

Vector meson dominance and $g_{\rho\pi\pi}$ at finite temperature from QCD sum rules

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Abstract

A Finite Energy QCD sum rule at non-zero temperature is used to determine the q^2 - and the T -dependence of the $\rho\pi\pi$ vertex function in the space-like region. A comparison with an independent QCD determination of the electromagnetic pion form factor F_π at $T \neq 0$ indicates that Vector Meson Dominance holds to a very good approximation at finite temperature. At the same time, analytical evidence for deconfinement is obtained from the result that $g_{\rho\pi\pi}(q^2, T)$ vanishes at the critical temperature T_c , independently of q^2 . Also, by extrapolating the $\rho\pi\pi$ form factor to $q^2 = 0$, it is found that the pion radius increases with increasing T , and it diverges at $T = T_c$. © 1997 Elsevier Science B.V.

One of the popular reactions proposed for probing the quark-gluon plasma is dilepton production in high energy heavy ion collisions [1]. An important piece of information required to calculate the dilepton production rates, in the hadronic phase, is the temperature variation of the electromagnetic pion form factor, $F_\pi(q^2)$. In these calculations it has been usually assumed that Vector Meson Dominance (VMD) remains valid at non-zero temperature and, with a few exceptions [2], that the rho-meson mass and width are temperature independent. It has been argued long ago, though, that all hadronic widths should increase with increasing temperature, and presumably diverge at the critical temperature for deconfinement [2–4]. This is expected to hold also for particles which are (hadronically) stable at $T = 0$, e.g. nucleons and pions. Actual calculations in various frameworks do support such a scenario [5,6]. In this sense, the imaginary part of a hadronic Green's function, i.e. the width, may be viewed as a phenomenological signal for the occur-

rence of deconfinement. In addition, a QCD sum rule determination of the pion form factor at finite temperature clearly shows that it depends on T in such a way that it vanishes at the critical temperature, while the pion radius diverges there [7]. This determination of $F_\pi(Q^2, T)$ does not rely on any form of VMD, as it is based on the three-point function associated to the electromagnetic current and two axial-vector currents, thus projecting directly the electromagnetic pion form factor (in the space-like region $q^2 = -Q^2 < 0$).

In this paper we study Finite Energy QCD sum rules (FESR) at $T \neq 0$ for the three-point function involving the rho-meson interpolating current plus two axial-vector divergences. This allows us to determine the Q^2 - and the T -dependence of the $\rho\pi\pi$ coupling, as well as to gauge the validity of VMD at finite temperature. We begin with the determination of $g_{\rho\pi\pi}(Q^2)$ at zero temperature (for an earlier analysis using Laplace sum rules see [8]), in order to establish normalizations, as well as to check VMD here. This can be ac-

complished by using $g_{\rho\pi\pi}(Q^2)$ determined from the sum rules, together with VMD, and comparing the resulting pion form factor with the data. At finite temperature, we can also compare it with the direct determination [7], i.e. with a theoretical result not relying on VMD.

Since the latter does fit the data very well at $T = 0$, we can adopt it as the benchmark $F_\pi(Q^2, T)$ in the absence of experimental data at $T \neq 0$. Agreement between the two expressions could be taken as evidence in support of VMD at finite T .

We consider first the $T = 0$ correlator

$$\begin{aligned} \Pi_\mu(q) &= i^2 \int \int d^4x d^4y e^{-iq \cdot y} e^{ip' \cdot x} \\ &\times \langle 0 | T(j_\pi^\dagger(x) J_\mu^\rho(y) j_\pi(0)) | 0 \rangle \\ &= \Pi_1(q^2) P_\mu + \Pi_2(q^2) q_\mu, \end{aligned} \quad (1)$$

where $J_\mu^\rho(y) = \frac{1}{2} : [\bar{u}(y) \gamma_\mu u(y) - \bar{d}(y) \gamma_\mu d(y)] :$, $j_\pi(x) = (m_u + m_d) : \bar{d}(x) i \gamma_5 u(x) :$, $q_\mu = (p' - p)_\mu$, and $P_\mu = (p' + p)_\mu$. Calculating the imaginary part of the above three-point function in perturbative QCD to leading order in α_s and the quark masses gives the result

