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ESSAYS IN INFORMATION ACQUISITION AND CAMPAIGN FINANCE

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To my mother, my sister and my brother, who have always shared their unconditional love and care. This journey would not have been the same if it wasn't for your support. It is true that one does not choose one's family, but I was lucky enough to be born in the best.

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Chapter I

Optimal Information Disclosure with Rationally Inattentive Receivers

Optimal Information Disclosure with Rationally Inattentive Receivers

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Abstract

A privately informed receiver is being persuaded by a sender to take a given course of action. After the persuasion stage, the receiver may decide how much information to purchase from a private information technology. With this information at hand, the receiver must then choose between following the sender's recommended action or not. In this context, I characterize the receiver's information acquisition strategy and the sender's disclosure policy. I show that when the receiver is rationally inattentive, the sender will completely crowd out information acquisition. However, information acquisiton may arise in equilibrium when the receiver's information structure is restricted.

1 Introduction

An uninformed decision maker, the receiver, must choose between two mutually exclusive actions. Before choosing his action, he can acquire information from two different sources in a predetermined order. First, he receives information from an interested party, the sender. The sender's and the receiver's interests are not aligned. The receiver may also acquire information from a private but costly information technology. After the acquisition of information is concluded, the receiver decides what action to follow.

The sender's utility is dependent only on the receiver's decision, and she will try to persuade him to take her most preferred action. We will assume commitment power on the sender's behalf, meaning that she can commit to a communication strategy before being approached by the receiver. The receiver, however, has a state dependent utility function. Without loss of generality, we will assume that the state space consists of two elements, each one yielding a higher utility to a different element in the action space. Preference misalignment implies that the receiver can be doubtful of the information disclosed, given him incentives to acquire information on his own.

There are numerous contexts that fit this model's description. Consider, for example, a setting where a lobbyist group is trying to persuade a politician to take a particular action. A well informed politician will be aware that the disclosed information is intended to influence his decisions and may, therefore, doubt it. Even when the lobbyist's claims are verifiable, the politician is only aware of the information that has been disclosed to him. He knows, however, that there might be information that the lobbyist has strategically chosen not to disclose. For example, two separate studies might have found contradictory implications of increasing public funding in medical care. If the lobbyist group supports state-financed treatment, they will only disclose the study which presents positive evidence in favor of public-funded medical care.

On this understanding, the politician may want to acquire more information by himself. There are several technologies from which the politician might learn. For example, he could hire an independent researcher to investigate and shed further light on the issue, or he could browse the web himself. His willingness to pay for information will depend on how convincing was the information disclosed by lobbyists. The more convinced he is, the less he'll be willing to pay for information. In that sense, the disclosure policy chosen by the lobbyist group can influence the flow of information between the politician and his private information technology.

The goal of this paper is to study the implications of the accessibility to a private technology on the sender's optimal disclosure policy. My main findings concern with the benefits acquired by a privately informed agent, in comparison to one without access to private information. Interesting enough is that, if no restrictions are imposed on the information technology, the value of information is not derived from its usage, but solely from its existence. When the receiver is capable of learning by his own means, the probability of finding that the disclosed information wasn't accurate is a sufficient threat to the sender. She will, then, find it optimal to increase the amount of information disclosed until the receiver decides not to resort to the private technology.

When the receiver has restricted access to information technologies, equilibria with positive amounts of information acquisition may arise. Moreover, the amount of knowledge acquired depends on the set of experiments he may choose from. This suggests that special attention must be regarded to the way in which information acquisition is modeled, for our results are sensitive to these assumptions.

2 Related Literature

A growing literature has studied the optimal disclosure of information between agents with asymmetric information (Kamenica and Gentzkow (2011); Rayo and Segal (2010); Ostrovsky and Schwarz (2010)). Most of this work, however, model situations where the decision maker is uninformed. If the decision maker's utility is state-dependent, it is natural to assume that his desire of making the right decision can induce further information acquisition.

Extensions suggested in Kamenica and Gentzkow (2011) refer to cases where the receiver might be informed. In their setting, the receiver becomes informed after the observation of an exogenous signal. They show that the results of their model are robust to these specifications. Specifically, they argue that the sender's utility function is still characterized by the concave closure of her payoff function.

The value of information in persuasion games has been studied by Guo and Shmaya (2019), Kolotilin (2018), Kolotilin et al. (2017), and Gentzkow and Kamenica (2017). In the first three, private information is held by the receiver. Contrary to the models presented in the following sections, the access to this information is exogenous and free. In Section 5.2, we solve a model with an information technology similar to that in Kolotilin (2018). This technology is characterized by the precision of its signal. We will assume that the decision maker has the ability to choose, at a cost, the precision of the signal.

Our work is also tightly related to the rational inattention literature. This approach finds the answer to rational mistakes not in the availability of information, but in the fact the individuals have a limited capacity of processing it. Inattentive decision-makers make their decisions based not on the total amount of information available, but on the information they choose to attend[1]. When there are monetary or cognitive costs associated with information gathering, agents will filter some information prior to their decision making and will, sometimes, make rational but mistaken decisions.

Agents that choose not to attend to a specific piece of information are said to be rationally inattentive. Beginning with Sims (2003), a growing literature has studied the consequences of this information processing behavior on economic outcomes in macroeconomics (Sims (2006), Acharya and Lin (2019)) and finance (Huang and Hong (2007), Mondria (2010)). Mostly related to the work presented in this paper is the work by Matejka and McKay (2015) who characterize the optimal information-processing behavior of a consumer who

[1](Prasad et al., 2013) delivers evidence of 146 commonly practiced medical procedures that offer no net benefit to patients.

must choose among a discrete set of actions. In this context, information acquisition results in probabilistic choices: agents will determine the probability of choosing a specific action in a way that resembles the logit formula.

The rest of this paper is organized as follows. In Section 4, we solve the model with the standard rational inattention assumptions. We assume the receiver has the ability to shape signals in any manner he chooses. In Section 5 we solve the model for two particular information technologies. According to the first one, Receiver observes a signal with probability ϵ, which he may increase at a cost. However, the signal's precision (p) is fixed. The second technology allows him to increase p at a cost, but he has no control over ϵ. It turns out that, under certain parametrizations of our model, this technologies allow for on-path information acquisition. Finally, in Section 6 we develop a model where the rceiver may acquire information from heterogenoeus experts. This framework also rationalizes on-path information acquisition.

3 The Model

Overview.– There are two players in this game, a sender (she) and a receiver (he). The receiver must choose between two mutually exclusive actions in the action space $A \equiv \{0, 1\}$. The state of nature is an element of the set $\Omega \equiv \{(0, b), (b, 0)\}$, where the first entry of each vector corresponds to the receiver's payoff if he chooses action $a = 0$ and the second entry is the payoff of choosing action $a = 1$. When the receiver approaches the seller, she perfectly observes the state of nature and chooses how much information to disclose based on this observation. After the reception of this information, he must decide between taking the recommended action or using a private information technology to acquire further knowledge about the state of nature.

Player's payoffs.– Let v_a denote the sender's payoff when the receiver takes action $a \in A$. We will normalize $v_0 = 0$ and let $a = 1$ be the action that yields the highest payoff, $v_a = 1$.

The receiver has a state dependent utility function. Let $u_a(\omega)$ denote his payoff when he takes the action a and the state of nature is ω. Finally, assume $b < 0$. As a result, if the receiver knew the true state of nature, his preferred action would be $a = 1$ whenever $\omega = (b, 0)$ and $a = 0$ otherwise.

Information.– The receiver is initially uncertain about the true state of nature ω which is relevant for his decision. Before approaching the sender, he has an initial probability distribution over the possible states of nature. Let $\boldsymbol{\mu_0} \in \Delta(\Omega)^2$ denote these prior beliefs, where $\boldsymbol{\mu_0} \equiv (\mu_0, 1-\mu_0)$. Before the two players are matched, the sender commits to a specific mechanism of information disclosure. She must choose a signal $\pi \in \Delta(R_1 \times \Omega)$, which consists of a finite realization space R_1 and a family of distributions $\{\pi(\cdot|\omega)\}_{\omega\in\Omega}$. After observing the signal realization $r_1 \in R_1$, the receiver updates his beliefs to $\boldsymbol{\mu_1}$ according to Bayes's law and decides wether or not to pay for additional information.

Information Costs.– After the persuasion stage, the receiver can acquire further knowledge about the state of nature with his private information technology. This technology will deliver a second signal $r_2 \in R_2$. An information strategy consists in choosing a joint distribution $F(r_2, \omega) \in \Delta(R_2 \times \Omega)$ such that the marginal distribution over states is consistent with the belief $\boldsymbol{\mu}_1$, this condition implies that he is only free to choose the conditional distribution $F(r_2|\omega)$, for $F(\omega|r_2)$ is given by Bayes's law[3].

Let $\boldsymbol{\mu_2} \equiv F(\cdot|r_2)$ denote the receiver's posterior beliefs after receiving signal r_2. Following the rational inattention literature, we will assume that the cost of any information strategy (F) will be proportional to its expected reduction in entropy:

$$\hat{C}(F) \equiv \lambda \left(H(\boldsymbol{\mu_1}) - \mathbb{E}_{r_2}\left[H\left(F(\cdot|r_2)\right)\right]\right) \tag{1}$$

where λ is a positive parameter that represents the unit cost of information and the function

[2] ΔX denotes the set of all probability distributions on X.

[3] Kamenica and Gentzkow (2011) show that for each of these distributions over posterior beliefs, there exists a signal structure that induces it. Given that the proof is constructive, it allows us to recover the exact signal structure, $\pi(s|\omega)$, that induces these beliefs.

H is the Entropy function.

Definition 1 (Entropy) *The entropy $H(\boldsymbol{\mu})$ of a given belief $\boldsymbol{\mu}$ is defined by*

$$H(\boldsymbol{\mu}) = -\left[\mu log(\mu) + (1-\mu)log(1-\mu)\right] \tag{2}$$

Solution Concept.– To solve our model we proceed by backward induction. We begin by solving the receiver's information acquisition problem after he has been persuaded by the sender. Being able to anticipate the receiver's behavior, the sender then determines her optimal persuasion policy. Proceeding in this way guarantees that the sender's and receiver's strategies, together, constitute a perfect bayesian equilibrium.

4 Rational Inattention

According to the rational inattention literature, the receiver may choose signals and beliefs in any manner he wishes to. We follow an argument similar to that in Matejka and McKay (2015) to simplify the oprimization problem. In this framework, the receiver may choose any distribution over posterior beliefs. Once these beliefs are set, he then chooses amongst any type of experiment that can induce these beliefs. Rationality requires him to choose the experiment that induces the chosen beliefs at a lower cost.

Under this circumstances, the receiver solves:

Remark 1 (Receiver's Problem)

Let $P_i(\omega)$ denote the conditional probability of choosing action i in state ω and ω_i denote the payoff of taking action i when the state is ω. A rationally inattentive receiver solves:

$$\begin{aligned} &\max_{P_i(\omega)} \sum_{i\in\{0,1\}} \sum_{\omega\in\Omega} \omega_i P_i(\omega)\boldsymbol{\mu_1}(\omega) - \tilde{C}(\mathcal{P}, \boldsymbol{\mu}_1) \\ &s.t{:}\ P_i(\omega) \geq 0 \ for\ all\ \omega, \quad \sum_{i\in\{0,1\}} P_i(\omega) = 1 \end{aligned} \tag{3}$$

$$Where\ \tilde{C}(\mathcal{P}, \boldsymbol{\mu_1}) = \lambda \left[-\sum_{i\in\{0,1\}} P_i^0 ln\left(P_i^0\right) + \sum_{\omega\in\Omega} \sum_{i\in\{0,1\}} \left[P_i(\omega) ln\left(P_i(\omega)\right)\right] \boldsymbol{\mu}_1(\omega) \right]$$

Proof. This is a standard result in the literature. ■

One of the advantages of working with this formulation of the problem is that the benefits of information acquisition are linear in the choice probabilities. Even when the cost function is not, linearity in benefits allows for tractability and analytic solutions for the choice variables. With this in mind, we may now proceed to find the receiver's information acquisition policy.

4.1 Information Acquisition

Theorem 1 (Receiver's optimal strategy)

Consider a rationally inattentive receiver with beliefs μ_1 who faces the optimization problem in (3). His optimal information strategy is characterized by two cutoff points, $\underline{\mu}$ and $\overline{\mu}$, such that information is bought only if $\underline{\mu} < \mu_1 < \overline{\mu}$.

Proof. See Appendix C ■

The receiver pays for additional information whenever his beliefs lie in the interval $\left(\underline{\mu}, \overline{\mu}\right)$, otherwise, he will inmediatley take the sender's recommended action.

The receiver will benefit from information aquisition whenever one signal realization induces him to change the decision he would otherwise had taken without the additionl knowledge. For example, if $\mu_1 > \frac{1}{2}$, he will be willing to pay for information only if the signal $s = \ell$ induces beliefs $\mu_2 < \frac{1}{2}$. Depending on how far μ_1 is from $\frac{1}{2}$, this may require buying a very costly experiment. Hence, for a given unit cost of information, the incentives to buy information vanish as μ_1 diverges from $\frac{1}{2}$. Claim 3 in Theorem 1's proof states that the measure of the information acquisition interval is decreasing in λ. When the unit cost of information increases, even small martingales become sufficiently expensive for them to be worth buying. The measure of the interval where the receiver actively seeks for information is, therefore, strictly decreasing in λ.

Claim 4 in the proof of Theorem 1 states that the conditional probability of taking action $i = 1$ is higher whenever the state of nature is the one in which this action yields the highest utility. If this wasn't the case, the receiver could increase his utility not by following but by taking a course of action that contradicts the recommendations obtained from the information technology.

The sender can perfectly anticipate the receiver's information acquisition policy when she decides how much information to disclose in the persuasion stage. The following section uses this observation to construct the disclosure policy that maximizes her expected utility.

4.2 Information Disclosure

The sender knows that whenever her message places the receiver's beliefs in the region $(\underline{\mu}, \overline{\mu})$ he will resort to his private information technology. Otherwise, he will follow her recommended action without further verification. Her payoff as a function of the receiver's beliefs is:

$$\Pi(\mu_1) = \begin{cases} 0 & \text{if } \mu_1 < \underline{\mu} \\ \frac{\mu_1 - e^{\frac{b}{\lambda}} \cdot (1-\mu_1)}{1-e^{\frac{b}{\lambda}}} & \text{if } \underline{\mu} \leq \mu_1 < \overline{\mu} \\ 1 & \text{if } \overline{\mu} \leq \mu_1 \leq 1 \end{cases} \tag{4}$$

The sender's payoff corresponds to the unconditional probability that the receiver takes action $i = 1$. It's worthwhile noting that for beliefs smaller than (but sufficiently close to) $\frac{1}{2}$, the acquisition of information increases her payoff when compared to the case where the receiver doesn't have this option. If we were to ignore the fact that she is susceptible to the receiver's verification of her messages, then she will receive no profits whenever $\mu_1 < \frac{1}{2}$. However, when the receiver is rationally inattentive, these beliefs imply that there is value in acquiring information; for a sufficiently small martingale may induce him to change his course of action. Hence, the sender's payoff increases from 0 to the probability of receiving signal $s = h$.

Nonetheless, this increase in utility comes with a cost. Using a similar argument for beliefs greater than (but sufficiently close to) $\frac{1}{2}$, information acquisition results in a reduction of the sender's payoff that is proportional to the probability of receiving the signal $s = \ell$.

Figure 1 illustrates this trade-off by plotting the sender's expected payoff with and without information acquisition.

Once again, when the sender is a monopolist, the maximum utility that she can attain is determined by the concave closure of the profit function in Figure 1. The strategy that allows

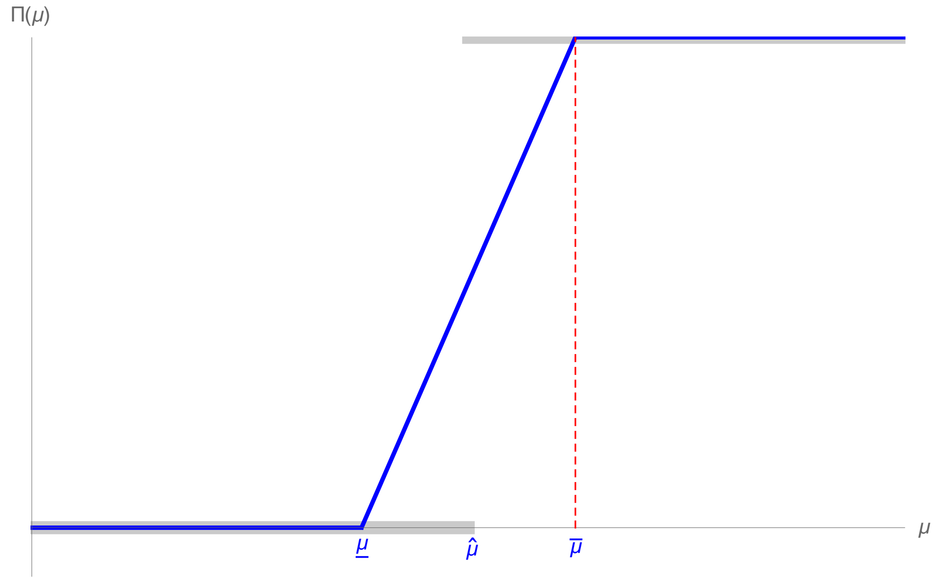


Figure 1: Sender's expected payoff function
(Gray corresponds to no information acquisition)

her to achieve this utility is well known: when the receiver approaches with beliefs $\mu_0 \in (0, \overline{\mu})$, the disclosure policy is such that with probability $\frac{\mu_0}{\overline{\mu}}$, the sender will communicate signal h, in which case the receiver's posterior beliefs will be $\mu_1 = \overline{\mu}$. With the remaining probability, the sender reveals signal ℓ. In this last case, bayesian updating implies $\mu_1 = 0$, in which case the receiver will be persuaded to take the action $a = 0$. The next proposition formalizes this argument.

Proposition 1 (Disclosure Policy)

Suppose that a sender is matched with a receiver with beliefs μ_0. The optimal disclosure policy satisfies:

1. *If $\mu_0 \in (0, \overline{\mu})$, the sender communicates $s = h$ with probability $\mu_0 \cdot \frac{1}{\overline{\mu}}$ and $s = \ell$ with the remaining probability.*

2. *If $\mu_0 \geq \overline{\mu}$, she discloses no information at all.*

Proposition 1 implies that we may write the monopolist's optimal strategy as follows:

$$\mu_1 = \begin{cases} \delta_{\{\mu_0\}} & \text{if } \mu_0 \geq \overline{\mu} \\ \left(1 - \frac{\mu_0}{\overline{\mu}}\right) \delta_{\{0\}} + \left(\frac{\mu_0}{\overline{\mu}}\right) \delta_{\{\overline{\mu}\}} & \text{if } 0 < \mu_0 < \overline{\mu} \end{cases} \tag{5}$$

Corollary 1

In equilibrium, the sender completely crowds out the receiver's information acquisition.

Proof. For any $\mu_0 \in [0, 1]$, her disclosure of information is such that his updated beliefs, μ_1, always lie outside the interval $(\underline{\mu}, \overline{\mu})$ According to receiver's information strategy, at this beliefs he will opt not to purchase additional information. ■

5 Restricted Information Technologies

In this section, we develop an information technology that depends on two parameters. When the receiver consults this technology, he receives a signal with probability $\epsilon \in [0, 1]$, with the remaining probability the technology fails and produces no signal. Conditional on receiving a signal, its precision is given by the parameter $p \in \left[\frac{1}{2}, 1\right]$. Conditional probabilities induced by this technology are depicted in the following table:

	$\omega = H$	$\omega = L$
$s = h$	$\epsilon \cdot p$	$\epsilon \cdot (1 - p)$
$s = \ell$	$\epsilon \cdot (1 - p)$	$\epsilon \cdot p$
$s = \emptyset$	$1 - \epsilon$	$1 - \epsilon$

Table 1: Restricted Technology's Signal Structure

In what follows, we solve our model by first assuming that the receiver has control over ϵ while p remains fixed. We then switch control from ϵ to p.

5.1 Choosing ϵ_i

Let the receiver have control over the probability with which he will receive a signal from the information technology. Let $\epsilon \in [0, 1]$ denote this probability and let $p \in \left[\frac{1}{2}, 1\right]$ be the (fixed) precision of such signal.

Given that he must decide between two mutually exclusive actions, it is without loss of generality to restrict attention to experiments that result in only one of two possible signals. This property comes as a consequence of our entropic cost assumption: the fact that two or more signals lead to the same action implies that information is being received but not acted upon. This simplification of the signal realization space implies that we can interpret each one of them as an *action recommendation*; e.g, when the signal realization induces an increase in the probability assigned to state H, we may interpret this as the recommendation *"take action $a = 1$"*. Furthermore, given that information is costly, in equilibrium we will require that action recommendations are followed.

