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Unemployment Effects of the Chilean Basic-Income Scheme in a Labor Search and Matching Framework with Precautionary Savings

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Abstract

In this thesis, I incorporate a minimum wage and a basic-income scheme to a standard model of labor search-frictions with endogenous separations. Increasing the minimum wage generates a negative spillover on bargained wages, dampening the increase of unemployment through job creation in this framework. The government-guaranteed minimum income for workers is found to altogether counteract the equilibrium effects of the minimum wage by subsidizing low productivity matches. Numerical exercises find that the policies have a quantitatively small effect on labor outcomes in this context. The model is then extended to include risk-averse consumers and Bewley-Huggett-Aiyagari incomplete asset markets, where computational experiments suggest that the policy effects can be amplified through wage dependence on wealth. A numerical exercise suggests that the minimum wage has detrimental, albeit small, welfare effects on workers and that the basic income policy could be welfare improving on average for both employed and unemployed workers.

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1 Introduction

This past year, the Chilean government issued a law that provides formal workers a basic income level, above that of the minimum wage, in the form of a wage subsidy. Said subsidy amounts to a fiscal transfer equal to the difference between the worker's net wage and the guaranteed basic income level. In particular, it guarantees formal workers whose gross wage is less than \$380.000 (Chilean pesos) a net income of \$300.000.[1] As such, this policy introduces a wedge between the paid and received wages in matches involving a low-income employee, and could therefore have non-trivial equilibrium consequences in the labor market. Moreover, if workers are risk-averse and face liquidity constraints, it could also have a direct impact on asset accumulation due to consumption smoothing motives.

The purpose of this work is to shed light on the qualitative and quantitative effects of this type of basic-income subsidy on labor market outcomes and potential individual welfare effects. To this extent, a model that generates equilibrium unemployment is required. The standard framework of labor search frictions is adopted here, considering endogenous separations, as in Mortensen and Pissarides (1994), in order to evaluate the extent to which the basic income scheme acts to mitigate job destruction. Although the policy is a transfer to the worker, it is found in this context that it effectively also acts as a subsidy to low productivity firms, decreasing job separation.

The chosen wage-setting mechanism is constrained Nash-bargaining, as in Flinn (2006), which adds a minimum wage constraint to the otherwise standard bargaining problem. This constraint also serves to impose a subsidy upper bound, as is the case with the actual basic income policy. Given the wage bargaining assumption, the minimum wage is found to have a negative spillover on the overall level of wages, and the basic income policy the opposite effect. Furthermore, in the model, the wage subsidy generates a jump discontinuity in the wage schedule and job's value function, which enhances the pass-through effect to low productivity firms. As discussed below, the channel that generates this is the expected effect the policies have on the match's surplus, as wages are the mechanism that split the job's rents between the participating parties. Note that given the nature of the basic income policy to be evaluated, the comparison case would be that of an economy with a binding minimum wage.

Regarding the labor market effects of the minimum wage, there doesn't seem to be a clear consensus in the literature, as reviewed by Neumark and Wascher (2008). Flinn (2006) builds a matching model with endogenous contact rates and estimates the effects of a minimum wage on unemployment and welfare, finding that it could be welfare improving for both the firm and worker despite the effect on unemployment being ambiguous in principle. In particular, it is emphasized that the minimum wage can act as an instrument that can be used to approach the Hosios (1990) efficiency condition, allocating a surplus share to the searches closer to their respective matching function elasticity. Navarro and Tejada (undated) build a public-private sector with skilled and unskilled workers that also features endogenous contact rates, which is estimated for Chile. One of their key findings is that the public sector setup counteracts the negative minimum wage effect implied by the model. Although through a different channel, I also find that the basic income policy buffers the minimum wage effects. In line with the results of Costain and Reiter (2008), the numerical exercise performed under the Shimer (2005) calibration suggests that these effects are quantitatively small in the linear utility model. However, upon the introduction of risk aversion for the worker and incomplete markets, computational experiments suggest that the effects are non-negligible.

The introduction of a minimum wage to the framework of Mortensen and Pissarides (1994) is found to increase unemployment as a result of greater match separation: low productivity firms that would have only been profitable at a wage bargained below the legal minimum close. Regarding job creation, the wage setting mechanism here dampens the effect of the minimum wage on market tightness. As firms anticipate the lost surplus share when productivity drops due to the minimum wage, bargained wages in unconstrained matches fall, further compressing the wage distribution. Given the assumption that productivity is large enough in new matches for the wage to be unconstrained, the minimum-wage spillover smooths the effect on job creation by increasing profitability in new matches, which counteracts the fall in the job's value implied by its shorter expected duration.

On the other hand, the basic income policy is found to act as a subsidy to low productivity firms, allowing them to bargain the wage down to the minimum wage when employing a worker eligible to receive the basic income subsidy. This results in a decrease of marginal job destruction, implying a longer match duration. The increase in the match's continuation value results in a positive spillover to bargained wages. Although this would put downward pressure on a new job's profitability, numerical exercises find that in equilibrium the minimum wage effect prevails on job creation, resulting in a higher market tightness.

[1] All the details regarding this law can be found in Ministerio de Desarrollo Social y Familia (2020).

In a context of incomplete asset markets, a new channel for welfare effects of labor market policy is introduced: it could potentially serve as insurance for the consumer against otherwise non-insurable idiosyncratic labor income shocks. In a model with incomplete markets and exogenous separations, Krusell, Mukoyama, and Şahin (2010) find that the negative welfare on unemployment generated by an increase in unemployment insurance trumps the gains obtained through consumption smoothing. These authors highlight the stark contrast to the optimal insurance policy obtained in a standard Bewley-Huggett-Aiyagari model, where a complete replacement rate would be desired as the consumer's labor income is exogenous to the policy. In their model, the extra insurance provided by the tax-financed UI for the consumers is insufficient to overcome the negative effect on unemployment spells caused by the fall in vacancy creation, which is caused by the decrease in the job's profitability. They also find that the model generates very little wage dispersion, due to the fact that only heterogeneity across assets is considered. Bils, Chang, and Kim (2011) address the question of whether introducing asset heterogeneity helps matching models explain business cycle fluctuations by constructing a model with incomplete asset markets and endogenous separations. They find that the model faces a trade-off between generating realistic wage growth dispersion and cyclical fluctuations. As I restrict my analysis to that of a steady state, their framework with greater wage dispersion provides a useful basis for studying the effects of labor policy on layoffs. The work of Lalé (2019) also highlights the importance of considering incomplete markets when evaluating welfare effects. The author builds a life-cycle model with endogenous separations and finds that, in contrast to the risk-neutral setting, severance payments have non-neutral effects on welfare, which generates negative welfare effects on consumers through a bargained wage schedule that counters consumption smoothing.

To introduce incomplete asset markets, I follow the treatment and benchmark calibration of Bils et al. (2011) for the baseline model, and then include the policies analogously to the linear model. In this context, the effect of the minimum wage on unemployment is amplified through the wage dependence on wealth. Since the worker's outside option is weakened when savings are low, given that this reduces consumption smoothing possibilities, wages fall at an increasing rate as wealth approaches the debt limit at all productivity levels. As long as separations depend on assets for some productivity level, an increase in the minimum wage can then lead to expected profitability losses in firms matched to workers just below that asset threshold, increasing the separation sensitivity to the policy.

A numerical exercise suggests, once again, that the basic income works to undo the negative labor effects the minimum wage implies in this context. By financing low productivity firms, it is found that job destruction here too falls. Furthermore, the subsidy acts as an insurance device through three channels here: decreased job-loss risk, driven by a lower separation rate; decreased wage-risk, as a result of the compression of the worker's wage distribution; and decreased unemployment risk, given that the subsidy increases job creation and therefore decreases unemployment duration. A preliminary welfare analysis exercise is conducted, and it is found that the minimum wage decreases average welfare among both employed and unemployed workers, albeit at a quantitatively small level, and that the basic income has the opposite effect at a larger scale.

The current work is organized as follows: Section 2 presents the model under the standard risk-neutrality assumption predominant in matching models and discusses the equilibrium effects of the minimum wage and basic income policies. Section 3 extends the model to include consumer risk-aversion and incomplete asset markets in order to study the insurance and welfare effects of the policies. Finally, Section 4 summarizes the main findings. The resolution of the equilibrium conditions is self-contained in the appendix.

2 Risk-neutral model

The model in this section builds on the framework of Mortensen and Pissarides (1994). A minimum wage is introduced as in Flinn (2006) and the taxation setup draws on ideas from Mortensen and Pissarides (2003). Labor search frictions are assumed in that it potentially takes time for an unemployed worker to find a job and for a firm to fill a vacancy, which generates frictional unemployment in equilibrium as long as some jobs end in any given period. For tractability, I will restrict the entirety of the analysis to that of a steady-state throughout this work, so I drop time indexes in most of what follows.

2.1 Model framework

There exists a continuum of workers of measure one and a continuum of firms, both sharing a common discount factor $\beta \equiv \frac{1}{1+r}$, where r denotes the interest rate. In any given period, workers can be employed or unemployed. Workers that are employed receive a gross wage w_e and unemployed workers search for jobs, obtaining an unemployment benefit b. Firms have a technology that specializes in either production or vacancy, the former requiring the firm to be matched to a worker. Each firm in this economy is assumed to employ one worker, and can therefore be thought of as a job position that is either vacant or filled. Firms that undertake production pay their employee a wage w and those that are searching for a worker pay a vacancy flow cost c.

The distinction between the wages w_e and w is made in order to accommodate the basic income policy. It is assumed that there exists an exogenous minimum wage $\underline{w}$ and a government-guaranteed basic income BI for workers. The latter policy is materialized as a subsidy to the worker's wage, bridging the gap between the wage w paid by the firm and the basic income level BI. In order to finance this policy, it is assumed that there exist payroll taxes τ_e and τ_f, paid respectively by the employee and firm.

The job search process in this economy is characterized by a matching function that embodies labor search-frictions and determines the job finding and filling rates. In any period, an unemployed worker meets a firm with probability p and a vacant firm is matched to a searching worker with probability q. Let u denote the number of unemployed workers and v the number of job vacancies. The amount of matches in any period is assumed to be determined by the following matching function:

$$m(u, v) = \chi u^{\eta} v^{1-\eta}, \tag{1}$$

where χ is a scale parameter. As matches are assumed to occur at random, the probability that an unemployed worker finds a job is given by $p = \frac{m}{u}$ and, similarly, the probability that a vacancy is filled is $q = \frac{m}{v}$. Letting $\theta \equiv \frac{v}{u}$ denote the market tightness, the job filling rate q and finding rate p are respectively described by:

$$q(\theta) = \chi\theta^{-\eta}, \quad p(\theta) = \theta q(\theta). \tag{2}$$

It is assumed that a matched worker-firm pair produces a stochastic output of value x, corresponding to the firm's idiosyncratic productivity or match quality. All new jobs are assumed to be formed at a productivity level x_0. In any given period, a new productivity is drawn with probability ϕ from a distribution characterized by a cumulative distribution function $G(x)$ with a bounded support $[\underline{x}, \overline{x}]$. Note that under these assumptions, match quality x is a persistent but memoryless process. The wage paid by a firm in this context will be assumed to be a productivity contingent contract $w(x)$. This is equivalent to the assumption that wages are renegotiated whenever a new productivity shock hits the firm.

Match separation in this economy is assumed to be endogenous and occurs whenever either party finds it in their interest to break up the match, that is, whenever their outside option is more valuable to them than the job. Once a match ends, a new round of search is triggered for both the respective worker and firm. As shown below, separations satisfy a reservation property: only matches of quality $x > R$ are continued, where R denotes the reservation productivity. Since a proportion ϕ of firms draw a new productivity each period, the inflow to unemployment is then given by $\phi G(R)(1-u)$, where $1-u$ is the amount of employed workers. Similarly, a proportion $p(\theta)$ of unemployed workers find a job every period, and therefore the outflow of unemployment is simply $p(\theta)u$. The steady state unemployment rate that equates these two flows is given by:

$$u = \frac{\phi G(R)}{p(\theta) + \phi G(R)}. \tag{3}$$

Let $w_e(x)$ denote the wage an employee obtains in a productivity x match and $w(x)$ the wage paid by the firm. Note that under the basic income policy, the worker's wage is described by $w_e(x) = \max\left\{w(x), \frac{BI}{1-\tau_e}\right\}$, where BI and τ_e denote the basic income level and payroll tax, respectively. Given the previous assumptions, the value of employment is then defined by:

$$W(x) = (1-\tau_e)w_e(x) + \beta\left[(1-\phi)W(x) + \phi\left(G(R)U + \int_R^{\bar{x}} W(z)\, dG(z)\right)\right], \tag{4}$$

where U denotes the value of unemployment and R the reservation productivity. The above expression accounts for the fact that a match receives a productivity shock with probability ϕ, in which case a new match quality is randomly drawn from the distribution characterized by G. Given the separation reservation property, whenever a match receives a productivity shock that generates a match quality below R, the worker enters unemployment. The value of a job to a productivity x match to a firm is analogously defined by:

$$J(x) = x - (1+\tau_f)w(x) + \beta\Big[(1-\phi)J(x) + \phi\Big(G(R)V + \int_R^{\bar{x}} J(z)\,dG(z)\Big)\Big], \tag{5}$$

where x denotes the firm's output, V the value of vacancy and τ_f the firm's payroll tax. Let x_0 denote the productivity level of new jobs. Since unemployed workers meet a vacant firm with probability $p(\theta)$ each period, the value of unemployment is defined by:

$$U = b + \beta\Big[p(\theta)W(x_0) + (1-p(\theta))U\Big], \tag{6}$$

where b denotes the worker's unemployment insurance. Firms that search for workers incur a vacancy cost c each period and meet a worker with probability $q(\theta)$, and the value of vacancy is similarly defined by:

$$V = -c + \beta\Big[q(\theta)J(x_0) + (1-q(\theta))V\Big]. \tag{7}$$

Given that posting vacancies is costly, jobs must entitle rents in equilibrium, or otherwise there would be no incentive for firms to enter the market. In equilibrium, free-entry of firms occurs until it is no longer profitable to post a vacancy; that is, $V = 0$ holds in equilibrium. Since jobs generate a positive surplus and the worker-firm pair constitutes a bilateral monopoly, a surplus sharing must be specified.