$$\begin{aligned} \text{Im } \Pi_\mu|_{\text{QCD}} &= \frac{3}{4} \frac{(m_u + m_d)^2}{[(s + s' + Q^2)^2 - 4ss']^{3/2}} \\ &\times [-Q^2 ss' P_\mu + ss'(s - s') q_\mu], \end{aligned} \quad (2)$$

where $s = p^2$, $s' = p'^2$, and $Q^2 = -q^2 \geq 0$. The hadronic counterpart of this correlator may be obtained after saturation with the pion intermediate state, and using the current-field identity

$$j_\mu^a = \frac{M_\rho^2}{f_\rho} \rho_\mu^a \quad (a = 1, 2, 3), \quad (3)$$

where j_μ^a is the isospin current, ρ_μ^a is the rho-meson field, and the experimental value of the coupling is $f_\rho = 5.0 \pm 0.1$, as obtained from the decay rate of the rho-meson into $e^+ e^-$ [9]. The result for the hadronic spectral function, e.g. $\text{Im } \Pi_1$ is

$$\begin{aligned} \text{Im } \Pi_1(s, s', Q^2)|_{\text{HAD}} &= -2f_\pi^2 \mu_\pi^4 \\ &\times \frac{M_\rho^2}{M_\rho^2 + Q^2} \frac{g_{\rho\pi\pi}(Q^2)}{f_\rho} \pi^2 \delta(s - \mu_\pi^2) \delta(s' - \mu_\pi^2) \\ &+ \theta(s - s_0) \theta(s' - s'_0) \text{Im } \Pi_1(s, s', Q^2)|_{\text{QCD}}, \end{aligned} \quad (4)$$

where $f_\pi = 93.2$ MeV and $g_{\rho\pi\pi}(M_\rho^2) = 6.06 \pm 0.03$ [9]. In principle, $g_{\rho\pi\pi}$ is a form factor, i.e. a function of q^2 . In the simpler version of VMD, i.e. single rho-meson dominance, this coupling would be strictly constant. However, there are radial excitations of the rho-meson (the $\rho(1450)$, $\rho(1700)$, etc.) which make a non-negligible contribution and turn the coupling into a form factor. For instance, naive VMD applied to the electromagnetic pion form factor predicts $g_{\rho\pi\pi}/f_\rho = 1$, while the experimental value is 20% higher. Hadronic models, such as e.g. the dual model [10], account for this difference by incorporating the rho-meson radial excitations; this gives $g_{\rho\pi\pi}(0)/f_\rho = 1$, but $g_{\rho\pi\pi}(M_\rho^2)/f_\rho \simeq 1.2$, in agreement with experiment. At the same time, naive VMD does not fit the data too well in the space-like region [10]; much better fits are obtained by allowing for a q^2 -dependence of $g_{\rho\pi\pi}$. In addition to the pion pole contribution in Eq. (4), there are additional terms from the pionic radial excitations, the a_1 meson, etc. However, we shall include these in the hadronic continuum, which is supposed to be well approximated by the perturbative QCD expression Eq. (2), provided that the thresholds $s_0 \simeq s'_0 > 1-3$ GeV².

At this point one can invoke Cauchy's theorem which leads to Finite Energy Sum Rules (FESR); the one of lowest dimension in the present case is

$$\begin{aligned} \int_0^{s_0} \int_0^{s'_0} \text{Im } \Pi_1(s, s')|_{\text{HAD}} ds ds' \\ = \int_0^{s_0} \int_0^{s'_0} \text{Im } \Pi_1(s, s')|_{\text{QCD}} ds ds', \end{aligned} \quad (5)$$

where s_0, s'_0 are the continuum thresholds, i.e. the onset of perturbative QCD. After substitution of the QCD and hadronic spectral functions in the FESR one obtains

$$\frac{g_{\rho\pi\pi}(Q^2)}{f_\rho} = \frac{3}{8\pi^2} \frac{f_\pi^2}{\langle \bar{q}q \rangle^2} \frac{Q^2}{M_\rho^2} (Q^2 + M_\rho^2) I(Q^2), \quad (6)$$

