PONTIFICIA UNIVERSIDAD CATOLICA DE CHILE
SCHOOL OF ENGINEERING

PARENTS PARTICIPATING IN THE ADOLESCENT STUDENTS' LEARNING PROCESS: A TEXT MESSAGES' INTERVENTION IN A MATH CLASS CONTEXT

MACARENA PAZ SANTANA SEPÚLVEDA

Thesis submitted to the Office of Graduate Studies in partial fulfilment of the requirements for the Degree of Doctor in Engineering Sciences.

Advisor:
MIGUEL NUSSBAUM VOEHL

Santiago, Chile, January, 2019

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To every child or student who had once had a learning experience leaded or framed by an adult.

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ABSTRACT

There is a consensus that parents' involvement in student's school life is key to academic achievement, but it can be very challenging for parents to participate. Even when parents have the time required to support their children's education, they can increase their children's anxiety about school when they try to help, especially if parents feel ill equipped to do so. The purpose of this thesis is to develop a teacher-driven intervention to involve parents from low-income communities in the student's learning process and evaluate its impact on achievement in mathematics among adolescent children. To this aim, quantitative and qualitative evidence of a series of intervention studies and field experiments is provided. To help parents feel at ease and limit the amount of stress they may feel, non-technical parental involvement was promoted using activities for the parents to complete with the adolescent student that did not include any formal curricular content and that were designed to be short, simple and playful. The teacher then connected these activities to the curricular content in class to make the activities useful and, hopefully, meaningful for both parents and students.

Through the design-based research method, our research team explored whether developing a meeting space outside the school environment between parents and the adolescent student would increase the child's subsequent academic performance in mathematics. Successive small and medium scale studies were conducted (between 21 and 422 students per iteration). This work includes: (i) Two intervention studies to improve the

wording and readability of the messages and to explore initial reactions to the intervention of both parents and students and (ii) Two field experiments to assess the effect of the intervention on students' academic achievement and to explore whether the effect of parental involvement in simple and playful activities would be moderated by the students' negative emotions around the subject, i.e. math anxiety, prior to the intervention. It also explored parents' perceptions about completing activities that were designed to be simple and playful.

Based on these studies, this thesis comprises several findings. The first field experiment revealed that for a small group of Chilean students (treatment=28; control=28) the 5-weeks text-messaging intervention increased their math GPA by 0.488 standard deviations ($p<0.05$) more than students whose parents only received test and homework reminders. This effect remained over time, extending into the following school year. This thesis later confirmed that these students suffered from elevated levels of math anxiety at the outset of the intervention.

The second field experiment explored whether math anxiety moderated the relationship between being assigned to the treatment and the student's post-intervention performance. To this aim, our research team conducted a field experiment with parents of 422 Chilean students in 9^{th} and 10^{th} grade to receive text messages over the course of 12 weeks. Half of the participating parents received weekly assignments for non-academic activities to complete with their children; the other half received text messages informing them of their children's upcoming math tests. The study found that students whose parents were assigned to do non-academic homework with their children performed significantly better on their math tests, and that the subset of students in that group who suffered from higher levels of math anxiety at the outset of the study demonstrated decreased math anxiety after treatment. The overall effect of the activities was positive, but not significant.

Semi-structured phone interviews with parents from low-income schools explored their view of the relationship with their child and the school, two weeks after the intervention. Half of the interviewed parents found the activities were an opportunity to spend time with their child, an opportunity that was valued from two different perspectives. As an opportunity to strengthen their relationship with their child and to become more

involved in their child's school life Many parents appreciated the opportunity to communicate, approach, empathize with and get to know their adolescent child better.

The contribution of this thesis is showing teachers can use simple and low-cost technology to improve the performance of students that have math anxiety, students that represent a relevant sample of the worldwide population. These findings highlight the importance of offering parents accessible ways to involve themselves in their children's school lives and also highlight the power of behavioral nudges that encourage positive parent-student exchanges. Future studies should explore further in the role of parents' math anxiety and develop new ways to engage parents in positive ways into the student's learning process.

Keywords:

Parental involvement; Parent-teacher collaboration; Mathematics; Math anxiety; Mathematics teaching; Adolescence; SMS; Text message

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1. INTRODUCTION

1.1. The problem

The performance of Chilean adolescent students in reading and mathematics literacy has not changed or improved significantly for more than 10 years, shown in both national and international assessments (Ministry of Education, 2018; Rivas & Scasso, 2017). The results in Mathematics SIMCE test in 10th grade (see Figure 1, Ministry of Education, 2018), corresponding to the national assessment system, acknowledge there is a need to make deep changes in the educational system gears, such as school's educational projects, teachers teaching strategies or students' individual learning process.

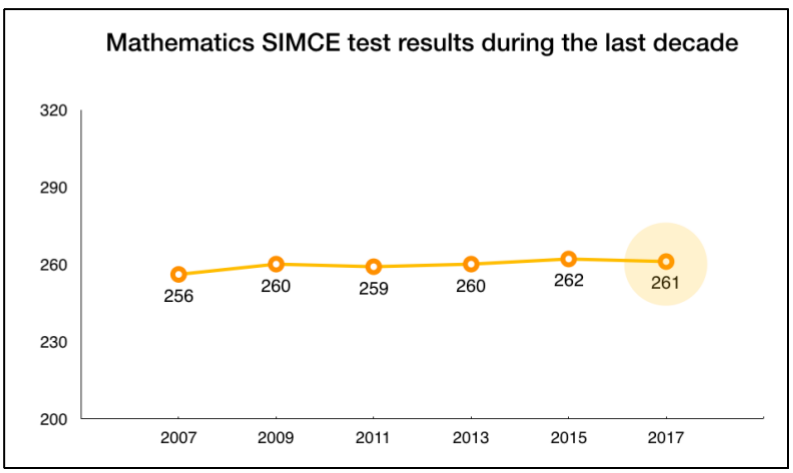

Figure 1—1. Mathematics SIMCE test results in 10th grade (yearly). Adapted from Ministry of Education (2018). Available in: https://goo.gl/wKfkqi

These trends are overall very difficult to modify, however, in low-income backgrounds, the situation is even more challenging. Chilean low-income schools, on average, have consistently performed poorly according to national standards (Libertad y Desarrollo, 2016; Ministry of Education, 2018). In Chile, school SES is highly correlated with individual socioeconomic level (for details see section 9.1.3. Chilean Context, Appendix 1—A). More than 70% of the Chilean students belonging to the lowest socio-economic levels who applied to college could not be admitted in a recent selection process, due to their insufficient academic performance (BBCL Educación superior, 2015). Developing solutions for adolescent students who belong to low socioeconomic status (SES) schools is especially necessary because most of their families do not have the resources to help their child bridge the gap between what they know and what they should know (Crouter & Booth, 2014). To explore further the Chilean mathematics achievement gap, see La Tercera (2014).

Adolescent learning outcomes and development are a result of several factors. From a bioecological theoretical perspective, human development is the result of a series of interactions between themselves and the very different environments surrounding them, e.g. classroom experience, school environment, family environment, non-familial expieriences outside of schools, among others (Bronfenbrenner & Morris, 1998). Therefore, it is crucial to try new strategies and different approaches related to the interactions that occur around the student's learning process.

It is widely accepted that parents' socioeconomic level and their parenting practices play a crucial role for the students' developmental and learning process (e.g. Blums, Belsky, Grimm, & Chen, 2016; Epstein & Sanders, 2002; Hill & Tyson, 2009; Jeynes, 2005; Sirin, 2005), however, it is challenging for parents to participate in the adolescent students' school life (for details see Chapter 3). Even with available time, parents may have difficulty communicating with teens, as many parents lack the skills required for positive and productive interactions with their children (Cabus & Ariës, 2017; Dumont, Trautwein, & Nagy, 2012; Wilder, 2014).

During a casual conversation a friend of mine, the principal of a low-income school in Santiago, Chile, pointed out: "We all know it's important to engage parents in the adolescent learning process, but none of us have a clear idea of how to do this". Despite of its importance, the schools have not yet found a way to make parents play a clear and fruitful role in their child's learning process. In fact, UNICEF (2018) recently pointed that "fathers are one of the best, yet most underutilized child development resources".

1.2. The general proposal

Considering the opportunity that parents' engagement represent for the student, the general research question for this thesis is the following*: "Can fruitful parent-student interactions be fostered by the school institution?"* Consequently, the aim for this study is to design an intervention that allows improving academic performance of adolescent students in a simple, binding and scalable way in schools from low socioeconomic context.

1.2.1. Why the Mathematics Subject?

As described in further Chapter 2 and 3, the math subject was the focus of this work because we identified three main opportunities for developing fruitful parent-student interactions around this subject. Firstly, the mathematics subject is often taught by a subject teacher who is often more distant to parents compared to the head teacher. Parents of middle school students often lack of a clear point of contact with the school as the number of teachers and subjects increases, therefore, their level of communication with subject teachers decreases (Eccles & Harold, 1993; Hill & Tyson, 2009; Simons-Morton & Crump, 2003). This decrease may lead to conflict between parents and students by the lack of clarity that parents have when it comes to the effort made by their children in school (Bergman, 2015). Secondly, the complexity of the subject often leads students and their parents to perceive math as difficult and, consequently, to avoid it or have negative attitudes towards it (Ashcraft & Krause, 2007; Ashcraft & Moore, 2009; Dowker, Sarkar & Looi, 2016). The adverse emotional reactions to math or the prospect of doing math are known as math anxiety (Maloney & Beilock, 2012). Thirdly, parents often feel unprepared to help students complete their math schoolwork, conditions under which parental involvement could negatively impact students' learning process (Maloney, Ramirez, Gunderson, Levine, & Beilock, 2015; Mellon & Moutavelis, 2011; Vukovic, Roberts, & Green Wright, 2013).

1.2.2. Intervention Design

As described in Chapter 3, interactions between teenagers and their parents may be particularly hard for parents and detrimental for students especially if parents are not confident in their own abilities. To limit the potential of anxiety, the activities included in this work were defined as non-academic, because despite they were related to the math curriculum, they did not require any math knowledge to be completed. These activities were short in duration, considering parents' time constraints (see Chapter 3). Parents also received recommendations about how to engage in this activities and how to motivate the student at school. In this work, parents' participation was defined as non-technical parental involvement, as non-technical skills were required to complete any of the suggested tasks.

Consequently, the math teacher invited parents to complete short, non-academic activities delivered via text message. As described in Chapter 2, this work includes text-messages as the channel to communicate with parents, because text-messages interventions are increasing in popularity due to their low-cost and scalability.

It may be the case that these activities open a meeting space to talk about school-related issues which may, in turn, increase the student's mathematics performance. It has been shown that students whose parents monitor their performance in school have higher academic achievement than their unsupervised peers (Coley & Hoffman, 1996; Fulton & Turner, 2008; Hill & Wang, 2015).

Considering that the preparedness parents feel to face a challenge related to their child influences the way they communicate with them, it is important to explore the type of parent-student exchanges that are fostered when a teacher encourages low-income parents to work with their adolescent children.

1.2.3. Thesis Analytical Approach

In order to understand the complexities of the implementation of the program in the educational context, we used Design-Based Research. This methodology allows to have a much more finished understanding of the problem through a series of iterations in the investigative process (Anderson & Shattuck, 2012, Barab & Squire, 2004). At every iteration we sought to have a wider understanding of the agents participating in the intervention, i.e. parents, teachers and students and re-design the intervention accordingly. We summarize this process in two main stages:

First stage: A series of three small scale studies were conducted, re-designing the intervention at every iteration. As described in Chapter 2, in the first two study interventions (2015) we tested the content of the text messages and consequently re-wrote them, we tested extrinsic incentives for have parents replying text messages and we assessed the overall reactions of the students through informal class observation, parents text messages responses and informal interviews. The third study, a small-scale (n=56) field experiment conducted in 2016, showed positive effects of the intervention on students' mathematics performance. Results from phone interviews showed that the parents perceived the moment where they completed the activities with their student was fun. However, we needed to generalize whether this intervention would work for a broader population of parents and students and, therefore, to explore further the mechanisms that drive the effect that was found.

Second stage: as described in Chapter 3, a middle scale field study (12 teachers in 8 schools, involving aa students) was conducted in 2017 and we evaluated whether the effect of the intervention varied depending on the students' initial level of math anxiety. Participant parents of the study were interviewed by phone with aim to explore their perceptions and behaviours during the intervention for several reasons. Firstly, their perceptions and behaviours may influence the student's emotions around the subject. As pointed out in Chapter 4, a potential treatment for students' math anxiety is to change how adults in their lives frame the subject (Dowker, Sarkar, & Looi, 2016). Secondly, parents' behaviors may or may not allow the creation of a space where the parent and the student can share a warmth moment (e.g. laugh together). Parental warmth has been widely shown to be a key factor for the child's development and academic performance (e.g. Kim & Rohner, 2002). Thirdly, the type of parent-student exchanges nudged by the intervention may or may not strengthen the relationship between them. Considering that many students suffer an important emotional deprivation in the infantile period due to a lack of a parent-child relationship or a serious break in the parent-child relationship (Bender, 1947), it is important to explore whether the intervention can bridge the communication between the parents and their child. The quality of the relationship between the child and their parent contributes significantly to the commitment with the school, in low income contexts (Murray, 2009).

1.3. Research hypotheses

The following hypotheses were used to frame the work carried out for this thesis:

H1. The development of a meeting space outside the school environment between parents and the adolescent student, consisting of simple and playful activities, will increase the child's subsequent academic performance.

H2. The effect of parental involvement in simple and playful activities will be moderated by the students' level of mathematics anxiety, prior to the intervention.

H3. Parents can have several perceptions about the experience of completing activities that were designed to be simple and playful with their adolescent student.

1.4. Research questions

The research questions that guided the work that was carried out for this thesis are the following:

Q1. What effect does this intervention have on the students' academic performance in mathematics?

Q2. What type of parent-student exchanges are fostered when a teacher encourages low-income parents to work with their adolescent children on completing short, non-academic activities delivered via text message?

Q3. Do text messages encouraging parental involvement in school-related work affect students' mathematics performance?

Q4. Does the impact of these text messages vary depending on the initial level of student's mathematical anxiety?

Q5. Can the intervention support students with high math anxiety by reducing their anxiety?

Q6. In terms of the relationship with their child and the school, how do parents perceive the experience of working with their child on non-academic math assignments?