As in Flinn (2006), it is assumed that wages are determined by constrained Nash-bargaining over the match's surplus. Let $H(x) \equiv W(x) - U$ and $J(x)$ respectively denote the worker's and firm's surplus obtained from a job. Note that since $V = 0$, the job's value and surplus to a firm are equivalent in equilibrium, and therefore these terms will be used interchangeably in what follows. The wage schedule is a productivity-contingent contract that solves the following problem:

$$\max_{w(x)} \; H(x)^{\gamma} J(x)^{1-\gamma} \quad \text{s.t.} \;\; w(x) \geq \underline{w}, \tag{8}$$

where $\gamma \in [0,1]$ denotes the worker's bargaining power and $1-\gamma$ that of the firm. In order to characterize the wage schedule, two important points should be noted. First, since workers eligible to receive the basic income subsidy have a guaranteed constant wage, these employees are indifferent over the wage the firm pays them. As the job's value is decreasing in wages, the firm will in turn pay eligible workers the minimum wage. Second, since the solution to the unconstrained problem is a wage schedule that is increasing in productivity, the minimum wage acts solely as a side constraint to the problem and is paid in low productivity matches. In particular, the minimum wage will be paid by the firm up to a productivity threshold x^*, at which the bargained wage coincides with the gross basic income.

2.2 Equilibrium

2.2.1 Match surplus, wages and value of unemployment.

Free-entry of firms requires that the value of vacancy be $V = 0$. The value functions (4) and (5) respectively imply that the worker's and firm's surplus can be written as:

$$H(x) = \frac{1+r}{r+\phi}\Big[(1-\tau_e)w_e(x) - \frac{r}{1+r}U + \frac{\phi}{1+r}\int_R^{\bar{x}} H(z)\,dG(z)\Big] \text{ and} \tag{9}$$

$$J(x) = \frac{1+r}{r+\phi}\Big[x - (1+\tau_f)w(x) + \frac{\phi}{1+r}\int_R^{\bar{x}} J(z)\,dG(z)\Big]. \tag{10}$$

Recall that the wage schedule will be constrained by the minimum wage up to a productivity level x^*. The above expressions imply that when the wage is an interior solution to the bargaining problem (8), the worker and firm respectively obtain a constant fraction ς and $1-\varsigma$ of the match's surplus. That is, for $x > x^*$, the following condition is satisfied:

$$\frac{H(x)}{\varsigma} = S(x) = \frac{J(x)}{1-\varsigma}, \tag{11}$$

where $S(x) = H(x) + J(x)$ denotes the match's surplus and the constant ς is defined by:

$$\varsigma \equiv \frac{\gamma}{\gamma + (1-\gamma)\frac{1+\tau_f}{1-\tau_e}}.$$

Therefore, an increase in payroll taxes has the equivalent effect to a decrease in the worker's effective bargaining power, as a lower amount of the surplus can be captured by the employee at any productivity level. Note that in absence of payroll taxes, the optimality condition (11) reduces to the standard case in which each party obtains a share of the surplus equal to their bargaining power. Furthermore, note that in absence of a binding minimum wage, this rule also implies that separations are privately efficient: the worker and firm agree to destroy the match when its total surplus is negative. However, the introduction of a minimum wage renders separations privately inefficient.

To see this, consider the case of a continuing low quality match in which the bargained wage would have been below the minimum wage in absence of this policy. Upon introducing a binding minimum wage, this worker is clearly willing to continue the match case, as he would receive a higher wage than the previously bargained one. However, as the firm would incur greater costs in this case, it could potentially lead to losses that result in the termination of the job. That is, a binding minimum wage increases marginal job destruction in a privately inefficient way.

To determine the equilibrium wage schedule, recall from the previous discussion that the wage will be an interior solution to the bargaining problem whenever $x > x^*$. As shown in Appendix 5.1.3, assuming that both the minimum wage and basic income are binding, the surplus functions (9) and (10) and surplus sharing rule (11), along with the fact that the firm pays employees eligible for the basic income the minimum wage, imply that the equilibrium wages $w(x)$ for productivity levels $x \in (x^*, \bar{x}]$ are given by the following expression:

$$w(x) = \underbrace{\frac{\gamma}{1+\tau_f}x + \frac{1-\gamma}{1-\tau_e}\frac{r}{1+r}U}_{\substack{\text{endogenous separations}\\ \text{wage curve}}} - \underbrace{\frac{\phi}{r+\phi}\frac{\gamma}{1+\tau_f}\int_R^{x^*}(x^*-z)\,dG(z)}_{\text{minimum wage spillover}} + \underbrace{\frac{\phi}{r+\phi}\gamma\int_R^{x^*}\left(\frac{BI}{1-\tau_e} - \underline{w}\right)dG(z)}_{\text{basic income spillover}}, \tag{12}$$

which singles out the effects of each policy on the bargained wages. Note that it is assumed that $\frac{BI}{1-\tau_e} \geq \underline{w}$, nesting the case of a non-binding basic income policy when this condition is satisfied with equality, implying that $\tau_e = \tau_f = 0$, as there would be no subsidy to finance. Also note that, by definition, bargained wages are an interior solution to (8) for all productivity $x > x^*$, and when the minimum wage is not binding, it holds that the separation thresholds R and x^* are equal, reducing the wage schedule to that of a standard endogenous separations model. In the general case, for matches of quality $x \in [R, x^*]$, the firm pays the minimum wage $\underline{w}$ and the worker receives the basic income $\frac{BI}{1-\tau_e}$.

Consider first the case without labor market policies. As noted in Pissarides (2000), the bargained wage is increasing in both productivity and in the worker's outside option, which is given by the value of unemployment. As the wage is simply a surplus assigning mechanism, an increase in the firm's bargaining power drives the wage down towards the worker's outside option, and likewise, an increase in γ results in a higher bargained wage. In equilibrium, a determinant of the value of unemployment is its expected duration. An increase in market tightness shifts the wage schedule up as it strengthens the worker's outside option, by making the exit from unemployment more likely. As can be seen in (12), these results extend to bargained wages in matches with productivity $x \in (x^*, \bar{x}]$ under the minimum wage and basic income policies.

Turning now to the case with policies, note that for productivity levels in $[R, x^*]$, the relevant gross wages to the firm and worker are constant, given by $\underline{w}$ and $\frac{BI}{1-\tau_e}$, respectively. It follows directly from (10) that both the job's value and match surplus are increasing in productivity over that range. Therefore, the minimum wage imposition implies that firms obtain a decreasing share of the surplus as productivity falls. As future productivity may fall in a continuing match, bargained wages internalize the expected surplus share loss that firms would incur in the case of a future binding minimum wage. Appendix 5.1.3 shows that the second bracketed term precisely accounts for this expected loss, which occurs with probability $\phi[G(x^*) - G(R)]$.

The model therefore implies that a minimum wage has a negative spillover on the level of bargained wages, decreasing the overall wage schedule if market tightness is held constant. This implies that the minimum wage also compresses the wage distribution: on the one hand, a binding minimum wage implies a larger lower bound on the wage schedule, and on the other, the spillover to bargained wages implies a decrease in the upper bound, generating lower wage dispersion overall. The basic income acts opposite to the minimum wage by a similar mechanism. The policy effectively acts as a subsidy for a group of firms with mid to low productivity levels. In particular, firms that would have otherwise bargained a wage above the minimum but below the guaranteed basic income level now pay their workers the minimum wage, increasing their profitability. In addition, the surplus of employees in low productivity matches increases, as they directly benefit from the government's transfer. As these effects lead to an increase in the surplus of a continuing match, part of this additional profitability is captured by workers in high productivity matches, raising the wage level. That is, the basic income policy counteracts the effects of the minimum wage on the bargained wage schedule.

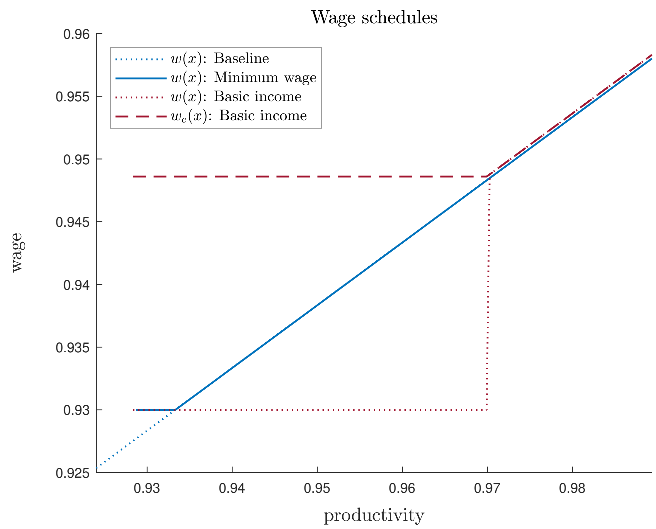

Figure 1: Equilibrium wages

The equilibrium wage schedule obtained is depicted in Figure 1. Note that, given the nature of the bargaining process, the introduction of a basic income in this context produces a jump discontinuity in the wage schedule at $x = x^*$, which in turn produces a jump discontinuity in the job's value function at that point. As it is clear from the wage schedule and the job's value function (10), the job's value is monotonically increasing in productivity, except at x^* where the jump discontinuity occurs. In particular, Appendix 5.1.3 shows that the job's value function at that point can be written as:[2]

$$J(x^*) = \frac{1+r}{r+\phi}\left[(x^* - R) - \underbrace{(1+\tau_f)\left(\frac{BI}{1-\tau_e} - \underline{w}\right)}_{\substack{\text{Job's value function}\\ \text{jump discontinuity}}}\right]. \tag{13}$$

The underlying assumption made throughout this section is that this jump discontinuity is not large enough to imply a negative job's value at x^*. That is, in order for the reservation property to be satisfied and for separations to occur only at match quality $x < R$, the size of the basic income must not exceed the minimum wage by a large amount. I limit the analysis throughout to this case, as with reasonable calibrations this assumption is satisfied.

To determine the value of unemployment, note that when a new job's productivity x_0 is high enough for the wage to be an interior solution to the bargaining problem (8), the surplus is shared according to (11) in a new job. For

[2] Strictly speaking, this is the limiting job's value as x approaches x^* from the right. As Appendix 5.1.3 shows, the job's value at productivity levels $x \in [R, x^*]$, for which the minimum wage is binding, is given by:

$$\underline{J}(x) = \frac{1+r}{r+\phi}(x - R).$$

Therefore, the job drops $\Delta J(x^*) = (1+\tau_f)\left(\frac{BI}{1-\tau_e} - \underline{w}\right)$ in value as the employee crosses the basic-income eligibility threshold.

simplicity, assume that $x_0 \geq x^*$, so that this will be the case. The general statement of the value of unemployment is provided in the appendix. The free-entry condition $V = 0$ and value function of vacancy (7) imply the following expression the value of a new job:

$$\frac{c}{q(\theta)} = \frac{1}{1+r} J(x_0). \tag{14}$$

As $\frac{1}{q(\theta)}$ corresponds to the expected duration of vacancy, the above equation implies that in equilibrium, vacancies are posted until their expected cost equates the discounted job's value. This expression, combined with the surplus sharing rule (11) and the value function of unemployment (6), implies that U is determined by:

$$\frac{r}{1+r} U = b + \frac{1-\tau_e}{1+\tau_f} \frac{\gamma}{1-\gamma} c\theta. \tag{15}$$

As mentioned previously, the worker's outside option is increasing in θ, as it becomes more likely for an unemployed worker to be matched to a firm when market tightness is high. It follows directly from (15) that increases in market tightness, unemployment insurance, or other variables that increase U result in higher bargained wages. This link between wages and market tightness turns out to be a key mechanism for explaining the equilibrium curves derived below.

2.2.2 Equilibrium curves

To focus on the standard equilibrium curves, suppose that payroll taxes are exogenous. In the baseline framework of Mortensen and Pissarides (1994), a job creation and job destruction curve pin down the equilibrium market tightness and reservation value. Here, an additional condition is needed in order to determine the equilibrium triple (θ, R, x^*): the threshold at which bargained wages are an interior solution to the bargaining problem (8).

Given the assumptions made thus far, it is clear that it is the firm who determines match separation when the minimum wage is binding. As discussed previously, the job's value is increasing in productivity almost everywhere, satisfying a reservation property. The reservation productivity R is defined by a null job's value, i.e., $J(R) = 0$. Note that, by definition, the minimum wage is binding at all productivity $x \leq x^*$ and therefore $w(R) = \underline{w}$. As shown in Appendix 5.1.3, the free-entry condition, job's value function (10) and wage schedule (12) imply that the reservation productivity is determined by the following partial **job destruction curve**:

$$0 = R - (1+\tau_f)\underline{w} + \frac{\phi}{r+\phi}\left(\int_R^{\bar{x}} (z-R)\,dG(z) - \int_{x^*}^{\bar{x}} \left[\gamma(z-x^*) + (1+\tau_f)\left(\frac{BI}{1-\tau_e} - \underline{w}\right)\right] dG(z)\right). \tag{JD}$$

The term partial is used to highlight the fact that the above equation does not represent a direct relationship between the reservation productivity x^* and market tightness θ like it does in the baseline model derived in appendix 5.1.1. Therefore, an additional condition linking the threshold x^* with market tightness will be required to represent the job destruction curve in the (R, θ) plane. Although the (JD) curve differs from its analogue in the baseline model, it's worthwhile mentioning the economic mechanisms driving job destruction in the baseline model, as much of the same intuition carries over to the case with policies. As shown in Appendix 5.1.1, the baseline job destruction curve is given by:

$$0 = R - \frac{r}{1+r} U + \frac{\phi}{r+\phi} \int_R^{\bar{x}} (z-R)\,dG(z).$$

Note that when there exist no binding policies, i.e., when $\frac{BI}{1-\tau_e} = \underline{w}$, $x^* = R$ and $\tau_e = \tau_f = 0$, the (JD) condition reduces to the above expression as the bargained wage would satisfy (12) at the reservation productivity R. As shown in Appendix 5.1.1, the baseline job destruction curve represents an increasing relationship between R and θ. The reasoning behind this is as follows: since the bargained wage is increasing in the worker's outside option, more marginal jobs are destroyed when market tightness is high, as this strengthens the worker's outside option by increasing the probability of finding a job. Given that productivity is stochastic, the above expression also implies that there exists labor hoarding: some matches that incur current losses are maintained as they have a positive expected continuation value, captured by the integral term.

