PONTIFICIA UNIVERSIDAD CATÓLICA DE CHILE
ESCUELA DE INGENIERÍA

ON THE EFFECTS OF SHORT TERM MAINTENANCE SERVICE CONTRACTS ON RELATED DECISION MAKING

GABRIEL ALEJANDRO SANTELICES VOLANTE

Thesis submitted to the Office of Research and Graduate Studies
in partial fulfillment of the requirements for the degree of
Master of Science in Engineering

Advisor:
RODRIGO PASCUAL JIMENEZ

Santiago de Chile, January 2015

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Gratefully to my family

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First and foremost, I have to thank my parents for their love and support throughout this journey. Thank you both for giving me strength to reach for the stars and chase my dreams. My siblings and grandparents deserve my wholehearted thanks as well.

I would like to sincerely thank my supervisor, Professor Pascual, for his guidance and support throughout this study, and especially for his confidence in me.

Thank you, Lord, for the wonderful gift of life and for always being there for me.

This thesis is just the beginning. I am uploading another step to fulfill the dream I had since I was little, a dream that was not always easy to answer when people asked me ”what do you want to be when you grow up?” Today I know that the answer is not what, but who.

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ABSTRACT

This article studies the positive and negative effects that short term maintenance contracts may have on related decision making. We present an original model to estimate such effects and select the optimal preventive maintenance intervals and contract terms for pieces of equipment which are serviced by an external party. In the context of the contract, the intention of each party is in general to maximize its own profit, which usually leads to unaligned interests and decisions. To resolve this issue, we propose incentive schemes to assure the contract sustainability by achieving channel coordination between the client and its service vendor. Special focus is put on how the project net present value analysis of both parties affects decision-making regarding equipment maintenance. Our model considers a new alternative of negotiating contracts with non-constant maintenance intervals. The proposed model helps to identify conditions that justify maintenance deferrals with its associated negligence, in terms of lifecycle reduction and performance deterioration, when no channel coordination is promoted. Additionally, we present a simple procedure to settle an optimal contract duration, benefiting both parties. The proposed methodology is tested using a baseline case study from the literature. It illustrates how return on investment analysis may significantly impact optimal maintenance intervals during the contract for both parties. Accordingly, incentives need to be reevaluated to achieve channel coordination. The suggested approach can be easily implemented in commercial spreadsheets, facilitating sensitivity analyses.

RESUMEN

Este trabajo estudia los efectos positivos y negativos que pueden tener los contratos de mantenimiento de corto plazo en la toma de decisiones. Para la situación en la que el dueño de un equipo externaliza los servicios de mantenimiento a un contratista, se presenta un modelo original para estimar dichos efectos, los términos contractuales adecuados y encontrar el intervalo óptimo entre mantenciones preventivas. En un contrato de este tipo, generalmente la intención de ambas partes es maximizar su propio beneficio, lo que usualmente resulta en intereses y decisiones desalineadas. Para resolver este problema, se propone un sistema de incentivos que asegure la sustentabilidad del contrato a través de la coordinación entre el cliente y su contratista. Se hace énfasis especial en cómo el ánalisis del valor presente del contrato afecta las decisiones asociadas al mantenimiento del equipo. El modelo considera una nueva alternativa de negociación con intervalos no constantes entre mantenciones preventivas. El modelo ayuda también a identificar condiciones que justifican la postergación de las mantenciones, con sus respectivas consecuencias en términos de reducción de la vida del equipo y deterioro de su rendimiento, cuando no se promueve la coordinación de la cadena. Adicionalmente se presenta un procedimiento simple para determinar la duración óptima del contrato, beneficiando a ambas partes. Se realizaron pruebas usando un caso de estudio base presente en la literatura. En él se muestra cómo el análisis del valor del dinero en el tiempo puede impactar significativamente a ambas partes en la selección del intervalo óptimo entre mantenciones preventivas. De acuerdo a esto, la solución consiste en proponer incentivos para lograr la coordinación de la cadena. Este modelo puede ser implementado fácilmente en planillas de cálculo comerciales, que facilitan además los análisis de sensibilidad.

1. INTRODUCTION

Nowadays, outsourcing of certain functions in equipment intensive organizations is a common practice. This is specially true for equipment maintenance. Criticality, technical complexity, and uniqueness, to mention only a few criteria, may turn equipment servicing by external parties an appealing choice. As manufacturers seek to improve process performance and gain competitive advantages, contractors may offer lower costs, increased reliability and maintainability and higher levels of service, making it even more attractive to outsource maintenance (Yang et al., 2007). Outsourcing helps companies focus more on their core business (Kersten et al., 2007). In this context, it is frequent to observe that both parties try to maximize their own expected profits and do not coordinate efforts to maximize gains for them as part of their service supply chain. Such non-alignment of interests may result in a gap between the delivered service level and the one expected by clients (Wong & Jaya, 2008). In order to resolve this issue, a number of constraints can be added to the contract, as the trust levels between the parties may be reduced by non-sustainable attitude from one or both parties during the contract span.

In the above mentioned context, a problem that may arise in equipment intensive organizations is the common practice of deferring equipment's maintenance due to a myopic vision. This practice reduces equipment lifecycle and considerably deteriorates their performance. Furthermore, the costs of maintenance also increase with deferral, mostly as a result of accelerated deterioration rates as well as heightened labour and material costs (Zhang et al., 2009). For example, regarding infrastructure assets De Sitter concludes that if maintenance is deferred, then the maintenance cost may be expected to be up to five times higher (De-Sitter, 1984).

In a highly competitive and variable business environment, efficient coordination in the supply chain arises as the best solution for these common problems. In the past years, research has shown that a correct choice of incentives provides flexibility, creates new value, generates extra profit and leads to a win-win situation for both parties involved (Tarakci, Tang, Moskowitz, & Plante, 2006). Despite this evident need for coordination, efforts to implement it are still in the early stages. Kumar states that supply chains can create systems that integrate instant visibility and entire dynamic supply chains on an as-needed basis. Those chains are more likely to reach competitive advantages with respect to those that do not adopt such systems (Kumar, 2001). In this matter, Kanda and Deshmukh review various perspectives on supply coordination issues, coordination mechanisms, and the gaps in the literature (Kanda & Deshmukh, 2008). Li provides a review of coordination mechanisms of supply chain systems, highlighting behavioral aspects and information needed in the coordination (Li & Wang, 2007). In the literature, two main perspectives for contract channel coordination have been offered: analytical models and game theory approaches.

