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Beyond Means:
Distributional Analysis of Gender Pay Gaps

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Beyond Means: Distributional Analysis of Gender Pay Gaps*

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Abstract

This paper extends the canonical two-way fixed effects model proposed by Abowd et al. (1999) and Card et al. (2016) using unconditional quantile regressions (UQR) to analyze how firms and individual attributes influence pay dispersion at different points of the earnings distribution. Leveraging administrative employer-employee data from Chile, I find that individual effects account for most of the wage dispersion at upper quantiles, while firm effects explain a much more significant portion of wage dispersion at lower quantiles. Turning to gender inequality, I document that the gender pay gap increases as we move up on the earnings distribution. It is documented that sorting is more important than bargaining to understand firm-driven gaps at the median and top of the distribution, increasing the gender pay gap. However, the bargaining dimension is more relevant at the bottom of the earnings distribution, reducing the gender pay gap. We connect this finding with evidence on gender-based unionization.

Keywords: unconditional quantile regressions, decomposition methods, gender pay gap

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†Any errors or omissions in this work are my own responsibility. Comments at: vandrade@uc.cl

1 Introduction

A linear regression primarily models the first moment of the conditional distribution of Y given X, specifically the conditional expectation $E(Y \mid X)$. This method offers limited insights into the conditional distribution of $Y \mid X$, particularly when outcomes vary significantly across the distribution. A classical application of distributional regressions is in the analysis of wage gaps. In such regressions, like quantile regressions, the covariates are expected to explain the wage dispersion in varying proportions.

Quantile regressions become particularly interesting when examining the gender pay gap (GPG), especially in the context of Chile. The GPG in Chile shows significant variations across quantiles. As illustrated in Figure 1, the gender wage gap increases monotonically across these quantiles. For instance, for individuals in the 20th quantile, the difference between men and women is approximately 7.25% higher (about \$27 more per month), while for those in the 80th quantile, the gap reaches around 22.14% (equivalent to \$210 more per month). This finding starkly contrasts with the estimated average, where men earn 16.18% more (or \$98 more per month). In this paper, we aim to delve into and identify the key factors that explain these varying GPG differences across quantiles.

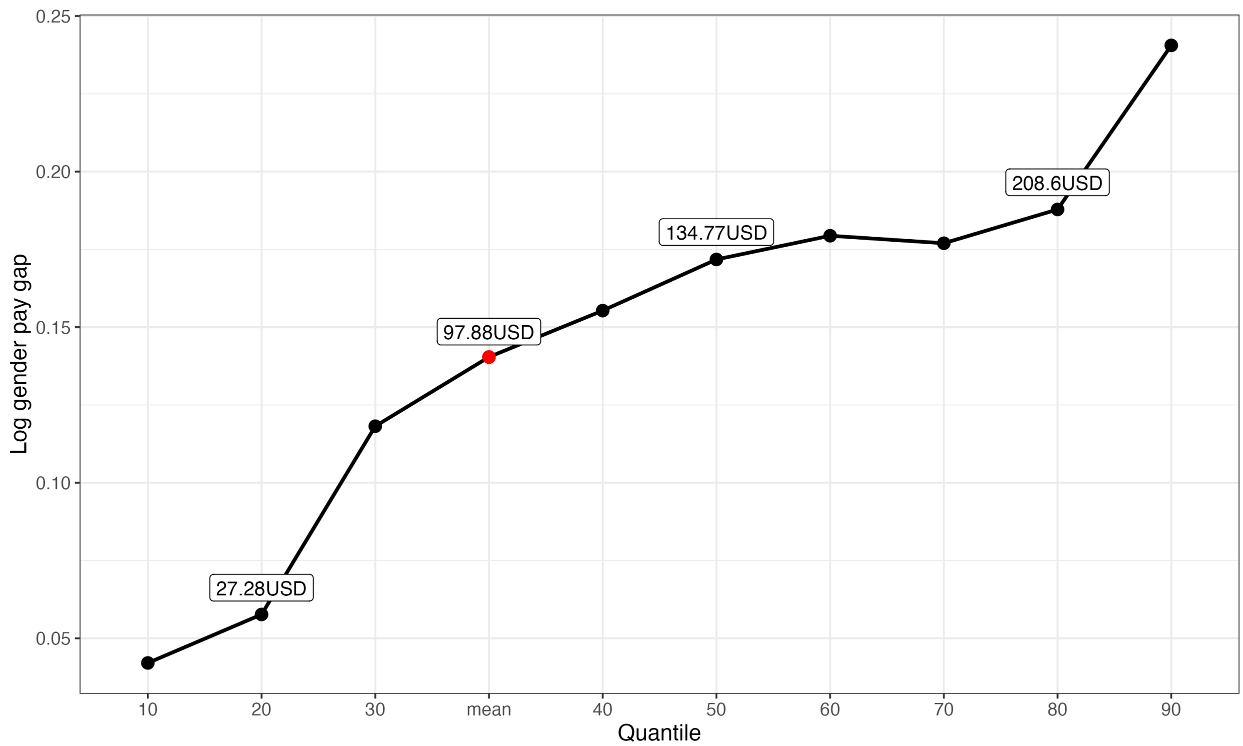


Figure 1: Distributional Gender Pay Gap in Chile (2013-2018). Note: The gender pay gap is the difference in log earnings percentiles between males and females. The mean gender pay gap is computed as the mean log earnings of males minus the log earnings of females.

Despite existing literature consistently highlighting substantial variations in changes across the wage distribution (Antonczyk et al., 2010; Arellano & Bonhomme, 2017; Gallego-Granados & Wrohlich, 2020; Maasoumi & Wang, 2019), the determination of wage dispersion across the distribution, especially concerning gender, has yet to be addressed. A significant contribution to understanding wage distribution analysis comes from Firpo et al. (2018), who employ Unconditional Quantile Regressions (UQR) to demonstrate that wage changes in the United States between 1988 and 2016 were concentrated among individuals with higher levels of education and high-wage occupations. However, this analysis solely focuses on the wage distribution of men and does not provide a clear understanding of the role of firms in this wage gap.

In contrast, the literature examining gender wage inequality has more explicitly differentiated between the roles of firms and worker characteristics in wage determination. This is done using the model developed by Abowd et al. (1999) (AKM). Studies by Card et al. (2016) and Cruz and Rau (2022) in Portugal and Chile have demonstrated that while the individual fixed effect explains more than the firm fixed effect, the latter is still significant. It indicates that matching individuals with high-wage firms plays a crucial role in understanding the Gender Pay Gap (GPG). Additionally, they break down the firm wage premium into an explained and an unexplained component. The first component, known as sorting, arises from the wage differences that occur *between* firms. The second, referred to as bargaining, results from the wage differences *within* firms. However, this explanation is based on mean-based models, leaving uncertainty about how these components vary across the wage distribution.

This paper aims to contribute to the existing literature in three significant ways. First, it proposes an extension of the traditional linear model introduced by Abowd et al. (1999) and Card et al. (2013, 2016) to the Unconditional Quantile Regression (UQR) framework, building upon the foundational work by Firpo et al. (2009b). This expansion offers a comprehensive examination of the influence of firms, covariates, and person effects on wage dispersion, as well as the gender pay gap across various quantiles of the earnings distribution.

Second, we extend the analysis of variance and decomposition using a distributional approach, incorporating UQR as our analytical tool. This approach enhances our ability to assess the distributional aspects of factors contributing to wage disparities, deepening our understanding of the mechanisms underlying these differences.

Third, decomposition analysis holds significant relevance in policy evaluation (Fortin et al., 2011) as it allows us to disentangle differences in wage distributions between genders. This separation encompasses a discrimination effect, resulting from pay disparities between men and women with equivalent characteristics, and a composition effect, stemming from differences in characteristics between the two groups (Blinder, 1973; Oaxaca, 1973). Under specific conditions we discuss, such analyses enable us to infer causality by comparing observed and counterfactual distributions (Chernozhukov et al., 2013).

Utilizing administrative employer-employee data from the Chilean Unemployment Insurance Registry, our findings reveal a significant distributional difference. In lower quantiles, the effect of firms (e.g., due to their payment policies) on wage dispersion is more pronounced than the person-fixed effect (approximately 30%). In higher quantiles, the individual fixed effect (skills) explains more wage variation (approximately 70%).

Regarding gender pay inequality, it is observed that firms contribute to reducing the gap at the bottom of the wage distribution while exacerbating it at the top. Furthermore, decompositions suggest this phenomenon is attributed to bargaining aiding in narrowing the gap at the bottom of the distribution, whereas both sorting and bargaining contribute to its widening at the top. This finding contrasts with the traditional literature, where mean regressions consistently emphasize the role of sorting over bargaining. Consequently, we delve into factors that could help shed light on this result and find a gender-based distributional differentiation in unionization: women are more unionized than men at the bottom of the distribution, whereas men exhibit higher unionization rates at the top. We connect this outcome to gender-specific employment differentiation to gain a deeper understanding of the price discrimination that occurs across various segments of the wage distribution.

The rest of the paper is as follows. In Section 2, we review the recent literature on the GPG. In Section 3, we explore the econometric derivations we need in order to understand the methods employed in this paper. Following that, we provide an explanation of the data (Section 4), present our results (Section 5), and offer insights into the findings (Section 6). Section 7 concludes.

2 Gender wage gap overview

Recent research on gender inequalities in earnings has produced a meaningful set of findings, focusing on substantial changes over the decades. For instance, in the United States, the median full-time equivalent gender wage gap decreased from 37.6% in 1975 to 18.8% in 2010. Similarly, Sweden exhibited a decline from 18.3% in 1975 to 14.3% in 2010, while Chile demonstrated a gender wage gap of 9.1% in 2010, indicative of a broader trend toward reduced gender disparities, i.e., a convergence between female and male wages (Kunze, 2018).

Initially, researchers directed their attention towards human capital accumulation and the competitive labor market model to comprehend these shifts. They utilized straightforward logarithmic wage regressions incorporating the individual fixed effect (as in the model by Katz and Murphy (1992)). The results indicate that as the gender gap convergences, disparities in human capital capabilities between men and women have significantly diminished and, in some cases, been eliminated (Goldin, 2014). This wage convergence between males and females has been observed with the relative increase in work experience, often playing a more substantial role than education (Blau & Kahn, 2017). While numerous countries have witnessed a decline in the gender wage gap since the 1970s, variations persist across nations and periods, requiring further exploration (Kunze, 2018).

In line with this, the study of gender wage disparities sought explanations for other factors contributing to the observed differences in male and female wages. Research has aimed to determine how these disparities are attributable to differential treatment in the labor market versus differences in productive characteristics (Goldin, 2014). Additionally, researchers have examined how under-utilization and under-incentives for women's labor force participation may contribute to market inefficiencies (Goldin et al., 2017).

To gain a deeper understanding of these components of the gender wage gap, many studies incorporated two innovations into their research: first, highlighting the role of firms in wage-setting (Abowd et al., 1999; Card et al., 2013, 2016); second, integrating decomposition methods such as those proposed by Oaxaca (1973) and Blinder (1973). These methods allowed for the separation of the contribution of various components of wage setting (individual, firm, or covariates) into “explained” and “residual” portions. This separation aids researchers in identifying wage discrimination—the part of the gap that remains unaccounted for even when comparing observationally identical males and females (Mulligan & Rubinstein, 2008; O’Neill & Polachek, 1993).

An example of this new approach in gender wage gap research is evident when Blau and Kahn (2017) identify a ‘glass ceiling” effect. This concept highlights a significant gender gap at the upper end of the wage distribution, indicating that women face barriers when attempting to access top-level positions in the labor market. By 2010, the unexplained gender wage gap was more prominent at the 90th percentile than the 10th or 50th percentile, indicating a persistent challenge for highly skilled women. To the best of my knowledge, for the first time, decomposition methods has highlighted gender distributional differences as a stylized fact.

While previous research has primarily focused on factors associated with individual characteristics, such as female labor force participation selection (Blau et al., 2021; Gallego-Granados & Wrohlich, 2020) occupational selection, industry of employment, education, work experience, and the impact of maternity (H. Kleven et al., 2020; H. J. Kleven et al., 2015) this paper aims to delve into the less-explored role of firms in shaping gender wage inequality, especially due to their potential for distributional analysis.

2.1 The contribution of firms on gender inequality

The possibility of firms with some wage-setting power paying equally productive women and men differently was first suggested by Robinson (1969, p. 215) in her analysis of imperfect labor markets and the consequences of monopsony. Unlike traditional competitive labor market models, where wages are primarily determined by market-level supply and demand factors [1]. Robinson’s insight enables a closer examination of the wage-setting policies of

[1] With most gender wage inequality studies focusing on differences in observed and unobserved characteristics, particularly skills, of the labor supply

specific firms. Consequently, monopsony theory allows us to explore how discriminatory gender wage differences may emerge and persist when employers exert greater monopsony power over female workers compared to male workers. To support this notion, it is essential for women's labor supply to be less wage-elastic than that of men.

This insight marked a significant step toward building evidence linking changes in gender wage gaps to the control firms exert over the wages offered to certain workers. This firm-based approach facilitated the identification of two crucial factors contributing to wage inequality: composition effects between firms (explaining part of wage dispersion) and wage discrimination within firms (constituting the unexplained or residual wage dispersion).

The first factor, often referred to as "sorting" (Card et al., 2016), prompts us to inquire whether firms offering higher wages are more or less inclined to hire women. Sorting has been investigated through models that consider how women and men experience differential labor mobility due to firm-specific wage premiums (Barth et al., 2016; Song et al., 2019). This model introduces labor market frictions that affect wages through varying premiums offered by employers: firms with substantial wage premiums, reflecting shared surplus (high-wage firms), and those without surplus (low-wage firms) contribute to wage dispersion. High-wage establishments tend to employ a higher proportion of high-wage workers. Given that higher wage firms engage in more rent-sharing with their employees, increased sorting by wages has led to greater earnings inequality (Card et al., 2013, 2016). Empirically, these effects have been studied by examining earnings inequality across establishments (between inequality) and the increased sorting of workers by firms (Bonhomme et al., 2023; Song et al., 2019). In the context of gender wage inequality studies, Blau (1977) initially noted the potential significance of the between-firm sorting channel in contributing to the gender wage gap, observing that establishments with higher average wages tended to employ fewer women. Subsequent research, including investigations carried out by Groshen (1991),Petersen and Morgan (1995) , and Bayard et al. (2003), has pointed towards the idea that the varying distribution of women and men across workplaces with different wage levels contributes to a portion of the gender wage gap. However, these studies are not without a notable limitation – they often fail to adequately control for unobservable worker characteristics. This limitation can potentially lead to a conflation of segregation by ability with gender-based segregation.

To address this concern, some researchers have delved into the dynamics of inter-firm mobility. Studies by Loprest (1992), Hospido (2009), and Del Bono and Vuri (2011) in this domain reveal that women exhibit a similar likelihood to switch employers as men. However, women tend to experience smaller average wage increases with each job change compared to their male counterparts. Nevertheless, these studies have yet to definitively distinguish between two plausible hypotheses: one suggesting that women face greater difficulty securing positions at higher-paying firms, and the other proposing that women encounter relatively smaller wage gains when transitioning between firms compared to men.

In a separate investigation, Cardoso, Guimarães, and Portugal (2012) explored differential sorting using an AKM-style model, under the assumption that firm effects are equal for both men and women. More recently, Goldin et al. (2017) provided insights into the gender earnings gap, reporting that slightly over 40 percent of the widening gap can be attributed to men disproportionately shifting into higher-paying establishments.

The second factor, although not fully elucidated, is closely tied to the concept of bargaining power (Card et al., 2016). It seeks to explore whether firms offer distinct average wage premiums for men and women relative to the broader market dynamics. This gender-based wage differential in the marketplace has been attributed to the concept of bargaining power. Empirical insights into bargaining power have been revealed through models of search and matching, demonstrating that women and men possess differing relative bargaining power. Recent research indicates that approximately 60 percent of this phenomenon can be ascribed to women's reduced ability to enhance their earnings within firms (Goldin et al., 2017). More precisely, Blau and Kahn (2017) delves into how collective bargaining affects the gender wage gap. It is shown that the gender wage gap significantly diminishes when the gender gap in collective bargaining coverage is eliminated [2]

This provides a potential explanation for the unexplained portion of wage setting that contributes to the gender wage gap. Additionally, the concept of relative bargaining power finds support in social psychology and sociology, which posits that women are less likely to initiate negotiations with their employers (Bowles et al., 2007; Sinclair, 1995; Tomlinson, 2005) and women tend to be less successful negotiators (Andrade, 2021; Cox et al., 2007; Milkman, 1990).

