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Determination of the maximum global quantum discord via measurements of excitations in a cavity QED network

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Abstract

Multipartite quantum correlation is one of the most relevant indicators of the quantumness of a system in many body systems. This remarkable feature is in general difficult to characterize and the known definitions are hard to measure. Besides the efforts dedicated to solve this problem, the question of which is the best approach remains open. In this paper, we study the global quantum discord (GQD) as a bipartite and multipartite measure. We also check the limits of this definition and present an experimental scheme to determine the maximum of the GQD via the measurements of the system's excitations, during the time evolution of the present system.

Keywords: global, discord, measurement, cavity

(Some figures may appear in colour only in the online journal)

1. Introduction

Quantum correlations have been a hot topic over the last few years due to their powerful applications in quantum information and computational tasks [1, 2]. For bipartite states, different measures such as entanglement (E) [3] and quantum discord (QD) [4–6] are already well understood. Although, sometimes for multipartite systems, there are correlations which are not detected by the previous measurements. We could in principle have different ways of quantifying multipartite quantum correlations. One of them could be by measuring bipartite and tripartite correlations at the same time or by defining a genuine measure of multipartite correlations, without the bipartite ones. The second approach has received wide attention, probably because of the intuitive generalization of the bipartite case.

Many attempts to extend bipartite correlations to the multipartite case have been made [7–13], but still questions remain about these generalizations. The first was the tangle approach [7], which is related with E, but it is difficult to compute for mixed states. Also in [8], a generalization of the previous Wootters formula [7] was presented, having the same problem with mixed states, being a good measure only for bound estimation. Recently, a method for the construction of multipartite E witnesses was introduced in [12]. However, we are more interested in multipartite correlations related with QD, since it seems to be more robust than the E against decoherence [14]. Among the different endeavours to find a good measure of the correlations, is global quantum discord (GQD) [13]—the most promising one—because it is a straight extension from the bipartite to the multipartite case, it is symmetric and obeys monogamy properties. These unique advantages suggest that GQD is a resource for quantum information processing. More recently, much attention has been paid to the application of GQD and its connection with criticality [13, 15], as the detection of phase transitions [16, 17]. Nevertheless, some questions are open, for example: is it possible to measure GQD experimentally, or know when it reaches its maximum value? To answer this question, we first will study the distribution of excitations in the system, and see how this distribution can affect GQD. We also propose a model, which is a cavity QED system, where GQD has not been studied yet.

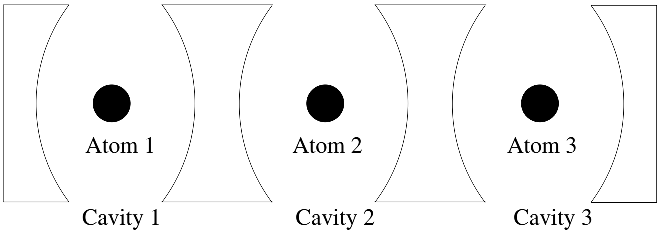

Figure 1. Three coupled cavity–atom systems.

Over the last few decades, cavity QED systems have been extensively researched, and several advantages, theoretical and experimental, are known about these systems [18–20]. The development of experimental techniques for their manipulation with an unprecedented level of control, as well as performing measurements inside the cavity are desirable features when choosing our model.

This paper is organized as follows: in section 2, we describe our system, the Hamiltonian and write a generalized master equation, where the Lindblad terms result from the coupling of each cavity to its own thermal reservoir at zero temperature. In section 3, we give a brief outline of GQD. In section 4, we present the main results of this paper, related to the applicability of GQD and we discuss our ability to gain, experimentally, information about this magnitude. Finally, section 5 is devoted to the conclusions.

2. The model

We have three coupled cavities, as shown in figure 1, where each cavity interacts with a single atom and its own reservoir. We choose Rydberg atoms with principal quantum numbers 51 and 50, where the transition is at 51, 1 GHz. The atom cavity strength coupling (g), corresponds to an interaction time of 1 μs. The photon lifetime inside the cavity is $T_{\text{cav}} = 1$ ms [21, 22]. The coupling between the cavities (J) is about $10^{-2}g$. We scale the time in the figures with $\gamma = 10^5$ Hz.