Q7. What sort of behavior is fostered among parents by school-driven, non-academic activities?

Q8. Given this kind of activity, what are the conditions that can promote changes in the parent-adolescent relationship?

1.5.Objectives

The objectives of this thesis are the following:

O1. Explore the impact that completing these activities has on the students' achievement in mathematics (as measured by their GPA in math).

O2. Explore the type of parent-student interaction promoted the activities (as perceived by the parents)

O3. Generalize the impact that completing these activities has on the students' achievement in mathematics (as measured by their GPA in math).

O4. Analyze whether students' mathematics anxiety moderates the relationship between being assigned to treatment and post-intervention mathematics performance.

O5. Study the parents' perception of working with their children math homework that exclude formal content of the subject and explore the impact that completing these tasks has on parent-student relationships (as perceived by the parents).

1.6. Results

The studies from this thesis have produced a series of results that are described below:

R1. The intervention improved the students' mathematics achievement for a small group of students in a low-income school in Santiago, Chile.

R2. The intervention promoted positive and inclusive interactions, as perceived by the parents.

R3. The intervention effect on average was positive but not significant in a larger sample of teachers and schools in Santiago, Chile.

R4. The intervention improved the students' mathematics performance for students who suffer from high levels of math anxiety.

R5. The subset of students in that group who suffered from higher levels of math anxiety at the outset of the study demonstrated decreased math anxiety after treatment.

R5. Half of the interviewed parents found the activities were an opportunity to spend time with their child, an opportunity that was valued from two different perspectives. As an opportunity to strengthen their relationship with their child and to become more involved in their child's school life

R6. Three groups of behavioral patterns were found based on 1) the parent's level of commitment to the intervention, 2) the level of communication between the parent and their child, and 3) the time they spent together (degree of encounter).

R7. To change the nature of the parent-student relationship, we saw that it is necessary that both the parent and the child (or at least the parent) show an interest in getting to know one another. This includes trying to motivate the child to participate in the activities or asking questions that go beyond the minimum requirements of the activity.

1.7. Theses outline

This thesis is based on studies that were carried out in order to meet the objectives detailed above. These studies constitute different research papers which are all currently at different stages of publication. Some have already been published, others have been accepted, while others have been submitted and are awaiting a response from the respective journals' referees. Below is a description for each chapter of its constituting paper, the topic that is addressed, the title of the paper, the authors, and the corresponding journal.

Chapter 2: The title of the paper included in this chapter is Having Fun Doing Math: Text Messages Promoting Parent Involvement Increased Student Learning, by the authors Santana, M., Nussbaum, M., Carmona, R. & Claro, S. from 2018. This paper, paper 1 (P1), has been accepted for publication by the Journal of Research for Educational Effectiveness (Impact Factor of 1.410), *in Press*. This study explores whether the intervention can have a positive effect on the students' mathematics performance. As this paper constitutes an exploratory work, it is a small-scale field study (n=56) and summarizes the two intervention studies conducted prior to developing the activities, including parents' reactions to the activities.

Chapter 3: The constituting paper for this chapter is titled Let's Spend Time Together: Text Messaging Parents Improved Performance of High Anxious Students, by the authors Santana, M., Nussbaum, M., Claro, S. & Loeb, S., from 2018. This paper, paper 2 (P2) has been sent to the Journal for Research in Mathematics Education. (Impact Factor of 2.5) and it is currently in the review process. This study aims to generalize thee effects of non-academic assignments parents complete with their teenage children and assessed whether student's math anxiety moderated the relationship between been assigned to the intervention and the students' performance in mathematics. Twelve math teachers (from 8 schools) invited the parents of 422 Chilean students in 9th and 10th grade to receive text messages over the course of 12 weeks. As the positive and significant effect on the students' mathematics achievement was confirmed for high-anxious students, but not for low-anxious students. This results highlights the importance of offering parents accessible ways to involve themselves in their children's school lives and that feel non-threatening to their children.

Chapter 4: This chapter includes the study Fostering Parent-Adolescent and Parent-School Relationships through Math Activities, by the authors Santana, M., Nussbaum, B., Piza, S., Imbarack, P., Perez, M., Alarcón, J., from 2018. This paper, paper 3 (P3), has been sent to the Journal Family and Marriage (Impact Factor of 1.2) and is currently in the review process. This paper explores the view that parents have of text messaging intervention. The main objective of the study is to explore the impact of the activities and suggestions sent to parents and whether it can potentially modify the relationship parents have with the school or with the student. The findings from this study reveal that many parents appreciated the opportunity to communicate, approach, empathize with and get to know their adolescent child better. Providing a space for family interaction that avoids conflicts, can lead to changes in how parents and adolescents interpret their relationship.

1.8. Thesis structure

The structure of this thesis is based on the hypotheses, research objectives and questions mentioned in section 1.3, 1.4 and 1.5. Figure 1—4 provides a model which shows the connections between each of these components.

Hypothesis 1 (H1) “The development of a meeting space outside the school environment between parents and the adolescent student, consisting of simple and playful activities, will increase the child's subsequent academic performance” relates to question 1 (Q1) “What effect does this intervention have on the students’ academic performance in mathematics? and to objective 1 (O1) “Explore the impact that completing these activities has on the students’ achievement in mathematics (as measured by their GPA in math)”. This question and objective are addressed in paper 1 (P1), upon which this thesis is based, “Having Fun Doing Math: Text Messages Promoting Parent Involvement Increased Student Learning”. The first result of this paper is “The intervention improved the students’ mathematics achievement for a small group of students in a low-income school in Santiago, Chile” (R1).

R1 leads to the formulation of the third objective, “Generalize the impact that completing these activities has on the students’ achievement in mathematics (as measured by their GPA in math)” (O3) which in turn leads to question 4 (Q4) “What effect does this intervention have on the students’ academic performance in mathematics?”, both described in paper 2 (P2) contained in this thesis, “Let’s Spend Time Together: Text Messaging Parents Improved Performance of High Anxious Students”. Results of this paper suggests that “The intervention effect on average was positive but not significant in a larger sample of teachers and schools in Santiago, Chile” (R3).

Hypothesis 2 (H2) “The effect of parental involvement in simple and playful activities will be moderated by the students’ level of mathematics anxiety, prior to the intervention", is made up of objective 4 "Analyze whether students' mathematics anxiety moderates the relationship between being assigned to treatment and post-intervention mathematics performance" (O4) which leaded to question 4 "Does the impact of these text messages vary depending on the initial level of student's mathematical anxiety?" (Q4) and question 5 "Can the intervention support students with high math anxiety by reducing their anxiety?" (Q5). This study showed that "the intervention improved the students' mathematics performance for students who suffer from high levels of math anxiety" (R4) and "The subset of students in that group who suffered from higher levels of math anxiety at the outset of the study demonstrated decreased math anxiety after treatment" (R5).

Finally, Hypothesis 3 (H3) "Parents can have several perceptions about the experience of completing activities that were designed to be simple and playful with their adolescent student" is made up of the objectives 2 and 5 (O2 and O5). Regarding objective 2 "Explore the type of parent-student interaction promoted the activities (as perceived by the parents)" (O2), it leads into question 2 "What type of parent-student exchanges are fostered when a teacher encourages low-income parents to work with their adolescent children on completing short, non-academic activities delivered via text message?" (Q2), which is covered by paper 1 (P1). This study suggested that "The intervention promoted positive and inclusive interaction, as perceived by the parents" (R2). This positive perception that included perceiving the activities as fun, also leaded to the objective 4 (O4), related to the student's math anxiety moderating effect. In relation to objective 5 "Study the parents' perception of working with their children math homework that exclude formal content of the subject and explore the impact that completing these tasks has on parent-student relationships (as perceived by the parents)”(O5), it leads into three different questions: “In terms of the relationship with their child and the school, how do parents perceive the experience of working with their child on non-academic math assignments?” (Q6), “What sort of behavior is fostered among parents by school-driven, non-academic activities?” (Q7) and “Given this kind of activity, what are the conditions that can promote changes in the parent-adolescent relationship?” (Q8), covered by the third paper “Fostering Parent-Adolescent and Parent-School Relationships through Math Activities”. The results from this paper suggests that “half of the interviewed parents found the activities were an opportunity to spend time with their child, an opportunity that was valued from two different perspectives. As an opportunity to strengthen their relationship with their child and to become more involved in their child’s school life” (R6), “Three groups of behavioral patterns were found based on 1) the parent’s level of commitment to the intervention, 2) the level of communication between the parent and their child, and 3) the time they spent together (degree of encounter)” (R7) and “to change the nature of the parent-student relationship, we saw that it is necessary that both the parent and the child (or at least the parent) show an interest in getting to know one another. This includes trying to motivate the child to participate in the activities or asking questions that go beyond the minimum requirements of the activity” (R8).

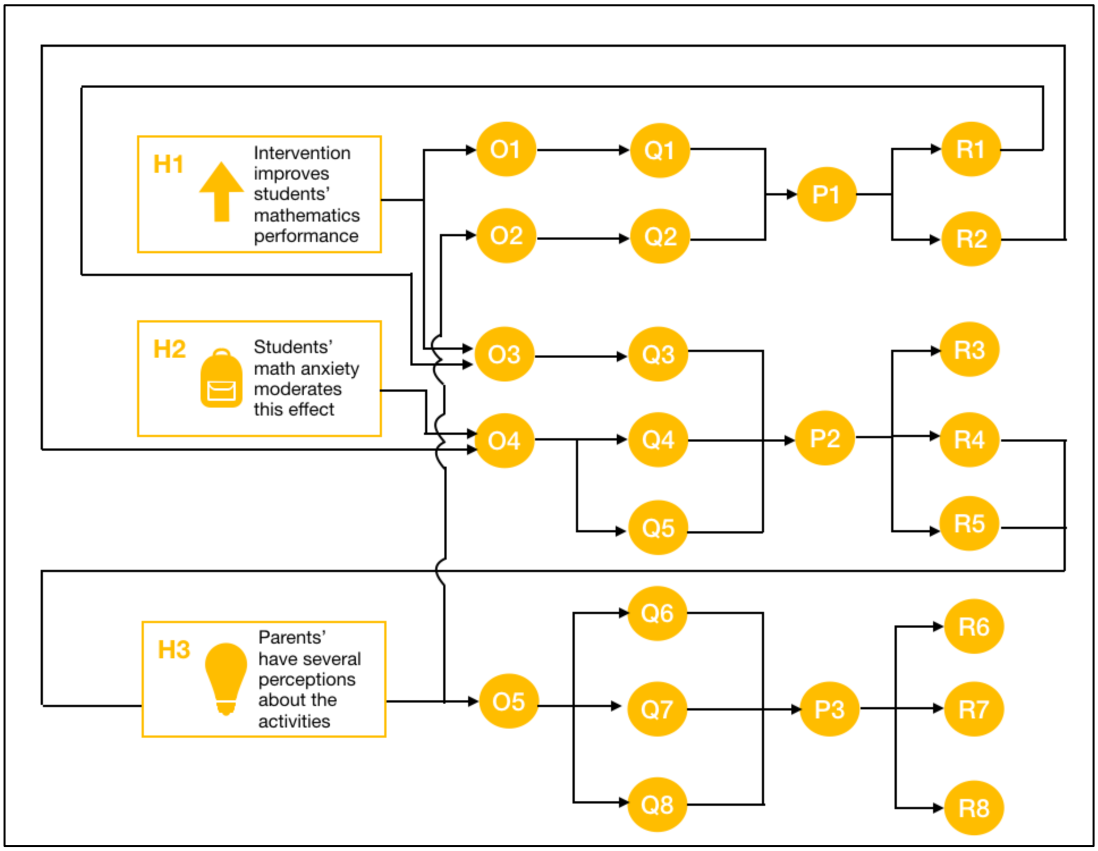

Figure 1—2 Connections between the hypotheses, research questions, objectives, papers and results.

2. HAVING FUN DOING MATH: TEXT MESSAGES PROMOTING PARENT INVOLVEMENT INCREASED STUDENT LEARNING

2.1. Background

International evidence reveals that a large percentage of the population does not achieve competency in mathematics, especially among lower socioeconomic groups (Duncan & Magnuson, 2013; Hanushek & Rivkin, 2006). At home, parents can help their children significantly improve their academic results by interacting with them and talking about school-related issues (Castro et al., 2015; Hattie, 2009; Jeynes, 2003, 2005). However, parental involvement in schoolwork is not consistently linked to a student's academic achievements (Castro et al., 2015; Hill & Tyson, 2009; Jeynes, 2005; Patall, Cooper & Robinson, 2008).

When parents try to help with conventional math homework, which are typically problem/question sets from textbooks or worksheets (Zhu, 2015), they often end up confusing the child even more (Balli, Demo & Wedman, 1998). In order to help with math homework, parents require a set of skills that can adapt to the student's needs (Ariës & Cabus, 2015; Doctoroff & Arnold, 2017; Pressman et al., 2015). However, Wilder (2014) explains that parents often fail to master the necessary concepts or are not familiar with the appropriate teaching methods. The scenario is complicated even further when adding factors such as the de-contextualization and complexity of the assignments and assessments (Lyons, 2006; Nitko, 1996). Given the possible lack of knowledge and skills among parents, it has been proposed that homework assignments set by the teacher should not require any knowledge of the subject (Vukovic, Roberts & Green Wright, 2013). For instance, parents could be asked to work with the student and count how many cars of different colors pass by in five minutes. This experience could then be used in the classroom to work on formal mathematical concepts, such as range and frequency. It is also suggested that such assignments should be written in a language that is accessible to the parents (Bull, Brooking & Campbell, 2008). Furthermore, assignments should be designed to explicitly involve the parents (Tam & Chan, 2016) and be structured in such a way that the parents can act as a source of support, rather than as instructors (Donaldson-Pressman, Jackson & Pressman, 2014; Pressman et al., 2015).

There are a huge number of factors that can influence the impact of conventional math assignments on the learning process (Zhu, 2015). The design and effectiveness of such assignments has therefore been a constant source of debate. The time taken and interest shown by students in this type of homework assignment have been positively associated with academic achievement (Singh, Granville & Dika, 2002; Xu, Yuan, Xu & Xu, 2016). However, conventional homework assignments are also one of the main sources of conflict between parents and students (Bernedo, Fuentes & Fernández, 2005; Bosma et al., 1996; Del Valle, 1994). The support offered by parents to their children is not always perceived as such (Moroni, Dumont, Trautwein, Niggli & Baeriswyl, 2015). When students feel that their parents are interfering with their homework, or when the homework leads to a conflict, this can result in poorer academic performance (Dumont et al., 2012). During early adolescence, children start to question their parents' authority. Parent-student conflict therefore tends to increase during this time (McGue, Elkins, Walden & Iacono, 2005), as roles and expectations are realigned (Hill & Tyson, 2009).