2.2 Setting of decomposition analysis

Let $\mathcal{X}_D$ denote the support of population characteristics, and D represent the existence of groups. We aim to analyze the wage differences (Y_i) between women ($D = 1$) and men ($D = 0$), given that we have X_D characteristics in $\mathcal{X}$, where $\mathcal{X} \subseteq \mathcal{X}_1 = \mathcal{X}_0$ [3].

It is expected that these differences behave differently across various parts of the distribution of Y_i. To analyze this, we introduce the concept of *overall* $\nu-$*difference* in wages between the two groups (women and men) with respect to the statistic ν. This overall $\nu-$difference can be expressed as:

$$\Delta_o^\nu = \nu(F_{Y_1|D_1}) - \nu(F_{Y_0|D_0}) \tag{1}$$

Here, the distributional statistic of interest is denoted as $\nu(F_{Y_g|D_s})$, where $\nu : \mathcal{F}_\nu \to \mathbb{R}$ is a real-valued function, and $\mathcal{F}_\nu$ is a set containing the distribution functions of the groups, i.e., $F_{Y_g|D_s} \in \mathcal{F}_\nu$. In general, the distribution function $F_{Y_g|D_s}$ represents the potential outcome of Y_g for workers in group s. For our empirical application, we represent the observed distributions for women and men as $F_{Y_1|D_1}$ and $F_{Y_0|D_0}$, respectively. Using the law of iterated

[2] This convergence in collective bargaining coverage between men and women results from a combination of a greater decline in men's private sector coverage and an increase in women's public sector coverage, with men's public sector coverage remaining stable.

[3] This assumption is more restrictive. However, for the current problem, it is sufficient that $\mathcal{X}_1 \subseteq \mathcal{X}_0$.

probabilities [4], we obtain:

For women

$$F_{Y_1|D_1}(y) = \int_{\mathcal{X}_1} F_{Y_1|X,D_1}(y \mid X = x) dF_{X|D_1}(x)$$

For men

$$F_{Y_0|D_0}(y) = \int_{\mathcal{X}_0} F_{Y_0|X,D_0}(y \mid X = x) dF_{X|D_0}(x)$$

In the context of potential outcomes, when $g = s$, we refer to it as an observed distribution, while when $g \neq s$, we have a counterfactual distribution. The counterfactual distribution, denoted as $F_{Y_1|D_0}$, represents the p.d.f of wages that women would have if they faced the wage structure of men. This distribution does not arise from any observable population, but it can be expressed as an integration of the conditional distribution of wages for men with respect to the distribution of women's characteristics [5]:

$$F_{Y_1|D_0}(y) = \int_{\mathcal{X}_1} F_{Y_0|X,D_0}(y \mid X = x) dF_{X|D_1}(x)$$

Here, $F_{Y_0|X_1}(y \mid x)$ is the conditional distribution of wages for men given women's characteristics x, and $F_{X|D_1}(x)$ is the distribution of women's characteristics. This integral represents the counterfactual distribution of wages that would prevail if women were paid like men.

Under simple counterfactual, overlapping support and ignorability assumptions (see Appendix 9.1), the overall ν- difference can be identify as

[4] The statement simply indicates that the unconditional cumulative distribution of Y can be obtained by integrating the marginal cumulative $F_{Y_g|X,D_s}(y \mid X = x)$ over all possible realizations of X

[5] This assumes that conditional wage distribution of men can be extrapolated for every $x \in \mathcal{X}$, ie, $\mathcal{X}_1 \subseteq \mathcal{X}_0$. More formally, marginal distribution of women can be replace by men. This is known as Invariance of Conditional Distribution (Fortin et al., 2011)

$$\Delta_O^\nu = \nu(F_{Y_1|D_1}) - \nu(F_{Y_0|D_0}) = \underbrace{\nu(F_{Y_1|D_1}) - \ \nu(F_{Y_1|D_0})}_{\Delta_S^\nu} + \underbrace{\nu(F_{Y_1|D_0}) - \nu(F_{Y_0|D_0})}_{\Delta_X^\nu} \tag{2}$$

where

- $\Delta_S^\nu = \nu(F_{Y_1|D_1}) - \nu(F_{Y_1|D_0})$, captures the structural difference between the wage-setting functions for women and men, often referred to as the composition effect.
- $\Delta_X^\nu = \nu(F_{Y_1|D_0}) - \nu(F_{Y_0|D_0})$, signifies the impact resulting from the variation in the distribution of characteristics between women and men, commonly known as the wage structure component.

2.3 Wage setting model

The decomposition of the entire difference depends on establishing a meaningful counterfactual wage distribution. This implies that the hypothetical states of the world can be crafted to simulate the probable configuration of wage distributions in the event that workers were subject to different remuneration for their observed attributes. In the context of this paper, our specific interest centers on the exploration of the scenario where women's earnings align with those of men.

As this inquiry elucidates, counterfactual analyses employed in decompositions involve the consideration of both observable and unobservable characteristics of workers in relation to their wages for both genders. This entails the formulation of structural wage-setting functions that interlink these two aspects. This relationship is depicted by the functions denoted as f_1 and f_0, as elaborated further in Section 9.1. These functions depend on observable elements (Φ) as well as unobservable components (ε). The relationship is expressed as:

$$Y_{ig} = f_g(\Phi, \varepsilon) \quad \text{where} \quad g = 1, 0 \tag{3}$$

where $f_g(\cdot,\cdot)$ is a unknown real-valued mappings $f_g : \mathcal{X} \times \mathbb{R}^m \to \mathbb{R} \cup \{0\}$. Through an in-depth analysis and decomposition of this functional form, we can identify the primary drivers contributing to the GWG. Specifically, the model we adopt for our investigation is derived from the work of Abowd et al. (1999) (henceforth AKM), which was extended for a gender-specific analysis by Card et al. (2016) (henceforth CCK). The functional form takes the following shape:

$$Y_{it} = \underbrace{\alpha_{it} + \gamma^{D_i} S_{iJ(i,t)t}}_{f_g(\Phi,\varepsilon)} \tag{4}$$

Where α_{it} represents the outside option available to worker i in period t, often associated with self-employment income; $S_{iJ(i,t)t} \geq 0$ denotes the match surplus between worker i and firm $J(i,t)$ during period t; γ^{D_i} signifies the gender-specific share of the surplus captured by the worker's gender (1 for female, 0 for male); $D_i \in \{1, 0\}$: represents the gender indicator.

The framework extends further with the introduction of firm-fixed effects:

$$S_{iJ(i,t)t} = \bar{S}_{J(i,t)} + \phi_{J(i,t)t} + m_{iJ(i,t)} \tag{5}$$

Where $\bar{S}_{J(i,t)}$ encompasses time-invariant factors like market power associated with firm $J(i,t)$; $\phi_{J(i,t)t}$: represents time-varying factors that influence the match surplus; $m_{iJ(i,t)}$ captures the person-specific component of surplus for worker i, accounting for idiosyncratic skills or attributes valuable to the job

Furthermore, the person-fixed effect can be described by:

$$a_{it} = a_i + X'_{it}\beta^{D_i} + \varepsilon_{it} \tag{6}$$

Where a_{it} denotes the permanent component, such as inherent ability or general skills; X'_{it}: represents time-varying components, encompassing observed attributes, labor market experience, and changing returns to education; ε_{it}: accounts for the transitory component.

By integrating these elements, the comprehensive model with two-way fixed effects emerges as:

$$Y_{it} = \underbrace{a_i + \Psi^{D_i}_{J(i,t)} + X'_{it}\beta^{D_i} + r_{it}}_{f_g(\Phi,\varepsilon)} \tag{7}$$

Where $\Psi^{D_i}_{J(i,t)} \equiv \gamma^{D_i}\bar{S}_{J(i,t)}$; $r_{it} = \gamma^{D_i}(\phi_{J(i,t)} + m_{J(i,t)}) + \varepsilon_{it}$; The average wage effect of transitioning from firm j to firm k is given by $\psi_k - \psi_j$

This model, an extension of Abowd et al. (1999) adapted to a gender-specific framework (Card et al., 2016), serves as the basis for our investigation into the gender pay gap.

2.4 Extending the linear case to quantiles regressions

We have defined our theoretical model, namely one that emphasizes the significance of firms in determining wages. Now, as we have outlined in this study, our interest lies in understanding how this component has changed across the distribution. One approach to recovering the entire distribution involves extending the canonical model to Unconditional Quantile Regressions (UQR). This method was developed by Firpo et al. (2009a, 2009b) to address the limitations in interpretation of the Conditional Quantile Regression (CQR) model (Borah & Basu, 2013). For a more thorough understanding, additional details and demonstrations can be found in Appendix .

Considering equation 29 in the context of quantiles, we can elucidate the concept as follows:

Property 1 - Integral relationship in case of quantiles

$$q_\tau = \int RIF(y; q_\tau) dF_y(y) \tag{8}$$

Now, building upon the preceding result and regardless of the specific functional form, we can express a distinct version of Theorem 1 from Firpo et al. (2009b) in the following manner:

Theorem 1 . *Integration of Marginal effect of a change in the distribution*

$$\begin{aligned} q_\tau &= \int E\left(RIF\left(y; q_\tau\right) \mid \Phi = \phi\right) dF_\Phi(\phi) \\ &= E_\Phi\{E\left(RIF\left(y; q_\tau\right) \mid \Phi = \phi\right)\} \end{aligned} \tag{9}$$

This theorem underscores that the quantile q_τ can be seen as the integration of the expected Recentered Influence Function ($E[RIF(y; q_\tau) \mid \Phi = \phi]$) over the distribution of $\Phi \subseteq \mathcal{X}$. In other words, it articulates how the cumulative distribution function of Φ (denoted by F_Φ) influences the quantile q_τ through the conditional expectation of the Recentered Influence Function.

Taking into consideration assumptions 5 and 6 (see Appendix 9.3), we can delve into the conditional expectation of the RIF-regression function $E[RIF(y; q_\tau) \mid \Phi = \phi]$ and express it as a linear function of explanatory variables. In our specific context (AKM model), this can be formulated as:

$$\begin{aligned} RIF(y; q_\tau) &= f_g(\Phi, \varepsilon) \\ RIF(y; q_\tau) &= a_i + \Psi^{D_i}_{J(i,t)} + X'_{it}\beta^{D_i} + r_{it} \end{aligned} \tag{10}$$

$$\begin{aligned} E[RIF(y; q_\tau) \mid \Phi = \phi] &= E[f_g(\Phi, \varepsilon)] \\ E[RIF(y; q_\tau) \mid \Phi = \phi] &= E[a_i + \Psi^{D_i}_{J(i,t)} + X'_{it}\beta^{D_i} + r_{it} \mid \Phi = \phi] \end{aligned} \tag{11}$$

Let us look at the idea behind the average derivative in unconditional quantile regression. This concept, denoted as $E_\Phi\{\frac{df_g(\Phi,\varepsilon)}{d\Phi}\}$, helps us understand how a slight shift in the distribution of covariates influences the τ-th unconditional quantile of Y. Imagine we are looking at how changing firms' characteristics affect wage distribution. We focus on that specific point, like the 50th percentile, while keeping all other factors steady. This concept gives us a way to measure this impact in a precise and controlled manner.

As a result, the coefficients $\gamma_\tau^{D_i}$ can be straightforwardly interpreted in an unconditional manner: $E\left[\mathrm{RIF}\left(y; q_\tau\right)\right] = E_\Phi\left[E\left(\mathrm{RIF}\left(y; q_\tau\right) \mid X\right)\right] = E(\Phi)\gamma_\tau^{D_i}$. This means that we can confidently say that a change in a particular variable, firm characteristics, directly relates to a change in the τ-th unconditional quantile of Y, representing, for example, a certain wage level.

This idea connects with Unconditional Quantile Regression (UQR). In this method, we estimate the coefficients $\gamma_\tau^{D_i}$ using Ordinary Least Squares (OLS). The formula for calculating these coefficients, $E\left[\mathrm{RIF}\left(y; q_\tau\right)\right] = E_\Phi\left[\mathrm{RIF}\left(y; q_\tau\right) \mid \Phi\right] = E(\Phi)\gamma_\tau^{D_i}$, shows how this process captures the effect of covariates on different quantiles of the outcome.

$$q_\tau = E\left[\mathrm{RIF}\left(y; q_\tau\right)\right] = E_\Phi\left[\mathrm{RIF}\left(y; q_\tau\right) \mid \Phi = \phi\right] = E(\Phi)\gamma_\tau^{D_i} \tag{12}$$

3 Distributional analysis

To develop the equation 2, we express the RIF regressions as defined by equation , that is, $f_g(\Phi) \equiv E[RIF(Y_g; \nu(F_{Y_g|D_s})) \mid \Phi, D = s]$ for $s = 0, 1$, and $f_C(\Phi) \equiv E[RIF(Y_1; \nu(F_{Y_1|D_0})) \mid \Phi, D = 0]$. Therefore, we have

$$\begin{aligned} \nu(F_{Y_g|D_s}) &= E[f_g(\Phi) \mid D = s] \quad , s = 1, 0 \\ \nu(F_{Y_1|D_0}) &= E[f_c(\Phi) \mid D = 0] \end{aligned} \tag{13}$$

Building upon the result from 2, the ν-overall decomposition can be written as:

$$\begin{aligned} \Delta_S^\nu &= E[f_1(\Phi) \mid D = 1] - E[f_c(\Phi) \mid D = 0] \\ \Delta_X^\nu &= E[f_c(\Phi) \mid D = 0] - E[f_0(\Phi) \mid D = 0] \end{aligned} \tag{14}$$

$$\begin{aligned} \Delta_O^\nu &= \mathbb{E}[Y \mid D = 1] - \mathbb{E}[Y \mid D = 0] \\ &= \mathbb{E}[\mathbb{E}(Y \mid \Phi, D = 1) \mid D = 1] - \mathbb{E}[\mathbb{E}(Y \mid \Phi, D = 0) \mid D = 0] \\ &= \left(\mathbb{E}[\Phi \mid D = 1]'\gamma_1^\nu + \mathbb{E}\left[\varepsilon_1 \mid D = 1\right]\right) - \left(\mathbb{E}[\Phi \mid D = 0]'\gamma_0^\nu + \mathbb{E}\left[\varepsilon_0 \mid D = 0\right]\right), \end{aligned} \tag{15}$$

where $\mathbb{E}\left[\varepsilon_s \mid D = s\right] = 0$ because $\mathbb{E}\left[\varepsilon_s \mid \Phi, D = s\right] = 0$, so the expression reduces to $\Delta_O^\nu = \mathbb{E}[\Phi \mid D = 1]'\gamma_1^\nu - \mathbb{E}[\Phi \mid D = 0]'\gamma_0^\nu$. Thus, by adding and subtracting $\mathbb{E}[\Phi \mid D = 1]'\gamma_0^\nu$ we get

$$\Delta_O^\nu = \underbrace{\mathbb{E}[\Phi \mid D = 1]'\left(\gamma_1^\nu - \gamma_c^\nu\right)}_{\Delta_{S,OB}^\nu} + \underbrace{\left(\mathbb{E}[\Phi \mid D = 1] - \mathbb{E}[\Phi \mid D = 0]\right)'\gamma_0^\nu}_{\Delta_{X,OB}^\nu}.$$

where

$$\gamma_s^\nu = \left(\mathbb{E}\left[\omega_s(D)\Phi\Phi' \mid D=s\right]\right)^{-1} \cdot \mathbb{E}\left[\omega_s(D)\,\mathrm{RIF}\left(Y_g;\nu(F_{Y_g|D_s})\right)\Phi \mid D=s\right], \quad s=0,1,$$
$$\gamma_c^\nu = \left(\mathbb{E}\left[\omega_c(D,\Phi)\Phi\Phi' \mid D=1\right]\right)^{-1} \cdot \mathbb{E}\left[\omega_c(D,\Phi)\,\mathrm{RIF}\left(Y_0;\nu(F_{Y_g|D_s})\right)\Phi \mid D=1\right]. \tag{16}$$

3.1 Variance decomposition

Assuming that the linearity assumption of the RIF regression holds, we propose the following equation as represented by :

$$\begin{aligned}\mathbb{V}(\mathrm{RIF}(Y_g;\nu(F_{Y_g|D_s}))) = \mathbb{V}(\alpha_i) + \mathbb{V}(\Psi_{i,t}^{D_i}) + \mathbb{V}(X_{i,t}'\beta^{D_i}) + \mathbb{V}(r_{it}) + \\ 2\mathbb{C}(\Psi_{i,t}^{D_i}, X_{i,t}'\beta^{D_i}) + 2\mathbb{C}(\Psi_{i,t}^{D_i}, \alpha_i) + 2\mathbb{C}(\alpha_i, X_{i,t}'\beta^{D_i})\end{aligned} \tag{17}$$

A key aspect in the discussion of wage gaps is to descriptively understand which factors have a greater influence on wage dispersion. To address this, we perform a variance decomposition presented in equation 17.One particularity is that the dependent variable is transformed by a recentred influence function (RIF), which implies that the distribution of this variable is more sensitive to extreme values in that distribution.