Let $\mu_2(j)$ denote the receiver's updated belief after he receives signal $r_2 = j$. Then:

1. If $j = h$, which happens with probability $\mathbb{P}(h) = \mu_1 * p + (1 - p) * (1 - \mu_1)$:

$$\mu_2(h) = \frac{\mu_1 * p}{\mu_1 * p + (1 - p) * (1 - \mu_1)}$$

2. If $j = \ell$, which happens with probability $\mathbb{P}(\ell) = \mu_1 * (1-p) + p * (1-\mu_1)$:

$$\mu_2(\ell) = \frac{\mu_1 * (1-p)}{\mu_1 * (1-p) + p * (1-\mu_1)}$$

3. If no signal is received, the the buyer's beliefs are not updated:

$$\mu_2(\emptyset) = \mu_1$$

5.1.1 Utility Functions Without Information.

Consider the scenario where the receiver cannot acquire information on his own, nor can he be persuaded by the sender to follow one particular action. If he takes action $a = 0$, his expected payoff is $u(\mu|0) = \mu * b$. On the other hand, if $a = 1$, his expected payoff is given by $u(\mu|1) = b \cdot (1-\mu)$. These two lines intersect at $\hat{\mu} = \frac{1}{2}$ which corresponds to the prior beliefs where he is indifferent between both actions. His utility function is described by the following equation:

$$U_R(\mu) = \max\left\{b \cdot \mu, b \cdot (1-\mu)\right\} \tag{6}$$

Given that the receiver takes the action $a = 1$ if and only if $\mu > \hat{\mu}$, the sender's utility can be described as follows:

$$U_S(\mu) = \begin{cases} 0 & \text{if } \mu < 0.5 \\ 1 & \text{if } \mu \geq 0.5 \end{cases} \tag{7}$$

5.1.2 Information Acquisition

When he can purchase information, the receiver's expected utility is described by the following equation:

$$V_R(\mu_1) = \max_{0 \leq \epsilon \leq 1} \quad (1-\epsilon)U_R(\mu_1) + \epsilon \left[\mathbb{P}(t) * U_R(\mu_2(t)) + \mathbb{P}(h) * U_R(\mu_2(h)) \right] \dots$$
$$- \lambda \left[H(\boldsymbol{\mu_1}) - \mathbb{E}_{r_2}(H(F(\cdot|r_2))) \right] \tag{8}$$

Proposition 2 (Receiver's optimal strategy)

The problem described in Equation (8), and its solution, can be characterised as follows:

1. *The objective function is linear in the choice variable* ϵ.

2. *The optimal strategy consists of two cutoff points* $\underline{\epsilon}$ *and* $\overline{\epsilon}$ *such that* $\epsilon^*(\mu_1) = 1$ *whenever* $\mu_1 \in [\underline{\epsilon}, \overline{\epsilon}]$ *and* $\epsilon^*(\mu_1) = 0$ *otherwise.*

3. *If* $b > \frac{\lambda}{(1-2p)} \left[ln(4) + 2(1-p)ln(1-p) + 2pln(p) \right]$, *then* $\underline{\epsilon} = \overline{\epsilon}$ *and additional information will never be bought.*

4. $\underline{\epsilon}$ *(*$\overline{\epsilon}$*) is weakly increasing (decreasing) in the unit cost of information (*λ*).*

Proof. See Appendix A: Proofs ■

The receiver's information policy allows the sender to modify her disclosure of information in such a way that he will prefer not to recur to this private technology. That is, the sender may crowd out his information acquisition. As we will argue further on in this section, crowding out information is costly. However, under certain conditions (for example, when information is sufficiently expensive), the sender's disclosure policy will completely crowd out information acquisition. To prove this, we'll begin by finding her optimal disclosure policy.

5.1.3 Information Disclosure

Proposition 2 tells us that, if $\mu_1 < \underline{\epsilon}$ or $\mu_1 > \overline{\epsilon}$, no additional information is bought. In the former case, the receiver will choose action $a = 0$ (yielding no payoffs for the sender) and in the latter he will choose action $a = 1$ (yielding a payoff of 1). When $\mu_1 \in [\underline{\epsilon}, \overline{\epsilon}]$, he will take action $a = 1$ if and only if he receives signal t. The probability of this event is $\epsilon \cdot \mathbb{P}(t)$. Given that $\epsilon^* = 1$ in this interval, the probability with which action $a = 1$ is taken is given by $\mathbb{P}(h) = 2p\mu_1 + 1 - \mu_1 - p$. Taking this into account, we can write the sender's expected payoff function as:

$$\Pi(\mu_1) = \begin{cases} 0 & \text{if } 0 < \mu_1 < \underline{\epsilon} \\ [2p\mu_1 + 1 - \mu_1 - p] & \text{if } \mu_1 \in [\underline{\epsilon}, \overline{\epsilon}] \\ 1 & \text{if } \overline{\epsilon} < \mu_1 < 1 \end{cases} \tag{9}$$

Comparing equations (7) and (9) allows us to understand how does the receiver's private acquisition of information modifies the sender's disclosure policy. If the receiver pays for additional information, the sender's utility increases in the interval $(\underline{\epsilon}, \hat{\mu})$. This happens because, without information acquisition, he would have chosen action $a = 0$ with probability 1 in this interval. However, when he buys information, the reception of signal h will make him change his decision to action $a = 1$. In that case, the sender's expected payoff is given by $\mathbb{P}(h) > 0$. A similar interpretation allows us to conclude that, when compared to the no information scenario, her expected payoff decreases in the interval $(\hat{\mu}, \overline{\epsilon})$ because the probability with which the buyer takes action $a = 1$ decreases. Her task is to find a signal structure such that her expected payoff is maximized. Following Kamenica and Gentzkow (2011), her utility function corresponds to the concave closure[4] of $\Pi(\mu_1)$. Proposition 3 states that, depending on the values of the relevant parameters, her optimal disclosure policy might fully or partially crowd out information .

Proposition 3 (Disclosure Policy) *Let $f : [0, 1] \mapsto \mathbb{R}$ be the probability with which the*

[4]Let $f : \mu \to \mathbb{R}$ be a real function. The concave closure of f, C, is defined by:

$$C(\mu) \equiv \sup\{z | (\mu, z) \in co(f)\}$$

where $co(f)$ denotes the convex hull of the graph of f

receiver receives signal h when $\epsilon = 1$; that is $f(\mu) = 2p\mu + 1 - \mu - p$. Furthermore, let $m : [0,1] \mapsto \mathbb{R}$ be a function[5] *defined by $m(\mu) = \frac{f(\mu)}{\mu}$. Then, the sender's optimal disclosure policy satisfies:*

1. *If $m(\underline{\epsilon}) < (\overline{\epsilon})^{-1}$, the optimal disclosure policy is such that the induced posterior beliefs on the receiver will take one of two values: $\mu_1 \in \{0, \overline{\epsilon}\}$. If δ corresponds to Dirac's measure, the optimal policy is given by:*

$$\mu_1 = \begin{cases} \delta_{\{\mu_0\}} & \text{if } \mu_0 \geq \overline{\epsilon} \\ \left(1 - \frac{\mu_0}{\overline{\epsilon}}\right) \delta_{\{0\}} + \left(\frac{\mu_0}{\overline{\epsilon}}\right) \delta_{\{\overline{\epsilon}\}} & \text{if } 0 < \mu_0 < \overline{\epsilon} \end{cases}$$

2. *If $m(\underline{\epsilon}) > (\overline{\epsilon})^{-1}$, the optimal disclosure policy is such that the induced posterior beliefs will take one of three values: $\mu_1 \in \{0, \underline{\epsilon}, \overline{\epsilon}\}$. In this case, the optimal policy is given by:*

$$\mu_1 = \begin{cases} \delta_{\{\mu_0\}} & \text{if } \mu_0 \geq \overline{\epsilon} \\ \left(1 - \frac{f(\underline{\epsilon})(\overline{\epsilon}-\mu_0)+(\mu_0-\underline{\epsilon})}{\overline{\epsilon}-\underline{\epsilon}}\right) \delta_{\{\underline{\epsilon}\}} + \left(\frac{f(\underline{\epsilon})(\overline{\epsilon}-\mu_0)+(\mu_0-\underline{\epsilon})}{\overline{\epsilon}-\underline{\epsilon}}\right) \delta_{\{\overline{\epsilon}\}} & \text{if } \underline{\epsilon} < \mu_0 < \overline{\epsilon} \\ \left(1 - \frac{f(\underline{\epsilon})}{\underline{\epsilon}} \cdot \mu_0\right) \delta_{\{0\}} + \left(\frac{f(\underline{\epsilon})}{\underline{\epsilon}} \cdot \mu_0\right) \delta_{\{\underline{\epsilon}\}} & \text{if } 0 < \mu_0 < \underline{\epsilon} \end{cases}$$

Proof. See Appendix A: Proofs. ■

Corollary 2 *In equilibrium, the sender crowds out information acquisition only when the receiver has low incentives to acquire information. Otherwise, on-path information acquisition will arise.*

Proposition 3 tells us how much information will be transmitted in the optimal disclosure policy. The condition for partial crowding out of information is $m(\underline{\epsilon}) > \frac{1}{\overline{\epsilon}}$ which can be rewritten as $(1-p) > \underline{\epsilon}\left[\frac{1}{\overline{\epsilon}} + 1 - 2p\right]$. When the incentives to acquire information are low, e.g., if the signal is very expensive or precise, Proposition 2 suggests that $\underline{\epsilon} \precsim \overline{\epsilon}$. In that case,

[5]The value $m(\mu)$ corresponds to the slope of a straight line that goes through the origin and the point $(\mu, f(\mu))$

the condition reduces to $p \cdot (2\underline{\epsilon} - 1) > \underline{\epsilon}$, which is not satisfied. On the other hand, when the incentives to acquire information are high, $\underline{\epsilon} \approx 0$ and $\overline{\epsilon} \approx 1$. The condition, in this case, simplifies to $(1-p) > 2 \cdot \underline{\epsilon} \cdot (1-p)$ which is satisfied because $2 \cdot \underline{\epsilon} < 1$.

When the receiver has low incentives to acquire information, the sender can fully crowd out the endogenous acquisition of information (this is, for every possible signal that he can receive, he will always choose $\epsilon = 0$). In this context, crowding out of information is not necessarily bad news for the receiver for it implies more disclosure of information at no cost. However, when incentives are high, the sender can only partially crowd out information.

5.2 Choosing Signal's Precision

We now turn to the case where the probability of recieving a signal, ϵ is fixed, but the receiver may control its precision. Precision values are restricted to the interval $[\frac{1}{2}, 1]$[6].

5.2.1 Information Acquisition

Contrary to the model in the last section, in this case bayesian updating of beliefs is sensitive to the receiver's choice of precision. This setting justifies the existence of a lower bound ($\underline{p}$) on the chosen precision such that, if any agent is willing to pay for additional information, the selected precision should satisfy $p^* > \underline{p}(\mu)$. Suppose, initially, that $\mu_1 < 0.5$. The reception of signal ℓ implies $\mu_2 < \mu_1 < 0.5$. On the other hand, signal h will imply that $\mu_2 > \mu_1$. For the signal to change the receiver's decision, $\mu_2 > 0.5$ must hold. This will happen whenever $p > (1 - \mu)$. An analogous argument allows to conclude that, when $\mu_1 > 0.5$, utility maximization requires $p > \mu$.

Given a belief μ_1, and precision p, the receiver's utility is described by equation (10). In this setting, $\mu_2(s_2)$ varies with p in a non linear fashion. This makes the problem harder to solve because the benefit function isn't linear in the choice variable any more. Even when there is no analytic solution for $p^*(\mu)$, comparative statics on optimal precision can still be

[6] This guarantees that $\mu_2(h) > \mu_1$

found. These are the contents in Proposition 4

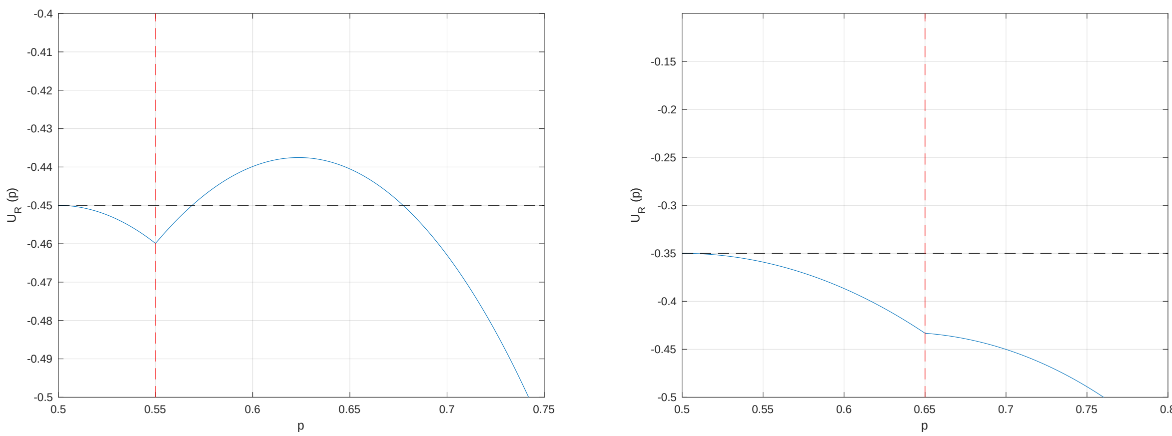
$$\begin{aligned} U_R(p|\mu_1) = (1-\epsilon)U_R(\mu_1) + \epsilon\left[\mathbb{P}(h) * U_R(\mu_2(h)) + \mathbb{P}(\ell) * U_R(\mu_2(\ell))\right]\dots \\ -\lambda\left[H(\boldsymbol{\mu_1}) - \mathbb{E}_{r_2}(H(F(\cdot|r_2)))\right] \end{aligned} \tag{10}$$

Figure 2 represents the receiver's utility as a function of signal precision. Panel (a) considers a receiver with beliefs $\mu_1 = 0.55$, while Panel (b) considers one with $\mu_1 = 0.65$. Vertical dotted lines correspond to lower bounds on the signal precision, $0,55$ and 0.65 respectively. Horizontal dotted lines correspond to $u^0 \equiv U_R(0)$. When $p \in \left[0.5, \underline{p}\right]$ the receiver's utility is decreasing in the signal's precision because he is paying for a positive amount of information that is not valuable for him. In the event that he receives information, this particular knowledge will not make him change the decision he would have made without information. Therefore, in this interval, information is not valuable but it is costly, implying that utility is decreasing. When the chosen precision surpasses the lower bound, information starts being valuable. The amount of information acquired is determined not only by the unit cost of precision but by the resulting beliefs after the persuasion stage. Receivers with beliefs close to $\hat{\mu}$ will find it optimal to acquire information. However, if more information is disclosed in the persuasion stage, the receiver might find it optimal to take the recommended action without pursuing further knowledge of the state of nature.

Let $\Gamma(\mu)$ denote the set of possible precision values (other than 0.5); this is:

$$\Gamma(\mu) = \begin{cases} [1-\mu, 1] & \text{if } \mu \leq 0.5 \\ \\ [\mu, 1] & \text{if } \mu \geq 0.5 \end{cases}$$

The receiver's optimization problem consists in choosing $p \in \{0.5\} \cup \Gamma(\mu)$ in order to maximize Equation 10. When deciding how much precision to buy, he compares his utility without information (u^0) with the one obtained when he chooses the optimal value for p in the set



(a) Informed Receiver

(b) Uninformed Receiver

Figure 2: Receiver's Utility

$\Gamma(\mu)$. Hence, his utility function can be re-written as follows:

$$V_R(\mu) = \max\left\{u^0, \max_{p \in \Gamma(\mu)} U_R(p|\mu)\right\} \tag{11}$$

Proposition 4 (Receiver's optimal strategy)

Consider the utility maximization problem described in Equation 11. The solution to this problem satisfies the following properties:

1. *The optimal strategy consists of two cutoff points $\underline{\mu}$ and $\overline{\mu}$ such that $p^*(\mu) > 0.5$ whenever $\mu \in [\underline{\mu}, \overline{\mu}]$ and $p^*(\mu) = 0.5$ otherwise.*
2. *$p^*(\mu)$ is decreasing in the interval $[\underline{\mu}, \hat{\mu}]$*
3. *$p^*(\mu)$ is increasing in the interval $[\hat{\mu}, \overline{\mu}]$*

Proof. See Appendix B ■

Figure 3 illustrates the receiver's precision choice for as a function of his beliefs. Optimal precision is not monotone in beliefs, μ_1. Incentives to acquire information are at their peak whenever $\mu_1 = 0.5$, the belief where he is indifferent between any of the two possible actions. However, when beliefs diverge from the center, a higher degree of precision is required to modify his uninformed action. This results in the U- shaped policy function. Given that the cost of information is increasing in its precision, there are well-defined boundaries $\underline{\mu}$ and $\overline{\mu}$ such that when $\mu \notin [\underline{\mu}, \overline{\mu}]$, information costs are prohibitive and no information is bought.

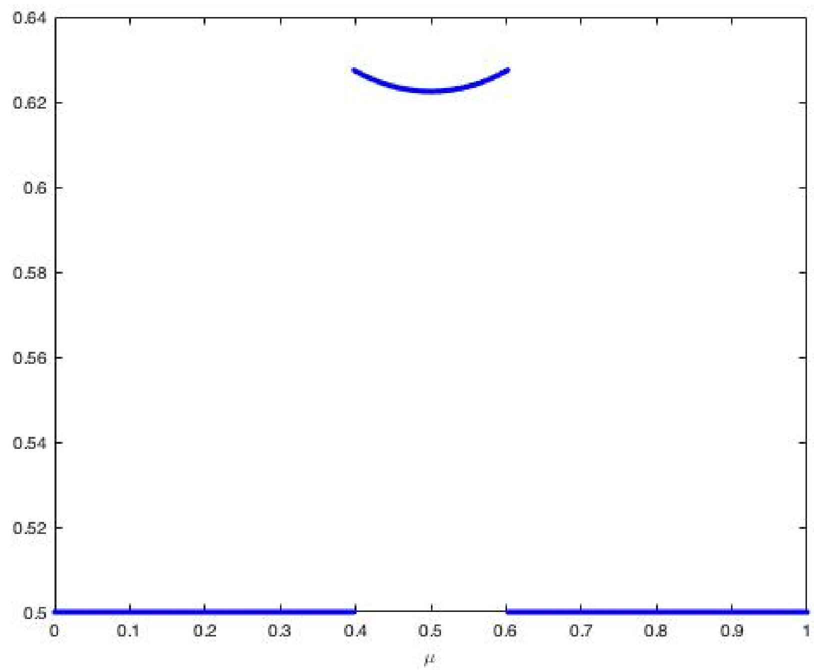


Figure 3: Receiver's choice of precision.

5.2.2 Information Disclosure

When deciding how much information to disclose, the sender will take into account the receiver's subsequent acquisition of information and its implications for her utility. Proposition 5 states how much information she will disclose in equilibrium. This is a similar result to the one presented in Proposition 3. Namely, the receiver's incentives to acquire information depend on the probability that information is received and the unit cost of information precision. When incentives for information acquisition are low, complete crowding out of information acquisition is achieved. Nonetheless, complete crowding out is not possible when incentives to buy information are high.

Proposition 5 (Disclosure Policy)

Let $f : [0,1] \mapsto \mathbb{R}$ denote the probability with which the reciver receives signal h in the interval $[\underline{\mu}, \overline{\mu}]$. Let $m : [0,1] \mapsto \mathbb{R}$ be as in Proposition 3. Then, the optimal disclosure policy satisfies:

1. *If $m(\hat{\mu}) < (\overline{\mu})^{-1}$, the optimal disclosure policy is such that the induced posterior beliefs on the buyer will take one of two values: $\mu_1 \in \{0, \overline{\mu}\}$*
2. *If $m(\hat{\mu}) > (\overline{\mu})^{-1}$, the optimal disclosure policy is such that the induced posterior beliefs will take one of three values: $\mu_1 \in \{0, \hat{\mu}, \overline{\mu}\}$*

Proof. The argument followed here is analogous to the one in Proposition 3 ■

Corollary 3 *On-path information acquisition may arise whenever ϵ is high or when the unit cost of information is low.*

6 Learning from Multiple Experts

In the previous section, we saw how $\mathcal{R}$'s ability to learn limits $\mathcal{S}$'s ability to persuade him. If $\mathcal{R}$ resorts to highly informative experiments in the sense of Blackwell (1951), $\mathcal{S}$ must disclose more information than he would otherwise do with less informative technologies. This disclosure of information is enough to convince $\mathcal{R}$ to take the suggested action, thus making his private technology unprofitable. This result corresponds to the Never-Learning Lemma in Matyskova (2018).

These findings suggest an information acquisition structure that does not seem to hold in some decision-making contexts, particularly, when stakes are high. Wagner and Wagner

(1999) find that, in the health market, out of every five people who visit the doctor, one seeks a second opinion. The proportion increases to 50% when patients are diagnosed with cancer.

In what follows, we will develop a model that rationalizes the acquisition of information in equilibrium.

