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A new treatment of the in-medium chiral condensates

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A new formalism to calculate the in-medium chiral condensate is presented. At lower densities, this approach leads to a linear expression. If we demand a compatibility with the famous model-independent result, then the pion-nucleon sigma term should be six times the average current mass of light quarks. QCD-like interactions may slow the decreasing behavior of the condensate with increasing densities, compared with the linear extrapolation, if densities are lower than twice the nuclear saturation density. At higher densities, the condensate vanishes inevitably.

The behavior of chiral condensates in a medium has been an interesting topic in nuclear physics [1]. A popular method to calculate the in-medium quark condensate is the Feynman-Helmann theorem. The main difficulty, however, is the assumptions we have to make on the derivatives of model parameters with respect to the quark current mass.

To bypass this difficulty, we will apply a similar idea as in the study of strange quark matter [2–5] by defining an equivalent mass. A differential equation which determines the equivalent mass will be derived. At lower densities, the new formalism leads to a linear decreasing condensate. A comparison with the result in nuclear matter implies that the pion-nucleon sigma term should be six times the average current mass of light quarks. At higher densities, it turns out that the decreasing speed of the condensate with increasing densities is lowered, compared with the linear extrapolation.

The QCD Hamiltonian density can be schematically written as

$$H_{\mathrm{QCD}} = H_{\mathrm{k}} + \sum_i m_{i0}\bar{q}_i q_i + H_{\mathrm{I}}, \tag{1}$$

where H_{k} is the kinetic term, m_{i0} is the current mass of quark flavor i, and H_{I} is the interacting part of the Hamiltonian. The sum goes over all flavors involved.

The basic idea of the mass-density-dependent model of quark matter is that the system energy can be expressed as in a noninteracting system, where the strong interaction implies a variation of the quark masses with density. In order not to confuse with other mass concepts, let us call such a density-dependent mass as an equivalent mass. It can be separated into two parts, i.e.,

$$m_i = m_{i0} + m_{\mathrm{I}}, \tag{2}$$

where the first term is the quark current mass and the second part is a flavor independent interacting part. Therefore, we will have a Hamiltonian density of the form

$$H_{\mathrm{eqv}} = H_{\mathrm{k}} + \sum_i m_i \bar{q}_i q_i. \tag{3}$$

We require that the two Hamiltonian densities H_{eqv} and H_{QCD} should have the same expectation value for any state $|\Psi\rangle$, i.e.,

$$\langle\Psi|H_{\mathrm{eqv}}|\Psi\rangle = \langle\Psi|H_{\mathrm{QCD}}|\Psi\rangle. \tag{4}$$

Applying this equality to the state $|n_{\mathrm{B}}\rangle$ with baryon number density n_{B} and to the vacuum state $|0\rangle$, we have

$$\langle H_{\mathrm{eqv}}\rangle_{n_{\mathrm{B}}} - \langle H_{\mathrm{eqv}}\rangle_0 = \langle H_{\mathrm{QCD}}\rangle_{n_{\mathrm{B}}} - \langle H_{\mathrm{QCD}}\rangle_0. \tag{5}$$

Here we use $\langle A\rangle_{n_{\mathrm{B}}}\equiv\langle n_{\mathrm{B}}|A|n_{\mathrm{B}}\rangle$ and $\langle A\rangle_0\equiv\langle 0|A|0\rangle$ for an arbitrary operator A.

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We restrict ourselves to systems with uniformly distributed particles where we can write $\langle\Psi|m(n_\mathrm{B})\bar{q}q|\Psi\rangle = m(n_\mathrm{B})\langle\Psi|\bar{q}q|\Psi\rangle$. Accordingly we can solve Eq. (5) for m_I, getting

$$m_\mathrm{I} = \frac{\epsilon_\mathrm{I}}{\sum_i(\langle\bar{q}_i q_i\rangle_{n_\mathrm{B}} - \langle\bar{q}_i q_i\rangle_0)}, \tag{6}$$

where $\epsilon_\mathrm{I} \equiv \langle H_\mathrm{I}\rangle_{n_\mathrm{B}} - \langle H_\mathrm{I}\rangle_0$ is the interacting energy density.