The Hamiltonian of the system, in the basis of the dressed states (polaritonic) [23], is given by

$$H = \sum_{i=1}^{3}(\omega_i - g_i)|E\rangle_i\langle E| + + \sum_{i=1}^{2}\frac{J_i}{2}\left(L_i^{\dagger}L_{i+1} + L_i^{-}L_{i+1}^{\dagger}\right) \quad (1)$$

where $|E_i\rangle = \frac{1}{\sqrt{2}}(|1, g\rangle_i - |0, e\rangle_i)$ and $|G_i\rangle = |0, g\rangle_i$ are the dressed states, corresponding to excited and ground states, respectively. The other operators $L_i^{\dagger} = |E_i\rangle\langle G_i|$ and $L_i^{-} = |G_i\rangle\langle E_i|$ are to create or destroy those states. So we can consider polaritons as two-level systems. We can just have one photon, at most, because due to photon blockade, double or higher occupancy of the polaritonic states is prohibited [24, 25].

The main source of dissipation originates from the leakage of the cavity photons due to imperfect reflectivity of the cavity mirrors. A second source of dissipation, corresponding to atomic spontaneous emission, will be neglected assuming long atomic lifetimes.

An approach to modelling the above mentioned losses, in the presence of a single mode quantized cavity field, is using the microscopic master equation, which goes back to the ideas of Davies on how to describe the system–reservoir interactions in a Markovian master equation [26]. For a three-cavity system at zero temperature, the master equation is [14, 27]

$$\dot{\rho}(t) = -\mathrm{i}\,[H, \rho(t)] + \sum_{n=1}^{3}\sum_{\omega>0}^{\infty}\gamma_n(\omega) \times \left(A_n(\omega)\rho(t)A_n^{\dagger}(\omega) - \frac{1}{2}\left\{A_n^{\dagger}(\omega)A_n(\omega), \rho(t)\right\}\right) \quad (2)$$

where A_n correspond to the Davies operators. The n sum is over all the dissipation channels and the decay rates $\gamma_n(\omega)$ are the Fourier transform of the correlation functions of the environment [28].

The A_n operators are calculated as follows:

$$A_n(\omega_{\alpha\beta}) = |\phi\rangle_\alpha\langle\phi|a_n|\phi\rangle_\beta\langle\phi| \quad (3)$$

where $|\phi\rangle_\alpha$ are the eigenstates of the Hamiltonian, with λ_α their eigenvalues, and $\omega_{\alpha\beta} = \lambda_\beta - \lambda_\alpha$. The operator a_n is the destruction operator acting on the cavity mode. As we set the temperature to be zero, $\gamma_n(\omega)$ does not change significantly with the frequency and thus we assume it to be a constant (see [20] for thermal effects in equation (3)).

3. Global quantum discord

In the original proposal [4], QD was defined as a mismatch between quantum analogues of classically equivalent expressions of the mutual information

$$QD(\rho_{AB}) = I(\rho_{AB}) - J(\rho_{AB}). \quad (4)$$

The mutual information $I(\rho_{AB})$ of two subsystem can be expressed as

$$I(\rho_{AB}) = S(\rho_A) - S(\rho_A|\rho_B), \quad (5)$$

where $S(\rho) = -\mathrm{Tr}(\rho\log_2\rho)$ is the von Neumann entropy, and $S(\rho_A|\rho_B) = S(\rho_{AB}) - S(\rho_B)$.