The transformation of the random variable is monotonic (see Appendix), much like the commonly applied natural logarithm transformation. In this regard, both transformations alter the distribution of the random variable under consideration, but they do not change its interpretation as a measure of dispersion, as can be seen in the following result.

$$\begin{aligned}\mathbb{V}(RIF(\cdot)) &= \int (RIF(\cdot) - \nu(\cdot))^2 dF(y) \\ &= \int RIF(\cdot)^2 dF(y) - 2\nu(\cdot)\int RIF(\cdot) dF(y) + \nu(\cdot)^2 \int dF(y) \\ &= \int RIF(\cdot)^2 dF(y) - 2\nu(\cdot)^2 + \nu(\cdot)^2 \\ &= \int (RIF(y,\nu,F_y))^2 - (\nu(F))^2\end{aligned}$$

3.2 Decomposition using RIF-Regressions

In this derivation, we aim to extend the Oaxaca-Blinder decomposition framework to two groups using RIF-regressions. We will break down the differences in the quantile of interest between the groups into components related to individuals, covariates, and firms. This approach allows us to understand difference between groups across quantiles. Estimation procedures are explained in Appendix 9.4.

We start with the equation that expresses the quantile of interest, denoted as q_τ, in terms of various components using the RIF-regression framework (using 8 that is summarized in 31)

$$q_\tau = E\left(RIF\left(y_i; q_\tau\right)\right) = E(\alpha_{i\tau}) + E\left(X_i'\beta_\tau^{D_i}\right) + E(\Psi_{J(i,t),\tau}^{D_i}) + E\left(\varepsilon_{i\tau}\right) \tag{18}$$

Here $E(\alpha_{i\tau})$ represents the individual-specific component, $E(X_i'\beta_\tau^{D_i})$ represents the effect of covariates and $E(\Psi_{J(i,t),\tau}^{D_i})$ captures the firm-level impact.

To proceed, we apply the law of iterated expectations to further dissect the components. This involves taking the expectation of the conditional expectations with respect to different variables (as we show in equation 9, Theorem 1):

$$\begin{aligned} q_\tau = E\{E\left(RIF\left(y_i; q_\tau\right) \mid \cdot\right)\} = \\ E\{E(\alpha_{i\tau}) + E\left(X_i'\beta_\tau^{D_i}\right) + E(\Psi_{J(i,t),\tau}^{D_i}) + E\left(\varepsilon_{i\tau}\right) \mid \cdot\ \} \\ E\{E(\alpha_{i\tau}) \mid \cdot\ \} + E\{E\left(X_i'\beta_\tau\right) \mid \cdot\ \} + E\{E(\Psi_{J(i,t),\tau}) \mid \cdot\ \} \end{aligned} \tag{19}$$

With $E(\varepsilon_{i\tau}) = 0$ and $\cdot = \alpha_{i\tau}, X_i'\beta_\tau, \Psi_{J(i,t),\tau}$

Now, let us consider the difference in the quantile ($\Delta q_{\tau,g_i}$) between two groups (denoted as g_1 and g_2)

$$\Delta q_{\tau,g_i} = q_{\tau,g_1} - q_{\tau,g_2}$$

Substituting the expressions for q_{τ,g_1} and q_{τ,g_2} into the equation, we can further dissect the differences:

$$\Delta q_{\tau,g_i} = E\left[E\left(RIF\left(y_i;q_\tau\right) \mid \cdot, g_1\right)\right] - E\left[E\left(\mathrm{RIF}\left(y_i;q_\tau\right) \mid \cdot, g_2\right)\right]$$

This simplifies to:

$$\begin{aligned}\Delta q_{\tau,g_i} = E[&\underbrace{[E\left(\alpha_{i\tau}\right) \mid \cdot, g_1] - [E\left(\alpha_{i\tau}\right) \mid \cdot, g_2]}_{\text{Individual Effect}} + \underbrace{[E\left(X_i'\beta_\tau\right) \mid \cdot, g_1] - [E\left(X_i'\beta_\tau\right) \mid \cdot, g_2]}_{\text{Covariate Effect}} \\ &+ \underbrace{\left[E\left(\Psi_{J(i,t),\tau}\right) \mid \cdot, g_1\right] - \left[E\left(\Psi_{J(i,t),\tau}\right) \mid \cdot, g_2\right]}_{\text{Firm Effect}}]\end{aligned}$$

4 Identification assumptions

Before estimating the model formulated in equation , we need to formalize some of the conditions and assumptions necessary for model identification. The assumptions for the decomposition are detailed in Appendix 9.1.

Assumption 1. Double connected set

Lets $W_{it}^{j} \equiv 1[J(i,t) = j]$ is an indicator for employment at firm j in period t, with $j \in \{1, \ldots, J\}$ y $t \in \{1, \ldots, T\}$.The transition between the firms can be formulated with $T = 2$ as follows:

$$W_{i2}^{j} - W_{i1}^{j} = \begin{cases} 1 & \text{if } W_{i2}^{j} = 1 \wedge W_{i1}^{j} = 0 \text{ joiners} \\ -1 & \text{if } W_{i2}^{j} = 1 \wedge W_{i1}^{j} = 0 \text{ leavers} \\ 0 & \text{if } W_{i2}^{j} = 0 \wedge W_{i1}^{j} = 0 \vee W_{i2}^{j} = 1 \wedge W_{i1}^{j} = 1 \text{ stayers} \end{cases} \tag{20}$$

The first ones are classified within the category of “movers,” while the last case refers to “stayers.” According to Abowd et al. (1999), the fixed effects of the model are only identified in a ”connected set” of firms that are connected by worker mobility, i.e., by “movers.” We choose the largest connected dataset to increase statistical power. As indicated by (Card et al., 2016), since firm fixed-effects are gender-specific, we must construct the “dual-connected” set where we can find both women and men connected across firms.

A potential source of bias arises when mobility is limited, a source that has been widely discussed by Bonhomme et al. (2020, 2023). Since firm-specific parameters are only identified from workers moving across firms, if mobility is low, the dual-connected set will be very small. As shown by Bonhomme et al. (2023), this will result in an underestimation of the firm effect, primarily due to the bias that occurs in assortative matching. However, as demonstrated by (Cruz & Rau, 2022), in the case of Chile, this is not a concern because labor turnover is quite high compared to other OECD countries.

Assumption 2. Exogeneity

In order for the model estimated by OLS to obtain unbiased parameters, the following orthogonality condition, as stated by Card et al. (2016), must hold for all $jin\{1,...,J\}$:

$$E\left[(r_{it} - \bar{r}_i)\left(W_{it}^j - \bar{W}_i^j\right) \mid D(i)\right] = 0 \tag{21}$$

where bars over variables represent time averages. To understand what equation 21 represents, we develop the estimation in first differences with $T = 2$ in equation 22.

$$E\left[(r_{i2} - r_{i1})\left(W_{i2}^j - W_{i1}^j\right) \mid D(i)\right] = 0 \tag{22}$$

In simple terms, this condition tells us that the expected values of joiners and leavers must be the same, and therefore, in expectation, equation 22 equals zero.

$$\underbrace{E[r_{i2} - r_{i1} \mid W_{i2}^j = 1, W_{i1}^j = 0, D(i)] \times P(W_{i2}^j = 1, W_{i1}^j = 0, D(i))}_{\text{joiner}} =$$
$$\underbrace{E[r_{i2} - r_{i1} \mid W_{i2}^j = 0, W_{i1}^j = 1, D(i)] \times P(W_{i2}^j = 1, W_{i1}^j = 0, D(i))}_{\text{leaver}}$$

In that sense, the average bias associated with joiners and leavers would cancel out. If the mean biases are different, this condition is violated, resulting in the following expression:

$$r_{i2} - r_{i1} = \gamma^{D(i)}[\underbrace{\phi_{J(i,2)2} - \phi_{J(i,2)1}}_{1} + \underbrace{m_{J(i,2)} - m_{J(i,1)}}_{2}] + \underbrace{\varepsilon_{i2} - \varepsilon_{i1}}_{3} \tag{23}$$

Certainly, here is the text with the requested changes:

As shown by Card et al. (2013, 2016), this can imply three possible channels of bias. The first one is related to what the literature calls the "Ashenfelter dip." Let $\phi_{J(i,2)2} - \phi_{J(i,2)1} < 0$ be a negative shock in firm J at time t, then when $W^j_{i,2} - \bar{W}^j_{i,1} = -1$, which implies a decrease in wages denoted by w_{it} in leavers and an increase in wages w_{it} in joiners. This channel suggests that workers are more likely to leave firms experiencing negative shocks and join if there are positive shocks. The second one is when mobility is related to idiosyncratic match effects. Card et al. (2016) search and matching research assumes wage gain asymmetry in movers and joiners may overstate. The third one arises when the direction of firm-to-firm is correlated with the transitory wage shock, meaning that if a worker is performing well, they may be more likely to move to a higher-wage firm.

One way to empirically test if this condition is being met is by checking for symmetry since symmetry is consistent with non-selective mobility. In this regard, gains and losses when workers move between quantiles of the distribution would be similar but opposite in sign. However, given that the dependent variable corresponds to a transformation of the log of wages, the interpretation of this test is not straightforward [6]. Therefore, the verification of orthogonality and the econometric formalization of this exercise remain as future work for this study.

Assumption 3. Additive separability

The additive structure of the worker-firm model 3 is a specification that has been discussed in research using two-way fixed effects (Card et al., 2013), in terms of whether this structure provides a good approximation to the wage-setting for each gender.

One way to descriptively explore whether this holds true is to examine the mean residuals for subgroups of observations classified by the decile of the estimated person effect and the decile of the estimated firm effect. As seen in Appendix 9.2, the mean residuals are very small across all deciles, suggesting that equation 3 provides a good approximation to the

[6] In the case of quantiles $q_\tau = E(RIF(Y, q_\tau))$, we have two cases to consider. The first one is when workers move within a quantile, and the second one is when they move between quantiles. The first case appears to be trivial because it is similar to the mean model, as we can obtain the same orthogonality expression, but not so for the case of moving between quantiles.

wage structure. Additionally, since there are no discernible patterns among different deciles for firm fixed effects and person fixed effects, there don't appear to be other pay components that could be interfering with additive separability.

Assumption 4. Normalization

Firm's premium for each gender are non-negative, and will be zero at firms that do not pay surplus and that are above the worker's outside option. In this sense, the firm premium is only identified relative to a firm or set of firms as a reference, which is defined based on rent-sharing. That's why, just as (Card et al., 2016) did, we normalize the firm fixed effect by defining a set of firms as "low-surplus" with a value of 0 in the firm effect.

Formally, let $\bar{S}_j^o$ denote the mean surplus per worker at firm j, and τ is the threshold below which firms pay zero rents on average. Then the normalization rule is formulated as follows:

$$E\left[\psi^g_{J(i,t)} \mid \bar{S}^o_{J(i,t)} \leq \tau\right] = 0, g \in \{F, M\}$$

In concrete terms, the normalization is performed after estimating the firm fixed effects by gender. In particular, this normalization becomes even more important because, being a high-dimensional fixed effects estimation, the fixed effects have an expected value of zero (Correia, 2019). Additionally, we assume that the firms below the threshold are those in the hotel and restaurant sector due to a vast literature that shows this sector has the smallest premiums Card et al., 2016; Cruz and Rau, 2022; Krueger and Summers, 1988. Therefore, it is assumed that there is no rent-sharing in relative terms in this sector.

5 Empirical application

This section presents an empirical application to illustrate how distributional analysis works in practice, both for variance analysis and decomposition. Specifically, we start by showing results that compare mean estimates (OLS) with those from different distribution percentiles (UQR), the differences in explained variances for each of the AKM model parameters; and finally, we focus on decomposing the firm effects for different quantiles.

5.1 Data

We utilized extensive data comprising 42,796,782 observations, representing a panel of individuals per month from January 2013 to December 2018 ($t = 60$). These data encompass monthly information on taxable wages.

The data corresponds to 20% of the sample obtained from matched employer-employee data and originates from the Unemployment Insurance (UI) Registry. Accessible since 2002, this public administrative record mandates firms to report employee jobs every month (for contributors to the system). It is important to note that this data solely covers the formal sector covered by labor code [7], which constitutes approximately 73% of the workforce according to the National Institute of Statistics (2021). [8].

Although the data is sizable, the UI Registry presents certain issues. Furthermore, three challenges emerge in the form of censorship and non-random selection of the sample: (i) Hourly wage data is unavailable due to the absence of information on working hours or the classification of employment as full-time or part-time in our dataset. To mitigate potential ambiguity arising from salaries that could potentially correspond to part-time positions, we have chosen to limit our sample to individuals whose earnings meet or exceed the minimum

[7]This means that public sector workers are not included in this analysis

[8]Berniell et al. (2021) findings suggest that, on average, men exhibit a 4.9 percentage point lower inclusion rate. As reported by Sehnbruch and Carranza (2015), men constituted around 50% of the total in 2005, a proportion that surged to 80% among formal Chilean wage earners in 2012. Sánchez et al. (2022) indicates the figure as 86% for 2019.

wage requirement for each respective period [9] ; (ii) earnings are top-coded at the upper limit in the registry. However, only 3% of our sample's earnings are subject to right-censorship, which does not significantly impact the analysis [10]; and (ii) female selection arising from lower female participation in the labor force. [11].

[9] It is essential to note that while some individuals in the sample may indeed work part-time, those whose earnings fall below the minimum wage threshold are inherently considered part-time workers, as this legal record does not permit compensation below the established legal minimums. Also, we report the results without minimum wage restriction on Appendix 9.4

[10] Card et al. (2013) encountered a severe top-censoring issue (up to 10%) and used imputation techniques for censored observations, resulting in insignificant changes in their analysis. We will perform a similar imputation as a robustness test.

[11] The Chilean National Employment Survey (ENE) recorded a female labor force participation rate of 48.3% in the last quarter of 2021, compared to the male rate of 70.2%. This trend has remained relatively stable since 2010 (excluding the pandemic). We discuss this selection issue in Appendix 9.4

6 Results

6.1 Goodness of fit

In Table 1, we present a summary of the parameter estimates using the AKM model by each quantile for the entire sample and for men and women. All samples were constructed from the double largest connected set of workers. As indicated in equation 7, the model includes fixed effects for individuals, firms, and covariates. These covariates consist of the year, education (no education, primary, secondary, and tertiary), and age (in years) in quadratic and cubic terms. Additionally, we include an interaction between years and education and age[12] terms with education dummies (a specification similar to that of Card et al. (2016) and Cruz and Rau (2022)).

Two results stand out from the table. Firstly, the mean models exhibit a better fit when compared to the quantile models. In particular, the adjusted R-squared is around 0.88 (for all three samples). Secondly, the goodness of fit increases across quantiles, indicating that the explained variance of the model is much higher in the upper quantiles.

This means that the proposed model explains a greater proportion of the variability of the response variable ($RIF(F_y, q_\tau)$) at the top of the distribution, compared to the bottom. In this sense, a part of the data generating process is not being captured by the model in the lower quantiles, leaving an unexplained residual portion. In the discussion, we provide some hypotheses about the mechanism underlying this result.

[12]We adjust the quadratic and cubic terms by shifting their center to age 40. As age and year variables exhibit perfect collinearity when individual effects are accounted for, we omit the linear age term. This adjustment allows for more straightforward interpretations of year and individual effects, assuming a flat age profile at age 40, which seems to be the case.