Therefore, considering the quarks as a free system, while keeping the system energy unchanged, they should acquire an equivalent mass corresponding to the current mass plus the common interacting part shown in Eq. (6). The equivalent mass is a function of the quark current mass and of the density. Note that quark confinement implies the following natural requirement:

$$\lim_{n_\mathrm{B}\to 0} m_\mathrm{I} = \infty. \tag{7}$$

Because the Hamiltonian density H_eqv has the form of a system of free particles with equivalent masses m_i, the energy density of quark matter can be expressed as

$$\begin{aligned}\epsilon &= \sum_i \frac{g}{2\pi^2}\int_0^{k_\mathrm{f}} \sqrt{k^2+m_i^2}\,k^2dk \\ &= 3n_\mathrm{B}\sum_i m_i F\left(\frac{k_\mathrm{f}}{m_i}\right),\end{aligned} \tag{8}$$

where $g = 3(\mathrm{colors})\times 2(\mathrm{spins}) = 6$ is the degeneracy factor, and

$$k_\mathrm{f} = \left(\frac{18}{g}\pi^2 n_\mathrm{B}\right)^{1/3} \tag{9}$$

is the Fermi momentum of the quarks. The function $F(x)$ is

$$F(x) \equiv \frac{3}{8}[x\sqrt{x^2+1}\,(2x^2+1) - \mathrm{sh}^{-1}(x)]/x^3. \tag{10}$$

For convenience, let us define the function $f(x)$

$$\begin{aligned}f(x) &\equiv -x^2 d[F(x)/x]/dx \\ &= \frac{3}{2}\left[x\sqrt{x^2+1} - \mathrm{sh}^{-1}(x)\right]/x^3.\end{aligned} \tag{11}$$

On the other hand, the total energy density can also be expressed as

$$\begin{aligned}\epsilon &= \sum_i \frac{g}{2\pi^2}\int_0^{k_\mathrm{f0}} \sqrt{k^2+m_{i0}^2}k^2dk + \epsilon_\mathrm{I} \\ &= \frac{3}{N_\mathrm{f}}n_\mathrm{B}\sum_i m_{i0}F\left(\frac{k_\mathrm{f0}}{m_{i0}}\right) + \epsilon_\mathrm{I},\end{aligned} \tag{12}$$

where $N_\mathrm{f} = 2$ is the number of flavors, the first term is the energy density without interactions, and the second term is the interacting part. Because $n_\mathrm{B}/N_\mathrm{f}$ is the baryon number density for each flavor, the Fermi momentum k_f0 here is

$$k_\mathrm{f0} = \left(\frac{18}{g}\pi^2\frac{n_\mathrm{B}}{N_\mathrm{f}}\right)^{1/3}. \tag{13}$$

It looks similar to the non-interacting system. However, the Fermi momentum k_f in Eq. (9) is bigger. It has been boosted because of the Fermi momentum dependence on density through the equivalent mass. In the appendix A, we will give a proof for the boosting of the Fermi momentum.

Combining Eqs. (12) and (8) we identify

$$\frac{\epsilon_\mathrm{I}}{3n_\mathrm{B}} = \sum_i\left[m_i F\left(\frac{k_\mathrm{f}}{m_i}\right) - \frac{m_{i0}}{N_\mathrm{f}}F\left(\frac{k_\mathrm{f0}}{m_{i0}}\right)\right]. \tag{14}$$

The Hellmann-Feynman theorem gives

$$\langle\Psi|\frac{\partial}{\partial\lambda}H(\lambda)|\Psi\rangle = \frac{\partial}{\partial\lambda}\langle\Psi|H(\lambda)|\Psi\rangle, \tag{15}$$

where $|\Psi\rangle$ is a normalized eigenvector of the Hamiltonian $H(\lambda)$ which depends on a parameter λ.

On application, in Eq. (15), of the substitutions $\lambda \to m_{i0}$ and $H(\lambda) \to \int d^3x H_\mathrm{QCD}$, one gets $\langle\Psi|\int d^3x\bar{q}_i q_i|\Psi\rangle = \frac{\partial}{\partial m_{i0}}\langle\Psi|\int d^3x H_\mathrm{QCD}|\Psi\rangle$ for each flavor i. Applying this equality, respectively, to the state $|n_\mathrm{B}\rangle$ (quark matter with baryon number density n_B) and to the vacuum $|0\rangle$, one obtains

$$\langle\bar{q}_i q_i\rangle_{n_\mathrm{B}} - \langle\bar{q}_i q_i\rangle_0 = \frac{\partial\epsilon}{\partial m_{i0}}, \tag{16}$$