Regarding contract modelling using an analytical approach, Tarakci et al. (Tarakci et al., 2006) consider the use of several incentive policies to achieve channel coordination. They also extend these policies for multiple contractors with different maintenance capabilities. In a more recent works, Wang (Wang, 2010) presents different contract designs for maintenance service, based on the level of outsourcing of maintenance activities. He also discusses the consequences of information asymmetry between the parties. Tarakci et al. (Tarakci, Ponnaiyan, & Kulkarni, 2013) extend previous research and consider the risk mitigation context in which backup machines exist to reduce the loss of revenue due to machine failures. Such mitigation implies extra investment in capital assets which may be quite expensive to acquire and/or operate. Their model considers those machines as multi-component systems. They showed that the backup machine reduces the profit difference when incentives are offered to align interests of the supply service chain. Pascual et al. (Pascual et al., 2013) extend previous analyses by considering imperfect maintenance and contracts with a known duration at the same time. Their study considers profit and non-profit oriented organizations. Asgharizadeh and Murthy (Murthy & Asgharizadeh, 1999) consider a game theory approach to determine the optimal pricing structure, the number of customers to service and the number of service channels. Jackson and Pascual (Jackson & Pascual, 2008) address a problem where the contractor determines the price of maintenance and the client decides the preventive maintenance interval. They exploit a Nash arbitration scheme to distribute the expected profit between the parties. Nagarajan and Sosic (Nagarajan & Sošić, 2008) provide a survey of cooperative game applications in the context of supply chain management. They place special emphasis on profit allocation and stability. Hamidi et al. (Hamidi et al., 2014) present two game theory approaches for a joint decision-making contract. In this study, the equipment owner and the service agent need to establish a fair service contract by jointly determining the preventive replacement and part ordering times considering uncertain equipment failures.

In the case of maintenance contracts, other important issues that must be considered are the time horizon of the contract and the intervals between interventions. Although most contracts have a finite horizon and its duration is set by the client as a single decision maker (Tayur et al., 1999), previous channel coordination studies have only focused on infinite horizon contracts. These limitations could be overcome in order to achieve a more realistic model in the current operational context. This finite horizon consideration should be taken into account because it can strongly influence the optimal policies of a contract, the contractor's amortization of investments, and the aging process of the assets. In this line, there is a lack of literature noted by Lugtigheid et al. (Lugtigheid et al., 2007). Consequently, they present a model to improve repair and replacement of critical components in a finite horizon. However, their model focuses on a component level and does not have a systemic view. Regarding the intervals between interventions, current studies only consider maintenance contracts with constant maintenance intervals. There is no literature on optimization of maintenance contracts with non–constant intervals.

We also found other important gaps in the literature of maintenance contracts that are worth filling. There are no previous works that consider the effects of net present value in the context of maintenance contracts. Clearly this is an important gap, specially in long-term contracts arrangements because they are more sensitive to variability in profits and changes in interests rates (Cummins & Santomero, 1999). Other interesting topic that has not been addressed is the determination of the optimal duration of a maintenance contract. A maintenance outsourcing contract in equipment intensive organizations typically involves a long-term relationship between the client and its vendor. However, in some cases clients prefer to avoid committing to a unique vendor for a long time, so they rather sign short-term renewable contracts, increasing the flexibility of the contract and reducing its associated risks. This happens because companies rarely have a defined policy for contracts duration. For example, Jet Airlines, Indias premier international airline, signed in 2010 a 10-year contract with the company ST Aerospace for engines maintenance and engineering support. On the other hand, Jet Airlines extended recently its contract for just one additional year with the company Jordan Aircraft for the maintenance, repair and overhaul of four Airbus A330 aircrafts. Therefore, the net present value consideration and the optimal duration of maintenance contracts are practical and important issues that need to be studied.

It is important to address the relevance of maintenance outsourcing and overcome the limitations of the existing chain coordination models. This paper studies the positive and negative effects of short-term maintenance contracts on the related decision making. The remaining chapters of this work are structured as follows. Chapter 2 presents the model formulation for finite horizon contracts and continuous discount rate. Section 3 a case study for the previous model. Chapter 4 describes the cost subsidization coordination mechanism for the proposed model. Chapter 5 presents the model formulation for contracts with non-constant intervals between interventions and continuous discount rate. Finally, chapter 6 provides the conclusions and future developments of the work.

2. MODEL FORMULATION

Let us consider a finite horizon contract over a time period T_m, which is a decision variable decided by the client. The purpose of the decision makers is to set a maintenance interval $T \geq 0$ such that the discounted costs over the contract period are minimized. For tractability reasons, we initially consider the following constraint:

$$T_m = (n+1)(T + T_p), n \in \mathbb{Z} \tag{2.1}$$

where n is the number of preventive maintenance (PM) interventions, T is the interval between PMs, and T_p is the time duration of a PM. According to this model, PM interventions are performed at the end of each interval. Both n and T are also model's decision variables.

The expected profit for the client during the interval between PMs depends on the revenue of the client per unit time of uptime c_f, the equipment's availability $A(T)$ and the PM's price p, charged by the vendor:

$$(c_f A(T) - p)(T + T_p)$$

with

$$A(T) = \frac{T - N(T)T_r}{T + T_p}$$

$$N(T) = \int_0^T \frac{\beta}{\eta}\left(\frac{t}{\eta}\right)^{\beta-1} dt$$

where $N(T)$ is the expected number of failures during T, assuming minimal repair for $[0, T]$, and T_r is the time taken to repair the equipment. $\eta > 0$ and $\beta \geq 2$ are the parameters of equipment reliability.

For the vendor, the expected maintenance costs during the interval between PMs is:

$$C_i(T) = C_{ip} + C_{ir}N(T)$$

where C_{ip} is the PMs cost for the vendor and C_{ir} is the repair cost.

Then, the profit for each interval between PMs is:

$$p(T + T_p) - C_i(T) = p(T + T_p) - C_{ip} - C_{ir}N(T)$$

This model considers the following assumptions:

- There is one single client and one single vendor.
- There is complete and perfect information on both client and vendor sides.
- The vendor is free to select the PM interval. In this contracts, where maintenance services are outsourced, the selection of the PM interval is delegated to the vendor.
- The parameters of equipment reliability follow a power law NHPP process in between PMs, with an increasing failure rate. This process is commonly used to describe a failure mechanism in the maintenance and reliability literature because it can model many distributions forms by changing its parameters.
- Perfect preventive is considered. This assumption is commonly used in the maintenance and reliability literature, and the results of this work remain unchanged as long as the failure rate remains constant over time.
- Minimal repair is considered for corrective maintenance, with a relatively small repair time. This means a CM restores the process back to operation and the deterioration of the process remains during CM.
- Costs remain constant over time.
- There is fully disclosed equipment revenue of the client. This assumption may be a little impractical in some situations, but in the case of capital intensive industries, like the mining industry, this information is well known by vendors due to the high mobility of workers between companies.

Comparing the profit of both parties, it is clear that the client focuses on maximizing the uptime level, while the vendor emphasizes cost minimization. If we consider the entire contract horizon and discount the flows, we obtain the Net Present Value (NPV) for the client during the contract:

$$\Pi_m = \sum_{i=1}^{n+1} \left(\int_0^T c_f e^{-\theta t} dt - \int_0^T \frac{\beta}{\eta} \left(\frac{t}{\eta}\right)^{\beta-1} c_f T_r e^{-\theta t} dt - \int_0^{T+T_p} p e^{-\theta t} dt \right) e^{-\theta(T+T_p)(i-1)}$$

where $\theta > 0$ is the continuous discount rate.

