different compartment sizes may lead to different results. This is a compelling future extension.

Other many possible extensions of this work include the location of milk collection points to accumulate milk from small and distant farmers. Additionally, another extension involves the consideration of the random nature of each producer's daily milk availability. Other extensions of interest involve speeding up solutions by using heuristics that solve the whole problem at once by not requiring subroutines, and the analysis of different blending rules that are less conservative than the current rules used at the actual firm.

3. THE BLENDED MILK COLLECTION PROBLEM USING MILK COLLECTION POINTS

A novel problem for the collection of raw milk from a network of farms supplying a dairy is specified and solved. The proposed approach incorporates milk blending and the delivery of production to collection points by small, distant farms. The milk is collected by, and blended in, a homogeneous fleet of trucks and classified according to the lowest quality product included in the blend. Optimization criteria are used to determine where the collection points should be located and which producers are allocated for delivery to them, with all other production picked up directly at the farm. The approach is built around an integer programming model and two implementation strategies, one using a branch-and-cut algorithm for small instances and the other a heuristic procedure combining both exact and approximated methods to handle large instances within a practical time period. A real case study involving 500 farms and 112 possible collection points is solved and the results compared. The impact on the solutions of dividing the real instance into zones is also explored.

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3.1. Introduction

This chapter presents, models and solves a real-world raw milk collection problem facing a dairy products company in southern Chile. The company must collect the milk from a set of producer farms distributed across a wide geographical area and transport it to a dairy processing plant. All the milk produced must be collected given that the farms all belong to a cooperative.

Since different farms produce different qualities of milk, their collection by tanker trucks is normally carried out using either different vehicles or the same vehicles but with segregated compartments. In Paredes-Belmar et al. (2016), however, it is assumed the various qualities of milk can be mixed in one compartment of a single truck. Upon arriving at the plant, the mixed product is then classified according to the lowest quality milk included in it, thus reducing its commercial value and therefore revenue on the final products. On the other hand, combining different qualities in this way considerably lowers transport costs, thereby increasing profit.

The simplest method of collecting the milk is for the trucks to visit each producer directly at the farm (door-to-door collection). If the collection routes are long and have multiple stops, transport costs will tend to be high. Direct truck pickup may therefore be inefficient in cases where there are many small producers located far from the dairy processing plant, especially with segregated collection of different milk qualities. In such situations, the collection process would be facilitated by setting up collection points where the output of small producers located far from the plant could be stored in tanks (Anquez & Tiersonnier, 1962; Bylund, 2003). That way, the trucks could collect the total production from a group of such producers, whether of one or more milk qualities, in one single stop, thus reducing the time and cost of transport.

This alternative is addressed in the present chapter, which generalizes an earlier problem formulated and solved in Paredes-Belmar et al. (2016) by incorporating the task of specifying the location of milk collection points. A model is developed that determines the number and location of these storage centres where farms can deliver their production. The general aim is to cut the tanker trucks' route lengths, which will result in lower truck fleet operating costs to the benefit of the dairy company. Our formulation also includes an access cost reflecting the expense to the producer of delivering to a collection point, part of which is refunded by the dairy. This cost is considerably smaller than the transport cost of serving small, distant producers.

To the best of our knowledge, defining the milk collection problem to include the twin factors of mixed milk qualities and milk collection points, although clearly of significant utility to many agents in the dairy industry, has not previously been undertaken in the specialized literature. Our first contribution in this paper is therefore to introduce what we call the milk blending with collection points problem (MBCP) and design a mixed integer linear programming model for solving it. We use both new and known cuts with their respective separation algorithms and present a branch-and-cut algorithm to solve small instances of up to 40 nodes (farms or collection points). For each truck serving this network, the model determines the farms and collection points to be visited, the sequence of the visits and the amount of milk delivered to the points by small farms without truck visits.

Our second contribution is to develop an ad hoc three-stage procedure that solves the MBCP for large instances. This algorithm is applied to the real case of a Chilean dairy company collecting milk from 500 producers with a fleet of 80 trucks and 112 candidate collection points. The first of the three stages optimally solves a covering problem that allocates small producers to collection points; the second stage generates feasible routes using the ant colony metaheuristic; and finally, the third stage chooses the best routes from those generated by the metaheuristic for direct visits to farms and collection points.

The objective of the proposed formulation is to maximize profit from the milk collection operation on the basis of: (i) revenue obtained from the milk delivered to the plant, (ii) truck transport cost, (iii) collection point cost (i.e., operating plus pro-rated installation cost), and (iv) farms' collection point access cost. Since with a single milk quality the MBCP reduces to the Capacitated *m*-Ring-Star problem, which is known to be NP-hard (Baldacci, Dell'Amico, & González, 2007), the MBCP must be NP-hard as well.

Note also that although the milk must be collected every day, variations in daily production due to weather, cattle disease, etc. are relatively insignificant (Dayarian et al., 2015a). Collection routes do not therefore have to be rescheduled on a daily basis.

The remainder of this chapter is organized into four sections. Subsection 3.2 reviews the related literature, subsection 3.3 formally introduces our proposed problem and sets out its mathematical formulation, subsection 3.4 discusses various solution approaches to the problem, subsection 3.5 reports on a case study, and finally, subsection 3.5 presents our conclusions and some suggestions for future research.

3.2. Literature review

This survey of the literature is divided into two parts: other milk collection problems and problems having a similar structure to our proposed problem.

3.2.1. Milk collection problems

Sankaran & Ubgade (1994), Igbaria et al. (1996), Butler et al. (1997) , Basnet et al. (1999), Prasertsri & Kilmer (2004), Butler et al. (2005), Hoff & Løkketangen (2007), Claassen & Hendriks (2007), Dayarian et al. (2013), Dayarian et al. (2015a), Dayarian et al. (2015b) and Masson et al. (2015) report on real-world milk collection instances with a single quality of milk. Dooley et al. (2005), Caramia & Guerriero (2010), Lahrichi et al. (2015) and Sethanan & Pitakaso (2016) study cases with more than one milk quality. In none of these cases is mixing of different milk qualities in the same truck or truck compartment permitted.

Recently, Paredes-Belmar et al. (2016) presented a milk collection problem for a real instance in Chile with three milk qualities that the dairy allows to be blended in the same truck provided it has a positive impact on company profit. The article discusses the benefits of blending and reports increases in profit. The results are compared with the dairy’s current collection procedure and with segregated collection by quality (no blending), demonstrating major improvements. As was noted here in the introduction, the present study departs significantly from Paredes-Belmar et al. (2016) in that as well as considering the blending of milk qualities, it addresses the collection point number and location decision. The resulting formulation thus combines a routing problem with an allocation problem and a location problem, and in this sense both the model and the solution method for different-sized instances are quite novel.

Mumtaz, Jalil & Chatha (2014) present a routing and location problem for the collection of a single milk quality. Their model determines the dispatch points that mark the start and end of the routes travelled by the trucks to pick up milk exclusively at collection centres where it must be dropped off by the producers. The model then decides the collection centre sequence for each truck collecting milk from these centres. The milk is brought to the dispatch points from where it is brought to the dairy plant in larger capacity tankers. The authors develop a heuristic to solve the collection problem for small instances.

3.2.2. Problems with a similar structure

The milk collection problem with collection points is a particular case of the location-routing problem (LRP) with assignment decisions and multiple product types. A number of surveys of the LRP literature have appeared in recent years. Laporte & Rodríguez-Martin (2007) review problems of locating cycles in transport networks. Nagy & Salhi (2007) give an extensive overview of the LRP. Current & Schilling (1994) introduce the median tour problem (MTP) and the maximal covering tour problem (MCTP). Two other similar problems, the ring star problem (RSP) and the median cycle problem (MCP), are studied by Labbé et al. (2004) and Labbé et al. (2005). Moreno-Pérez et al. (2003), Renaud et al. (2004), Liefooghe, Jourdan, Basseur, Talbi & Burke (2008), Kedad-Sidhoum & Nguyen (2010) and Calvete, Galé & Iranzo (2013) propose exact and approximate methods for solving the MCP and the RSP.

Baldacci et al. (2007) develop an extension of the RSP known as the Capacitated *m*-Ring-Star problem (C*m*RSP). They design *m* routes for visiting a set of customers who supply/demand a single product. Customers not located on the routes are assigned to customers who belong to some route. The maximum number of customers visited and assigned limits the maximum capacity of each route. The solution approach minimizes the cost of the *m* routes and the assignment costs of the customers not visited. Other authors such as Hoshino & de Souza (2009), Naji-Azimi, Salari & Toth (2010), Baldacci & Dell’Amico (2010), Berinsky & Zabala (2011) and Naji-Azimi, Salari & Toth (2012) suggest different solution methods for the C*m*RSP, some of them approximate and others exact.

A problem with a structure similar to MCBP for transporting a single product type is the vehicle routing-allocation problem (VRAP), introduced by Beasley & Nascimento (1996). It consists in identifying routes for a number of vehicles that must service a subset of customers. Those not visited can either be assigned to a customer who is, or simply excluded but with a penalty. The solution minimizes the routing costs for the customers visited and the assignment costs and penalties for those not on the routes. Vogt et al. (2007) study the single vehicle routing-allocation problem (SVRAP), a particular case of the VRAP, using a tabu search to solve large instances. Ghoniem et al. (2013) present a distribution problem for a food bank based on the VRAP. Vehicles transport the food to a set of distribution points that the patrons of the service must travel to. The model minimizes the vehicle transport and patrons' travel cost. Extensions of the VRAP are applied to school bus routing, where the problem is to assign students to stops and design the stop sequence of each bus so as to minimize such factors as the number of bus routes, total route length, load variation, and total walking distance to the nearest stop over all students (Bowerman et al., 1995; Riera-Ledesma & Salazar-González, 2012; P Schittekat et al., 2006; Patrick Schittekat et al., 2013).

3.3. The blended milk collection problem with collection points

The milk blending with collection points problem (MBCP) as developed here accommodates any number of qualities of raw milk, although in the examples we use three qualities, classified as A, B and C in decreasing order of quality. The product of any mix of different qualities is discrete. Thus, if quality A is mixed with quality C, the result is classified as quality C regardless of the two quality' respective proportions in the blend. This classification rule is set by the dairy and reflects the requirements of the processing plant's final products. Each farm produces a single milk quality and is paid accordingly. Thus, the unit cost of each quality is constant and does not influence the modelling. The tanker trucks are homogeneous and do not have segregated compartments.

As noted in the introduction, the milk producing area in our problem has a network of collection points with storage tanks to improve the efficiency of collection. Producer farms in remote locations often deliver their production to these points and charge the haulage to the dairy. This expense, which we call access cost, is included in the modelling. As might be expected, this delivery system is used if and when it is cheaper than a direct visit to the producer by a tanker truck; if this is not the case, a truck picks up the milk directly at the farm. Since the trucks do not have compartments, all the milk collected is necessarily mixed in the trucks, never at the collection points.

A quantitative example illustrating the potential advantage of the proposed approach compared to others, existing models is shown in Figure 3-1. The network in the example has 8 nodes representing six farms numbered 1 to 6; the dairy, numbered 0; and a collection point denoted CP. Indicated above or below each farm node is the amount (in litres) and quality of milk produced. The links connecting up the nodes are labelled according to the cost of travelling them in monetary units. In the case of links (CP,5) and (CP,6) between the collection point and nodes 5 and 6, two cost figures are shown: 50 for the truck cost and 10 for the producer access cost. The unit revenue by milk quality is 1.0, 0.7 and 0.3 units for qualities A, B and C, respectively. There are three trucks, each with a load capacity of 520 litres. As many collection problems, this problem is solved over an auxiliary, fully connected network, whose arcs have the length of the shortest routes between every pair of nodes.

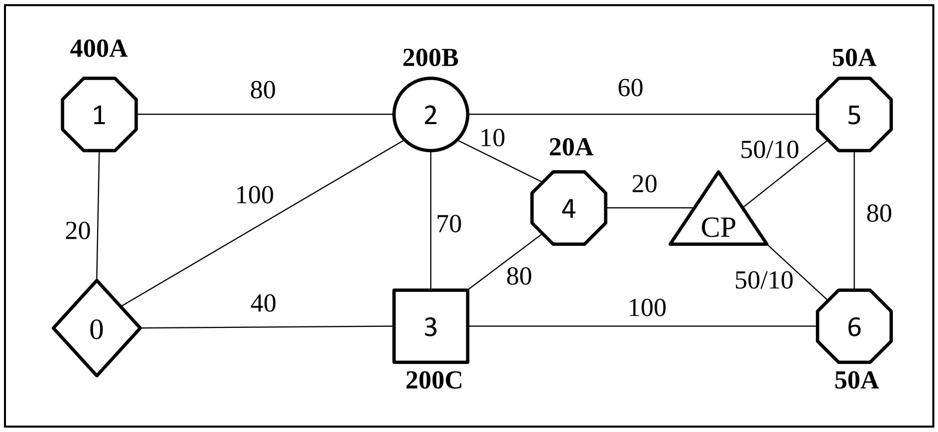


Figure 3-1. Numerical example to compare the solutions of the CVRP, MB, C*m*RSP and MBCP. An octagonal shape denotes milk A, a circle milk B, and a square, milk C.

If neither blending nor collection points are permitted, separate trucks for each milk quality must be used. The resulting situation is similar to the capacitated vehicle routing problem (CVRP) (Irnich et al. 2014). There are three truck routes: 0-1-2-4-2-5-6-3-0 (quality A, collection sequence: 1-4-5-6), 0-2-0 (quality B) and 0-3-0 (quality C). Total truck transport costs in this case are **680 units** while revenues are **720 units**, thus producing a profit of **40 units**. If blending is permitted but not collection points, we have the simple milk blending problem (MB). Three trucks are needed for this case and the routes are 0-1-0 (quality A), 0-2-4-2-5-6-3-0 (quality B, collection sequence 2-4-5-6) and 0-3-0 (quality C). One of the trucks collects and mixes 200 litres of quality A milk and 120 litres of quality B milk, which according to the blending rule counts as 320 litres of quality B at the plant. Total truck costs are **480 units**, revenues are **684 units** and profit is **164 units.**

If collection points are permitted but not blending, the problem is similar to the C*m*RSP (Baldacci et al. 2007). The three truck routes are in this case are 0-1-2-4-CP-4-2-1-0 (quality A, collection sequence 1-2-CP), 0-2-0 (quality B) and 0-3-0 (quality C). Total truck transport costs for this situation are **560 units** while revenues are **720 units**, for a profit of **160 units**. Finally, under our MBCP approach with both blending and collection points, the 3 routes are 0-1-0 (quality A), 0-2-4-CP-4-2-0 (quality B, collection sequence 2-4-CP) and 0-3-0 (quality C). This means transport costs of **400 units** and revenues of **684 units**. Thus, the profit is **284 units**, which is 120 units higher than with blending alone, 124 higher than C*m*RSP and 204 higher than CVRP.

As we noted earlier, our approach is a generalization of other problems in the literature. If, for example, there is little advantage in milk blending, the MBCP solution will be the same as that of the C*m*RSP. And if under MBCP the access and collection point costs are high, the solution will reduce to the MB approach, that is, milk blending but not collection points. Finally, if neither collection points nor blending is advantageous, the solution of MBCP will be the same as that generated by the CVRP.

The proposed problem thus attempts to find the collection point sequence for each truck setting out from and returning to the dairy plant in such a way as to maximize profit on the milk collection while satisfying all of the conditions indicated above.