Table 1: Summary of Estimated TWFE Models by method for all sample, females, and males (2013-2018)

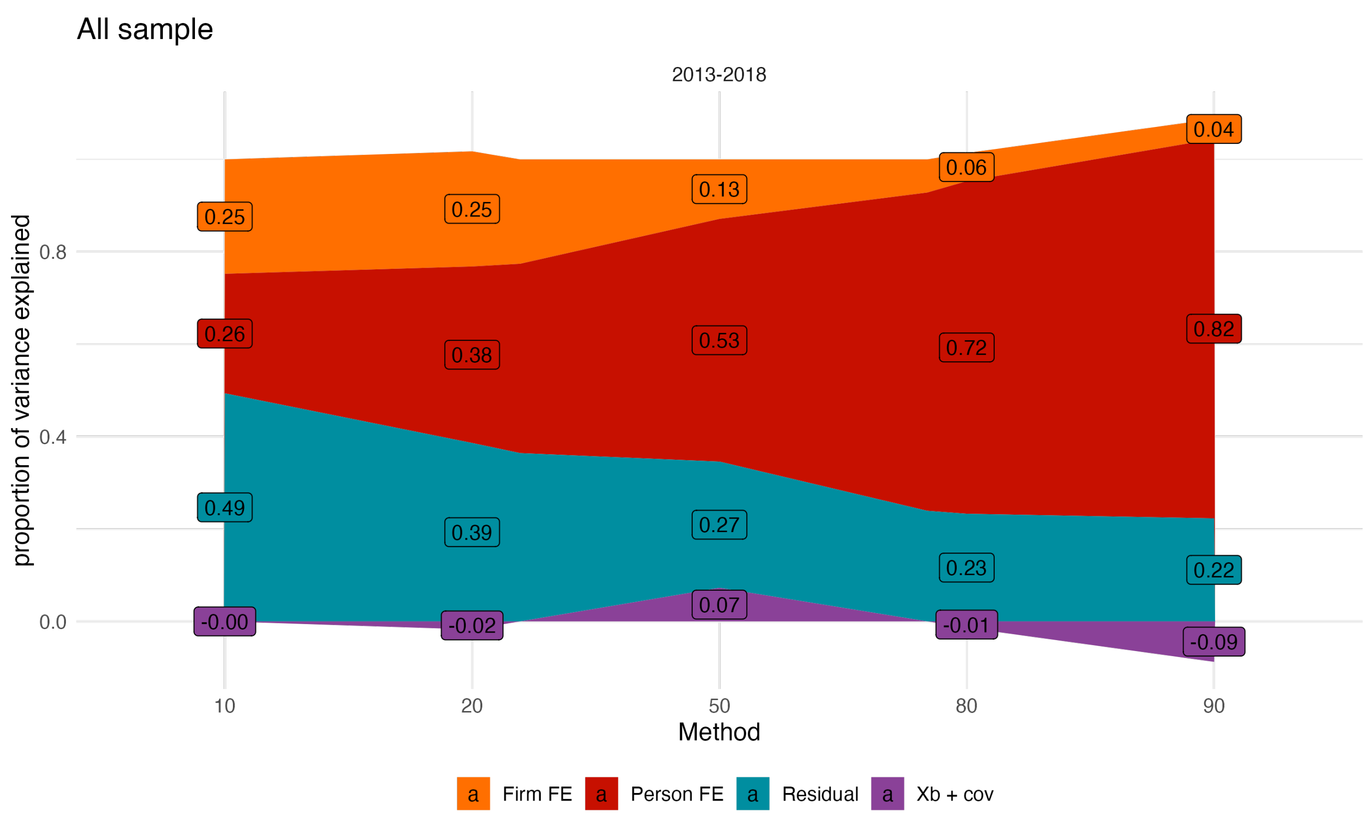
	Indicator	Mean	10	20	50	80	90
all	N	42,786,782	42,786,782	42,786,782	42,786,782	42,786,782	42,786,782
	Person FE (θ)	1,198,798	1,198,798	1,198,798	1,198,798	1,198,798	1,198,798
	Firm FE (ϕ)	80,417	80,417	80,417	80,417	80,417	80,417
	R^2 adjusted	0.88	0.49	0.60	0.72	0.76	0.77
	RMSE	0.22	0.34	0.39	0.46	0.65	0.85
F	N	16,542,868	16,542,868	16,542,868	16,542,868	16,542,868	16,542,868
	Person FE (θ)	486,794	486,794	486,794	486,794	486,794	486,794
	Firm FE (ϕ)	80,417	80,417	80,417	80,417	80,417	80,417
	R^2 adjusted	0.89	0.50	0.59	0.72	0.77	0.76
	RMSE	0.21	0.31	0.29	0.42	0.59	0.83
M	N	26,237,430	26,237,430	26,237,430	26,237,430	26,237,430	26,237,430
	Person FE (θ)	711,681	711,681	711,681	711,681	711,681	711,681
	Firm FE (ϕ)	80,417	80,417	80,417	80,417	80,417	80,417
	R^2 adjusted	0.88	0.50	0.61	0.71	0.76	0.77
	RMSE	0.23	0.34	0.44	0.46	0.66	0.79

6.2 Variance decomposition

To descriptively understand which factors have a greater influence on wage dispersion. We conducted a variance decomposition presented in equation 3.1, along with the particularity of the transformation of the random variable.

Figures 17 and 18 show the variance decomposition resulting from the AKM in the mean (as a benchmark) and at different quantiles. This represents the proportion of variance in the dependent variable explained by each model parameter. More specifically, it shows how much the fixed effect of the individual, fixed effect of the firm, residual, and covariates (plus covariances) explain the response of each quantile to marginal changes in the data (given that an influence function transforms wages).

Figure 2: Variance decomposition by method for all the samples. Note: We collapse the variance contribution of $X\beta$ and covariances in a single component.



From a broader perspective, the Recentered Influence Function for the highest quantiles

(q_{90} and q_{80}) of the earnings distribution is strongly determined by the individual effect and not by the firm effect (82% vs. 4%, in the case of q_{90}). In this regard, in the upper tail of the distribution, the workers' skills explain a more significant proportion of RIF(log earnings) dispersion.

In contrast, the Recentered Influence Function for the highest quantiles (q_{10} and q_{20}) of the earnings distribution is more determined by the firm effect if we compare with the highest quantiles(25% in q_{10} vs. 4% in q_{90}). As a result, at the bottom of the distribution, the wage policies of the firms explain a higher proportion of RIF(log earnings) dispersion.

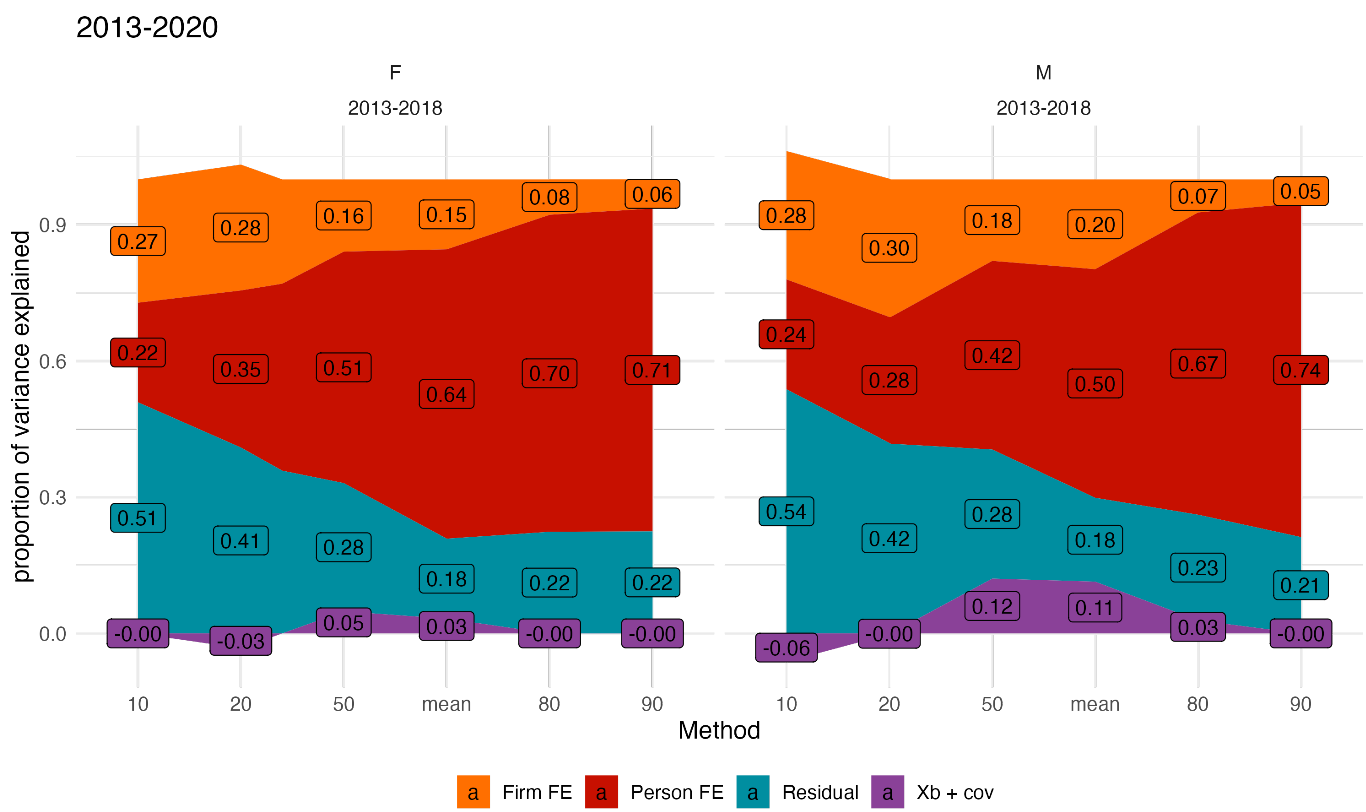
Both results—the importance of skills for the wealthiest individuals and the importance of firms for the less rich—are relevant in understanding wage dispersion for both women and men. Figure 17 shows that while the wage variation for men in the lower quantiles is more explained—in proportion—by the firm effect, this difference with women is relatively low.

Another significant result relates to how much wage variation is explained by the other components. Connected to the goodness-of-fit result, in the lower quantiles, the residual explains much more of the wage variance (around 50%) than in the case of the higher quantiles (around 20%) [13].

Finally, it is essential to mention that the results in the mean are similar to those obtained by other studies that have conducted AKM with Chilean unemployment insurance data (Aldunate et al., 2020; Cruz & Rau, 2022; Huneeus et al., 2022; Muñoz S et al., 2018). In addition, the results for the median are closer to the results of the lower quantiles, which is expected given the skewed wage distribution.

[13] Finally, note that the shaded area in purple has no relevant interpretation, as it corresponds to the contribution of the covariates plus covariances (generally negative).

Figure 3: Variance decomposition by method for male and female. Note: We collapse variance contribution of $X\beta$ and covariances in a single component



6.3 Decompositions

Table 2: CCK (2016) decompositions for Firm FE (2013-2018)

Method	Gender Pay gap	Male premium	Female premium	Gender premium gap	Sorting	Bargaining
10	0.091	0.125	0.177	-0.052	0.0260	-0.0780
20	0.151	0.178	0.199	-0.021	0.0485	-0.0700
50	0.241	0.244	0.144	0.100	0.0720	0.0280
80	0.276	0.206	0.114	0.092	0.0575	0.0345
90	0.314	0.242	0.098	0.144	0.0610	0.0825

Note: Sorting and Bargaining are calculated using the mean effect if we use female or male distribution.

Now, Table 2 provides us with information on the Oaxaca-Blinder decomposition for each of the quantiles. Our goal is to examine, across quantiles, whether sorting or bargaining is more relevant in understanding the gender gap. Before proceeding, it is crucial to remember that the decompositions are in terms of expected value, and, according to the result of Theorem 9, sorting and bargaining represent the contribution to the gender wage gap in a specific quantile.

The gender premium gap indicates how much the firm's wage premium benefits men and women. In this analysis, we observe that the premium benefits women for lower quantiles, while for higher quantiles and above the 50th percentile, it benefits men (where wage gaps are higher). The proportion of the wage gap explained by the firm follows an S-shaped curve, where in the lowest quantile, it helps reduce the gap, and in the highest quantile, it contributes to its increase.

Focusing on the lower quantiles, the wage gap is reduced due to the wage distribution within firms, with bargaining playing a crucial role (as it is negative). Nevertheless, sorting counteracts this effect, benefiting men. In these quantiles (20th and 10th), bargaining proves to be more relevant than sorting, explaining 46.4% of the wage gap in the 20th quantile and

12.5% in the 80th quantile.

At the 50th percentile, the gender premium gap reverses in favor of men. At this point in the distribution, sorting becomes more relevant than bargaining. However, unlike the lower quantiles, both components contribute to increasing the wage gap in favor of men. This result is similar to the study by Cruz and Rau (2022), as the mean and median results exhibit similarities in how much sorting explains the wage gap. Nevertheless, in contrast to the mean model, bargaining represents a slightly more significant proportion, even though the wage gap and other components are similar.

On the other hand, in the higher quantiles, the monotonically increasing relationship of the gender premium gap persists, and sorting becomes more important than bargaining for explaining the wage gap. Both components benefit men in terms of wages. For example, in the 80th quantile, 20.8% of the gap is attributed to sorting, and 12.5% to bargaining.

Table 3: CCK decompositions across age groups

Age	Method	Gender Pay gap	Male premium	Female premium	Gender premium gap	Sorting	Bargaining
< 30	10	0.023	0.122	0.177	-0.054	0.0195	-0.0745
$30 - 40$	10	0.112	0.125	0.186	-0.060	0.0250	-0.0850
> 40	10	0.127	0.126	0.169	-0.043	0.0325	-0.0750
< 30	20	0.022	0.172	0.199	-0.027	0.0365	-0.0635
$30 - 40$	20	0.192	0.181	0.213	-0.032	0.0480	-0.0805
> 40	20	0.217	0.180	0.187	-0.007	0.0590	-0.0660
< 30	50	0.063	0.223	0.140	0.083	0.0495	0.0335
$30 - 40$	50	0.256	0.264	0.162	0.102	0.0770	0.0250
> 40	50	0.363	0.244	0.132	0.112	0.0860	0.0260
< 30	80	0.097	0.171	0.104	0.067	0.0340	0.0330
$30 - 40$	80	0.285	0.235	0.132	0.103	0.0670	0.0360
> 40	80	0.392	0.209	0.107	0.102	0.0690	0.0335
< 30	90	0.198	0.195	0.089	0.106	0.0345	0.0705
$30 - 40$	90	0.253	0.278	0.113	0.166	0.0690	0.0965
> 40	90	0.424	0.248	0.092	0.156	0.0760	0.0800

Note: Sorting and Bargaining are calculated using the mean effect if we use female or male distribution.

Table 3 displays the decomposition results for each age group. In general, we can observe that the wage gap increases over time, and this is primarily explained by the firm effect in the early stages of the work life, not so much in the case of adults over 40 years old.

Secondly, it is noticeable that bargaining remains more relevant for the lower quantiles. To a lesser extent, it is evident that sorting becomes more relevant for the 50th quantile. Moreover, as individuals progress through the life cycle, we can observe that sorting becomes increasingly important. In this regard, as people age, men appear to have a more effective match with high-wage firms compared to women. This aligns with the evidence presented by Bertrand et al. (2010), Goldin (2014), and Manning and Swaffield (2008), suggesting that male and female wages are similar at the start of their careers, with differences primarily emerging during early career stages.

Table 4: CCK decompositions across education groups

Education	Method	Gender Pay gap	Male premium	Female premium	Gender premium gap	Sorting	Bargaining
Less than high school	10	0.093	0.129	0.168	-0.039	0.0320	-0.0710
High school	10	0.093	0.124	0.175	-0.052	0.0260	-0.0780
University	10	0.118	0.121	0.203	-0.083	0.0145	-0.0975
Less than high school	20	0.144	0.181	0.186	-0.005	0.0560	-0.0610
High school	20	0.153	0.177	0.196	-0.019	0.0500	-0.0690
University	20	0.221	0.177	0.242	-0.065	0.0320	-0.0975
Less than high school	50	0.232	0.235	0.129	0.106	0.0740	0.0325
High school	50	0.258	0.244	0.140	0.104	0.0750	0.0285
University	50	0.296	0.274	0.200	0.074	0.0650	0.0100
Less than high school	80	0.173	0.185	0.101	0.084	0.0495	0.0345
High school	80	0.286	0.204	0.110	0.094	0.0610	0.0335
University	80	0.640	0.277	0.164	0.113	0.0790	0.0345
Less than high school	90	0.221	0.208	0.086	0.122	0.0505	0.0720
High school	90	0.320	0.239	0.094	0.145	0.0640	0.0815
University	90	0.668	0.351	0.143	0.208	0.0945	0.1130

Note: Sorting and Bargaining are calculated using the mean effect if we use female or male distribution.