where $\epsilon \equiv \langle H_\mathrm{QCD}\rangle_{n_\mathrm{B}} - \langle H_\mathrm{QCD}\rangle_0$ is the total energy density. Now let us substitute Eq. (8) into Eq. (16), carry out the corresponding derivative, and sum over the flavor index. We get

$$\begin{aligned}&\sum_i[\langle\bar{q}_i q_i\rangle_{n_\mathrm{B}} - \langle\bar{q}_i q_i\rangle_0] \\ &\quad= 3n_\mathrm{B}\sum_i f\left(\frac{k_\mathrm{f}}{m_i}\right)[1+\nabla m_\mathrm{I}]\end{aligned} \tag{17}$$

with $\nabla \equiv \sum_i \partial/\partial m_{i0}$. Note that ∇ is a differential operator in mass space.

Comparing this equation with Eq. (6) we have

$$\nabla m_{\mathrm{I}} = \frac{\epsilon_{\mathrm{I}}/(3n_{\mathrm{B}})}{m_{\mathrm{I}} \sum_i f\left(k_{\mathrm{f}}/m_i\right)} - 1. \tag{18}$$

Replacing $\epsilon_{\mathrm{I}}/(3n_{\mathrm{B}}$ here by the right hand side of Eq. (14) we get a first order differential equation for the interacting equivalent mass.

Such a mass really exists, and we can prove that it can be expressed in terms of the interacting energy density ϵ_{I} formally as

$$m_{\mathrm{I}} = \frac{\epsilon_{\mathrm{I}}/(3n_{\mathrm{B}})}{\frac{\nabla \epsilon_{\mathrm{I}}}{3n_{\mathrm{B}}} + \frac{1}{N_{\mathrm{f}}} \sum_i f\left(\frac{k_{\mathrm{f}0}}{m_{i0}}\right)}. \tag{19}$$

In the flavor symmetric case, i.e., $m_{u0} = m_{d0} = \cdots = m_0$, we have $m_u = m_d = \cdots = m$, $\langle \bar{q}_u q_u \rangle = \langle \bar{q}_d q_d \rangle = \cdots = \langle \bar{q}q \rangle$, and $\nabla = \partial/\partial m_0$. In this case

$$\frac{\partial m_{\mathrm{I}}}{\partial m_0} = \frac{mF\left(k_{\mathrm{f}}/m\right) - \frac{m_0}{N_{\mathrm{f}}} F\left(k_{\mathrm{f}0}/m_0\right)}{m_{\mathrm{I}} f\left(k_{\mathrm{f}}/m\right)} - 1, \tag{20}$$

$$\frac{\langle \bar{q}q \rangle_{n_{\mathrm{B}}}}{\langle \bar{q}q \rangle_0} = 1 + \frac{1}{N_{\mathrm{f}} \langle \bar{q}q \rangle_0} \frac{\epsilon_{\mathrm{I}}}{m_{\mathrm{I}}}, \tag{21}$$

$$mF\left(\frac{k_{\mathrm{f}}}{m}\right) - \frac{m_0}{N_{\mathrm{f}}} F\left(\frac{k_{\mathrm{f}0}}{m_0}\right) = \frac{\epsilon_{\mathrm{I}}}{3N_{\mathrm{f}} n_{\mathrm{B}}}. \tag{22}$$

Since $\lim_{x\to 0} F(x) = 1$, Eq. (22) becomes at lower densities

$$m = m_0 + \frac{\epsilon_{\mathrm{I}}}{3N_{\mathrm{f}} n_{\mathrm{B}}}. \tag{23}$$

This means $m_{\mathrm{I}} = \epsilon_{\mathrm{I}}/(3N_{\mathrm{f}} n_{\mathrm{B}})$, i.e., $\epsilon_{\mathrm{I}}/m_{\mathrm{I}} = 3N_{\mathrm{f}} n_{\mathrm{B}}$. Substituting this ratio into Eq. (21), we get

$$\frac{\langle \bar{q}q \rangle_{n_{\mathrm{B}}}}{\langle \bar{q}q \rangle_0} = 1 - \frac{n_{\mathrm{B}}}{n^*} \tag{24}$$

with

$$n^* \equiv -\frac{1}{3} \langle \bar{q}q \rangle_0 = \frac{m_\pi^2 f_\pi^2}{6m_0}, \tag{25}$$

where $m_\pi \approx 140$ MeV is the pion mass and $f_\pi \approx 93.2$ MeV is the pion decay constant.