The classical correlation $J(\rho_{AB})$ is defined as the maximum information that one can obtain from A by performing a measurement on B, and in general this definition is not symmetric:

$$J(\rho_{AB}) = \max_{\{\Pi_B^k\}}[S(\rho_A) - S(\rho_{AB}|\{\Pi_B^k\})], \quad (6)$$

where $\{\Pi_B^k\}$ is a complete set of projectors performed on subsystem B and $S(\rho_{AB}|\{\Pi_B^k\}) = \sum_k p_k S(\rho_A^k)$. The reduced density operator ρ^k associated with the measurement result k is

$$\rho^k = \frac{1}{p_k}(I\otimes\Pi_B^k)\rho(I\otimes\Pi_B^k) \quad (7)$$

with I the identity operator.

Notice that $I(\rho_{AB})$ can be rewritten in terms of the relative entropy, $S(\rho\|\sigma) = \mathrm{Tr}(\rho\log_2\rho - \rho\log_2\sigma)$, as

$$I(\rho_{AB}) = S(\rho_{AB}\|\rho_A\otimes\rho_B). \quad (8)$$

Also, by symmetrizing the definition through the introduction of bilateral measurements, and after some algebra we get a new definition of QD, given by

$$\mathrm{GQD}(\rho_{AB}) = \min_{\{\Pi_A^j\otimes\Pi_B^k\}}[S(\rho_{AB}\|\Phi_{AB}(\rho_{AB})) - S(\rho_A\|\Phi_A(\rho_A)) - S(\rho_B\|\Phi_B(\rho_B))] \quad (9)$$

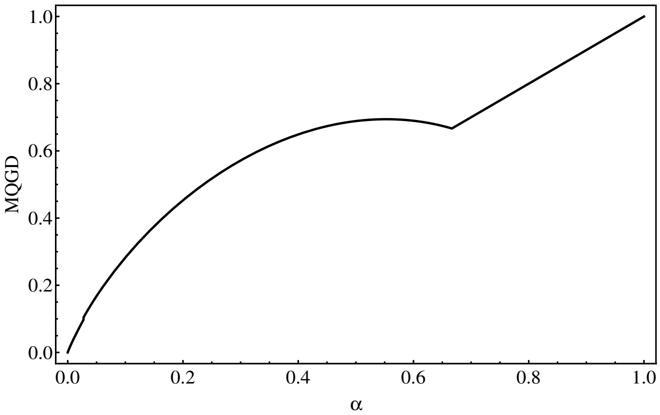

Figure 2. Genuine tripartite GQD increases from zero to one when the initial state goes from the Bell to GHZ state.

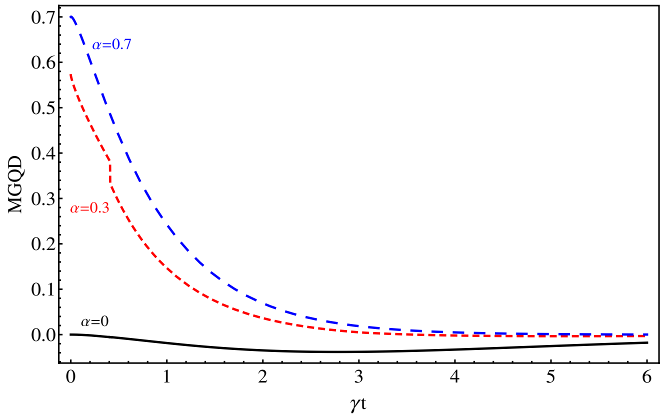

Figure 3. For $\alpha = 0$ the MGQD becomes negative, indicating that the GQD does not include separately bipartite and tripartite correlations. $T_{\text{cav}} = 10\ \mu$s.

with $\Phi(\rho_{AB}) = \sum_{j,k}(\Pi_A^j \otimes \Pi_B^k)\rho_{AB}(\Pi_A^j \otimes \Pi_B^k)$. From equation (9) the generalization to multipartite discord is evident,

$$\mathrm{GQD}(\rho_{A_1\ldots A_N}) = \min_{\{\Pi^k\}}\left[S(\rho_{A_1\ldots A_N}\|\Phi(\rho_{A_1\ldots A_N})) - \sum_{j=1}^{N} S(\rho_{A_j}\|\Phi(\rho_{A_j}))\right] \tag{10}$$

where $\Phi(\rho_{A_j}) = \sum_k \Pi_{A_j}^k \rho_{A_j} \Pi_{A_j}^k$ and $\Phi(\rho_{A_1\ldots A_N}) = \sum_k \Pi^k \rho_{A_1\ldots A_N} \Pi^k$, with $\Pi^k = \Pi_{A_1}^{k_1} \otimes \ldots \otimes \Pi_{A_N}^{k_N}$ and k denoting the index string $(j_1 \ldots j_N)$.