Regarding the decompositions for different educational levels (Table 4), we can not observe changes in sorting and bargaining relevance for the gender gap as academic grades increase). Interestingly, in addition to widening wage gaps at higher educational levels, the gender wage gap for university graduates in the 80th quantile is primarily determined by their match with high-wage firms. This tells us that even if women attain the same level of education as men, their ability to move to higher-paying firms and positions is more readily facilitated by men, likely due to family responsibilities Goldin et al. (2017).

Tables 4 and 6, which presents decompositions by economic activity sector, roughly indicates that in some sectors, the firm contribution moves in conjunction with bargaining. In this scenario, the sectors of agriculture, commerce, finance, industry, mining, transportation, and utilities are included.

Table 5: CCK decompositions across economic sectors (part 1)

Econ Sectors	Method	Gender Pay gap	Male premium	Female premium	Gender premium gap	Sorting	Bargaining
Agricultural	10	0.081	0.130	0.182	-0.051	0.0215	-0.0730
Mining	10	0.134	0.121	0.266	-0.145	0.0030	-0.1475
Industry	10	0.131	0.148	0.192	-0.044	0.0275	-0.0720
Utilities	10	0.084	0.123	0.203	-0.079	0.0085	-0.0885
Commerce	10	0.082	0.110	0.160	-0.050	0.0265	-0.0765
Transport	10	0.097	0.119	0.214	-0.094	0.0000	-0.0940
Hotels and Restaurants	10	-0.009	0.119	0.163	-0.044	0.0015	-0.0460
Finance	10	0.118	0.109	0.227	-0.119	0.0015	-0.1200
Administrative	10	0.064	0.096	0.113	-0.017	0.0335	-0.0505
Educ. and Public Serv.	10	0.087	0.088	0.193	-0.105	-0.0110	-0.0945
Others	10	0.054	0.153	0.210	-0.058	0.0255	-0.0835
Agricultural	20	0.127	0.209	0.227	-0.018	0.0390	-0.0575
Mining	20	0.257	0.201	0.409	-0.208	0.0045	-0.2125
Industry	20	0.226	0.212	0.238	-0.026	0.0450	-0.0710
Utilities	20	0.149	0.169	0.275	-0.107	0.0145	-0.1210
Commerce	20	0.123	0.161	0.154	0.007	0.0440	-0.0370
Transport	20	0.184	0.184	0.274	-0.089	0.0165	-0.1060
Hotels and Restaurants	20	0.023	0.143	0.170	-0.028	0.0055	-0.0330
Finance	20	0.209	0.184	0.280	-0.096	0.0075	-0.1035
Administrative	20	0.074	0.130	0.121	0.009	0.0505	-0.0420
Educ. and Public Serv.	20	0.130	0.111	0.207	-0.096	-0.0130	-0.0825
Others	20	0.116	0.209	0.248	-0.038	0.0465	-0.0855
Agricultural	50	0.205	0.270	0.093	0.176	0.0520	0.1240
Mining	50	0.300	0.465	0.588	-0.123	0.0245	-0.1475
Industry	50	0.316	0.283	0.156	0.126	0.0800	0.0460
Utilities	50	0.173	0.311	0.289	0.022	0.0215	0.0005
Commerce	50	0.259	0.219	0.086	0.133	0.0685	0.0645
Transport	50	0.182	0.256	0.186	0.069	0.0260	0.0430
Hotels and Restaurants	50	0.257	0.131	0.108	0.023	0.0145	0.0090
Finance	50	0.192	0.307	0.257	0.051	0.0190	0.0320
Administrative	50	0.253	0.170	0.081	0.089	0.0650	0.0240
Educ. and Public Serv.	50	0.160	0.130	0.177	-0.047	-0.0110	-0.0360
Others	50	0.134	0.276	0.183	0.094	0.0655	0.0285

Note: Sorting and Bargaining are calculated using the mean effect if we use female or male distribution.

Table 6: CCK decompositions across economic sectors (part 2)

Econ Sectors	Method	Gender Pay gap	Male premium	Female premium	Gender premium gap	Sorting	Bargaining
Agricultural	80	0.259	0.091	0.053	0.038	0.0100	0.0280
Mining	80	0.381	0.766	0.795	-0.030	0.0790	-0.1085
Industry	80	0.263	0.236	0.117	0.119	0.0570	0.0620
Utilities	80	0.074	0.343	0.244	0.099	0.0065	0.0930
Commerce	80	0.408	0.183	0.081	0.103	0.0710	0.0320
Transport	80	-0.057	0.141	0.127	0.014	-0.0235	0.0375
Hotels and Restaurants	80	0.260	0.083	0.078	0.004	0.0140	-0.0090
Finance	80	0.407	0.349	0.259	0.090	0.0380	0.0515
Administrative	80	0.274	0.135	0.053	0.082	0.0450	0.0370
Educ. and Public Serv.	80	0.223	0.124	0.110	0.015	-0.0020	0.0170
Others	80	0.046	0.223	0.144	0.078	0.0345	0.0435
Agricultural	90	0.338	0.073	0.041	0.032	0.0110	0.0220
Mining	90	0.395	1.230	0.850	0.379	0.0325	0.3465
Industry	90	0.220	0.277	0.105	0.171	0.0510	0.1205
Utilities	90	-0.005	0.488	0.232	0.256	0.0150	0.2400
Commerce	90	0.443	0.219	0.066	0.154	0.0725	0.0805
Transport	90	0.049	0.128	0.060	0.068	-0.0030	0.0710
Hotels and Restaurants	90	0.337	0.084	0.055	0.029	0.0155	0.0145
Finance	90	0.354	0.411	0.244	0.167	0.0395	0.1275
Administrative	90	0.319	0.150	0.048	0.102	0.0365	0.0655
Educ. and Public Serv.	90	0.383	0.075	0.073	0.002	0.0080	-0.0055
Others	90	-0.028	0.247	0.129	0.118	0.0040	0.1145

Note: Sorting and Bargaining are calculated using the mean effect if we use female or male distribution.

7 Discussions

The question of what can explain the residual portion of the gender wage gap that remains and why it varies significantly across different quantiles is a complex and central issue in the analysis of wage disparities between men and women.

This issue recurs throughout the results of this paper. Firstly, the residual or unexplained proportion is much larger in the lower quantiles compared to the higher ones. From a purely descriptive stage, we can notice that the extent to which the variation in the RIF of wages is not explained is much more noticeable at the bottom of the distribution. Secondly, when estimating counterfactuals, we can see that, unlike previous literature, the *within* or residual component (wage structure) is more relevant at the lower end of the distribution than the *between* or structure component.

What mechanisms can explain this difference across quantiles in the residual part?As Goldin (2014) suggests, there are several contenders when it comes to explaining this residual portion of the wage gap. Some argue that it may be due to actual discrimination, while others attribute it to differences in women's ability to negotiate and their willingness to compete. Additionally, differential employer promotion standards based on gender differences in the probability of leaving the workforce could also play a role [14].

Until now, we have no sources of variation that could be connected with any of the mechanisms previously mentioned. However, when Card et al. (2016) sought to explain this residual part, they revisited the interpretation of Robinson (1969) regarding imperfect competition, indicating that wage differentials within firms are due to the bargaining power of worker groups.

Data on unionization can provide suggestive insights into what is happening in this residual part, as suggested by Card et al. (2016), which is attributed to bargaining.

[14] Others argue that the gender wage gap can be explained by factors such as labor market equilibrium with compensating differentials and endogenous job design. As women improve their productivity-related skills and become more similar to men in the workforce, the role of human capital in explaining wage differences diminishes. What seems to remain is largely linked to how firms compensate individuals with differing preferences for workplace amenities, especially those related to flexibility.

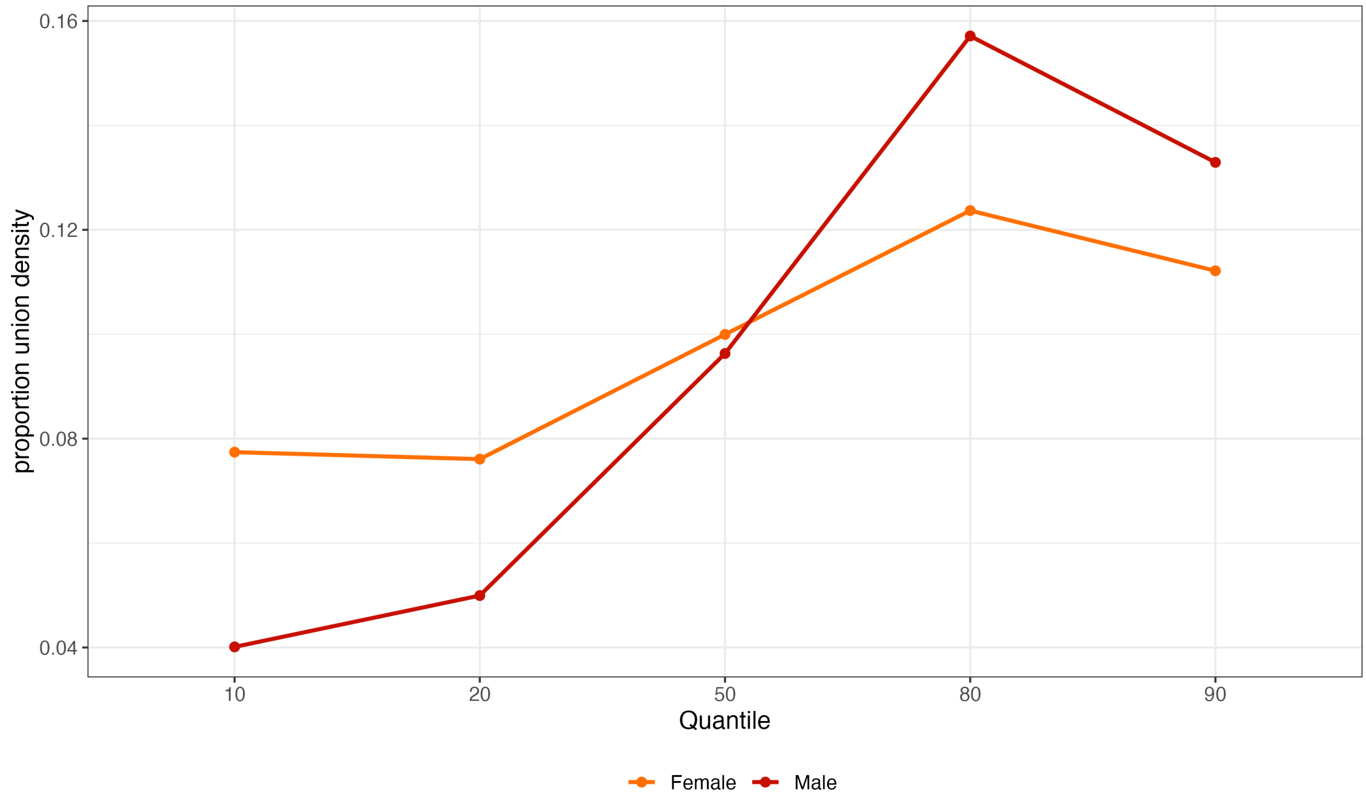


Figure 4: Union density by sex and quantiles of labor income.
Source: CASEN (2017) using private sector employees who meet the criteria of earning at least the minimum wage or more

Figure 4 on unionization by gender across the wage distribution aligns with the bargaining explanation proposed by Card et al. (2016). Firstly, as seen in Figure 4, unionization is lower in the lower quantiles of the distribution and higher in the upper quantiles, indicating potentially greater bargaining power for employees earning more, such as professionals and supervisors.

Secondly, women have higher unionization rates than men in the lower income quantiles, a trend that reverses roughly at the income median and becomes evident in the 80th quantile. This corresponds to the result in Table 1, where bargaining helps reduce the gender wage gap (favoring women) in the lower part of the distribution, while at the top of the distribution, men benefit from it in terms of wages.

It is interesting to note that a similar explanation was put forth in the writings of Robinson (1969) when trying to explain how wage discrimination occurred in the labor market between men and women. Robinson (1969) sought to understand what happened when men and women were equally efficient at work but had different supply conditions. They were paid the same wage within each group, but the wages of women and men were different. Robinson (1969) explained that when men are organized in unions that enforce a minimum wage, and women are not, "Then the supply of men is perfectly elastic, and the supply of women is less than perfectly elastic" (p. 303).

While this point was first proposed in the 1930s, where the primary role of unions was to advocate for the enforcement of minimum wages, a similar explanation can be extrapolated to the differential supply conditions that arise for men and women across the distribution. In particular, for Robinson (1969), this can be clearly reflected in the differentiation of employment by gender, with the dominant gender ultimately determining the demand price for labor.

Figure 5 illustrates something apparent but connects with the previously discussed topic. Unskilled workers are primarily found at the bottom of the distribution, while managers/professionals are at the top of the distribution. As we can observe, they are also differentiated by gender. Women are predominantly concentrated in the lower part of the distribution for both groups, but in a higher proportion among unskilled workers. Similarly, men are primarily in the upper tail of the distribution, but in a higher proportion in managerial and professional occupations.

In this regard, those engaged in bargaining in the lower quantiles are mainly found in female-dominated and less skilled sectors, such as sales workers, domestic service, and service operatives. They are the ones who manage to push for higher wages, thus reducing the gender wage gap. In contrast, at the top of the distribution, those engaging in bargaining are workers in male-dominated and skilled sectors, for example, through professional and supervisory unions.

As a corollary to the explanation by Robinson (1969), this residual part, which is propor-

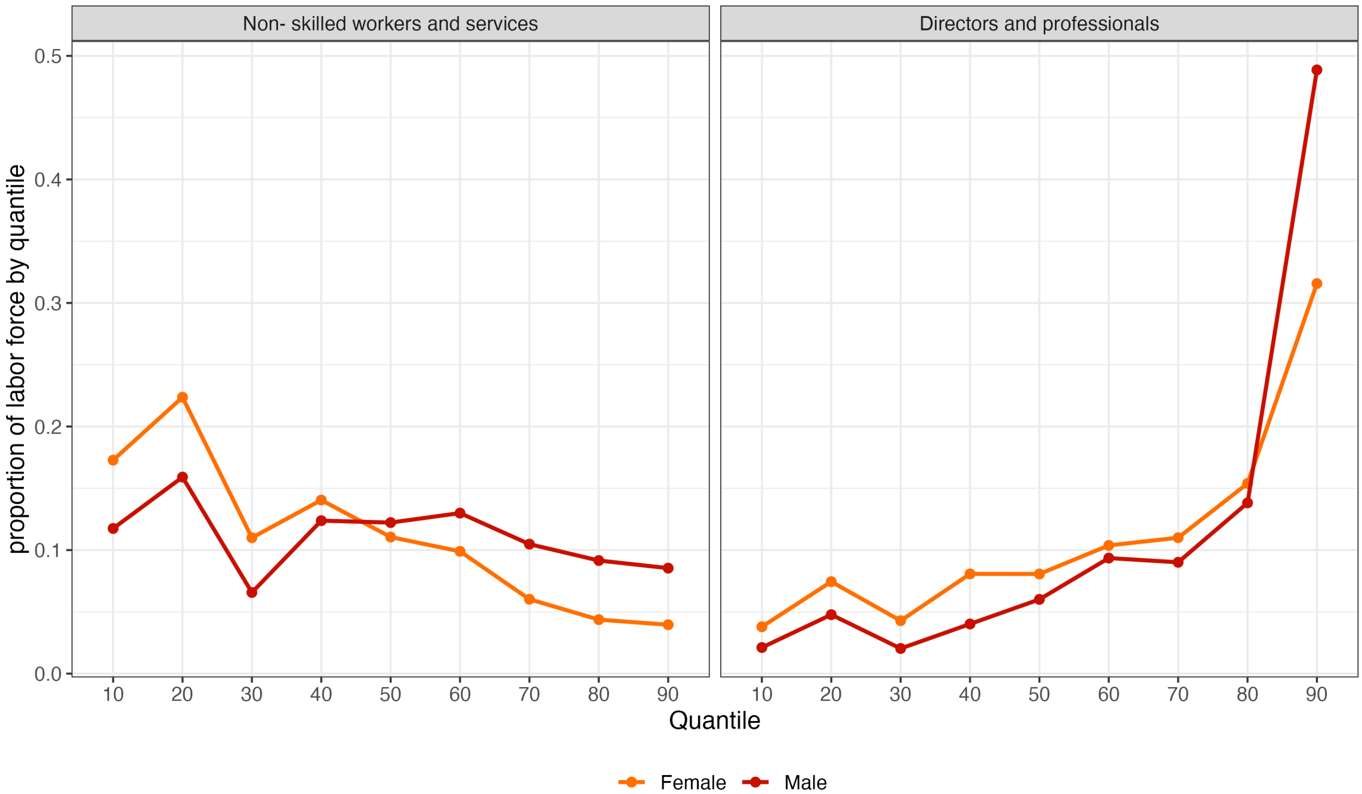


Figure 5: Labour force participation by sex and quantile.
Source: CASEN (2017) using private sector employees who meet the criteria of earning at least the minimum wage or more

tionally larger at the bottom of the distribution, could also be attributed to the consequences of the minimum wage. In particular, as we discussed earlier, employment conditions may differ for both genders across the distribution. Specifically, the minimum wage could be playing a significant role in the data generation process for the bottom of the distribution. In this sense, this institutional arrangement would lead to a lower gender wage gap at the lower end of the distribution and within firms (as shown by the counterfactual results).