It is not only the students and parents who are responsible for conflicts over school assignments; it also depends on the input provided by teachers to promote exchanges between the two. In addition to conventional homework assignments, parents also receive information from the teacher on other school-related issues, such as behavior (Oinas, Vainikainen & Hotulainen, 2017) and attendance (Bodén, 2017), among others. This information can often damage the relationship between the student and their parents as it can trigger supervisory and controlling practices (Doctoroff & Arnold, 2017; Itkin, 1955). This is particularly relevant in difficult contexts, where the parent-student relationship is often highly negative (Hagan, Roubinov, Adler, Boyce & Bush, 2016; Suldo, 2009). Therefore, conventional math homework often has a negative connotation as it acts as a source of conflict between the student and their parents.

For many low-income parents, getting involved in the learning process at home is hard. Some of the most frequent challenges include their schedule at work and workload (Crouter & Booth, 2014; Gracia & Kalmijn, 2016; Tubbs, Roy & Burton, 2005). Homework assignments should therefore be short enough to increase the likelihood of parents and students finding the necessary time to work together. However, it has been shown that simply finding the time to get involved is not enough. Cabus and Ariës (2017) reveal that even with the same level of parental involvement (i.e. helping with homework and talking about school-related issues), students from low-income families performed worse than the average student. The researchers explain that this situation may be due to the "the differential effectiveness of parents teaching or helping strategies" (Ariës & Cabus, 2015), while another study suggests it may be linked to "parental competence to help with homework" (Dumont et al., 2012). It is therefore important to ensure that homework assignments are kept simple, i.e. that they do not require the sort of knowledge or skills that may be lacking among low-income parents.

Considering the above, if the math teacher at a low-income school provides parents with quick, simple tasks that do not involve any formal curricular mathematics content (i.e. are non-academic), it may lead to positive parent-student exchanges. Furthermore, by using examples in class that are related to the homework assignment, the teacher may enrich the students' learning experience and, consequently, improve their achievement in math.

To the best of our knowledge, there are only a few examples in the literature of studies that examine the effects of encouraging parents to complete activities with their children at home. However, none of these studies describe the interactions that are fostered by these activities, nor the possible effects that they may have on the parent-student relationship. Furthermore, the homework activities in these studies have been designed for early childhood education or elementary education. Berkowitz et al. (2015, 2016) use tablets to allow parents and first graders to read a story together that focuses on number problems. Using the same technology, they then answer questions on topics such as counting, geometry, arithmetic, fractions and probability. This is particularly beneficial for students with higher levels of anxiety in mathematics.

Mayer, Kalil, Oreopoulos and Gallegos (2015) use tablets and text message reminders to increase how much parents read to their children. Positive effects have also been observed on reading habits when text messages are sent to parents encouraging them to complete early literacy activities with their young children (Doss, Fahle, Loeb & York, 2018; York, Loeb & Doss, 2018). Overall, contacting parents via text message (SMS) without the need for an internet connection has been shown to be cost-efficient and has become increasingly popular for parent interventions (Bergman, 2015; Berlinski, Busso, Dinkelman & Martinez, 2016; Castleman & Page, 2015; Maloney, Converse, Gibbs, Levine & Beilock, 2015).

Given this, our first research question is therefore the following:

- What type of parent-student exchanges are fostered when a teacher encourages low-income parents to work with their adolescent children on completing short, non-academic activities delivered via text message?

We present an intervention in which parents receive non-academic activities via text message that are to be completed before the following math class. Parents also receive a reminder and a suggestion of how to encourage the student. Consequently, our second research question asks:

- What effect does this intervention have on the students' academic performance in mathematics?

2.2. Method

This research is based on a field experiment in which parents were chosen at random to receive a set of text messages encouraging them to complete short and simple non-academic activities with their adolescent children. We use a mixed-methods approach for data collection and analysis. The study combines quantitative and qualitative data (Creswell & Clark, 2007) to gain a better understanding of the impact that completing these activities has on parent-student relationships (as perceived by the parents) and the students' achievement in mathematics (as measured by their GPA in math).

2.2.1. Participants

The field experiment was conducted at a Chilean school in a low-socioeconomic neighborhood and ran from the end of June to the beginning of August, 2016 (i.e. the Winter and beginning of the Spring term). Fifty percent of students at this school came from families whose household income was considered to be lower than the cost of covering the families' basic needs. The school has high levels of absenteeism and dropouts, while its academic results are below average even when adjusting for socioeconomic factors (Agencia de Calidad de la Educación, 2016).

The adults who participated in this study were parents and guardians, i.e. the people who are responsible for ensuring that the students fulfill their duties at school. In Chile, while the guardians are often the parents, this role is sometimes played by other relatives. Therefore, for simplicity's sake, guardians and parents are hereby referred to as parents throughout the paper. Participating students were from three different grades: 8th, 9th and 10th grades. All 56 parent-student dyads from these courses agreed to participate in the study. Of the 56 parents involved in the study, 55.8% had graduated from high school, with only 4.2% having gone on to higher education. The study involved students aged between 14 and 16 years old. The participating classes had 23 students (14 girls and 9 boys), 17 students (10 girls and 7 boys) and 16 students (9 girls and 7 boys), respectively. All math lessons in these three grades were taught by the same teacher during 2016 (the year of the intervention), while another teacher taught and assessed the students the following year (2017).

The sample size was determined by feasibility. We decided to work with just one teacher and 28 students in the treatment group and 28 students in the control group. This meets the “12 per group” rule of thumb for fist-time trials (Julious, 2005). Billingham, Whitehead & Julious (2013) state that for pilot-trials “a formal sample size calculation may not be appropriate”, since a larger sample size may entail exposing participants to unnecessary risks. Moreover, our analyses include **11** controlling variables. Results from their Monte Carlo simulation suggest that two subjects per variable allows for a reasonable estimation of the regression coefficients, standard errors and confidence intervals (Austin & Steyerberg, 2015).

2.2.2. Measurements

End-of-term grade point average in mathematics: To answer the research question regarding the effect of the intervention on academic achievement, our main variable of interest was academic performance in mathematics. This was measured on two occasions in order to monitor the impact over time. The first measurement corresponds to the end-of-term math GPA during the year of the intervention (i.e., 2016 Spring term). This *Spring 2016 Math GPA* corresponds to the average grade achieved by a student on the four tests that were given during the three months of the Spring term. Grades can range from 1 (the lowest grade possible in Chile) to 7 (the highest grade). The Spring 2016 Math GPA was collected for all 56 students (M=5.60, SD=0.82). To help interpret the regression coefficients, this outcome was standardized within the sample to have a mean of 0 and standard deviation of 1.

The second measurement is the *Fall 2017 Math GPA*, which corresponds to the end-of-term math GPA collected the following academic year. This represents the average grade on the four tests that were given during the 3 months of the Fall 2017 term (M= 4.96, SD=1.26). This measurement is available for the 51 students (91% of participants) who continued at the school. The students who did not continue were distributed evenly across the two conditions (i.e. 2 students from the treatment group and 3 students from the control group). As with the previous outcome, this variable was also standardized within the sample to have a mean of 0 and a standard deviation of 1.

The Chilean grading system allows each test to be graded to 1 decimal place. Therefore, an example grade would be a 5.4. As a result, the Spring 2016 and Fall 2017 Math GPA outcomes include 27 and 32 distinct decimal values, respectively (see Figures Appendix 2—E1 and Appendix 2—E2, Appendix 2—E). Given the number of different values for each of the outcomes, we modelled these as continuous variables.

Previous performance: The students' performance in mathematics during the period leading up to the intervention (Fall 2016) was considered, as well as their overall performance at school and their performance in mathematics in previous years (2013, 2014 and 2015). This information was collected from school records or from the Chilean Department of Education (Ministerio de Educacion, 2016). The *Fall 2016 Math GPA* corresponds to the average grade achieved by each student on the four tests that were given during the three months of the Fall term (M=4.94; SD=1.13). The *Overall Grade Point Average* corresponds to the average grade achieved by the student across all subjects in 2013 (M=5.55, SD=0.54), 2014 (M=5.58, SD=0.63), and 2015 (M=5.39, SD=0.60). The *Math Grade Point Average* corresponds to the average grade achieved by a student in mathematics by the end of the year in 2013 (M=4.72, SD=0.85), 2014 (M=4.84, SD=0.99), and 2015 (M=4.86, SD=0.10). Each variable was standardized to have a mean of 0 and standard deviation of 1. Multiple imputation (Rubin, 2004) was used for 6 cases in which the school records were not available. This calculation was made using the Fall 2016 GPA, as well as the student's class and gender. Imputation does not change the results of the analysis (when students with missing records are not included in the analysis the estimated effect size is larger).

Gender: All of the students were classified as male (1) or female (0). There was no missing information for this variable, while 41% of the students were male.

2.2.3. Intervention

To determine the effect of engaging parents in non-academic activities with their children, two groups were randomly assigned as the control group and treatment group. Parents in the treatment group were encouraged via text message to participate in non-academic activities, while parents in the control group were assigned to receive only receive administrative information, e.g. test dates. A description of the design of the activities that were included in the intervention can be found below (for a list of the text messages included sent to the control group, see Appendix 2—C).

In order to foster parent-student exchanges through non-academic activities, a series of text messages (SMS) was designed using the Design Thinking approach (Brown, 2008). The Design Thinking approach has been shown to be useful when looking to solve problems creatively (Tschimmel, 2012). This approach has also been used to develop instructional material and psychological interventions (Yeager et al., 2016). This process consists of three stages: inspiration, which involves understanding the problem and its context; ideation, which is the process of synthetizing what was observed in the field and developing and testing competing ideas; and implementation, which is the development of the solution with real end-users (in this case, students, parents and teachers) (Howard & Davis, 2011). Therefore, the "final" design of the intervention was the result of an iterative process, which is described briefly in Table 2—1.

Table 2—1 Summary of the iterative process.

Phase N°: Design Thinking Scope	Participants	Phase Description
Phase 2: Ideation	Research team	As a result of discussing several ideas, we thought of involving parents through short and understandable activities (i.e. avoid complex vocabulary and formal knowledge of mathematics). We developed the first set of activities.
Phase 3: Implementation[a]	16 students (8th grade)	We tested our first set of activities in a real-life setting with parents and students, before then testing them in a classroom. Students who completed the activities with their parents showed enthusiasm and willingness to participate in class activities related to the homework. It was difficult to track whether parents were receiving the text messages as only a few of them answered our messages.
Phase 4: Ideation	Research team	We re-evaluated our proposition. We decided to include extrinsic motivators in the next implementation phase (Phase 7) in order for parents to answer our messages. This resulted in adding phone credit if they answered our messages.
Phase 5: Inspiration	4 parents	Starting as early as early childhood, there are significant differences in vocabulary based on socioeconomic status (Hart & Risley, 2003). Therefore, to avoid problems with reading, the content and phrasing of the messages was piloted with parents of students from a similar sociocultural background to those involved in the study. We learnt that even when using what we considered as simple vocabulary, parents could relate a word with a different meaning than what was intended (e.g. a "cube" was understood as a typical Chilean ice-cream instead of the geometrical figure). The understandability of the text messages should always be tested if possible.
Phase 6: Ideation	Research team	The messages were rewritten and checked by three members of the research team based on the parents' feedback.
Phase 7: Implementation[a]	44 students (9th grade)	We decided to test our new set of messages for a second time in real-life classroom conditions. However, this time we included extrinsic motivators. At the end of the intervention, some parents expressed feeling a little bit lost about why they were receiving the activities. Adding phone credit increased the response rate for those parents who do not have a phone plan. However, we realized that just because the parents answered more messages, it did not necessarily mean that they completed more activities.
Phase 8: Ideation:	Research team	Based on the results of Phase 7, we decided to include a brief introduction to the series of activities (see Appendix 2—B, Table Appendix 2—B1) and to drop the extrinsic motivators. We redefined our solution and decided to include a text message suggesting that the parents encourage their child. We decided to measure the impact of the intervention on the students' achievement.
Phase 9: Implementation		Present study intervention.

Note. [a]The design process also included informal conversations with students and parents in order to get feedback during the first two implementation phases.

The activities did not include formal curricular mathematics content, nor did they require any previous knowledge. Therefore, how these activities related to mathematics was not evident to the students or their parents. Nevertheless, the activities were aligned to the objectives defined in the national curriculum for high-school math. These simple tasks drew on aspects of the students' daily life, were written in simple language and required only basic knowledge of mathematics (e.g. counting). They were also designed to be completed in 5 to 15 minutes. In the classroom, the math teacher then indirectly connected these activities to the learning objectives by referring to the same topics (Hindin & Mueller, 2016). All students could participate in the classroom activities as they did not require the homework assignments to have been completed. Parents were encouraged to complete the activities with their child through two introductory text messages (See Table Appendix 2—B1, Appendix 2—B).

To show how the non-academic homework was connected to the learning objectives in the following class (usually on the Tuesday), we will analyze one of these activities (see Table Appendix 2—B2, Appendix 2—B), namely the "container activity". This activity required parents to talk with their children about the largest container in which they had stored water or any other liquid. In the same text message, they were also invited to discuss how much liquid they thought there was in said container. The message was as follows: "Hi, It's Miss Cami. Talk about the largest container, pot, or jar that you have ever filled with water or any other liquid. How much liquid did it hold?" Later, in class, the teacher started the session with the following question: "What's the largest container you have ever filled with any liquid?" This question was for all 10th grade students, regardless of whether or not their parents had received the text message or they had done the activity. The teacher then used the students' answers to this question in order to create exercises in which the students had to calculate the dimensions of different bodies of water (e.g., a swimming pool filled with 10,000 liters of water). Weekly activities were sent to the parents via text message over a period of five weeks. Each week, the parents received three messages:

1. Friday: The first message of the cycle described the activity that was to be completed with the student (e.g., the "container activity").

2. Monday: The second message was a reminder, asking the parents whether they had completed the task and how it had gone.