3.4. Mixed integer programming model

We now formulate our proposed mixed integer linear programming model for solving the MBCP. Let N be the set of all nodes in the milk collection network excluding the dairy plant, and let $N_0 = N \cup \{0\}$, where 0 is the plant node, be the set of all nodes including the plant. Also, let $G(N_0, A)$ be a complete graph of the network where A is the set of links defined by $A = \{(i, j) \in N_0 \times N_0 \mid i \neq j\}$. The set of milk qualities is T and according to the classification rule stated earlier. The set of producer farm nodes are denoted $N^t : t \in T$. The milk quality as delivered to the dairy is $D^t = \{r \in T \mid \text{a blend of } r \text{ and } t \text{ gives } r\text{. Includes } r = t\}$. The candidate collection points belong to set *CP*. Thus, $N = \left(\bigcup_{t \in T} N^t \right) \cup CP$. Also, *K* is the set of available trucks; *IT* is the set of ordered pairs (*i*,*t*) that associates node $i \in N$ with milk quality $t \in T$; $CN \subseteq N$ is the set of farms that are candidates for delivering their milk to collection points instead of truck visits for direct pickup, either because of their low production volume or their closeness to at least one candidate collection point; $CN^t \subseteq CN$ is the set of farms in *CN* that produce quality *t* milk; *AP* is the set of links that connect collection points *CP* and the candidate farms *CN* such that $AP \subseteq A$; q_i^t $(i,t) \in IT$ is the quantity of quality *t* milk produced by farm $i \in N \setminus CP$; *Q* is the load capacity of each truck; c_{ij} is the truck transport cost of travelling link $(i, j) \in A$; α^t is the unit revenue for quality *t* milk; β_{ij} is the access cost to farm $i \in CN$ for delivering milk to collection point located at node $j \in CP$ such that $(i, j) \in AP$; and C_i is the collection point cost of node $i \in CP$.

Decision variables

$$x_{ij}^k = \begin{cases} 1 & \text{If truck } k \text{ travels directly from node } i \text{ to node } j : (i, j) \in A \\ 0 & \text{otherwise} \end{cases}$$

$$y_i^{kt} = \begin{cases} 1 & \text{If truck } k \text{ loads milk quality } t \text{ from node } i : (i, t) \in IT \\ 0 & \text{otherwise} \end{cases}$$

$$g_{ij}^{kt} = \begin{cases} 1 & \text{If the farm } i \text{ delivers milk } t \text{ to point } j : (i, j) \in AP, \text{visited by truck } k \\ 0 & \text{otherwise} \end{cases}$$

$$z^{kt} = \begin{cases} 1 & \text{If truck } k \text{ delivers milk } t \text{ to the plant} \\ 0 & \text{otherwise} \end{cases}$$

w^{kt} = Volume of milk t that truck k delivers to the plant

The problem itself can now be specified, beginning with the objective function (3.1):

$$Max \sum_{t \in T} \sum_{k \in K} \alpha^t w^{kt} - \sum_{i,j \in N_0} \sum_{k \in K} c_{ij} x_{ij}^k - \sum_{i \in CN} \sum_{k \in K} \sum_{t \in T} \sum_{j \in CP} \beta_{ij} g_{ij}^{kt} - \sum_{i \in CP} \sum_{t \in T} \sum_{k \in K} C_i y_i^{kt} \quad (3.1)$$

The objective (3.1) maximizes profit earned on milk collection, deducting from revenue the transport, access and collection point costs.

The constraints are as follows:

$$\sum_{t \in T} \sum_{i \notin \{CP\}} q_i^t y_i^{kt} + \sum_{t \in T} \sum_{i \in CN} \sum_{j \in CP} q_i^t g_{ij}^{kt} \le Q \qquad \forall k \in K \qquad (3.2)$$

$$\sum_{k \in K} y_i^{kt} = 1 \qquad \forall i \in N \setminus \{CN, CP\},\ t \in T : (i,t) \in IT \qquad (3.3)$$

$$\sum_{i \in N_0} x_{ij}^k = y_j^{kt} \qquad \forall k \in K,\ j \in N \setminus \{CN, CP\},\ t \in T : (j,t) \in IT \qquad (3.4)$$

$$\sum_{j \neq 0} x_{0j}^k \le 1 \qquad \forall k \in K \qquad (3.5)$$

$$\sum_{i \in N_0} x_{ij}^k = \sum_{h \in N_0} x_{jh}^k \qquad \forall k \in K,\ j \in N_0 \qquad (3.6)$$

$$\sum_{i \in S, j \in S} x_{ij}^k \le |S| - 1 \qquad \forall S \subseteq N, k \in K \qquad (3.7)$$

The first six constraints, (3.2) - (3.7), are those typically found in some form in traditional vehicle routing problems. Constraint (3.2) sets the load capacity of the individual tanker trucks for milk collected either at the farm gate or at collection points. Constraints (3.3) impose a single visit to each node as long as it is neither a collection point nor a candidate farm for allocation to a collection point. Constraints (3.4) require that if a truck *k* visiting a farm node *j* that produces quality *t* has travelled there from any node *i*, the truck collects all of the quality *t* milk produced at node *j*. Constraints (3.5) ensure that for every truck there must be only one route starting from the origin node. Constraints (3.6) are the flow conservation constraints for each farm and truck. Constraints (3.7) eliminate any subtours that may appear in a given solution.

$$z^{kt} \leq 1 - y_i^{kr} \qquad \forall k \in K, i \in N, t, \; r \in T, r \in D^t \setminus t \tag{3.8}$$

$$\sum_{t \in T} z^{kt} \leq 1 \qquad \forall k \in K \tag{3.9}$$

$$w^{kt} \leq z^{kt} Q \qquad \forall k \in K, t \in T \tag{3.10}$$

$$w^{kt} \leq \sum_{\substack{r:t \in D^r \\ h \in N^r \setminus CP}} q_h^r y_h^{kr} + \sum_{\substack{r:t \in D^r \\ h \in N^r \setminus CP}} \sum_{j \in CP} q_h^r g_{hj}^{kr} \qquad \forall k \in K, t \in T \tag{3.11}$$

$$\sum_{k \in K} \sum_{t \in T} w^{kt} = \sum_{(i,t) \in IT} q_i^t \tag{3.12}$$

The next five constraints, (3.8) - (3.12), establish the rules related to milk blending. Constraint (3.8) ensures that every truckload of milk delivered to the dairy is counted as having a quality no higher than the lowest quality collected by the truck, reflecting the milk quality classification rule. Constraint (3.9), also reflecting this rule, imposes that each truck can deliver only one quality of milk to the plant. Constraints (3.10) impose that truck *k* delivers quality *t* milk to the dairy whenever a decision to collect that quality of milk is made. Constraints (3.11) counts the amount of milk collected from farms and collection points and delivered at the plant by each truck. Constraint (3.12) guarantees that the available trucks collect all of the milk produced by the farms.

$$\sum_{k\in K} y_i^{kt} \le 1 \qquad \forall i \in CP, t \in T : (i,t) \in IT \qquad (3.13)$$

$$\sum_{k\in K} y_i^{kt} + \sum_{k\in K}\sum_{j\in CP} g_{ij}^{kt} = 1 \qquad \forall i \in CN, t \in T : (i,t) \in IT \qquad (3.14)$$

$$g_{ij}^{kt} \le \sum_{h\in N_0} x_{hj}^{k} \qquad \forall k \in K, i \in CN, j \in CP, t \in T, (i,j) \in AP \qquad (3.15)$$

$$\sum_{j\in CP} g_{ij}^{kt} \le 1 \qquad \forall k \in K, i \in CN, t \in T, (i,j) \in AP \qquad (3.16)$$

$$y_i^{kt}, z^{kt}, g_{ij}^{kt} \in \{0,1\} \qquad \forall k \in K, i, j \in N, t \in T, (i,j) \in AP \qquad (3.17)$$

$$x_{ij}^{k} \in \{0,1\} \qquad \forall k \in K, i, j \in N_0, (i,j) \in A \qquad (3.18)$$

$$w^{kt} \ge 0 \qquad \forall k \in K; t \in T \qquad (3.19)$$

Constraint (3.13) is explained below. Constraints (3.14) stipulate that each farm can be either allocated to a collection point or directly visited by a truck. Constraints (3.15) ensure that a truck visits each collection point. Constraints (3.16) impose that a candidate farm for assignment to a collection point can be allocated to only one such point. Finally, constraints (3.17), (3.18) and (3.19) define the nature of the variables.

Note here that every candidate collection point is expanded to three nodes, each one representing a storage tank for a quality of milk and all three having the same geographical coordinates. Constraint set (3.13) imposes that each of the three can be visited only by one truck.

The number of variables and constraints in the model depends on the number of nodes and trucks in the network. A small instance with 20 farms of which 10 are candidates for producer delivery to collection points ($|CN| = 10$), 6 collection points to be located by the model (i.e., 2 collection points with 3 tanks each, as just explained), 3 qualities of milk and 3 trucks would have 2,571 variables and 720 constraints.

3.5. Solution approaches

Two solution approaches are proposed. The first one is based on a branch-and-cut algorithm using new as well as known cuts to solve small instances; the second one consists of a procedure that combines exact routines and heuristics to solve large cases.

3.5.1. Branch-and-cut approach for small instances

For this approach we employ the model presented in subsection 3.1. Given that the number of individual constraints in (3.7) grows exponentially with the size of the instance, the model cannot be used directly with this constraint set. We therefore relax (3.7) as well as the integrality constraints (3.17)-(3.18). Once the solution is found, a support graph $G_s^k\left(N^k, A_s^k\right)$ is constructed for each truck k that includes every link whose associated variables in the solution are different from zero, that is, $x_{ij}^k > 0$ and $g_{ij}^{kt} > 0$. Ad-hoc separation algorithms are used to detect violated cuts in the support graphs.

We propose the following cuts in the branch-and-cut algorithm:

Proposition 1

The cut

$$x_{ij}^k \leq \sum_{h \neq i} x_{jh}^k \qquad \forall (i, j) \in A : i, j \neq \{0\}, k \in K \qquad (3.20)$$

is valid for the MBCP. **Proof:** Dror et al. (1994). □

In the support graph, the separation algorithm searches for each link (i,j) related to the variables $x_{ij}^{k} > 0$ such that $i, j \neq 0$. For each link it checks for violation of cut (3.20). The order of the algorithm is $O(|K||N|^3)$.

Proposition 2

The cut

$$\sum_{i \in S} x_{ij}^{k} \leq \sum_{\substack{h \in S^c \\ m \in S : h \neq j}} x_{hm}^{k} \qquad \forall S \subseteq N, j \in S^c, j \neq \{0\}, k \in K \tag{3.21}$$

is a valid cut for the MBCP.

Proof: For a set of nodes $S \subseteq N$, if a truck k uses a link (i,j) where $i \in S, j \in S^c \subseteq N \setminus S$, there must exist a link (h,m) where $m \in S$. Otherwise, there would be no connectivity in the route of k. Note that $h \neq j$; if this were not the case, there would be a subtour including j and the nodes in S. □

For each link (i,j) in every support graph, the algorithm finds the set of nodes S that are connected to j. After a node is inserted in S, the algorithm checks for a cut violation (3.21) and adds a cut if necessary. The order of the algorithm is $O(|K||N|^4)$.

Proposition 3

The following inequality:

$$\sum_{i \in S^c, j \in S} x_{ij}^{k} \geq y_{h}^{kt} \qquad \forall S \subseteq N, h \in S, k \in K, t \in T : (h,t) \in IT \tag{3.22}$$

is a valid cut for the MBCP. **Proof:** Toth & Vigo (2001)□

The algorithm constructs a set $\bar{S}$, starting with node 0. All of the nodes connected to $\bar{S}$ through a link in the support graph are added one by one until no more connected nodes are found. A set S in $\bar{S}^c$ is then constructed using the same procedure, starting with any node $l \notin \bar{S}^c$. The order of the algorithm is $O(|K||N|^2)$.

Proposition 4

The following inequality:

$$\sum_{i \in S} g_{ji}^{kt} \leq \sum_{\substack{h \in N \setminus S \\ l \in S : h \neq j}} x_{hl}^{k} \qquad \forall S \subseteq CP,\ j \in S^c \subseteq CN,\ j \neq \{0\}, k \in K, t \in T \qquad (3.23)$$

is a valid cut for the MBCP.

Proof: If a link (j,i) is used by a producer to deliver milk from farm j to collection point i, that is, $g_{ji}^{kt} = 1$ where node $i \in S$, then i must be on the route of truck k. Since set S does not include node 0, there must be a link (h,l) that enters S, that is, $x_{hl}^{k} = 1$. This link cannot be (j,i) given that it may not be feasible. □

We use the same separation algorithm as in Proposition 3.

3.5.2. Approach for large instances

An MBCP with more than 50 nodes is difficult to solve to optimality due to its size and complexity. In such cases, therefore, we propose a heuristic procedure (HP) that produces good quality solutions. This approach in fact combines heuristics with mathematical programming models, consisting of a meta-heuristic stage followed by two stages of exact optimization. A flowchart of the three HP stages is shown in Figure 3-2.

The procedure begins with the collection points location-allocation stage, which locates the collection points and identifies the farms allocated to them using an ad hoc covering model. The farms so assigned are eliminated from the instance and their milk aggregated at their corresponding collection points. These points thus become the equivalent of farm nodes in the simple MB problem (which has no collection points), but with three milk qualities and a "production" equal to the sum of the milk produced by the farm nodes allocated to them. The resulting set of nodes is the union of the set of farms and the located collection points. Then, in the feasible route generation stage of the procedure, an ant colony metaheuristic identifies feasible routes. Finally, in the route selection stage, the best routes are chosen for the definitive solution.

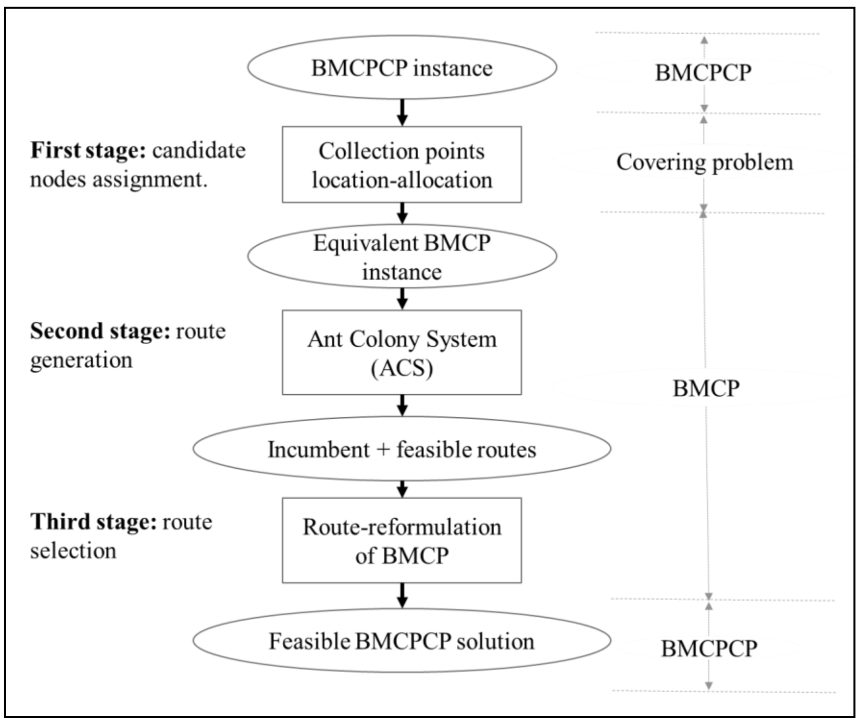


Figure 3-2. Flowchart of the proposed heuristic procedure (HP).