In order to define the local measurements $\Pi^{\pm} = |\pm\rangle\langle\pm|$, we considered rotations in the directions of the basis vectors of the subsystem j,

$$\begin{aligned} |+\rangle_j &= \cos\left(\frac{\theta_j}{2}\right)|E\rangle_j + \mathrm{e}^{\mathrm{i}\varphi_j}\sin\left(\frac{\theta_j}{2}\right)|G\rangle_j \\ |-\rangle_j &= -\,\mathrm{e}^{-\mathrm{i}\varphi_j}\sin\left(\frac{\theta_j}{2}\right)|E\rangle_j + \cos\left(\frac{\theta_j}{2}\right)|G\rangle_j. \end{aligned} \tag{11}$$

4. Results

4.1. Genuine tripartite measure

It has been shown that GQD is a multipartite measurement [13, 15], that not only measures tripartite quantum correlations, as the tangle defined by Wootters [7], but also bipartite correlations. This statement can be illustrated with the following example. If we prepare our system initially in a mixture of a genuine tripartite correlated state (GHZ) and a bipartite Bell state,

$$\rho(0) = \frac{\alpha}{2}(|EEE\rangle\langle EEE| + |GGG\rangle\langle GGG|) + \frac{(1-\alpha)}{2}(|\Psi\rangle\langle\Psi| \otimes |G\rangle_2\langle G|) \tag{12}$$

with $|\Psi\rangle = (|E_1G_3\rangle + |G_1E_3\rangle)$, as α increases from zero to one, the system goes from bipartite to tripartite correlations, but $\mathrm{GQD} = 1$ for all α. The question is, what happens when we eliminate all the bipartite QD? In figure 2 we plot the function $\mathrm{MGQD} = \mathrm{GQD}_{123} - \mathrm{GQD}_{12} - \mathrm{GQD}_{13} - \mathrm{GQD}_{23}$, for the same initial state in equation (12). Notice that for $\alpha = 0$ there is no multipartite correlation and for $\alpha = 1$ the MGQD is one, as expected from a GHZ state. Near to $\alpha = 0.7$ the function has a point where the derivative does not exist, this is because of the change in the angles during the numerical minimization.

At this point, it seems that there is no problem with the new definition of genuine multipartite correlation. However, when we analyse the time evolution of the state (12) using the master equation (2) for MGQD, particularly for $\alpha = 0$, the function becomes negative at certain times, see figure 3. We also tried the Werner state, obtaining similar results. This negative behaviour of MGQD is enhanced when the initial condition is near a pure bipartite correlated state.

A first approach to solve this problem can be the use of the monogamy restrictions [29, 30], where the exact solution is lost, but at least we can estimate a upper bound for the genuine tripartite correlations. From [29] and [30], we write two monogamy relations:

$$\mathrm{GQD}(\rho_{ABC}) \geqslant \mathrm{GQD}(\rho_{AB}) + \mathrm{GQD}(\rho_{AC}) \tag{13}$$

$$\mathrm{GQD}(\rho_{ABC}) \geqslant \mathrm{GQD}(\rho_{AB}) + \mathrm{GQD}(\rho_{BC}). \tag{14}$$

The authors of these two papers define a ‘residual GQD’ (D_R) as the difference between the left-hand and right-hand side of the above equations. The problem with the definition of D_R is that is non symmetric with respect to the pairwise combinations. Instead, we define a new D_R, based on the above equations, getting

$$\mathrm{GQD}(\rho_{ABC}) \geqslant \tfrac{2}{3}(\mathrm{GQD}(\rho_{AB}) + \mathrm{GQD}(\rho_{BC}) + \mathrm{GQD}(\rho_{AC})). \tag{15}$$