3. Wednesday: The third message included a suggestion of what the parent could say to the student to encourage them. For example, these suggestions included communicating high expectations to the child (Hattie, 2009; Hill & Tyson, 2009), or fostering a growth mindset (Dweck, 2008). In the latter case, this suggestion was made because recent studies show that a growth mindset was particularly lacking among low-income students in Chile when compared to high-income students (Claro, Paunesku & Dweck, 2016). Furthermore, a growth mindset has also been shown to be important for achievement in mathematics (Dweck, 2008). An example of such a message is the following: "During this weekend's activity, we recommend telling your child that you

are proud of her when she tries to get better at math and that you value her effort" (see Table Appendix 2—B2, Appendix 2—B). The teacher did not mention this message to the students in the classroom.

The Monday and Wednesday messages were the same for all three grades. However, the activities were different for each grade depending on the math topics covered that week. The full list of text messages that were sent to the three groups are listed in Table B2, Appendix 2—B.

2.2.4. Protocol

Within each class, parents of each student were invited to attend a parent-teacher conference, where they signed an informed consent form. The parents who did not attend the meeting were later contacted by telephone and agreed to participate in the study. None of the parents or students refused to participate in the study. After this, the students from each class were randomly assigned to the control and treatment groups (randomized within class). Table 2—2 shows that there was an even distribution between the treatment and control groups. There were no significant differences in terms of the percentage of boys assigned to each group for the three classes (8^{th}, 9^{th} and 10^{th} grade). There were also no significant differences in terms of their previous achievement in Fall 2016. This was also the case with their GPAs for 2015, 2014, and 2013, as well as their Math GPAs for 2015 and 2014. The only exception to this was achievement in math for 2013 (three years before the intervention). This was also true for the 51 students for whom there was information available on their achievement the following academic year, as reported in Table Appendix 2—A1 (Appendix 2—A).

Baseline differences were assessed using a fixed effects model with robust standard errors in order to compare the mean scores between the treatment and control groups and ensure that they were comparable (Table 2—2). Furthermore, two independent sample t-tests were also carried out (Williams, Grajales & Kurkiewicz, 2013), with the results leading to the same conclusion.

Table 2—2 Breakdown of students involved in the intervention

	Control			Treatment			Sig. diff.	
Variable	N	Mean	SD	N	Mean	SD	p[a]	Robust SE
Grade	28			28				
8th	28	.36		28	.46		0.42	0.133
9th	28	.32		28	.29		0.78	0.125
10th	28	.32		28	.25		0.56	0.123
Male	28	.32		28	.50		0.20	0.136
Previous academic achievement[b,c]								
2013 Math GPA	23	4.60	0.94	27	4.82	0.79	0.04*	0.195
2013 Overall GPA	23	5.50	0.53	27	5.60	0.55	0.25	0.176
2014 Math GPA	23	4.89	0.93	27	4.80	1.06	0.79	0.166
2014 Overall GPA	23	5.56	0.63	27	5.60	0.64	0.43	0.166
2015 Math GPA	23	4.89	0.93	27	4.80	1.06	0.98	0.161
2015 Overall GPA	23	5.56	0.63	27	5.60	0.64	0.69	0.124

Fall 2016 Math GPA	28	4.96	1.40	28	4.96	1.15	0.85	0.256

Notes. [a] The p-value is estimated using robust standard errors [b] The p-value is estimated using group fixed effects in addition to robust standard errors, i.e. using 8th, 9th and 10th grades as group dummies. * p<0.05. [c] The GPA means reported here correspond to the average using available GPAs of students from all the three grades.

This study was the first time that this particular set of messages had been tested in a real-life setting with parents and students (and subsequently in a classroom). As double-blind studies are not always the best for evaluating new treatments (Büller, Halperin, Bounameaux & Prins, 2008), the teacher was made aware of each student's study condition (i.e. control or treatment). This information allowed the teacher to detect whether the treatment had prompted any undesired behavior from the parents. For instance, one hypothetical case could have been a student telling the teacher that their parents had forced them to spend time with them when they did not want to. However, the treatment condition was not shared with the students themselves. Notwithstanding the above, the new math teacher (who taught the students during 2017, the year after the intervention) was completely unaware of whether the students had been in the control or treatment group in 2016.

Two weeks before the intervention began, two text messages were sent to the parents introducing the objective and characteristics of the activities (for the treatment group) or information (for the control group), as well as highlighting the importance of their participation (see Tables B1 and C1). From the week after this, the parents then received 3 weekly messages (treatment) or 1 weekly message (control) over a period of five weeks. At the end of the five weeks, the spring term then proceeded as usual.

2.2.5. Interviews

Interviews were used to explore the parents' perceptions of the effectiveness of the intervention and the characteristics of the parent-student exchanges that were triggered by the text messages. This approach is particularly valuable when addressing issues in complex educational contexts (Teddlie & Tashakkori, 2009). The following questions were used to guide the interview: "What was your perception of the activities and their implementation?" and "What prompted you to do the activities (or not do them)?" The aim of these questions was to understand 1) whether the text messages led to any parent-student exchanges, 2) the perception of the level of pleasure or displeasure produced by the activities, and 3) the obstacles faced when doing the activities (e.g. problems understanding the messages). These semi-structured interviews were conducted by the school teacher over the phone with twelve randomly-selected parents from the treatment group. Parents were told that the information they gave would not be shared with students, other parents or anyone else at the school. Each parent was contacted once during the course of study. The interviews lasted for approximately ten minutes and were conducted by the students' math teacher, in a friendly and informal tone. The participation by the classroom teacher as researcher in this study allowed for a wider and more integral view of the whole process (Marshall, 1996). As the teacher was part of the classroom dynamic, she had ample information on the students' behavior in class and a strong relationship with each parent. While we are aware that this relationship may have led the parents to try to please the teacher, the same relationship also has the potential for a deeper and more transparent conversation than would have taken place with an external researcher. The information held by the teacher, as well as the relationship she had with the parents, were both important factors when it came to examining the possible effects of this exploratory intervention.

To answer the research question regarding the quality of parent-student exchanges, the interview responses were analyzed qualitatively. A “concept map” was made in order to show the perceptions and actions of the subjects of the study, without developing any specific theory (Corbin & Strauss, 2014; Strauss & Corbin, 2002; Whittemore & Knafl, 2005). Doing so allowed us to answer whether well-perceived parent-student exchanges can be encouraged by the math teacher sending text messages to the parents. To study whether certain feelings, expectations or behaviors were fostered by the intervention, the parents’ responses were described textually. This includes any feelings, expectations or behaviors that can in theory constitute a relationship between two people (Sudhakar & Nellaiyapen, 2016). Following this, different descriptive categories were then generated (Peña, 2006). In order to integrate different pieces of data and give them meaning, some of the techniques listed by Miles and Huberman (1984) were adopted. The first approach was to “count” data segments containing similar information in order to get a sense of what was in the data (Morgan, 1993). This was followed by “noting patterns or themes” (by systematically searching for topics that are repeated), “seeing plausibility” (by checking whether conclusions make sense) and “clustering” (by assigning a word or phrase to represent an object, such as processes or events with similar characteristics) (Miles, Huberman & Saldana, 2014).

Quantitative analysis of the qualitative data was carried out in this way as it can help avoid bias (Gough & Scott, 2000).

2.2.6. Analytic Strategy

The following model was used to answer the research question regarding the impact of the intervention on the students' achievement in mathematics:

$$Y_{tic} = \beta_1 Treatment_{ic} + \beta_2 Y_{Fall\_2016\ ic} + GPA_{ic}\,\partial + \beta_3 Male_{ic} + \Omega_c + e_{ic} \quad (1)$$

Where Y_{tic} corresponds to the end of term Math GPA for either Spring 2016 or Fall 2017 of student i in class c (post intervention achievement), Y_{Fall_2016} corresponds to achievement in math at the start of the intervention, GPA_{ic} is a vector of previous achievement (end of year overall and math GPAs from 2013 to 2015), and Ω_c are class fixed effects. Error e_{ic} is modeled to be normally distributed with a mean of 0 and to be independent between students. The coefficient of interest is β_1, which represents the difference in achievement in mathematics between the treatment group and the control group, after controlling for previous achievement and other characteristics. Stata/SE 12.0 was used to estimate the coefficient of interest. Following the same strategy as Gershoff, Ansari, Purtell and Sexton (2016), a series of models were analyzed. Previous performance in mathematics (May 2016) was not initially included as a covariate in the first model. However, it was later included in subsequent models in order to increase the precision of the estimated effect. Following this, additional covariates were then added to the model so as to improve the **precision** when estimating the effect size. These covariates included the students' gender, as well as their overall GPA and their GPA in math for previous years. As explained in the measurements section, there was some attrition in the sample in order to analyze longer-term outcomes (Fall 2017 Math GPA). We therefore present an analysis of the effect on short-term achievement (i.e., Spring 2016 Math GPA) for both the full sample as well as the follow-up sample (Models 3 and 4). Finally, in order to confirm that the effect of the treatment was maintained in the medium term, the models were then repeated using follow-up achievement (i.e. Fall 2017 Math GPA) as the outcome (Models 5, 6 and 7). With regards to the covariates included in the models, the correlations between annual GPA and math GPA are not consistently strong for each grade (see Appendix 2—D, Table Appendix 2—D1). However, we also run two extra models considering a smaller subset of baseline controls. The first model**,** using the yearly average for the students' previous GPAs, i.e. changing from 11 covariates to 7 covariates. The second model, using a composite of the 2013-2015 covariates (completing 6 covariates). In this case, similar results were also found (see Appendix 2—D, Table Appendix 2—D2 and Table Appendix 2—D3).

Regardless of whether or not the parents or students received the treatment, all students are included in the analysis, i.e. we report intention-to-treat estimations (Little & Yau, 1996). In this sense, parents might have skipped activities if they changed phones and did not receive the message before their number was updated in the school records, or if they decided not to carry out the activity. Because it was not possible to systematically record who actually received the messages or completed the tasks, we are not able to estimate an effect exclusively for the treated families. Furthermore, as the interviews were conducted with parents before the end of the year, this may also be considered part of the treatment. We therefore explored whether students in the treatment group whose parents were interviewed had a higher GPA than the other students in the treatment group. This was done for the two periods that were assessed (Spring 2016 Math GPA and Fall 2017 Math GPA). The effect of the interviews on the outcomes was estimated as being close to zero for both periods (β =0.014, p=.96; β= -.146, p=0.63, respectively).

2.3. Results

In the following section, we first present the results in response to our first research question regarding the effect of the intervention on achievement in mathematics. We then present the qualitative results, which provide an insight into the type of parent-student exchanges that were triggered by the intervention.

2.3.1. What is the Effect on Achievement in Mathematics when Low-Income Parents are Sent Text Messages Encouraging them to Complete Short and Simple Non-Academic Math Activities with their Children?

To answer this question, the effect of the treatment on the students' GPA in math was measured at two different points in time (Table 2—3). The first four models present an analysis of short-term achievement in mathematics (i.e., the Spring 2016 math GPA). Models (5) to (7) present an analysis of the students' math GPA in Fall of the following year (6 months after the intervention).

Table 2—3 Effects of Non-Academic Tasks on Academic Achievement in Math for Spring 2016 and Fall 2017

	Model 1 No controls, only group dummies.		Model 2, controlling for previous achievement in math		Model 3, controlling for previous achievement in math and overall		Model 4, Model 3 using sample of Models 5,6,7.		Model 5, No controls, only group dummies.		Model 6, controlling for previous achievement in math		Model 7, controlling for all previous achievement	
	Effect on Math GPA, Spring 2016[a]								Effect on Math GPA, Fall 2017[b]					
	β	Robust SE	β	Robust SE	β	Robust SE	β	Robust SE	β	Robust SE	β	Robust SE	β	Robust SE
Treatment	0.416	0.257	0.388 †	0.222	0.448*	0.194	0.488*	0.206	0.339	0.255	0.415*	0.182	0.413*	0.167
N	56		56		56		51		51		51		51	
R^2	0.11		0.38		0.51		0.55		0.25		0.64		0.68	
Controls														
Fall 2016 Math GPA			X		X		X				X		X	
Male					X		X						X	
Annual Overall GPA[c] (years 2015, 2014, 2013)					X		X						X	
Annual Math GPA[c] (years 2015, 2014, 2013)					X		X						X	
Group dummies	X		X		X		X		X		X		X	

Note: †$p<0.10$; * $p<0.05$. Outcome variables and continuous covariates are standardized within sample (M=0; SD=1).
[a] Short term outcome is the grade point average between October and December, the third academic term in 2016.
[b] Long term outcome is the grade point average between March and May 2017, the first academic term in 2017.
[c] GPA at end of academic year (March and December) for 2013, 2014 and 2015, both overall and in mathematics. Imputed for 6 missing cases.

The first model in Table 2—3 shows the estimated effect of the treatment on Spring 2016 math GPA without factoring in student-level covariates (β=0.416, p=0.12). To improve precision, subsequent Models 2 and 3 controlled for student-level characteristics and previous academic achievement. The improved precision allows us to see that the effect is marginally significant ($p<0.10$) and of a similar size for both models (β=0.448, $p<0.1$). Model 4, Panel A, restricts the analysis to students who continued at the school (for whom we will analyze the long-term effects), confirming the positive and significant effect (β=0.488, $p<0.05$).

The final models in Table 2—3 reveal the estimated effects on the students' achievement in mathematics for the following academic year. The purpose of this was to analyze whether the effects persisted over time. The first model (Model 5) shows the estimated effect of the treatment without student-level covariates (β=0.339, p=0.19). To improve precision, subsequent Models 6 and 7 controlled for student-level characteristics and previous academic achievement. The improved precision presents a significant and considerably large effect (β=0.413, $p<0.05$). For the full specifications of Model 4 in each panel see Appendix 2—D, Table Appendix 2—D4.

We also ran the same models using the standardized outcomes for each grade, as opposed to the standardized outcomes within the sample. The aim of this was to explore how sensitive our results are to the standardization of the outcomes. Similar effect sizes were observed (see Appendix 2—D, Table Appendix 2—D5).

2.3.2. What Type of Parent-Student Exchanges are Fostered when a Teacher Encourages Low-Income Parents to Work with their Adolescent Children Completing Short, Non-Academic Activities Delivered via Text Message?