Returning to the case with policies, note that a first effect of the minimum wage, even when it turns out to be non-binding in equilibrium, is that it potentially imposes a lower bound on the admissible values of market tightness that can satisfy the job destruction condition. Whereas before, the wage could be bargained down when θ decreased, this is no longer possible past the legal minimum wage. This implies that when the minimum wage is high enough, low reservation productivity levels can no longer be sustained as they would imply a longer duration of a match that generates negative profits for the firm.

As Appendix 5.1.2 shows, the (JD) curve here represents a decreasing relationship between the productivity thresholds x^* and R. Recall that x^* denotes the productivity level at which wages become an interior solution to the bargaining problem (8). Since wages are increasing in productivity, if the wage schedule (12) were to shift down, due to a decrease in unemployment benefits b, for example, the productivity level at which the side constraint becomes binding would increase as there would be more low-earning employees. The generalized lower wages would increase the expected profitability of firms and, therefore, result in fewer marginal jobs being destroyed. Furthermore, since bargained wages are increasing in θ, the same mechanism implies that the job destruction curve slopes upwards in the (R, θ) plane.

The productivity threshold x^* is defined as the level at which wages are an interior solution to the bargaining problem (8). As previously discussed, the threshold x^* is implicitly defined by $w(x^*) = \frac{BI}{1-\tau_e}$. This **interior negotiation solution** curve, henceforth INS, is defined by the wage equation (12) as:

$$\frac{BI}{1-\tau_e} = \frac{\gamma}{1+\tau_f}x^* + \frac{1-\gamma}{1-\tau_e}\frac{r}{1+r}U - \frac{\phi}{r+\phi}\gamma\left[\frac{1}{1+\tau_f}\int_R^{x^*}(x^*-z)\,dG(z) - \int_R^{x^*}\left(\frac{BI}{1-\tau_e} - \underline{w}\right)dG(z)\right]. \tag{INS}$$

At a given market tightness, this expression represents a decreasing relationship between x^* and R since an increase in match duration is associated with lower wages, increasing the productivity range over which the labor market policies bind. On the other hand, when match duration is held fixed, an increase in market tightness has the opposite effect: the wage schedule shifts up as the worker's outside option is strengthened, diminishing the amount of employees eligible for the policy.

Analogously to the baseline model, Appendix 5.1.3 shows that the free-entry condition $V = 0$, the value function of vacancy (7), and the value of a new job $J(x_0)$ imply that the **job creation** is described by:

$$\frac{c}{q(\theta)} = \frac{1}{r+\phi}\left[(x_0 - R) - \gamma(x_0 - x^*) - (1+\tau_f)\left(\frac{BI}{1-\tau_e} - \underline{w}\right)\right]. \tag{JC}$$

This expression also nests the baseline job creation curve when there exist no labor market policies. Analogously to the baseline model, by holding x^* fixed one obtains a decreasing relationship between θ and R: when jobs have a long duration, their expected profitability is high and more vacancies are created. On the other hand, as shown in Appendix 5.1.2, when market tightness is held constant this expression implies an increasing relationship between the productivity thresholds. Given a high enough starting productivity x_0, the surplus is shared according to (11) in new matches. The positive relationship between the thresholds then simply reflects the fact that the match has an increasing terminal value in x^*, as separations are privately inefficient when $x^* > R$. When this is the case, it is clear that match duration falls.

The (JD) curve and the (INS) condition jointly determine the modified job destruction curve, which slopes upward in the (R, θ) plane, as argued previously. Similarly, the (JC) curve and the (INS) condition determine the analogue job creation curve.

Consider first the case of a non-binding basic income policy, i.e., $\frac{BI}{1-\tau_e} = \underline{w}$. The equilibrium curves for this case are depicted in Figure 2 under the calibration described in the following section. The dotted lines represent the equivalent curves in the baseline model, that is, when $\underline{w} = 0$. Note that, as discussed above, the job destruction curve no longer spans the whole productivity support, as the minimum wage impedes wages from being bargained down in low productivity matches, which would be required to sustain a longer match duration. Since a larger market tightness implies higher bargained wages, the minimum wage is not binding in low duration matches along the job destruction curve, merging this curve with its baseline counterpart. As can be seen in Figure 2, the job destruction curve becomes steeper when the minimum wage is binding, implying the termination of more marginal jobs. Turning to the job creation curve, note that the minimum wage spillover to bargained wages implies an increased profitability in new jobs, which dampens the fall in market tightness that would be implied by greater job destruction.

The baseline equilibrium and minimum-wage setting equilibrium are denoted respectively by (R_0, θ_0) and (R_m, θ_m) in Figure 2. As would be expected, match duration falls when the minimum wage is introduced, but market tightness

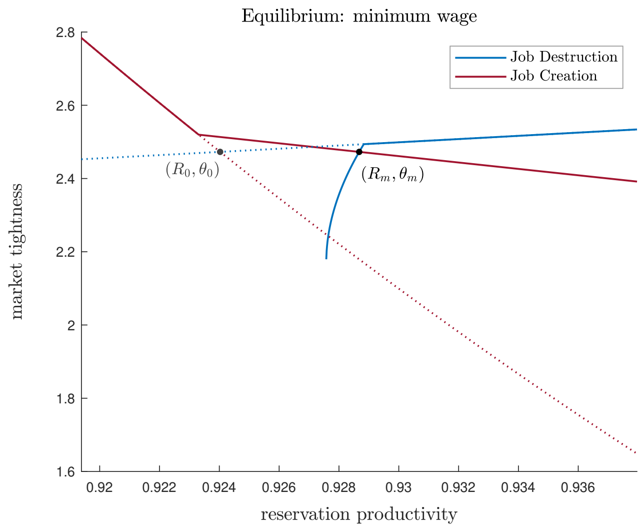

Figure 2: Equilibrium curves: minimum wage

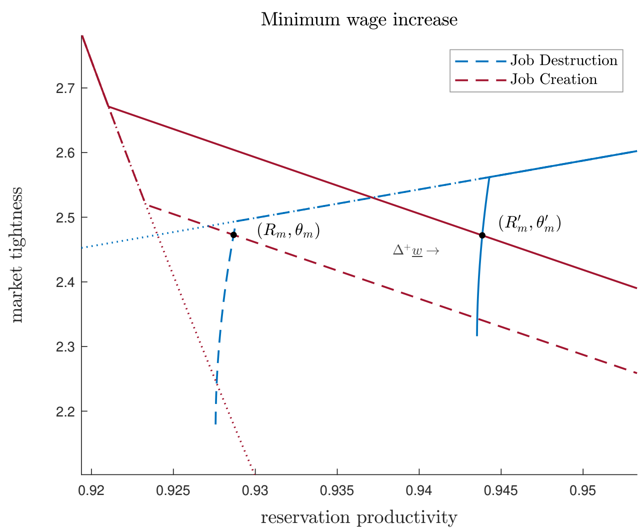

Figure 3: Equilibrium curves: minimum wage increase

reacts less due to the spillover on the bargained wage curve. Figure 3 depicts an increase in the minimum wage, which further strengthens the effects mentioned previously.

Turning now to the basic income policy, the equilibrium curves are depicted in Figure 4. First, note that the basic income decreases job destruction, as it acts effectively as a subsidy to low productivity firms. As before, a high market tightness is associated with high wages and, therefore, with a decrease in the measure of the employees eligible to receive the wage subsidy. The jump discontinuity following the spike in the job destruction curve corresponds to the point at which no workers are eligible for the subsidy, returning to the baseline case.

As the basic income has the opposite spillover effect on bargained wages to that of the minimum wage, there are no longer incentives to post more vacancies due to lower wages. However, as can be seen in Figure 4, this is not the case in equilibrium in the simulations performed, where the minimum wage effect on job creation is dominant. The net effect of the basic income policy is an increase in match duration and market tightness, as the increased lifetime of jobs compensates the fall in profitability due to the higher wage schedule. This result is linked to the assumption that the basic income is reasonably close to minimum wage, so as to not produce too large of a jump discontinuity in the job's value, as discussed previously. Therefore, as shown in (12), this implies that the minimum wage spillover remains relevant and drives the equilibrium job creation effect.

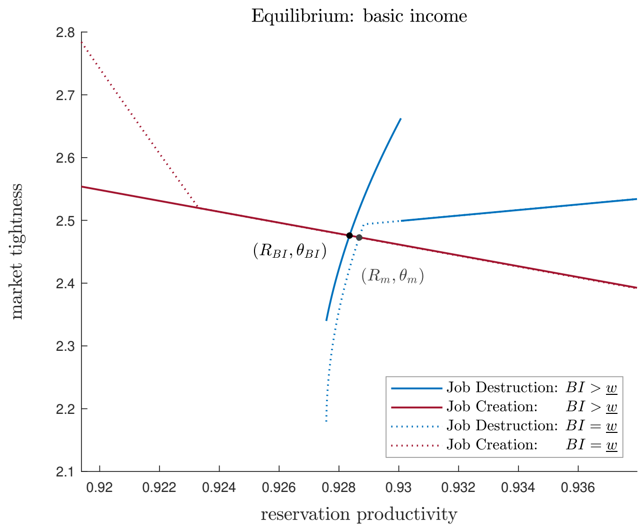

Figure 4: Equilibrium curves: basic income

2.2.3 Government

In order to endogenize the tax rates, it is assumed that the government maintains a balanced budget. Let $\Phi(x)$ denote the stationary distribution of matches over productivity x. From the above wage discussion, it follows that the budget balance condition is given by:

$$\int_R^{x^*} \left(\frac{BI}{1-\tau_e} - \underline{w} \right) d\Phi(z) = \int_R^{\bar{x}} \left[\tau_e w_e(x) + \tau_f w_f(x) \right] d\Phi(z), \tag{16}$$

where the left-hand side is the size of the basic income subsidy and the right-hand side the government's income. Note that an additional tax rule would be required to determine the payroll taxes τ_e and τ_f levels. For simplicity, in the numerical exercises that follow, only one of these tax instruments will be considered. In particular, τ_f will be assumed zero.

2.3 Numerical exercises

To evaluate the effect of the policies on labor market outcomes, a numerical solution to the model is provided here. The calibration of the baseline model borrows from Janiak and Wasmer (2014). Time frequency is taken to be monthly. The discount factor, defined as $\beta \equiv \frac{1}{1+r}$, is set to target an annualized interest rate of 4%. Matching elasticity η is taken to be 0.5 and the scale factor χ is chosen to target an equilibrium monthly job finding rate of 60%. The worker's bargaining power γ is set equal to the matching elasticity η, so as to satisfy the Hosios (1990) efficiency condition in the baseline model.

It is assumed that the match's productivity is drawn from a standard uniform distribution, and that all new matches start at the highest productivity $x_0 = 1$. This ensures that the bargained wage will be an interior solution to (8) in the following exercises when labor market policies are included. The shock arrival rate is set to $\phi = 4\%$ in order to target a monthly separation rate of 3.7% in equilibrium. Finally, the Shimer (2005) calibration of unemployment insurance $b = 0.4$ and vacancy cost $c = 0.213$ is adopted in order to generate a slightly larger wage dispersion.

Under this specification, the highest and lowest bargained wages are respectively $w(\bar{x}) = 0.9633$ and $w(R) = 0.9254$ in the baseline model. In line with the results of Hornstein, Krusell, and Violante (2011), the frictional wage-dispersion is very small, as the mean wage is only 2.05% larger than the lowest bargained wage $w(R)$. Figure 5 shows the effect of increasing the minimum wage on various labor market outcomes. As noted earlier, the minimum wage acts as a side constraint and therefore only has equilibrium effects when $\underline{w} > w(R)$. As can be seen in the right panel of Figure 5, an increase in the minimum wage results in a higher reservation productivity and therefore more marginal jobs are destroyed, resulting in a higher separation rate. This in turn reduces the expected duration of jobs and expected profitability falls, resulting in fewer vacancies opened. The reduction in market tightness

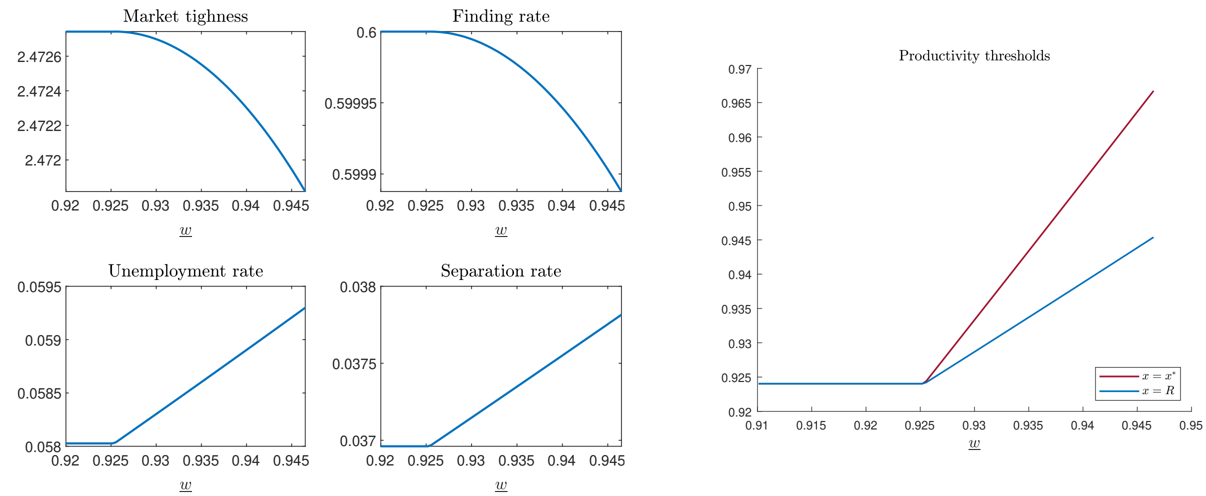

Figure 5: Minimum wage outcomes

and increase in separations therefore imply that steady state unemployment rises as the minimum wage increases. However, the quantitative effects are overall very small given the calibration used.