In this case,

$$\Pi_m = \left(c_f \frac{(1-e^{-\theta T})}{\theta} - p\frac{(1-e^{-\theta(T+T_p)})}{\theta} - \frac{c_f T_r \beta}{\eta^\beta} \int_0^T t^{\beta-1} e^{-\theta t} dt \right) \sum_{i=1}^{n+1} e^{-\theta(T+T_p)(i-1)}$$

but,

$$\int_0^T t^{\beta-1} e^{-\theta t} dt = \frac{1}{\theta^\beta} \int_0^{\theta T} \tau^{\beta-1} e^{-\tau} d\tau = \frac{1}{\theta^\beta} \Gamma_{inc}(\beta, \theta T)$$

where Γ_{inc} is the lower incomplete gamma function. Then,

$$\Pi_m = \left(c_f \frac{(1-e^{-\theta T})}{\theta} - p\frac{(1-e^{-\theta(T+T_p)})}{\theta} - \frac{c_f T_r \beta}{(\eta\theta)^\beta} \Gamma_{inc}(\beta, \theta T) \right) \alpha \tag{2.2}$$

with

$$\alpha = \frac{e^{\theta(T+T_p)} - e^{-\theta n(T+T_p)}}{e^{\theta(T+T_p)} - 1}$$

For the vendor, the NPV is

$$\Pi_c = \left(\int_0^{T+T_p} p e^{-\theta t} dt - C_{ip} e^{-\theta(T+T_p)} - C_{ir} \int_0^T \frac{\beta}{\eta}\left(\frac{t}{\eta}\right)^{\beta-1} e^{-\theta t} dt \right) \sum_{i=1}^{n+1} e^{-\theta(T+T_p)(i-1)}$$

then,

$$\Pi_c = \left(p\frac{(1-e^{-\theta(T+T_p)})}{\theta} - C_{ip} e^{-\theta(T+T_p)} - \frac{C_{ir}\beta}{(\eta\theta)^\beta} \Gamma_{inc}(\beta, \theta T) \right) \alpha \tag{2.3}$$

As *Lemma* 2.1 shows, an agreement between the client and the vendor is generally not achievable.

Lemma 2.1. *Define*

$$g_\theta(T) = \frac{1}{\eta^\beta}(\beta\theta^{1-\beta}\Gamma_{inc}(\beta, \theta T) - \theta^{1-\beta}\Gamma(\beta+1) + \beta T^{\beta-1}(e^{\theta T_p} - e^{-\theta T}))$$

Then,

(i) *The optimal solution T_m^* that maximizes the NPV of the client satisfies*

$$g_\theta(T_m^*) = \frac{e^{\theta T_p} - 1}{T_r} \tag{2.4}$$

(ii) *The optimal solution T_c^* that maximizes the NPV the vendor satisfies*

$$g_\theta(T_c^*) = \frac{\theta C_{ip}}{C_{ir}} \tag{2.5}$$

(iii) *The optimal solution T^* that maximizes the total NPV satisfies*

$$g_\theta(T^*) = \frac{c_f(e^{\theta T_p} - 1) + C_{ip}\theta}{c_f T_r + C_{ir}} \tag{2.6}$$

Because of the complexity of the NPV functions, there are not closed-form solutions for T_m^*, T_c^* and T^*. Nevertheless, as $g_\theta(T)$ is an increasing function of T, simple numerical procedures can be used to find the optimal solutions. It is important to note that this function is defined just for mathematical purposes and does not have any practical meaning.

The results of *Lemma* 2.1 are similar to the ones developed in the base case (Tarakci et al., 2006). However, g_θ depends also on the continuous discounting rate parameter. This consideration is important because it takes into account flow discounting and affects the optimal maintenance interval, as shown in Figure 2.1 for our first case study, which is further described in chapter 3.

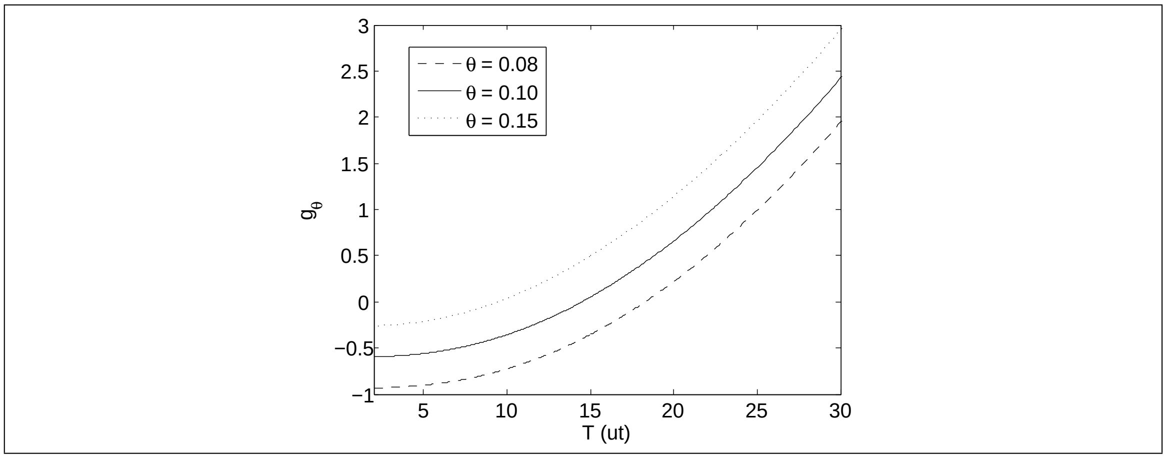

Figure 2.1. g_θ function for different values of θ

In the following Lemmas, we discuss the dependence of the function g_θ on the model parameters, the effects of the discount rate, and the effects of finite time horizon in setting the optimal PM intervals.

Lemma 2.2. *Holding other things constant,*

(i) *The optimal maintenance intervals for the client, the vendor and the supply chain decrease in scale and shape parameters of the process failure intensity function.*

(ii) *The optimal PM interval T_m^* for the client increases in the PM time T_p, while for the vendor T_c^* decreases in the PM time.*

Lemma 2.2 is intuitive and completely analogous to the one developed in the base case (Tarakci et al., 2006). Part (i) states that more frequent interventions are needed as the process failure intensity increases. However, part (ii) suggests that if an improvement in the PM time is made for the client, it will have a negative effect on the vendor's optimal time and channel coordination will be more difficult to reach.