3.5.2.1. Collection points location-allocation stage

The inclusion of the candidate collection points increments the size of the problem. We therefore choose a limited set of points using the solution of the following covering problem:

$$L_j = \begin{cases} 1 & \text{if collection point } j \in CP \text{ is located} \\ 0 & \text{otherwise} \end{cases}$$

$$A_{ij} = \begin{cases} 1 & \text{if node } i \in CN \text{ is allocated to } j \in CP : j \in N_i \\ 0 & \text{otherwise} \end{cases}$$

E: Maximum production that can be allocated to a collection point.

$$N_i = \left\{ j \in CP : d_{ij} \leq B \right\}, i \in CN, if \sum_{(i,t) \in IT} q_i^t \leq E$$

B: Maximum allocation distance.

The problem is formulated as follows:

$$Min \sum_{j \in CP} C_j L_j + \sum_{\substack{i \in CN: \\ |N_i| > 0}} \sum_{j \in N_i} \beta_{ij} A_{ij} \tag{3.24}$$

$$\sum_{j \in N_i} A_{ij} = 1 \qquad \forall i \in CN \tag{3.25}$$

$$A_{ij} \leq L_j \qquad \forall i \in N_i, j \in CP \tag{3.26}$$

$$L_j \in \{0,1\} \qquad \forall j \in CP \tag{3.27}$$

$$A_{ij} \in \{0,1\} \qquad \forall i \in N_i, j \in CP \tag{3.28}$$

The objective function (3.24) minimizes the sum of collection point costs and access costs. Constraints (3.25) ensure that every candidate node is allocated to a single collection point. Constraints (3.26) ensure the candidate node allocations are made only to points the procedure has already located. Lastly, (3.27) and (3.28) define the domains of the decision variables.

3.5.2.2. Feasible route generation stage: The Ant Colony System algorithm

This stage of the HP generates a set of reasonable routes *R*'. The method employed is ant colony optimization (ACO), a "meta-heuristic algorithm that mimics the communication between ants through pheromones that allow them to find better ways from their nest to the food sources" (Dorigo & Stützle, 2004). An ACO algorithm known as the ant colony system (ACS) has been used for discrete optimization with considerable success in routing applications given the similarity between them and the problem "solved" by ants searching for and carrying food (Bell & McMullen, 2004; Bullnheimer et al., 1999; Donati, Montemanni, Casagrande, Rizzoli, & Gambardella, 2008; Mazzeo & Loiseau, 2004; Montemanni et al., 2005; Yu et al., 2009). Applied to a routing problem, each trip by an ant corresponds to a route of a vehicle. The heuristic is used to generate feasible routes.

The method begins with a representation of the problem by a graph. The first step is to update the level of pheromone laid down by the ants, emulating its evaporation. The ants travel on the graph, generating one or more solutions. Optionally, these solutions can be improved using a local search procedure. The pheromones are then deposited on the links in the improved route, thus increasing the probability of obtaining good quality solutions. This cycle is iterated until a stopping criterion is satisfied. The pseudocode for the procedure is set out in Figure 3-3.

```
Initialize incumbent, ants and pheromone
While (Stopping criteria not met)
{
    Evaporate pheromone
    Build Solutions
    Calculate objective function value
    Improve solutions
    Store routes generated
    Deposit pheromone
    Update stopping status
}
Return Incumbent
```

Figure 3-3. Pseudocode of the proposed procedure

Each link begins with a quantity of pheromones *IniPhero*, which in our algorithm is a parameter. A lower bound (LB) for the objective function value (profit) is set at the level at which one truck serves each node. The main computation cycle to generate the solutions and update the pheromone level then begins. Every time a new feasible route is found, it is properly stored. The result of this heuristic is a set of *R*' routes. Each step of the heuristic is detailed in the Appendix.

3.5.2.3. Route selection stage based on the reformulation of MB

Let there by F identical trucks. Then MB can be reformulated as a route selection problem for a set of feasible routes. The notation for the necessary parameters and variables is as follows:

R: set of feasible routes for MB

S_i : set of routes that include $i \in N$ as a stop

u_r : net benefit of selecting route $r \in R$.

Decision variables:

$$\lambda_r = \begin{cases} 1 & \text{route } r \in R \text{ is selected in the solution of MB} \\ 0 & \text{otherwise} \end{cases}$$

MB can now be formulated as follows:

$$Min \sum_{r \in R} u_r \lambda_r \qquad (3.29)$$

$$\sum_{r \in S_i} \lambda_r = 1 \qquad \forall i \in N \qquad (3.30)$$

$$\sum_{r \in R} \lambda_r \leq F \qquad (3.31)$$

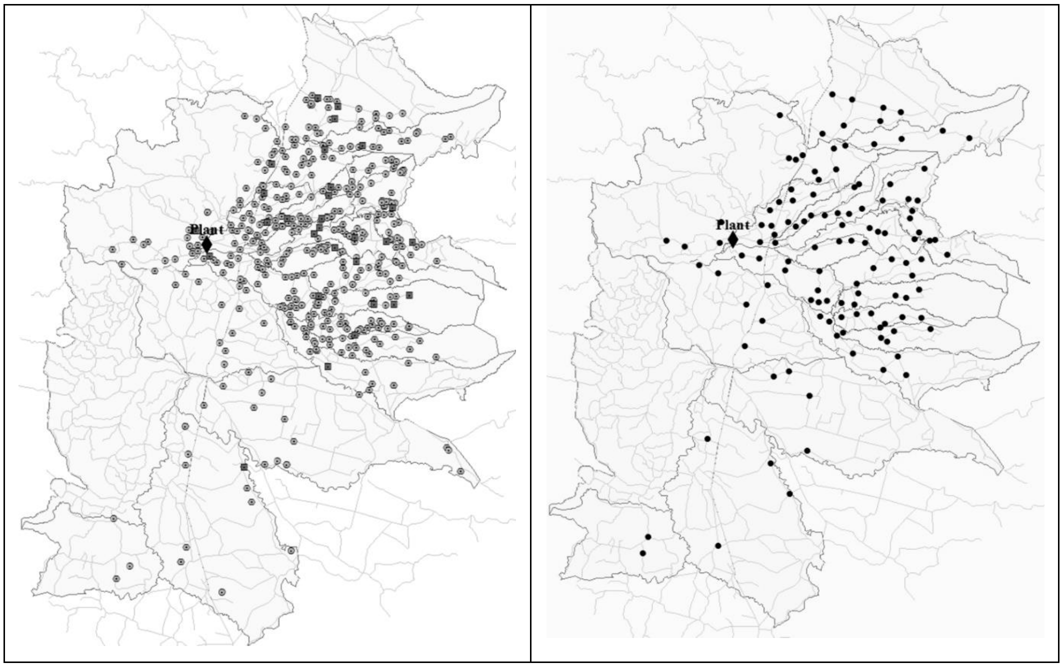
$$\lambda_r \in \{0,1\} \qquad \forall r \in R \qquad (3.32)$$

This model generates a large number of variables λ. Given that in the preceding stage we can construct a subset $R' \subset R$ of feasible routes, we solve a “truncated” version of (3.29)–(3.32) for R' to obtain the lower bound of the optimal value.

We then execute the route selection stage to obtain a feasible solution for the MB equivalent problem.

3.6. Case study

In our real case study, a dairy collects milk from 500 producer farms spread out over an area of 9,600 km^2. Of these farms, 313 produce quality A milk, 159 produce quality B and 28 quality C. Total production ranges anywhere from 60 and 25,000 litres per day, the average being 3,556 litres. The output of more than half of the producers is less than 2,500 litres. The dairy has a homogeneous fleet of 80 tanker trucks, each with a load capacity of 30,000 litres. The network is symmetric in the sense that transport costs $c_{ij} = c_{ji}$. These costs are proportional to the real distance d_{ij} between each node pair $(i, j) \in A$, such that $c_{ij} = d_{ij}$. A total of 112 candidate sites is added to locate the collection points. Site selection is based on the density of the geographical distribution of the farms, their distance from highways, and the location of highway intersections. A map of the area showing the distribution of the farms is shown in Figure 3-4a) and the same map displaying the network of candidate collection points is given in Figure 3-4b). On both maps the dairy plant is marked with a diamond.



a) b)

Figure 3-4**.** Distribution of farms and collection point network.

To estimate the access cost for producer deliveries to collection points we used the ACOTRAM software, provided by the Spanish government's road transport authority (Dirección General de Transporte Terrestre de España, 2014). The software estimated that the transport cost per kilometre for a direct farm visits by a tanker truck is 5 times the cost to a producer of making collection point deliveries using a typical farmer's pickup truck. In other words, the access cost for a farm $i \in CN$ delivering to a collection point $j \in CP$ is $a_{ij} = \left(\frac{1}{5} \right) c_{ij}$. This calculation is conservative given that in any given case draught animals, a tractor, etc. may actually make delivery from small farms. Note also that since collection point costs are small in relation to truck transport costs expressed in daily terms, we fixed the collection point costs at a baseline value of 1 unit.

To prevent spoilage, milk delivery to collection points must be carried out promptly after milking (Draaijer, 2002). With this in mind, the farms allocated to a given collection point are restricted to those situated within a radius of no more than 10 km. In addition, we assume that it is unlikely (and impractical) that a producer would be willing to deliver more than 2,000 litres per day to a collection point. Thus, for farms producing more than that amount, the only option is direct pickup by tanker truck.

Note that the problem would be similar if refrigerated centres were installed at the collection points. The associated costs would, however, be higher (due to the higher initial installation costs, although these would normally be pro-rated over various years) and storage capacity would be limited (Pérez, Maino, Agüero, & Pittet, 1994).

The results obtained in our case study are presented in the following two subsections for a small instance consisting of a subset of the 500 farms and a large instance that includes the complete set. The small instance is solved with the branch-and-cut algorithm, which we coded in AMPL version 20130109 and ran on a CPLEX 12.6.0 solver. As regards the large instance, models (3.24)-(3.28) and (3.29)-(3.32), that is, stages 1 and 3 of the heuristic procedure, were also executed using AMPL and CPLEX while stage 2, the ACS algorithm, was implemented in C++. The experimental parameters for the heuristic are detailed in the Appendix. All of the case study experiments were carried out on a PC with an Intel Core i7-2600 3.4 GHz processor and 16GB of RAM running the Ubuntu Server 12.04 LTS operating system.

3.6.1. Small instance (subset of farms)

The problem solved here is an instance consisting of a subset of 23 producer farms out of the 500 in the complete network. The idea is to contrast the efficiency of our MBCP model with the other solution approaches. We also compare the approximate solution of the HP, normally intended for large instances, to those of the other solutions, all of which are exact. The parameters for the instance are set out in Table 3-1. Node 0 is the dairy plant. Four trucks are available, each with a load capacity of 30,000 litres. The farms' milk production is 43,706 litres of quality A, 6,426 litres of quality B and 5,014 litres of quality C. Revenue for qualities A, B and C is 0.015, 0.0105 and 0.0045 monetary units, respectively. For each farm *i* and milk quality *t*, the table indicates the quantity *q* produced. *CN* = {99,162,258,107,133,140,403,367} is the set of candidate farms that can be allocated to a maximum of 5 collection points. They are indicated in bold quality on the graph of the physical node network in Figure 3-5.

Table 3-1. Farm parameters for small instance.

i	*t*	*q*	*i*	*t*	*q*
0	-	-	**258**	**A**	**1,407**
76	A	3,918	**99**	**B**	**87**
126	A	2,185	**107**	**B**	**205**
162	**A**	**1,972**	**133**	**B**	**324**
288	A	3,595	**140**	**B**	**362**
320	A	539	230	B	2,354
368	A	209	286	B	2,634
404	A	9,998	**403**	**B**	**460**
484	A	298	**367**	**C**	**791**
81	A	13,296	369	C	879
106	A	1,428	370	C	600
163	A	4,861	402	C	2,744

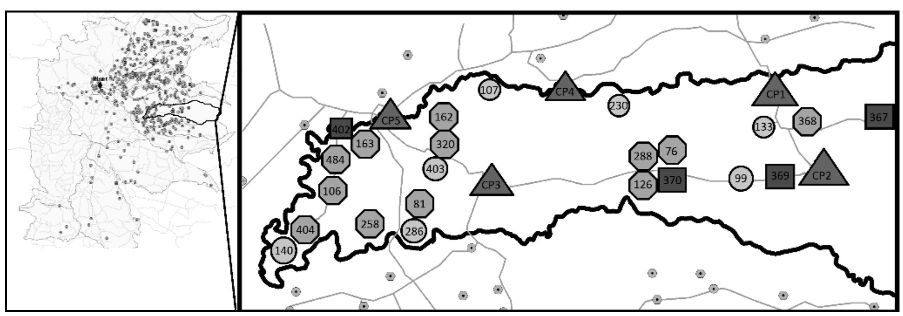


Figure 3-5. Farms and collection points for small instance.

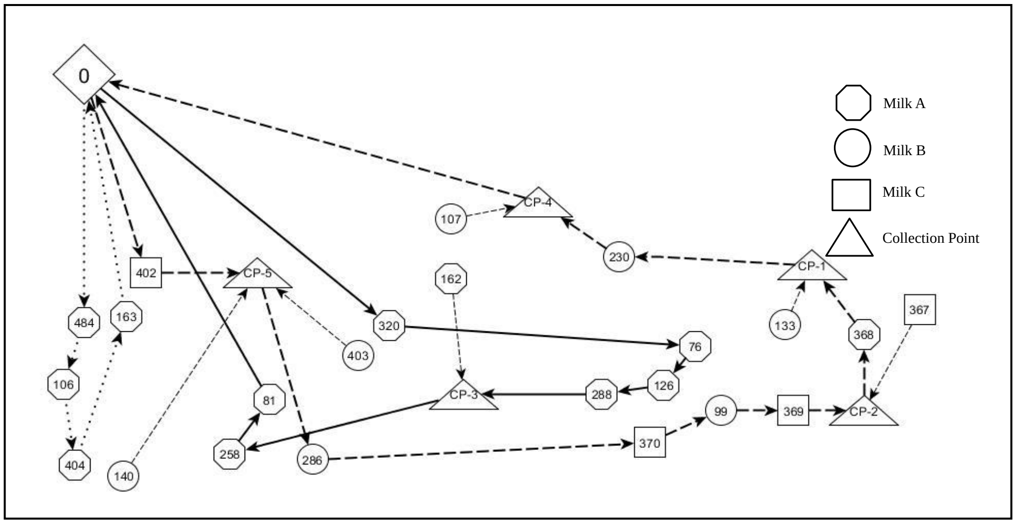


Figure 3-6. Truck routes for small instance.

When this instance is solved to optimality with the parameters just indicated, three truck routes are obtained as shown in Figure 3-6 by the solid, dashed and dotted lines, numbered 1, 2 and 3, respectively. Thus, three of the four available vehicles are required, the truck on routes 1 and 3 collecting milk of quality A and the truck on route 2 collecting quality C (blended with milk quality B).