In figure 4 we reported the comparison between D_{R1}, D_{R2} and D_{R3} from equations (13)–(15), respectively. Already from the initial state there are differences among the three curves. Notice that the residual global discord corresponding to equation (13) (red-dotted), seems to be the most restrictive one. Nevertheless, that can be easily changed by starting with a bipartite correlation of cavities 2 and 3, instead cavities 1 and 3, which will change D_{R2} to be the most restrictive one. But, our approach remains very well independent of the initial

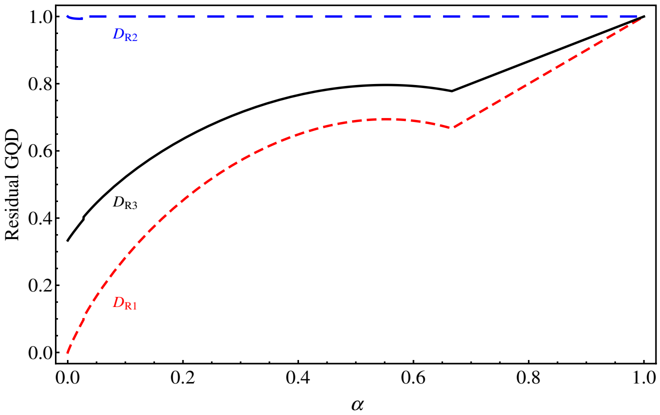

Figure 4. Residual GQD corresponding to our definition (D_{R3}) represents better the monogamy restriction, since it is a good approximation independent of the initial condition.

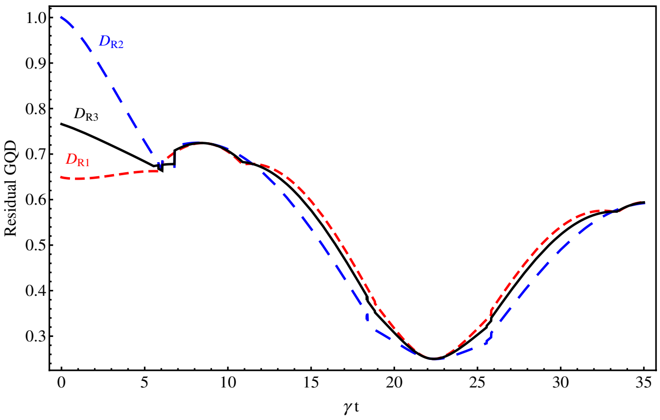

Figure 5. All definitions are close, during the time evolution of the system. We observe that D_{R3} remains between the other two, again showing more stability to variations of the initial conditions.

condition, as it includes all possible combinations of pairwise correlations.

Next, we analysed the time evolution of the above definitions for $\alpha = 0.4$. In figure 5 we show that certainly D_{R1} and our definition D_{R3} are close. However, D_{R1} is highly sensitive to the initial conditions, which is not the case for D_{R3}, so we conclude that D_{R3} is more suitable to describe the quantum correlations, for any initial condition. Notice that during the time evolution, far from the initial state, the behaviour of the three curves is quite similar. The reason is that for intermediate times, as the system gets more mixed, the correlations are more distributed among the subsystems. These bipartite correlations follow their own monogamy restrictions [31, 32], thus compensating for the differences between the various definitions of D_R.