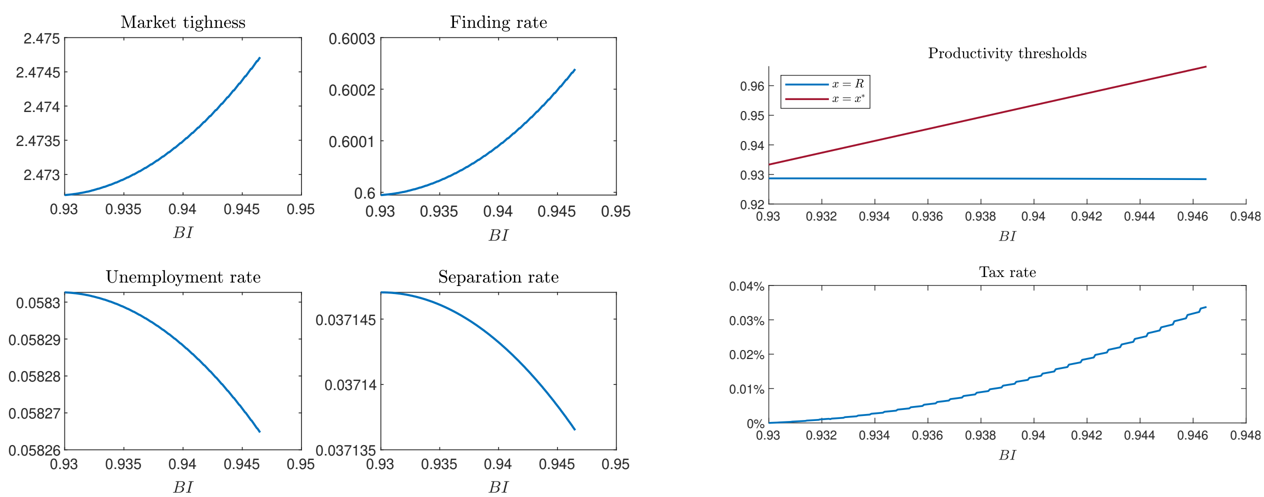

Figure 6: Basic income outcomes, $\underline{w} = 0.93$

Turning to the effect of the basic income policy, for illustrative purposes, consider the case of a minimum wage $\underline{w} = 0.93$. Figure 6 shows the effects produced by a basic income subsidy greater than the minimum wage. For simplicity, it is assumed that only the employee pays the payroll taxes τ_e. As can be seen in Figure 1, the basic income subsidy allows firms to pay eligible workers the minimum wage. This in turn increases their expected profitability and increases labor hoarding, as reflected by the slight decline in reservation productivity and separation rate.

As the guaranteed basic income rises, more vacancies are posted and the job finding increases. Therefore, fewer jobs are destroyed and more workers exit unemployment, both of which reduced equilibrium unemployment. Once again, the overall quantitative effects are modest given the current calibration. Note also that since the measure of eligible workers is small, the tax rate required to balance the government's balance is also low, increasing with the basic income level BI, as would be expected. Finally, as argued previously, it can be seen that the basic income counteracts the negative effects of a minimum wage on labor market outcomes.

The mechanism driving this result is the direct effect the policy has on job destruction, as it effectively subsidizes low productivity matches, increasing their profitability and resulting in fewer marginal jobs being destroyed. Recall that the minimum wage has a negative spillover on wages, which dampens its negative effect on labor aggregates, and that the basic income acts in an opposing direction through this channel, but is not as relevant in equilibrium. This quantitative exercise illustrates that the direct effect on job profitability is more important, ultimately increasing market tightness.

3 Incomplete asset markets

In order to highlight the insurance aspect of the aforementioned policies, this section introduces risk aversion for the consumer and incomplete asset markets, combining the imperfect insurance framework of Aiyagari (1994) and labor search frictions of Mortensen and Pissarides (1994) presented in the previous section. As the work of Bils et al. (2011) combines these features, I will use it as a basis to introduce the minimum wage and worker's basic-income subsidy scheme. The general setup assumptions and calibration here are therefore drawn from their work.

3.1 Model

In what follows, it is assumed that the firm's idiosyncratic productivity x follows a first order Markov process. In particular, the $\log x$ evolves according to:

$$\log x' = \xi + \rho \log x + \varepsilon, \quad \varepsilon \sim N(0, \sigma_\varepsilon), \tag{17}$$

where ξ denotes the process's unconditional mean and ε an innovation to idiosyncratic match quality. Note that, unlike in the previous section, this productivity process is by definition not memoryless. To compute the model, this process will be discretized to a finite state Markov chain using the method proposed by Tauchen (1986) and expectations will be calculated using the respective transition matrix. In this section, it is assumed that all new jobs are formed at mean productivity $x_0 = \bar{x}$. No additional changes are made to the matching framework, so the matching technology is still described by (1).

3.1.1 Workers

Workers can either be unemployed or matched to a firm of productivity x. Unlike in the previous section, it is assumed here that workers are risk averse, with preferences over consumption $c > 0$ and leisure $l \in \{0, 1\}$ described by:

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left[\frac{c_t^{1-\sigma} - 1}{1-\sigma} + B l_t \right], \tag{18}$$

where $\sigma > 0$ denotes the relative risk aversion coefficient and B the utility obtained from leisure. Note that this specification abstracts from the intensive labor supply margin, implying that employed workers obtain no leisure utility. As before, employed workers are paid a wage w by the firm and unemployed workers receive an exogenous income flow b, which can be interpreted as unemployment insurance.

It is assumed that there exist capital markets where assets can be traded at a constant and exogenous interest rate r. In order to introduce incomplete insurance, borrowing is limited to an amount $\underline{a} \leq 0$. The worker's problem is then to choose their asset holdings in order to maximize consumption, subject to said borrowing constraint and a budget constraint.

Recall that the wage bargaining assumptions in the preceding section implied that the model with linear utility exhibited privately-efficient separations when no policy was considered and firm-sided separations upon the inclusion of a minimum wage. As this constrained Nash-bargaining setup is maintained here, said separation feature also carries over as long as the bargained wage retains its non-decreasing structure over the worker's states, which indeed turns out to be the case. Whereas in the previous section there was a scalar separation threshold $x = x^*$, since workers are now characterized by the state pair (a, x), separations are now characterized by a threshold function $a^*(x)$, determined by the firm's problem. In particular, since the firm's value is decreasing over a for a given productivity level, a match of quality x is terminated by the firm whenever $a \geq a^*(x)$.

The employed worker's value function then solves the following problem:

$$\tilde{W}(w, a, x) = \max_{a'_e, c_e} \; u(c_e) + \beta \, \mathbb{E}_{z'|z} \left[U(a'_e) \cdot \mathbb{1}_{[a'_e > a^*(x')]} + W(a'_e, x') \cdot \mathbb{1}_{[a'_e \leq a^*(x')]} \right], \tag{19}$$

subject to

$$c_e + a'_e = (1+r)a + (1-\tau) \cdot \max\left\{w, \frac{BI}{1-\tau}\right\},$$
$$a'_e \geq \underline{a}, \quad c_e \geq 0,$$

where $\frac{BI}{1-\tau}$ is the government-guaranteed gross basic income, $\mathbb{1}_{[\cdot]}$ denotes the indicator function, and $a^*(x)$ the asset separation-threshold in a match of productivity x. The value function W takes into consideration that future wages $w' = \omega(a, x)$ are an equilibrium result of bargaining, that is, W is defined by:

$$W(a, x) \equiv \tilde{W}(\omega(a, x), a, x). \tag{20}$$

Note that when no labor policy is active, i.e., when neither the minimum wage nor basic income are binding, the problem reduces to that of Bils et al. (2011). Since separations are privately efficient in that case, the threshold function $a^*(x)$ is also consistent with the worker's endogenous separations, determined by $\max\langle W(a, x), U(a)\rangle$. On the other hand, the unemployed worker's value function U solves:

$$U(a) = \max_{a'_u, c_u} \; u(c_u) + \beta\Big[p(\theta) \cdot W(a'_u, \bar{x}) + (1 - p(\theta)) \cdot U(a'_u)\Big], \tag{21}$$

subject to

$$c_u + a'_u = (1+r)a + b,$$
$$a'_u \geq \underline{a}, \quad c_u \geq 0,$$

where p denotes the job finding probability and the assumption that all new matches start at the mean productivity $\bar{x}$ is reflected in the continuation value.

3.1.2 Firms

As in the previous section, firms are assumed to be risk-neutral and maximize the expected present value of profits:

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \left(\frac{1}{1+r}\right)^t \pi_t, \tag{22}$$

where $\pi = x - w_f$ denotes the period's profits and w_f the wage paid by the firm. Taking into account the fact that separations are determined by the firm, the value of a productivity x job to a firm matched with a worker of wealth a is defined by:

$$\tilde{J}(w, a, x) = x - \max\{w, \underline{w}\} + \frac{1}{1+r}\,\mathbb{E}_{z'|z}\Big[\max\langle J(a'_e, x'), V\rangle\Big], \tag{23}$$

where $\underline{w}$ denotes the minimum wage and J is defined analogously to the worker's problem:

$$J(a, x) \equiv \tilde{J}(\omega(a, x), a, x). \tag{24}$$

The above value function (24) implicitly defines the separation threshold a^* by $J(a^*, x) = 0$, which will differ from the worker's decision to separate once a binding minimum wage $\underline{w}$ is introduced. Turning to the value of vacancy to a firm, note that although all new matches are formed at mean productivity $\bar{x}$, a vacant firm is matched to a searching worker in the unemployment pool at random. Since equilibrium wages are a function of the worker's wealth, the expected value of vacancy depends on the unemployed worker distribution over assets, denoted by $\Phi_u(a)$. The value of vacancy is given by:

$$V = -\kappa + \beta\Big[q(\theta)\int J(a, \bar{x})\, d\Phi_u(a) + (1 - q(\theta))V\Big], \tag{25}$$

where q denotes the job-filling rate and κ the vacancy flow cost.

3.1.3 Wages

As before, wages are assumed to be Nash-bargained every period, subject to the minimum-wage constraint. That is, the equilibrium wage schedule $\omega(a, x)$ solves the following problem:

$$\max_{w} \left[\tilde{W}(w, a, x) - U(a)\right]^{\gamma} \left[\tilde{J}(w, a, x) - V\right]^{1-\gamma}, \quad \text{subject to } w \geq \underline{w}, \tag{26}$$

where γ denotes the worker's bargaining power. Once again, as baseline wages turns out to be a non-decreasing function over states (a, x), the minimum wage acts solely as a side constraint, as in Flinn (2006): the constraint is binding for state pairs (a, x) that would have otherwise resulted in a Nash-bargained wage below the allowed minimum $\underline{w}$. The basic income argument from the previous section also carries over: only workers that are paid a wage $w > \frac{BI}{(1-\tau)}$ care about the bargained wage, as the government here will guarantee them an income of BI regardless of the wage the firms pay them. Therefore, the firm will pay the worker the lowest amount possible, the minimum wage $\underline{w}$, whenever the Nash-bargained wage is below $\frac{BI}{1-\tau}$. Finally, an interior solution to the above bargaining problem (26) will satisfy the following optimality condition:

$$\frac{\gamma}{W(a, x) - U(a)} u'(c_e)(1 - \tau) = \frac{1 - \gamma}{J(a, x) - V}. \tag{27}$$

3.1.4 Distributions

Let $\mathcal{A}$ and $\mathcal{X}$ respectively denote the set of all possible realizations of assets a and productivity x. For all $A \subset \mathcal{A}$ and $X \subset \mathcal{X}$, the measures of unemployed workers Φ_u and employed workers Φ_e respectively evolve according to:

$$\Phi'_u(A) = (1 - p(\theta)) \int_A \int_{\mathcal{A}} \mathbb{1}_{[a'=a'_u]}\, d\Phi_u(a)\ da' + \int_A \int_{\mathcal{A},\mathcal{X}} \mathbb{1}_{[a'=a'_e, a'_e > a^*(x')]}\, dF(x'|x)\, d\Phi_e(a, x)\ da' \text{ and} \tag{28}$$

$$\Phi'_e(A, X) = \int_{A,X} \int_{\mathcal{A},\mathcal{X}} \mathbb{1}_{[a'=a'_e, a'_e \leq a^*(x')]}\, dF(x'|x)\, d\Phi_e(a, x)\ da' dx' + p(\theta) \int_A \int_{\mathcal{A}} \mathbb{1}_{[x'=\bar{x}, a'=a'_u]}\, d\Phi_u(a)\ da', \tag{29}$$

where $F(x'|x)$ denotes the productivity's conditional cumulative density function, described by (17). Note that movement across assets is a deterministic process, which is entirely determined by the worker's policy functions.

The first term of (28) reflects movement within the unemployment distribution due to a lack of meeting a firm. The second term describes the inflow to unemployment, which is given by the productivity draw an employed worker gets: future assets are chosen before observing future productivity, and all workers whose asset holdings exceed a^* are fired. It's worthwhile noting that it is plausible that when a match receives a substantial negative productivity shock, it will terminate regardless of the worker's asset holdings.

The employment distribution Φ_e has the obvious counterparts to the previously discussed Φ_u: transitions occur within the employment distribution, governed by the Markov chain when it comes to match quality, whenever the productivity shock is not large enough to lead to a separation. Separated workers constitute the unemployment inflow mentioned above. Finally, there's an employment inflow to mean productivity $\bar{x}$, inheriting the asset distribution from Φ_u as matching occurs at random.

3.1.5 Government

It is assumed that the government chooses the tax rate τ in order to maintain a balanced budget, which is respectively defined by:

$$\int_{\mathcal{A},\mathcal{X}} \tau \cdot \omega(a, x)\, d\Phi_e(a, x) = \int_{\mathcal{A},\mathcal{X}} (BI - \underline{w}) \cdot \mathbb{1}_{\left[\omega(a,x) \leq \frac{BI}{1-\tau}\right]}\, d\Phi_e(a, x). \tag{30}$$

It is worthwhile noting that under the bargaining assumptions made, the wedge between the wage bargained by the firm and the basic income results in a constant subsidy of $(BI - \underline{w})$ for all eligible workers.