Lemma 2.3. *The relationships among the optimal PM intervals for the client, the vendor and the supply chain are given by:*

(i) $T_m^* = T_c^* = T^*$ *if* $\frac{\theta C_{ip}}{C_{ir}} = \frac{(e^{\theta T_p}-1)}{T_r}$

(ii) $T_c^* > T^* > T_m^*$ *if* $\frac{\theta C_{ip}}{C_{ir}} > \frac{(e^{\theta T_p}-1)}{T_r}$

(iii) $T_c^* < T^* < T_m^*$ *if* $\frac{\theta C_{ip}}{C_{ir}} < \frac{(e^{\theta T_p}-1)}{T_r}$

Lemma 2.4. *If the time value of money is not considered, each of equations 2.2 and 2.3 becomes the expected profit per unit time, times the lengths of the contract period.*

$$\lim_{\theta \to 0} \Pi_m(T) = (c_f A(T) - p)T_m$$
$$\lim_{\theta \to 0} \Pi_c(T) = \left(\frac{p - (C_{ip} + C_{ir}N(T))}{T + T_p}\right) T_m$$

3. CASE STUDY

The parameters for our case study are similar to those in the base case of Tarakci et al (Tarakci et al., 2006). There are two additional parameters: the continuous discounting rate parameter (θ) and the contract duration (T_m), which is initially considered as a parameter and set at 5 times the characteristic life (η). Results compared with the base case profits are shown in Figure 3.1. One can see that the optimal maintenance interval for the base case is shorter and the profit curves decline faster than the NPV ones. This is because late flows reduce their present value due to discounting, so the optimum NPV is achieved with fewer maintenance interventions. Therefore, to perform fewer interventions, the maintenance interval must be longer. A study of g_θ function for the client, the vendor and the chain is shown in Figure 3.2.

Table 3.1. Case study parameters

Parameter	Value	Units
T_m	50	tu
θ	0.1	-
η	10	1/tu
β	3	-
T_p	1	tu
T_r	0.3	tu
C_{ip}	8	mu
C_{ir}	0.4	mu
c_f	15	mu/tu
p	2.5	mu/tu

According to *Lemma* 1, the optimal maintenance interval depends on the discount rate parameter and it can be strongly modified by the discount rate. As shown in Figure 3.3, the optimal maintenance interval grows linearly with the discount rate, while it does not depend on the contract's horizon. In this case, the NPV of the vendor is the most sensitive to the discount rate. Consequently, there are important differences in the NPV for different discount rates. For the case study parameters and the optimal maintenance interval of the chain, these differences are shown in Figure 3.4. Figure 3.5 shows the contour curves for

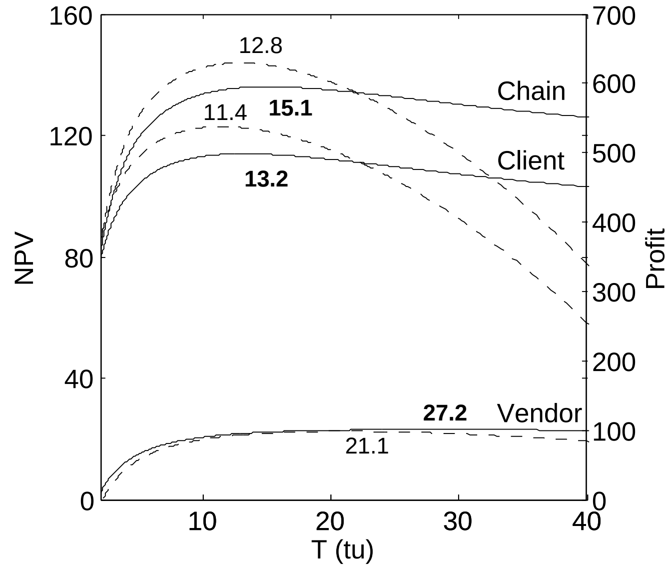

Figure 3.1. Expected NPV (solid line) and Expected Baseline Profit (dashed line) comparison for $\theta = 0.1$ and $T_m = 50$.

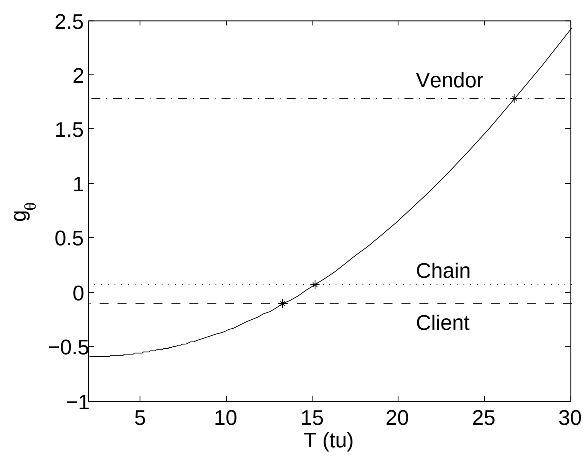

Figure 3.2. Study of $g_\theta(T)$ for $\theta = 0.1$ and $T_m = 50$, where * indicates each optimal value.

the NPV function. For higher discount rates the NPV of the contract is not very sensitive to the contract duration. On the other hand, the NPV depends strongly on the contract duration for lower discount rates. As a result, there are variables like the volatility and financial risks of the market that should be taken into account because they may be very influent in contracts duration related decision making. In both figures the contract duration

T_m is presented after being normalized by the characteristic life η of the equipment for practical purposes.

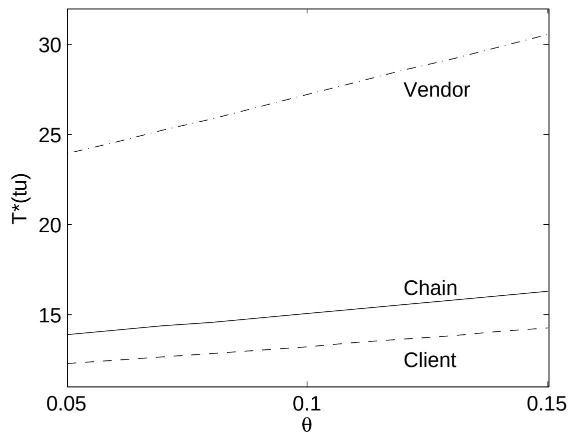

Figure 3.3. Optimal maintenance interval for different discount rates without incentives.

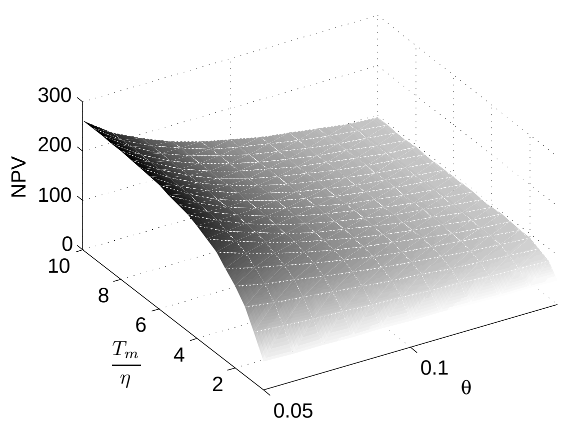

Figure 3.4. Chain's optimal NPV for different contract horizons and discount rates.