Since we have said nothing about the form of the interacting energy density, our result is model independent. Recalling that there is a model-dependent result in nuclear matter, i.e.,

$$\frac{\langle \bar{q}q \rangle_\rho}{\langle \bar{q}q \rangle_0} = 1 - \frac{\rho}{\rho^*} \text{ with } \rho^* \equiv \frac{M_\pi^2 F_\pi^2}{\sigma_{\mathrm{N}}}, \tag{26}$$

first proposed by Drukarev *et al.* [6], and later re-justified by many authors [7], we get, from the requirement $n^* = \rho^*$, the very interesting relation $\sigma_{\mathrm{N}} = 6m_0$, i.e., the pion-nucleon sigma term σ_{N} is six times the average current quark mass m_0. If one takes $\sigma_{\mathrm{N}} = 45$ MeV [8–10] and $m_0 = (m_{u0} + m_{d0})/2 = (5+10)/2 = 7.5$ MeV [11], we confirm this result.

The chiral condensate at higher densities can be calculated from Eqs. (20)-(22) if we know the interacting energy density ϵ_{I} from a realistic quark model. In the following, we consider a simple example.

Denoting the average distance between quarks by $\bar{r}$, the interaction between quarks by $\mathrm{v}(m_0, n_{\mathrm{B}})$, and assuming that each quark can only interact strongly with other N_0 nearest quarks at any moment, because of the saturation of strong interactions, the interacting energy density ϵ_{I} can be linked to density by

$$\epsilon_{\mathrm{I}} = \frac{3}{2} N_0 n_{\mathrm{B}} \mathrm{v}(m_0, \bar{r}). \tag{27}$$

The average inter-quark distance $\bar{r}$ is linked to density through $\bar{r} = \xi/n_b^{1/3}$. Here ξ is a geometrical factor related to the way in which we group the quarks together. In what follows, we have divided the system into sub cubic boxes, being then $\xi = 1/3^{1/3}$. We will take $N_0 = 2$ since a quark has a trend to interact strongly with other two quarks to form a baryon. The concrete value of N_0 as well as the value of ξ have only a marginal influence on the density behavior of the chiral condensate.

Substituting Eq. (27) into Eqs. (21) and (22), we have, respectively,

$$\frac{\langle \bar{q}q \rangle_{n_{\mathrm{B}}}}{\langle \bar{q}q \rangle_0} = 1 - \frac{N_0}{2N_{\mathrm{f}}} \frac{n_{\mathrm{B}}}{n^*} \frac{\mathrm{v}}{m_{\mathrm{I}}}, \tag{28}$$

$$mF\left(\frac{k_{\mathrm{f}}}{m}\right) - \frac{m_0}{N_{\mathrm{f}}} F\left(\frac{k_{\mathrm{f}0}}{m_0}\right) = \frac{N_0}{2N_{\mathrm{f}}} \mathrm{v}(\mathrm{m}_0, \mathrm{n}_{\mathrm{B}}). \tag{29}$$

If the parameter N_0 diverges faster than $k_{\rm f}$ or $n_{\rm B}^{1/3}$ at extremely higher densities, we have

$$\lim_{n_{\rm B}\to\infty} \mathrm{v}(m_0, \bar{r}) = 0. \tag{30}$$

which is consistent with asymptotic freedom.

To solve Eq. (20), we need an initial condition at $m_0 = m_0^*$. Let us suppose it to be

$$m(m_0^*, n_{\rm B}) = m(n_{\rm B}). \tag{31}$$

Usually, we will have

$$\mathrm{v}(m_0, n_{\rm B})|_{m_0=m_0^*} = \mathrm{v}(\bar{r}), \tag{32}$$

where $\mathrm{v}(\bar{r})$ is the inter-quark interaction for the special value m_0^* of the quark current mass m_0.

Eq. (20) is difficult to solve analytically. However, this can be done at lower densities.

Let's rewrite Eq. (19) as

$$m_{\rm I} = \frac{\mathrm{v}(m_0, \bar{r})}{\frac{2}{N_0} f\left(\frac{k_{\rm f0}}{m_0}\right) + \frac{\partial \mathrm{v}(m_0,\bar{r})}{\partial m_0}}. \tag{33}$$