The number of truck stops in this solution is 2 less than the stops required by the methods without collection points (CVRP and MB). Farm deliveries to these points by quality of milk are as follows: CP-1: 324 litres of quality B; CP-2: 791 litres of quality C; CP-3: 1,972 litres of quality A; CP-4: 205 litres of quality B; and CP-5: 822 litres of quality B. The total cost is 342 units and revenue is 705 units, generating a profit of **353** units.

The complete results for our model are displayed in Table 3-2 (column MBCP) where they are compared to those obtained for the same network with the other solution approaches mentioned earlier. With the exception of HP.MBCP, these solutions are optimal for their respective approaches and were obtained using the branch-and-cut algorithm. For CVRP, the results are for milk collection without blending or collection points, meaning different trucks for each milk quality. MB allows blending but not collection points while C*m*RSP represents the opposite (collection points but no blending). The HP.MBCP results are for the HP with both blending and collection points. All of the approaches use objective function (1).

Table 3-2. Results for small instance with different solution approaches.

	CVRP	**MB**	**C*m*RSP**	**HP.MBCP**	**MBCP**
No. of trucks used	4	3	4	3	3
Revenue	745	684	745	705	705
Transport cost	714	473	458	364	342
Access cost	-	-	8.5	12	5.4
Collection point cost	-	-	5	5	5
Profit	**32**	**211**	**274**	**324**	**353**
Milk A (Milk A)	43,706 (43,706)	41,525 (43,706)	43,706 (43,706)	43,497 (43,706)	43,497 (43,706)
Milk B (Milk B)	6,426 (6,426)	0 (6,426)	6,426 (6,426)	0 (6,426)	0 (6,426)
Milk C (Milk C)	5,014 (5,014)	13,621 (5,014)	5,014 (5,014)	11,649 (5,014)	11,649 (5,014)
Km per truck (mean)	179	158	115	121	114
% load (mean)	62	83	62	83	83
No. of stops	24	24	18	14	22
No. of coll. points used	-	-	3	5	5
Total milk, coll. Points	-	-	7,091	9,561	4,114
CPU time	50	60	1,500	3	4,737

As can be seen in the table, 4 trucks were required for the approaches that did not allow blending (CVRP and C*m*RSP). Profit under MBCP (353 units) is much higher than with CVRP (32 units). Blending and the consequent lowering of transport costs resulted in greater profit for both MBCP and MB, but the effect was much stronger for MBCP due to the additional cost reduction achieved by using collection points.

In Table 3-2, there are two sets of results for the different milk qualities. The lower figures in parentheses refer to the actual amounts produced and collected. The upper figures are the quantities delivered at the plant and are different wherever blending is allowed due to the rule on quality classification of blended milk. In the case of CVRP, the absence of blending means no downgrading of the classification and therefore higher revenues, but as noted above, transport costs are also high. With MB, blending results in a reduction of quality A milk at the plant, the difference (2,181 litres) being mixed with 6,246 litres of quality B and 5,014 litre of quality C, but transport costs are lower. Under C*m*RSP, revenue is the same as with CVRP but the use of collection points reduces the cost of transport.

Quality A milk delivered at the plant under our model is 43,497 litres, or 1,972 litres more than MB. All of this excess is from farm 162 (see Table 3-1), given that with the ability to use collection points, this producer's output can be picked up from CP-3. The 11,649 litres of milk classified quality C at the plant are the result of collecting and blending 6,426 litres of quality B, 5,014 litres of quality C and 209 litres of quality A. As these figures show, blending combined with collection points results in higher profit.

Although the use of collection points for C*m*RSP and MBCP is optional, Table 3-2 shows that while C*m*RSP uses only 3 points, MBCP uses 5, thus cutting mean truck distance travelled in kilometres from 179 to 114. Due primarily to blending, MB, MBCP and HP.MBCP achieve better average truck loads (83%) than CVRP and C*m*RSP (62%).

Finally, the solution time for MBCP is significantly longer than that required by CVRP. This is due to the former's greater complexity in terms of the numbers of variables and constraints. Also note that under HP.MBCP, profit was only 8.2% lower than MBCP's optimal value despite having a much shorter CPU time.

3.7. Complete instance

We now turn to the complete instance of our case study, with all 500 producer farms and 112 candidate collection points. Since finding an optimal solution within a reasonable time would be very difficult, we attempt to solve it by using a zone division strategy. Under this approach, the farm network is divided into 25 small zones based on geographical features such as rivers, lakes and roads as natural divisions. Each zone contains 10 to 40 farms.

For comparison purposes the results obtained with this strategy are set out in Table 3-3 for three methods: the HP approach without zone divisions (HP.MBCP), the HP approach for each zone separately (Z.HP.MBCP) and the branch-and-cut algorithm for each zone separately solved to optimality (Z.MBCP).

Table 3-3. Results for complete instance with different solution approaches.

	HP.MBCP (Approximate, without zones)	**Z.MBCP** (Optimal, with zones)	**Z.HP.M BCP** (Approximate, with zones)
No. of trucks used	70	77	78
No. of farms allocated	237	77	217
No. of points used	75	35	67
Revenues	24,312	23,501	23,201
Transport cost	8,021	8,998	9,296
Access cost	114	69	104
Collection point cost	103	50	95
Profit	**16,074**	**14,384**	**13,706**
CPU time (hh:mm:ss)	23:55:14	34:10:57	00:01:45

As these figures show, profit is highest for the HP solution without zone division. The approximate heuristic solution using zones (Z.HP.MBCP) averages a profit level 4.71% lower than the optimal solution using zones (Z.MBCP), which in turn averages 10.5% less than the heuristic solution without zones. The superior result for HP without zones may be attributed to the fact that zone borders do not limit its truck routing and collection point allocation solutions. Zoning, on the other hand, by definition excludes cases where the most efficient solution for a farm situated near a zone border might be a route or collection point in a neighbouring zone. Thus, without zoning the HP solution is likely to have shorter routes and fewer stops and will therefore make better use of the trucks, which translates into lower transport costs and more profit.

The solution time for HP with zones is far shorter than the times for the other two methods. Note that both HP approaches include execution times for the three stages of the heuristic procedure. With HP.MBCP, stage 1 takes less than one second, stage 2 (ACS) takes 238 seconds and stage 3 takes 85,875 seconds. Note, however, that HP without zones reaches an incumbent solution profit of 15,757 after only 9 minutes, with an upper bound of 16,266 and a gap of 3.1%. This is a good feasible solution that can be used where time is of the essence.

Not all of the 112 candidate collection points are used. The optimal MBCP solution only locates 35 while the HP with zones requires 67. The HP solution without zones uses 75 collection points, to which it allocates 237 farms. This means that 263 farms have direct truck visits and the trucks make 338 stops.

Some examples of the truck routes generated by HP are shown in Figure 3-7. The solid lines are the routes while the dashed lines represent farm allocations to collection points. The use of points is particularly noticeable where farms are densely concentrated. In such cases, the truck routes are shorter and have fewer stops. In Figure 3-7a) the truck collecting quality B milk makes 8 stops versus 12 for direct farm pickup only; in Figure 3-7b) the truck makes 6 stops collecting quality C; in Figure 3-7c) the truck makes 5 stops blending all three qualities; and finally, in Figure 3-7d) the truck makes 6 stops for quality B.

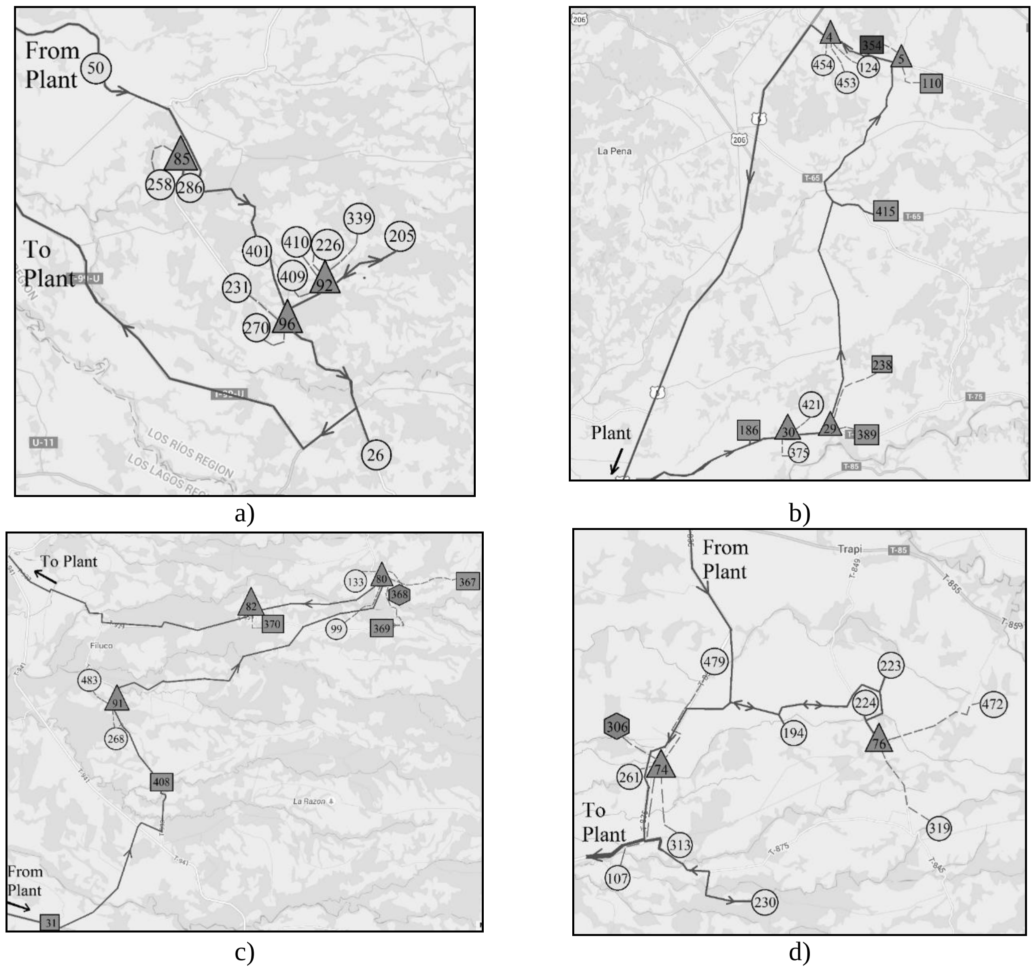


Figure 3-7. Some routes generated by the HP.

3.7.1. Sensitivity analysis

We conducted a sensitivity analysis to examine the behaviour of HP when certain of the parameters in the original case study are varied independently. The results of the analysis are summarized in Table 3-4 for seven different scenarios involving parameter variations as indicated in the top rows of the table. The base case results are those obtained with the case study parameters, repeated here for convenience: fleet size of 80 available trucks with a load capacity of 30,000 litres each, access cost $a_{ij} = \left(\frac{1}{5}\right)c_{ij}$, collection point covering within a maximum radius of 10 km, and a maximum production level of 2,000 litres for farms to be candidates for collection point allocation. As can be seen in the column headings, Scenario I increases access cost, Scenarios II and III vary the maximum collection point covering radius, Scenarios IV, V and VI increase the maximum production level for collection point allocation, and Scenario VII varies the truck fleet size and load capacity (the one farm with a production of more than 20,000 litres is served exclusively by two trucks).

Table 3-4. Sensitivity analysis of selected parameters.

		I	II	III	IV	V	VI	VII
	Base Case	**Access cost**	**Collection point covering radius**		**Maximum production for collection point allocation**			**Fleet size and capacity**
		$a_{ij} = \frac{1}{2} c_{ij}$	**5 *km***	**15 *km***	**3,000 *l***	**4,000 *l***	**5,000 *l***	**100 trucks – 20,000 *l***
No. of trucks used	70	70	71	72	69	68	68	93
No. of farms allocated	237	237	219	240	281	327	355	237
No. of points used	75	82	74	75	82	88	91	75
Revenues	24,312	24,332	24,237	24,364	24,372	24,361	24,167	24,354
Transportation Cost	8,012	8,182	8,258	8,070	8,110	8,102	8,031	10,097
Allocation Cost	114	245	80	123	137	154	173	114
Collection point cost	103	124	107	102	116	125	131	103
Profit	**16,074**	**15,781**	**15,792**	**16,069**	**16,009**	**15,980**	**15,832**	**9,880**
CPU time	23:55:14	23:54:15	23:52:58	23:56:12	23:56:53	01:53:03	02:03:51	23:54:09

When access cost is increased, profit declines relative to the base case and more use is made of collection points. When the collection point covering radius is reduced to 5 km, profit again declines, access cost decreases but transport cost rises. If, on the other hand, the covering radius is enlarged, the number of farms allocated to collection points increases and transport cost falls.

As for production levels, if farms producing up to 3000, 4,000 or 5,000 litres are allocable to collection points, more of them are assigned and the number of points located increases. This reduces transport cost and the number of farms with direct truck visits but access and collection point costs rise. In the 5,000 litre case, the number of farms allocated is 355, meaning only 145 are visited directly. The increase in farms allocated to collection points also reduces solution times given that more milk is delivered to each point so there are fewer direct truck visits to farms and ACS in turn does not have to generate as many routes.

Finally, if truck load capacity is lowered to 20,000 litres, profit declines due to the increase in transport cost since the individual vehicles cannot carry as much milk and more trucks will therefore be needed to collect total production in the network.

3.8. Conclusions and future work

A new approach was developed for solving a raw milk collection problem that arises in rural regions where there are many small producer farms scattered over a wide area and blending of different milk qualities is permitted. The proposed method reduces overall costs by locating collection points equipped with storage tanks where producers can deliver their milk for collection by tanker trucks if the cost of such producer deliveries is less than the cost of direct truck pickup at the farm. The problem, which we call the milk blending problem with collection points (MBCP), is solved using a mixed integer programming model.

The model was tested on a real-world case involving a large number of producers. Given the size of the instance and its consequent complexity, the farm network was divided into many small zones. Two solution approaches were applied: a branch-and-cut algorithm solving the problem to optimality and a three-stage heuristic procedure that included an ant colony system. For comparison purposes, the heuristic was also applied to the instance without using zone division.

The results showed that despite the division into zones, the algorithm was unable to solve the problem within a reasonable time period. Furthermore, the level of profit it generated on the milk collection operation was 10.5% less than the heuristic without zones, which also solved the instance significantly faster. When zones were used with the heuristic, it found a solution in mere seconds, although profit was 4.7% less than in the optimal algorithm solution. However, the heuristic without zone division also found a very satisfactory incumbent solution in terms of profit in a matter of minutes and would thus be a good feasible solution where time is a factor.

In addition, our case study findings further demonstrated that the use of collection points helps reduce transport costs and therefore boosts profit. If blending of different milk qualities is also allowed, the financial benefits are even greater.

This study could be extended in several directions. From a methodological standpoint, new exact methods are needed to solve the problem for medium and large instances. A hierarchical system could be incorporated with tanker trucks of different load capacities. Another interesting angle would be to consider refrigerated collection points where large quantities of milk, perhaps more than a single day's production, could be stored. A small fleet of trucks at each point would then collect milk from small farms, replacing producer deliveries. Finally, the proposed approach could be applied to other situations where products are mixed while in transit aboard a vehicle, such as the collection/distribution of dangerous materials or different classes of waste in a recycling system.