3.2 Equilibrium

The stationary recursive competitive equilibrium consists of a set of value functions $\tilde{W}(w,a,x)$, $W(a,x)$, $U(a)$, $\tilde{J}(w,a,x)$, $J(a,x)$ and V; consumption policy functions $c_e(a,x)$ and $c_u(a)$; asset holdings policy functions $a'_e(a,x)$ and $a'_u(a)$; a separation policy function $a^*(x)$; a wage schedule $\omega(a,x)$; a law of motion for the distributions $(\Phi'_u, \Phi'_e) = T(\Phi_u, \Phi_e)$; labor-market tightness θ; and tax rate τ, such that:

i) Given θ, ω, τ and a^*, the value function $\tilde{W}$, W and U solve the consumer's problem described in (19)–(21) with associated policy functions a'_e, c_e, a'_u and c_u.

ii) Given $\Phi_u, \theta, \omega, a'_e$ and a'_u, the value functions $\tilde{J}$, J and V solve the firm's problem described in (24)–(25) with associated separation threshold x^*.

iii) Given $\tilde{W}, U, \tilde{J}$ and V, ω solves the constrained Nash-bargaining problem (26).

iv) Given J and Φ_u, market tightness θ satisfies free entry condition $V = 0$.

v) Given ω and Φ_e, the government budget (30) is balanced with a tax rate τ.

vi) Given a'_e, a'_u, x^* and θ, Φ_u and Φ_e are stationary distributions described by T in (28)–(29).

3.3 Computation

In addition to a functional fixed-point solution to the worker's and firm's Bellman equations, solving the equilibrium requires obtaining a fixed point for the tax rate τ and market tightness θ and functional fixed points for the separation policy $a^*(x)$ and wages $\omega(a,x)$. To solve this problem, I extend the solution algorithm of Bils et al. (2011) to the requirements for the current setup.

The multiple required fixed points can be solved as follows: (i) guess on τ, θ, $a^*(x)$, and $\omega(a,x)$, (ii) compute the job finding $p(\theta)$ and filling $q(\theta)$ rates from the matching function, (iii) solve the worker's problem by using non-linear global methods, (iv) obtain the implied job's value from the Nash-bargaining first order condition, (v) solve the firm's problem using the worker's asset policy to perform the value function iteration, (vi) use the solution to the firm's problem and the current job's value implied by Nash-bargaining in the firm's value function definition to obtain the implied wage, (vii) update the wages $\omega(a,x)$ by dampening the update and repeat through (iii) until they converge, (viii) obtain the new separation policy implied by the converged wages, update the policy $a^*(x)$ and repeat (vii) and (viii) until the separation policy converges, (ix) compute the invariant worker distributions Φ_e and Φ_u, (x) obtain the value of vacancy using Φ_u and update market tightness accordingly, starting over at (ii) (some guess refinements can be made at this point), (xi) upon convergence of market tightness, perform a bisection on the government's budget balance, starting back at (i), with refined guesses, until convergence, (xii) verify converged elements are actually a solution to the problem.

To solve the worker's problem, value function iteration is performed with continuous policy methods. In particular, cubic spline interpolation is performed between grid points, as this solution method dominates the common alternatives for value function iteration (Heer & Maussner, 2011). To solve the invariant asset distribution, algorithm 5.2.2 from Heer and Maussner (2005) is followed.

3.4 Numerical exercises

The numerical exercise performed here uses the benchmark calibration of Bils et al. (2011). The authors set the consumer's relative risk aversion coefficient σ equal to one, the discount factor $\beta = 0.99477$ to target monthly time periods, and the monthly interest rate r to target a yearly rate of 0.06%. Following the calibration of Shimer (2005), they set b equal to 0.4 and the value of leisure to $B = 0.15$. The targeted unemployment and separation rates are respectively 6% and 2%, implying a job finding rate of $p = 31.33\%$. The authors chose to normalize steady-state market tightness to one, implying a match scale factor of $\chi = 0.3133$. Both the matching elasticity η and bargaining power γ are assumed to be 0.5. The persistence of log productivity is set to $\rho = 0.97$ and the innovation's standard deviation to $\sigma_\varepsilon = 0.13$. It is assumed that mean productivity is normalized to one, i.e., $\mathbb{E}[x] = 1$, which pins down the value of ξ. The productivity's process is approximated as a 15-state Markov chain using the method of Tauchen (1986). The credit limit is chosen to be $\underline{a} = -6$, which is roughly six months' worth of labor income. Finally, the above calibration and grid choice imply a vacancy flow cost of $\kappa = 0.524$ in the baseline steady-state.

The first numerical exercise performed is the evaluation of labor market aggregates as a function of the minimum wage. The lowest bargained wage in the baseline model turns out to be $w_{lb} = 0.8645$. Considering this, the steady states associated with a minimum wage up to 20% higher than w_{lb} are computed. The resulting labor aggregates are shown in Figure 7. Note that, given the calibration and discrete approximation for the productivity process,

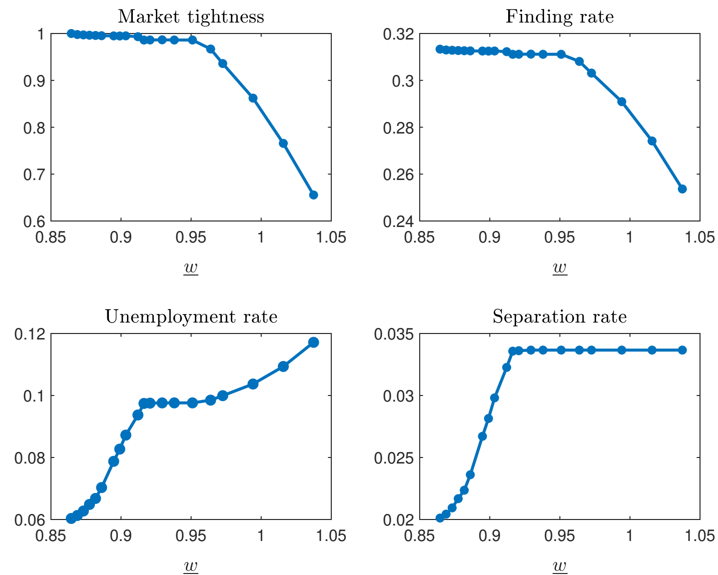

Figure 7: Minimum wage outcomes

the separation rate is the initial driver in the increase of unemployment as a response to increases in the minimum wage level. The intuition behind this can be understood by examining the baseline separation schedule, depicted in the left panel of Figure 9. As under the current calibration there is only one productivity level at which separations are determined by the worker's wealth, it is precisely workers with assets below that threshold that suffer a job loss, as the minimum wage impedes them from bargaining a lower wage which they would have otherwise accepted, driving the firm's profitability down.

The key driving force here is the existence of incomplete markets: consumers can't fully insure themselves against match quality shocks, and since job search takes time, the worker's outside option falls when his wealth is low, as there exists imperfect consumption smoothing due to the credit constraint. This allows firms to bargain lower wages when their employee's wealth is low, increasing profitability in those matches. This suggests that although the minimum wage might result in larger paid wages in low wealth matches, there may exist negative welfare effects on agents with low savings, as they are precisely the ones facing an increased separation rate risk were the minimum wage to increase, and are at an increased rate of facing a binding liquidity constraint when losing their job.

As the minimum wage increases, the extent to which a worker's wealth determines match separation falls and therefore market tightness falls, as jobs simply become less profitable and fewer vacancies are created, resulting in longer unemployment duration. The steep decrease in job creation, despite job duration being unaffected, is due largely to the assumption that new matches start at mean productivity and therefore are constrained by the minimum wage when it turns out to be large enough. This highlights the fact that here, unlike in the linear utility model, match duration is not the main driver of the increase of unemployment at any given minimum wage level. It is also worthwhile noting that, under the current framework, the model predicts that the minimum wage has strong quantitative effects on unemployment, unlike in Section 2.

The second numerical exercise performed is a steady state comparison of the baseline model, a minimum wage at which wealth is a determinant factor of separations and a high basic income level.[3] To highlight the effects of each

[3]It should be noted that the solution algorithm used is highly unstable around basic income levels that produce large jump discontinuities over assets in bargained wage for a given productivity level, requiring one to choose the policy calibration carefully to obtain

policy, the minimum wage is set 3.5% higher than the lowest bargained wage in the baseline model, and the basic income 5% higher than the minimum wage. The following table summarizes the steady-state results in each case:

Equilibrium	Baseline $w_{lb} = 0.8645$	Minimum wage $\underline{w} = 0.8948$	Basic income $BI = 0.9395$
Unemployment rate	6.03%	7.88%	6.91%
Separation rate	2.01%	2.67%	2.34%
Job finding rate	31.33%	31.25%	31.57%
Market tightness	1	0.995	1.0154

Table 1: Equilibrium labor-market aggregates

As in Section 2, the basic income policy works to counteract the negative effect of the minimum wage on labor market outcomes. In particular, it allows low wealth workers to retain their jobs in the current setup, even when facing a minimum wage. Note that vacancy creation actually increases above the baseline steady state level here, as the subsidy increases the job's profitability for low productivity and low wealth matches, amplifying the effect. Since the measure of eligible workers for the policy is small due to endogenous selection away from the critical separation threshold, a small tax rate of only approximately 0.5% is required to balance the government's budget, which suggests a small welfare impact of financing this policy. The resulting wage schedules under the considered policies are depicted in Figure 8. Note that, as in the baseline case, the wage schedule is increasing in assets as this

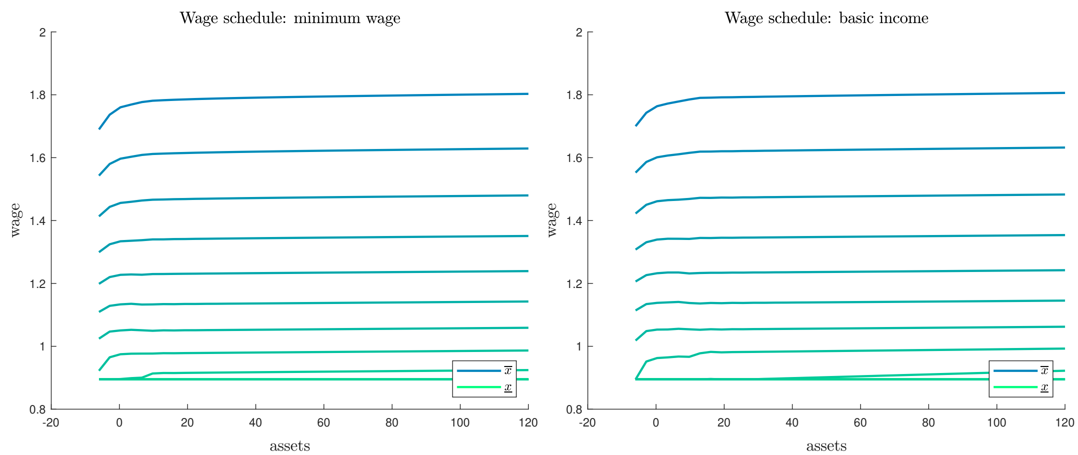

Figure 8: Wage schedules: policies

strengthens the worker's outside option by allowing for greater consumption smoothing via savings. The observed jumps in the wage function correspond to bargained wages that wouldn't be materialized in equilibrium, as these matches would separate. The respective separation thresholds are shown in Figure 9.

Summary statistics for the model under each policy are presented in Table 2. It is worthwhile noting that the minimum wage increases the right tail of the stationary distribution, as observed in Figure 10, reflecting the greater need for precautionary savings to insure against potential future unemployment. It is also the case that the asset separation threshold is lower with a minimum wage, leading workers to accumulate less assets in order to avoid this threshold. As can be observed in Table 2, this later selection mechanism dominates the overall effect on the worker's aggregate asset holdings.

It should be noted that the increase in unemployed worker's asset holdings is due to the greater separation rate implied by a lower asset threshold, as wealthier workers now enter unemployment when receiving negative productivity shocks. This can be seen in the existence of spikes around the asset cutoff in the stationary distribution of unemployed workers. The slight increase in aggregate consumption of unemployed workers simply reflects the fact that wealthier workers enter unemployment.

Once again, it can be seen that the basic income policy counteracts the effect of the minimum wage, working also to partially revert the invariant worker's distribution. There are various factors reinforcing this effect here: an increase

a convergent solution.

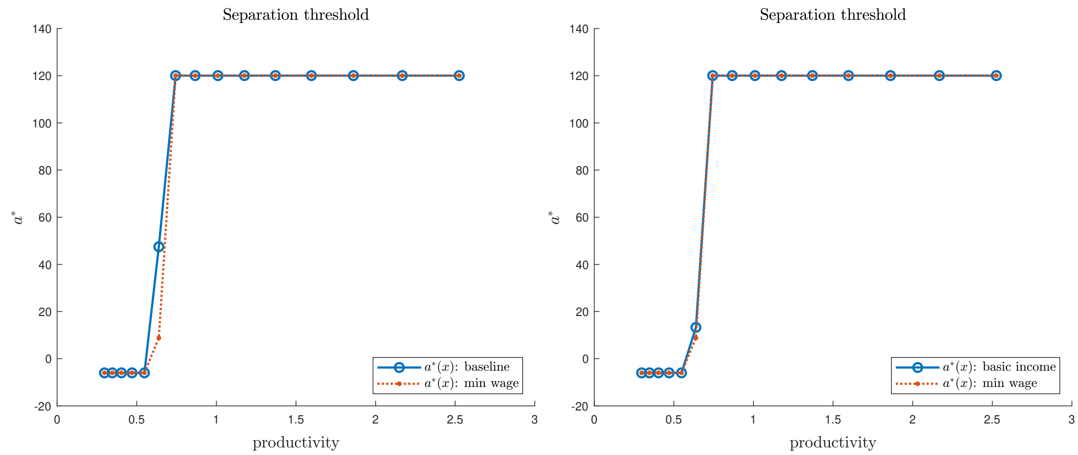

Figure 9: Separation thresholds

in the job finding rate, a fall in the separation rate, and a fall in wage dispersion. The overall effect on the asset distribution can be observed in Figure 11, where the basic income generates a lighter right-sided tail and a more skewed distribution towards lower wealth levels for unemployed workers, generating a decrease in the aggregate asset holdings, as observed in Table 2.