The Model

Let $\mathcal{R}$ have access to a set $\mathcal{I}$ of experts who can inform him about the state of nature. Experts are characterized by their type $\theta_i \in \{I, II\}$, and their relative efficiency in reducing type I and II errors, $\gamma_i \in [-1, 1]$. For each expert i, the agent can choose the probability with which his investigation is successful, $\epsilon_i \in [0, 1]$ and its precision, $p_i \in [\frac{1}{2}, 1]$. Formally, each expert is an experiment of the form:

	$\omega = H$	$\omega = S$
$s = h$	$\epsilon_i \cdot p_i$	$\epsilon_i \cdot \left(\frac{1}{2}(1 + \gamma_i) - \gamma_i \cdot p_i\right)$
$s = \ell$	$\epsilon_i \cdot (1 - p_i)$	$\epsilon_i \cdot \left(\gamma_i \cdot p_i + \frac{1}{2}(1 - \gamma_i)\right)$
$s = \emptyset$	$1 - \epsilon_i$	$1 - \epsilon_i$

(a) $\theta_i = I$

	$\omega = H$	$\omega = S$
h	$\epsilon_i \left(\gamma_i \cdot p_i + \frac{1}{2}(1 - \gamma_i)\right)$	$\epsilon_i \cdot (1 - p_i)$
ℓ	$\epsilon_i \left(-\gamma_i \cdot p_i + \frac{1}{2}(1 + \gamma_i)\right)$	$\epsilon_i \cdot p_i$
$\emptyset$	$1 - \epsilon_i$	$1 - \epsilon_i$

(b) $\theta_i = II$

Table 2: Private Information Technology

We will assume that $\mathcal{R}$ has a time restriction that prevents him from communicating with certainty with all the experts at once. Formally, equilibrium probabilities must satisfy:

$$0 \leq \sum_{i \in \mathcal{I}} \epsilon_i \leq 1$$

Table 2 gives a rationale for expert nomenclature. In panel (a), for example, increases in p_i decrease type I and II errors at rates $\gamma_i \cdot \epsilon_i$ and ϵ_i, respectively. Expert i with parameter $\gamma_i = -1$ is uninformative. Increases in γ_i imply greater ability of the expert to avoid type I

error. In turn, for any γ_i, choosing $\epsilon_i = 0$ or $p_i = \frac{1}{2}$ is equivalent to not acquiring information.

With the framework above in mind, we may proceed to define $\mathcal{R}'s$ information acquisition problem:

Definition 2 (Receiver's Restricted Problem I) *Let* $\Phi^i_j \equiv \mathbb{P}(s = j | p = p_i, \gamma = \gamma_i, \theta = \theta_i)$ *and* $\mu'_{ji} \equiv \mathbb{P}(\omega = H | s = j, p = p_i, \gamma = \gamma_i, \theta = \theta_i)$. *The receiver solves:*

$$\max_{\{(\epsilon_i, p_i)\}} \sum_{i \in \mathcal{I}} \epsilon_i \left(\sum_{j \in \{h, \ell\}} \Phi^i_j \cdot V \left[\mu'_{ji} \right] \right) + \left(1 - \sum_{i \in \mathcal{I}} \epsilon_i \right) V(\mu)$$

$$+\lambda \left[H(\mu) - \sum_{i \in \mathcal{I}} \epsilon_i \left(\sum_{j \in \{h, \ell\}} \Phi^i_j \cdot H \left[\mu'_{ji} \right] \right) + \left(1 - \sum_{i \in \mathcal{I}} \epsilon_i \right) H(\mu) \right]$$

$$s.t. \;\; \epsilon \in \Delta_{\mathcal{I}}, \quad \frac{1}{2} \leq p_i \leq 1$$

At this point, it should be clear that the information problem in our General Model in §5 is equivalent to the one stated above with the assumption that, for every type of expert, the set $\mathcal{I}$ contains one expert for every value of $\gamma_i \in [-1, 1]$.

Claim 1 *The solution to the Receiver's Restricted Problem I satisfies one and only one of the following:*

- $\epsilon^* = 0$
- $\epsilon_i = 1$ *for some* $i \in \mathcal{I}$ *and* $\epsilon_j = 0$ *for all* $j \neq i$

Proof. This proof is trivial on the observation that the receiver's problem is linear in the probabilities of receiving a signal. ■

Claim 1 has three crucial implications in the information acquisition structure. First, conditional on acquiring information, R will attend to only one expert. Second, when an expert i is consulted, R chooses a value of ϵ_i such that he never receives the signal $s = \emptyset$.

This result is in line with the behavior of a rationally inattentive agent. Finally, choosing $\epsilon_i = 1$ means that each signal is interpreted as an expert's action recommendation.

Let $Q_H \equiv \mathbb{P}(\text{accept}|\omega = H)$ and $Q_L \equiv \mathbb{P}(\text{accept}|\omega = L)$ denote the conditional probabilities of taking $\mathcal{S}$'s preferred action. Table 2 and Claim 1, together, imply that choosing an expert is analogous to choosing a given relation between conditional probabilities. For example, if $\mathcal{R}$ consults a Type I expert, conditional probabilities implied by the optimal information structure must satisfy the restriction $Q_H = \frac{1+\gamma_i}{2\cdot\gamma_i} - \frac{Q_L}{\gamma_i}$. On the other hand, when Type II experts are consulted, equilibrium conditional probabilities satisfy $Q_H = \frac{1}{2}(1+\gamma_i) - \gamma_i \cdot Q_L$. Claim 2 generalizes this observation.

Claim 2 *There is an biyective mapping between the set* $\{(\theta_i, \gamma_i)\}_{i\in\mathcal{I}}$ *and* $\mathbb{R}$.

Proof. We can think of each restriction as a line that goes through $(Q_H, Q_L) = \left(\frac{1}{2}, \frac{1}{2}\right)$. In that case, it suffices to show that every pair (θ_i, γ_i) has a unique intercept.

For Type I experts, the intercept is given by $\frac{1+\gamma_i}{2\cdot\gamma_i}$ which maps biyectively to $(-\infty, 0] \cup [1, +\infty)$. The relevant expression for Type II experts is $\frac{1}{2}(1+\gamma_i)$, which maps biyectively to $[0, 1]$. ■

Claim 2 has an important geometric value. It allows us to interpret each expert as a ray of the form $Q_H = \alpha_i - (1 - 2\cdot\alpha_i)Q_L$ in the (Q_L, Q_H) plane. Because the mapping is inyective, it thus allows us to identify each expert with a unique intercept on the plane's vertical axis as shown in Figure 4.

The receiver's problem can thus be simplified and interpreted as choosing an expert (or ray) in the feasible set, then choosing where to be located on that ray. The information acquisition problem can then be stated as follows:

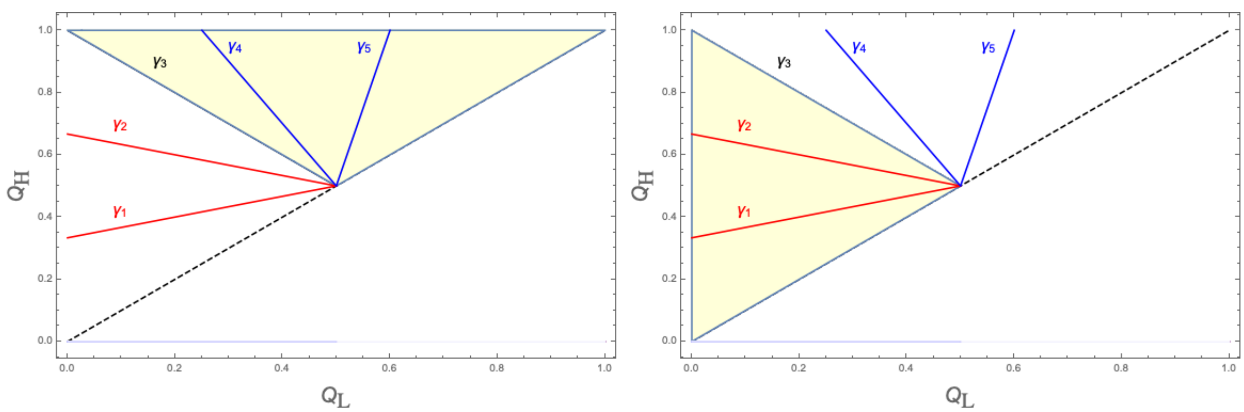
Definition 3 (Receiver's Restricted Problem II) *The optimal information strategy re-*

quires choosing an exper i^* and conditional probabilities (Q_h^*, Q_ℓ^*) that solve:

$$\max_{i \in \mathcal{I}} \left\{ \max_{(Q_H, Q_L)} -\mu(1-Q_H) - (1-\mu)Q_L - \mathcal{C}(Q_H, Q_L) \right\}$$
$$s.t. \quad (a)\ 0 \leq Q_H, Q_L \leq 1$$
$$(b)\ Q_H = \alpha_i + (1-2\alpha_i)Q_L$$

Where $Q \equiv \mu \cdot Q_H + (1-\mu) \cdot Q_L$ denotes the unconditional probability of taking $\mathcal{S}$'s preferred action and:

$$\mathcal{C}(Q_H, Q_L) = \lambda \left[-Q \cdot \ln(Q) - (1-Q) \cdot \ln(1-Q) \right] \ldots$$
$$+\lambda \sum_{\omega \in \{H,L\}} \left[Q_\omega \cdot \ln(Q_\omega) + (1-Q_\omega) \cdot \ln(1-Q_\omega) \right] \cdot \mathbb{P}(\omega)$$



(a) $\theta_i = I$ (b) $\theta_i = II$

Figure 4: Conditional Probabilities Restriction

Proposition 6 *Let i^* denote the chosen expert and $\alpha_i^*(\mu)$ his associated intercept. Let Q^* and $\hat{Q}$ denote the equilibrium unconditional probabilities for the restricted and unrestricted problems, respectively. Then:*

1. *$\exists \mu^*$ such that $Q^*(\mu^*, \alpha_i^*) = \hat{Q}(\mu^*)$*

2. *$\forall \mu \lesseqgtr \mu^*$, $\hat{Q}(\mu) \lesseqgtr Q^*(\mu, \alpha_i^*)$*

Proof. See Appendix D ■

Proposition 6 reveals considerable structure in the way $\mathcal{R}$ acquires information. The first item takes care of a situation where the optimization problem's constraint is not active. In these cases, the solution is equivalent to the one in our benchmark.

A more interesting dynamic arises when the set of available experts does not allow replicating the solution to our benchmark. In these cases, the optimal information acquisition strategy induces zones of over-optimism (where the unconditional probability of taking $\mathcal{S}$'s preferred action is higher than that in our benchmark) followed by zones of over-pessimism. It will be the case that $\mathcal{S}$ will take advantage of the over-optimism zones when constructing his persuading strategy.

$\mathcal{R}$ will turn to Type II experts whenever $\mu < \frac{1}{2}$. If he places a high probability on the product being of low quality, he would be inclined to reject it. Given that information is valuable only if it can make him change his mind, he should hire experts that are relatively good at identifying high quality products. This is exactly what Type II experts offer. As an example, suppose he chooses $p_i = 1$; then, only signal h would completely informative. In this sense, when $\mathcal{R}$ hires a Type II expert, he fishing for diamonds. On the other hand, when $\mu > \frac{1}{2}$, $\mathcal{R}$ will be fishing for lemons.

On-Path Information Acquisition

An essential difference between the benchmark and our model is the shape of the unconditional probability of choosing $a = 1$. In the former, this probability is a continuous and increasing function of $\mathcal{R}$'s beliefs. However, in the latter, the probability is a stepwise function where each step corresponds to one of the consulted experts. The magnitude of each jump depends on how "close" each expert is to the next. At the limit, when the availability of experts is such that $\mathcal{R}$ can choose any $\alpha_i \in \mathbb{R}$, our $Q^*(\mu)$ converges to $\hat{Q}(\mu)$.

In the unrestricted model, the optimal persuasion strategy induced posterior beliefs at the inattention zones' edges. This finding suffices to prove the crowding out of information in that model. If the disclosure strategy induces on-path information acquisition, it should better persuade $\mathcal{R}$ to buy with a greater probability than he otherwise would if his information acquisition is prevented. This is achieved as long as the induced posteriors lie at the leftmost edges of the over-optimism regions, as shown in Figure 5.

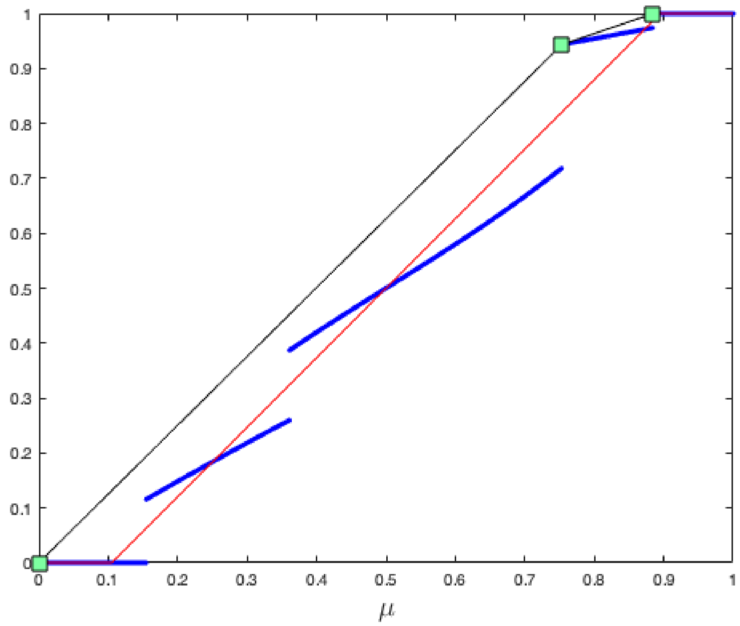


Figure 5: Persuasion with 3 experts
— $Q^*(\mu)$ — $\hat{Q}(\mu)$

7 Concluding Remarks

This paper proposes the solution to a strategic interaction model between a sender and a privately informed receiver.

Information comes to the receiver in a two-fold manner. At first, during the persuasion stage, information is free but is disclosed by an agent whose interests make him doubt about its veracity. In a second stage, receivers can acquire information from a private technology that is not biased by the sender's interests. This information is costly.

As we have shown, the second technology puts a limit on the profits that the sender can obtain in the first stage. In particular, an informed receiver obliges her to transmit more information than she would in the absence of private information acquisition. In equilibrium, information acquisition is completely crowded out by senders. That is, whenever receivers would optimally purchase information on their own, senders will prefer to provide this information themselves. Given that this information comes at no cost, the receiver's welfare can only increase. On this understanding it is clear that privately informed receivers find the information technology's value not in its use, but for the threat that this information poses on senders. Even when they do not resort to this technology in equilibrium, its existence allows them to prevent senders having an information advantage and extracting rents.

The three models compared in this work suggest that the way in which information acquisition is modeled has direct implications on the equilibrium strategies of both Sender and Receiver. In a world where information is now more abundant than ever, the rational inattention framework seems as a natural explanation of how agents become privately informed. This framework allows the agent to choose not only the amount of information purchased but the technology from which he desires to purchase it. In that sense, a rationally inattentive Receiver knows exactly what questions to ask in order to induce the sender to disclose a higher amount of information than she would if he was uninformed. A next step would be to find how do each agent's strategies change when the market is competitive and information may be obtained from multiple senders.

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8 Appendix A.

Proof of Proposition 1

• **Claim 1:** The objective function is linear in the choice variable, ϵ.

The objective function is:

$$\begin{aligned} f(\epsilon) =& (1-\epsilon)U_R(\mu_1) + \epsilon\left[\mathbb{P}(t) * U_R(\mu_2(t)) + \mathbb{P}(h) * U_R(\mu_2(h))\right] \dots \\ & - \lambda\left[H(\boldsymbol{\mu_1}) - \mathbb{E}_{r_2}(H(F(\cdot|r_2)))\right] \end{aligned} \tag{12}$$

Where :

$$\mathbb{E}_{r_2}(H(F(\cdot|r_2))) = (1-\epsilon)H(\boldsymbol{\mu}_1) + \epsilon\left[\mathbb{P}(h) * H(F(\cdot|h)) + \mathbb{P}(t) * H(F(\cdot|t))\right] \tag{13}$$

Substituting (13) in (12) and rearranging terms gives:

$$f(\epsilon,\mu_1) = U_R(\mu_1) + \dots$$

$$\epsilon\Big[\mathbb{P}(t)\big(U_R(\mu_2(t)) + \lambda H(F(\cdot|t))\big) + \mathbb{P}(h)\big(U_R(\mu_2(h)) + \lambda H(F(\cdot|h))\big) - U_R(\mu_1) - \lambda H(\boldsymbol{\mu_1})\Big] \tag{14}$$

Hence, as a function of ϵ, $f(\epsilon,\mu_1)$ is a straight line with intercept $U_R(\mu_1)$ and a slope given by the term between square brackets.

■

• **Claim 2:** The optimal strategy consists of two cutoff points $\underline{\epsilon}$ and $\overline{\epsilon}$ such that $\epsilon^*(\mu_1) = 1$ whenever $\mu_1 \in [\underline{\epsilon}, \overline{\epsilon}]$ and $\epsilon^*(\mu_1) = 0$ otherwise.

The term in square brackets in (14) takes a different functional form depending on wether $\mu_1 > \hat{\mu}$ or not. We will first consider the case when $\mu_1 < \hat{\mu}$.

Let us denote this term by $ML(\mu_1)$. It is straightforward to see that, given that the optimization problem is linear in ϵ, it is optimal to set $\epsilon = 1$ whenever $ML(\mu_1) > 0$ and $\epsilon = 0$ otherwise. Furthermore, let us notice that:

$$\lim_{\mu_1 \to 0} ML(\mu_1) = b(1-p) < 0$$

Finally, the following equation shows that $ML(\mu_1)$ is convex:

$$\frac{\partial^2 ML(\mu_1)}{\partial \mu_1^2} = \frac{-\lambda(1-2p)^2}{[-\mu_1 + p(2\mu_1 - 1)][1 - \mu_1 + p(2\mu_1 - 1)]} > 0 \tag{15}$$

Therefore, using the previous facts and that $ML(\mu_1)$ is continuous, we find that:

1. If $ML(\hat{\mu}) < 0$, $ML(\mu_1) < 0$ for all $\mu_1 \in [0, \hat{\mu}]$. Hence $\epsilon^*(\mu_1) = 0$ for all $\mu_1 \in [0, \hat{\mu}]$.
2. If $ML(\hat{\mu}) > 0$, there exists some value $\underline{\epsilon}$ such that $ML(\mu_1) < 0$ for all $\mu_1 \in [0, \underline{\epsilon}]$ and $ML(\mu_1) > 0$ for all $\mu_1 \in [\underline{\epsilon}, \hat{\mu}]$. This would mean that the optimal policy for ϵ is given by:
$$\epsilon^*(\mu_1) = \begin{cases} 0 & \text{if } \mu_1 \in [0, \underline{\epsilon}] \\ 1 & \text{if } \mu_1 \in [\underline{\epsilon}, \hat{\mu}] \end{cases}$$

Our condition for information acquisition is given by $ML(\hat{\mu}) > 0$ which translates to:

$$b < \frac{\lambda}{(1-2p)} \left[ln(4) + 2(1-p)ln(1-p) + 2pln(p) \right]$$

The analysis for the case where $\mu_1 > \hat{\mu}$ is analogous to the one above. In fact, the condition for information acquisition is exactly the same as the one just found.

■

• **Claim 3:** If $b > \frac{\lambda}{(1-2p)} \left[ln(4) + 2(1-p)ln(1-p) + 2pln(p) \right]$, then $\underline{\epsilon} = \overline{\epsilon}$ and additional information will never be bought.

This result follows directly from the proof of claim 2.

■

• **Claim 4:** $\underline{\epsilon}$ ($\overline{\epsilon}$) is weakly increasing (decreasing) in the unit cost of information (λ).

We will show that $\underline{\epsilon}$ is weakly increasing in λ. A similar argument shows that $\overline{\epsilon}$ is weakly decreasing.

We already know that the function $ML(\mu_1)$ is convex and that it satisfies the single crossing property. If we could show that $\frac{\partial ML(\mu_1)}{\partial \lambda} \leq 0$, our claim would follow.

$$
\begin{aligned}
\frac{\partial ML(\mu_1)}{\partial \lambda} =&(1-\mu_1)ln(1-\mu_1)+\mu_1 ln(\mu_1)+(p+\mu_1-1-p\mu_1)ln\left(\frac{(1-p)(1-\mu_1)}{1-p-\mu_1+2p\mu_1}\right)\\
&-p\mu_1 ln\left(\frac{p\mu_1}{1-p-\mu_1+2p\mu_1}\right)-p(1-\mu_1)ln\left(\frac{p-p\mu_1}{p+\mu_1-2p\mu_1}\right)\\
&-(1-p)\mu_1 ln\left(\frac{\mu_1-p\mu_1}{p+\mu_1-2p\mu_1}\right)
\end{aligned} \tag{16}
$$

Using (15) and Young's Theorem, we conclude that $\frac{\partial ML(\mu_1)}{\partial \lambda}$ is convex. Furthermore, we know that $\lim_{\mu_1\to 0^+} \frac{\partial ML(\mu_1)}{\partial \lambda}=0$, and:

$$
\frac{\partial ML(\mu_1)}{\partial \lambda}\Big|_{\mu_1=\hat{\mu}} = -ln(2)+\big[-(1-p)ln(1-p)-pln(p)\big]\leq 0
$$

(where the inequality follows from the fact that the term in square brackets is maximized at $p=0.5$ and its maximum value is $ln(2)$); hence, given $p\in(0.5,1)$, we have shown that $\frac{\partial ML(\mu_1)}{\partial \lambda}\leq 0$ for every $\mu_1\in[0,\hat{\mu}]$. We conclude that $\underline{\epsilon}$ is weakly increasing in λ.