3.9. Appendix. Ant Colony System (ACS) Routines

The Ant Colony System routines are described below.

'Evaporate pheromone' Routine

Updates the pheromone levels on the graph edges in imitation of the way ants use pheromones in nature, reducing the levels by a proportion ρ where $0 < \rho < 1$, that is,

$$\tau_{ij} \leftarrow (1-\rho)\tau_{ij} \qquad \forall (i, j) \in A \tag{3.33}$$

'Build Solutions' routine

Constructs *m* solutions for MBCP. The solution procedure is as follows. First, the routine chooses truck *k* with a probability proportional to its spare load capacity to travel along a given edge. It then randomly chooses the node *j* the truck will travel to next with a probability given by

$$p_{ij} = \frac{\tau_{ij}^{\alpha}\eta_{ij}^{\beta}}{\sum_{v \in V_i} \tau_{iv}^{\alpha}\eta_{iv}^{\beta}} \qquad \forall i \in N, j \in V_i \tag{3.34}$$

where i is the current node, V_i is the set of nodes that can be feasibly reached from i, j is a generic node in V_i, τ_{ij} is the current amount of pheromone on edge (i, j), and η_{ij} is a heuristic information parameter that represents the maximum revenue obtainable by travelling edge (i, j), that is, by visiting node j travelling from node i using only the information from those two nodes. In the case of a node representing a collection point, it is assumed that the milk production delivered there is the average level observed in the instance and that the rate of profit is given by the quality of milk delivered. Finally, α and β are parameters used to calibrate the search.

Once the route of truck *k* is determined, the locations of all the trucks are updated. If a truck is fully loaded or cannot make any further visits, its route is "closed" and it returns to the plant, its probability of being chosen then reset to zero.

This procedure is repeated until either all of the nodes have been visited or there are no more trucks with spare capacity to visit them. Then all of the open routes (whose trucks have not returned to the plant) are closed and the value of the objective function is calculated.

'Calculate objective function value' routine

This routine simply calculates the objective function value given the truck routes. Profit is obtained by first determining the milk quality, which depends on the different nodes visited, and then multiplying the profit rate for that quality by the total production collected. Truck transport costs are obtained directly from the routes while the access and collection point costs are calculated directly from the instance. If the solution is infeasible, the objective function is penalized by a factor uD times either the production not collected or the excess over truck capacity.

'Improve solutions' routine

The solutions obtained by the previous routine can be improved by a further two-stage routine. The first stage is a local intra-route search. For each truck route the routine identifies the collection points visited and then calculates the advantage of eliminating candidate farm nodes from the route and allocating them to collection points. If a point has no candidate farm nodes allocated to it, it is eliminated from the route. This stage is repeated until no further improvement to the current solution is possible.

The second stage of the routine is a local inter-route search consisting of *nTries* random attempts at obtaining an improvement. Two routes r_1 and r_2 and respective non-endpoint nodes n_1 and n_2 on those routes are chosen at random from the current solution. Node n_1 is then relocated immediately after n_2. If this results in an improvement in the OF value, the change is adopted; if not, node n_1

is relocated on route r_2 and node n_2 is relocated on r_1. If this results in an improvement in the OF value, the change is adopted; if not, it is abandoned. If this search produces an improvement at least once, the order of the route is optimized again by solving the associated TSP.

'Store routes generated' routine

The heuristic creates a text file indicating the routes it has generated in a format readable by the next routine. The purpose is to specify the nodes on each feasible route in the order they are visited and the profit each route contributes to the global solution.

'Deposit pheromone' routine

Let S_k be the set of solutions obtained in iteration k, $v(S)$ the objective function value of solution s and BKS the incumbent solution, that is, the best one obtained so far. For each solution $s \in S_k$ in iteration k, the routine follows the standard method of depositing $\gamma(v(s)–LB)$ units of pheromone on the solution's edges to encourage exploration of new areas in the feasible space of the problem. Also, in each iteration $\varepsilon(v(BKS)–LB)$ units of pheromone are deposited on the edges of the incumbent solution. This increases the probability the ants will explore solutions close to the incumbent. The γ and ε parameters determine the relative importance of the two aforementioned amounts of pheromone. In formal terms,

$$\tau_{ij} \leftarrow \tau_{ij} + \gamma\left(v\left(s\right) - LB\right) \qquad \forall s \in S_k, \left(i, j\right) \in s \tag{3.35}$$

$$\tau_{ij} \leftarrow \tau_{ij} + \varepsilon\left(v\left(BKS\right) - LB\right) \qquad \forall \left(i, j\right) \in BKS \tag{3.36}$$

'Update stopping status' routine

The main computation cycle is interrupted once either max*Iter* consecutive iterations have been run or the best solution obtained has not changed after max*UnsucIter* consecutive iterations. This routine also updates the various counters needed to track these two conditions.

Computational experiments were conducted using a matheuristic approach. After a brief preliminary tuning, the parameter values for the second stage of the heuristic procedure (i.e., the ACS) were set as follows: $\alpha = 2$ (high weighting of pheromone information), $\beta = 1$ (medium weighting of heuristic information), $\rho = 0.2$ (20% of pheromone evaporates at every iteration), $\varepsilon = 0.05$ (weight factor controlling the amount of pheromone deposited at each iteration as determined by the incumbent solution), $\gamma = 0.01$ (weight factor for pheromone deposited by each ant), $m = 10$ (i.e., ten ants are used), $\max Iter = 1{,}000$, $\max UnsucIter = 1{,}000$ (at most 1,000 iterations are run), $iniPhero = 10$ (initial pheromone level), $nTries = 1{,}000$ (each time the Improve Solutions routine is called, its second stage is run 1,000 times).

4. HAZARDOUS MATERIALS COLLECTION WITH MULTIPLE-PRODUCT LOADING

In this chapter we present a new hazardous material (HAZMAT) collection problem in which various industrial HAZMAT are transported using a homogeneous capacitated truck fleet. Different materials have different risk levels in terms of the size of exposed population. A truck can simultaneously carry different materials. The size of the population exposed to a loaded truck increases if a higher risk material is added to the load. We minimize the total exposed population and the total transportation cost. We present a case study in the City of Santiago in Chile to show practical application of our proposed approach.

This chapter was formatted as a manuscript titled “Hazardous materials collection with multiple-product loading”, and submitted for review to Journal of Cleaner Production in December, 2015. It was accepted in September, 2016 and published (Paredes-Belmar, Bronfman, Marianov, & Latorre-Núñez, 2017). This chapter contains the modifications done to the manuscript.

Complete reference: Paredes-Belmar G., Bronfman A., Marianov V., Latorre-Núñez G., (2017). Hazardous materials collection with multiple-product loading 141, 909-919. http://dx.doi.org/10.1016/j.jclepro.2016.09.163

4.1. Introduction

Some industries located in urban or semi-urban areas require large amounts of hazardous materials (HAZMAT) as supplies on a daily basis, which must be transported from their production sites. In turn, multiple hazardous wastes produced by industries must be transported to their final treatment and disposal sites. A few years ago, four billion tons of HAZMAT were estimated to have been annually transported worldwide (Zografos & Androutsopoulos, 2004).

The main risk of transporting HAZMAT or wastes comes from the possibility of accidents (fire, explosion, chemical leak, radiation, etc.) with significant consequences for human life and environment. Although the number of accidents may be low compared with the number of shipments of these materials, the danger to which the population is exposed can be very significant. This fact causes society to be particularly sensitive to HAZMAT transport and to clearly differentiate it from the shipment of other goods (Bianco, Caramia, Giordani, & Piccialli, 2013; Leal Junior & D'Agosto, 2011; Tarantilis & Kiranoudis, 2001). The general concern about the consequences of an accident involving HAZMAT has motivated researchers to develop multiple mathematical models to identify low-risk routes (Erkut, Tjandra, & Verter, 2007; Zografos & Androutsopoulos, 2008).

Generally, using low-risk routes tends to result in a high transportation cost. In contrast, low-cost routes can be riskier in terms of the potential consequences of an accident. Therefore, most of the literature on HAZMAT transportation considers the joint minimization of cost and risk, aimed at offering a set of efficient alternatives to decision makers (Bronfman et al., 2015; Das, Mazumder, & Gupta, 2012; Giannikos, 1998; Kremer et al., n.d.; K. G. Zografos & Androutsopoulos, 2004).

We propose a new approach to multi-product HAZMAT transportation that reduces cost and risk, which involves loading different types of HAZMAT or wastes in the same truck. This approach has been used in practice. In fact, standards exist that deal with compatibility of different HAZMAT for transportation (e.g., GPO, 2016). However, to the best of our knowledge, our approach has never been presented in the literature. Although some authors deal with the transportation of different types of HAZMAT (e.g., Nema & Gupta, 1999, 2003), they did not consider loading them on the same truck.

Some definitions are required to properly describe the advantages of our approach. Different authors use the term *risk* to express different indicators. We follow Tarantilis & Kiranoudis (2001), who defined risk as the *number of people exposed* to a certain danger or hazard (*population exposure risk*). We consider the fact that the distance that can potentially be reached by the negative effects of an accident involving HAZMAT depends on the material being carried, and this distance has been estimated (U.S. Department of Transportation, 2012). As a consequence, the exposed population also depends on the material, and it is defined as the population within the area that can be reached by a HAZMAT accident for a certain material. The population exposed to negative consequences by specific material *A* is associated to what we denote here as *material A risk*. A ranking can be made of all materials according to their risk and reachable area. We note that the population exposed to a riskier material also includes the population exposed to a less risky material.

By using our approach, both cost and risk decrease compared with the standard approach, which uses multiple trucks to carry a single material. The cost reduction comes from the economies of scope (different materials sharing the same trip), and the risk reduction is due to the fact that a truck loaded with multiple products exposes the same population that would similarly be exposed by the riskiest material being carried, in contrast to the condition when each material is separately transported, in which, the total exposed population is the sum of all the populations exposed to each material. In other words, the *truck risk* involves the riskiest material being carried by the truck irrespective of how many less risky materials are being simultaneously carried.

In our approach, a single truck risk varies along the route, and it is a function of the products transported by the truck on each section or link of the route. We assume that each customer or HAZMAT generator node produces HAZMAT with a single type of risk and that incompatible wastes or HAZMAT cannot be transported together. Because we deal with collection, the population exposed to a truck risk in successive sections of the route either remains the same or increases, depending on the risk associated with the last collected waste.

We also design a methodology to optimize the application of the proposed approach to real-size instances. Given an available capacitated truck fleet, the solution prescribes the least number of trucks to perform the collection as well as the routes that each truck must follow to collect the HAZMAT at the minimum weighted sum of costs and risks. The method includes constructing an auxiliary graph of the network and formulating an integer programming (IP) model that is solved using commercially available software (CPLEX). To determine an approximate set of non-dominated solutions, we use the weighting method where both objectives are normalized. Finally, a real HAZMAT collection case in a transportation network in the City of Santiago, Chile, is solved.

The remainder of the chapter is organized as follows. Subsection 4.2 contains the literature review. Subsection 4.3 provides more details on the approach, whereas subsection 4.4 describes the methodology. Subsection 4.5 presents the application to a real case in Santiago, Chile. Subsection 4.6 presents the conclusions and future work.

4.2. Literature review

In the specialized literature, most of the articles address the problem of transporting a single HAZMAT or product. Jacobs & Warmerdam (1994) simultaneously considered the routing and location of dumping sites. Giannikos (1998) located treatment or final disposal facilities and defined the transportation of hazardous wastes, thus minimizing operational cost, risk, difference in risk between population centers, and difference in disutility caused by placement of facilities. Leonelli, Bonvicini & Spadoni (2000) selected HAZMAT transportation routes that minimize costs and risk. The arcs had limited capacity. Tarantilis & Kiranoudis (2001) mitigated population exposure by designing the HAZMAT distribution routes. Zografos & Androutsopoulos (2002) presented two heuristics to solve the routing problem using time windows for HAZMAT distribution and studied the optimum deployment of emergency facilities for an adequate response in case of an accident. Shih & Lin (1999) and Shih & Lin (2003) studied the problem of infectious medical wastes in Taiwan. Zografos & Androutsopoulos (2004) defined the HAZMAT distribution problem as the vehicle routing and sequencing problem using time windows. Pradhananga et al. (2010) presented an ant colony algorithm to solve the HAZMAT transportation problem using time windows. Androutsopoulos & Zografos (2012) studied the HAZMAT distribution problem as a bi-objective vehicle routing and sequencing problem using time windows. Siddiqui & Verma (2015) proposed a linear IP model to route and sequence a heterogeneous fleet of ships for transportation of oil, thereby minimizing the cost and risk. Bronfman et al. (2015) proposed a new HAZMAT routing problem using a single type of product in an urban area between pre-selected origin and destination points.

Multiple products have been considered by many authors. Abkowitz & Cheng (1988) minimized risk and cost in HAZMAT transportation, and they focused on risk estimation. The levels of risk depended on the type of material, and each material was separately transported. Alternate routes to transport products between origin–destination pairs were established. Hu et al. (2002) presented a model to minimize costs for a reverse logistics system for several hazardous wastes. The model considered operational and governmental limitations in the design of an efficient and safe system. Ahluwalia & Nema (2006) developed a model for a multi-objective reverse logistics problem for the collection of several types of computational wastes. The costs and risks were minimized. Waste quantities varied with time, and these quantities were estimated using Monte Carlo simulation. Sheu (2007) developed a management system for the treatment of three types of hazardous wastes from multiple sources as a reverse logistics problem. The logistical costs and risk were minimized. The risk depended on the quantity of products being transported. The volume of wastes varied with time. Verter & Kara (2008) solved a network design for HAZMAT distribution by choosing among previously known paths. They considered various types of HAZMAT (gasoline, oil, petroleum, tar, and alcohol) that carried different levels of risk. Similar to our model, the risk imposed by a link of the route was estimated by the number of exposed people and depended on the type of material. No HAZMAT combinations were allowed in a truck. Samanlioglu (2013) determined the location of different technology treatment centers, recycling facilities, and disposal centers. The model was strategic and prescribed the total flow of waste between HAZMAT generators and the different types of facilities. The routes between generators and facilities and among facilities were preset, rather than designed for minimum risk and cost. Different types of wastes were transported in separate trucks. Nema & Gupta (1999) proposed a multi-objective integer model to optimally solve the problem of collection, treatment, and disposal of several hazardous wastes. Truck capacities were not considered. The risk imposed by each material depended on the volume of waste being transported, the hazard associated with it, the probability of an accident, and the exposed population. The model minimized costs and risk. Interactions among materials or the predominance of material risks was not considered. The authors applied the model to a test instance with 16 nodes and 20 arcs. Nema & Gupta (2003) improved the model proposed by Nema & Gupta (1999) using a goal programming model. Hamdi-Dhaoui, Labadi & Yalaoui (2010) studied the problem of routing and sequencing by transporting different compatible materials in a single truck but without considering the risk; only the transportation cost was minimized. They proposed a local search heuristic to solve the problem. Hamdi-Dhaoui, Labadi & Yalaoui (2011) continued the work of Hamdi-Dhaoui et al. (2010) by incorporating two heuristics: Iterated Local Search and greedy randomized adaptive search procedure–Evolutionary Local Search.