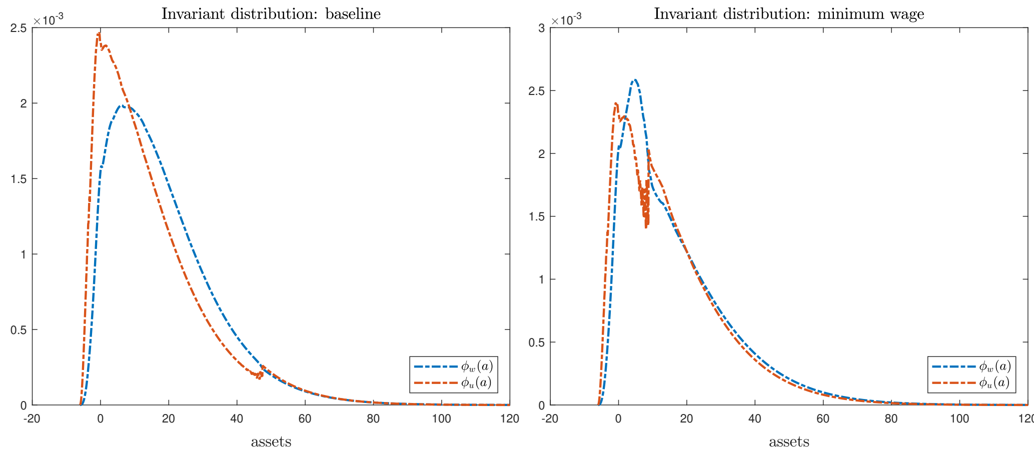

Figure 10: Invariant distributions: minimum wage

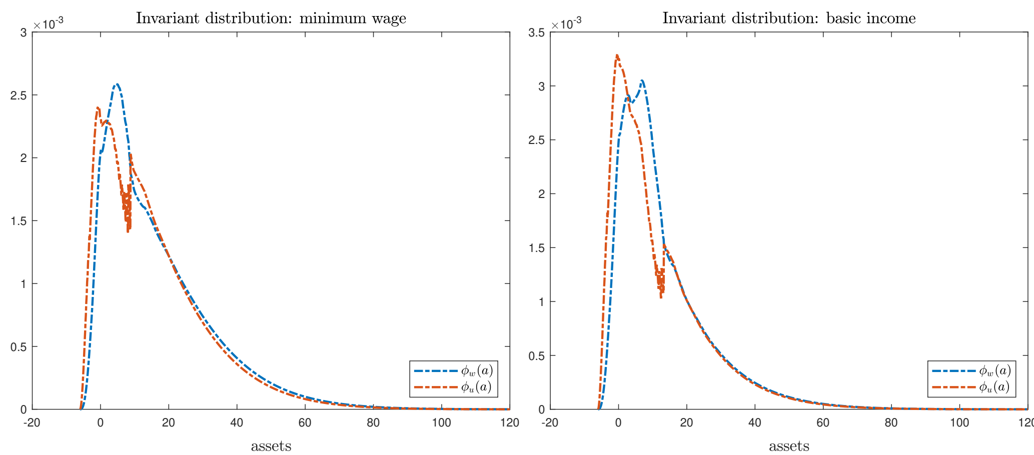

Figure 11: Invariant distributions: basic income

Equilibrium	Baseline $w_{lb} = 0.8645$	Minimum wage $\underline{w} = 0.8948$	Basic income $BI = 0.9395$
Average assets	17.9	16.6	12.8
Assets standard deviation	15.1	15.7	13.3
Average assets: employed	18.1	16.7	12.9
Average assets: unemployed	14.7	15.2	11.5
Average consumption: employed	1.19	1.18	1.17
Average consumption: unemployed	1.10	1.11	1.08
Average wage: firm	1.1422	1.1527	1.1473
Wage standard deviation: firm	0.2174	0.2144	0.2188
Average wage: worker			1.1549
Wage standard deviation: worker			0.2107

Table 2: Equilibrium summary statistics

It's also worthwhile noting that, although the average wages are higher with a minimum wage, the average bargained wages fall slightly from 1.1622 to 1.1606 over the unconstrained states upon the introduction of the minimum wage. Furthermore, as can be observed in Table 2, the same wage dispersion effects found in the linear model of Section 2 carry over: the wage distribution compresses when only the minimum wage is binding, and the basic income works in the opposite direction for the wages paid by the firm. Finally, it can be seen in Table 2 that, in the current case, the precautionary savings motive drives the redistribution effect on assets, which presents a lower dispersion when the risk of job loss falls.

3.5 Welfare effects

To discuss the policy's welfare effects,[4] I draw on the equivalent consumption variation from Krusell et al. (2010). Let $\{c_t, l_t\}_{t=0}^{\infty}$ denote an optimal allocation to the consumer's problem, where, for simplicity, shock history-dependence indexing has been omitted. The consumer's value function is then given by:

$$E(a, x, e) = \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left[\frac{c_t^{1-\sigma} - 1}{1 - \sigma} + Bl_t \right], \tag{31}$$

where $e \in \{\text{employed}, \text{unemployed}\}$ denotes the worker's employment state and $E \in \{W, U\}$ the respective value function. Let E_0 denote the value a worker obtains in the benchmark and E_1 that obtained in an alternative policy. As in Krusell et al. (2010), a measure of the policy's impact on welfare can be obtained in terms of an implied equivalent-consumption factor g: the amount of consumption $c(1 + g)$ necessary in the benchmark case to generate the same value as under the policy. In the case of log utility, as assumed by the calibration of the relative risk aversion coefficient $\sigma = 1$ in Section 3, this growth factor is determined by:

$$E_1(a, x, e) = \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t [\log c_t(1 + g) + Bl_t]. \tag{32}$$

The above expression implies that the equivalent-consumption factor g here is given by:

$$g(a, x, e) = \exp\big\{(1 - \beta)\big[E_1(a, x, e) - E_0(a, x, e)\big]\big\} - 1. \tag{33}$$

Figure 12 depicts the equivalent consumption variation g required to generate the utility level obtained with a binding minimum wage in the baseline model. As can be observed in this figure, the minimum wage generates losses for both employed and unemployed workers, with the greatest effect around the asset separation threshold. Therefore, the gain obtained from a higher wage in low productivity matches is completely offset and overturned by the increase in job loss risk. However, as Table 3 shows, the effect is quantitatively small. It is worthwhile recalling that the minimum wage is set 3% higher than the otherwise lowest bargained wage in the baseline, which in part accounts for the modest welfare effects.

In contrast, as observed in Table 3, the basic income policy is found to have larger and positive quantitative welfare effects for both the employed and unemployed workers, in comparison to both minimum wage and baseline scenarios.

[4] This section contains preliminary results, drawn only from the three simulated versions of the model.

Average equivalent consumption variation	Minimum wage / Baseline	Basic income / Minimum wage	Basic income / Baseline
Employed workers	-0.082%	0.235%	0.145%
Unemployed workers	-0.073%	0.297%	0.227%

Table 3: Welfare gains

The reason behind this, aside from the slightly larger magnitude for the considered policy, is that, as mentioned previously, it serves an insurance purpose by diminishing the wage risk, job loss risk, and unemployment duration. As workers are risk averse, the small tax rate of about 0.5% required to finance this is surpassed by the insurance effect generated, resulting in welfare gains for both unemployed and employed workers for the most part, the former benefiting in an equivalent consumption variation of 0.3% in comparison to the case with only a minimum wage. As would be expected, wealthier workers are the ones that would incur in welfare losses due to the taxation of their wages. On average, the numerical exercise suggests that the basic income policy would constitute a welfare improvement over the minimum wage policy.

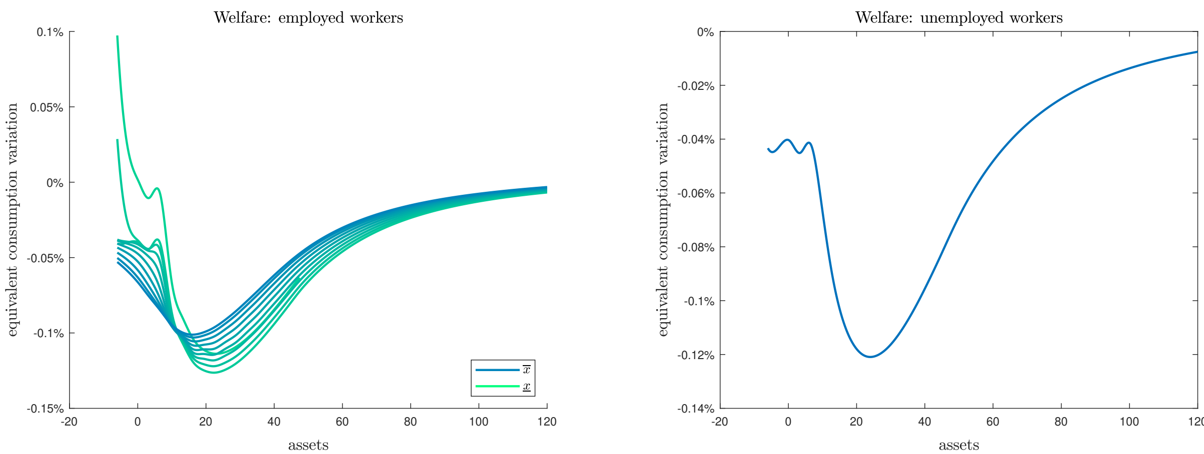

Figure 12: Welfare effects: minimum wage

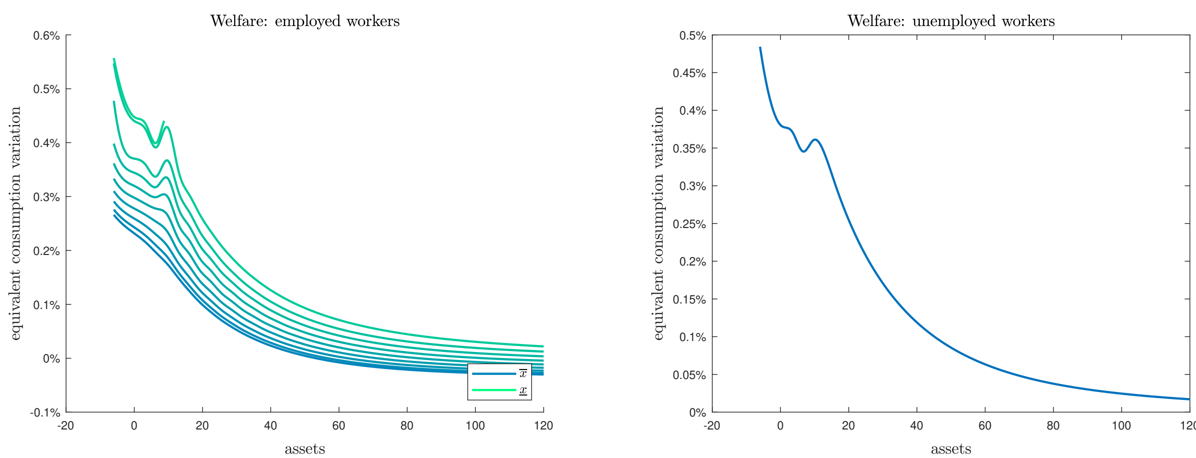

Figure 13: Welfare effects: basic income

4 Conclusions

I introduce a basic income scheme for the worker into the standard labor matching endogenous separations framework of Mortensen and Pissarides (1994). To achieve this, a minimum wage must be included to put a cap on the allowed size of the government's subsidy to workers. The minimum wage is introduced as a side constraint to the Nash-bargaining problem, as in Flinn (2006). I find that the model implies that the minimum wage increases unemployment through increased match separation, and that the basic income scheme counteracts this effect by effectively acting as a subsidy to low productivity firms. The minimum wage in this context is also found to generate a negative spillover on the bargained wage level, which dampens the effect on market tightness by increasing the profitability of new jobs. A numerical exercise is performed in order to quantify the effects of said policies on labor market outcomes. It is found that, in this context with risk-neutral agents, the quantitative effects generated by the policy are small. This would be in line with the lack of consensus in the literature over the effects of the minimum wage on labor outcomes.

As in Krusell et al. (2010) and Bils et al. (2011), the baseline model is then extended to include risk-averse workers and incomplete asset markets. In this context, the policies have non-negligible effect on labor market outcomes and associated job-loss and wage risks. It is noted that, in particular, workers with wealth levels close to the critical separation-asset threshold are the ones that suffer greater negative consequences upon the introduction of a minimum wage, as it impedes their possibility of bargaining a lower wage and therefore generates an increased chance of exiting into unemployment, where consumption smoothing is limited to savings due to the existence of incomplete asset markets. Given that the relevant asset cutoff is lower upon the inclusion of a minimum wage, it is low wealth workers who benefit the most from the basic income policy, as in addition to financing their employer, it decreases labor risks in the model. Preliminary welfare analysis also suggests that the basic income policy would potentially be welfare improving on average.

Future work is required in order to thoroughly quantify the effects of such policies on labor market outcomes and welfare. More extensive computational experiments under different calibrations would be an avenue to understanding under what circumstances the policy has significant effects on labor market outcome and is welfare improving altogether. An adequate estimation and calibration to the Chilean economy would also be required to quantify the effects the policy generates, which, as suggested by the model with incomplete markets, could turn out to be welfare improving.