■

Proof of Proposition Proposition 3

• **Claim 1:** If $m(\underline{\epsilon})<(\overline{\epsilon})^{-1}$, the optimal disclosure policy is such that the induced posterior beliefs on Receiver will take one of two values: $\mu_1\in\{0,\overline{\epsilon}\}$.

When $m(\underline{\epsilon})<(\overline{\epsilon})^{-1}$ the problem is identical to the one solved by Kamenica and Gentzkow (2011). The seller's utility function is given by the concave closure of $\Pi(\mu)$. Figure 6 provides an example of how might this function look like (vertical dashed lines correspond to $\underline{\epsilon}$ and $\overline{\epsilon}$)[7].

The function $f(\mu)$ corresponds to the straight line between $\underline{\epsilon}$ and $\overline{\epsilon}$.

If $m(\underline{\epsilon})<(\overline{\epsilon})^{-1}$, then the concave closure of the expected profit function is:

[7]Parameters are: $b=-1$, $\lambda=1$, $p=0.7$

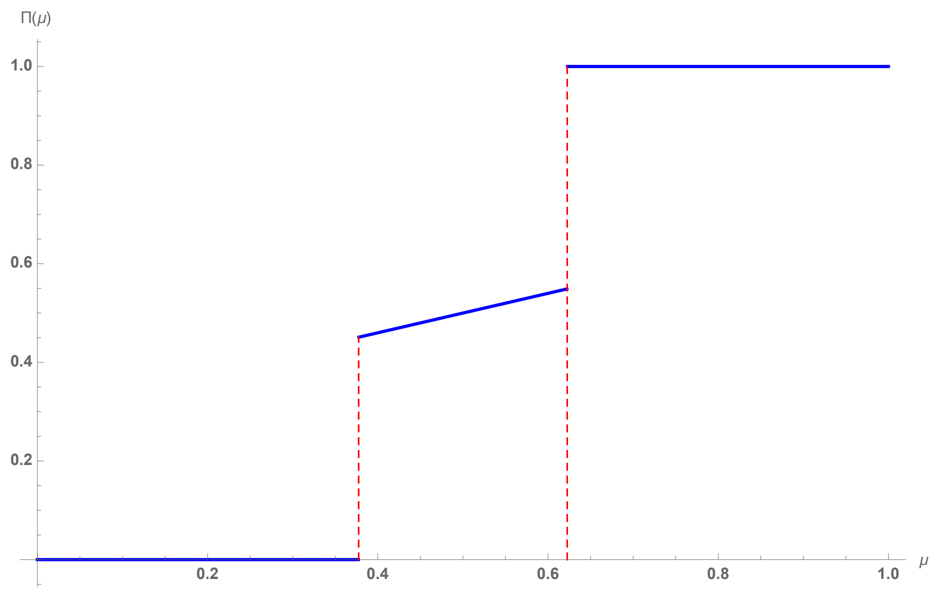


Figure 6: Sender's Expected Payoff Function

$$V_S(\mu) = \begin{cases} (\bar{\epsilon})^{-1} \cdot \mu & \text{if } 0 \leq \mu \leq \bar{\epsilon} \\ 1 & \text{if } \bar{\epsilon} \leq \mu \leq 1 \end{cases} \tag{17}$$

Figure 7 shows the convex cover of $\Pi(\mu)$ in light green, and its concave closure as a black line.

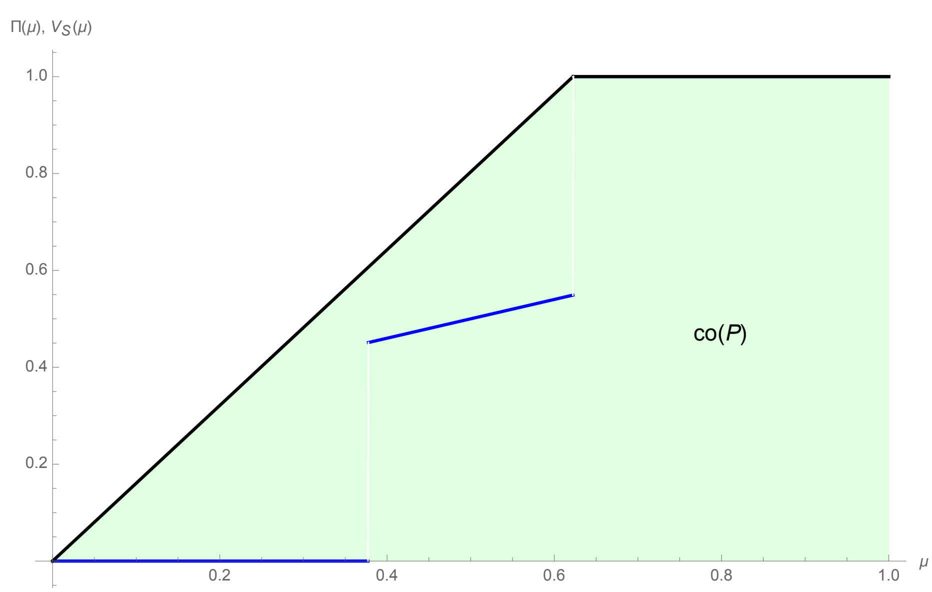


Figure 7: Concave Closure of Payoff Function in Figure 6

Then, when the sender is approached by a receiver with belief $\mu_0 \in (0, \overline{\epsilon})$, her optimal policy consists in placing him at belief $\mu_1 = \overline{\epsilon}$ with probability $\mu_o \cdot (\overline{\epsilon})^{-1}$ and placing him at belief $\mu_1 = 0$ with the remaining probability.

■

• **Claim 2:** If $m(\underline{\epsilon}) > (\overline{\epsilon})^{-1}$, the optimal disclosure policy is such that the induced posterior beliefs will take one of three values: $\mu_1 \in \{0, \underline{\epsilon}, \overline{\epsilon}\}$.

Before proving this claim we'll show that, in this case, $m(\underline{\epsilon}) > f'(\mu) \equiv 2p - 1$ must hold.

Suppose that $m(\underline{\epsilon}) = f'(\mu) \equiv 2p - 1$. Then, it must be true that:

$$m(\underline{\epsilon}) = \frac{f(\underline{\epsilon})}{\underline{\epsilon}} = 2p - 1 = \frac{f(\overline{\epsilon})}{\overline{\epsilon}}$$

Given that $1 > f(\overline{\epsilon})$, the equality above implies $\frac{1}{\overline{\epsilon}} > \frac{f(\overline{\epsilon})}{\overline{\epsilon}} = m(\underline{\epsilon})$ which yields a contradiction.

Now suppose that $m(\underline{\epsilon}) < f'(\mu)$. By definition of m, we know that $m(\underline{\epsilon}) \cdot \underline{\epsilon} = f(\underline{\epsilon})$. These two facts imply that $1 > m(\underline{\epsilon}) \cdot \overline{\epsilon}$. Given that $1 > f(\underline{\epsilon})$, the last inequality would imply that:

$$\frac{1}{\overline{\epsilon}} > \frac{f(\overline{\epsilon})}{\overline{\epsilon}} > m(\underline{\epsilon})$$

which, again, yields a contradiction.

Therefore, we conclude that $m(\underline{\epsilon}) > f'(\mu)$.

Moving on the the proof of the main claim, we will first show that the concave closure of Π must go through $(\underline{\epsilon}, f(\underline{\epsilon}))$.

It is straightforward to see that the concave closure cannot go through any point $(\mu, 0)$ with $\mu < \underline{\epsilon}$, for the resulting function wouldn't be concave.

If instead of $(\underline{\epsilon}, f(\underline{\epsilon}))$ we pick any other point $\mu^* \in (\underline{\epsilon}, \overline{\epsilon})$, it would be true that $\frac{f(\underline{\epsilon})}{\underline{\epsilon}} > \frac{f(\mu^*)}{\mu^*}$. Equivalently, $f(\underline{\epsilon}) > \frac{f(\mu^*)}{\mu^*} * \underline{\epsilon}$. The resulting function would, then, violate the definition of being the concave closure of $\Pi(\mu)$.

Finally, suppose that instead of $(\underline{\epsilon}, f(\underline{\epsilon}))$ we pick any other point $(\tilde{\mu}, 1)$, where $\tilde{\mu} > \overline{\epsilon}$. Given our hypothesis that $m(\underline{\epsilon}) > \frac{1}{\overline{\epsilon}}$, and given that $\frac{1}{\mu}$ is decreasing in μ, we have:

$$f(\underline{\epsilon}) > \frac{1}{\tilde{\mu}} * \underline{\epsilon}$$

Again, this inequality means that the proposed function would violate the definition of the concave closure.

Hence, when $0 < \mu < \underline{\epsilon}$, we conclude that the concave closure of Π is a straight line that goes through the origin and the point $(\underline{\epsilon}, f(\underline{\epsilon}))$.

A similar argument allows us to conclude that, when $\underline{\epsilon} < \mu < \overline{\epsilon}$, the concave closure is a straight line connecting the points $(\underline{\epsilon}, f(\underline{\epsilon}))$ and $(\overline{\epsilon}, 1)$.

Figure 8 and Figure 9 show an example of how might the expected payoff function and its concave closure look like for a particular set of parameters[8].

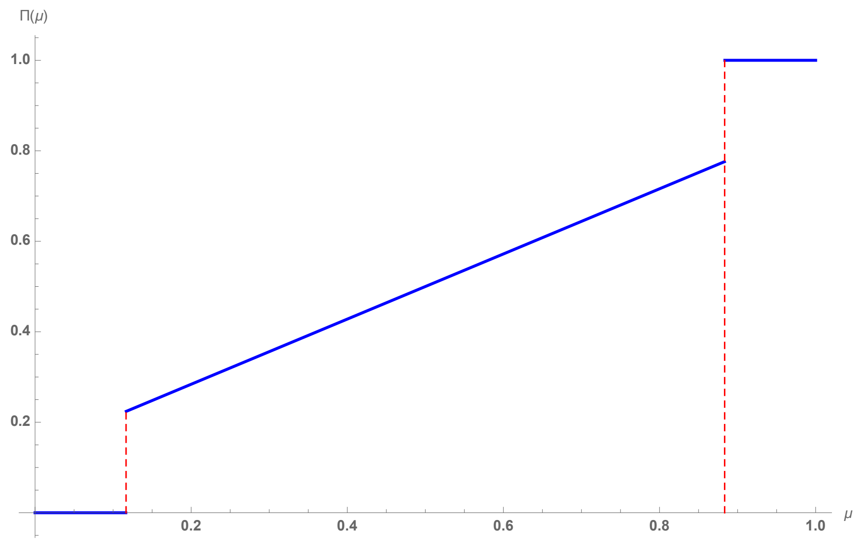


Figure 8: Sender's Expected Payoff Function

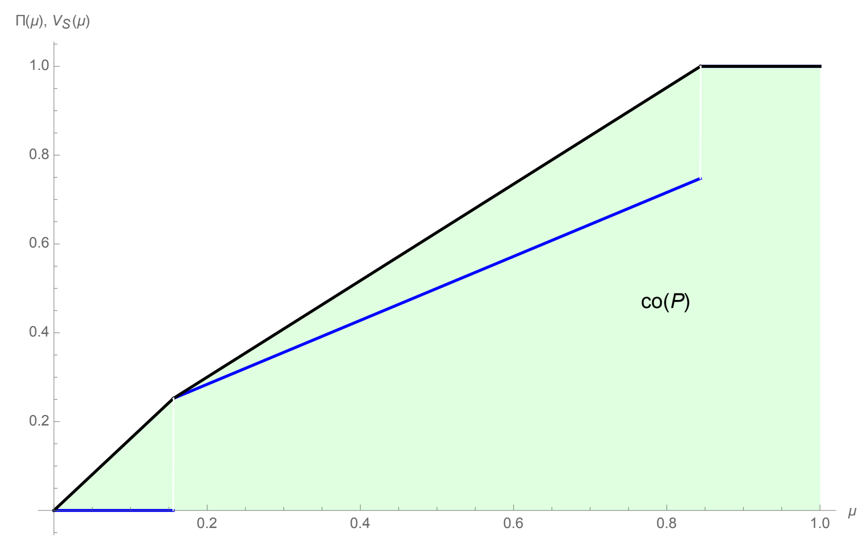


Figure 9: Concave Closure of Payoff function in Figure 8

[8]Parameters are: $b = -1$, $\lambda = 0.1$, $p = 0.86$

Thus, when Receiver approaches Sender with belief $\mu_0 \in (0, \underline{\epsilon})$, her optimal policy can be interpreted as placing him at beliefs $\mu_1 = \underline{\epsilon}$ with probability $\frac{f(\epsilon)}{\underline{\epsilon}} * \mu_0$ and placing him at belief $\mu_1 = 0$ with probability $1 - \frac{f(\epsilon)}{\underline{\epsilon}} * \mu_0$. On the other hand, if $\mu_0 \in (\underline{\epsilon}, \overline{\epsilon})$, then Sender will find it optimal to place him at belief $\mu_1 = \overline{\epsilon}$ with probability $\frac{f(\epsilon)(\overline{\epsilon}-\mu_0)+(\mu_0-\underline{\epsilon})}{\overline{\epsilon}-\underline{\epsilon}}$ and at belief $\mu_1 = \underline{\epsilon}$ with probability $1 - \frac{f(\epsilon)(\overline{\epsilon}-\mu_0)+(\mu_0-\underline{\epsilon})}{\overline{\epsilon}-\underline{\epsilon}}$.

■

9 Appendix B.

Proof of Proposition 4

Consider the utility function in Equation 10.

First order conditions wrt p imply that its solution, p^*, must satisfy the following:

$$-b + \lambda \left[(1-\mu) \ln \left(\frac{-(1-p^*)(1-\mu)}{p^* + \mu - 2p^*\mu} \right) - \mu \ln \left(\frac{-p^*\mu}{p^* + \mu - 1 - 2p^*\mu} \right) \right]$$

$$\lambda \left[(\mu - 1) \ln \left(\frac{p^*(1-\mu)}{p^* + \mu - 2p^*\mu} \right) + \mu \ln \left(\frac{\mu(1-p^*)}{p^* + \mu - 2p^*\mu} \right) \right] = 0$$

The former expression tends to 1 and $-\infty$ as p tends to $0,5$ and 1, respectively. Furthermore, differentiating it wrt p shows that it is strictly decreasing in p. Continuity of this expression means that there exists a value of precision in the interval $\left(\frac{1}{2}, 1\right)$ that maximizes Receiver's utility.

To show the existence of boundary $\underline{\mu}$, consider the case where $\mu < \hat{\mu}$. Differentiating the FOC wrt μ yields:

$$\frac{\partial^2 U_R(p|\mu_1)}{\partial p \partial \mu} = \lambda \left[(-1 + p + \mu - 2p\mu)^{-1} + (p + \mu - 2p\mu)^{-1} - \ln \left(\frac{(1-p)(1-\mu)}{(-1 + p + \mu - 2p\mu)} \right) \right]$$

$$+\lambda \left[-\ln \left(\frac{-p\mu}{p + \mu - 1 - 2p\mu} \right) + \ln \left(\frac{p - p\mu}{p + \mu - 2p\mu} \right) + \ln \left(\frac{\mu - p\mu}{p + \mu - 2p\mu} \right) \right]$$

If $\mu < \frac{1}{2} = \hat{\mu}$, the expression above is negative (U_R is submodular), meaning that maximizing values of p^* are decreasing with μ in this region. However, these candidates must satisfy the restriction $p^* > \underline{p}$ to be worth paying for. When $\mu \to \frac{1}{2}$, $\underline{p} = \frac{1}{2}$. For λ sufficiently big, $p^* = \frac{1}{2}$ and no information is bought. However, when the unit cost of information decreases, the optimal value of precision is strictly greater than $\frac{1}{2}$, implying a positive flow of information between the private technology and Receiver. We will focus on non prohibitive values for λ.

When μ decreases, both $\underline{p}$ and p^* increase. Boundary $\underline{\mu}$ is defined as the value of $\mu \in (0.5, 1)$ that satisfies $\underline{p}(\mu) \equiv (1-\mu) = p^*(\mu)$. If no such value exists, then $\underline{\mu} = 0$ and no information is bought. If it exists, Receiver will buy precision $p^*(\mu)$.

A similar argument for the case when $\mu > \hat{\mu}$ allows to find boundary $\overline{\mu}$. Supermodularity of U_R in that region implies that $p^*(\mu)$ is increasing.

■

10 Appendix C.

Proof of Theorem 1

• **Claim 1:** The optimal strategy consists of two cutoff points, $\underline{\mu}$ and $\overline{\mu}$ such that player P pays for a positive amount of information when $\underline{\mu} < \mu_1 < \overline{\mu}$ and searches for no further information when μ_1 is outside this interval.

As shown by Matejka and McKay (2015), the unconditional probabilities that solve Receiver's information acquisition problem are given by the solution to the following optimization problem:

$$\max_{P_1^0 \in [0,1]} \mu_1 \cdot \lambda \cdot ln\left(P_1^0 + (1-P_1^0)e^{\frac{b}{\lambda}}\right) + (1-\mu_1) \cdot \lambda \cdot ln\left(P_1^0 e^{\frac{b}{\lambda}} + (1-P_1^0)\right) \tag{18}$$

Standard optimization techniques imply that the optimal unconditional probability of taking action $a = 1$ is given by:

$$P_1^0 = \min\left\{\max\left\{0, \frac{e^{\frac{b}{\lambda}}(\mu_1 - 1) + \mu_1}{1 - e^{\frac{b}{\lambda}}}\right\}, 1\right\}$$

Furthermore, notice that

$$\frac{\partial P_1^0(\mu_1)}{\partial \mu_1} = \frac{1 + e^{\frac{b}{\lambda}}}{1 - e^{\frac{b}{\lambda}}} > 0$$

Then, when the solution for P_1^0 is interior, it is increasing in μ_1.

To find the cutoff values we need to find the values $\underline{\mu}$ and $\overline{\mu}$ such that $P_1^0(\underline{\mu}) = 0$ and $P_1^0(\overline{\mu}) = 1$. Given the monotonicity of P_1^0, this values will satisfy $\underline{\mu} > \overline{\mu}$. Straightforward algebra yields:

$$\underline{\mu} = \frac{e^{\frac{b}{\lambda}}}{1+e^{\frac{b}{\lambda}}} \qquad \overline{\mu} = \frac{1}{1+e^{\frac{b}{\lambda}}}$$

Finally, we wish to show that mutual information is equal to 0 whenever $\mu_1 \notin (\underline{\mu}, \overline{\mu})$.

One implication of Remark 1 is that mutual information may be written as:

$$MI(\boldsymbol{\mu_1}) = -\sum_{i\in\{0,1\}} P_i^0 \cdot ln\left(P_i^0\right) + \sum_{\omega\in\{S,H\}} \sum_{i\in\{0,1\}} \Big[P_i(\omega)\cdot ln\left(P_i(\omega)\right)\Big]\boldsymbol{\mu}_1(\omega) \tag{19}$$

For exposition simplification, define $P_i^j \equiv P_i(j)$ where $i \in \{0,1\}$ and $j \in \{T, H\}$. Expanding the summation above yields:

$$\begin{aligned} MI(\boldsymbol{\mu_1}) = &-\left(P_0^0 \cdot ln\left(P_0^0\right) + P_1^0 \cdot ln\left(P_1^0\right)\right) + \left(P_1^H \cdot ln\left(P_1^H\right) + P_0^H \cdot ln\left(P_0^H\right)\right) + \\ &\ldots\mu_1\left[\left(P_1^T \cdot ln\left(P_1^T\right) + P_0^T \cdot ln\left(P_0^T\right)\right) - \left(P_1^H \cdot ln\left(P_1^H\right) + P_0^H \cdot ln\left(P_0^H\right)\right)\right] \end{aligned} \tag{20}$$

If $P_1^0 \to 1$ then[9] $P_1^T, P_1^H \to 1$. Furthermore, if $P_1^0 \to 0$, then $P_1^T, P_1^H \to 0$. Using equation (20), this implies that, whenever $P_1^0 = 0$ or $P_1^0 = 1$, $MI(\boldsymbol{\mu}_1) = 0$. In other words, Receiver doesn't buy additional information when $\mu_1 \notin (\underline{\mu}, \overline{\mu})$.

■

- **Claim 2:** For a given λ, the amount of information bought is maximized at $\mu = \frac{1}{2}$.