In contrast to the described models, the approach presented in the present paper allows loading different materials in the same vehicle, which considers the vehicle capacity, costs, and population exposure that varies with the truck risk. The inclusion of capacity adds additional complexity because more than one vehicle is needed to satisfy all the customer demands. For each truck, the methodology determines the customers to be visited and the sequence to be followed in such a manner that all the products are collected. We note that the problem is NP-hard because for a single type of waste, the problem is reduced to a vehicle routing problem (VRP), which is NP-hard (Toth & Vigo, 2001).

4.3. Hazardous collection problem with multiple-product loading

Different hazardous wastes must be collected from a set of customers located in an urban area and transported to a single depot site. All wastes must be collected, and a customer must have all its wastes collected by one truck in one visit. The transportation network of the urban area is modeled as graph $G(N, A)$. The set N of nodes includes the depot, customers, and street intersection nodes. Set A of arcs represents the streets. Each customer generates a single type of hazardous waste. However, if a customer produces more than one type of waste, replicas of the node can be created: one for each type of waste. All replicas are co-located.

A fleet of trucks with equal capacities collects all the wastes, starting and ending their trips at the depot. The objective of the problem is to minimize both the exposed population and the transportation cost. The risk model is based on the population exposure (Giannikos, 1998; Wyman & Kuby, 1995). Each hazardous waste has an associated risk that corresponds to a maximum radius of reach in case of an accident, which is defined by the Emergency Response Guidebook (U.S. Department of Transportation, 2012). The population within reach of an accident that could happen on an arc in the route is considered as exposed, and its size depends on the risk of the waste being carried over that arc. Figure 4-1 shows this concept of arc (i, j).

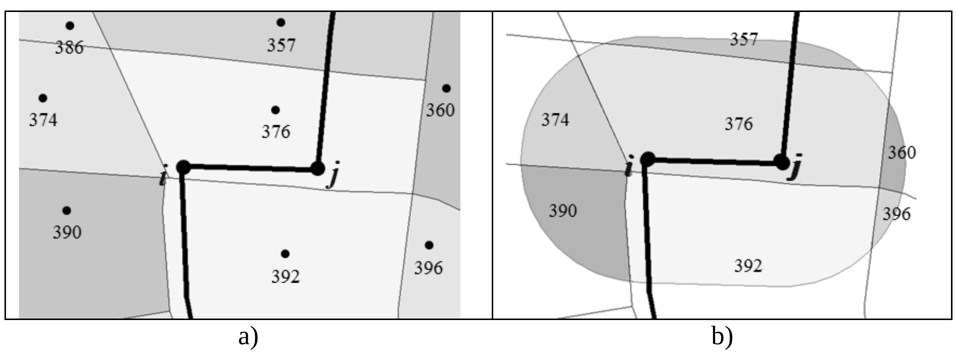


Figure 4-1. Computation of exposed population within each arc of the transportation network

The polygons shown in Figure 4-1a) represent census blocks with uniform population densities, as indicated by the gray shades. Figure 4-1b) shows the exposure zone ("stadium") around arc (i, j) based on a material with a known risk and its corresponding radius of reach. The total population inside the stadium is exposed if arc (i, j) is in the route of a truck. The size of the stadium depends on the risk of the material transported over the arc; for a riskier material, the size of the stadium increases and so does the exposed population. We note that the total exposed population is overestimated as the stadiums of successive arcs in the route intersect with one another. However, because this is not the focus of this study, we do not consider this error, which can be easily corrected as described in Kara, Erkut & Verter (2003).

Because the truck risk consists of the collected product with the highest risk, as the truck collects the wastes from a series of customers, its risk after each collection either remains the same or increases in discrete steps. Figure 4-2 shows an example of an instance and its solution for a single truck in which only the population exposure objective is minimized (and not the transportation cost). Figure 4-2a) shows the transportation network. The square (node 0) represents the depot, the circles represent the customers, and the framed letters show the risk of the material requiring collection at every customer node. The trucks can travel only over the arcs (streets). Three compatible materials are considered, namely, A, B, and C, with risk A < risk B < risk C. Because the truck risk represents the riskiest product in the truck, A + B = B, A + C = C, B + C = C, and A + B + C = C.

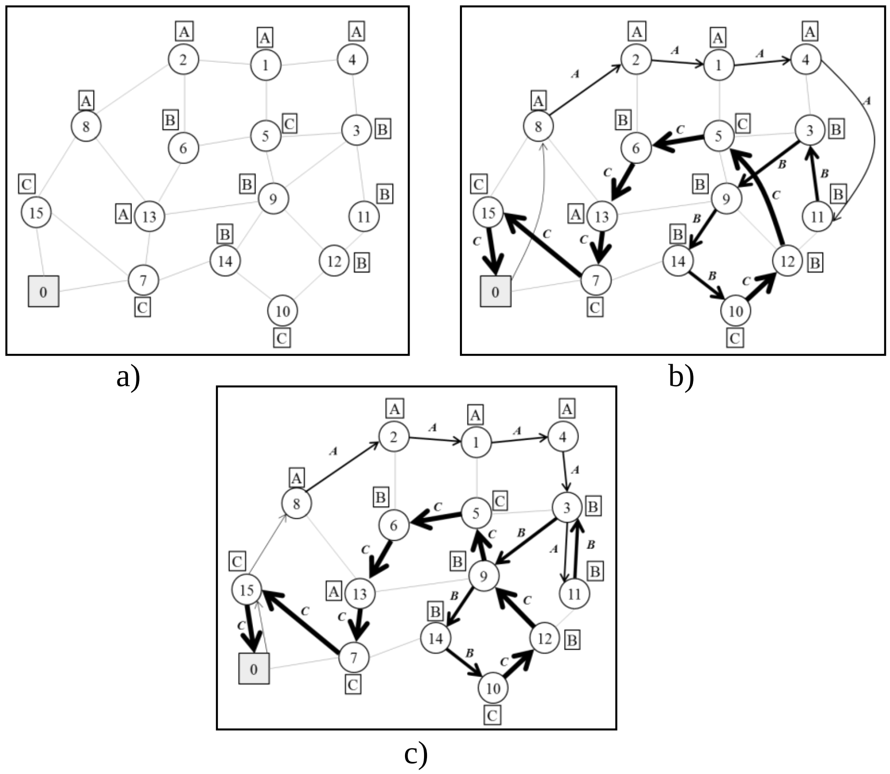


Figure 4-2. a) Transportation network. b) Sequence of customer node visits. c) Actual truck path.

Figure 4-2b) shows the sequence in which the customer wastes must be collected for minimum population exposure, and Figure 4-2c) shows the route over the actual transportation network that a truck must follow to collect the HAZMAT following the sequence shown in Figure 4-2b). The continuous arrows in Figure 4-2b) show the collection sequence, and the arcs in Figure 4-2c) represent the actual truck routes. Their thickness and the italics letters show the risk in which the population is exposed to in that particular step in the sequence or arc of the network. Each link (*i*, *j*) of the sequence in Figure 4-2b) must be implemented as the "best" path between successive customer nodes *i* and *j* shown in Figure 4-2c). By "best," we mean the path over the actual network that, among all possible paths between nodes *i* and *j*, contributes the most to minimize the objective, as will be explained in the next subsection.

We note that, to follow the optimal sequence shown in Figure 4-2b), the path over the actual transportation network [Figure 4-2c)] must visit some nodes twice without necessarily collecting waste, e.g., nodes 9 and 15. The hazardous waste in node 9 is collected on the first visit, whereas the reason for the second visit is that, among all possible paths between nodes 12 and 5 over the transportation network, the best one includes node 9. The same reason applies for node 15, whose HAZMAT is loaded on the second visit because if they were loaded on the first visit, the total exposed population would be much bigger. We note also that the truck travels along arcs (3, 11) and (11, 3). We consider risk A when the truck travels over arc (3, 11) and risk B when it travels over arc (11, 3) because the truck passes through these arcs at different times in its journey and causes exposure to different population sizes during each pass (some population is exposed twice).

To expose a smaller number of people to risk, the truck is loaded first with the least hazardous products (A) and later with the most hazardous ones (C) whenever possible. However, in some cases, loading the less dangerous products later in the route is convenient (such as in the case of node 13).

4.4. Proposed methodology

4.4.1. Description of the method

We initially assume that the least cost collection route is sought and that the travel cost over an arc is not necessarily proportional to its length. Such route could be found by solving a traveling salesman problem (TSP). The usual method consists of first constructing an auxiliary fully connected graph $G'(N',A')$ whose node set N' contains the depot and the customers. For each and every pair of nodes $i, j \in N'$, arc $(i, j)' \in A'$ exists with the same cost as the least cost path between i and j over the original graph $G(N, A)$. TSP is applied to this auxiliary graph, and the collection sequence is determined. This sequence is then transformed to a route over the original network by following the same collection sequence and replacing each arc $(i, j)' \in A'$ by the corresponding least cost path between i and j.

We now assume that the population exposure is minimized, instead of the cost, and that all materials have the same risk. In this case, the truck risk remains the same along the route after the first customer waste has been collected. Again, TSP can find the solution, and the procedure will be the same except that in this case, instead of the cost, each arc $(i, j)' \in A'$ will have the same population exposure as the least population exposure path between i and j over the original graph $G(N, A)$.

By now assuming that the materials have different risks, the population exposure of each arc is not known beforehand compared with the preceding case, and it depends on the materials that have been collected by the truck up to the time it reaches the arc. Consequently, it depends on the route between the depot and the node through which the arc is entered. The TSP cannot be applied, unlike that in the previous condition, and we propose a different procedure. By considering the same example shown in Figure 4-2, each arc has three possible population exposures corresponding to material risks A, B, and C. To solve this problem, we construct auxiliary graph $\bar{G}(\bar{N}, \bar{A})$ in the same manner as that in the preceding case (least population exposure paths) except that for each pair of nodes i and j, three "least exposure" arcs $\overline{(i, j)}^m$ that connect them are now possible, one for each material risk m. Once the auxiliary graph is constructed, an IP model will find the route, where arc $\overline{(i, j)}^m$ that corresponds to the risk m of the truck that travels from node i to j is chosen for each pair of nodes i, j. This auxiliary graph can be constructed for any number of material risks.

Finally, if the population exposure and cost are minimized, the procedure is similar except that the least exposure paths between node pairs now become the "least contribution to the objective" paths. Computing these paths is possible only if the full expression of the objective is known. Because the problem is bi-objective (cost and exposure objectives), the objective is known only if the problem is solved using the weighting method, i.e., minimizing a linear combination of population exposure and cost, with known weighting factors α and (1 - α) respectively. The objective would then be expressed as

$$Z = \alpha EP + (1-\alpha)C \tag{4.1}$$

where EP is the total exposed population, C is the total cost, and $0 \leq \alpha \leq 1$ is the weight of the population exposure objective.

4.4.2. Steps of the method

We now formally define the following sets, parameters, and variables.

Sets and parameters

R	Set of risks or waste types
K	Set of trucks
$N, \bar{N}$	Sets of nodes of the original and auxiliary graphs
$\bar{N}_C$	$\bar{N}_C = \bar{N} \setminus \{0\}$, i.e., the set of nodes including customers only
$A, \bar{A}$	Set of arcs of the original and auxiliary graphs
IR	Set of ordered pairs (i, m), meaning that customer i generates a material with risk m

D^m	Set of risks r such that the mixture of r and m is equal to r. It includes $r = m$
COM	Set of pairs (m, r) such that products with risks m and r are compatible
0	Depot node
α	Weight of the population exposure objective
EP_{lh}^m	Exposed population by a truck with risk m traveling over arc $(l, h) \in A$
c_{lh}	Travel cost over arc $(l, h) \in A$
c_{lh}^m	Contribution of arc $(l, h) \in A$ to the objective; $c_{lh}^m = \alpha EP_{lh}^m + (1-\alpha)c_{lh}$
$P^m(i, j)$	Least objective value path between i and $j \in A$ for a truck with risk m
$\overline{(i, j)}^m$	Arc of the auxiliary graph between i and j used for the route when the truck risk is m
a_{ij}^m	Attribute (weight) of arc $\overline{(i, j)}^m$
q_i	Amount of hazardous waste generated by customer $I \in V$. Note that $q_0 = 0$
Q	Capacity of a truck

Decision Variables

$$x_{ij}^k = \begin{cases} 1 & \text{If truck } k \in K \text{ goes from node } i \text{ to node } j : i, j \in \bar{N} \\ 0 & \text{otherwise} \end{cases}$$

$$y_i^k = \begin{cases} 1 & \text{If truck } k \in K \text{ loads product from customer } i \in V \\ 0 & \text{otherwise} \end{cases}$$

$$w_{ij}^{km} = \begin{cases} 1 & \text{If truck } k \in K \text{ transports material with maximum risk } m \in R \\ & \text{between nodes } i, j \in \bar{N} \\ 0 & \text{otherwise} \end{cases}$$

C_i^k:	Amount of waste transported by truck $k \in K$ before visiting node $i \in \bar{N}$. $C_0^k = 0$

First step

The first step of the solution approach consists in constructing auxiliary graph $\bar{G}(\bar{N}, \bar{A})$ as follows:

Initialization. $\bar{N}$ = {customers, depot node 0}. $\bar{A} = \phi$, the empty set. Pick a value for α.

i. For every possible risk *m*, compute the contribution of each arc $(l, h) \in A$ to the objective; $c_{lh}^{m} = \alpha EP_{lh}^{m} + (1-\alpha)c_{lh}$.

ii. For every pair of nodes ($i \in \bar{N}$, $j \in \bar{N}$) and every risk *m*

a. Find path $P^{m}(i, j) \in G(N, A)$ between *i* and *j* that contributes the least to the objective value for a truck with risk *m*. Use any shortest path algorithm with c_{lh}^{m} as the attribute of arc $(l, h) \in A$.

b. Add arc $\overline{(i, j)}^{m}$ to the set of arcs $\bar{A}$ whose attribute z_{ij}^{m} is the contribution of path $P^{m}(i, j)$ to the objective; $a_{ij}^{m} = \sum_{(l,h)\in P^{m}(i.j)} c_{lh}^{m}$.

Once the first step is finished, a fully connected auxiliary graph is obtained.