5 Appendix

5.1 Equilibrium: Linear utility

5.1.1 Baseline

This appendix contains the derivation of equilibrium conditions for the standard endogenous-separations model of Mortensen and Pissarides (1994). Match productivity x is assumed to follow a simple persistent but memoryless process: each period, x is drawn from a distribution $G(x)$ with probability ϕ. It is assumed that G is a continuous cdf with support $[\underline{x}, \bar{x}]$. The value of a job with productivity x to a firm $J(x)$ is given by:

$$J(x) = x - w(x) + \beta\left[\phi \int \max\langle J(z), V\rangle \, dG(z) + (1-\phi)J(x)\right], \tag{1}$$

where the continuation value reflects the firm's choice to close a job when it's not as profitable as vacancy. The flow cost of creating a vacancy is c and vacant firms meet a worker with probability $q(\theta)$. New jobs are created at productivity x_0, and therefore the value of vacancy V solves:

$$V = -c + \beta[q(\theta)J(x_0) + (1 - q(\theta))V]. \tag{2}$$

The value of a job with productivity x to the worker $W(x)$ is given by:

$$W(x) = w(x) + \beta\left[\phi \int \max\langle W(z), U\rangle \, dG(z) + (1-\phi)W(x)\right], \tag{3}$$

where the continuation value accounts for the possibility of quitting when unemployment is more valuable to a worker. Unemployed workers receive unemployment insurance b and meet a firm with probability $p(\theta)$, so the value of unemployment U satisfies:

$$U = b + \beta[p(\theta)W(x_0) + (1 - p(\theta))U]. \tag{4}$$

The wage schedule $w(x)$ is assumed to be a productivity-contingent contract that solves the following Nash-bargaining problem over the surplus:

$$\max_{w(x)} \; [W(x) - U]^{\gamma}[J(x) - V]^{1-\gamma}, \tag{5}$$

where γ denotes the worker's bargaining power and $(1-\gamma)$ that of the firm. It is assumed that there exists free entry of firms: in equilibrium, vacancies are created until they are of no value. Since it holds that $V = 0$ in equilibrium, the firm's surplus of a filled job is simply given by $J(x)$, which can be restated from (1) as:

$$J(x) = \frac{1}{1-\beta(1-\phi)}\left[x - w(x) + \beta\phi \int \max\langle J(z), 0\rangle \, dG(z)\right]. \tag{6}$$

Similarly, by conveniently rearranging terms in (3), one can obtain the following expression for the worker's surplus of employment:

$$H(x) = \frac{1}{1-\beta(1-\phi)}\left[w(x) - (1-\beta)U + \beta\phi \int \max\langle H(z), 0\rangle \, dG(z)\right], \tag{7}$$

where $H(x) \equiv W(x) - U$ denotes the worker's surplus. Let $S(x) \equiv H(x) + J(x)$ denote the match's total surplus. Noting that $\frac{\partial H(x)}{\partial w(x)} = -\frac{\partial J(x)}{\partial w(x)}$, the first order condition to the Nash-bargaining problem (5) can be written as:

$$\frac{W(x) - U}{\gamma} = S(x) = \frac{J(x) - V}{1-\gamma}. \tag{8}$$

That is, each party gets a share of the match's surplus $S(x)$, constant across productivity and equal to their bargaining power. Note that this sharing rule implies that separations are privately efficient: the firm and worker agree on destroying the match whenever its surplus is negative. Furthermore, as $S'(x) = \frac{1}{1-\beta(1-\phi)}$, it follows that the match's surplus is increasing in productivity and separations satisfy a reservation property: all matches of productivity $x < R$ separate, where R denotes the reservation productivity defined by $S(R) = 0$. The match's surplus is then given by:

$$S(x) = \frac{1}{1-\beta(1-\phi)}\left[x - (1-\beta)U + \beta\phi\int_R^{\bar{x}} S(z)\,dG(z)\right]. \tag{9}$$

Note also that since $S(R) = 0$ and $S'(x) = \frac{1}{1-\beta(1-\phi)}$, the match's surplus can be restated as:

$$S(x) = \frac{1}{1-\beta(1-\phi)}(x - R). \tag{10}$$

Using this fact and evaluating the surplus equation (9) at $x = R$ results in the **job destruction curve**, which implies that there exists labor hoarding as R is less than the worker's reservation value:

$$0 = R - (1-\beta)U + \frac{\beta\phi}{1-\beta(1-\phi)}\int_R^{\bar{x}} (z - R)\,dG(z), \tag{11}$$

where U is a function of market tightness θ. As wages are bargained in new matches, it follows from the value of vacancy (2), the free-entry condition $V = 0$, and the surplus sharing rule (8) that:

$$\frac{c}{q(\theta)} = \beta\frac{1-\gamma}{1-\beta(1-\phi)}(x_0 - R), \tag{12}$$

which is the **job creation curve**. To obtain the wage schedule, note that the wage sharing rule (8) and the job's surplus to a worker (6) and firm (7) imply the following:

$$w(x) = \gamma x + (1-\gamma)(1-\beta)U. \tag{13}$$

Finally, note that since wages are bargained in new matches, the value of unemployment (4), the value of vacancy (2), the free-entry condition $V = 0$, and the surplus sharing rule (8) imply that:

$$(1-\beta)U = b + \frac{\gamma}{1-\gamma}c\theta, \tag{14}$$

where the fact that $p(\theta) = \theta q(\theta)$ has been used. Therefore, the equilibrium reservation productivity and market tightness pair (R, θ) are pinned down by the following job destruction and job creation curves:

$$0 = R - b - \frac{\gamma}{1-\gamma}c\theta + \frac{\beta\phi}{1-\beta(1-\phi)}\int_R^{\bar{x}} (z - R)\,dG(z) \text{ and} \tag{JD}$$

$$\frac{c}{q(\theta)} = \beta\frac{1-\gamma}{1-\beta(1-\phi)}(x_0 - R). \tag{JC}$$

To differentiate the job destruction curve, note first that integration by parts in (JD) simplifies the equation to:

$$0 = R - b - \frac{\gamma}{1-\gamma}c\theta + \frac{\beta\phi}{1-\beta(1-\phi)}\int_R^{\bar{x}} (1 - G(z))dz. \tag{15}$$

Therefore, differentiation of (15) with respect to R and θ results in:

$$\frac{\gamma}{1-\gamma}cd\theta = \left[1 - \frac{\beta\phi}{1-\beta(1-\phi)}(1 - G(R))\right]dR, \tag{16}$$

which is an upward relationship between R and θ as both $\frac{\beta\phi}{1-\beta(1-\phi)}$ and $(1 - G(R))$ are bounded by 1. On the other hand, differentiation of (JC) with respect to R and θ results in:

$$\eta\frac{c}{p(\theta)}d\theta = -\frac{\beta(1-\gamma)}{1-\beta(1-\phi)}dR, \tag{17}$$

which is a downward relationship between R and θ.

5.1.2 Minimum wage

Given that the solution to the unconstrained Nash-bargaining problem (5) is an increasing wage schedule, the introduction of a minimum wage $\underline{w}$ acts simply as a side constraint on the bargained wages. As would be expected, a binding minimum wage implies greater destruction of marginal jobs: a firm's profitability decreases as low productivity shocks can't be absorbed by bargaining a lower wage, resulting in shorter jobs. It is still the case here that the job's value is non-decreasing in productivity and, therefore, a reservation property is once again satisfied. It is important to note that when the minimum wage is binding, separations are determined by the firm at $J(R) = 0$, i.e, when jobs are no longer valuable to them, and not by a null joint surplus.

Using the reservation value property, the surplus of a job to a firm $J(x)$ and worker $H(x)$ are respectively given by:

$$J(x) = \frac{1}{1-\beta(1-\phi)}\left[x - w(x) + \beta\phi\int_R^{\bar{x}} J(z)\,dG(z)\right] \text{ and} \tag{18}$$

$$H(x) = \frac{1}{1-\beta(1-\phi)}\left[w(x) - (1-\beta)U + \beta\phi\int_R^{\bar{x}} H(z)\,dG(z)\right]. \tag{19}$$

Wages $w(x)$ are assumed to be determined by the following constrained Nash-bargaining problem:

$$\max_{w(x)} \; H(x)^{\gamma} J(x)^{1-\gamma}, \;\; \text{s.t. } w(x) \geq \underline{w}. \tag{20}$$

Recall from Section 5.1.1 that unconstrained wages are increasing in productivity x. The imposition of a binding minimum wage therefore acts solely as a side constraint: wages are constrained at the minimum $\underline{w}$ until a productivity threshold $x = x^*$, at which freely bargained wage and minimum wage coincide. That is, the wage schedule is given by $w(x) = \max\langle w^i(x), \underline{w}\rangle$, where $w^i(x)$ denotes an interior solution to the bargaining problem (20), satisfying the following first order condition:

$$\frac{H(x)}{\gamma} = S(x) = \frac{J(x)}{1-\gamma}. \tag{21}$$

This works out to the same surplus sharing rule as encountered in the previous section for all productivity $x \in [x^*, \bar{x}]$. Proceeding as before, the bargained wage for all $x \in [x^*, \bar{x}]$ can then be written as:

$$w(x) = \gamma x + (1-\gamma)(1-\beta)U + \beta\phi\int_R^{x^*}\Big(\gamma J(z) - (1-\gamma)H(z)\Big)\,dG(z). \tag{22}$$

To evaluate the above integral, note first that for productivity $x \in [R, x^*]$, the worker receives the minimum wage $\underline{w}$ and therefore his surplus from employment is constant over that range. Furthermore, as the minimum wage and the bargained wage coincide at $x = x^*$, the surplus sharing rule (21) implies that the worker's surplus $H(x)$ for all $x \in [R, x^*]$ satisfies:

$$H(x^*) = \frac{\gamma}{1-\gamma} J(x^*). \tag{23}$$

Secondly, the firm pays the minimum wage $\underline{w}$ in low productivity matches, which is constant over x. Therefore, the job's value (18) implies that $J'(x) = \frac{1}{1-\beta(1-\phi)}$ for $x \in [R, x^*]$, and since job destruction is given by $J(R) = 0$, it follows that the job's value over said productivity range can be written as:

$$\underline{J}(x) = \frac{1}{1-\beta(1-\phi)}(x - R), \tag{24}$$

where $\underline{J}(x)$ denotes the job's value function when the minimum wage $\underline{w}$ is paid. Using these results, one can rewrite the bargained wages (22) as:

$$w(x) = \gamma x + (1-\gamma)(1-\beta)U \underbrace{- \frac{\phi}{r+\phi}\gamma\int_R^{x^*}(x^* - z)\,dG(z)}_{\text{minimum-wage spillover}}. \tag{25}$$

The above expression reflects the fact that firms internalize the lost surplus share in low productivity matches generated by a binding minimum wage. To this effect, there exists a negative spillover on bargained wages, which increases whenever it becomes more likely that the firm pays the minimum wage in the future. Note that when match duration is shorter, fewer low-productivity matches survive, decreasing the probability of having a binding minimum wage. As in the previous section, the surplus equations (18) and (19) imply that the match's total surplus is given by:

$$S(x) = \frac{1}{1-\beta(1-\phi)}\left[x - (1-\beta)U + \beta\phi \int_R^{\bar{x}} S(z)\, dG(z)\right]. \tag{26}$$

However, unlike in the previous section, matches are destroyed even when their total surplus is positive. This difference arises as firms face lower profitability in low productivity matches when there exists a minimum wage. In particular, a binding minimum wage implies that the worker's share of the surplus increases as productivity falls. Note that since $J(R) = 0$ and the worker's surplus $H(x)$ is constant over $x \in [R, x^*]$, the total match's surplus at the reservation productivity $S(R) = H(R)$ satisfies:

$$S(R) = \frac{\gamma}{1-\gamma}\frac{1}{1-\beta(1-\phi)}(x^* - R), \tag{27}$$

which is strictly positive assuming a binding minimum wage. As minimum wages become more likely to be binding, the inefficiency in match separation rises. Noting from equation (26) that $S'(x) = \frac{1}{1-\beta(1-\phi)}$, it follows from the surplus equation (27) that:

$$S(x) = \frac{1}{1-\gamma}\frac{1}{1-\beta(1-\phi)}\Big((x-R) - \gamma(x - x^*)\Big). \tag{28}$$

As shown below, the job destruction curve implies that, in equilibrium, $\frac{dx^*}{dR} < 0$. Shortening the duration of the match then also decreases its terminal value and the associated negative spillover on wages. Using this surplus equation, one can verify that the wage spillover in equation (25) is precisely the expected share of the surplus the firm loses due to the minimum wage, which is described by the following expression:

$$\underline{J}(x) - (1-\gamma)S(x) = -\frac{\gamma}{1-\beta(1-\phi)}(x^* - x), \tag{29}$$

which is always negative over $[R, x^*]$. Turning to the determination of the job destruction curve, recall that matches separate when $J(R) = 0$ in presence of a binding minimum wage. Note that (24) implies that $\underline{J}'(x) = \frac{1}{1-\beta(1-\phi)}$ over $[R, x^*]$ and that $J(x) = (1-\gamma)S(x)$ for all productivity $x \in [x^*, \bar{x}]$ by virtue of the surplus sharing rule (21). Using these results, the job's value (18) can be restated as:

$$J(x) = \frac{1}{1-\beta(1-\phi)}\left[x - w(x) + \frac{\beta\phi}{1-\beta(1-\phi)}\left(\int_R^{\bar{x}}(z-R)\,dG(z) - \gamma\int_{x^*}^{\bar{x}}(z-x^*)\,dG(z)\right)\right]. \tag{30}$$

Recognizing that the minimum wage $\underline{w}$ is paid at the reservation productivity R when the constraint is binding in the wage bargaining problem, the **job destruction equation** can be written as:

$$0 = R - \underline{w} + \frac{\beta\phi}{1-\beta(1-\phi)}\left(\int_R^{\bar{x}}(z-R)\,dG(z) - \gamma\int_{x^*}^{\bar{x}}(z-x^*)\,dG(z)\right). \tag{JD}$$

A new equilibrium condition is required here in order to pin down the value of x^*. This condition is given by the threshold x^* at which the **bargained wages** coincides with the minimum wage: $w^i(x^*) = \underline{w}$. Using the wage equation (25), this condition, the interior negotiated solution condition (hereon INS), is given by:

$$\underline{w} = \gamma x^* + (1-\gamma)(1-\beta)U - \gamma\frac{\beta\phi}{1-\beta(1-\phi)}\int_R^{x^*}(x^* - z)\,dG(z). \tag{INS}$$

To obtain the value of unemployment, note that two potential cases arise given a binding minimum wage, determined by the wage-setting in new matches. Whenever starting productivity is high enough to satisfy $x_0 \geq x^*$, the surplus is shared according to (21) in a new job and the value of unemployment is therefore given by the same equation of the previous section. On the other hand, if a new job is formed at a low productivity $x_0 < x^*$, the worker is paid the minimum wage and his surplus is given by (23). The value of unemployment is then described by:

$$(1-\beta)U = \begin{cases} b + \dfrac{\gamma}{1-\gamma}c\theta & \text{if } x_0 \geq x^*. \\[2ex] b + \dfrac{\gamma}{1-\gamma}\dfrac{\beta p(\theta)}{1-\beta(1-\phi)}(x^* - R) & \text{if } x_0 < x^*. \end{cases} \tag{31}$$

Analogously, since the firm pays the minimum wage over $x \in [R, x^*]$, valuing the job at $\underline{J}(x)$ defined by (24), and shares the surplus according to (21) otherwise, the **job creation condition** here is given by:

$$\frac{c}{q(\theta)} = \begin{cases} \frac{\beta}{1-\beta(1-\phi)}\Big((x_0 - R) - \gamma(x_0 - x^*)\Big) & \text{if } x_0 \geq x^*. \\ \frac{\beta}{1-\beta(1-\phi)}(x_0 - R) & \text{if } x_0 < x^*. \end{cases} \tag{JC}$$

Note that when the minimum wage is more likely to bind, the negative spillover caused on the wage schedule implies that new matches with negotiated wages have increased profitability, which induces more job creation.