To answer this research question, the responses from the interviews conducted with treated parents were categorized so as to reflect their perspectives and behavior during the intervention. Three main characteristics emerged from this analysis of the interviews:

2.3.2.1. The activities led to useful parent-student exchanges

Nine parents (75%) expressed that they found the activities to be useful, stating things such as: “It was good for bringing the family together”, “It allows us to analyze (situations)”, and “It let us break our routine”. For instance, in one of the activities they were asked to think of a place near their home and to try to measure the distance between the two places (see the “nearby place activity” in Table Appendix 2—B2, Appendix 2—B). One parent mentioned that they had never talked before about how long it took her to get to work. Other comments also highlighted that the activities provided an opportunity to express their concerns to their child and to spend time with them (“I like it because it gives us an opportunity to talk”, “It lets [my child] realize that there are people there for them”), as well as the opportunity to learn more about what their child is studying at school: “The [non-academic] activities weren’t a problem for me, they helped me get to know what topics [my child] was studying at school", "[the non-academic activities] helped me know what [my child] was doing".

2.3.2.2. The parents perceived these exchanges as either positive or neutral

Nine parents (75%) said that, in general, the tasks were fun. This was evident from phrases such as: "I thought it was really fun, I read it to everyone", "How ridiculous! How fun!" For instance, one parent commented that it was fun to talk about the ages of the people who live in their house (the "relatives' ages activity" required them to talk about the age of their relatives. See Table Appendix 2—B2, Appendix 2—B). The parents also suggested that it caused laughter "I'd give it a 10, we laughed a lot". As an example, on one occasion they were asked to go with the student to a bus stop and to count how many cars passed by in five minutes (see the "bus stop activity" Table Appendix 2—B2, Appendix 2—B). One parent explained that this situation led to some laughter, because they felt people laughed when they saw them sitting at the bus stop without taking the bus. Although the majority of parents said that the activities were fun, three parents (25%) suggested that their child was indifferent, with expressions such as: "I had to be on his case to make sure he participated" and "He found it to be like any other homework assignment".

2.3.2.3. Half of the parents interviewed thought that the activities were simple and easy to complete.

Six parents (50%) felt that the activities let "everyone participate". This inclusive nature of the activity was justified from various perspectives, as expressed through phrases such as "You didn't have to think much", "They were easy", "[What the teacher asked for] was very simple, yet also educational", "We were able to do this simple task together", "I liked that [the messages] came when we were at the dinner table (lunch and dinner) because we were all together". The parents did not highlight any specific activity as being simple and easy, although they did characterize the experience as such.

2.3.2.4. Parents faced some obstacles for completing some of the activities. Five parents (42%) said that they had difficulty completing the activities. Some of the difficulties that were identified included: finding time to be with the student, due to the parent's workload and household chores (3 parents); understanding the task and a feeling of uncertainty, not knowing whether they were doing it correctly (1 parent); when the activity involved leaving the house (1 parent); and lack of interest from the child (1 parent). Parents did not refer to specific activities when expressing these difficulties, but to the activities in general.

2.4. Discussion, limitations and future work

Our study reveals that simple, non-academic activities between parents and students can have an impact on a student's performance in mathematics. The context of the study was a school in a low-income neighborhood in Chile. Students whose parents received text messages with ideas for parent-student activities improved their performance in mathematics significantly when compared to students whose parents only received text messages with administrative information. This improvement also lasted over time: the effect that was observed 3 months after the last text message was sent was 0.488 standard deviations (p=.040), while after 9 months it was 0.413

standard deviations (p=.040). According to Hattie (2009), this is slightly larger than the typical effect of initiatives that are implemented by teachers in the classroom (estimated to be 0.40 standard deviations). This typical effect size was calculated by evaluating more than 800 meta-analyses (over 50,000 small, medium and large-scale studies) studying different influences on student achievement. Despite Hattie (2009) provides a fair idea of the effect size that teacher led interventions can have, it is important to stress out that it does not classify interventions by its length, intensity or study design. Therefore, it does not allow us to compare, for instance, whether this intervention is "lighter-touch" than the interventions covered by Hattie (2009). In terms of triggering positive parent-student exchanges, the majority of parents who were interviewed confirmed that the activities led to a positive interaction with their child, which would not necessarily have existed spontaneously otherwise. These results may be explained by Rice et al.'s findings (2013), which showed that when students feel that their parents support them and encourage them to work on mathematics their academic performance improves.

This positive effect adds to the growing body of literature studying the effects of text messages on educational outcomes (Bergman, 2015; Berlinski, Busso, Dinkelman & Martinez, 2016; Bodén, 2016; Castleman & Page, 2015; Doss, Fahle, Loeb & York, 2018; Groot, Sanders, Rogers & Bloomenthal, 2017; Mayer, Kalil, Oreopoulos & Gallegos, 2015; York, Loeb & Doss, 2018). The effect on student performance is also in line with the results of encouraging a first-grade student and their parent to solve simple number problems together (Berkowitz et al., 2015, 2016). Our findings are particularly relevant given the low cost of implementing the project.

Carrying out the activities allowed the parents and students to communicate with each other, as well as to laugh and play. This is particularly relevant as the literature has shown that laughter fosters the development of positive relationships and good health among children (e.g. Raddy et al., 2002). Despite some of the children not showing particular interest in the activities, the parents expressed that the activities brought their family together and allowed them to show their affection. Generating exchanges that foster a positive parent-student relationship is important as it improves the child's commitment at school, not only in terms of behavior, but also cognitively and emotionally (Mo & Singh, 2008; Sudhakar & Nellaiyapen, 2016). Furthermore, in low-income contexts the quality of the relationship significantly explains the student's general commitment at school (Murray, 2009).

While we do not have any data to explain the mechanisms that may be driving the effects that were observed (roughly .40 SD), the treatment may impact student learning in one of two ways. The first way is through the completion of pre-class activities, i.e. students completing the activities with their parents at home. These pre-class activities enable the teacher to expand the class experience by asking the parents to share a real-world experience related to the subject matter with the student at home. Jeynes (2012) found a significant effect size (0.35) for interventions designed to help parents and teachers collaborate with each other in order to improve the children's academic or behavioral outcomes. Our results are slightly higher, which may be due to the influence of the second possible mechanism: the encouragement of parent-student communication combined with the potential transformation of the students' aspirations and/or mindset. The activities may increase the time a parent spends with the student in a playful activity, which may in turn act as a trigger for sending empowering messages, which was one of the goals of the text messages. The parents mentioned in the interviews that the activities provided them with an opportunity to meet with the student. They stated that the positive environment established by the activities opened the door to parents communicating with the student about school-related issues. This meeting may also have encouraged parents to deliver messages, such as how much they value the student's effort or promoting a growth mindset. Interventions that aim to build on the students' social and emotional development, such as setting goals or the capacity to maintain positive relationships, have an average effect size of 0.26 on achievement in mathematics (Corcoran, Cheung, Kim & Xie, 2017). In particular, previous interventions that aim to change the students' mindset in a short space of time have already estimated a significant, positive effect (d=0.1) on the end of term GPA for low-achieving students (e.g., Yeager et al., 2016). Therefore, the effect sizes we observe in this study may be a combination of the two explanations described above. Further research that includes a broader range of post-intervention measures has to be carried out in order to validate these hypotheses.

The treatment effects reported in this study are somewhat larger than those expected, based on the results of previous text messaging interventions promoting parental engagement in the learning process (ranging from 0.1 to 0.3 SD) (Bergman, 2015; Berlinski et al., 2016; Kraft & Monti-Nussbaum, 2017; York et al., 2018). However, it is important to acknowledge that this is an exploratory study and, therefore, the results cannot be generalized to other populations. In this sense, the sample may correspond to a group of students with specific characteristics or from a homogeneous population, for which the intervention was particularly beneficial. Based on the teacher's knowledge and their perception of the students in their classroom, we venture to suggest that students in this study had very low levels of previous knowledge in math, were largely demotivated with the subject and highly anxious towards it (or had negative emotions regarding mathematics). However, we did not analyze these characteristics in this study. For future work, we are studying the replication of this intervention in different schools and, therefore, with a wider variety of student experiences regarding mathematics. We also assess the qualitative factors in order to characterize the different samples.

There are some limitations to this study. Firstly, the data that is available does not allow us to identify which student or parent traits help these text messages lead to an increase in academic performance in mathematics. For example, mathematical anxiety (both from parents as well as students) has an effect on academic performance in this subject (Berkowitz et al., 2015; Casad, Hale & Wachs, 2015; Wang et al., 2015). In addition, the anxiety that a parent feels about mathematics also affects the child's own anxiety (Maloney, Ramirez, Gunderson, Levine & Beilock, 2015). Given this, it is therefore important to study the effect of anxiety and other characteristics of a child's upbringing in future interventions. In this sense, these are all elements that may moderate the influence of the treatment on the parent-student relationship and the students' achievement (Okagaki & Luster, 2005; Suldo, 2009). Future research should therefore take into consideration various contexts and a large enough sample so as to ensure the representativeness of different parenting styles

(López, Valenzuela, Nussbaum & Tsai, 2015). Furthermore, given that the activities were not designed to be funny, the laughter described by the parents may be attributed to the fact that the activities get them out of their routine or, perhaps, to the joy that comes from getting involved in mathematics in a simple way. Confirming the origin of this behavior is important as enjoying while studying mathematics has a positive impact on the students' interest and performance in said subject (Schukajlow & Krug, 2014; Schukajlow & Rakoczy, 2016). It is not clear whether the activities would become routine if the intervention were to be extended, thus losing the sense of joy. We therefore recommend studying the origin of this behavior and its sustainability in greater depth for future interventions.

Secondly, the design of the study does not allow us to distinguish whether the effect of the treatment is due to spending time with a parent or to completing the activity outside of class, regardless of whoever is encouraging the activities. For example, it is possible that similar effects may be found by nudging an after-school mentor to encourage students to complete the activities, or some other adult who is valued by the student. While we cannot discount this hypothesis, we can only highlight that our results represent the effect of nudging parents to complete activities.

Thirdly, the current sample size has two limitations. Firstly, it does not allow for subgroup analysis, while secondly it makes it difficult to generalize the results to other populations. However, one of the strengths of this setup was that one teacher was responsible for all of the events. We could therefore be sure that the teacher's treatment would be the same in each classroom. Therefore, despite a small sample size, the evidence collected in this study provides us with a fair insight and suggests that it is worth replicating this study with a larger sample. Implementing a larger scale randomized controlled trial should test whether the intervention would be effective in different low-income contexts and with different teachers so as to be able to generalize the results with greater confidence. A larger-scale randomized controlled trial would also allow researchers to test the effect of different treatment arms and therefore isolate the effect of the activities themselves (as opposed to the effect of the teacher's script suggestions about growth mindset and valuing the student's effort).

Additionally, since the teacher was aware of each student's experimental condition, we cannot reject the existence of potential assessment bias or other unconscious differences that may come from the teacher-student relationship. However, this issue is addressed by including the long-term analysis, where the dependent variable was provided by a teacher who was not aware of the experiment (e.g. grades from Spring 2017). Now that we have initial evidence of potential benefits to students, we suggest running a larger scale, double-blind study of the intervention. As explained in the method section, we decided that the teacher should conduct the interviews. However, this had a limitation of possible bias, given the parents' potential desire to please the teacher. Therefore, we also suggest including non-school related interviewers for future experimental designs. Finally, because we conducted interviews with parents from the treatment group during the intervention and before the students were assessed, we cannot determine whether the effect that was observed was influenced in any way by these interviews. In this sense, it is worth noting that only half of the parents from the treatment group were interviewed.

2.5. Conclusion

In response to the first research question: “What type of parent-student exchanges are fostered when a teacher encourage low-income parents to work with their adolescent children on short, non-academic activities delivered via text message?”, the qualitative study suggests that the messages can lead to inclusive exchanges. Such exchanges not only take place between the parent and the student, but can also include the wider family. In this sense, a parent who reads the activity over dinner may include siblings or other family members in the conversation.

Our second research question asked: “What effect does this intervention have on the students’ academic performance in mathematics?”. We showed that performance in mathematics improved significantly for those who received non-academic activities, compared with those who only received administrative updates.

This study contributes to the growing body of literature suggesting that low-touch, text message-enabled interventions can have a meaningful impact on parental engagement and student learning. In our particular case, the study focused on low-income Chilean students from highly disadvantaged backgrounds. Given the small sample size, ours is merely an exploratory study. However, when considered in conjunction with other studies on this topic, it lends support to the idea that simple forms of ICT can play a key role in the family-student-teacher ecosystem.

3. LET'S SPEND TIME TOGETHER: TEXT MESSAGING PARENTS IMPROVED PERFORMANCE OF HIGH ANXIOUS STUDENTS

3.1. Introduction

Parental involvement in teenagers' academic lives can improve students' opinion of their own abilities and their attitudes towards learning (Rice et al., 2013). Studies have shown that adolescent students' academic performance improves when their parents communicate with them about their schoolwork (Affuso, Bacchini & Miranda, 2017). Students whose parents monitor their performance in school— even a little— demonstrate higher academic achievement than their unsupervised peers (Coley & Hoffman, 1996; Fulton & Turner, 2008; Hill & Wang, 2015).

However, involving parents and guardians (hereafter referred to as "parents") in teenagers' school lives has several challenges. One of the most common obstacles to parental participation in school activities is the parents' time constraints; many parents have inflexible work schedules and find school meeting times inconvenient or impossible to accommodate (Crouter & Booth, 2014; Gurria, 2016; Tubbs, Roy, & Burton, 2005; Turney & Kao, 2009). Even with available time, however, parents may have difficulty communicating with teens, as many parents lack the skills required for positive and productive interactions with their children. For example, some parents believe that they lack the technical knowledge required to help their children with math (Cannon & Ginsburg, 2008; Dauber & Epstein, 1993), and many parents do indeed lack the knowledge and pedagogical know-how to help their child with high-school-level math (Cabus & Ariës, 2017; Dumont, Trautwein & Nagy, 2012; Wilder, 2014). Interactions between teenagers and their parents may be particularly difficult for parents and counter-productive for students if they concern topics for which the parents are already ill at ease.