Second step

Solve the IP problem over $\bar{G}(\bar{N}, \bar{A})$.

$$Min\, Z = \sum_{i\in \bar{N}_C, j\in \bar{N}} \sum_{m\in R} \sum_{k\in K} a_{ij}^{m} w_{ij}^{km} + \sum_{j\in \bar{N}_C} \sum_{m\in R} \sum_{k\in K} a_{0j}^{m} w_{0j}^{km} \tag{4.2}$$

$$\sum_{i \in \bar{N}_C} y_i^k q_i \leq Q \qquad \forall k \in K \tag{4.3}$$

$$\sum_{k \in K} y_i^k = 1 \qquad \forall i \in \bar{N}_C : (i,m) \in IR \tag{4.4}$$

$$\sum_{j \in \bar{N}_C} x_{0j}^k \leq 1 \qquad \forall k \in K \tag{4.5}$$

$$\sum_{i \in \bar{N}} x_{ij}^k = \sum_{h \in \bar{N}} x_{jh}^k \qquad \forall k \in K, j \in \bar{N} \tag{4.6}$$

$$\sum_{h \in \bar{N}} x_{hi}^k = y_i^k \qquad \forall k \in K, i \in \bar{N}_C : (i,m) \in IR \tag{4.7}$$

$$\sum_{j \in \bar{N} \setminus i} w_{ij}^{km} \leq 1 - \sum_{h \in \bar{N}_C \setminus i} w_{hi}^{kr} \qquad \forall k \in K, i \in \bar{N}_C, m, r \in R : r \in D^m \setminus m \tag{4.8}$$

$$\sum_{j \in \bar{N} \setminus i} w_{ij}^{km} \leq 1 - y_i^k \qquad \forall k \in K, i \in \bar{N}_C, m, r \in R \tag{4.9}$$

$$: r \in D^m \setminus m, (i,r) \in IR$$

$$\sum_{m \in R} w_{ij}^{km} \geq x_{ij}^k \qquad \forall k \in K, i, j \in \bar{N} \tag{4.10}$$

$$\sum_{m \in R} w_{ij}^{km} \leq 1 \qquad \forall k \in K, i, j \in \bar{N} \tag{4.11}$$

$$y_i^k + y_j^k \leq 1 \qquad \forall k \in K, i, j \in \bar{N}_C, (m,r) \in COM \tag{4.12}$$

$$C_j^k \geq C_i^k + q_i - Q(1 - x_{ij}^k) \qquad \forall k \in K, j \in \bar{N}_C, i \in \bar{N} \tag{4.13}$$

$$x_{ij}^k, y_i^k, w_{ij}^{km} \in \{0,1\} \qquad \forall k \in K, i, j \in \bar{N}, m \in R \tag{4.14}$$

$$C_j^k \geq 0 \qquad \forall k \in K, j \in \bar{N} \tag{4.15}$$

Objective function (4.2) minimizes the exposed population and the total transport cost, weighted by α and $(1 - \alpha)$, respectively. Constraints (4.3) – (4.7) and (4.13) are capacitated VRP constraints. Constraint (3) limits the capacity of each truck. Constraint (4.4) indicates that each customer is attended to using a single truck. Constraint (4.5) indicates that each active truck only leaves the depot once. We note that the equality in constraint (4.5) would force all trucks being used even if not needed. The forced use of all trucks in any situation could result in a suboptimal solution in terms of population exposure and cost. Constraint (4.6) establishes the flow balance in all nodes and all trucks. Constraint (4.7) indicates that if truck k travels from node h to node i, then a load is added to the truck, which comes from customer i.

Constraints (4.8) – (4.11) allow following and updating the risk associated with the trucks along the arcs and nodes in their route over $\bar{G}(\bar{N},\bar{A})$. Because the truck risk can only increase when collecting HAZMAT, if truck k exits node i with risk m, constraint (4.8) indicates that the same truck could not have entered node i with a risk higher than m. We note that none of the constraints preclude a truck from loading a product with lower risk m at node j if the truck goes from i to j with higher risk r. Constraint (4.9) guarantees that if a truck loads a product with a type r risk from a customer, then the risk of the next arc cannot be lower than risk m, i.e., the truck cannot leave node i with a risk lower than r. Constraints (4.10) and (4.11) indicate that if truck k goes from node i to node j, its load will only have one type of risk. Constraint (412) avoids incompatible material combinations. Constraint (4.13), also known as the Miller–Tucker–Zemlin constraint (Miller, Tucker, & Zemlin, 1960), records the volume of products after each customer collection of the truck and avoids the appearance of sub-tours; Constraints (4.14) and (4.15) define the nature of the decision variables.

If p is the number of hazardous wastes, n is the number of nodes (including the depot), and k is the number of trucks, then the model has $(n^2k(1 + p) - nkp + n)$, i.e., $O(n^3kp)$ decision variables and $(k + np + 2kn + kp^2\,(2(n - 1) + (n^2 - 1)) + 3k(n^2 - n)) = O(n^3k^2)$ constraints, excluding domain constraints (4.14) and (4.15).

Instead of using the absolute values of the exposure and cost objectives f_E and f_C, respectively, we normalize them so that they become comparable. We use the normalization in Bronfman et al. (2015).

$$Z_i = \left[\frac{f_i - I_i}{AI_i - I_i} \right] \tag{4.16}$$

where Z_i is the normalized objective (cost or population exposure), f_i is the objective function before normalization, I_i is best value of f_i, and AI_i is its worst value.

We use the following *ad hoc* cuts to tighten the model and accelerate the problem solution.

$$y_i^m = \sum_{j \in \bar{N}, r \in D^m} w_{ij}^r \qquad \forall k \in K, i \in \bar{N}_C, (i,m) \in IR \tag{4.17}$$

Constraint (4.17) indicates that if truck k collects a type m product from customer i, the outgoing arc can only have a risk that is the same or higher than that from customer i.

Third step

The solution of the IP formulation is a sequence of arcs $\overline{(i,j)}^m$ for each truck k (described by variables $w_{ij}^{km} = 1$) that prescribes the order in which the customer waste must be collected to minimize the objective. The third step consists of finding the route over the actual transportation network, represented by graph $G(N, A)$, which is found by replacing each arc $\overline{(i,j)}^m$ with its corresponding path $P^m(i, j) \in G(N, A)$.

Figure 4-2b) and Figure 4-2c) show this concept. Figure 4-2b) shows the visit sequence of a truck (arcs $\overline{(i,j)}^m$), whereas Figure 4-2c) shows the corresponding route over the original transportation network.

Finally, the total transportation cost is the sum of the costs of all arcs in the routes, and the total exposed population is the sum of all exposed populations in all routes.

4.4.3. Other Mixing Rules

Some particular cases exist in which combining the transport of different types of hazardous wastes can generate risks that are different from that associated with the most hazardous waste. Usually, these combinations are prohibited by incompatibility restriction (4.12). However, our model can also solve this special situation by adding new constraints. For example, if the mixture of a risk A material with a risk B material results in a type C risk (which may have a higher or lower risk level than type B), the constraints take the following form:

$$\sum_{j\in\bar{N}\setminus i} w_{ij}^{kB} \le 1 - \sum_{h\in\bar{N}_C\setminus i} w_{hi}^{kA} \qquad \forall k\in K, i\in\bar{N}_C \tag{4.18}$$

$$\sum_{j\in\bar{N}\setminus i} w_{ij}^{kB} \le 1 - y_i^k \qquad \forall k\in K, i\in\bar{N}_C, (i,A)\in IR \tag{4.19}$$

$$\sum_{j\in\bar{N}\setminus i} w_{ij}^{kC} \ge \sum_{h\in\bar{N}\setminus i} w_{hi}^{kB} + y_i^k - 1 \qquad \forall k\in K, i\in\bar{N}_C, (i,A)\in IR \tag{4.20}$$

$$\sum_{j\in\bar{N}_C\setminus i} w_{ij}^{kC} \ge \sum_{h\in\bar{N}_C\setminus i} w_{hi}^{kA} + y_i^k - 1 \qquad \forall k\in K, i\in\bar{N}, (i,B)\in IR \tag{4.21}$$

Constraint (4.18) indicates that if a truck travels along arc (*h*,*i*) carrying risk A wastes, then the same truck cannot leave node *i* with risk B because although it loads a type B material at node *i*, the combination of A and B does not result in risk B. Constraint (4.19) states that a truck that loads a type A material at node *i* cannot exit that node carrying a type B material because if the truck has entered node *i* carrying a type A material, the output risk would be A; if it has entered *i* carrying a type B material, the output risk would be C. Constraint (4.20) establishes that if a truck travels along arc (*h*,*i*) with risk B and collects a risk A material at node i, then the truck will leave that node with a type C risk. The same idea is replicated in constraint (4.21).

4.5. Case Study

4.5.1. Description

We applied the proposed methodology to a hypothetical case based on a network of streets, roads, and industrial areas in Santiago, Chile. The city has an area of 641 km^2. The transportation network has 6,231 arcs and 2,205 nodes; in other words, this is a very large network. We must mention that the city has a high population density, reaching 446.9 inhabitants per square kilometer (INE, 2010). Without losing generality, we assumed that cost c_{ij} of using arc $(i, j) \in A$ is proportional to its length. Of the sample 3,500 industries in the Metropolitan Region of Santiago, as reported by Cisternas-Véliz (2003), we selected 167 factories whose industrial activities correspond to manufacturing of chemical substances and products. The factories are distributed throughout the city. Five types of wastes with risk notations A, B, C, D, and E were considered based on the production activity in the sample of selected industries. The types and quantities of wastes generated from each factory were randomly assigned.

Article 87 of the Chilean legislation concerning the handling of hazardous wastes (MINSAL, 2004) stipulates that certain wastes cannot be transported in a single vehicle because they may produce violent reactions such as explosions or fire. We defined 14 groups of hazardous wastes, designated *A*-1 to *A*-7 and *B*-1 to *B*-7. The wastes included in these groups were corrosive liquids, explosives, pesticides, alcohols, and other chemicals.

Table 4-1 lists the detailed information of the five types of hazardous wastes. It was created based on information from the following sources: Chilean Code 382 Hazardous Substances—Terminology and General Classification (INN, 1998), Decree 298 on the Regulation of Hazardous Shipments through Streets and Roads in Chile (MINTRATEL, 1995), Decree 148 on the Regulation of Safe Handling of Hazardous Wastes in Chile (MINSAL, 2004), and the Emergency Response Guidebook (ERG) (U.S. Department of Transportation, 2012). The first column in the table lists the identifier for HAZMAT types. A is an aqueous waste, B is a hydride, C is a mineral acid, D is a waste derived from chloride, and E is a metallic waste. The second column determines the compatibility group of the wastes for transportation and storage. The third column indicates the number of emergency response guide (ERG number) for each material. The information in the ERG considers inherent potential risks of the material in terms of fire, explosion, and health effects and information about immediate isolation of the incident location and evacuation distances for small and large spills, as well as emergency response actions, including first-aid activities. Finally, the fourth column shows the isolation distances (hazard radius) established in case of spill incidents involving small quantities of the materials during daytime.

Table 4-1. Initial isolation and protective action (U.S. Department of Transportation, 2012)

Id.	**Compatibility group**	**ERG number**	**Hazard radius (m)**
A	*A*-4	129	**50**
B	*B*-4	137	**100**
C	*B*-7	131	**200**
D	*A*-7	156	**300**
E	*B*-4	156	**400**

For this case study, we considered that the five waste classes are transported during a single day. lists the risk dominance and compatibility of the five types of waste being studied when these were transported by a single truck. To determine the risk when two or more different risk levels were transported together in a same truck, we used both Table 4-1 and Table 4-2.

Table 4-2. Waste–waste compatibility rules and risk dominance

Id\Id	**A**	**B**	**C**	**D**	**E**
A	A	-	C	D	-
B	-	B	C	D	E
C	C	C	C	-	E
D	D	D	-	D	E
E	-	E	E	E	E

The region was divided into eight collection zones (see Figure 4-3). Zone 8 is a residential area with no factories. This regional division is frequently used to distribute products in the City of Santiago because the neighborhoods grouped in Figure 4-3 have similar municipal regulations. Table 4-3 lists the regional information. *#H* is the number of factories in each zone, and q_A, q_B, q_C, q_D, and q_E are the quantities of waste types A, B, C, D, and E, respectively, generated in each of the zones. Each zone has two trucks with sufficient capacity to carry the wastes from each subdivision. The problem is independently solved for each zone.

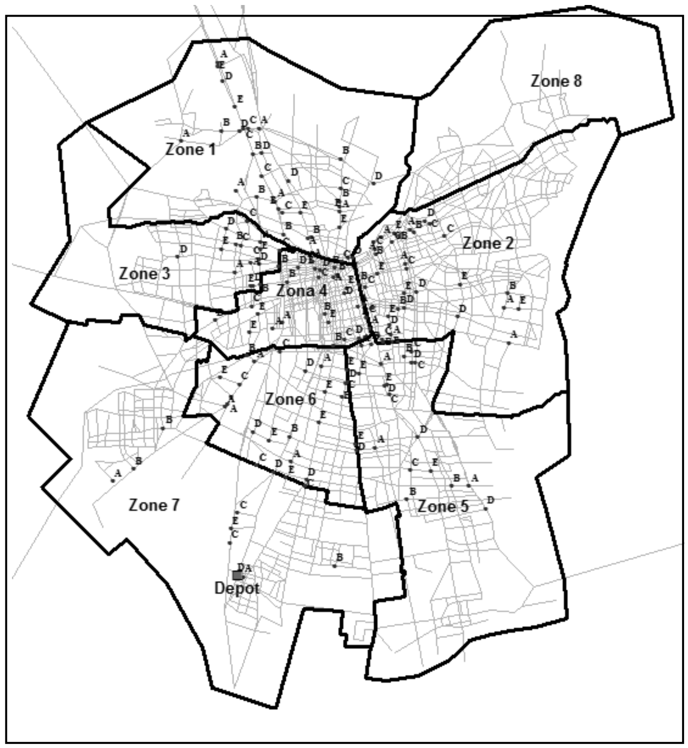


Figure 4-3. Collection zones in Santiago, Chile

Table 4-3. Detailed information on the study zones

	#H	q_A	q_B	q_C	q_D	q_E
Zone 1	32	7640	6680	3040	4260	5720
Zone 2	36	5440	10890	14420	20540	11740
Zone 3	16	8040	2170	4170	4120	1970
Zone 4	30	12440	1840	3560	5940	3390
Zone 5	21	1560	4370	1820	2760	2810
Zone 6	21	5600	3270	6060	2310	3770
Zone 7	11	3840	2490	2170	4980	2230

4.5.2. Results

The case study was solved using the commercial solver CPLEX Version 12.6 and coded with AMPL version 20130109 on a PC with Intel processor i7-2600, 3.4 GHz, 16-GB RAM, and an Ubuntu Server 12.04 LTS.

Figure 4-4 shows the solutions for minimum cost and minimum exposed population when the wastes from the 36 customers in Zone 2 are collected. The route of each truck is separately shown. The population density is indicated by shades; darker shades indicate higher population density. Figure 4-4a) and Figure 4-4b) show the routes that result in minimum exposed population for the two trucks used (disregarding transportation costs).

Figure 4-4a) shows that the truck begins its route unloaded at the depot until it loads hazardous product A (indicated by the thinnest line). The truck then collects type A wastes from several customers (indicated by segmented line sections). Finally, it loads type D wastes, indicated by changes in the type and thickness of the line (continuous double lines). The truck risk remains at D until it reaches the depot. We note that in the last section, the truck also collects type A wastes, with the dominance between A and D being maintained. Figure 4-4b) shows that the truck begins its route at the depot. It then loads risk B products followed by C products, changing the truck risk from risk B to risk C. The truck risk remains constant (indicated by segmented lines) until it loads risk E products. The truck risk remains at E on the return trip to the depot (indicated by continuous double lines). Figure 4-4c) and Figure 4-4d) show the minimum cost routes for both trucks. In these two figures, the line thickness is the same for all routes given that there is no interest in counting the exposed population or the change in risk of the trucks. Figure 4-4 shows that the solutions for the minimum exposed population tend to be longer and therefore costlier. These routes avoid transit through areas with high population densities. On the other hand, the routes with the minimum cost are shorter but expose a larger population. In both sample cases (cost and exposed population), incompatibility conditions are observed. Figure 4-4a) shows how special care is taken regarding the loading of type D risk wastes at the end of the route to avoid exposure of the population to higher risk. Similarly, Figure 4-4b) shows that the collection of type E wastes is done at the end of the route.

If transportation cost is the objective, HAZMAT loading order is not a concern, except in not loading incompatible materials on the same truck. Therefore, Figure 4-4c) shows that the first truck collects type A and D wastes from 14 customers, and Figure 4-4d) shows that the second truck collects types B, C, and E wastes (22 customers), both in any order.