4.2. Estimation of the GQD by means of the excitation probabilities of the subsystems

Quantum correlation measurements are very important for quantum information and quantum computation, and even now are difficult to perform [33], especially for higher correlations such as the tripartite one. However, there is a connection

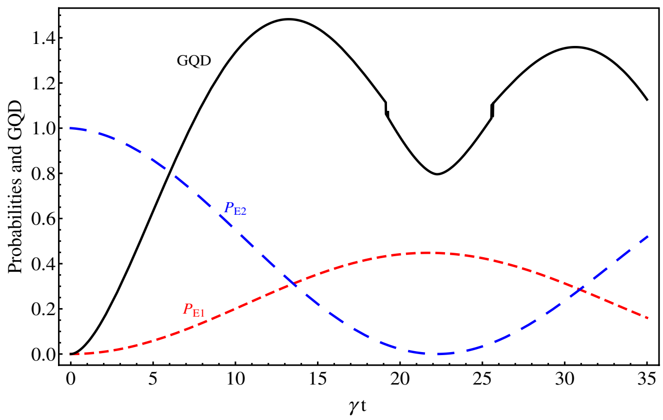

Figure 6. GQD reaches its maximum when the three probabilities cross at a certain time. Due to symmetry, $P_{E3} = P_{E1}$.

between the localization of the excitations throughout the system and the quantum correlations of its parts. To illustrate this, we first consider a typical bipartite Bell state $|\phi\rangle = \frac{1}{\sqrt{2}}(|10\rangle + |01\rangle)$. It is well known that this state is maximally correlated, but we also notice that the probability of finding an excitation in each subsystems is $1/2$. In other words, we could say in this example, that when the subsystems are highly correlated, the excitations are equally distributed through them.

In our system, things are more complicated, since we have three cavities and we could have up to three excitations. Nevertheless, the same rule applies. For example, let us assume that initially we have one excitation in cavity 2, and let P_{E1}, P_{E2} and P_{E3} be the probabilities of finding the polariton in cavities 1, 2 and 3, respectively. In figure 6, we plot the time evolution of GQD and these three probabilities. We can readily see that when the three probabilities cross at a certain time, the GQD reaches its maximum value, as in the case of two qubits. Thus we believe that the GQD is associated with disorder or equal distribution of the excitations among the three cavities. Since the coincidence of the probabilities of finding an excitation is not a proof per se, of the presence of quantum correlations (one could find examples of fully mixed uncorrelated states with the same characteristics), it is quite clear that only a combination of the present observation and the full tomography will be the final proof of the presence of these correlations.

Similar results are also observed for the state in equation (12), see figure 7. Here we show the matrix elements of the density operator for $\alpha = 0.1$ and $\alpha = 0.5$. We used the standard basis: $|1\rangle = |EEE\rangle$, $|2\rangle = |EEG\rangle$, $|3\rangle = |EGE\rangle$, $|4\rangle = |GEE\rangle$, $|5\rangle = |EGG\rangle$, $|6\rangle = |GEG\rangle$, $|7\rangle = |GGE\rangle$, $|8\rangle = |GGG\rangle$. Each graphic corresponds to the maximum of the GQD. Notice that again the three probabilities, associated to the $|5\rangle\langle 5|$, $|6\rangle\langle 6|$ and $|7\rangle\langle 7|$ matrix elements, are equal. It is worthwhile noticing that for the long time evolution these elements, as well as the correlations tend to disappear because of the losses.

As we saw, one of the advantages of GQD is that for any mixed initial bipartite and tripartite state, with only one measurement we can estimate how correlated the subsystems are. Then, one could experimentally detect when the maximum

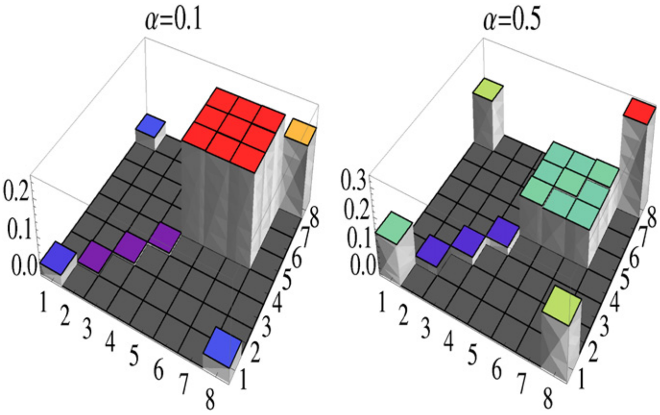

Figure 7. Density operator's elements for the initial state in equation (12). The maximum of GQD is reached when $P_{E1} = P_{E2} = P_{E3}$ and the off-diagonal components do not vanish.