where

$$\begin{aligned} I(Q^2) &= \frac{s_0}{16} \left(3 + \frac{s_0}{Q^2} \right) \\ &+ \frac{1}{8} (s_0 + \frac{3}{4} Q^2) \ln \left(\frac{Q^2}{Q^2 + 2s_0} \right), \end{aligned} \quad (7)$$

and use was made of the Gell-Mann, Oakes and Renner (GMOR) relation [11]

$$f_\pi^2 \mu_\pi^2 = -(m_u + m_d) \langle \bar{q}q \rangle. \quad (8)$$

The result for $I(Q^2)$ above was obtained after a double integration in the s, s' plane. The region of integration is a triangle in this plane, but use of other shapes doesn't introduce appreciable differences in the numerical results for $g_{\rho\pi\pi}(Q^2)$. An advantage of using FESR as opposed to e.g. Laplace transform QCD sum rules, is that the latter requires knowledge of the vacuum condensates of all dimensions. In contrast, at most one condensate contributes to a FESR of a given dimension. In the present case, since the dimension of Π_1 is $d = 2$, there are no condensates appearing in the lowest dimensional FESR. Invoking now Extended Vector Meson Dominance (EVMD), i.e. VMD but with allowance for a possible Q^2 -dependence of $g_{\rho\pi\pi}$, leads to a well known expression for the electromagnetic pion form factor

$$F_\pi(Q^2)|_{\text{EVMD}} = \frac{M_\rho^2}{M_\rho^2 + Q^2} \frac{g_{\rho\pi\pi}(Q^2)}{f_\rho}. \quad (9)$$

Substituting the FESR result Eq. (6) into Eq. (9) gives

$$F_\pi(Q^2)|_{\text{EVMD}} = \frac{3}{8\pi^2} \frac{f_\pi^2}{\langle \bar{q}q \rangle^2} Q^2 \left[\frac{s_0}{16} \left(3 + \frac{s_0}{Q^2} \right) + \frac{1}{8} (s_0 + \frac{3}{4} Q^2) \ln \left(\frac{Q^2}{Q^2 + 2s_0} \right) \right]. \quad (10)$$

This result can be compared with that based on a three-point function involving the electromagnetic current and two axial-vector currents [12], which projects the pion form factor directly, and hence makes no use of VMD, i.e.

$$F_\pi(Q^2) = \frac{1}{16\pi^2} \frac{1}{f_\pi^2} \frac{s'_0}{(1 + Q^2/2s'_0)^2}. \quad (11)$$

Although Eqs. (10) and (11) look structurally very different, they are numerically very similar for $Q^2 \geq 0.5 \text{ GeV}^2$, if $s_0 \simeq 2.18 \text{ GeV}^2$, and $s'_0 \simeq 1 \text{ GeV}^2$. The latter value leads to a very good fit of the data above 1 GeV^2 [12]. On the other hand, with $s_0 \simeq 2.18 \text{ GeV}^2$, the extrapolation of Eq. (6) to $Q^2 = 0$ gives $g_{\rho\pi\pi}(0)/f_\rho \simeq 1$. It should be stressed that the onset of the continuum need not be the same in the

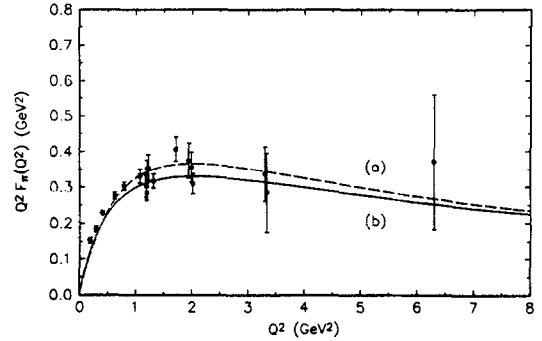


Fig. 1. The electromagnetic pion form factor at $T = 0$, determined from the QCD-FESR, Eq. (11), i.e. without invoking VMD (dashed curve (a)), compared with the result of the independent QCD-FESR for $g_{\rho\pi\pi}$ plus VMD, Eq. (10) (solid curve (b)). Experimental data is from [20].