After determining the optimal maintenance interval, we defined a methodology to determine the appropriate duration of the contract from an economic point of view and define what should be considered as short-term. It consists on determining the contract duration where the optimal NPV for all parties reaches a stability zone. As shown in Figure 3.5, in this zone the NPV no longer increases with a longer duration of the contract. Hence,

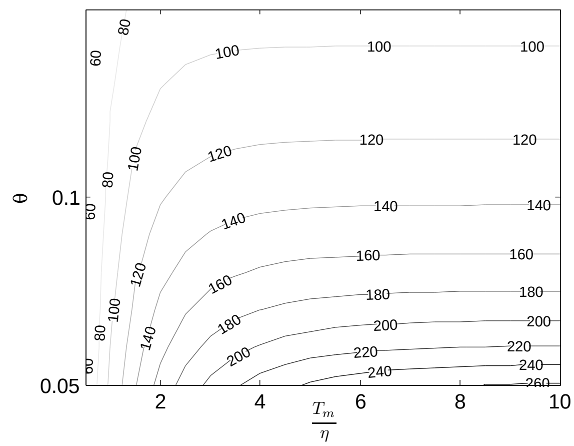

Figure 3.5. Chain's optimal NPV for different contract horizons and discount rates.

for longer contracts, as the NPV of both parties is not sensitive to the contract duration, the coordination analysis may be approached by any model with an infinite horizon assumption, according to the Renewal Reward Process Theory. To determine the contract duration D where the NPV stabilizes, we calculated the asymptotic NPV value π for each party and determined the contract duration where the NPV reaches the 99% of this value:

$$\Pi(T^*, D) = 0.99\pi$$

where,

$$\pi = \lim_{T_m \to \infty} \Pi(T^*, T_m)$$

The contract's normalized duration before being stable is shown in Figure 3.6. For the case study parameters, a short term contract should be defined within a duration of 4.6 characteristic life of the equipment. According to the model formulation and the previous definition of a short-term contract, the optimal contract duration is the same for all the contract participants, further encouraging the coordination. This strategy not only maximizes the joint NPV, but also provides suitable and convenient contract terms for both parties involved.

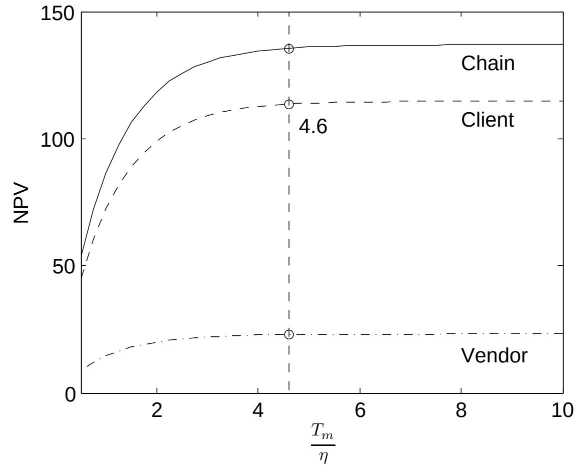

Figure 3.6. Expected NPV stabilization.

A particular situation of this model is the alternative of not doing preventive maintenance ($n = 0$). This may be an option which the preventive maintenance costs are not worthwhile, when there is a high exposure to risks like human errors or infant mortality, or when the contract duration is too short (Worsham, 2000). In the case study, as shown in Figure 3.7 the optimal maintenance interval for each party is better than this alternative, so maintenance policies should be included in the contract. For a general case, if the conditions mentioned above are not met, the alternative of not doing preventive maintenance will not be preferred unless the optimal maintenance interval is the same as the contract duration.

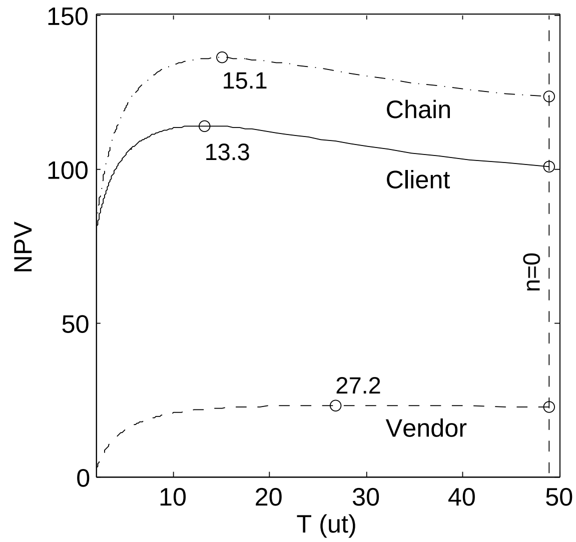

Figure 3.7. Comparison of Expected NPV with the situation without doing PMs.

4. COST SUBSIDIZATION COORDINATION MECHANISM

If $T_c^* > T^*$ the client may subsidize a portion of the PM cost of the vendor in order to achieve the channel coordination. This means that the client pays the vendor ΔC_{ip} each time the vendor performs a PM act. Then, the effective cost for the vendor is $C'_{ip} = C_{ip} - \Delta C_{ip}$. According to *Lemma* 2.1, to achieve channel coordination the vendor's optimal PM interval T_c^* must satisfy:

$$g_\theta(T_c^*) = \frac{\theta C'_{ip}}{C_{ir}}$$
$$g_\theta(T_c^*) = \frac{c_f(e^{\theta T_p} - 1) + C_{ip}\theta}{c_f T_r + C_{ir}}$$

Consequently,

$$C'_{ip} = \frac{C_{ir}}{\theta}\left(\frac{c_f(e^{\theta T_p} - 1) + C_{ip}\theta}{c_f T_r + C_{ir}}\right) \tag{4.1}$$

and

$$\Delta C_{ip} = C_{ip} - C'_{ip} \tag{4.2}$$

Lemma 4.1. *Channel coordination can be achieved using a cost subsidization mechanism with* $p \in [p_1, p_2]$, *where*

$$p_1 = \pi + \frac{C_i(T^*)}{T^* + T_p} - \frac{\Delta C_{ip}}{T^* + T_p} \geq 0$$

and

$$p_2 = c_f A(T^*) + \pi - \left(c_f A(T_c^*) - \frac{C_i(T_c^*)}{T^* + T_p}\right) - \frac{\Delta C_{ip}}{T^* + T_p} \geq p_1$$

where π *is the the vendor's unit-time minimum profit to participate in the contractual relationship.*

If $T_c^* < T^*$ the client may subsidize the cost of the corrective maintenance in order to achieve the channel coordination. This means that the client pays the vendor ΔC_{ir} each time the vendor performs a CM act. Then, the effective cost for the vendor is $C'_{ir} =$