The amount of information bought by Receiver is given by the mutual information between prior and posterior beliefs. Remark 1 implies that we can rewrite mutual information as follows:

$$MI(\mu;\lambda) = -\sum_{i\in\{0,1\}} P_i^0 ln\left(P_i^0\right) + \sum_{\omega\in\Omega}\sum_{i\in\{0,1\}} \Big[P_i(\omega) ln\left(P_i(\omega)\right)\Big]\boldsymbol{\mu}_1(\omega)$$

Furthermore, differentiation of this function indicates that mutual information is strictly

[9]See proof of Claim 3 below for the explicit solutions for P_1^T and P_1^H.

concave in beliefs ($\mu \in [0,1]$):

$$\frac{\partial^2 MI(\mu;\lambda)}{\partial \mu^2} = \frac{-1}{\mu \cdot (1-\mu)} < 0$$

Finally,

$$\frac{\partial MI(\mu;\lambda)}{\partial \mu}\Big|_{\mu=\frac{1}{2}} = 0$$

Hence, the amount of information bought is maximized at $\mu = \frac{1}{2}$

■

- **Claim 3:** $\underline{\mu}(\overline{\mu})$ is increasing (decreasing) in λ.

The following two derivatives suffice to prove our claim:

$$\frac{\partial \underline{\mu}}{\partial \lambda} = -\frac{b \cdot e^{\frac{b}{\lambda}}}{\left(1+e^{\frac{b}{\lambda}}\right)^2 \cdot \lambda^2} > 0$$

$$\frac{\partial \overline{\mu}}{\partial \lambda} = \frac{b \cdot e^{\frac{b}{\lambda}}}{\left(1+e^{\frac{b}{\lambda}}\right)^2 \cdot \lambda^2} < 0$$

■

- **Claim 4:** The optimal conditional probabilities satisfy $P_1(H) \leq P_1(T)$ for all μ_1.

By Theorem 1 in Matejka and McKay (2015), when $\lambda > 0$, the conditional choice probabilities are given by:

$$P_1(T) = \frac{P_1^0}{P_1^0\left(1-e^{\frac{b}{\lambda}}\right)+e^{\frac{b}{\lambda}}} \qquad P_1(H) = \frac{P_1^0 \cdot e^{\frac{b}{\lambda}}}{P_1^0\left(e^{\frac{b}{\lambda}}-1\right)+1}$$

$P_1(T) \geq P_1(H)$ is satisfied whenever $P_1^0 \leq 1$. The last inequality always holds by definition of P_1^0.

■

Proof of Proposition 1

The proof follows an analogous argument to the one in the proof of Proposition 3. In particular, Sender's utility function corresponds to the concavification of the profit function in equation (4). This utility function is attained when she follows the strategy described in equation (5).

■

11 Appendix D.

Proof of Proposition 6

This result hinges on the fact that the marginal benefit of Q_H and Q_L are always constant, while the marginal costs aren't. Let $F(Q_H, Q_L)$ denote the receiver's objective function.

($\gamma > 0$) Consider a situation where the unconstrained solution (Q_H^*, Q_L^*) lies below the ray describing the chosen expert in the constrained solution $(\hat{Q}_H, \hat{Q}_L)$. We want to show that this values satisfy $\hat{Q}_H > Q_H^*$ and $\hat{Q}_L > Q_L^*$. In the unconstrained solutions, the first order conditions for a maximum must hold. Let us consider the vertical projection of this solution to the relevant ray. At this point, we can show that:

$$\frac{\partial F}{\partial Q_L}\bigg|_{Q_L^*, Q_H(Q_L^*)} + \frac{\partial F}{\partial Q_H} \cdot \frac{\partial Q_H}{\partial Q_L}\bigg|_{Q_L^*, Q_H(Q_L^*)} > 0$$

The first term of the equition above corresponds to the direct effects of increasing Q_L. At this point, marginal costs excedes marginal benefits, therefore an increase in Q_L improves the overall utility by increasing entropy and reducing costs. The reverse argument applies for the first derivative of the second term in the equation (increasing Q_H reduces utility by increasing costs even more.) Finally, $\gamma > 0$ implies that the second derivative of the second term is negative. An analogous argument can be used to prove that when the projection is made horizontally, increasing Q_H results in an overall increase in utility.

A similar argument as the one presented above can be used to prove that, whenever the unconstrained solution lies above the ray describing the chosen experts, the optimal conditional probabilities satisfy $\hat{Q}_H < Q_H^*$ and $\hat{Q}_L < Q_L^*$

($\gamma < 0$) When the receiver faces a trade-off between precision in the high and low states, using only the signs of the above expressions is not sufficient to prove our result. However, direct calculations of the derivatives can solve this issue.

Suppose we are dealing with a type II expert. In that case, we know that $\frac{\partial Q_H}{\partial Q_L} < 1$. First order conditions for a maximum require:

$$\mu = \underbrace{-\mu[\ln(Q) - \ln(1-Q)] + \mu[\ln(Q_H) - \ln(1-Q_H)]}_{(1)}$$

and

$$-(1-\mu) = \underbrace{-(1-\mu)[\ln(Q) - \ln(1-Q)] + (1-\mu)[\ln(Q_L) - \ln(1-Q_L)]}_{(2)}$$

For our result to hold, we need to show that

$$\frac{-\partial(2)}{\partial Q_H} > \frac{\partial(1)}{\partial Q_H}$$

Which holds if and only if $Q_H(1-Q_H) > Q(1-Q)$. Given that $Q < Q_H < \frac{1}{2}$, the condition is satisfied. An analogous argument can show that this result holds when we consider type I experts for which $\frac{\partial Q_H}{\partial Q_L} > 1$

■

Chapter II

Crowding Out of Consumer's Information Acquisition in Search Markets

Crowding Out of Consumer's Information Acquisition in Search Markets

Mauricio Sauma Webb

November 3, 2021

Abstract

In a context where rational consumers are being persuaded to buy a product, it is natural for them to search for more information apart from that disclosed by sellers. Acquisition of information may imply accessing a private technology, or searching for another seller who can further inform the consumer. I apply the rational inattention framework to study how consumers benefit from access to a private information technology in a model of sequential search. It turns out the value of information is derived not from its use but from its availability. Private information technologies and competition among sellers limits their capacity to extract rents from information asymmetries. Even when sellers can coordinate on their disclosure policies, informed consumers may induce full revelation of private information.

sectionIntroduction

A consumer (he) is looking forward to buy a product. He would like to purchase the product only if its quality is high, and must pay a penalty for making the wrong decision; that is, he is penalized whenever (i) he buys a low quality product, or (ii) he rejects a high quality product. The product's quality is not an observable characteristic and is unknown to the consumer. Apart from information a seller (she) might disclose about her product, a consumer may become privately informed about its quality. His capacity to process information is costly. Moreover, there is a group of sellers that he can sequentially visit. If after being persuaded to buy a product he is not convinced to take this action, he may visit another seller.

Products are manufactured in a competitive market. Whenever a consumer buys the product from a seller, she receives strictly positive profits. When the product is not sold, she receives the status quo profit of 0. Sellers are, thus, not concerned about their product's quality but are only interested in selling it. They will strategically disclose information about the product's quality in order to persuade consumers to buy it.

Consumers receive information from two different sources. On a first stage, a persuading seller discloses information at no cost. Knowledge of the seller's incentives might make a consumer doubt about the information disclosed. In such cases, he may recur to a costly private technology to gain further knowledge about the product's quality. Whenever the two sources of information don't coincide on their assessment of the product, consumers may leave the current store and randomly sample another seller.

In this setting, I study how do the disclosure policies adopted by sellers depend on the consumer's ability to acquire information. Given that consumers are utility maximizers, their decisions to purchase information depend on the marginal benefits and costs of information acquisition. The marginal cost of information is always positive. Marginal benefits, however, are positive only if its reception can induce him to modify his action. I show that sellers can use this observation at their favor. If a seller's disclosure of information convinces the consumer that the product's quality is high[1], then changing his mind would require processing a large amount of information. The costs required by this mental processing and filtering of information are such that the consumer would prefer not to attend to the his information technology and buy the product.

I characterize the optimal disclosure policy followed by sellers in a symmetric equilibrium. To characterize this, I consider sellers who follow γ-strategies. According to these strategies, a seller will send only one of two possible signals: one of them induces interim beliefs $\mu_1 = 0$ while the other induces $\mu_1 = \gamma$. First, I will show that if γ induces information acquisition, then a seller can deviate and adopt a more profitable strategy that induces no information acquisition. Therefore, if a γ-strategy does not preclude information acquisition, it cannot be an equilibrium. Furthermore, the equilibrium disclosure policy must be such that, even when information acquisition is precluded, no individual seller may find a profitable deviation. The main theorem characterizes these equilibrium strategies.

Finally, I present comparative statics on equilibrium disclosure policies. Naturally, as the costs of acquiring information increase, consumer's incentives to become privately informed decrease. This allows sellers to adopt less informative policies. If information costs are prohibitive, sellers' equilibrium policies allows them to obtain monopoly profits. As information costs decrease, sellers's are required to disclose more precise information. Surprisingly, full revelation of sellers' private information may arise even when information costs are strictly positive.

This result may appear trivial at first sight. However it is in stark contrast to the results obtained from a similar model without the information acquisition technology. In Board and Lu (2018), for example, consumers face a similar problem to the one presented here. Their inability to become privately informed implies that if sellers follow symmetric strategies, no

[1] By convincing I mean that the consumer's posterior beliefs are "sufficiently" close to 1.

additional information will be disclosed to the consumer if he searches for a new seller. He must, however, pay a fixed cost of searching. According to this setting, if sellers are capable of coordinating their strategies, they can extract monopoly rents. Surprisingly, this result holds even for search costs that are arbitrarily close but greater than 0.

Related Literature – This work is directly related to the theories of persuasion and rational inattention. Beginning with the former, extensive literature has studied the optimal disclosure of information between agents with asymmetric information (Kamenica and Gentzkow (2011); Rayo and Segal (2010); Ostrovsky and Schwarz (2010)). All these models share the following assumptions: (1) decision makers have state dependent utility functions, (2) there is a preference mis-alignment between sellers and buyers, and (3) senders are the decision maker's only source of information. Under assumptions (1) and (2), it is natural to assume that the decision makers may benefit from additional sources of information.

Following this line, Guo and Shmaya (2019), Kolotilin (2018) and Kolotilin et al. (2017), study the value of private information held by the decision maker. In all these models, the decision to acquire information is not modeled; instead, they assume consumers have access to an exogenous and costless technology that provides information in the form of a signal. Extensions suggested in Kamenica and Gentzkow (2011) refer to cases where the decision maker can be informed. Independent of the timing of the signal's reception, they show that the results of their model are robust to these specifications. Most importantly, that the sender's utility function is still characterized by the concave closure of her payoff function.

Board and Lu (2018) study the disclosure policies adopted by sellers when they offer their product in a competitive market and buyers have no access to private information technologies. Their results imply that if sellers can observe the buyer's beliefs, equilibrium strategies disclose the "monopoly level" of information. This result is strongly dependent on the amount of information available to the buyer after he visits any seller. When sellers use symmetric strategies, a buyer that searches again will pay a positive fixed cost but will receive no information from the next seller. Hence, it is optimal for them to buy immediately. Furthermore, if sellers use the monopoly strategy, no seller has incentives to deviate and so, in equilibrium, they receive monopoly profits. The model I present builds on this framework but adds the asumption that consumers are rationally inattentive. It turns out that this latter assumption has strong implications on the model's equilibrium. In particular, even when considering sellers that can perfectly coordinate their disclosure strategies, monopoly profits can be attained only when information costs are prohibitive.

The inattention approach to information acquisition finds the answer to rational mistakes not in the availability of information, but in the fact the individuals have a limited capacity of processing it. Inattentive decision-makers make their decisions based on information they choose to attend, not on its availability[2]. Advantages of using the rational inattention framework include allowing the decision maker to have an unrestricted choice of the information processing technology. When there are monetary or cognitive costs associated with gathering of information, agents will take an information filtering process prior to their decision making and will, sometimes, make rational but mistaken decisions.

Agents that choose not to attend to a specific piece of information are said to be rationally inattentive. Beginning with Sims (2003), a growing literature has studied the consequences of this information processing behavior on economic outcomes in macroeconomics (Sims (2006), Acharya and Lin (2019)) and finance (Huang and Hong (2007), Mondria (2010)). Mostly related to the work presented in this paper is the work by Matejka and McKay (2015) who characterize the optimal information-processing behavior of a consumer who must choose among a discrete set of actions. In this context, information acquisition results in probabilistic choices: agents will determine the probability of choosing a specific action in a way that resembles the multinomial logit formula.

[2](Prasad et al., 2013) delivers evidence of 146 commonly practiced medical procedures that offer no net benefit to patients.

The rest of this paper is organized as follows. Section 2 presents the models's assumptions. Section 3 formalizes the buyer's optimization problem and solves for equilibrium information acquisition policies. Section 4 builds on these results and characterizes equilibrium disclosure policies. Section 5 concludes.

1 The Model

Our game is populated by one consumer and an infinite sequence of identical sellers. The consumer must decide between buying the product or not, $a \in A = \{1, 0\}$. The state of nature is drawn from the set $\Omega \equiv \{(0, b), (b, 0)\}$, where the first and second entries of each vector correspond to the consumer's payoffs if he chooses actions $a = 0$ or $a = 1$, respectively. Each state, thus, represents the benefit to the consumer if he purchases the product. Let $\omega^L \equiv (0, b)$ denote the state where the product is of low quality and $\omega^H \equiv (b, 0)$ where quality is high. Furthermore, let ω^i_a denote the payoff of choosing action a in state ω^i, $i = L, H$. We assume $b < 0$ for payoffs to be consistent with this definition. As opposed to consumers, sellers have state independent utility functions. We assume that a seller earns a payoff of 1 if her product is bought. Otherwise, she receives the status quo payoff of 0.

The consumer is initially uncertain about a product's quality. Both him and the seller share a common prior over the state of nature. Let $\boldsymbol{\mu_0} \in \Delta\Omega$[3] denote these beliefs, where $\boldsymbol{\mu_0} \equiv (\mu_0, 1 - \mu_0)$. Before being visited by a consumer, each seller commits to a specific mechanism of information disclosure. When a consumer is matched to a seller, the latter observes the state of nature and discloses information. Her strategy is a signal $\pi \in \Delta(R_1 \times \Omega)$. This signal consists of a finite realization space R_1 and a family of conditional distributions $\{\pi(\cdot|\omega)\}_{\omega\in\Omega}$. After observing the signal realization $r_1 \in R_1$, consumers update their beliefs to $\boldsymbol{\mu_1}$ according to Bayes's law and decide wether to pay for additional information or not.

[3] ΔX denotes the set of all probability distributions over X.

When a consumer acquires additional information, his technology will deliver a second signal $r_2 \in R_2$. His information strategy is a distribution $F(r_2, \omega) \in \Delta(R_2 \times \Omega)$ such that the marginal distribution over states is consistent with the belief $\boldsymbol{\mu}_1$. This latter condition implies that he is only free to choose the conditional distribution $F(r_2|\omega)$, for $F(\omega|r_2)$ is given by Bayes's law.

Let $\boldsymbol{\mu_2} \equiv F(\cdot|r_2)$ denote his updated belief after receiving signal r_2. Following the rational inattention literature, we will assume that the cost of any particular information strategy (F) is proportional to the expected reduction in entropy[4]:

$$\hat{C}(F) \equiv \lambda \left(H(\boldsymbol{\mu_1}) - \mathbb{E}_{r_2} \left[H \left(\boldsymbol{\mu_2} \right) \right] \right) \tag{21}$$

where λ is a positive parameter that represents the unit cost of information and the function H is corresponds to Shannon's Entropy:

Definition 1 (Entropy) *The entropy $H(\boldsymbol{\mu})$ of a given belief $\boldsymbol{\mu}$ is defined by*

$$H(\boldsymbol{\mu}) = - \left[\mu log(\mu) + (1-\mu) log(1-\mu) \right] \tag{22}$$

Conditional on acquiring information, the game proceeds as follows. If the information acquired validates the one disclosed by the seller, he buys the product and leaves the market. However, if the new signal and the seller's disclosed information don't coincide in the product's assessment, the consumer pays the search cost, samples a new seller and the sequence of events starts over.

[4]One could think more generally in any function C that is posterior separable and concave. For a discussion on this cost functions, see **?**

2 The Consumer's Problem

According to the rational inattention literature, the consumer may design signals and beliefs in any manner he wishes to. We follow the standard approach to further simplify his optimization problem. Given that information acquisition is costly, no two signals can lead to the same action, for information would be received but not acted upon. It is without loss of generality, then, to assume that what the consumer chooses is the probability with which he receives one of two signals conditional on the state. Each of this signals, in turn, corresponds to an action recommendation.

There are technical difficulties which come from including competition among sellers. When a seller is a monopolist, the expected payoff achieved by a consumer who follows his technology's recommendations is a sum of two parameters weighted by the likelihood of each state, according to posterior beliefs. However, this doesn't hold when consumers can search for new sellers. When there is competition between sellers, payoffs received from following recommendations must now be determined in equilibrium, for the posterior beliefs after searching depend on the disclosure policies adopted by sellers.

In order to formulate the consumer's information problem, we first define what will be meant when we say a seller follows a γ-strategy.

Definition 2 (Seller's Strategies)
We say that a seller follows a γ-strategy whenever he chooses two signals such that one of them induces belief 0 and the other induces belief γ. In this sense, these signals can be interpreted as the action recommendations to "not buy" and "buy", respectively.

According to Definition 2, when a consumer with belief μ is matched with a seller who follows a γ-strategy, the latter will recommend action "buy" with probability $\frac{\mu}{\gamma}$. Our next is to understand the consumer's decision after being persuaded to buy the product.

Consider a consumer who visits a seller who follows a γ-strategy and whose posterior belief after the persuasion stage is $\mu_1 = \gamma$ ($\mu_1 = 0$ implies he exists the market and has no further decisions to make). At this point, he has an expected utility of $V(\gamma)$. That is, $V(\gamma)$ corresponds to his value of standing at beliefs γ before he has the option to acquire additional information.

After acquiring information, a consumer will buy the product only if the received signal supports the information disclosed by the seller. If he does, he receives a payoff of $(1-\mu_2)\cdot b$. However, if his technology suggests the product is of low quality, he pays a fixed cost and arrives at a new seller. Our focus on symmetric strategies means that this new seller will also follow a γ-strategy, implying that the expected utility of this course of action is $\mu_2 \cdot \frac{V(\gamma)}{\gamma}$. This observation imposes a new complication on the framework presented in Matejka and McKay (2015), where there is a fixed payoff of following any action. We show that conditional choice probabilities are still determined by equations that resemble the multinomial Logit formula.

2.1 Consumer's Information Acquisition

From here on, we denote $Q^H \equiv \mathbb{P}(r = h|\omega = \omega^H)$ and $Q^L \equiv \mathbb{P}(r = h|\omega = \omega^L)$ as the conditional probabilities of buying the product in states ω^H and ω^L, respectively; and $Q \equiv \mu \cdot Q^H + (1-\mu)\cdot Q^L$ as the unconditional probability of taking this same action.

Suppose all sellers follow a γ-strategy and consider a consumer with belief μ who must decide how much information to acquire. This consumer's problem is defined by:

$$V(\mu) = \max_{\mu_2^h, \mu_2^\ell} \left\{ Q \cdot b \cdot \left[1 - \boldsymbol{\mu_2^h}\right] + (1-Q) \cdot \left[-c + \frac{\boldsymbol{\mu_2^\ell}}{\gamma} \cdot V(\gamma)\right] - \tilde{C}(\boldsymbol{\mu_2^h}, \boldsymbol{\mu_2^\ell}, \boldsymbol{\mu}) \right\}$$

$$\text{s.t } \mu = Q \cdot \mu_2^h + (1-Q)\cdot \mu_2^\ell$$

where μ_2^i denotes the posterior belief after receiving signal i and $\tilde{C}(Q^H, Q^L, \boldsymbol{\mu})$ is defined by:

$$\lambda\left[-Q\ln(Q)-(1-Q)\ln(1-Q)+\sum_{H,L}\left\{\mu\cdot Q^i\log(Q^i)+(1-\mu)(1-Q^i)\ln(1-Q^i)\right\}\right]$$

To solve this problem we must first find the value of $V(\gamma)$, the value of a consumer standing at beliefs γ. Let us then consider a consumer who has already been persuaded by a seller and whose interim beliefs are $\mu=\gamma$. In this case, the consumer's problem can be rewritten as:

$$V(\gamma)=\max_{\mu_2^h,\mu_2^\ell}\left\{Q\cdot b\cdot\left[1-\boldsymbol{\mu_2^h}\right]+(1-Q)\cdot\left[-c+\frac{\boldsymbol{\mu_2^\ell}}{\gamma}\cdot V(\gamma)\right]-\tilde{C}(\boldsymbol{\mu_2^h},\boldsymbol{\mu_2^\ell},\boldsymbol{\gamma})\right\} \tag{23}$$
$$\text{s.t } \gamma=Q\cdot\mu_2^h+(1-Q)\cdot\mu_2^\ell$$

Finding the consumer's information acquisition policy requires solving for the probability of receiving each signal conditional on the state. This probabilities are characterized by the following proposition.