Given the academic benefits of parental involvement, teachers might improve their students' performance by prompting parents to spend more time engaging with their children's academic lives. For example, math teachers could design homework for adolescent students to complete with their parents, which gives both children and parents the opportunity to communicate about what the student is learning in math class (Balli, Demo & Wedman, 1998; Epstein & Van Voorhis, 2001). However, in keeping with the challenges that parents face when interacting with the teenagers, increasing parents' participation in homework does not necessarily yield gains in student achievement (Ariës & Cabus, 2015; Cabus & Ariës, 2017; Castro et al., 2015; Hanna Dumont et al., 2012; Pinquart, 2015; Pomerantz, Moorman & Litwack, 2007). Several studies have shown that parents can actually cause harm to students by interfering with their schoolwork if they, the parents, are ill equipped to do so. For example, parents who are less prepared to help their children with academic content may be more likely to use negative reinforcement to convey the importance of education (Mellon & Moutavelis, 2011). Parents who feel anxious about their own proficiency in mathematics have been shown to negatively influence students' attitudes towards the subject (Maloney, Ramirez, Gunderson, Levine & Beilock, 2015; Vukovic et al., 2013).

One mechanism by which parent involvement, particularly in math, could negatively impact students is through increasing student anxiety. “Math anxiety” refers to adverse emotional reactions to math or the prospect of doing math (Maloney & Beilock, 2012). Students’ math anxiety tends to increase dramatically throughout middle and high school (Suárez-Pellicioni, Núñez-Peña & Colomé, 2016). The complexity of the subject often leads students and their parents to perceive math as difficult and, consequently, to avoid it (Ashcraft & Krause, 2007; Ashcraft & Moore, 2009; Dowker, Sarkar & Looi, 2016). Parents who have anxiety about math but nevertheless help their child with mathematics homework sometimes transmit their personal fear of the subject to their children, making the child in turn more nervous when he or she faces math tasks (Vukovic et al., 2013). Parental involvement in math homework under such circumstances is associated with diminished gains in academic achievement as compared to students whose parents were less anxious about math (Maloney, Ramirez, Gunderson, Levine & Beilock, 2015). Thus, initiatives promoting parental involvement in students’ math education may backfire if they assume parents understand the content covered in math classes (Vukovic, Roberts & Green Wright, 2013).

While encouraging parent participation, particularly in high school math, may not be beneficial in all cases, the overall benefit of parent participation suggests that well-targeted encouragement may be useful. In particular, interactions that relate to the math curriculum but do not require parents to have particular skills may reap the gains of interaction without the drawback of parent stress and increased student anxiety. For example, a PISA report showed that having personal experience with bank accounts was positively correlated with students' performance on the financial literacy assessment (Gurria, 2016), providing evidence that non-technical but related experiences can improve students' technical performance. Parent-child assignments can take myriad forms. In a supermarket, for example, a parent might ask his or her child to think about the number of sandwich types that could be made with a certain set of ingredients, thereby prompting the child to learn about combinatorics in everyday life (Hindin & Mueller, 2016). When a student completes non-academic activities with a parent or other adult who is close to them, that student may become more invested in classroom content. Students' learning is made meaningful "when the abstract is made concrete and personal" (Brown, Roediger & McDaniel, 2014).

To the best of our knowledge, only three programs have evaluated the academic effects of engaging parents and adolescent children in activities that do not presuppose that the parent possesses special skills or knowledge of class content. The first program, Teachers Involve Parents in Schoolwork (TIPS), provided middle-school students with activities that involved the students' parents. For example, students were assigned to work through a math problem and then explain it to their parents; for their science classes, students were asked to act like a scientist by writing up a report and discussing their results and the implications of those results with an adult; and for their language classes, students were asked to read their papers aloud to their parents and listen for their reactions. TIPS increased parental involvement in homework (Balli, Demo & Wedman, 1998; J. Epstein, Simon & Salinas, 1997; Van Voorhis, 2003). Parents reported that they had fun completing the activities with their children and that the assignments opened up opportunities to discuss school with the students (Van Voorhis, 2001). However, TIPS did not have a significant effect on students' performance in math (Balli, Demo & Wedman, 1998).

The second study assessed the impact of a five-week text messaging program that nudged parents to complete short non-academic activities with their adolescent children (Author, 2018). These activities were non-academic insofar as they did not require parents to have any particular skills or content proficiency. The texts also sometimes suggested that the parents encourage their children to invest effort in the school subject at hand. As was the case with TIPS, participating adults reported enjoying themselves while completing the activities. Unlike in TIPS, however, the study found a positive treatment effect on students' GPA in math (~.40 SD). This result, though exciting, is difficult to generalize because the treatment group consisted of only 51 students. Moreover, the study did not isolate potential mechanisms underlying the observable improvement in math performance, nor did the study evaluate whether the effects of the treatment on students' achievement varied depending on students' pre-existing traits, such as their level of math anxiety.

The third program also nudged parents and students to engage in conversations around schoolwork, although it did not involve shared parent-child home activities. Kraft and Rogers (2015) asked teachers to send information to the parents of adolescent students about what each student could improve or what they were doing well, once a week during a summer school program. The treatment group proved more likely to pass the program and less likely to drop out. The researchers concluded that the treatment group's improved performance was attributable to the positive effects of parent-teacher communication. While there is a growing literature on the benefits of using text messages to communicate with parents (e.g., Berkowitz et al., 2015; Doss, Fahle, Loeb & York, 2018; Mayer, Kalil, Oreopoulos & Gallegos, 2015; York, Loeb & Doss, 2018), only the three studies mentioned above have explored the effects on adolescent students in particular.

While each of these studies assess the effects of non-technical parent involvement, they do not focus on mechanisms by which the programs benefit students. In particular, they do not address how these programs affect students' math anxiety or are differentially effective for students with greater or lesser math anxiety. Studies have found that math anxiety affects the degree to which parental involvement impacts young students' performance in math (Berkowitz et al., 2015; Vukovic et al., 2013). Dowker et al. (2016) suggest that a potential treatment for students' math anxiety is to change how adults in their lives frame the subject; parents should demonstrate positive attitudes towards mathematics and avoid negative ones when communicating with their children. Math activities that are fun for parents and children alike could therefore help to diminish students' math anxiety by weakening the association between math and difficulty.

This study designs and explores the effects of a text-messaging program that promotes parent-student interactions around mathematics without requiring parents to have prior knowledge of mathematical content and with a particular focus on math anxiety. Using a randomized control trial design, we sent text messages to treatment-group parents on behalf of their children's high-school math teacher inviting them to engage in non-academic, math-focused activities with their teenagers. We also randomized complementary suggestions concerning how the parent might encourage students' academic efforts. These exchanges offer opportunities for parents to communicate that math is valuable and that they have high expectations for their children's performance in the subject—both sentiments that correlate with students' achievement (Hill & Tyson, 2009; Jeynes, 2005). The present study aims to answer the following research questions:

- Do text messages encouraging parental involvement in non-academic schoolwork affect students' performance in math?
- Does the impact of this intervention vary depending on the student's initial level of math anxiety?
- Can the intervention support students with high math anxiety by reducing their anxiety?

3.2. Methods

3.2.1. Experimental design

3.2.1.1. Intervention. The treatment was a 12-week text-messaging program for parents of adolescents in low-income schools. We designed this treatment in hopes of encouraging parents to engage with their children at home around math. Each week, the parents received an activity assignment that had no obvious relationship to math. However, the students' math instructors later drew on the home activity in the classroom. For example, in the "Soccer Players" activity (Appendix 3—A), parents were asked to discuss with their child who their child's favorite soccer player was. Besides providing an excuse for parents and students to spend time together, this non-academic homework later supported a math lesson in factoring in which students were tasked with identifying what factors (players) are shared in common by different algebraic terms (teams).

Each activity was built around a learning objective drawn from the math curriculum, but each was designed in such a way that participants would not need knowledge of math in order to complete them. Table Appendix 3—A1 and Table Appendix 3—A2 in Appendix 3—A list the learning objectives attached to each activity. The at-home activities were not compulsory and were not graded. They did not involve correct (or incorrect) answers. The activities were intended to be playful, and were designed to be completed by the student and their parent together over a short period of time (15 minutes). The activities were simple enough to be communicated via SMS (each assignment prompt was less than 160 characters) and most of the texts included a personal greeting from the teacher. To find the complete list of activities sent by each participating teacher, see Table A1 (9^{th} grade) and Table A2 (10^{th} grade) in Appendix 3—A.

On Fridays at 8:00pm, treatment parents received an SMS message with activity instructions. On Saturday at noon, they received a second SMS message encouraging them to complete the assignment. This second message had three aims: 1) to alleviate academic pressures (e.g., it reminded parents that the assignment was not obligatory, had no right answers, and was not going to be graded), 2) to motivate parents to complete the activities (e.g., by reminding parents that the assignment would support the student's learning process), and 3) to encourage the parents to enjoy the opportunity to bond with their children (e.g., explicitly indicating that “the only requirement of the activity is that you share a nice moment together”).

Parents received assignments that dovetailed with what would be taught in class the following week. Each Wednesday, researchers would ask the teachers to list their learning objectives for the class in the coming week and to note any upcoming tests or additional homework. If teachers were unsure of the content they would cover in the coming week, they were contacted again on the following days (Thursday and Friday) so their lesson plans could solidify. Once the teacher provided the requisite information, the research team created a corresponding home activity for parents to complete with their children.

The class materials developed by the researchers were sent over e-mail and in paper form to each teacher on Monday so the teachers could incorporate the materials into lesson plans throughout the week as they saw fit. We helped the teachers connect the parent-child homework activity to the in-class learning goal by offering the teachers two one-page handouts, one for personal pedagogical use and one to distribute to students. Examples of these handouts can be found in Figures Appendix 3—B1 and Appendix 3—B2 in Appendix 3—B. Teachers were especially encouraged to make use of the student handout (Figure Appendix 3—B2, section 9.3.4, Appendix 3—B); researchers reminded teachers that otherwise the parent-child work at home might seem useless to the participating families. At the end of the trial the teachers were asked how many of the handouts they distributed to their class; on average, they reported using them in 9.7 of the 12 activities (SD=2.3).

The handouts were designed so each student could complete them, regardless of whether he or she belonged to the treatment or control group. However, these handouts drew on examples that recalled the parent-child activity completed the weekend before. Thus, the home activity was connected to in-class work, even as students from both experimental condition received identical instruction (every student received the class handout). This study design ensured that any difference in performance outcomes between experimental groups would be attributable to the students' supplementary experiences at home and not to variability in classroom instruction. Treatment students may have experienced the classroom instruction differently because of their association between the class content and time with their parents, but the math instruction itself was the same across the board. The treatment effect measures the effects of parent-child communication around math schoolwork, not the everyday contextualization of mathematics, which was built into the lesson plans delivered to treatment and control groups alike.

We divided the treatment group into two tracks, an Activities Track and a Script Track. These tracks differed only in the content of some of the messages sent to treatment parents on Saturdays. The Saturday SMS sent to the Activities Track parents included further explanations of how the activity was going to be used in class (although the SMS never went into any mathematical detail) (see Appendix 3—A, Table Appendix 3—A3 (column A)). The Saturday SMS's sent to the Script Track, by contrast, included suggestions that the parents encourage students' efforts in school and help to foster growth mindset (see Appendix 3—A, Table Appendix 3—A3, column B). For a comparison between the Activities and Script Track messages sent on Saturdays, see Appendix 3—A, Table Appendix 3—A3, column C. We piloted a version of the Script Track with 56 students and their parents in a five-week trial; the treatment group of students in that study demonstrated a significant improvement in their quarterly math GPA (~.40 SD) (Author, 2018).

Parents of students in the control group received text reminders of tests or additional assignments due in the coming week. These reminders were sent on Fridays at 8:00pm. A control text might read, for example, "Hi, next Wednesday there is a math test." A complete list of messages sent to parents in the control group appears in Appendix 3—A, Table Appendix 3—A4.

Students in each participating class were randomly assigned to one of the two conditions (control or treatment); each treatment student was randomly assigned to the Activities or Script Track. Teachers were not informed of the experimental condition of each student. While the treatment group received 12 messages over the course of the 12-week study, the control group received an average of 2.48 messages (SD=2.12).

3.2.2. Data and Sample

Math teachers from Chilean high schools were recruited for this study through two channels. The first was an open call for participants sent by email to teachers who belonged to an NGO. This approach successfully recruited nine teachers, each from different schools, who were teaching 9^{th-} and/or 10th-grade classes in 2017. We contacted the principals overseeing the interested teachers, and all of them agreed to allow their teachers and families to participate. The second recruitment channel was a direct invitation to colleagues of the already recruited teachers. Three teachers from two schools responded to this invitation and registered to participate. All eight treatment schools served low-income populations; at least 50% of the schools' families fell below Chile's poverty line (i.e., income was insufficient to meet basic family needs) (Ministry of Education, 2018).

We invited the parents of the 623 students in the 12 participating teachers' classes to participate in the study through school meetings held in March and April of 2017. 422 of the 623 sets of parents or guardians (67.5%) agreed to participate in the study and were randomized in treatment and control conditions. When enrollment concluded, the 12 teachers were randomly assigned to one of three start dates for the trial. In Chile, the academic year usually runs from the end of February to the beginning of December. The first group of teachers and their student-parent participants received treatment texts and materials from early April to early June 2017. The second group received treatment from late April to mid-July 2017. The third received treatment from early May to mid-August 2017. The research team elected to stagger implementation because this was the first time the intervention and the text-messaging platform were being implemented at a large scale. Rolling out the intervention in stages allowed the research team to detect and address logistical problems early. The second and third groups of participants experienced a two-week interruption in the text treatment due to winter break.

Table 3—1 provides statistics describing each of the analyzed sample groups (columns 2 to 4), which are smaller in number than the randomization sample groups (column 1) because our research team chose not to include all the participants' results in our final analysis (see below Balance check and attrition analysis section). In order to answer to our first research question, we analyzed two sample groups. Sample A is comprised of students whose post-intervention Mathematics GPAs were available. Sample B is comprised of students whose post-intervention Language GPAs were available (see columns 2 and 3, Table 3—1). In order to answer our second research question, we further restricted each of these samples to those students who had completed the pre-intervention survey and, therefore, had a value on pre-intervention math anxiety (n=223 for mathematics GPA and n=204 for language GPA, respectively). In addition to addressing our initial research questions, we explored the effects of the treatment on reported math anxiety using data from the post-intervention survey. The pertinent sample group for this question, Sample C, demonstrated post-intervention math anxiety (column 4, Table 3—1).

The randomized sample consisted of 422 students who belonged to 17 different 9th- and 10th-grade classrooms (n=187 and n=235, respectively) during the 2017 academic year. As presented in Table 1, the average age of these students and their parents was 14.9 and 42.9 years, respectively, at the beginning of the intervention. Students also reported living on average with 4.72 people at home and being sometimes anxious about math (see below, Measures section). As shown in Table 3—1, the characteristics of each of the analytic samples is similar in composition to the randomization sample.