The result that the minimum wage establishes conditions for the non-existence of wage gaps (since no group can earn less than this) is mechanical. However, above this point in the distribution, there is no institutional mechanism to prevent it. Moreover, our results show no variation with and without the minimum wage constraint.

In summary, what happens in the residual part at the bottom of the distribution is an open task in this research. We present some suggestive explanations for what might be happening in the lower quantiles.

8 Conclusion

In this study, we have expanded upon the canonical two-way fixed effects model (Abowd et al., 1999; Card et al., 2016) using unconditional quantile regressions (Firpo et al., 2009b) to analyze how firm and individual attributes impact wage dispersion across different points of the earnings distribution.

One of our most salient findings is that individual characteristics account for the majority of wage dispersion in the upper quantiles, whereas firm attributes play a much more substantial role in wage dispersion in the lower quantiles. This underscores the importance of considering both individuals and firms when addressing wage inequalities.

Concerning the gender pay gap, we found that this gap widens as we move up the income quantiles. Furthermore, we discovered that the bargaining dimension contributes to decrease the firm-driven gender pay gaps at the lower end of the income distribution, but not at the upper end.

Our approach to variance decomposition also yielded intriguing results. In the lower quantiles, firm effects have a more pronounced impact on wage dispersion than firm fixed effects, emphasizing the significance of firm wage policies in this segment. Conversely, in the upper quantiles, it is the individual fixed effect (workers' skills) that explains a greater share of wage variation.

The discussion surrounding the residual portion of the gender wage gap, particularly its variation across different quantiles, is a complex issue with various potential explanations. While some suggest it may be attributed to discrimination or differences in negotiation skills, others emphasize factors like compensating differentials, endogenous job design, and workplace amenity preferences. Importantly, the role of bargaining and unionization emerges as a critical element in understanding this residual gap.

Figure 4 highlights the significance of unionization, with lower quantiles seeing higher unionization among women, particularly in female-dominated and less skilled sectors like sales, domestic service, and service work. In these lower quantiles, bargaining plays a substantial role in reducing the gender wage gap. However, as we move up the income distribution, men dominate in managerial and professional roles, where unionization is higher. Here, bargaining benefits men, contributing to the wage gap's persistence.

This pattern aligns with the historical idea proposed by Robinson (1969), where gender wage differences result from differences in supply conditions and the presence of unions. While this explanation dates back to the 1930s, it remains relevant today, highlighting the importance of addressing gender disparities not only in terms of skills and qualifications but also in terms of negotiating power and unionization across various sectors and income levels.

In summary, this paper contributes to the existing literature in several crucial ways. We have extended the traditional model using unconditional quantile regressions, providing a more comprehensive understanding of the influence of both firms and individual characteristics on wage dispersion. Additionally, we have offered detailed insights into how gender wage gap components vary across different parts of the pay distribution, which can have substantial implications for public and firm policies.

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9 Appendix

9.1 Assumptions for Oaxaca-Blinder Decomposition

In this subsection, we outline the key assumptions necessary for conducting the Oaxaca-Blinder (OB) decomposition, which aims to disentangle the sources of wage differences. These assumptions establish the foundation to identify the underlying parameters of interest within the framework.

Assumption 1 ***Disjoint Groups.*** *The population can be partitioned into mutually exclusive groups. Here, we consider two groups denoted as g, with $D_{gi} = \mathbb{1}\{i \text{ is in } g\}$ where $g = 1, 0$, and $\mathbb{1}(\cdot)$ is the indicator function.*

Assumption 2 ***Structural Form.*** *Each worker i belonging to group 1 (women) or 0 (men) is compensated according to the wage structure f_1 and f_0, respectively. These separable functions are defined by both observable (Φ) and unobservable (ε) components.*

Furthermore, we introduce a specific functional property, namely *Additive Separability*, which serves as a condition in Card, Cardoso, and Klein (CCK), and can be established rigorously.

Assumption 3 ***Counterfactual Treatment.*** *This assumption focuses on counterfactual wage structures. Specifically, f^C corresponds to a simple counterfactual treatment if $f^C(\cdot, \cdot)$ aligns with $f_1(\cdot, \cdot)$ for group 0 or $f^c(\cdot, \cdot)$ aligns with $f_0(\cdot, \cdot)$ for group 1.*

Within the potential outcomes framework introduced by Rubin and Rosenbaum (1983, CITE), consider $Y_{g|D_s}$, where $g = 1, 0$ denotes potential outcomes, and $s = 1, 0$ signifies group membership. For group 1, the observed wage is represented as $Y_{1|D_1}$, while $Y^C_{0|D_1}$ symbolizes the counterfactual wage. In the context of group 0, $Y_{0|D_0}$ pertains to the observed wage, and the counterfactual wage is $Y^C_{1|D_0}$. The superscript C *emphasizes counterfactuals. For instance, in the case of group 0 comprising males and group 1 comprising females, the binary variable D_0 signifies worker gender.*

For a female worker i $(D = 1)$, her observed wage under gender treatment is $Y_{1|D_1,i} = f_1(\Phi, \varepsilon_i)$; counterfactually, if male: $Y^C_{0|D_1,i} = f^C(\Phi, \varepsilon_i) = f_0(\Phi, \varepsilon_i), i \in 1$. Another counterfactual scenario explores the wage of a male worker j as if they were female: $Y^C_{1|D_0,i} = m^C(\Phi, \varepsilon_i) = f_1(\Phi, \varepsilon_i), i \in 0$. The choice of counterfactual parallels the reference group selection in the OB decomposition.

Assumption 4 ***Overlapping Support.*** *Let the support of all wage-setting factors [Φ_0; "ε_0"] be $\mathcal{X} \times \mathcal{E}$. For all [$\phi_0$; ε_0] in $\mathcal{X} \times \mathcal{E}$, it is ensured that $0 < Pr[D_1 = 1|\Phi = \phi; \varepsilon = \varepsilon_0] < 1$.*

This assumption plays a pivotal role in gender wage differential decomposition. However, it's worth noting that there are instances where explanatory variables exist that do not satisfy this condition.

Assumption 5 ***Ignorability****. Let (Φ, D, ε) have a joint distribution. For all x in $\mathcal{X} : \varepsilon$ is independent of D given $X = x$.*

It is also called unconfoundedness and allows identification of the treatment effect on the treated sub-population.

Under overlapping support and ignorability assumption, we can identify the parameters of the counterfactual distribution of $F_{Y_1|D_0}$. To show how identification works, Firpo et al. (2018) defines the following weighting functions

- Reweighting functions transform features of the marginal distribution of Y into features of the conditional distribution of Y_1 given $D = 1$

$$\omega_1(D) \equiv \frac{D}{p} \tag{24}$$

- Reweighting functions transform features of the marginal distribution of Y into features of the conditional distribution of Y_1 given $D = 1$

$$\omega_0(D) \equiv \frac{1 - D}{1 - p} \tag{25}$$

- Reweighting functions transform features of the marginal distribution of Y into features of the counterfactual distribution of Y_0 given $D = 1$

$$\omega_C(D, \Phi) \equiv \left(\frac{p(X)}{1 - p(X)}\right) \cdot \left(\frac{1 - D}{p}\right). \tag{26}$$

9.2 Assumptions for AKM identification

Assumption 3. Additive Separability

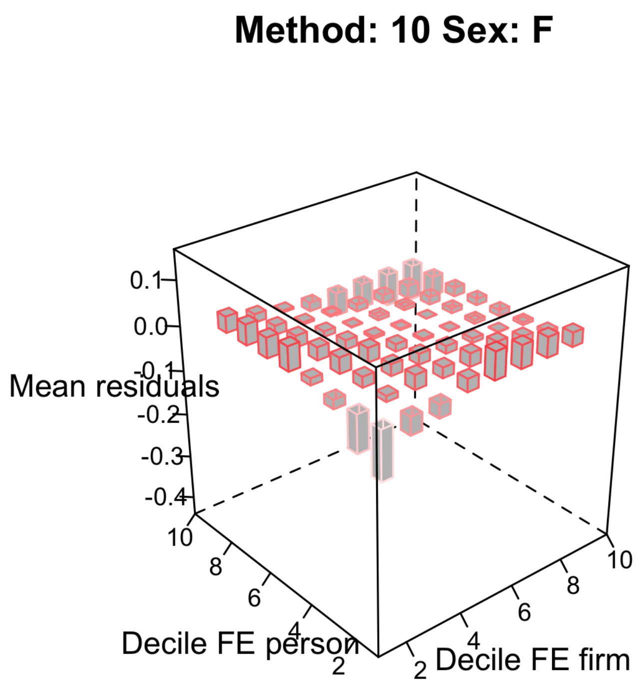

Figure 6: Figure plots residuals from $RIF(Y, q_{10})$ for female workers, classified by decile estimated firm effect and decile of estimated worker effect

Method: 10 Sex: M

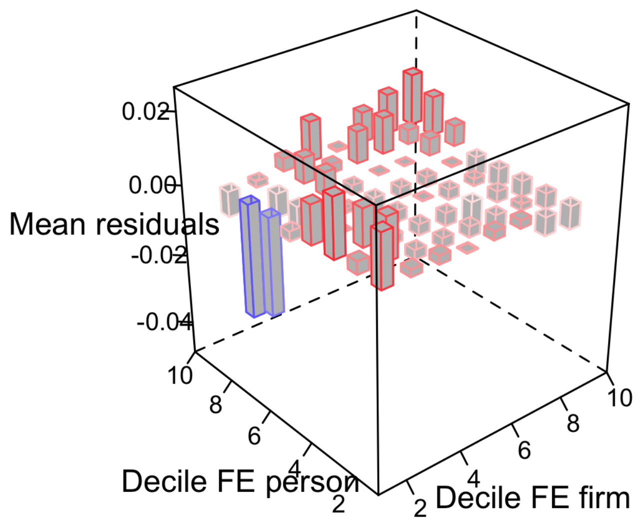

Figure 7: Figure plots residuals from $RIF(Y, q_{10})$ for male workers, classified by decile estimated firm effect and decile of estimated worker effect

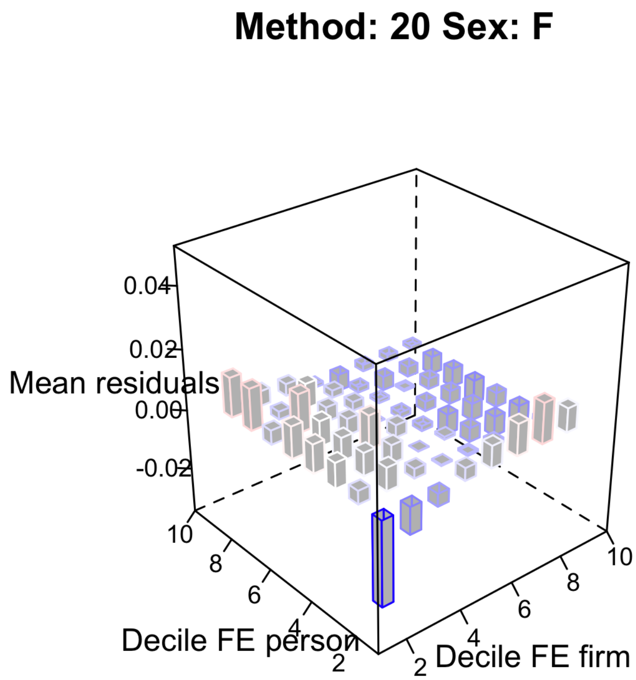

Figure 8: Figure plots residuals from $RIF(Y, q_{20})$ for female workers, classified by decile estimated firm effect and decile of estimated worker effect

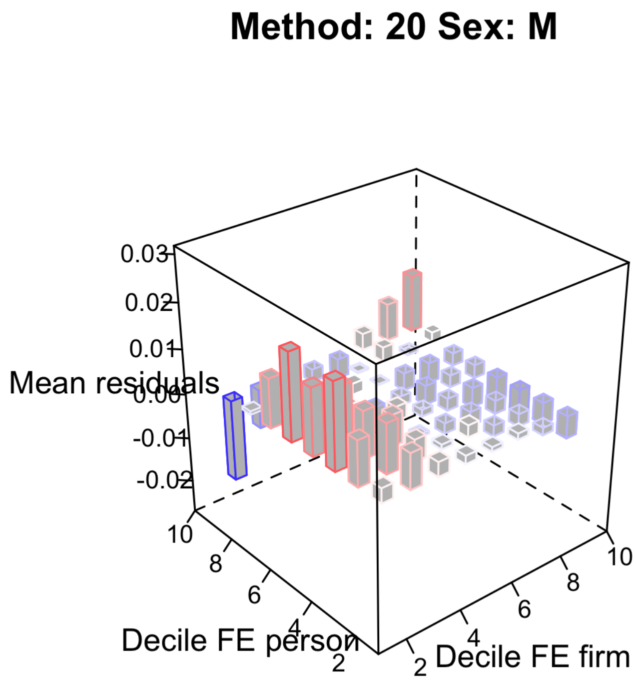

Figure 9: Figure plots residuals from $RIF(Y, q_{20})$ for male workers, classified by decile estimated firm effect and decile of estimated worker effect

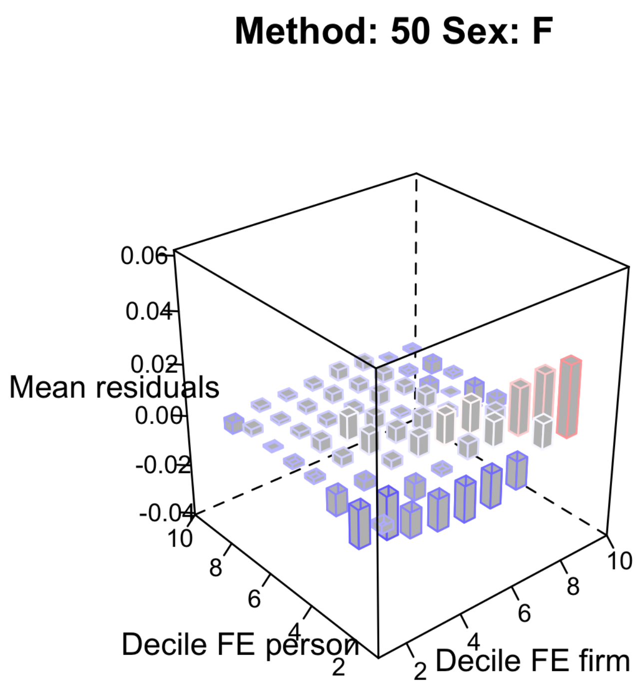

Figure 10: Figure plots residuals from $RIF(Y, q_{50})$ for female workers, classified by decile estimated firm effect and decile of estimated worker effect

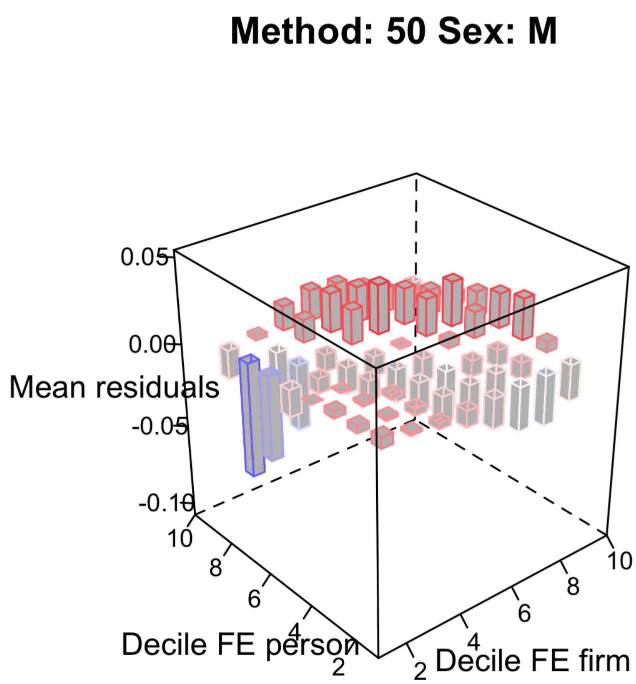

Figure 11: Figure plots residuals from $RIF(Y, q_{50})$ for male workers, classified by decile estimated firm effect and decile of estimated worker effect