$C_{ir} - \Delta C_{ir}$. In order to achieve channel coordination C'_{ir} is given by:

$$C'_{ir} = C_{ip}\theta\left(\frac{c_f T_r + C_{ir}}{c_f(e^{\theta T_p} - 1) + C_{ip}\theta}\right) \tag{4.3}$$

and

$$\Delta C_{ir} = C_{ir} - C'_{ir}$$

Lemma 4.2. *Channel coordination can be achieved using a cost subsidization mechanism with* $p \in [p_1, p_2]$*, where*

$$p_1 = \pi + \frac{C_i(T^*)}{T^* + T_p} - \frac{N(T^*)\Delta C_{ir}}{T^* + T_p} \geq 0$$

and

$$p_2 = c_f A(T^*) + \pi - \left(c_f A(T_c^*) - \frac{C_i(T_c^*)}{T^* + T_p}\right) - \frac{N(T^*)\Delta C_{ir}}{T^* + T_p} \geq p_1$$

Lemma 4.3. *If flow discounting is not considered, equations 4.1 and 4.3 are the same as the vendor's effective cost developed in the base case (Tarakci et al., 2006).*

$$\lim_{\theta \to 0} C'_{ip} = C_{ir}\left(\frac{c_f T_p + C_{ip}}{c_f T_r + C_{ir}}\right)$$
$$\lim_{\theta \to 0} C'_{ir} = C_{ip}\left(\frac{c_f T_r + C_{ir}}{c_f T_p + C_{ip}}\right)$$

4.1. Case Study

Using the same parameters of the case study in chapter 3, the proposed cost subsidization mechanism can be applied. Because $T_c^* > T^*$, the client pays the vendor ΔC_{ip} each time a PM act is performed to increase the frequency of PM. Using equations 4.1 and 4.2 we obtain that $\Delta C_{ip} = 6.06$. A comparison of the results with the case without incentives is shown in Figure 4.1.

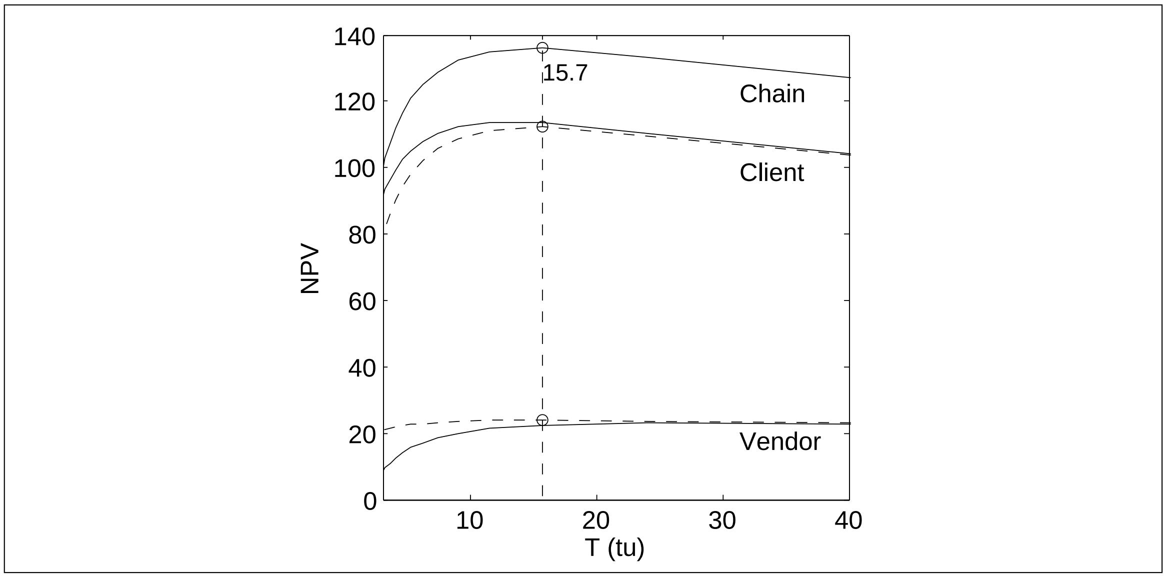

Figure 4.1. Expected NPV without incentives (continuous line) and CS mechanism (dashed line) comparison.

5. NON-CONSTANT INTERVAL VARIATION

Non-constant maintenance intervals may be a smart option to reduce costs in finite span maintenance service contracts, especially because most models assume constant intervention intervals. Also, non-constant maintenance intervals arise because most equipment undergo an aging process that decreases their remaining useful life and increases the need for overhaul. When the components are not replaced, their health after preventive maintenance improves, although it is not as new. Hence, as time passes equipment need to be repaired more and more often. To address this consideration, we propose a variation of the previous model with a linear constraint for simplicity:

$$T_i = T_0(1 + \delta i), \quad \forall i \in \{1, ..., (n+1)\}$$

with

$$n \in \mathbb{Z}, \quad (1 + \delta n) > 0$$

$$\begin{aligned} T_m &= \sum_{i=1}^{n+1} T_i + T_p \\ &= \sum_{i=1}^{n+1} T_0(1 + \delta i) + T_p \end{aligned}$$

Then,

$$T_m = (n+1)\left(T_0\left(1 + \frac{1}{2}\delta(n+1) + \frac{1}{2}\delta\right) + T_p\right)$$

and

$$T_0 = \frac{\frac{T_m}{(n+1)} - T_p}{\left(1 + \frac{1}{2}\delta(n+1) + \frac{1}{2}\delta\right)}$$

The NPV for the client is:

$$\Pi_m^{'} = \sum_{i=1}^{n+1} \left(\int_0^{T_i} c_f e^{-\theta t} dt - \int_0^{T_i} c_f T_r \frac{\beta}{\eta} \left(\frac{t}{\eta}\right)^{\beta-1} e^{-\theta t} t - \int_0^{T_i+T_p} p e^{-\theta t} dt) \right) e^{-\theta(T_i+T_p)(i-1)}$$

$$= \sum_{i=1}^{n+1} \left(\int_0^{T_0(1+\delta i)} c_f e^{-\theta t} dt - \int_0^{T_0(1+\delta i)} c_f T_r \frac{\beta}{\eta} \left(\frac{t}{\eta}\right)^{\beta-1} e^{-\theta t} dt + \right.$$

$$\left. - \int_0^{T_0(1+\delta i)+T_p} p e^{-\theta t} dt \right) e^{-\theta(T_0(1+\delta i)+T_p)(i-1)}$$