Proposition 1 (Conditional choice probabilities)

The problem in Equation 23 has optimal choice probabilities given by:

$$Q^H=\frac{\gamma e^{\frac{b+c}{\gamma}}+(1-\gamma)e^{\frac{V(\gamma)+\gamma(2b+c)}{\lambda\gamma}}-e^{\frac{V(\gamma)+b\gamma}{\lambda\gamma}}}{(1-\gamma)\left(e^{\frac{V(\gamma)+b\gamma}{\lambda\gamma}}-1\right)\left(e^{\frac{b+c}{\lambda}}-1\right)} \tag{24}$$

$$Q^L = \frac{\gamma e^{\frac{b+c}{\gamma}} + (1-\gamma) e^{\frac{V(\gamma)+\gamma(2b+c)}{\lambda\gamma}} - e^{\frac{V(\gamma)+b\gamma}{\lambda\gamma}}}{\gamma \left(e^{\frac{V(\gamma)+b\gamma}{\lambda\gamma}} - 1 \right) \left(e^{\frac{V(\gamma)+b\gamma}{\lambda\gamma}} - e^{\frac{b+c}{\lambda}} \right)} \tag{25}$$

Proof. See Appendix A ■

It's important to note that $V(\gamma)$ remains an unknown in equations (24) and (25). Substituting the optimal choice probabilities back into equation (23) produces an expression of the form $V(\gamma) = G\big(V(\gamma)\big)$, where G is non-linear function of $V(\gamma)$. The consumer's utility is, therefore, a fixed point of the operator G. $V(\gamma)$ and, consequently, Q^H and Q^L can be solved for numerically. The higher γ is, the more informative are sellers' disclosure policies. As Figure 10 illustrates this implies that consumers enjoy higher levels of utility.

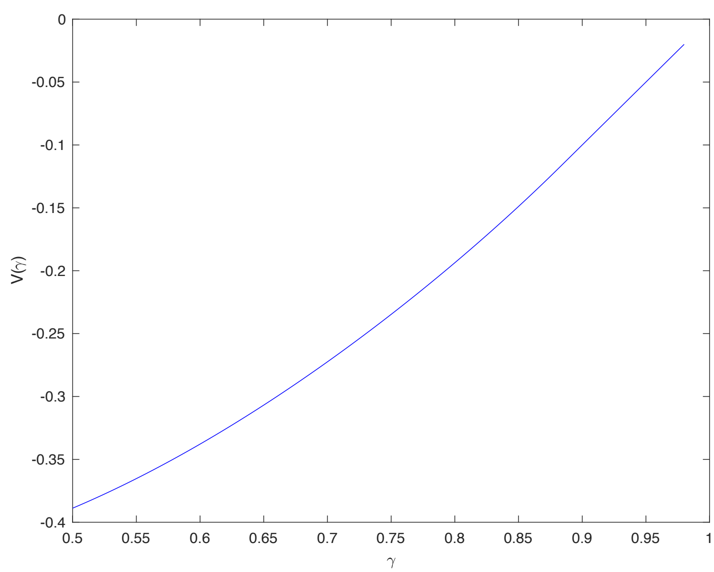


Figure 10: Consumer's Expected Utility at γ

$\big(b = -1, c = 0.1, \lambda = 0.9\big)$

Once $V(\gamma)$ is found, we may use this information to solve for the consumer's problem with off-path beliefs.

The consumer's information acquisition policy is an important input to the seller's optimization problem. In particular, equations (24) and (25) allow the seller to guess with what probability will he sell the product if he follows a γ-strategy. By increasing γ, sellers can increase the probability with which the product is sold after recommending this action. However, sellers face a tradeoff between the strength of their action recommendations and the frequency with which this recommendation is given. The following section takes into account this tradeoff to characterize the equilibrium disclosure policy, γ^*.

3 Seller's Disclosure Policy

Consider a consumer who, after acquiring information, receives a signal that supports the information disclosed by the seller. If this is the case, there is nothing preventing him to recur to his private technology once again. Indeed, if the experiment is sufficiently informative, he may benefit from purchasing information a second time. We assume that when consumers receive a good signal they buy the product immediately, instead of acquiring more information. Lemma 2 in Appendix B shows that this is without loss of generality.

The set of beliefs where Shannon's Mutual Information takes positive values will define the region where the consumer purchases a positive amount of information. Finding this region reduces to characterizing the interval where $Q^L < Q^H$ (for $Q^H = Q^L$ implies the technology is uninformative). In Figure 12, values of γ for which the green line (posterior beliefs after information acquisition) is above the blue one (posterior beliefs after the persuasion stage), means that information is being acquired. Lemma 1 characterizes the set of beliefs where this is true.

Lemma 1

When sellers follow γ-strategies, consumers actively search for information whenever their beliefs lie in the interval $\mathcal{I} = \Big(\underline{\mu}(\gamma), \overline{\mu}(\gamma)\Big)$, where:

$$\underline{\mu}(\gamma) = \frac{e^{\frac{V(\gamma)+b\gamma}{\lambda\gamma}} - e^{\frac{V(\gamma)+\gamma(2b+c)}{\lambda\gamma}}}{e^{\frac{b+c}{\lambda}} - e^{\frac{V(\gamma)+\gamma(2b+c)}{\lambda\gamma}}} \tag{26}$$

$$\overline{\mu}(\gamma) = \frac{e^{\frac{b+c}{\gamma}} - 1}{e^{\frac{V(\gamma)+b\gamma}{\lambda\gamma}} - 1} \tag{27}$$

Proof. See Appendix A. ■

It can be shown that $\gamma \in \mathcal{I}$. Therefore, for information acquisition to be precluded, sellers' disclosure policies must imply that $\mathcal{I}$ has empty interior. The smallest γ that satisfies this property can be seen in Figure 12 as the first point where the green and blue lines intersect. Theorem 1 states that this value of γ corresponds to the equilibrium disclosure policy.

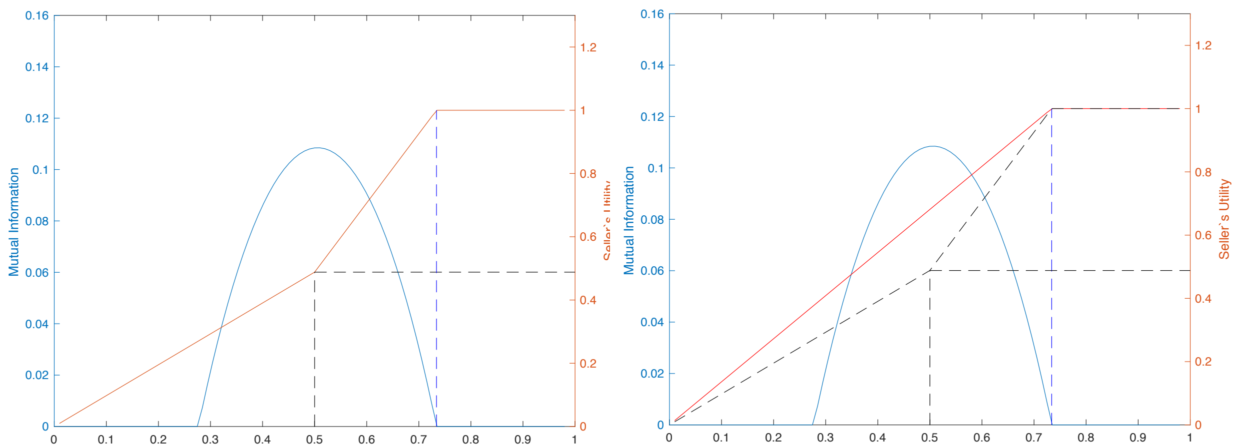
Theorem 1 *A situation where every seller follows a γ^*-strategy is an equilibrium if and only if:*

$$\gamma^* = \underline{\mu}(\gamma^*) = \overline{\mu}(\gamma^*)$$

Proof. See Appendix A. ■

The constructive approach in Theorem 1's proof provides clarifying intuition behind this result. Suppose every seller follows a γ-strategy. If, at $\mu = \gamma$, a consumer searches for more information, a profitable deviation from this strategy can be found. When the consumer isn't self informed, standing at beliefs γ would yield a profit of 1 to the seller. However, when he searches for information, the signal realization might make him doubt the current seller and go to the next one. This means her expected profit is less than 1 whenever her disclosure policy incentives information acquisition. Figure 11a illustrates a seller's utility function when $\underline{\mu}(\gamma) < \gamma < \overline{\mu}(\gamma)$. The convexity of this function in the interval $\big(0, \overline{\mu}(\gamma)\big)$ means that she can increase her expected utility by deviating to a $\overline{\mu}(\gamma)$-strategy. Figure 11b compares the seller's utility before (dotted line) and after (red line) the strategy deviation.

Let γ' denote this deviation. If $\underline{\mu}(\gamma') < \gamma' < \overline{\mu}(\gamma')$, the same argument can be reproduced to show that she can further increase her utility by modifying her disclosure policy. However, if γ' is such that information acquisition is prevented, then, no profitable deviation can be found, and γ' will characterize the equilibrium disclosure strategy. Intuitively, whenever consumers are willing to pay for additional information, and given that sellers can disclose information at no costo, it is profitable for a seller to provide this knowledge herself.



(a) Utility with informed consumer

(b) Optimal Deviation

Figure 11: Crowding out of Information Acquisition

Corollary 1 *A seller's optimal disclosure policy completely crowds out information acquisition. The product is, thus, sold without delay.*

Proof. This results follows directly from the proposition above. On the understanding that consumers will never acquire additional information, there will never be a signal that contradicts sellers' recommendations. Hence, after being persuaded, he will either buy the product or leave the market, but will never search again. ■

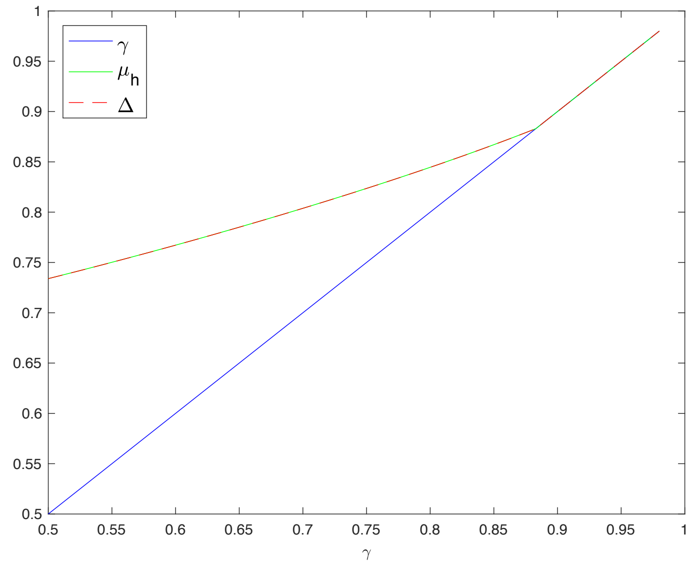


Figure 12: Δ and $F(H|h)$

$(b = -1, c = 0.1, \lambda = 0.9)$

Equilibrium disclosure policies can be solved for by iterating over profitable deviations. Figure 12 shows, for every value of $\gamma \in \left[\frac{1}{2}, 1\right]$, the optimal deviations (green line). For expositional purposes, the identity function has been plotted in blue. For low values of γ, the optimal deviation is greater than γ. As γ increases, the difference between its value and the optimal deviation converges to zero. Given that sellers are utility maximizers, the optimal disclosure policy is defined as the smallest value of γ that precludes information acquisition. The equation that characterizes this solution is presented in Theorem 2.

Theorem 2

Let $c, \lambda > 0$. A seller's optimal disclosure policy, γ^* is implicitly defined by:

$$e^{\frac{b+c}{\lambda}} - 1 = \gamma^* \left[e^{\frac{b}{\lambda \cdot \gamma^*}} - 1 \right]$$

Furthermore, γ^* is weakly decreasing in the unit cost of information, λ, and in the fixed search cost, c.

Proof. We are looking for γ^* such that $\gamma^* = \overline{\mu}(\gamma^*)$. Crowding out of information acquisition implies that $V(\gamma^*) = b \cdot (1 - \gamma^*)$, therefore, the optimal persuasion strategy is implicitly defined by:

$$\frac{e^{\frac{b+c}{\lambda}} - 1}{e^{\frac{b}{\gamma^* \lambda}} - 1} = \gamma^* \tag{28}$$

Rearranging equation (28) gives us our desired result. ■

Figure 13 shows the value of γ^* for pairs $(c, \lambda) \in [0, 1]^2$. Cooler (warmer) colors correspond to low (high) levels of information disclosure. Once again, this picture shows that there is a smooth and continuous reduction in the amount of information disclosed by sellers as information costs increase. Compared to a model without information acquisition, where sellers attain monopoly profits, in this setting the possibility to extract monopoly rents is limited by the consumer's private information technology. By perturbating their beliefs, a consumer can always learn something from visiting a new seller. This erodes the rents extracted by sellers.

A better informed consumer, therefore, induces the seller to disclose more information that he would otherwise do in a setting without information acquisition. The amount of information disclosed is just enough to preculde additional information acquisition by consumers. The ability to learn empowers the consumer and harms the sellers in such a way that sellers preferr to disclose this information, at no cost, in order to preculde consumer's acquisition of information. This prevents the consumer from leaving the current seller with no profit.

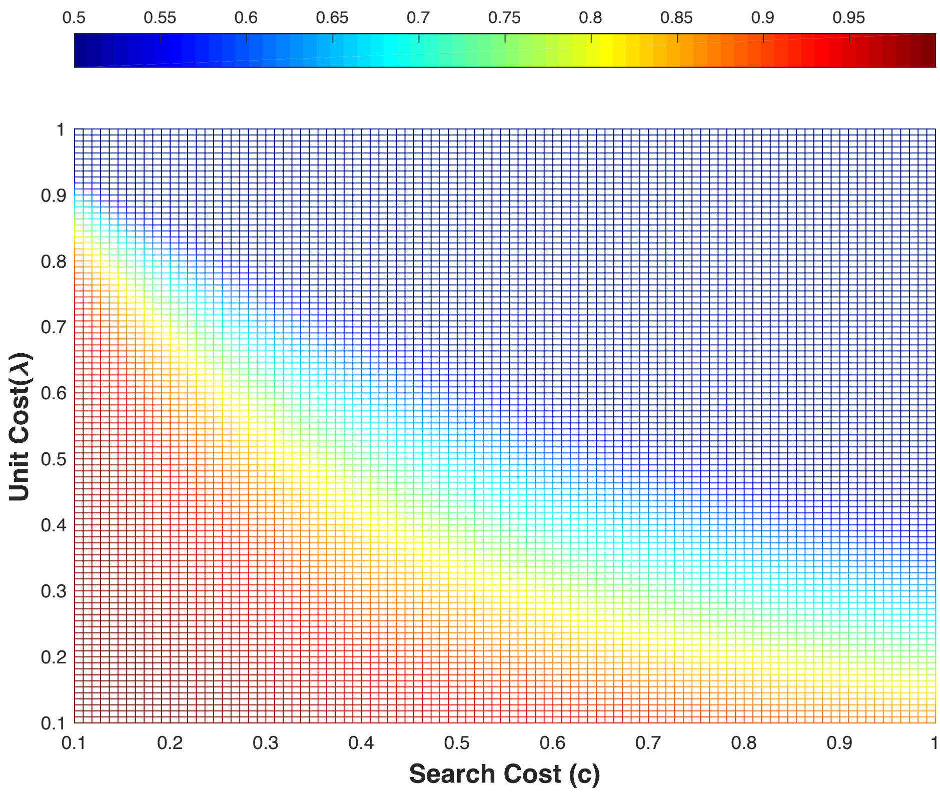


Figure 13: Equilibrium disclosure policy (γ^*)
($b = -1$)

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4 Appendix A: Proofs

Proof of Proposition 1

First order conditions w.r.t Q^H and Q^L, respectively, imply:

$$\frac{V(\gamma)}{\lambda \cdot \gamma} - \frac{c}{\lambda} = \ln\left(\frac{1-Q^H}{Q^H}\right) + \ln\left(\frac{P_h^0}{1-P_h^0}\right) \tag{29}$$

$$\frac{b+c}{\lambda} = \ln\left(\frac{Q^L}{1-Q^L}\right) + \ln\left(\frac{1-P_h^0}{P_h^0}\right) \tag{30}$$

Together, equations (29) and (30) imply:

$$Q^H = \frac{Q^L}{Q^L + (1-Q^L)\cdot e^z} \tag{31}$$

where $z = \frac{1}{\lambda \cdot \gamma}\left[V(\gamma) + b \cdot \gamma\right]$. Substituting (31) in (29) yields our expression for Q^H (equation (24)). Using (31) again yields equation (25).

■

Proof of Lemma 1

Suppose sellers follows γ-strategies. The consumer will acquire a positive amount of information whenever $Q^H > Q^L$. The interval where this condition is satisfied is defined by the 2 values of beliefs where $Q^H = Q^L$ holds. In that sense, $\underline{\mu}(\gamma)$ and $\overline{\mu}(\gamma)$ are defined, respectively, as the smallest and largest roots of $Q^H - Q^L$. These roots correspond to the expressions in equations (26) and (27).

■

Proof of Theorem 1

Suppose that the Seller follows strategy $0, \gamma$ and that $\underline{\mu}(\gamma) < \gamma < \overline{\mu}(\gamma)$. Following Lemma 2, the consumer will acquire a positive amount of information after he is persuaded to buy the product. We will show that, in this case, the Seller's preferred deviation is $\gamma' = \overline{\mu}(\gamma)$. Let μ define the consumer's prior beliefs when being matched with the seller and consider the following scenarios:

$\boldsymbol{\mu \in (\overline{\mu}(\gamma), 1]}$: In this interval there is no information acquisition nor any information disclosed by the seller. The consumer buys the product with certainty and the seller gets a payoff equal to 1.

$\boldsymbol{\mu \in (\gamma, \overline{\mu}(\gamma)]}$: Here, there is no information disclosed by the consumer but the seller does acquire information on his own. The seller's payoff equals the unconditional probability that the consumer receives signal h (P_h^0). This probability is linear in μ and satisfies:

$$\frac{\partial P_h^0(\mu)}{\partial \mu} = \frac{e^{\frac{c}{\lambda}} - e^{\frac{V(\gamma)+(b+c)\gamma}{\gamma \cdot \lambda}}}{\left(e^{\frac{b+c}{\lambda}} - 1\right)\left(e^{\frac{V(\gamma)}{\gamma \cdot \lambda}} - e^{\frac{c}{\lambda}}\right)}$$

$\boldsymbol{\mu \in [0, \gamma]}$: In this interval, the seller's disclosure policy persuades the consumer to acquire the product with probability $\frac{\mu}{\gamma}$. At beliefs γ, the consumer acquires information and receives signal h with probability $P_h^0(\gamma)$. Therefore, the seller's expected payoff is given by the expression $\frac{\mu}{\gamma} \cdot P_h^0(\gamma)$. It is, therefore, linear in μ with a slope of $\frac{1}{\gamma} \cdot P_h^0(\gamma)$

The assumption that $|b| > |c|$ guarantees that $\frac{\partial P_h^0(\mu)}{\partial \mu} > \frac{1}{\gamma} \cdot P_h^0(\gamma)$ and, therefore, the seller's utility function is convex in $[0, \overline{\mu}]$. It is easy to verify that this function is continuous in its complete domain. Following Kamenica and Gentzkow (2011), we know that the seller can increase his utility by following a strategy characterized by the concavification of his expected utility function. This means that $\gamma' = \overline{\mu}(\gamma)$. See Figure 11, panel (b).

If $\gamma' = \overline{\mu}(\gamma')$, the seller has no profitable deviations left and, hence, γ' characterises his optimal persuasion strategy. However, when $\gamma' < \overline{\mu}(\gamma')$ we can apply the argument above again to show that there exists yet another profitable deviation.

The observations that $\overline{\mu}(\gamma)$ is a continuous function and that $\lim_{\gamma \to 1} \overline{\mu}(\gamma) < 1$, together, imply that there exists a value γ^* such that $\gamma^* = \overline{\mu}(\gamma^*)$.

■

5 Appendix B. Secondary Results

Lemma 2

Let $\Delta \in (\gamma, 1)$ *be the belief where a consumer is indifferent between buying information or buying the product. That is,* Δ *satisfies:*

$$(1 - \Delta) \cdot b = V(\Delta)$$

The consumer's equilibrium strategy implies $\Delta = F(H|h)$*.*

Chapter III

Policy Diverging Effects of Public Funding in Political Elections

Policy Diverging Effects of Public Funding in Political Elections.

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Abstract

We consider a model in which two candidates compete for office in political elections. Candidates do not have their own revenue sources, hence they may resort to seek funding for their campaign expenditures by either acquiring debt or by taking money from an interest group. After the election takes place, each candidate receives governmental transfers proportional to the electoral results. In this context, we examine the optimal fundraising strategies chosen by each candidate and its implications on the type of equilibria that may arise. We show that there is an equilibrium in which increases in the public budget, designated to finance campaign spending, increases the divergence in the competing candidate's proposed policies. This result is stronger the more uninformed the electorate is about candidates' proposed policies.