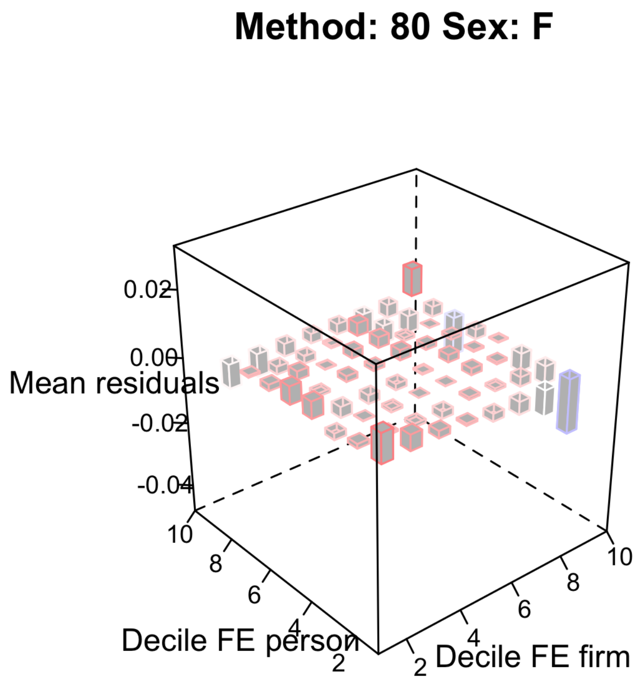

Figure 12: Figure plots residuals from $RIF(Y, q_{80})$ for female workers, classified by decile estimated firm effect and decile of estimated worker effect

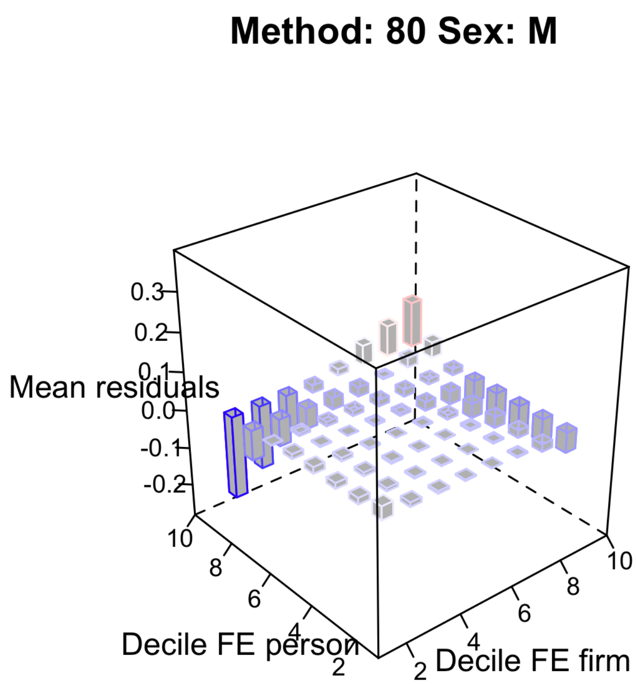

Figure 13: Figure plots residuals from $RIF(Y, q_{80})$ for male workers, classified by decile estimated firm effect and decile of estimated worker effect

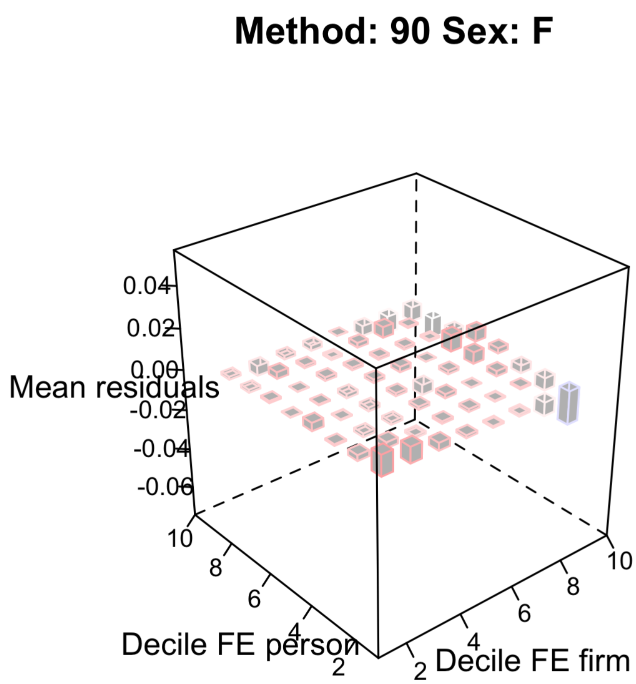

Figure 14: Figure plots residuals from $RIF(Y, q_{90})$ for female workers, classified by decile estimated firm effect and decile of estimated worker effect

Method: 90 Sex: M

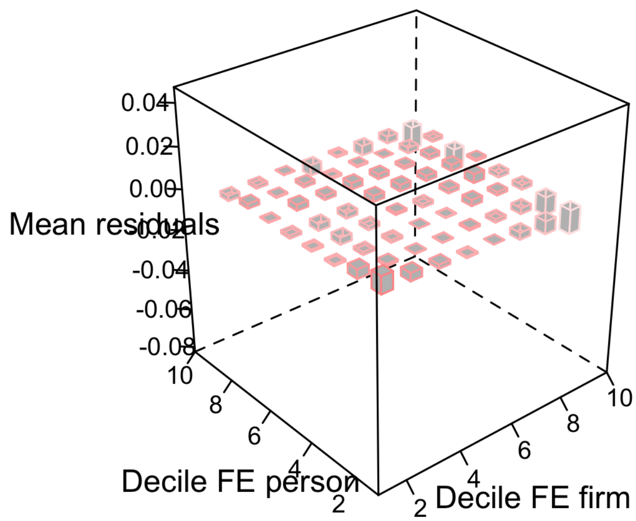

Figure 15: Figure plots residuals from $RIF(Y, q_{90})$ for male workers, classified by decile estimated firm effect and decile of estimated worker effect

9.3 Unconditional Quantile Regressions

In response to the limitations inherent in the Conditional Quantile Regression (CQR) model (Borah & Basu, 2013), a novel technique named Unconditional Quantile Regression (UQR) was recently introduced by Firpo et al. (2009a, 2009b).

In the context of CQRs, the interpretation of the conditional mean remains valid: $q_\tau(Y \mid X) = X\beta_\tau^{CQR}$, where β_τ^{CQR} represents the influence of X on the τ-th Conditional Quantile (CQ) of Y given X. However, the use of the Law of Iterated Expectations (LIE) doesn't work in this case. This means that $q_\tau \neq E_X\left[q_\tau(Y \mid X)\right] = E(X)\beta_\tau^{CQR}$, where q_τ signifies the quantile τ. As a result, β_τ^{CQR} cannot be interpreted as the impact of raising the average value of X in the q_τ.

Basics

An Influence Function (IF) acts as a valuable analytical tool that helps us understand how the addition or removal of an observation affects the value of a statistic, indicated as $\nu(F)$ (Hampel, 1974). In simpler terms, the IF can be seen as a measure of how sensitive the distributional statistic ν is to small changes in the distribution of F_Y (Cowell & Flachaire, 2015). It provides a way to grasp the direction and magnitude of the change in ν that would occur due to minor alterations in the distribution of the variable of interest.

$$\mathrm{IF}(y; \nu(F_Y)) = \lim_{\varepsilon \to 0} \frac{\left[\nu\left((1-\varepsilon)\cdot F_Y + \varepsilon \cdot \delta_y\right) - v(F_Y)\right]}{\varepsilon} = \frac{\partial \nu(\nu F_Y)}{\partial \varepsilon} \tag{27}$$

With $0 \leq \varepsilon \leq 1$ and where:

- F signifies the cumulative distribution function of variable Y.
- δ_y is a distribution concentrated solely at the value y.

However, instead of directly employing the IF, the authors suggest using a recentered version of the statistic known as the Recentered Influence Function (RIF). Since $\int IF(y;\nu,F_Y)dF_Y(y)$ is obtained by integrating the statistic augmented with its corresponding IF:

$$
\begin{aligned}
\mathrm{RIF}(y;\nu,F_Y) = \\
\nu(F_Y) + \int IF(y;\nu,F_Y) \cdot dF_Y(y) \\
\nu(F_Y) + \mathrm{IF}(y;\nu)
\end{aligned}
\tag{28}
$$

An advantageous property of the RIF is that its expectation aligns with $\nu(F)$.

Property 1.

$$
\begin{aligned}
\nu(F_Y) = \int RIF(y;\nu,F_Y)dF_Y(y) \\
= \int\int RIF(Y;\nu,F_Y) \cdot dF_{Y|X}(y \mid X=x) \cdot dF_X(x) \\
= \int E[RIF(Y;\nu,F_Y) \mid X=x] \cdot dF_X(x)
\end{aligned}
\tag{29}
$$

This demonstrates that when we seek to understand the impact of a variable X on a functional $\nu(F_Y)$ (such as a quantile), the procedure involves integrating over $E[RIF(Y;\nu,F_Y) \mid X=x]$, which can be accomplished using regression methods. This connection holds significant importance when utilizing decomposition methods.

The case of quantiles

For the case where the statistic of interest is a particular quantile τ of the outcome distribution:

$$
\mathrm{IF}\left(y;q_\tau\right) = \frac{\tau - \mathbb{1}\left\{Y \leq q_\tau\right\}}{f_Y\left(q_\tau\right)}
$$

Here:

- q_τ denotes the τ-th quantile of the unconditional distribution of Y.
- $f_Y(q_\tau)$ represents the probability density function of Y evaluated at q_τ.
- $\mathbb{1}\{Y \leq q_\tau\}$ functions as an indicator variable to ascertain if an outcome value is less than q_τ.

The RIF is linked to the quantile by the equation:

$$\mathrm{RIF}(y; q_\tau) = q_\tau + \mathrm{IF}(y; q_\tau)$$

Following the proposition 1 in the paper Firpo et al. (2009b), the conditional expectation of RIF $(y; q_\tau)$ is modeled as a function of explanatory variables:

$$E[\mathrm{RIF}(Y;\tau) \mid X = x] = f_\tau(x) \tag{30}$$

Under the following assumptions, the Recentered Influence Function with Ordinary Least Squares (RIF-OLS) can identify the parameters of interest (Heckley et al., 2016).

Assumption 6 ***Additive linearity.*** *Assuming a linear functional form with an additive error term for the regression model, the RIF equation 30 can be expressed as:*

$$E[\mathrm{RIF}(Y;\tau) \mid X = x] = X'\Phi + \varepsilon \tag{31}$$

Assumption 7 ***Zero conditional mean.*** $E[\varepsilon \mid X] = 0$*, implying that under the assumption of error term independence from the conditional mean,* Φ *holds a meaningful interpretation.*

The linearity assumption implies that marginal effects remain constant across the entire distribution of X. Consequently, the derivative of equation 31 with respect to the covariates X can be written as

$$\frac{dE[\mathrm{RIF}(Y;\tau) \mid X=x]}{dx} = \frac{d[X'\Phi + \varepsilon]}{dx} = \Phi \tag{32}$$

Therefore, a RIF regression aligns with the concept of Unconditional Quantile Regression (UQR). This linkage stems from the fact that $E_X E[RIF(Y;\tau) \mid x] = q_\tau$, and $E_Z\left(dm_\tau(x)/dX\right)$ represents the marginal effect of a slight shift in the distribution of covariates on the τ-th unconditional quantile of Y, with other factors held constant.

9.4 OB Estimation steps

1. First step - Estimating the Weights and distributional statistics

As demonstrated by Firpo and Pinto (2016) and Firpo et al. (2018), the initial step involves estimating weights, denoted as ω, which are generally functions of the joint distribution of (D, Φ). Three weighting functions under consideration are $\omega_1(D)$, $\omega_0(D)$, and $\omega_C(D, \Phi)$. The first two weights can be estimated using sample analogs:

$$\widehat{\omega}_1(D) = \frac{D}{\widehat{p}} \quad \text{and} \quad \widehat{\omega}_0(D) = \frac{1-D}{1-\widehat{p}}$$

where $\widehat{p} = \frac{1}{N}\sum_{i=1}^{N} D_i$. The weighting function $\omega_C(D, \Phi)$ can be estimated as:

$$\widehat{\omega}_C(D, \Phi) = \frac{1-D}{\widehat{p}} \cdot \left(\frac{\widehat{p}(\Phi)}{1-\widehat{p}(\Phi)}\right), \tag{33}$$

where $\widehat{p}(\cdot)$ is an estimator of the true proportion under the two approaches, ensuring that the weights sum up to one.

Subsequently, we estimate distributional statistics ν_1, ν_0, and ν_C using the calculated weights:

$$\widehat{\nu}_s = \nu\left(\widehat{F}_s\right), \quad s = 0, 1; \quad \widehat{\nu}_C = \nu\left(\widehat{F}_C\right), \tag{34}$$

where replacing the cumulative distribution function (CDF) with the empirical distribution function leads to the estimators of interest:

$$
\begin{aligned}
\widehat{F}_s(y) &= \sum_{i=1}^{N} \widehat{\omega}_s^* (D_i) \cdot \mathbb{I}\{Y_i \leq y\}, \quad s = 0, 1 \\
\widehat{F}_C(y) &= \sum_{i=1}^{N} \widehat{\omega}_C^* (D_i, \Phi_i) \cdot \mathbb{I}\{Y_i \leq y\}
\end{aligned} \tag{35}
$$

This procedure allows us to estimate the distributional characteristics of interest using appropriately calculated weights and empirical distribution functions.

In the case of **quantiles**, the first stage is obtained by reweighting as $\hat{q_{\tau,s}} = argmin_q \sum_{i=1}^{N} \hat{\omega}_s(D_i)\rho_\tau(Y_i - q)$ for $s = 0, 1$ and $\hat{q_{\tau,c}} = argmin_q \sum_{i=1}^{N} \hat{\omega}_c(D_i, \Phi_i)\rho_\tau(Y_i - q)$. The mapping $\rho(\cdot)$ is the check function suggested by Koenker and Bassett Jr (1978)

2. Second step - Regression coefficients and decompose effect

Now that we have the weights and functionals, we can calculate the coefficients $\gamma_1^\nu, \gamma_0^\nu, \gamma_c^\nu$

$$
\begin{aligned}
\widehat{\gamma}_s^\nu &= \left(\sum_{i=1}^{N} \widehat{\omega}_s^* (D_i) \Phi_i \Phi_i' \right)^{-1} \cdot \sum_{i=1}^{N} \widehat{\omega}_s^* (D_i) \widehat{\mathrm{RIF}} \left(Y_i; \nu(F_{Y_g|D_s}) \right) \Phi_i, \quad s = 0, 1 \\
\widehat{\gamma}_C^\nu &= \left(\sum_{i=1}^{N} \widehat{\omega}_C^* (D_i, \Phi_i) \Phi_i \Phi_i' \right)^{-1} \cdot \sum_{i=1}^{N} \widehat{\omega}_C^* (D_i, \Phi_i) \widehat{\mathrm{RIF}} \left(Y_i; \nu(F_{Y_g|D_s}) \right) \Phi_i
\end{aligned} \tag{36}
$$

Using the functionals $\nu(\cdot)$ we can get the $RIF(\cdot)$. For quantiles, the RIF can be estimated as $\widehat{\mathrm{RIF}}(y; q_{\tau 1}, F) = \widehat{q_{\tau 1}} + \left(\widehat{f}_1 (\widehat{q}_\tau) \right)^{-1} \cdot (\tau - \mathbb{I}\{y \leq \widehat{q_{\tau,1}}\})$ where $\widehat{f}_1(\cdot)$ is a consistent estimator for the density of $Y_1 \mid T = 1, f_1(\cdot)$ [15].