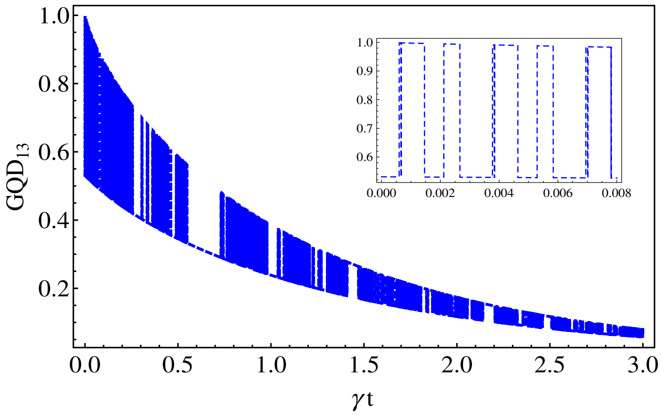

Figure 8. Sudden changes in the bipartite GQD for the cavities 1, 3. $T_{cav} = 10\ \mu$s.

GQD is reached by measuring the polaritons in the cavities [34]. To summarize, the GQD can provide us with valuable information about any class of multipartite correlations and furthermore, this can be experimentally observed by measuring the excitations of our system.

5. Summary and conclusions

We analysed the global quantum discord, as a measure of the joint bipartite and tripartite correlations. We showed its limitations in detecting a genuine tripartite correlation, since negative values show up. However, we presented an upper bound which turned out to be a good estimation, valid for any initial condition. Then we studied the relation between the disorder of the system and the GQD. Our goal was to associate the GQD with some experimentally measurable quantity, such as the degree of excitation of each sub-system. We found that when excitations were nearly equally distributed, among the various sub-systems, the GQD reached its maximal value.

Moreover, the sensitivity of this measure, which is certainly related with the bilateral projection and the minimization process, seems very interesting for its different applications. In order to illustrate this feature, we focus on the sudden transition effect [35, 36]. This effect depends strongly on the initial conditions, and it can be seen only when some restrictions are fulfilled. For example, we start with the initial state proposed in [35] for the cavities 1 and 3, and assume for the second cavity to be in an excited or ground state

$$\rho(0) = \begin{pmatrix} (1+c_3) & 0 & 0 & (c_1-c_2) \\ 0 & (1-c_3) & (c_1+c_2) & 0 \\ 0 & (c_1+c_2) & (1-c_3) & 0 \\ (c_1-c_2) & 0 & 0 & (1+c_3) \end{pmatrix} \otimes |i\rangle_2\langle i| \qquad (16)$$

where the matrix is in the basis: $|EE\rangle$, $|EG\rangle$, $|GE\rangle$, $|GG\rangle$, and $|i\rangle = \{|E\rangle, |G\rangle\}$.

In figure 8 we plotted the quantum discord, defined in equation (9) between cavities 1 and 3, when cavity 2 is initially in the state $|G\rangle$, and weakly coupled to the other two cavities. The parameters are: $c_1 = 1$, $c_2 = -c_3$ and $c_3 = 0.8$. The inset corresponds to a zoom at the beginning of the curve. We observe rapid oscillations that have not been reported before, for this particular measure, and also abrupt changes in the derivative, which is quite unusual. The change of different sets of angles in the minimization procedure is responsible for the sudden changes in the derivatives of the GQD (inset of figure 8). With respect to the rapid oscillations, we chose a set of angles corresponding to the lower branch, eliminating the minimization procedure, and observed that the oscillations subsided but without the sudden changes in the derivatives. On the other hand, if we took the upper branch, we observed no oscillations. We do not yet have a full understanding of such an odd behaviour. We did the same for the QD defined in equation (4), following two different approaches [37, 38], and we did not find such effects in either case. This proves that GQD, proposed in [13], is more sensitive than the others.

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