1 Introduction

There is a widely held view that the use of public funds to finance political campaigns is central to a functional democracy. Among other reasons, their advocates argue that it is a fundamental instrument for leveling the playing field and increasing competition during electoral contests. Further arguments in favor of public funding provisions include avoiding the influence of illegal funding on politics and limiting the possibility of wealthy interest groups conditioning their support to political favors or contracts once candidates are elected. Following this line, it is commonly argued that public funding enhances the democratic functioning of the government, as it promotes candidates with views that are more in line with those of the median voter as opposed to those of interest groups. Yet, there seems to be no consensus of the effects of public funds on political outcomes.

On the other hand, those who support less use of public funds for campaign finance argue that it may incentive the abuse of state resources. There is also the populist view that condemns the use of taxpayers' money to finance the campaign expenses of a few. When referring to the discontent caused by the use of these funds in electoral contests, Casas and Zovatto (2015) argue: "*(. . .) the introduction of state subsidies has been full of controversy not only for the cost imposed on taxpayers (...) but also for the uncertainty about its effects. (. . .) detractors have long maintained its inefficiency as an instrument for protection against the influence purchase of political actors.* "

Even when this is true, there has been an almost universal implementation of some version of public funding since its adoption by Uruguay back in 1928. For example, every OECD member (except Switzerland) provides public funding for campaign elections. Data from the ACE Electoral Knowledge Network shows that, up to date, at least 70% of countries provide some type of public funding to finance candidate's or party's campaign expenses. (See Figure 14[1]).

Figure 14 may lead to misleading conclusions, for even when a country's Political Financing System (PFS) allows for state provisions to pay for candidate's expenses, this does not assure that every political party will opt in and make use of these funds. In that sense, Figure 14 addresses the availability of public funds, not its adoption. For example, consider the general elections in the United States. Its PFS provides public funding in both direct and indirect ways. Data from the Federal Election Commission[2] shows that former president, Barack Obama, set a precedent in the 2008 presidential campaign by specializing in private fundraising. Out of the $84 million in public funds spent in the general election, 100% were allocated to the Republican candidate, John McCain. In subsequent elections, no public funds have been spent to finance presidential political campaigns. Although some parties may prefer not to receive public funding, there is a large set of countries where candidates do receive these funds. Table 3 illustrates the adoption of public funding in selected countries of the European region. Greece and Turkey are at the top of the board with public funding

[1] Missing values in gray.

[2] www.fec.gov - accessed on June 14, 2019.

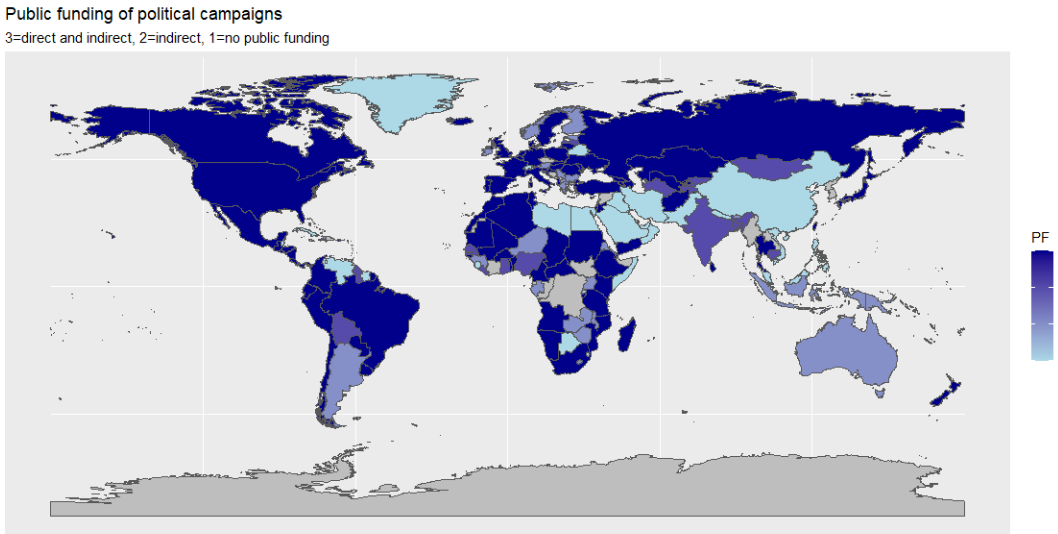

Figure 14: Public Funding Worldwide
(Data: Electoral Knowledge Network)

amounting to 90% of total funding. Norway (67.5%) and Hungary (60%) have a more balanced funding regime; the United Kingdom ranks last with 35%.

30% – 50%	50% – 70%	70% – 80%	80% – 100%
France	Hungary	Denmark	Belgium
Germany	Norway	Finland	Greece
Netherlands		Iceland	Italy
United Kingdom		Poland	Portugal
		Sweeden	Slovak Republic
			Spain
			Turkey

Table 3: Public funding as a percentage of political parties' total funding.
(Source: Greco Third Evaluation Round)

Mechanisms to allocate public funds amongst competing candidates vary from one country to another. Examples of allocating mechanisms include a flat rate distributed equally among competing candidates and the allocation of funds proportional to some performance measure in elections (e.g votes or seats received in the previous or current election). Countries such as Germany and Netherlands establish additional rules that define an upper bound on state contributions proportional to private funds raised by candidates. These rules, sometimes referred to as "matching funds", are said to encourage citizen participation and, more importantly, strengthen the link between parties and voters[3].

[3]https://www.loc.gov/law/help/campaign-finance/germany.php

Our model will consider a mechanism such that public funds are distributed in proportion to the vote share obtained in the immediately previous election. Variations of this funding mechanism have been adopted in a vast amount of countries in Europe (France, Germany, Spain, Sweden), America (Canada, Chile, Costa Rica), Asia (Indonesia), Africa (Mauritania, Namibia), and Oceania (Australia, New Zealand) [4].

Our model's predictions will shed some light over the prevailing uncertainty concerning the politic implications of public funding. We find that the arguments presented both for and against public funds are all sound depending on the configuration of the electoral contest, in particular, the heterogeneity in interest groups's support and the candidates' preferences for being elected. More precisely, we find that in a model in which parties compete for funds in order to convince uninformed voters, a government that provides public financing of campaigns may end up, counter-intuitively, promoting policy divergence and polarization.

Our emphasis on policy polarization stems from the observation that, when voter's preferences are distributed uniformly along the political line, polarization will reduce the average voter's welfare. Nevertheless, it is worth noting that policy polarization hasn't always been thought of as a welfare decreasing phenomenon. Back in the 1950's, the American Political Science Association (APSA) published a document titled "Towards a more responsible two-party system" calling for more polarization in the political system. According to their views, the heterogeneity of political ideas within each party allowed them to work together too easily. This ideology overlap meant that the voters struggled to make a decision: "*When there are two parties identifiable by the kinds of action they propose, the voters have an actual choice. On the other hand, the sort of opposition presented by a coalition that cuts across party lines, as a regular thing, tends to deprive the public of a meaningful alternative*".

In the model we present, parties face a trade-off between convincing informed voters (who know candidates' platform positions) and obtaining funds from policy-motivated interest groups in order to finance campaign advertising and capture uninformed agent's votes. When the government introduces its public campaign funding rule, it enhances the electoral competition. However, when there are asymmetries in interest groups support, intermediate levels of public funding may induce polarization as higher competition makes the strong-interest-group candidate more prone to distorting it's policy position in order to guarantee a favourable electoral result.

We build on the extensive literature that studies the role of advertising in presidential elections. Prat (2002) considers elections as signalling games (indirectly informative advertising) and Coate (2004) models campaign advertisement as a way of conveying truthful information (directly informative advertising). Coate (2004) offers a comprehensive model in which candidates can use advertising to provide voters with hard information about their policy positions, ideologies, or qualifications for office. However, his approach focuses solely on the "matching-funds" technology which is not broadly adopted by governments in reality. Contrary to our model, Desai and Duggan (2020) develop a model where campaign advertising might benefit or hurt candidates. They model a situation where one challenger is competing

[4] For further information, visit: http://aceproject.org/.

against the incumbent. Each candidate is supported by one interest group,

More importantly, this paper contributes to the much less explored literature (despite it's important relevance and widespread adoption) on public funding of electoral contests. Notably, Coate (2004) and Ashworth (2006) develop models with public funding provision according to the matching funds rule. However, their models focus on the effects on voters' well-being when parties carry out informative advertising (direct or indirect), rather than on imminent candidate polarization.

Among the first attempts to study this phenomenon is Ortuno-Ortin and Schultz (2005). Their approach, however, does not allow for private financing strategies. Their public funds allocation rule is such that both parties receive a fixed amount of money plus a quantity that depends on the proportion of votes received in the election. Parties are policy motivated and each party's preferred policy is more extreme than the median voter's preference. The authors show that public funding of campaigns make policies converge when there is a positive amount of impressionabel voters.

Our model is similar to that of Baron (1994) and Grossman and Helpman (2001), who include voters that may be informed or uninformed about candidates' policy positions and interest groups with policy preferences that offer private funding in exchange of the candidate's commitment to depart from the median towards the interest group's position. As in our framework, parties face a trade-off between winning the votes of informed voters and reaching to the uninformed voters trough advertising. In particular, Baron (1994) extends his model to account for public financing of elections in a lump-sum fashion. With this technology, he argues that public funding results in equilibrium policy positions that are closer to the median. Our model, however, shows that this is in general not true when considering alternative mechanisms of public financing.

Closely related to our work is Hall (2014) who addresses, empirically, the issue of the polarizing effects of public funds. His results are based on a regression discontinuity design using data on U.S state legislature elections for Arizona, Conneticut, Maine, Minnesota and Winsconsin. He shows that an increase in public funding for running candidates results in a decrease of the electoral incumbency advantage and an increase in overall legislature's polarization.

Given that public funds seek a closer connection between parties platforms and voters' preferences, its implementation is regularly accompanied by limits on interest group donations. Public funding, then, reorganizes the electoral portfolio of candidates, reducing the relative importance of interest group money and increasing the share of individual donations. If individual donors tend to support more extreme candidates, the result will be one of higher competing power of extreme ideological parties. This last argument is supported by Barber (2016) who shows that polarization will decrease if further limits are set on the individual level donations.

Once a state provision of funds is set in place, candidates still need to take a strategic decision whether to accept this funds or not. Contrary to the pattern observed at state level elections, at the federal level parties have opted not to use public funds and increase their dependence on interest groups and private donors, avoiding the donation limits imposed by the public fund provision. When it comes to fundraising by parties, the 2020 election campaigns were record-breaking. Together, Joe Biden and Donald Trump raised more than 7 billion dollars from private donors[5], more than double the amount in each of the previous two presidential elections.

The remainder of this paper is organized as follows. Section 1 presents a simple model of electoral competition. Section 3 characterizes three possible equilibrium outcomes of the electoral fundraising strategies. Section 4 presents our main result; the increasing polarization effect of public funds in presidential elections. Finally, Section 5 concludes.

2 The Model

Overview.– A country's population consists of two groups of agents. One group is comprised of two political candidates competing for office. The second group is integrated by a continuum of voters of mass 1.

Political candidates must choose what policy to implement if they are elected. Let $p_i \in [0, 1]$ denote the platform location chosen by candidate i. Without loss of generality, we will assume that candidates 1 and 2 have leftist and rightist ideologies, respectively. This assumption implies that platform locations must satisfy $p_1 \leq \frac{1}{2} \leq p_2$.

A fraction k of the constituents are *impressionable* voters. The voting decisions of these agents are based on media and advertising. The rest of the constituents are *informed* voters. These agents make their voting decisions based on political candidates' platform locations.

Political candidates are allowed to raise public and private funds in order to finance their advertising campaign. These advertisements are set to persuade impressionable agents to vote for a given candidate. We assume that an impressionable voter will vote for candidate i if he receives a piece of advertisement from him. Furthermore, no voter can receive two pieces of advertisement.

Following Coate (2004) we assume private funds are distributed by two interest groups, each supporting one candidate[6]. Candidates may increase private donations by proposing platforms closer to the interest group's bliss point. As in Desai and Duggan (2020), interest groups are located in each end of the political line and are, therefore, completely polarized.

[5] http://followthemoney.org/

[6] Even when one interest group may, in practice, support candidates from opposing ideological stances, the assumption is in line with empirical evidence from state level elections: "[After public funding is put in place] . . . almost every Democratic candidate received all of her donations from donors that *only* gave to Democrats and almost every Republican candidate received all of her donations from donors that *only* gave to Republicans" (Hall, 2014).

Let α_i be a measure of each interest group size (or fund availability). The amount of private funds received by candidate i when he chooses platform p_i, C_i^p, are determined by the following rules[7]:

$$C_1^p = \alpha_1 \cdot (1 - p_1) \tag{32}$$

$$C_2^p = \alpha_2 \cdot p_2 \tag{33}$$

Candidates may also finance their advertising campaign with public funds. Each candidate can borrow an amount t_i from the government. After the election takes place, the government will distribute a total budget of T between the two candidates according to the total vote share obtained by each one. Let $v \in [0, 1]$ denote candidate 1's expected vote share, then, the amount of funds received by candidate i is determined by the following rule:

$$C_1^g = v \cdot T \tag{34}$$

$$C_2^g = (1 - v) \cdot T \tag{35}$$

Total funds raised by candidate i (C_i) are:

$$C_i = C_i^p + C_i^g \tag{36}$$

Candidates' outstanding debt with the government at the end of the election is:

$$\max \left\{0, t_i - C_i^g\right\}$$

Voters' Decisions.– Voters' utilities are a function of the policy implemented by the elected candidate. Each voter j is characterized by his location (or bliss point) in the political line, x_j. The utility of a voter with bliss point x is given by the following expression:

$$u(x_j, p) = \left|x_j - p\right| \tag{37}$$

where p is the platform chosen by the elected official.

We suppose that informed constituents are uniformly distributed along the political line. If

[7] α_i corresponds to the marginal increase in private funds when each candidate moves it's platform further from the center of the political line. Therefore, α_i can also be interpreted as the marginal willingness to pay for a more extremist policy.

p_1 and p_2 are the proposed platforms by candidates, the location of the informed voter who is indifferent between voting for either candidate, $\overline{x}$, is determined by:

$$\overline{x} = \frac{p_1 + p_2}{2} \tag{38}$$

Hence, given p_1 and p_2, the amount of informed agents voting for candidate 1 is given by $(1-k) \cdot \frac{p_1+p_2}{2}$.

Impressionable constituents make their voting decisions based on advertisements received. Let $\lambda_i : \mathbb{R}^2_+ \mapsto [0,1]$ be a function that determines the fraction of impressionable agents that vote for agent i. Then, given C_1 and C_2:

$$\lambda_i(C_i, C_j) = \frac{C_i}{C_i + C_j} \tag{39}$$

Clearly, $\lambda_1 + \lambda_2 = 1$, meaning that every impressionable agent receives advertisement from one candidate. We believe this technology captures an empirically relevant characteristic of political campaign advertising. When λ_i is a function of the relative amount of money spent by agent i, and not on its absolute amount, it allows competition for media slots which results in wasteful advertising. Formally, for any given equilibrium, there is another equilibrium that replicates the election's outcome with less spending on advertising by each candidate.

Candidates' payoffs.– Each presidential candidate is worried about the percentage of votes he receives and how indebted he is at the end of the election. Let $\theta_i > 0$ denote the relative importance that candidate i assigns to his vote share. Candidate's utilities after the election has taken place are:

$$U_1(p_1, p_2, t_1, t_2) = [\theta_1 + T]\left[\lambda_1(C_1, C_2) \cdot k + (1-k) \cdot \overline{x}\right] - t_1 \tag{40}$$

$$U_2(p_1, p_2, t_1, t_2) = [\theta_2 + T]\left[\lambda_2(C_1, C_2) \cdot k + (1-k) \cdot (1-\overline{x})\right] - t_2 \tag{41}$$

Candidates 1 and 2 choose their platform locations and the amount of public money borrowed in order to maximize (40) and (41), respectively.

3 Equilibria Characterization.

We begin our analysis of the model with a characterization of the fund raising strategies implemented in equilibrium.

Proposition 1 (Equilibrium Fundraising Strategies) *In equilibrium, a candidate will borrow a positive amount moeny if and only if he places his platform in the center of the political line:* $\left(p_i \neq \frac{1}{2}\right) \iff (t_i = 0)$ *Furthermore, candidate* i *will set* $p_i = \frac{1}{2}$ *whenever the following condition is met:*

$$\frac{\theta_i + T}{\alpha_i} \geq \frac{2}{1-k} \tag{42}$$

Proof. See Appendix 1 ■

Proposition 1 implies that candidates will specialize in only one source of fundraising for advertisement. This does not mean that he will only receive funds from the source he specializes in. In fact, in every possible equilibrium, presidential candidates receive both public and private funds.

To see this notice that, even when candidate i sets $t_i = 0$, the amount of private money spent in advertising will imply that his vote share is strictly positive. As a result, public funds received after the election will be strictly positive too. Furthermore if he sets $p_i = \frac{1}{2}$, equations (32) and (33) imply that he will raise a strictly positive amount of private funds whenever $\alpha_i \neq 0$[8].

The condition for public fundraising will be met whenever candidates assign a high priority to their vote share, when the total funds allocated by the government are sufficiently high or whenever interest groups are small. Furthermore, given T, θ_i and α_i, a higher proportion of impressionable voters increases the probability that candidates recur to interest groups for fundraising.

Corollary 1 *Equilibrium fundraising strategies induce one of the three following equilibria:*

1. ***Publicly Financed Elections:*** *Both candidates locate their platforms at the center of the political line and* $t_i > 0$ *for* $i = 1, 2$.
2. ***Privately Financed Elections:*** *Platform positions diverge from* $\frac{1}{2}$. *Candidates propose equally extremist policies. Finally,* $t_i = 0$ *for* $i = 1, 2$.
3. ***Mixed Financed Elections:*** *Candidate* i *chooses* $p_i = \frac{1}{2}$ *and* $t_i > 0$. *Candidate* j *chooses* $t_j = 0$ *and possibly interior values for* p_j.

Proof.

The realization of each equilibrium depends on the comparison between the marginal cost of raising public funds and that of raising private funds. The former is lower than the latter whenever (42) is satisfied.

[8]For one example of this phenomenon in the 2020 US elections, refer to https://www.opensecrets.org/news/2019/08/2020-democrats-get-help-from-big-money-super-pacs/

When public funds are cheaper for both candidates, campaign advertising will be publicly financed. On the other hand, if private funds are cheaper for both candidates, advertising will be privately financed. Finally, if private funds are cheaper for one candidate but not for the other, the elections will be publicly financed for the former and privately financed for the latter; this is what we refer to as a mixed financed election.

Table 4 summarizes the observations above:

	$\frac{\theta_2+T}{\alpha_2} \geq \frac{2}{1-k}$	$\frac{\theta_2+T}{\alpha_2} < \frac{2}{1-k}$
$\frac{\theta_1+T}{\alpha_1} \geq \frac{2}{1-k}$	Publicly Financed	Mixed Financed
$\frac{\theta_1+T}{\alpha_1} < \frac{2}{1-k}$	Mixed Financed	Privately Financed

Table 4: Fundraising Equilibirum Strategies

■

A convenient way to write the condition in equation (42) is:

$$k \leq 1 - \frac{2 \cdot \alpha_i}{\theta_i + T}$$

This inequality partitions the unit interval into three cells, each of them corresponding to one type equilibrium as shown in Figure 15.

When the proportion of impressionable voters is low, placing a platform near the political line's far ends is very costly. In this case, presidential candidates will compete for informed voters. These agents are not susceptible to advertising. Hence, there is a reduction in the benefit of raising private funds (in the limit, when $k = 0$, there is no use for advertising).

As the proportion of uninformed voters grows, fundraising gains relevance for competition. At the limit, the incentives to place the platforms at the extremes of the political line rise. To see this, notice that taking a centrist position will be of no use to any candidate because the amount of private funds received would be the minimum possible. Furthermore, the amount of public funds received vanishes as ignorance of platform positions grows among the electorate.

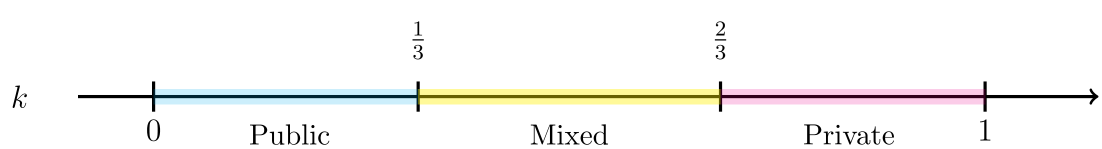

Figure 15: Proportion of Impressionable Voters and Resulting Equilibrium
(Parameter Values: $\alpha_1 = \alpha_2 = 5$, $\theta_1 = 25$, $\theta_2 = 10$,$T = 5$)

Publicly Financed Elections

In order to guarantee a publicly financed election, this section assumes that the following condition holds:

$$k < \min_{i \in \{1,2\}} \left\{ 1 - \frac{2 \cdot \alpha_i}{\theta_i + T} \right\}$$

Presidential candidates will locate their platforms at the middle of the political line, which in turn implies that the indifferent informed agent has a bliss point given by $\overline{x} = \frac{1}{2}$. Consequently, each candidate will receive support from half of the informed voters.