At lower densities, the Fermi momentum $k_{\rm f}$ is small, so the function $F(x)$ approaches to 1. Accordingly, from Eq. (29) we get $m_{\rm I} = \frac{N_0}{2N_{\rm f}} \mathrm{v}(m_0, n_{\rm B})$. Replacing the left hand side of Eq. (33) with this expression, and integrating the resulting equation under the initial condition given in Eq. (32), we have

$$m_{\rm I}(m_0, n_{\rm B}) = \frac{N_0}{2N_{\rm f}} \mathrm{v}(\bar{r}) + \int_{m_0^*}^{m_0} \left[1 - \frac{1}{N_{\rm f}} f\left(\frac{k_{\rm f0}}{m_0}\right)\right] dm_0. \tag{34}$$

In general, an explicit analytical solution for the condensate is not available, and we have to perform numerical calculations. For a given inter-quark interaction $\mathrm{v}(\bar{r})$, we can first solve Eq. (29) to obtain the initial condition in Eq. (31) for the equivalent mass, then solve the differential Eq. (20), and finally calculate the quark condensate through Eq. (28).

There are various expressions for $\mathrm{v}(\bar{r})$ in literature, e.g., the Cornell potential [12], the Richardson potential [13], the so-called QCD potentials [14,15], etc. They are all flavor-independent. Let's take a QCD-like interaction of the form

$$\mathrm{v}(\bar{r}) = \sigma \bar{r} - \frac{4}{3} \frac{\alpha_s(\bar{r})}{\bar{r}}. \tag{35}$$

The first term $\sigma\bar{r}$ is the long-range confining part. The second term incorporates perturbative effects. To second order in perturbation theory, one has [14,15]

$$\alpha_s(\bar{r}) = \frac{4\pi}{b_0 \lambda(\bar{r})} \left[1 - \frac{b_1}{b_0^2} \frac{\ln \lambda(\bar{r})}{\lambda(\bar{r})} + \frac{b_2}{\lambda(\bar{r})}\right] \tag{36}$$

where [16]

$$\lambda(\bar{r}) \equiv \ln[(\bar{r}\Lambda_{\overline{ms}})^{-2} + b] \tag{37}$$

and $b_0 = (11N_{\rm c} - 2N_{\rm f})/3$, $b_1 = [34N_{\rm c}^2 - N_{\rm f}(13N_{\rm c}^2 - 3)/N_{\rm c}]/3$, and $b_2 = (31N_{\rm c} - 10N_{\rm f})/(9b_0) + 2\gamma_{\rm E}$ for SU($N_{\rm c}$) and $N_{\rm f}$ flavors. $\gamma_{\rm E}$ is the Euler constant.

Besides these constants, there are three free parameters, i.e. σ, $\Lambda_{\overline{ms}}$, and b. The QCD scale parameter is usually taken to be $\Lambda_{\overline{ms}} = 300$ MeV. The value for the string tension σ from potential models varies in the range 0.18—0.22 GeV^2 [17], and we take $\sigma = 0.2$ GeV^2. As for the parameter b, we take three values i.e. 10, 20, and 30, in the reasonable range [16]. The value of m_0^* in Eq. (32) is taken to be 7.5 MeV. The numerical results are plotted in Fig. 1.

In Fig. 1, the straight line is the linear extrapolation of Eq. (24). It does not depend on the form of the inter-quark interaction $\mathrm{v}(\bar{r})$, and so is 'model-independent'. The other three lines are for $m_0 = 7.5$ MeV, but for different b values. At lower densities, the chiral condensate decreases linearly with increasing densities. When the density increases, being less than two times the nuclear saturation density, the decreasing speed is slowed. However, for even higher densities, it can be shown that the condensate vanishes rapidly.

It should be noted that if the Fermi momentum in Eqs. (8) had not been boosted, the main Eq. (21) is still valid while Eqs. (20) and (22), and accordingly Eqs. (23), (29), (33)–(34), and especially the important equation (24) and the relation between the pion-nucleon sigma term and the current quark masses would be different by a factor, as has been formulated in the first part of Ref. [18].

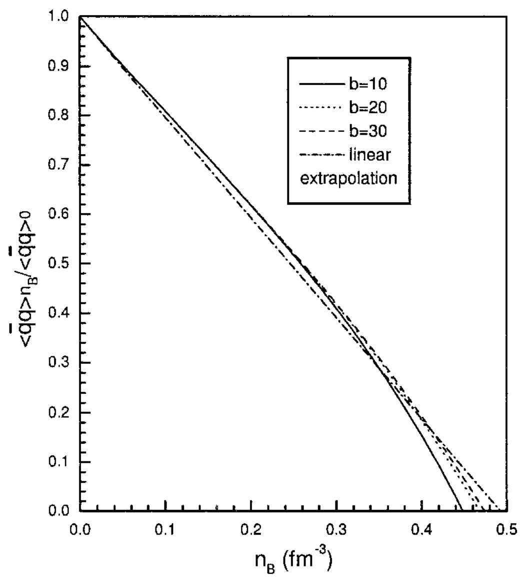

Figure 1. Density dependence of the quark condensate in quark matter.