When there exists a binding minimum wage, the job destruction curve (JD) implies a negative relationship between the reservation threshold R and interior wage solution threshold x^*. To see this, first note that integration by parts allows one to rewrite equation (JD) as:

$$0 = R - \underline{w} + \frac{\beta\phi}{1-\beta(1-\phi)}\bigg(\int_R^{\bar{x}}(1-G(z))dz - \gamma\int_{x^*}^{\bar{x}}(1-G(z))dz\bigg). \tag{32}$$

Given a minimum wage $\underline{w}$, differentiation of this equation implies:

$$0 = \Big[r + \phi G(R)\Big]\, dR + \Big[\phi(1 - G(x^*))\Big]\gamma\, dx^*. \tag{33}$$

It follows that $\frac{dx^*}{dR} < 0$ along the job destruction curve. The underlying reason is that when negotiated wages are high, fewer jobs pay the minimum wage and the interior solution threshold x^* falls. If wages are high, the firm's expected profitability falls and more marginal jobs are destroyed, resulting in an increase of the reservation threshold R. As can be seen in the wage condition (25), an increase in market tightness strengthens the worker's outside option by making exit from unemployment more likely, which results in higher bargained wages. As before, the job destruction curve is then an upward sloping curve in the (R, θ) plane, with an associated decreasing relationship over (R, x^*).

Turning now to the job creation curve, it can be seen that the relationship between the productivity thresholds depends on the market tightness. To examine this relationship, observe first that differentiating the (INS) condition at a given market tightness θ results in:

$$0 = \Big[\phi(x^* - R)G'(R)\Big]\, dR + \Big[r + \phi\big(1 - [G(x^*) - G(R)]\big)\Big]\, dx^*, \tag{34}$$

which implies that $\frac{dx^*}{dR} < 0$ at constant θ. Note that an increase in the reservation productivity decreases the probability of having a binding minimum wage, given that fewer low productivity jobs survive. Therefore, as the job's expected duration shortens, the minimum-wage spillover on bargained wages also falls, shifting the entire wage schedule up. This results in an even lower chance of paying the minimum wage, decreasing x^*. Recall that an increase in market tightness strengthens the worker's outside option, decreasing the productivity at which bargained wages are an interior solution to (20) and, therefore, shifting the INS curve down in the (R, x^*) plane.

On the other hand, differentiating the free-entry condition (JC) when market tightness is taken to be constant results in an increasing relationship between R and x^*:

$$0 = -dR + \gamma\, dx^*. \tag{35}$$

That is, a shorter job duration would have to be compensated by greater profitability in order for there to be no incentive to decrease vacancies. This would require an increase in the minimum wage spillover on bargained wages. An increase in market tightness θ extends the duration of vacancy and therefore calls for greater expected profits at a constant job duration, shifting the job creation curve up in the (R, x^*) plane as a higher x^* is associated with a lower wage schedule.

It follows from the previous discussion that there exists a decreasing relationship between R and θ along the job destruction curve, but that the relationship between R and x^* is ambiguous. To see this, differentiating of the INS condition and the job creation curve results in:

$$0 = \Big[p(\theta) - \eta\phi(x^* - R)G'(R)\Big]\frac{c}{p(\theta)} + \Big[1 - \frac{\beta\phi}{1-\beta(1-\phi)}\big(G(x^*) - G(R)\big) + \gamma\frac{\beta\phi}{1-\beta(1-\phi)}(x^* - R)G'(R)\Big]\frac{\partial x^*}{\partial\theta}, \tag{36}$$

which implies an ambiguous sign for $\frac{\partial x^*}{\partial \theta}$ due to the first term. On the one hand, when job duration falls, fewer vacancies are posted in response to the lower profitability of new jobs. This makes it harder for unemployed workers to find a job, which results in lower bargained wages and therefore in an increase in the probability of receiving the minimum wage. On the other hand, a fall in job duration is associated with fewer low productivity matches surviving, which are the ones constrained by the minimum wage. Under the calibration used, the first effect dominates and the productivity threshold x^* increases along the job creation curve.

5.1.3 Basic income subsidy

Building on the previously laid out minimum-wage framework in Section 5.1.2, this section introduces the basic income policy. It is assumed that the government provides workers a monetary transfer equal to the gap between their wage and a guaranteed basic-income level BI. In order to maintain a balanced budget, the government finances this subsidy scheme with payroll taxes τ_f and τ_e, paid by the firm and worker, respectively. The assumption that wages solve constrained Nash-bargaining over the surplus is maintained, and, therefore, the wage paid by the firm is once again given by $w(x) = \max\{w^i(x), \underline{w}\}$, where $w^i(x)$ denotes an interior solution to the bargaining problem (20). Workers in low productivity matches receive an effective wage of BI, so their gross income at any productivity is now given by $w_e(x) = \max\left\{\frac{BI}{1-\tau_e}, w(x)\right\}$.

Noting that separations once again satisfy the reservation property described by $J(R) = 0$, the worker's and firm's surplus of a job respectively solve the following:

$$H(x) = \frac{1}{1-\beta(1-\phi)}\left[(1-\tau_e)w_e(x) - (1-\beta)U + \beta\phi\int_R^{\bar{x}} H(z)\,dG(z)\right] \text{ and} \tag{37}$$

$$J(x) = \frac{1}{1-\beta(1-\phi)}\left[x - (1+\tau_f)w(x) + \beta\phi\int_R^{\bar{x}} J(z)\,dG(z)\right]. \tag{38}$$

To determine the wage schedule, first note that workers eligible for the basic income policy are indifferent over the wage they receive from the firm, as the government guarantees them a constant income. Therefore, firms employing said eligible workers will pay them the minimum wage $\underline{w}$. This coupled with the fact that wages are increasing in the unconstrained case implies that the wage schedule has a jump discontinuity at x^* whenever $\frac{BI}{1-\tau_e} > \underline{w}$.

Secondly, note that in the presence of payroll taxes, an interior solution to the wage bargaining problem $w^i(x)$ satisfies the following first order condition:

$$\gamma(1-\tau_e)J(x) = (1+\tau_f)(1-\gamma)H(x). \tag{39}$$

As in the previous section, this surplus sharing rule and the surplus functions (37) and (38) imply that for productivity $x \in [x^*, \bar{x}]$, the wage $w(x)$ solves:

$$w(x) = \frac{\gamma}{1+\tau_f}x + \frac{1-\gamma}{1-\tau_e}(1-\beta)U + \beta\phi\left(\frac{\gamma}{1+\tau_f}\int_R^{x^*} J(z)\,dG(z) - \frac{1-\gamma}{1-\tau_e}\int_R^{x^*} H(z)\,dG(z)\right). \tag{40}$$

Note that the worker receives a constant income of BI for $x \in [R, x^*]$. Furthermore, since the worker is indifferent over the bargained wage in this productivity interval, which allows the firm to pay the minimum wage $\underline{w}$, the surplus sharing rule (39) holds for $x \in (x^*, \bar{x}]$. Recognizing that the wage schedule has a jump discontinuity at x^* when $\frac{BI}{1-\tau_e} > \underline{w}$, the worker's surplus from employment for $x \in [R, x^*]$ is given by:

$$H(x^*) = \lim_{z \downarrow x^*} \frac{1-\tau_e}{1+\tau_f}\frac{\gamma}{1-\gamma}J(z). \tag{41}$$

To determine the job's value, note first that since low productivity firms pay the minimum wage, (38) implies that the job's value for $x \in [R, x^*]$ is given by:

$$\underline{J}(x) = \frac{1}{1-\beta(1-\phi)}(x - R), \tag{42}$$

where the fact that $J(R) = 0$ has been used. It is also convenient to note that by using $J(R) = 0$ in (37), one can rewrite the job's value for $x \in (x^*, \bar{x}]$ as:

$$J(x) = \underline{J}(x) - \frac{1+\tau_f}{1-\beta(1-\phi)}\big(w(x) - \underline{w}\big). \tag{43}$$

As $\lim_{z \downarrow x^*} w(x) = \frac{BI}{1-\tau_e}$ by construction, the above expression implies that the job's value as productivity approaches x^* from above is given by:

$$J(x^*) = \frac{1}{1-\beta(1-\phi)}\left[(x^* - R) - \underbrace{(1+\tau_f)\left(\frac{BI}{1-\tau_e} - \underline{w}\right)}_{\substack{\text{Job's value function}\\ \text{jump discontinuity}}}\right]. \tag{44}$$

By inspecting the job's value functions (42) and (44), it is clear that the jump discontinuity in the job's value is $\Delta J(x^*) = (1+\tau_f)\left(\frac{BI}{1-\tau_e} - \underline{w}\right)$, which reflects the fact that the firm's wage is no longer effectively subsidized at productivity levels marginally greater than x^*, as the worker becomes non-eligible for the basic income subsidy. Clearly, when the government doesn't guarantee additional income to the worker, the problem reduces to that of the previous section (as no taxes would be required either). Using the job's value (42) in the first integral of the wage equation (40) and substituting the job's value limit (44) in the worker's surplus (41) for the second integral, one can write the wage schedule $w(x)$ for $x \in (x^*, \bar{x}]$ as:

$$w(x) = \frac{\gamma}{1+\tau_f}x + \frac{1-\gamma}{1-\tau_e}(1-\beta)U - \frac{\phi}{r+\phi}\frac{\gamma}{1+\tau_f}\int_R^{x^*}\left[(x^* - z) - \underbrace{(1+\tau_f)\left(\frac{BI}{1-\tau_e} - \underline{w}\right)}_{\text{subsidy spillover}}\right] dG(z). \tag{45}$$

The introduction of the basic income scheme counteracts the minimum-wage spillover on bargained wages, as the firm's profits increase over the productivity range where they would have otherwise paid $w(x) \in \left(\underline{w}, \frac{BI}{1-\tau_e}\right]$.

To determine the equilibrium curves, first note that by using the wage schedule (45) and the limiting job's value (44), one can rewrite the job's surplus (38) for productivity $x \in (x^*, \bar{x}]$ as:

$$J(x) = \frac{1}{1-\beta(1-\phi)}\left[x - (1+\tau_f)w(x) + \frac{\beta\phi}{1-\beta(1-\phi)}\left(\int_R^{\bar{x}}(z-R)\,dG(z) - \int_{x^*}^{\bar{x}}\left[\gamma(z-x^*) + (1+\tau_f)\left(\frac{BI}{1-\tau_e} - \underline{w}\right)\right]dG(z)\right)\right]. \tag{46}$$

Since, by definition, the reservation productivity R satisfies $J(R) = 0$ and the minimum wage is paid at said productivity, evaluation of the above expression at R results in the following **job destruction curve**:

$$0 = R - (1+\tau_f)\underline{w} + \frac{\beta\phi}{1-\beta(1-\phi)}\left(\int_R^{\bar{x}}(z-R)\,dG(z) - \int_{x^*}^{\bar{x}}\left[\gamma(z-x^*) + (1+\tau_f)\left(\frac{BI}{1-\tau_e} - \underline{w}\right)\right]dG(z)\right). \tag{JD}$$

As the worker becomes indifferent over the wage paid by the firm over $x \in [R, x^*]$, the INS condition here is given, as in the analogous case in the previous section, by:

$$\frac{BI}{1-\tau_e} = \frac{\gamma}{1+\tau_f}x^* + \frac{1-\gamma}{1-\tau_e}(1-\beta)U - \frac{\beta\phi}{1-\beta(1-\phi)}\frac{\gamma}{1+\tau_f}\int_R^{x^*}\left[(x^*-z) - (1+\tau_f)\left(\frac{BI}{1-\tau_e} - \underline{w}\right)\right]dG(z). \tag{INS}$$

Also, similar to the case in the previous section, the value of unemployment is given by:

$$(1-\beta)U = \begin{cases} b + \dfrac{1-\tau_e}{1+\tau_f}\dfrac{\gamma}{1-\gamma}c\theta & \text{if } x_0 \geq x^*. \\ b + \dfrac{1-\tau_e}{1+\tau_f}\dfrac{\gamma}{1-\gamma}\dfrac{\beta p(\theta)}{1-\beta(1-\phi)}\left[(x^*-R) - (1-\tau_f)\left(\dfrac{BI}{1-\tau_e} - \underline{w}\right)\right] & \text{if } x_0 < x^*. \end{cases} \tag{47}$$

Finally, the job creation condition is given by:

$$\frac{c}{q(\theta)} = \begin{cases} \dfrac{\beta}{1-\beta(1-\phi)}\left[(x_0-R) - \gamma(x_0-x^*) - (1+\tau_f)\left(\dfrac{BI}{1-\tau_e} - \underline{w}\right)\right] & \text{if } x_0 \geq x^*. \\ \dfrac{\beta}{1-\beta(1-\phi)}(x_0-R) & \text{if } x_0 < x^*. \end{cases} \tag{JC}$$

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