Table 3—1 Sample Summary Statistics - Means and Standard Deviations

	Randomized Sample	Sample A: Math GPA Available	Sample B: Language GPA Available	Sample C: Math Anxiety Reporting Available
	Panel A: Children Characteristics			
Administrative Data				
Age (in years)[a,e]	14.94	14.84	14.79	14.81
	(0.95)	(0.98)	(0.97)	(0.98)
Missing age (%)	13	2.8	1.5	2.4
Female (%)	43	45	45	46
In 9th grade (%)	44	49	52	49
In 10th grade (%)	56	51	48	51
2016 Overall GPA [a,b,f]	5.34	5.33	5.34	5.33
	(0.60)	(0.61)	(0.61)	(0.62)
Missing 2016 Overall GPA (%)	1.4	1.1	0.9	1.0
2016 Mathematics GPA[,b,f]	5.07	5.06	5.05	5.07
	(0.86)	(0.85)	(0.85)	(0.87)
Missing 2016 Mathematics GPA (%)	1.7	1.4	1.2	1.4
Pre-Intervention Student Survey Data				
Number of people living at home	4.72	4.70	4.69	4.84
	(1.81)	(1.80)	(1.77)	(1.75)
Math Anxiety (MA)[c,e]	2.97	2.95	2.97	2.91
	(0.72)	(0.77)	(0.75)	(0.74)
Math Motivation				
Intrinsic Motivation (IM)[c,e]	3.65	3.65	3.66	3.66
	(0.76)	(0.80)	(0.76)	(0.78)
Self Efficacy (SE)[c,e]	3.42	3.43	3.44	3.44
	(0.70)	(0.76)	(0.72)	(0.73)
Self-Determination (SD)[c,e]	3.33	3.34	3.36	3.35
	(0.58)	(0.61)	(0.59)	(0.61)
Parental Motivation (PM)[c,e]	3.68	3.68	3.70	3.66
	(0.73)	(0.77)	(0.73)	(0.79)
Personal Relevance (PR)[c,e]	3.74	3.73	3.75	3.75

	(0.66)	(0.70)	(0.66)	(0.69)
Grade Motivation (GM)[c,e]	4.33	4.33	4.37	4.33
	(0.52)	(0.56)	(0.51)	(0.55)
Career Motivation (CM)[c,e]	4.10	4.10	4.13	4.11
	(0.66)	(0.70)	(0.66)	(0.69)
Missing Student Survey (%)	43	37	38	38
	Panel B: Parental Characteristics			
Pre-Intervention Parents Survev Data				
Age (in years)[c]	42.90	42.85	42.90	42.98
	(5.48)	(5.28)	(5.48)	(5.17)
Missing age	0.58	0.61	0.58	0.61
Female (%)[c]	84	86	87	85
Missing Gender (%)	54	56	54	57
Relationship to child[d]				
Respondent is Mother (%)	82	82	85	82
Respondent is Father (%)	13	12	11	12
Respondent is Other (%)	4.6	4.6	4.0	5.7
Unknown relationship to child (%)	54	57	54	58
Parenting Style[c]				
Authoritative Score[e]	4.33	4.33	4.33	4.33
	(0.30)	(0.30)	(0.30)	(0.30)
Authoritarian Score[e]	2.20	2.20	2.20	2.20
	(0.37)	(0.36)	(0.37)	(0.35)
Missing Parent Survey (%)	51	53	51	54
Observations (N)	422	352	328	290

Notes: Numbers in parentheses are standard deviations for continuous variables. A. Missing data values imputed to be the classroom mean. b Chilean school year typically runs from early March to middle December. c Missing data values imputed to be the grade level mean. d Missing data coded as zero e. Likert scale ranging from 1 (never) to 5 (always). f. Chilean GPAs range from 1 to 7. e. Students' age was calculated using their birthdays as documented in school records; in cases in which their birthday was unregistered, researchers relied on self-reported age.

3.2.3. Measures

Students' academic performance and students' mathematics anxiety were measured to explore the effects of the text-messages intervention. With the aim of improving power of calculation, we added some extra measures of students' characteristics that are related to their academic performance. Descriptive statistics for these pre-intervention measures were provided in Table 3—1.

(a) **Students academic performance.** We examined two academic outcomes: mathematics GPA 2017 and language GPA 2017. The treatment condition was not expected to have an effect on language GPA, so we included it as a reference outcome. These two measures were modelled as continuous variables as Chilean students can be graded by the teacher from 2 to 7, with a 1-point decimal accuracy (37 different values). For a graph of the distribution of the two raw outcomes see Figure Appendix 3—D1 and Appendix 3—D2, Appendix 3—D. As baseline measures, we included mathematics GPA 2016 and overall GPA 2016.

(b) Math anxiety. To examine how students' level of math anxiety influence the treatment effect on their learning outcomes, we extracted six questions from the Spanish version of the short Mathematics Anxiety Rating Scale (sMARS) (Núñez-Peña, Suárez-Pellicioni, Guilera, & Mercadé-Carranza, 2013) that capture the anxiety provoked by different situations related mathematics and the school, shown in Appendix 3—E. We surveyed the students before their parents received the first text message (α=0.80) and two weeks after the last message (α=0.81). This score considered three different constructs of anxiety: (a) math test anxiety (3 items, e.g. "I'm nervous about how I'm going to do math tests") (b) numerical task anxiety (2 items, e.g. "It makes me nervous or anxious to have to solve a series of additions or subtractions") and (c) course anxiety (1 item. "I get nervous or anxious when entering a math class"). We averaged the 6 to get an overall anxiety measure.

(d) Quality of family relationships. Following the strategy used by Meier & Musick (2014), we examined two highly reliable measures of quality of family relationships that relates to adolescent well-being: (a) parent–child relationship quality (father/mother) (α_{mother}=0.90 and α_{father}=0.95, 5 items) that captures several dimensions of their relationship such as closeness, warmth and communication, and (b) global family relationship quality (α=0.85, 3 items) that indicates to what extent the student feels her family understand her, pays attention to her and whether they have fun together. We additionally measured in what extent the student spent time with each of their parents, because it was not included in the previous two constructs (α_{mother}=0.82 and α_{father}=0.92, 3 items) but has been previously conceptualized as part of the parent-adolescent relationship (Furstenberg et al., 1999). Items translated into English are available in Appendix 3—E.

(e) Motivation questionnaire. With the aim of controlling for students' mathematics motivational traits, we adapted the Science Motivation Questionnaire II (Glynn, Brickman, Armstrong, & Taasoobshirazi, 2011; Glynn, Taasoobshirazi, & Brickman, 2009). The students were surveyed about five components of their motivation to learn mathematics at school: (a) Self-Efficacy (α=0.88, 4 items) (b) Self-Determination (α=0.74, 5 items), (c) Intrinsic Motivation (α=0.84, 4 items), (d) Career Motivation (α=0.82, 4 items) and (e) Grade Motivation (α=0.68, 2 items). We also developed a scale to measure in what extent students' motivation to learn and to perform well at math is related with their family (α=0.83, 4 items). To see the items and criteria used to include items in this instrument see Appendix 3—E.

(h) Parenting styles. With the aim of controlling for parents' perceived parenting style, the Spanish version of the Parenting Styles and Dimensions Questionnaire (SPSDQ; Velásquez & Villouta, 2013; PSDQ; Robinson, Mandleco, Olsen, & Hart, 2001) was completed by parents. We used as control covariates the authoritative pattern (α=0.86, 27 items) and the authoritarian pattern (α=0.81, 20 items). The original items are available in English in Appendix 3—E.

3.2.4 Analytical Strategy

Estimating treatment effects. To answer our first research question regarding the impact of the intervention on students' math performance, we estimate a set of models using the following model specification:

$$y_{icst} = \alpha + \beta_1 \, T_{ics}^{t} \; + \delta X_{ics} + \theta \, Z_{ics}^{(t-1)} + \gamma_c + \varepsilon_{ics} \; (1)$$

where y_{icst} is the outcome of interest of student i in classroom c at school s, at the end of the year t. The student outcomes are the final mathematics GPA and language GPA, standardized to have standard deviation one and mean zero. T_{ics}^{t}, X_{ics} , γ_c and ε_{ics} represent the treatment assignment, baseline covariates, classroom fixed effects, and a student-level error-term. $Z_{ics}^{(t-1)}$ is a vector that includes student achievement the previous year $(t-1)$, both overall GPA and mathematics GPA.

We begin by estimating the effect of being assigned to the treatment. In the second model, we include classroom fixed effects because the randomization was blocked by classroom. The third model accounts for the initial imbalances found in the randomization checks. For the fourth model, we add extra controls to improve the precision of the treatment estimate calculation.

To explore the effects of each treatment track, we use a similar approach, first regressing students' achievement on being assigned to each of the treatment arms (model 5) and then adding the additional controls. The specification for the final model is the following:

$$y_{icst} = \alpha + \beta_1 \cdot AT_{ics} + \beta_2 \cdot ST_{ics} + \delta \cdot X_{ics} + \theta \cdot Z_{ics(t-1)} + \gamma_c + \varepsilon_{ics} \quad (2)$$

the coefficients of interest are β_1 and β_2, which represent the effect of being part of the Activities Treatment track and the Script Treatment track, respectively, compared to the control group. AT_{ics} and ST_{ics} are binary variables representing which of these two treatment arms the student were assigned to. The control group is the omitted reference group.

Estimating heterogeneous main treatment effects. To answer our second research question, we explore treatment effect heterogeneity based on the initial level of the students' math anxiety. With this purpose, we split the sample in two groups and run the same set of models described above. One of the groups represented "high anxious students," students whose math anxiety levels prior to the intervention were over the mean. The other group "low anxious students" included the students with math anxiety levels below the mean. Splitting the sample allows the effects of predictors to vary for each group. Models using interactions produce similar results (see Table Appendix 3—C4, Appendix 3—C).

Math anxiety effects. To better understand mechanisms, we also assess the effect of the intervention on math anxiety using the same set of models. We run this for the full sample and split by students based on their initial math anxiety.

3.2.4. Balance Check and Attrition Analysis

Some students left the analytical samples for several reasons. One of the teachers of the second group decided to leave the study at the beginning of the intervention, as a result, two classrooms left the study after the second week and no outcomes were reported (46 students of the 422 students, 11%). This attrition is independent of the treatment, since all control and treated students of the aforementioned classrooms were taken out of the analytical samples. Additionally, twenty-four students (5.7%) left their corresponding school during the school year (11 treatment and 13 control). One of the schools did not provide information on students' language achievement i.e. the language sample was reduced by a whole class of 24 students (13 treatment and 11 control). The math anxiety sample lost 23% of students because they were absent on the day when teachers applied the post intervention survey (48 treatment and 38 control). Additionally, three (0.7%) families opted out from receiving text messages (two control and one treatment), but they were not taken out from the analytical samples.

To check the assumption that we could expect equal outcomes for the treatment and for the control group, we examined the equivalence of baseline measures for the randomization sample and the analytic samples. We tested 17 covariates for each sample and Table Appendix 3—C2 in Appendix 3—C shows the results. While most of the variables were balanced between treatment and control for each sample, the students belonging to treatment group were slightly older and live with a fewer number of people at their home compared to students in the control group (Samples A, B and C). We also evaluated imbalance across treatment arms and the control group (see Table Appendix 3—C3, Appendix 3—C). When compared to the control group, students in the Activities group were older in samples A and B, and lived with a fewer number of people in samples B and C. Students in the Script group lived with fewer people compared to control group in sample C. All these baseline covariates were listed as controls (see below, Analytical Strategy section).

Whether the treatment had an effect on attrition of the sample, could lead to bias in the results interpretation. Table 2 shows the results of regressing binary variables that indicate the availability of the outcomes for each student on the treatment status, controlling by classroom fixed effects. The availability of none of the outcomes resulted significantly affected by the treatment (see coefficients of the treatment status in Table 2), i.e. evidence suggest that students from the control group did not attrite differently to students in the treatment group, from the randomization sample. Additionally, initial characteristics combined with treatment may have an effect on the attrition of the sample. For example, imagine high math anxious students hat completed the activities were less likely to answer the questionnaire. The treatment group would reflect a decrease in their math anxiety. Therefore, we evaluate whether any of the measured characteristics of students together with the treatment had an effect on the attrition of the sample (compared to the control group). To this aim we conducted regressions that included treatment interacted with the baseline covariate and controlled by class fixed effects. In Appendix 3—C, Table

Appendix 3—C1, shows that younger students and students with a higher number of people living at their home were more likely to attrite from the randomization sample.

Table 3—2 Treatment Effects on Study Attrition

Panel A: Students - Whether the Student Outcome is Missing

Measure	*Treatment*	Adj. R^2
Mathematics GPA 2017	-0.011	0.040
	(0.025)	
Language GPA 2017	-0.000	0.446
	(0.026)	
	(0.023)	
Student Survey (Post-Intervention)	0.046	0.059
	(0.049)	
N=422		

Notes: Each row represents a separate regression model (only the coefficients of the treatments status are reported). All regressions include classroom fixed effects. All analyses are based on the randomization sample. Statistical significance levels: *p<0.05.

3.3. Results

We do not find an overall effect of the treatment or the treatment arms on student GPA in either math or language. Table 3—3 presents the overall treatment effect on the students' math and language GPA, measured at the end of the academic year. None of the five proposed models reveal a significant average effect of the treatment on students' math (model 4, $\beta=0.09$, $p=0.324$) or language GPA (model 4, $\beta=0.02$, $p=0.792$).

Table 3—3 Treatment Effects on Standardized Students' Outcomes

	Treatment Effect Estimates			
	Model 1	Model 2	Model 3	Model 4
Panel A: Students Math Performance				
GPA 2017 mathematics	0.023	0.044	0.115	0.086
	(0.11)	(0.10)	(0.10)	(0.09)
N=	352	352	352	352
Panel B: Students Academic Performance	0.060	0.047	0.092	0.022
GPA 2017 language	(0.11)	(0.10)	(0.10)	(0.08)
N=	328	328	328	328
Model Inclusions				
Classroom Fixed Effects		X	X	X
Imbalance pre-treatment covariates[a]			X	X
Additional pre-treatment covariates[b]				X

The omitted reference group in all regressions is the control group. Standard errors in parentheses. [a]Included students age, missing dummies indicating whether there was a missing on pre-intervention mathematics or language GPA (2016) or for pre-survey family quality relation scores, number of people living in the student's home. [b]Included gender and pre-intervention mathematics or language GPA (2016). It also included pre-intervention measures for students' math traits, such as: anxiety, self-efficacy, self-determination, intrinsic motivation, personal relevance, parental motivation, grade motivation and career motivation and parenting styles measures: authoritative and authoritarian. Dummy variables were included to indicate missingness in parents’ survey or students survey.