A. Why should the effective Fermi momentum be boosted

In this appendix, we show that the effective Fermi momentum in the equivalent mass approach should be boosted to a higher value.

We start from

$$d(VE) = Td(VS) - PdV + \mu d(Vn), \tag{38}$$

which is the combination of the first and second laws of thermodynamics. Here n is the particle number density, E is the energy density, and S is the entropy density. Because the system is uniformly distributed, the corresponding extensive quantities are, respectively, Vn, VE, and VS. μ is the chemical potential. From this expression we can get

$$T = \left.\frac{dE}{dS}\right|_n, \tag{39}$$

$$P + E - TS - \mu n = -V \left.\frac{dE}{dV}\right|_{S,n} = 0, \tag{40}$$

$$\mu = \left.\frac{dE}{dn}\right|_S. \tag{41}$$

At zero temperature, the entropy becomes zero, Eqs. (40) and (41) become, respectively,

$$P = -E + \mu n, \tag{42}$$

$$dn = dE/\mu. \tag{43}$$

In our equivalent mass model, the energy density is given with the equivalent mass as

$$E = \frac{g^*}{2\pi^2}\int_0^{k_f} \sqrt{p^2+m^2}p^2 dx \tag{44}$$

$$= \frac{g^*}{16\pi^2}\left[k_f\sqrt{k_f^2+m^2}\,(2k_f^2+m^2) - m^4\mathrm{sh}^{-1}\left(\frac{k_f}{m}\right)\right], \tag{45}$$

where the Fermi momentum k_f satisfies $k_f \equiv \sqrt{\mu^2 - m^2}$. The degeneracy factor g^* is $N_f(\text{flavor})\times 2(\text{spin})\times 3(\text{color})$.

From Eq. (45) we get $dE = \frac{\partial E}{\partial k_f}dk_f + \frac{\partial E}{\partial m}dm$. Substituting this into Eq. (43) then gives

$$dn = \frac{g^* k_f^2}{2\pi^2}dk_f + \frac{g^* m}{4\pi^2}\left[k_f - \frac{m^2\mathrm{sh}^{-1}(k_f/m)}{\sqrt{k_f^2+m^2}}\right]dm. \tag{46}$$

If the mass does not depend on the density or Fermi momentum, the second term vanishes. One then has

$$k_f = \left(\frac{6\pi^2}{g^*}n\right)^{1/3}. \tag{47}$$

Eq. (47) is the well-known expression for the non-interacting system. However, in the mass-density-dependent case where interactions are treated non-pertutbatively by defining an equivalent mass, the quark number density should be given by integrating over both sides of Eq. (46):

$$n = \frac{g^* k_f^3}{6\pi^2} + \frac{g^*}{4\pi^2}\int\left[k_f - \frac{m^2\mathrm{sh}^{-1}(k_f/m)}{\sqrt{k_f^2+m^2}}\right]mdm \tag{48}$$

Usually the equivalent mass is a big quantity, much larger than the current mass. Therefore, the ratio k_f/m is small if the densities are not too high. Let us then expand the integrand of the second term on the right hand side of Eq. (48) with respect to k_f/m, taking then the lowest order term. We get:

$$n = \frac{g^*}{6\pi^2}k_f^3 + \frac{g^*}{6\pi^2}\int \frac{k_f^3}{m}dm. \tag{49}$$

Because of quark confinement and asymptotic freedom, m increases with decreasing k_f. Therefore, the simplest parametrization should be

$$m = m_0 + \frac{C}{k_f^Z}. \tag{50}$$

with C being a constant. To be consistent with the linear confinement, the exponent Z is equal to 1 [1]. However, to reproduce the presently accepted value for the pion-nucleon term (about 45 MeV), Z should be about 3/2. Substituting Eq. (50) into Eq. (49) then gives

$$k_f = \left(\frac{6}{g^*}\pi^2\frac{n}{1-Z/3}\right)^{1/3}. \tag{51}$$

Comparing Eqs. (47) and (51), it is obvious that, for the same density, the Fermi momentum of the interacting system is different from that of the non-interacting case. When taking $Z = 3/2$, Eq. (51) becomes Eq. (9).

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