We study the effects of changing the relative weights of both objectives, i.e., changing the value of α, and obtaining an approximation to the Pareto-optimal (or non-inferior) solution set. Table 4-4 and Table 4-5 list the non-inferior solutions obtained for each in the study zones. In the tables, Z_1 is the value of the exposed population, Z_2 is the transportation cost, and T is the CPU time in seconds. The tables show a conflict between the two objectives. For example, in Zone 1, when the value of total risk Z_1 is optimized, the distance covered by both trucks is 307,576 km, which is 1.96 times the distance covered when Z_2 is minimized. In addition, the exposed population measured in terms of the total number of people exposed by the routes is 482,214, which is approximately half of the extreme solution that minimizes cost. This effect is observed in all the study zones.

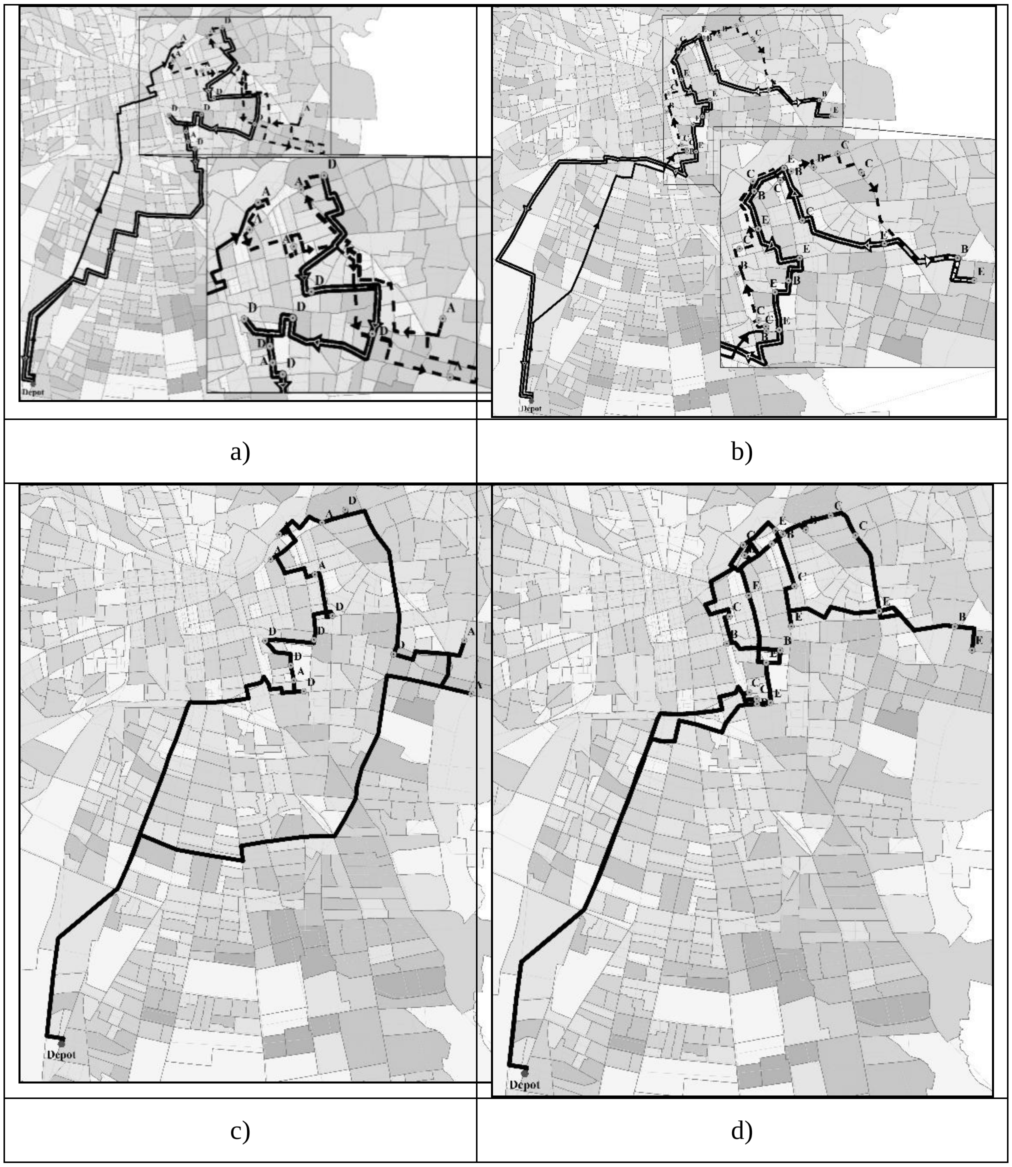


Figure 4-4**.** (a) and (b) Minimum risk solutions and (c) and (d) minimum cost solutions in Zone 2

Table 4-4. Non-inferior solutions: Zones 1 to 4

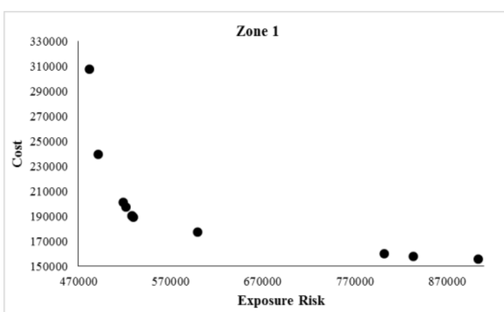
	Zone 1			Zone 2			Zone 3			Zone 4		
α	**Z_1**	**Z_2**	**T**	**Z_1**	**Z_2**	**T**	**Z_1**	**Z_2**	**T**	**Z_1**	**Z_2**	**T**
0.0	903547	155629	306	1402070	126354	14	623614	89565	1	1061540	98155	14
0.1	833642	157974	23973	1241740	127333	16	494322	89919	2	1047880	98184	7
0.2	801631	160202	8737	1188350	129013	31	442571	91277	2	997973	99136	17
0.3	599435	177142	50875	1113960	130272	38	388960	93681	2	932783	101475	34
0.4	529482	189183	6514	1039610	134074	50	352480	96819	7	810527	111129	43
0.5	527862	190300	3875	995684	138140	411	352480	96819	4	807328	112124	43
0.6	527861	190300	1545	985787	140171	288	330726	98466	7	805208	112602	33
0.7	521715	197604	13601	940303	151982	14656	330726	98466	8	805208	112602	23
0.8	518628	201312	9481	939033	154250	2239	330726	98466	9	805208	112602	100
0.9	491501	239727	3767	931954	165060	23456	330726	98466	7	804982	118679	29
1.0	482214	307576	3894	926804	166340	24836	325137	106926	15	795138	129177	27

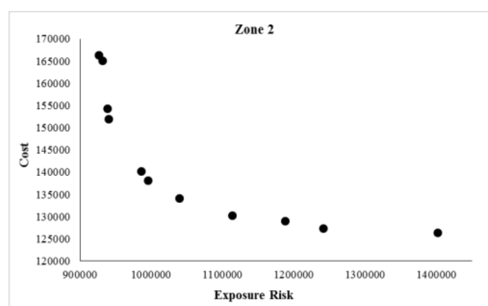
We can also see that the computation times are higher in Zones 1 and 2 because the associated graphs are highly asymmetrical and there are many solutions with the same objective value. In these two zones, a wider spectrum of efficient solutions is present given the large differences in the magnitude of their extreme solutions. We note that the model is particularly sensitive to the number of nodes in each zone.

Table 4-5 Non-inferior solutions: Zones 5 to 7

	Zone 5			Zone 6			Zone 7		
α	Z_1	Z_2	T	Z_1	Z_2	T	Z_1	Z_2	T
0.0	1314110	91416	52	506311	76038	20	528548	73727	1
0.1	750240	93549	93	465917	76492	14	308804	74024	1
0.2	667934	94980	57	441919	77395	4	177098	74747	1
0.3	658548	95595	32	441241	77461	4	173410	74971	1
0.4	637343	96600	37	441241	77461	4	160615	76052	1
0.5	618596	97714	18	440194	77574	5	160615	76052	1
0.6	618596	97714	26	432915	79370	4	154897	77119	1
0.7	618596	97714	16	432915	79370	4	150658	79311	1
0.8	604930	104584	31	428229	82316	5	127131	84378	1
0.9	603127	107253	19	426224	89092	11	127131	84378	1
1.0	580803	148435	16	425712	98510	13	125158	90324	1

Figure 4-5 shows the efficient boundary (Pareto optimal solutions) for each of the first two zones. In each case, we see the same conflict of objectives. Each point in the figures represents a different combination of weights of the objectives and a different route configuration.





a) b)

Figure 4-5**.** Trade-off curve of the first two zones

Because this is the first time that a problem of multi-product collection with mixed load on the same truck is dealt with, no other method is available for comparison with our works. Any comparison would mean simplifying the problem in such a manner that our approach would not make sense anymore. However, the results can be compared with those using separate trucks for each material, which is the problem solved in the literature. Using five available trucks in all cases, Table 4-6 list the comparison of a single product with multiple-product loading in two extreme cases: minimization of exposed population only and minimization of cost only.

The results are shown in terms of the number of required trucks, exposed population objective (EP), and cost objective (Cost). The results listed in Table 4-6 show how the solutions with multi-product loading completely dominate those of the separate loading in the sense that both objectives are better (by far, in most cases) when multiple-product loading is used, irrespective of what objective is being minimized. The only exception is Zone 5 where, when the cost is minimized, the exposed population increases compared with that of the single-product loading. However, by slightly changing the weights of the objectives, a solution that dominates the single-product solution is found with a cost of 96,732 and an exposed population of 732,601. Further, the number of trucks is always smaller: at most four in one case, three in three cases, and two in three cases.

Table 4-6. Comparison between the single and multiple product loading

	Zone 1 (min EP)		Zone 1 (min cost)	
	Single	Multiple	Single	Multiple
#Trucks	5	2	5	2
EP	**616045**	**482214**	1237880	903547
Cost	511759	307576	**312198**	**155629**

	Zone 2 (min EP)		Zone 2 (min cost)	
	Single	Multiple	Single	Multiple
#Trucks	5	2	5	2
EP	**1069150**	**926804**	1587930	1402070
Cost	333838	166340	**263279**	**126354**

	Zone 3 (min EP)		Zone 3 (min cost)	
	Single	Multiple	Single	Multiple
#Trucks	5	2	5	2
EP	**460922**	**325137**	790990	623614
Cost	227717	106926	**202895**	**89565**

	Zone 4 (min EP)		Zone 4 (min cost)	
	Single	Multiple	Single	Multiple
#Trucks	5	3	5	2
EP	**1074320**	**790773**	1479860	1061540
Cost	245761	162004	**210350**	**98155**

	Zone 5 (min EP)		Zone 5 (min cost)	
	Single	Multiple	Single	Multiple
#Trucks	5	3	5	2
EP	**754824**	**575217**	1170300	1314110
Cost	232946	173088	**194901**	**91416**

	Zone 6 (min EP)		Zone 6 (min cost)	
	Single	Multiple	Single	Multiple
#Trucks	5	3	5	2
EP	**509814**	**405792**	671858	506311
Cost	197635	125485	**159891**	**76038**

	Zone 7 (min EP)		Zone 7 (min cost)	
	Single	Multiple	Single	Multiple
#Trucks	5	4	5	2
EP	**153917**	**114281**	183309	174312
Cost	110789	92182	**103743**	**73727**

4.6. Conclusions

We have proposed a new approach to solve the problem of hazardous waste collection in a transportation network. The proposed work minimizes the risk of exposure of the population, expressed quantitatively as the total population affected by the route in case of an accident, and minimizes transportation costs.

In contrast to other contributions in the literature, we propose loading multiple products in the same trucks, which reduces both the cost and the exposed population. We design a methodology to appropriately use the multi-product loading, which includes precomputation of the best paths among all relevant nodes in the network for different waste risks and the formulation of an IP model that monitors risks along the route followed by the capacitated trucks, keeping track of the changes in the risk that each truck exposes the population to as new waste is added. The risk associated with each truck tends to increase when products with higher risk are loaded. We suggest an extension of the model that considers other mixing conditions.

The model is applied to HAZMAT collection over the transportation network in Santiago, Chile. The results indicate significant differences between the extreme minimum cost and minimum risk solutions. Therefore, determining a set of intermediate non-dominated solutions that allow decision makers to select the best alternative becomes necessary.

This study can be extended to several directions. From the multi-objective perspective, we could incorporate criteria such as exposure time, total hazard, expected consequence, and other factors. Although a distribution model can be trivially formulated as a variant of the collection model, a model of pickup and delivery would be an interesting extension in which the risk associated with the trucks may increase or decrease when the customers supply or require HAZMAT. Preliminary tests with such a model indicate that designing an efficient heuristic would be necessary to solve large instances in short times.

5. CONCLUSION

This thesis develops a new vehicle routing problem considering product mixing in the case in which there are interactions between the different products loaded in the same vehicle or truck compartment. Three real applications are addressed, modeled and solved. The contributions are both theoretical and practical.

In Chapter 2 we state, formulate and solve a new milk-blending problem. A firm has to collect raw milk to manufacture final products. The firm has a heterogeneous truck fleet. There is a set of farms spread in a large rural area. There are three qualities of milk and each farm produces a single quality of milk. The farms are grouped as a cooperative, so all milk must be collected on a daily basis. Minimum amounts to each quality of milk need to be met at the plant. The blending of different qualities of milk is allowed, but the quality of resulting blending becomes equal to the lesser quality loaded. We optimize this approach using an integer programming model to solve small instances with a branch-and-cut algorithm.

A real instance we use as a case study has 500 farms and it is hard to solve. So, we develop an approximate procedure with three stages: 1) Build clusters to divide the large area into districts; 2) Allocate trucks and minimum amounts of each quality of milk to satisfy the quotas; 3) Solve the small areas with the integer model.

The results show that the milk blending is convenient in terms of the profit, when is compared with both the current intuitive procedure and the use of exclusive trucks (VRP).

In Chapter 3, the blended milk collection problem using collection point is addressed. We allow milk collection points to accumulate milk produced by small and distant farms. The vehicles can collect milk from big farms and from the collection points. We formulate the problem with an integer programming model, and we solve it with a heuristic procedure. This procedure has three stages: 1) solve optimally an ad-hoc covering problem, to allocate the small and distant farmers to a collection points; 2) generate feasible routes with an Ant Colony meta-heuristic and 3) select optimally the best routes with a set-partitioning formulation.

The results emphasize the advantage of the milk collection allowing blending and collection points. The vehicles travel lesser distances and the number of stops is decreased, saving costs and time. The profit is better using our procedure when is compared with other approaches in the literature.

Finally, in Chapter 4, a new HAZMAT collection problem is presented, modeled and solved. In this case, a set of hazardous wastes needs to be collected using a homogeneous truck fleet. Each waste has his exposure radius. We count the total population exposed by trucks considering that the exposed population changes when new and different waste is added to the truck.

We solve an instance in the city of Santiago, Chile using the commercial software CPLEX. We compare our results with the use of exclusive trucks for each type of waste. The results show the effectiveness of the proposed approach. There are economies of scope when different products are transported together in a same truck.

Future work along this line includes the construction of more sophisticated heuristics to solve the different instances of the problems in short time. Another extension consists in the development of a general framework to analyze and solve the vehicle routing problem with product mixing, addressing any type of mix in the vehicles.

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