The following proposition characterizes the equilibrium amount of money borrowed and the vote share achieved by each candidate.

Proposition 2 *Publicly Financed Equilibrium*

In a publicly financed election, equilibrium fundraising strategies imply:

1. *There is no policy polarization.*
2. *Candidate i will win the election whenever $\theta_i > \theta_j$.*
3. *The debt acquired by candidate i (t_i) is increasing in θ_i and T.*
4. *t_i is increasing in θ_j whenever $\theta_j < \theta_i$.*

Proof. See Appendix 1 ■

Let us note, first, that the equilibrium vote share received by candidate 1 is a convex combination of $\frac{1}{2}$ and $\frac{\mu_1}{\mu_1+\mu_2}$. This means that the candidate with highest support from the electorate will be the one who has a stronger preference for office. It also suggests that Candidate i will be the one who receives a higher amount of the ex-post allocation of public funds whenever $\theta_i > \theta_j$ holds.

In order to determine how much money to borrow, each candidate determines the total amount of funds to be spent for advertising purposes, C_i . Once this decision is made, the amount of money borrowed from the government is just a residual variable (the difference between the desired total budget and the amount of funds raised from interest groups).

Privately Financed Elections

Whenever the proportion of impressionable voters is high, it is far more profitable to raise funds from interest groups. Informed voters are negligible, bringing down the cost of locating platforms at the political line's boundaries.

Given that $t_1 = t_2 = 0$, we only need to find the equilibrium platform locations to fully characterize the elections results. This characterization is the content of the following proposition.

Proposition 3 *Privately Financed Equilibrium*

Suppose the following condition holds:

$$k > \max_{i \in \{1,2\}} \left\{ 1 - \frac{2 \cdot \alpha_i}{\theta_i + T} \right\}$$

Then, elections are privately financed and, in equilibrium:

1. *Policy polarization is weakly increasing in the proportion of impressionable voters (k).*
2. *Both candidates choose equally extremist or moderate platform locations. That is:*

$$\left| \frac{1}{2} - p_1^* \right| = \left| p_2^* - \frac{1}{2} \right|$$

3. *Candidate i will win the election whenever he is supported by a larger interest group than his opponent's.*

Proof. See Appendix 1. ■

The first statement in the proposition above can be proved by straightforward calculations. Depending on the proportion of impressionable voters, platform locations may be interior or border solutions. When solutions are interior, a further increase in this population's size increases the marginal benefit of spending in advertising. In a privately funded equilibrium, candidates have no alternative but to move closer to their interest group bliss point in order to obtain more funds. Policy polarization is, therefore, increasing in k. However, when interest groups are small, it is likely that platform locations will be a border solution. In this case, further increases in k have no effect on the candidate's optimal policy.

One interesting fact about this equilibrium is that, even when candidates may be supported by heterogeneous interest groups, the proposed policies are equally moderate. Even when the disadvantaged candidate knows for certain he's bound to loose the contest, he has the motivation to compete for the public funds allocated after the election. If both parties take equally extremist positions, the disadvantaged candidate finds it optimal to remain where he is. If he takes a step further away from the center, his opponent can raise the same amount of funds with a smaller deviation. If this is the case, then the proportion of impressionable voters supporting each candidate will not change. However, the advantaged candidate will now receive a higher support from informed voters. By all means, the disadvantaged candidate has no incentives to persue this course of action. Furthermore, given that the proportion of informed voters is small, a similar argument allows us to conclude that he won't move closer to the center either.

The last statement in Proposition 3 comes from the observation that each candidate will receive support from half of the total population of informed voters. The impressionable voters are, therefore, the sole determinants of the election's winner. Furthermore, $p_2^* = 1 - p_1^*$ implies that the candidate supported by a larger interest group will be the one with highest advertising spending. In that sense, Candidate 1 wins the election whenever $\alpha_1 > \alpha_2$.

Mixed Financed Elections

In a mixed financed equilibrium, one candidate turns to public fundraising while the other one resorts to private fundraising. Given that we have two candidates, there are two possible equilibria to analyze. Without loss of generality, we will solve the case where candidate 1 resorts to public fundraising. That is, we assume:

$$1 - \frac{2 \cdot \alpha_2}{\theta_2 + T} < k < 1 - \frac{2 \cdot \alpha_1}{\theta_1 + T}$$

Given this combination of parameters, we know beforehand that in equilibrium:

$$t_2^* = 0 \quad \text{and} \quad p_1^* = \frac{1}{2}$$

This, in turn, means that Candidate 1 will concentrate his campaign strategy to gather the highest support from the informed voters. On the other hand, Candidate 2's optimal strategy is to focus on the impressionable voters. He will take a more extreme position compared to his adversary in order to raise a higher amount of private funds and flood the media with advertising.

4 Policy Polarization in Mixed Financed Elections

In this section, we will study the effects of increasing the government's funding budget (T) on the equilibrium strategies adopted by both political parties.

A first effect of an increase in T is admittedly straightforward: it decreases the attractiveness of private funds by increasing its relative cost. Given the threshold values obtained for each of the three types of equilibria described above, the public finance equilibrium is now more likely. Increasing state transfers also raises the stakes for every candidate by raising the marginal benefit of each vote received (public funds allocated to each candidate are an increasing function of votes received), hence increasing competition for office. However, the strategic decisions taken by each candidate when competition intensifies depends on the fundraising sources selected by each.

We will show that in a mixed financing equilibrium, increased competition in the electoral contest can polarize the platform locations selected by candidates.

For the sake of exposition, consider a situation where Candidate 2 has access to interest group funding while Candidate 1 doesn't. This is, $\alpha_2 = 1$ and $\alpha_1 = 0$. Let both parties have the same intrinsic preferences for holding office: $\theta_1 = \theta_2 = 1$.

In this context, and in the absence of public funding, Candidate 1 can only hope to capture votes from informed agents by setting $p_1 = 1/2$. However, as soon as the government starts raising the amount of public funds that it distributes from $T = 0$ to a positive value $T' > 0$, he will try and capture some of these funds by acquiring debt to finance campaing advertising.

In the absence of public funding, the size of the interest group supporting Cantdidate 2 allows him to place his platform at the center of the political line and still win the election. As T increases, each candidate is affected by the two effects described above. The competition effect makes Candidate 1 increase the amount of debt acquired and improve his position with impressionable voters. When the amount of funds to allocate are sufficiently small in comparison to his interest group size, Candidate 2 is still wealthy enough to remain at the center of the political line and profit from its advantageous interest group differential (See Figure 16).

When T crosses a well defined threshold, the competition effect steps in and Candidate 2 starts competing by distorting its position in order to obtain more private funding. If $k = 0.6$, then as long as $T < 4$, increasing the public budget will create policy polarization. This effect is reversed whenever $T > 4$. A sufficiently big decrease in the relative price of raising funds from debt induces Candidate 2 to specialize in debt fundraising, reducing policy polarization to 0.

Theorem 1 below formalizes this result.

4.1 Policy polarizing effect of public funding.

Theorem 1 *If $\theta_2 > \theta_1$ and $\left[2 \cdot (\alpha_2 - \alpha_1) > (\theta_2 - \theta_1) \cdot (1 - k)\right]$. Then, in a mixed financed equilibrium, policy polarization is increasing in T.*

Intuitively, given the utility specification for each candidate, it is clear that an increment in T raises the marginal benefit of each vote received. This increase is the same for both

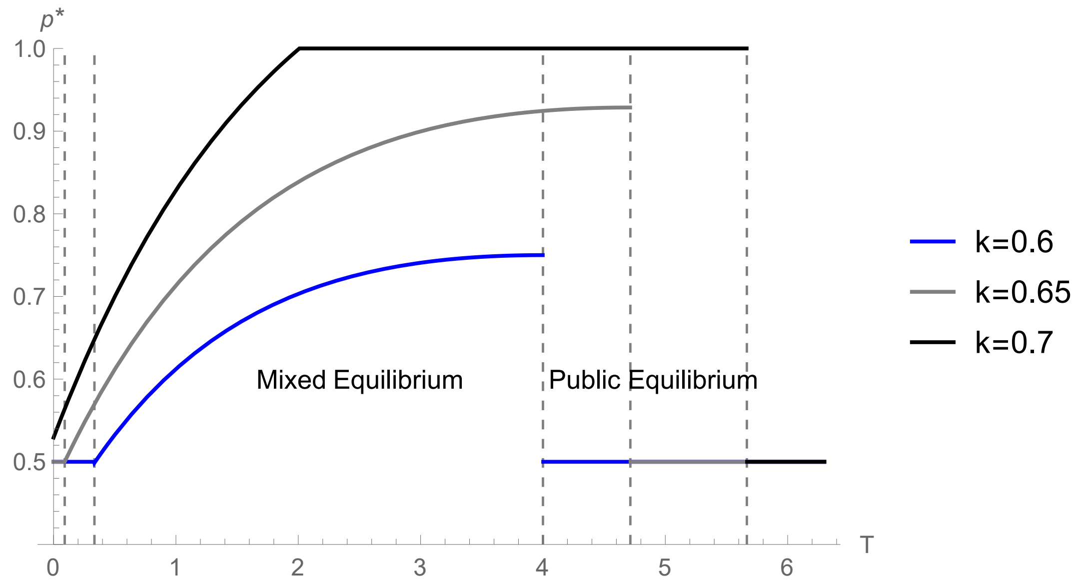

Figure 16: Candidate 2's platform locations.

parties. Hence, a first effect of raising the amount of public funds available is to increase the degree of competition for voters.

Recall that each candidate's optimal strategy is determined as follows: at first, candidates decide the number of funds spent in advertising. Subsequently, depending on the relative prices, they determine the optimal source to raise these funds.

Equations (58) and (59) together (see Appendix 1), determine the budget each candidate must raise for advertising purposes. On the observation that these two functions are strictly increasing in T, we conclude that the optimal reaction of each candidate is to increase the total funds allocated to campaign advertising.

Conditional on being in a mixed funding equilibrium, and given that the interest group supporting Candidate 1 is wealthier than the one supporting his opponent, we know that the relative price of financing via debt is higher for Candidate 2 than for Candidate 1. Hence, the privately funded candidate will opt to substitute votes between informed and impressionable voters. That is, he will move his platform further from the center of the political line, receive more funds from his supporting group, and use them to compete for impressionable voters. On the other hand, Candidate 1 finds it optimal to finance the increase in his campaign spending with further debt. Summarizing, increases in T imply increases in t_1 and p_2.

This polarization increasing effect of public funds comes with a limit. Bearing in mind that increasing the public budget implies further increases in the relative price of raising private funds, there is a threshold value for T that induces the privately funded candidate to resort to debt funding. Specifically, Candidate 2 will turn to debt whenever

$$T > \frac{2 \cdot \alpha_2}{1-k} - \theta_2$$

5 Concluding Remarks

This paper develops a theory of electoral competition that sheds light on counterintuitive effects of public funding of elections. There is a common understanding that private fundraising of political parties has significant costs on voter's welfare when this funds are traded for policy favors by elected candidates. With this understanding, encouraging the use of public funds has been a popular mechanism to solve policy pressures by interest groups.

Our work presents a first step towards understanding a seemingly unnoticed effect of the use of public funds in political elections. The model considers political candidates who care not only for the probability of being elected but also on the outstanding debt at the end of the campaign. Increases in the amount of funds allocated distort the candidates preferences and turns the election into a money making enterprise. Candidates who initially had little incentives to compete for office will now be strong competitors, not for their desire for office but for the profitability of holding it. If this is the case, competition considerations may incentive politicians to resort to interest group money, increasing the equilibrium policy polarization.

The polarization effect of public funds comes to an end when public budget reaches a sufficiently high threshold. When this happens, private funding costs are higher than public funding costs for both candidates. Consequently, they will optimally avoid the use of private funds, eliminating the polarizing effects of these groups. This observation has significant policy implications when deciding budget availability for elections. In particular, when policy-makers increase public funds availability to avoid political favors, they should take into account a the existence of a lower bound on the total budget that should be set in order achieve the desired results. When increases in public funding are below this bound, the policy will have an undesired effect.

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Appendix 1: Proof of propositions 1-4.

Proof of Proposition 1:

Candidate 1 solves:

$$\max_{t_1,p_1} \quad [\theta_1 + T] \cdot \left[\left(\frac{\alpha_1(1-p_1)+t_1}{\alpha_1(1-p_1)+t_1+\alpha_2 p_2 + t_2} \right) \cdot k + (1-k) \cdot \left(\frac{p_1+p_2}{2} \right) \right] - t_1 \tag{43}$$

$$\text{subject to: } t_1 \geq 0, \quad p_1 \in \left[0, \frac{1}{2}\right]$$

Our claim is that there cannot be interior solutions for both t_1 and p_1. First order conditions for interior solutions require:

$$[p_1] \quad \frac{C_2}{(C_1+C_2)^2} = \frac{(1-k)}{2 \cdot k \cdot \alpha_1} \tag{44}$$

$$[t_1] \quad \frac{C_2}{(C_1+C_2)^2} = \frac{1}{(\theta_1+T) \cdot k} \tag{45}$$

First order conditions require that the marginal benefit of money spent in campaign advertisment (LHS) must match its marginal cost (RHS). However, given that the marginal cost of raising money from either source is constant, equations (44) and (45) cannot be met at the same time.

Each candidate will specialize in raising money from one single source (the one with the lowest marginal cost). The optimality condition is, therefore:

$$\frac{C_2}{(C_1+C_2)^2} = \min \left\{ \frac{(1-k)}{2 \cdot k \cdot \alpha_1}, \frac{1}{(\theta_1+T) \cdot k} \right\} \tag{46}$$

An analougous argument allows us to find the optimality conditions for Candidate 2:

$$\frac{C_1}{(C_1+C_2)^2} = \min \left\{ \frac{(1-k)}{2 \cdot k \cdot \alpha_2}, \frac{1}{(\theta_2+T) \cdot k} \right\} \tag{47}$$

■

Proof of Proposition 2:

If $k < \min\left\{1 - \frac{2\alpha_1}{\theta_1+T}, 1 - \frac{2\alpha_2}{\theta_2+T}\right\}$, first order conditions for an optimum imply that the following equality holds:

$$C_1 = \frac{\theta_1 + T}{\theta_2 + T} \cdot C_2 \tag{48}$$

Let $\mu_i \equiv \theta_i + T$. Plugging (48) back in equation (46) and solving for t_2 yields:

$$t_2^* = \max\left\{0, \frac{\mu_2^2 \cdot \mu_1 \cdot k}{(\mu_1 + \mu_2)^2} - \frac{\alpha_2}{2}\right\} \tag{49}$$

Using the last two equations we can recover the optimal amount of money borrowed by candidate 1:

$$t_1^* = \max\left\{0, \frac{\mu_1^2 \cdot \mu_2 \cdot k}{(\mu_1 + \mu_2)^2} - \frac{\alpha_1}{2}\right\} \tag{50}$$

Differentiating the expressions above we get

$$\frac{\partial t_i}{\partial \theta_i} = \frac{2 \cdot (\theta_i + T)(\theta_j + T)^2}{(\theta_1 + \theta_2 + 2T)^3} \cdot k > 0$$

and:

$$\left(\frac{\partial t_i}{\partial \theta_j} = \frac{(\theta_i + T)^2 \cdot (\theta_i - \theta_j)}{(\theta_i + \theta_j + 2T)^3} \cdot k > 0\right) \Leftrightarrow \left(\theta_i > \theta_j\right)$$

Equations (49) and (50) combined with the knowledge that $p_1^* = p_2^* = \frac{1}{2}$, allows us to find the equilibrium vote share for candidate 1:

$$v^* = \frac{k \cdot (\frac{\alpha_1}{2} + t_1^*)}{(\frac{\alpha_1}{2} + t_1^*) + (\frac{\alpha_2}{2} + t_2^*)} + (1-k) \cdot \left[\frac{\frac{1}{2} + \frac{1}{2}}{2}\right] = k \cdot \left(\frac{\mu_1}{\mu_1 + \mu_2}\right) + \frac{(1-k)}{2} \tag{51}$$

The last expression shows that v^* is a convex combination of $\frac{1}{2}$ and $\frac{\mu_1}{\mu_1+\mu_2}$. Consequently, $\left(v^* > \frac{1}{2}\right) \Leftrightarrow (\theta_1 > \theta_2)$.

■

Proof of Proposition 3:

In this section, we assume the following holds:

$$k > \max_{i\in\{1,2\}} \left\{1 - \frac{2\cdot\alpha_i}{\theta_i + T}\right\}$$

Given that asking for unconditioned funds is to expensive, in this equilibrium $t_1^* = t_2^* = 0$. Furthermore, for candidate i, first order conditions require:

$$\frac{C_j}{(C_i + C_j)^2} = \frac{(1-k)}{2\cdot k\cdot\alpha_i}$$

These two equations imply that the relative spending in advertising is proportional to the supporting interest group size:

$$\frac{C_1}{C_2} = \frac{\alpha_1}{\alpha_2} \tag{52}$$

Plugging this back in first order conditions yields:

$$C_2 = \left[\frac{2\cdot k\cdot\alpha_1}{1-k}\right]\cdot\left[\frac{\alpha_2}{\alpha_1+\alpha_2}\right]^2 \tag{53}$$

Using (52) we get:

$$C_1 = \left[\frac{2\cdot k\cdot\alpha_2}{1-k}\right]\cdot\left[\frac{\alpha_1}{\alpha_1+\alpha_2}\right]^2 \tag{54}$$

Finally, given that $C_1 = \alpha_1\cdot(1-p_1)$ and $C_2 = \alpha_2\cdot p_2$, we can retrieve equilibrium platform locations:

$$p_1^* = 1 - \left[\frac{2k}{1-k}\right]\cdot(\alpha_1\cdot\alpha_2)\cdot(\alpha_1+\alpha_2)^{-2} \tag{55}$$

$$p_2^* = \left[\frac{2k}{1-k}\right]\cdot(\alpha_1\cdot\alpha_2)\cdot(\alpha_1+\alpha_2)^{-2} \tag{56}$$

It is straightforward to see that $p_1^* + p_2^* = 1$.

From (55) and (56) we can deduce that an interior equilibrium in platform locations will exist whenever $k \in \mathcal{K}$, where:

$$\mathcal{K} = \left(\frac{(\alpha_1+\alpha_2)^2}{(\alpha_1+\alpha_2)^2 + 4\cdot\alpha_1\cdot\alpha_2}, \frac{(\alpha_1+\alpha_2)^2}{(\alpha_1+\alpha_2)^2 + 2\cdot\alpha_1\cdot\alpha_2} \right)$$

In that case, we may re-define platform locations as follows:

$$p_2^* = 1 - p_1^* \quad \text{and} \quad p_1^* = \begin{cases} \frac{1}{2} & \text{if } k \leq \inf(\mathcal{K}) \\ 1 - \left(\frac{2k}{1-k}\right)\cdot\left(\frac{\alpha_1\cdot\alpha_2}{(\alpha_1+\alpha_2)^2}\right) & \text{if } k \in \mathcal{K} \\ 0 & \text{if } k > \sup(\mathcal{K}) \end{cases}$$

Finally, for every possible equilibrium value of platform locations, the vote share received by Candidate 1 is given by:

$$v^* = \frac{C_1}{C_1+C_2}\cdot k + \frac{(1-k)}{2} = \frac{\alpha_1}{\alpha_1+\alpha_2}\cdot k + \frac{(1-k)}{2} \tag{57}$$

■

Proof of Proposition 4:

In this section, we assume the following holds:

$$1 - \frac{2\cdot\alpha_2}{\theta_2+T} < k < 1 - \frac{2\cdot\alpha_1}{\theta_1+T}$$

The proof resembles those in Propositions 2 and 3. On one hand, we know that $p_1^* = \frac{1}{2}$ and $t_2^* = 0$. On the other hand, first order conditions imply:

$$C_1 = \frac{(1-k)\cdot\mu_1}{2\cdot\alpha_2}\cdot C_2 \tag{58}$$

Substituting back into first order conditions yields:

$$C_2 = \frac{4\cdot\mu_1\cdot k\cdot\alpha_2^2}{[(1-k)\cdot\mu_1 + 2\cdot\alpha_2]^2} \tag{59}$$

Using the fact that $C_2 = \alpha_2\cdot p_2$ yields the desired expression for p_2^*. Furthermore, taking the optimal value for C_2 and substituting in (58) gives the optimal value for C_1. The fact that $C_1^p = \frac{\alpha_1}{2}$ yields the desired expression for t_1^*.

Finally, restrictions on p_2 must be imposed. Specifically, we require $\frac{1}{2} \leq p_2^* \leq 1$. The conditions obtained from this restrictions complete the characterization of the equilibrium outcomes.

$\blacksquare$