Then,

$$\Pi_m^{'} = -\frac{c_f}{\theta} \sum_{i=1}^{n+1} (e^{-\theta T_0(1+\delta i)} - 1) e^{-\theta(T_0(1+\delta i)+T_p)(i-1)} +$$

$$- \sum_{i=1}^{n+1} \frac{c_f T_r \beta}{\eta^\beta} \left(\int_0^{T_0(1+\delta i)} t^{\beta-1} e^{-\theta t} dt \right) e^{-\theta(T_0(1+\delta i)+T_p)(i-1)} +$$

$$- \frac{p}{\theta} \sum_{i=1}^{n+1} (e^{-\theta(T_0+T_0\delta i+T_p)} - 1) e^{-\theta(T_0(1+\delta i)+T_p)(i-1)}$$

Because

$$\int_0^{T_0(1+\delta i)} t^{\beta-1} e^{-\theta t} dt = \frac{1}{\theta^\beta} \int_0^{\theta T_0(1+\delta i)} \tau^{\beta-1} e^{-\tau} d\tau = \frac{1}{\theta^\beta} \Gamma_{inc}\left(\beta, \theta T_0(1+\delta i)\right)$$

we have,

$$\Pi_m^{'} = -\frac{c_f}{\theta} \sum_{i=1}^{n+1} (e^{-\theta T_0(1+\delta i)} - 1) e^{-\theta(T_0(1+\delta i)+T_p)(i-1)} +$$

$$- \sum_{i=1}^{n+1} \frac{c_f T_r \beta}{(\eta\theta)^\beta} \Gamma_{inc}\left(\beta, \theta T_0(1+\delta i)\right) e^{-\theta(T_0(1+\delta i)+T_p)(i-1)}$$

$$- \frac{p}{\theta} \sum_{i=1}^{n+1} (e^{-\theta(T_0+T_0\delta i+T_p)} - 1) e^{-\theta(T_0(1+\delta i)+T_p)(i-1)}$$

or,

$$\Pi_m^{'} = \sum_{i=1}^{n+1} \Bigg(- \frac{c_f(e^{-\theta T_0(1+\delta i)} - 1)}{\theta} - \frac{c_f T_r \beta}{(\eta\theta)^{\beta}} \Gamma_{inc}\left(\beta, \theta T_0(1+\delta i)\right)$$
$$- \frac{p(e^{-\theta(T_0(1+\delta i)+T_p)} - 1)}{\theta} \Bigg) e^{-\theta(T_0(1+\delta i)+T_p)(i-1)}$$

The NPV for the vendor is:

$$\Pi_c^{'} = \sum_{i=1}^{n+1} \Bigg(\frac{p(e^{-\theta(T_0(1+\delta i)+T_p)} - 1)}{\theta} - \frac{C_{ir} \beta}{(\theta\eta)^{\beta}} \Gamma_{inc}\left(\beta, \theta T_0(1+\delta i)\right)$$
$$- C_{ip} e^{-\theta T_0(1+\delta i)} \Bigg) e^{-\theta(T_0(1+\delta i)+T_p)(i-1)}$$

For the case study parameters, the chain's optimal NPV for different number of interventions and negative linear variations is shown in Figure 5.1. Figure 5.2 shows the chain's NPV for negative linear variations and the optimal number of interventions. Positive linear variations were not considered because they produced sub-optimal results. As seen in figure 5.1, the function has some peaks, due to the existence of local maximums for each number n of interventions. The global maximum is reached with a variation of $\delta = -0.22$.

Figure 5.2 shows the chain's optimal NPV for different negative linear variations. The optimal number of maintenance interventions is the same as the constant maintenance intervals' case. The difference is that the frequency between interventions varies, prioritizing more interventions at the beginning of the contract. Results show that the contract's NPV increases significantly by considering the alternative of performing the first preventive maintenances more often and retarding the last ones. In this line, other variations of non-constant maintenance intervals may be considered in order to achieve even better results.

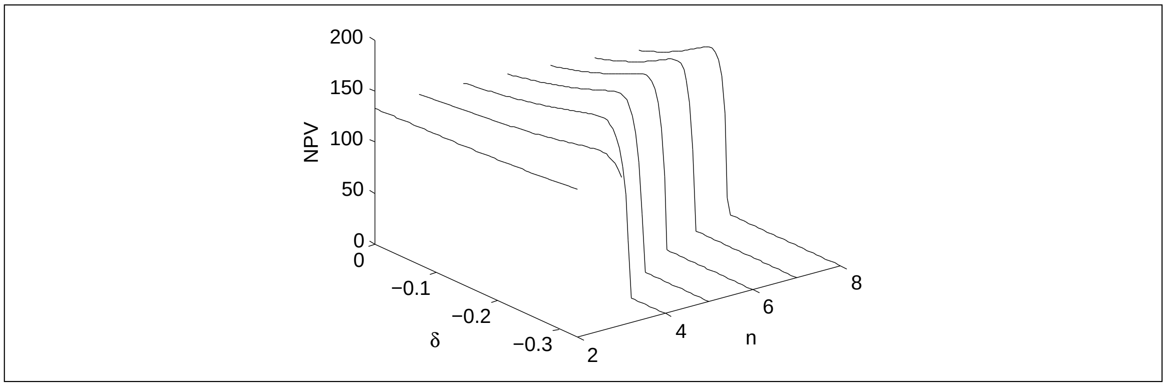

Figure 5.1. Chain's optimal NPV for different number of interventions and linear variations with $\theta = 0.1$ and $T_m = 30$.

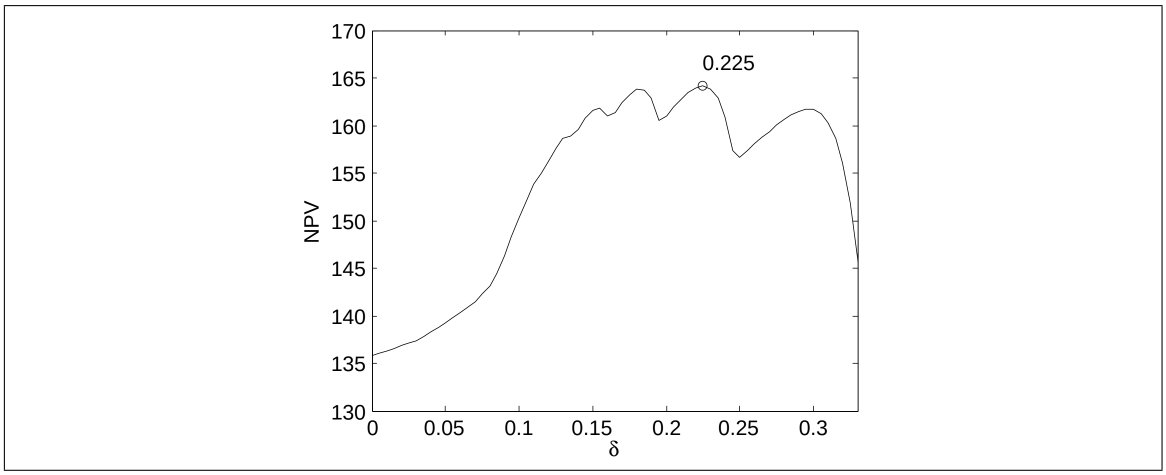

Figure 5.2. Chain's optimal NPV for different linear variations.