From equation 36 we can break down the effect into the distributional statistic for men $D = 0$ and women $D = 1$ as:

[15] We can estimated this using the kernel method

$$\widehat{\Delta}_S^{\nu} = \left(\sum_{i=1}^{N} \widehat{\omega}_1^*(D_i)\,\Phi_i\right)'(\widehat{\gamma}_1^{\nu} - \widehat{\gamma}_C^{\nu})$$
$$\widehat{\Delta}_X^{\nu} = \left(\sum_{i=1}^{N} \widehat{\omega}_1^*(D_i)\,\Phi_i\right)'\widehat{\gamma}_C^{\nu} - \left(\sum_{i=1}^{N} \widehat{\omega}_0^*(D_i)\,\Phi_i\right)'\widehat{\gamma}_0^{\nu} \tag{37}$$

Then, for the quantiles, the estimators for the gaps are:

$$\widehat{\Delta}_O^{q\tau} = \widehat{q}_1 - \widehat{q\tau}'; \quad \widehat{\Delta}_S^{q\tau} = \widehat{q\tau}_1 - \widehat{q\tau} \quad \text{and} \quad \widehat{\Delta}_X^{q\tau} = \widehat{q\tau}_C - \widehat{q\tau}_0. \tag{38}$$

9.5 Results

With minimum wage restriction

*Goodness of fit

Table 7: Summary of Estimated TWFE Models by method for all sample, females, and males (2007-2012)

	Indicator	Mean	10	20	50	80	90
all	N	37,871,330	37,871,330	37,871,330	37,871,330	37,871,330	37,871,330
	Person FE (θ)	1,127,355	1,127,355	1,127,355	1,127,355	1,127,355	1,127,355
	Firm FE (ϕ)	83,777	83,777	83,777	83,777	83,777	83,777
	R^2 adjusted	0.81	0.41	0.54	0.68	0.73	0.74
	RMSE	0.35	1.91	0.62	0.53	0.70	0.97
F	N	14,045,023	14,045,023	14,045,023	14,045,023	14,045,023	14,045,023
	Person FE (θ)	462,463	462,463	462,463	462,463	462,463	462,463
	Firm FE (ϕ)	83,777	83,777	83,777	83,777	83,777	83,777
	R^2 adjusted	0.81	0.43	0.51	0.68	0.75	0.75
	RMSE	0.33	1.57	0.46	0.46	0.72	0.93
M	N	23,819,413	23,819,413	23,819,413	23,819,413	23,819,413	23,819,413
	Person FE (θ)	664,679	664,679	664,679	664,679	664,679	664,679
	Firm FE (ϕ)	83,777	83,777	83,777	83,777	83,777	83,777
	R^2 adjusted	0.80	0.38	0.52	0.68	0.72	0.75
	RMSE	0.35	1.11	0.42	0.54	0.73	1.02

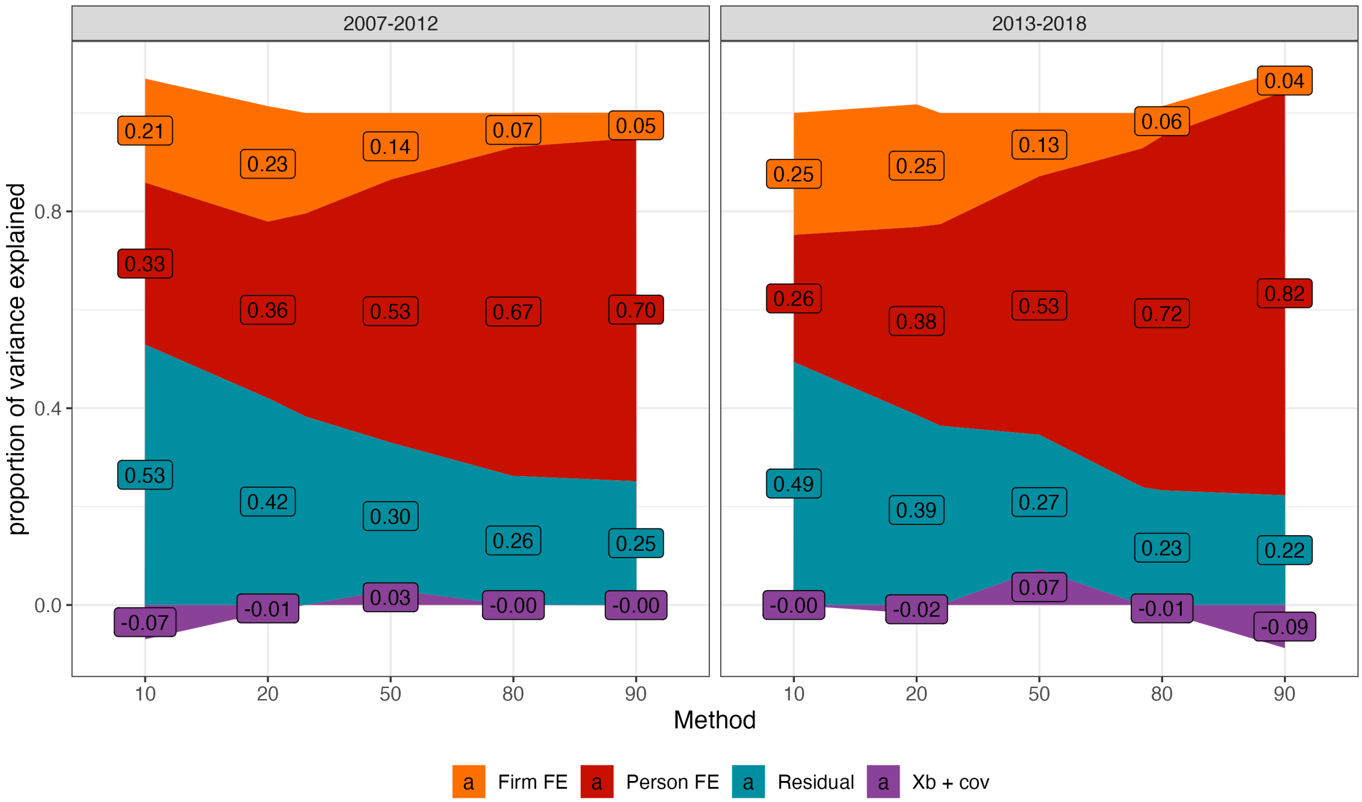

Figure 16: Variance decomposition by method and each period for all the sample. Note: We collapse variance contribution of $X\beta$ and covariances in a single component.

With all sample (no minimum wage restriction)

Goodness of fit

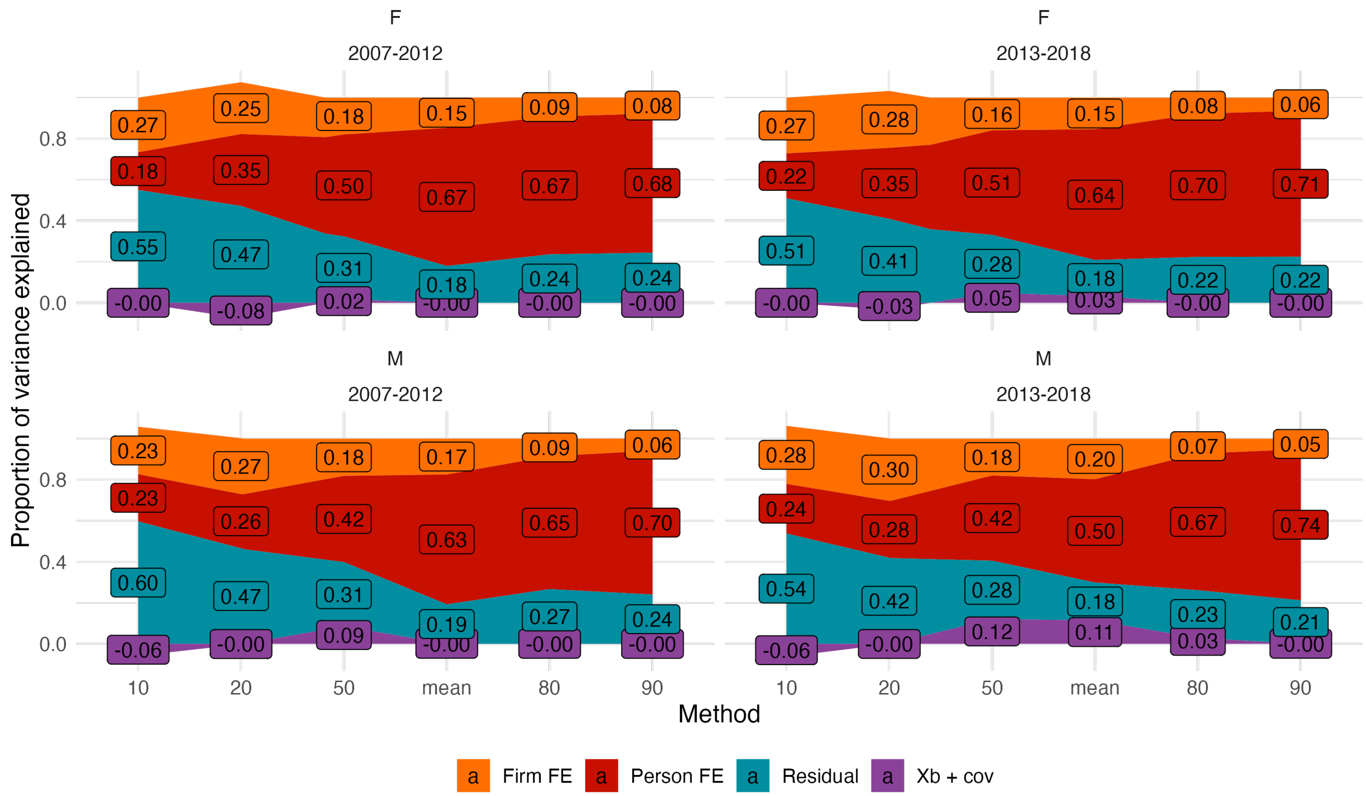

Figure 17: Variance decomposition by method and each period for male and female. Note: We collapse variance contribution of $X\beta$ and covariances in a single component

Table 8: Summary of Estimated TWFE Models by method for all sample, females and males (2007-2013)

	Indicator	**Mean**	**10**	**20**	**50**	**80**	**90**
all	N	37,871,330	37,871,330	37,871,330	37,871,330	37,871,330	37,871,330
	Person FE (θ)	1,127,355	1,127,355	1,127,355	1,127,355	1,127,355	1,127,355
	Firm FE (ϕ)	83,777	83,777	83,777	83,777	83,777	83,777
	R^2 adjusted	0.81	0.41	0.54	0.68	0.73	0.74
	RMSE	0.35	1.91	0.62	0.53	0.70	0.97
F	N	14,045,023	14,045,023	14,045,023	14,045,023	14,045,023	14,045,023
	Person FE (θ)	462,463	462,463	462,463	462,463	462,463	462,463
	Firm FE (ϕ)	83,777	83,777	83,777	83,777	83,777	83,777
	R^2 adjusted	0.81	0.43	0.51	0.68	0.75	0.75
	RMSE	0.33	1.57	0.46	0.46	0.72	0.93
M	N	23,819,413	23,819,413	23,819,413	23,819,413	23,819,413	23,819,413
	Person FE (θ)	664,679	664,679	664,679	664,679	664,679	664,679
	Firm FE (ϕ)	83,777	83,777	83,777	83,777	83,777	83,777
	R^2 adjusted	0.80	0.38	0.52	0.68	0.72	0.75
	RMSE	0.35	1.11	0.42	0.54	0.73	1.02

Table 9: Summary of Estimated TWFE Models by method for all sample, females and males (2013-2018)

	Indicator	Mean	10	20	50	80	90
all	N	49,583,608	49,583,608	49,583,608	49,583,608	49,583,608	49,583,608
	Person FE (θ)	1,359,831	1,359,831	1,359,831	1,359,831	1,359,831	1,359,831
	Firm FE (ϕ)	101,026	101,026	101,026	101,026	101,026	101,026
	R^2 adjusted	0.81	0.45	0.56	0.71	0.76	0.77
	RMSE	0.34	1.29	0.51	0.50	0.65	0.90
F	N	19,751,633	19,751,633	19,751,633	19,751,633	19,751,633	19,751,633
	Person FE (θ)	576,244	576,244	576,244	576,244	576,244	576,244
	Firm FE (ϕ)	101,026	101,026	101,026	101,026	101,026	101,026
	R^2 adjusted	0.82	0.47	0.58	0.71	0.77	0.77
	RMSE	0.33	1.66	0.59	0.46	0.60	0.86
M	N	29,824,795	29,824,795	29,824,795	29,824,795	29,824,795	29,824,795
	Person FE (θ)	783,444	783,444	783,444	783,444	783,444	783,444
	Firm FE (ϕ)	101,026	101,026	101,026	101,026	101,026	101,026
	R^2 adjusted	0.81	0.45	0.57	0.71	0.76	0.78
	RMSE	0.35	1.16	0.45	0.49	0.67	0.84

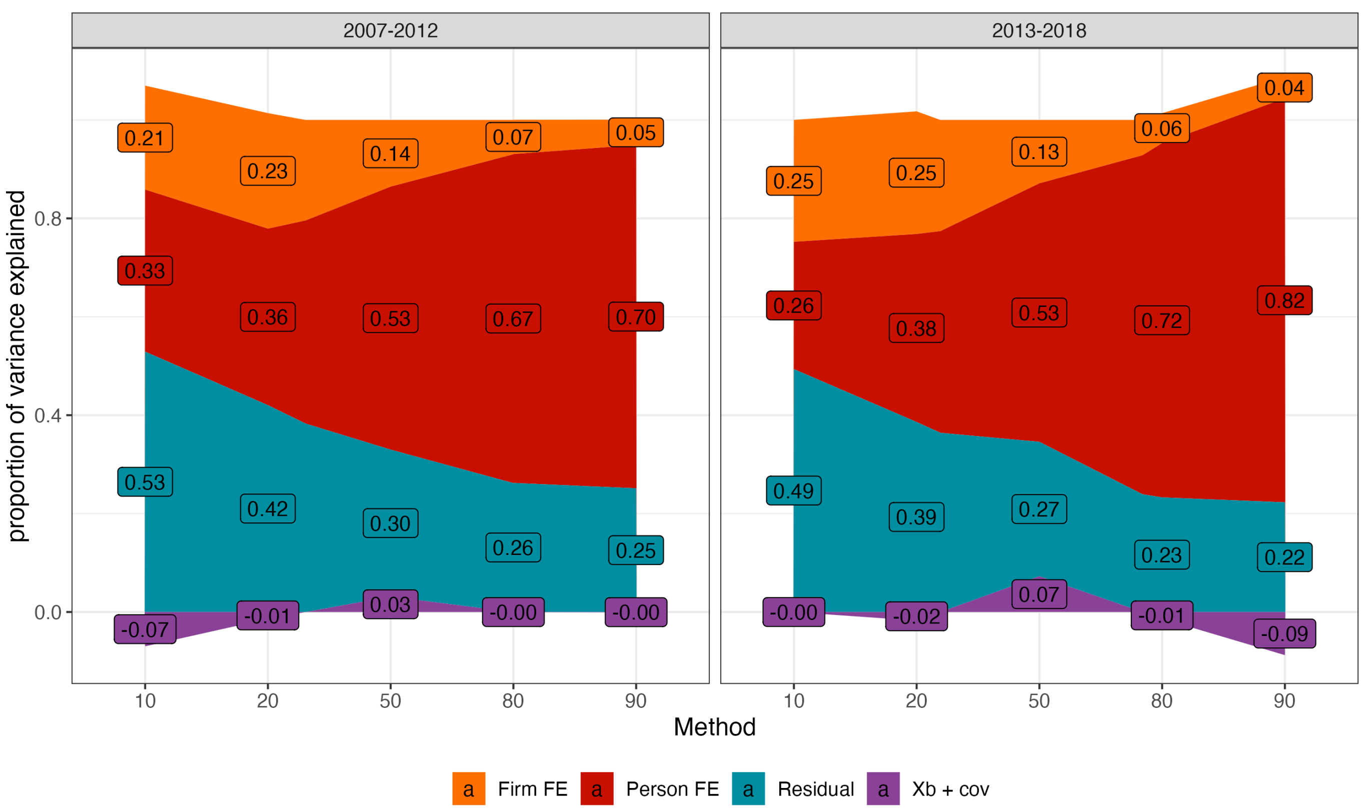

Figure 18: Variance decomposition by method and each period for all the sample. Note: We collapse variance contribution of $X\beta$ and covariances in a single component.