two cases, as the correlators are different. A priori, all one knows is that s_0 (s'_0) should roughly be in the region where the resonances lose prominence, and the hadronic continuum takes over, i.e. somewhere in the interval 1–3 GeV^2 . On the other hand, the extrapolation to $Q^2 = 0$ of any of the above results should not be taken too seriously, as the Operator Product Expansion, the backbone of QCD sum rules, diverges at the origin. The good numerical agreement between Eqs. (10) and (11), as shown in Fig. 1, may be seen as a reflection of the validity of EVMD. In any case, our main purpose here is not another fit to the data at $T = 0$, but rather the non-zero temperature behaviour of the $\rho\pi\pi$ form factor. Since it is actually the ratio $g_{\rho\pi\pi}(Q^2, T)/g_{\rho\pi\pi}(Q^2, 0)$ which counts, modestly reasonable $T = 0$ results are normally sufficient.

Next, we reconsider the above FESR at finite temperature. Thermal corrections to $\Pi_1(q^2)|_{\text{QCD}}$ can be calculated in the standard fashion [13–15], and we find for the imaginary part

$$\begin{aligned} \text{Im } \Pi_1(s, s', Q^2, T) \\ = \text{Im } \Pi_1(s, s', Q^2, 0) F(s, s', Q^2, T), \end{aligned} \quad (12)$$

where

$$\begin{aligned} F(s, s', Q^2, T) = 1 - n_1 - n_2 - n_3 \\ + n_1 n_2 + n_1 n_3 + n_2 n_3, \end{aligned} \quad (13)$$

$$n_1 \equiv n_2 \equiv n_F \left(\left| \frac{1}{2T} \sqrt{\frac{x+y}{2}} \right| \right), \quad (14)$$

$$n_3 \equiv n_F \left(\left| \frac{Q^2 + (x - y)/2}{2T \sqrt{\frac{x+y}{2}}} \right| \right), \quad (15)$$

$n_F(x) = (1 + e^x)^{-1}$ is the Fermi thermal factor, and $x = s + s'$, $y = s - s'$. On the hadronic side, both f_π and $\langle \bar{q}q \rangle$ will develop a temperature dependence, and so will s_0 . The latter follows from the notion that as the resonance peaks in the spectral function become broader, the onset of the continuum should shift towards threshold [14–16]. The temperature behaviour of this asymptotic freedom threshold has been obtained from the lowest dimension FESR associated to the two-point function involving the axial-vector currents [14–16], with $f_\pi(T)$ as a known input (for a recent updated discussion see [17]). For temperatures not too close to the critical temperature T_c , say $T < 0.8T_c$, the following scaling relation has been found to hold to a very good approximation [17]

$$\frac{f_\pi^2(T)}{f_\pi^2(0)} \simeq \frac{\langle \bar{q}q \rangle_T}{\langle \bar{q}q \rangle} \simeq \frac{s_0(T)}{s_0(0)}. \quad (16)$$

We shall make use of this relation in the sequel, together with the results of [18] for $f_\pi(T)$ and $\langle \bar{q}q \rangle_T$, in the chiral limit, as well as for $m_q \neq 0$. In addition, we shall invoke the GMOR relation at finite T , which only gets modified at next to leading order in the quark masses [17].

The $T \neq 0$ FESR now reads

$$\begin{aligned} & \frac{g_{\rho\pi\pi}(Q^2, T)}{f_\rho} \\ &= \frac{3}{8\pi^2} \frac{f_\pi^2(T)}{\langle \bar{q}q \rangle_T^2} \frac{Q^2}{M_\rho^2} (Q^2 + M_\rho^2) I(Q^2, T), \end{aligned} \quad (17)$$

where

$$\begin{aligned} I(Q^2, T) &= \frac{1}{8} \int_0^{s_0(T)} dx \int_{-x}^x dy \frac{(x^2 - y^2)}{(Q^4 + 2xQ^2 + y^2)^{3/2}} \\ &\times F(x, y, Q^2, T), \end{aligned} \quad (18)$$

and the integration in Eq. (18) must now be done numerically. The rho-meson mass was assumed to be temperature independent; the modest increase of M_ρ near T_c as obtained from QCD sum rules [19], and other methods [6], does not change qualitatively the conclusions. The results for the ratio $g_{\rho\pi\pi}(Q^2, T)/g_{\rho\pi\pi}(Q^2, 0)$ are shown in Fig. 2 for