6. CONCLUSIONS AND FUTURE RESEARCH

This paper introduces a model that defines contractual terms in a preventive maintenance contract to coordinate the supply chain. This is achieved by setting a strategy for the client and its vendor that maximizes the total expected profit. A cost subsidization mechanism is proposed in order to align both interests and prevent that each one maximizes its own profit. In particular, the method finds the optimal interval between PMs for the client, vendor and service chain, respectively, along with the optimal cost subsidy.

The proposed model generalizes previous work because it considers finite horizon service contracts and the effect of the return on investment analysis. We show how these conditions may affect decision making for both parties and may increase maintenance deferrals. We also consider the alternative of maintenance contracts with non-constant time intervals between interventions.

For the finite-horizon contracts with constant maintenance intervals, we found that the optimal interval strongly depends on the discount rate, so it is very sensitive to the volatility and financial risks of the market, as well as the coordination subsidy. Also, we show that a short-contract can be defined according to the duration of the contract where the NPV reaches a stable zone. For longer contracts, the profit of both parties is not very sensitive, so the coordination analysis may be studied using an infinite horizon approach. Conversely, short-term contracts require another kind of analysis, and in some cases preventive maintenance may be not even necessary. For the finite-horizon contracts with non-constant maintenance intervals we found that the joint NPV can be considerably improved if the equipment is repaired more and more often as time passes. This suggests that the client and the vendor should also consider in the contract terms the alternative of performing non–periodic maintenances during the contract horizon for mutual benefit.

We demonstrate that the model maximizes the overall profit and achieves win–win coordination of the supply chain through the proposed cost subsidization coordination mechanism. In every case studied, coordination between the client and its vendor results in a Nash Equilibrium from which none of the parties has incentives to deviate. These serves as an encouragement for the client and the vendor to continually improve the policies of their maintenance contracts, as profits increase relative to those obtained when no coordination occurs. It also helps both to maintain a lasting and sustainable relationship over time.

The present model may be expanded considering contract negotiations with asymmetric information. When one of the parties owns some of the information it is common not to share it to take advantage of the situation. Also future research may be done to explore other coordination mechanisms, such as uptime bonus or combined strategies, and how these mechanisms may be affected in non-profit centered organizations. Finally, the model may be expanded by considering imperfect maintenance or considering other types of contracts in order to achieve a full realistic implementation.

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APPENDIX

A. PROOFS

Proof of Lemma 1. Without the loss of generality, we prove that T_c^* maximizes the NPV of the vendor. The first derivative of Π_c with respect to T is:

$$\frac{\partial \Pi_c(T)}{\partial T} = \left(e^{\frac{\theta T_m}{(n+1)}} - e^{\frac{n\theta T_m}{(n+1)}}\right) \frac{C_{ir}(\beta\theta^{1-\beta}\Gamma_{inc}(\beta, \theta T) - \theta^{1-\beta}\Gamma(\beta+1)}{n^2\eta^\beta T\left(1 - e^{\frac{\theta T_m}{(n+1)}}\right)}$$
$$+ \left(e^{\frac{\theta T_m}{(n+1)}} - e^{\frac{n\theta T_m}{(n+1)}}\right) \frac{\beta T^{\beta-1}(e^{\theta T_p} - e^{-\theta T}) - C_{ip}\theta T}{n^2\eta^\beta T\left(1 - e^{\frac{\theta T_m}{(n+1)}}\right)}$$

Taking $\frac{\partial \Pi_c(T)}{\partial T} = 0$, the condition for optimality can be expressed as:

$$C_{ir}(\beta\theta^{1-\beta}\Gamma_{inc}(\beta, \theta T) - \theta^{1-\beta}\Gamma(\beta+1) + \beta T^{\beta-1}(e^{\theta T_p} - e^{-\theta T})) - C_{ip}\theta T = 0$$

or

$$C_{ir} g_\theta(T) = \theta C_{ip}$$

which is the same as Equation (5). As $\theta > 0$, $g_\theta(T)$ is positive and increases in T. It is straightforward to verify that for $\beta \geq 2$, the second derivative of $\Pi_c(T)$ with respect to T is always negative. Therefore, T_c^* must be the global solution. Parts (i) and (iii) are analogous for the client and the chain to obtain T_m^* and T^* respectively.

Proof of Lemma 2. As $g_\theta(T)$ is an increasing function of T, it can be also verified that $g_\theta(T)$ increases for larger values of β or η. Hence, if the equipment's reliability parameters become larger, the optimal PM intervals T_m^*, T_c^*, and T^* need to be reduced for Equations (4), (5), and (6) to hold. Dividing Equation (4) by $(e^{\theta T_p} - 1)$, an increasing function of T_p for $\theta > 0$, we obtain

$$\frac{\beta\theta^{1-\beta}\Gamma_{inc}(\beta, \theta T) - \theta^{1-\beta}\Gamma(\beta+1)}{\eta^\beta(e^{\theta T_p} - 1)} + \frac{\beta T^{\beta-1}}{\eta^\beta}\frac{(e^{\theta T_p} - e^{-\theta T})}{(e^{\theta T_p} - 1)} = \frac{1}{T_r}$$

If T_p becomes larger, T has to increase in order for the above equation to hold, since $e^{-\theta T} \leq 1$ for $\theta > 0$. Therefore, T_m increases in T_p. On the contrary, since $g_\theta(T)$ increases for larger values of T_p and the vendor's optimal solution does not depend on T_p, if T_p

becomes larger T needs to be reduced for Equation (5) to hold. Hence, T_c decreases in T_p.

Proof of Lemma 3. The relationships between T_m^*, T_c^*, and T^* simply follow from comparing the right-hand side values of Equations (4), (5), and (6).

Proof of Lemma 4. These results can be obtained directly by calculating the client's and the vendor's NPVs, respectively, as $\lim_{\theta\to 0}$.

Proof of Lemma 5. With the subsidization ΔC_{ip}, the vendor's total expected profit is given by $p - \frac{C_i(T^*)}{T^*+T_p} + \frac{\Delta C_{ip}}{T^*+T_p}$ and should be greater or equal to π. Solving for p, the lower bound p_1 is obtained. On the other hand, we know that the client's total expected profit is given by $c_f A(T^*) - p - \frac{\Delta C_{ip}}{T^*+T_p}$ and should be greater or equal to the client's profit $(c_f A(T_c^*) - C_i(T_c^*)) - \pi$. Solving for p, the upper bound p_2 is obtained. Finally, with simple algebra it is easy to show that $p_2 - p_1 \geq 0$.

Proof of Lemma 6. The proof is similar to that of Lemma 5.

Proof of Lemma 7. These results can be obtained directly by calculating the respective $\lim_{\theta\to 0}$ of Equations (7) and (9).