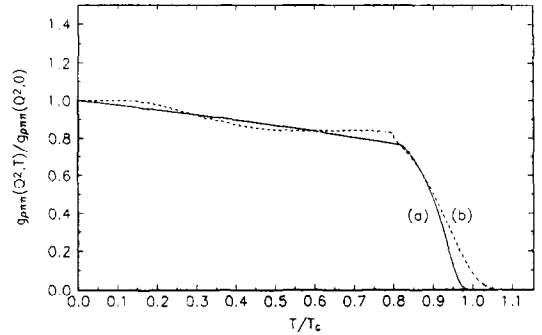


Fig. 2. The ratio of the $\rho\pi\pi$ form factors at finite and zero temperatures as a function of T/T_c . Curve (a) uses $f_\pi(T)$, $\langle \bar{q}q \rangle(T)$, and $s_0(T)$ in the chiral limit, and curve (b) away from this limit ($m_q \neq 0$). Both curves are for $Q^2 = 1 \text{ GeV}^2$, while other values of Q^2 give essentially the same ratios.

$f_\pi(T)$ and $\langle \bar{q}q \rangle_T$ in the chiral limit (curve (a)), as well as for $m_q \neq 0$ (curve (b)). Although $Q^2 = 1 \text{ GeV}^2$ was used in this figure, higher values of Q^2 give similar results. Particularly, and importantly, the vanishing of the ratio at or near the critical temperature is basically Q^2 -independent. This provides analytical evidence for deconfinement. The good agreement between the pion form factor using $g_{\rho\pi\pi}(Q^2)$ plus VMD and that obtained directly without invoking VMD, persists at $T \neq 0$. An extrapolation of these results to $Q^2 = 0$ allows for a determination of the $\rho\pi\pi$ root mean squared radius. Although this is divergent at any temperature because of mass singularities, the ratio $\langle r^2 \rangle(T)/\langle r^2 \rangle(0)$ is well defined; it increases with increasing T until the critical temperature where it becomes infinite, thus signalling deconfinement. The temperature behaviour of this ratio is shown in Fig. 3.

To conclude, we obtained direct evidence supporting Extended VMD at $T = 0$ from a comparison of the $\rho\pi\pi$ vertex function determined from QCD sum rules and the experimental data on the electromagnetic pion form factor. At finite temperature, and in the absence of data, we compared our determination with a benchmark pion form factor obtained from independent QCD sum rules and not using VMD. The close agreement found between the two expressions may be interpreted as supportive of (Extended) VMD at $T \neq 0$. At the same time, our determination provides additional analytical evidence for deconfinement, as $g_{\rho\pi\pi}(Q^2, T)$ decreases with T , vanishing at the critical temperature, while the ratio of the $\rho\pi\pi$ root mean

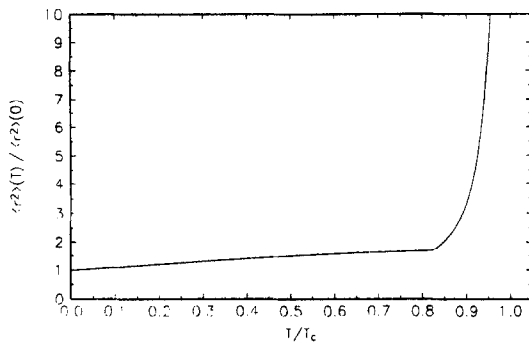


Fig. 3. The ratio of the (strong) pion radius at finite and at zero temperatures as a function of T/T_c .

square radii $\langle r^2 \rangle(T) / \langle r^2 \rangle(0)$ increases with temperature and becomes infinite at $T = T_c$.

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