** p<0.01, * p<0.05, † p<0.1

Table 3—4 presents the effects for the two treatment tracks: Activities and Script. Neither of the two showed consistent significant effects on any of the students' learning outcomes. However, the magnitude of the revealed effects was marginally different. On the one hand, the magnitude of the effect of the Activities track was consistently close to zero (model 8, $\beta=0.027$, $p=0.802$), with the direction of the effect on math varying varies from negative to positive when including extra covariates to the models, i.e. when adding precision to the estimates. On the other hand, the magnitude of the effect of the Script track on math GPA stays positive across the models (~0.15 standard deviations; model 8, $\beta=0.127$, $p=0.239$), while the magnitude of its effect on language GPA moves closer to zero as the models increase their precision (model 8, $\beta=-0.004$, $p=0.966$).

Table 3—4 Treatment Arms Effects on Standardized Students' Outcomes

	Treatment Tracks Effect Estimates							
	Model 5		Model 6		Model 7		Model 8	
	Activities Track	Script Track	Activities Track	Script Track	Activities Track	Script Track	Activities Track	Script Track
Panel A: Students Math Performance[a]								
Std. GPA 2017 mathematics	-0.081	0.089	-0.067	0.127	0.015	0.183	0.027	0.127
	(0.13)	(0.13)	(0.13)	(0.13)	(0.13)	(0.13)	(0.11)	(0.11)
N=	352		352		352		352	
Panel B: Students Spanish Performance[b]								
Std. GPA 2017 language	0.050	0.042	0.020	0.066	0.077	0.099	0.040	-0.004
	(0.14)	(0.13)	(0.12)	(0.12)	(0.13)	(0.12)	(0.10)	(0.10)
N=	328		328		328		328	
Classroom Fixed Effects			X		X		X	
Imbalance pre-treatment covariates[c]					X		X	
Additional pre-treatment covariates[d]							X	

The omitted reference group in all regressions is the control group. Standard errors in parentheses. [a] Activities and Script treatment tracks comprised by 87 and 92 students, respectively. [b] Activities and Script treatment tracks comprised by 81 and 85 students, respectively. [c]Included students age, missing dummies indicating whether there was a missing on pre-intervention mathematics or language GPA (2016) or for pre-survey family quality relation scores, number of people living in the student's home. [d] Included gender and pre-intervention mathematics or language GPA (2016). It also included pre-intervention measures for students' math traits, such as: anxiety, self-efficacy, self-determination, intrinsic motivation, personal relevance, parental motivation, grade motivation and career motivation and parenting styles measures: authoritative and authoritarian. Dummy variables were included to indicate missingness in parents’ survey or students survey.

** $p<0.01$, * $p<0.05$, † $p<0.1$

We do, however, find a positive effect of the treatment for students who began with high math anxiety. As shown in Table 3—5, the high-anxiety students exhibit consistently positive (~0.4 standard deviations) and significant ($p<0.05$) treatment effects on students' math GPA across models. The point estimates for low-anxiety students are negative for math GPA, though the estimates are not significantly different than zero when additional controls are included (model 4, β=-0.117, $p<0.437$).

Figure 3—1 graphs the estimated effect of the treatment at each level of student pre-treatment math anxiety. The graph shows that the treatment effect is positive and significant for students who feel anxious about math between "often" or "always".

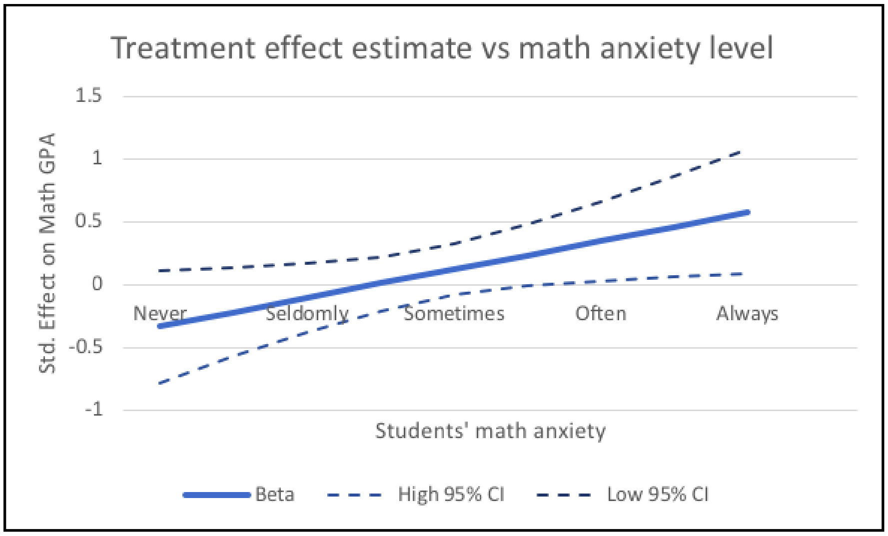

Figure 3—1 The treatment effect increases as the students' math anxiety increase

We assess the effect of the treatment on students' math anxiety. Table 3—6 provides the results. The average effect of the overall treatment is not significantly different from zero (β=-0.021, p=0.817). Similarly, the treatment arms do not show a significant effect on math anxiety (Activity track: β=-0.133, p=0.219; Script track: β=0.094, p=0.459) and the effect of the treatment arms do not significantly differ from each other, F(1, 245)=2.29, p=0.132.

However, some students may not have math anxiety and, as a result, no program could reduce their anxiety. When splitting the sample by initial anxiety (see model 8 in Table 3—6), the treatment appears to decrease math anxiety for highly anxious students, with marginal significance (β=-0.333, p<0.10). The magnitude of the treatment effect on math anxiety appears negative for these students regardless of the treatment track, but the sample size per track is too small to draw any conclusion.

Table 3—5 Treatment Heterogeneous Effects on Math GPA

	Treatment Effect Estimates							
	Model 1		Model 2		Model 3		Model 4	
	Model 1a	Model 1b	Model 2a	Model 2b	Model 3a	Model 3b	Model 4a	Model 4b
	Low Anxious Students	High Anxious Students	Low Anxious Students	High Anxious Students	Low Anxious Students	High Anxious Students	Low Anxious Students	High Anxious Students
Panel A: Students Math Performance								
GPA 2017 mathematics	-0.379†	0.429*	-0.351†	0.435*	-0.150	0.560**	-0.117	0.358*
	(0.20)	(0.17)	(0.20)	(0.19)	(0.19)	(0.19)	(0.15)	(0.16)
N=	112	111	112	111	112	111	112	111
Panel B: Students Spanish Performance								
GPA 2017 language	-0.197	0.343†	-0.250	0.352*	-0.193	0.418*	-0.128	0.231
	(0.21)	(0.18)	(0.20)	(0.17)	(0.21)	(0.19)	(0.18)	(0.16)
N=	101	103	101	103	101	103	101	103
Model Inclusions								
Classroom Fixed Effects			X		X		X	
Imbalance pre-treatment covariates[a]					X		X	
Additional pre-treatment covariates[b]							X	

The omitted reference group in all regressions is the control group. Standard errors in parentheses. A. Included students age and number of people living in the student's home. B. Included gender and pre-intervention mathematics or language GPA (2016). It also included pre-intervention measures for students' math traits, such as: anxiety, self-efficacy, self-determination, intrinsic motivation, personal relevance, parental motivation, grade motivation and career motivation and parenting styles measures: authoritative and authoritarian. Dummy variables were included to indicate missingness in parents' survey or students survey.

** p<0.01, * p<0.05, † p<0.1

Table 3—6 Treatment and Treatment Arms Effects on Math Anxiety

	Model 4	Model 8		
Sample	Treatment	Activities Track	Script Track	N
Post-intervention Math Anxiety Sample	-0.021	-0.133	0.094	290
	(0.10)	(0.13)	(0.13)	
Pre-Intervention Low-Anxious Students[a]	0.102	-0.089	0.349	93
	(0.20)	(0.25)	(0.28)	
Pre-Intervention High-Anxious Students[b]	-0.334†	-0.380	-0.283	86
	(0.19)	(0.24)	(0.25)	

Notes: Only the coefficients of the treatments and treatment arm status are reported. All regressions include the full specification models. The omitted reference group in all regressions is the control group. [a]Include students that answer both the pre and the post intervention survey, and their pre-intervention math anxiety measure was over the mean. [b]Include students that answer both the pre and the post intervention survey, and their pre-intervention math anxiety measure was below the mean. All analyses are based on analytical sample.

Statistical significance levels: ** $p<0.01$, * $p<0.05$, † $p<0.1$

3.4. Discussion and conclusions

High-school math can be difficult for adolescent students. Parents may reduce this difficulty by providing appropriate supports and encouragement, but they may also increase students' anxiety and inhibit their success. Parents' own math anxiety and discomfort with the subject can bleed over to students. As a result, programs that aim to increase parent involvement in high-school math may not benefit students. Yet, substantial research points to the potential benefits of parent involvement, which might apply to involvement in high school math, if it can be done without eliciting parental or student math anxiety.

To assess the potential to involve parents in high-school math, we conducted a randomized controlled trial of a text-messaging program that invite parents to complete pre-class activities with the adolescent student at home in the context of mathematics class. To limit the potential of anxiety, the activities were related to the math curriculum but did not require any math knowledge. The treatment did not show significant effects on students' mathematics performance (β=0.09, p=0.298) or language performance (β=0.02, p=0.792). These findings are similar to those in Balli et al. (1998), which evaluates the effect of having the student complete post-class activities with their parents (TIPS program) and does not show a significant effect on mathematics performance. They contrast with Van Voorhis (2001, 2003), which does show significant effects of parent-student activities on students' academic performance in language and arts and science.

While we do not find an effect of the intervention for the sample as a whole, we do find a positive effect for students with math anxiety. Math anxiety is a common affliction for high school students in which they have adverse emotional reactions to math or the prospect of doing math even if they have the scaffolding to understand the work (Maloney & Beilock, 2012). Students' math anxiety can be particularly strong in high school (Suárez-Pellicioni, Núñez-Peña & Colomé, 2016). The positive findings for students with math anxiety may explain the difference in overall effects between this study and author (2018), which implements a similar treatment and estimated an effect of 0.4 SD, much bigger than the ~0.1-0.2 SD effects in this study. Students in Author 2018, had substantially greater mathematics anxiety than students in our sample. The findings of positive effects for math-anxious students are encouraging as math anxiety affects numerous students of many countries (Lee, 2009).

We find some evidence that the intervention helps high math anxious students by diminishing their math anxiety. While only marginally significant, the estimates show a negative relationship between the treatment and student math anxiety for students who were initially anxious. Vukovic et al. (2013) found that, when it comes to the relationship between parental involvement and students' mathematic performance, the role of math anxiety depended on the mathematical task and the type of involvement. They found that the parental support in schoolwork and the communication of their expectations for students' performance influence directly young students' performance in whole numbers arithmetic and indirectly in word problems and arithmetic reasoning through mathematics anxiety (Vukovic et al., 2013). Authors argue that math anxiety plays a moderator role between young students' performance and parental involvement only for more difficult tasks, i.e. that are more anxiety provoking for students and parents. Our results extend Vukovic et al.'s hypothesis to adolescent students, considering that high-school math content and assessments are often perceived as difficult by the students (Lyons, 2006).

Additionally, Vukovic et al. (2013) found that the role of math anxiety depended on the type of parental involvement, reporting that some kind involvement such us involvement in education (e.g. helping in academic homework) and parental valence toward school were not significantly correlated with students' levels of mathematics anxiety. Our analysis of the effect of the intervention on students' math anxiety complement their findings because activities seems to consistently decrease students' anxiety regardless of their initial level of anxiety, but its effect on students' math anxiety when combined with a script related to schoolwork it's not clear (see Table 8). However, the magnitude of the Script track showed a steady positive effect on students' academic performance which may not be driven by a change in students' math anxiety. This evidence builds on parent-teacher collaboration on upbringing along student adolescence, which has not been an often studied topic in the last decade (e.g, Stroetinga, Leeman, & Veugelers, 2018). As our main conclusion, we encourage mathematics teachers to invite parents to complete non-academic activities with their students who are highly anxious, in the context of subject.

The significance of the study demonstrating the potential for low-cost and scalable approaches to affect parent-teenager interactions and improve the math performance of students with math anxiety, students that represent a relevant sample of the worldwide population.

3.5. Limitations and future work

This study has several limitations. First, the communication between teachers and parents was not allowed in our intervention (a text message was included to inform parents that teacher could not read their text messages responses, see Table Appendix 3—A3,

Appendix 3—A), which might have reduced the effect of the intervention compared to author 2018 who included a third message prompting parent-teacher communication during 5 weeks. Future studies may include a treatment arm that allow this communication to measure whether there is an effect due to this parent-teacher communication. Second, the magnitude of the effect of complementing these activities with prompts for parents to promote students' effort value and growth mindset (Script track) showed a steady magnitude across the models (~0.1 SD), however, it was not significant (see model 7, p=0.146, Table 3—4). These results suggest that the combination between non-academic pre-class activities and providing parents with suggested script to share with the student, could work as an opportunity for parents to provide support in schoolwork and communicate their expectations for students' performance. Previous studies show a positive and significant correlation between parents' expectations and students' performance (Hill & Tyson, 2009; Jeynes, 2005, 2007), however, differences between the treatment arms were not found to be significantly different. Future studies might explore the effects of an intervention that exclusively aims to support parents' expectations for students' performance, without the activities. Thirdly, results of our analysis suggest that the Script track may increase math anxiety for low anxious students' math (see Table 3—6). While these activities were intended to avoid provoking math anxiety to parents or students, it may be the case that parents' math anxiety was conveyed to students (Maloney et al., 2015). Future studies could assess the moderator role that parents' math anxiety may play in this type of interventions. Fourth, the perceived difficulty of the mathematics assessments could vary between schools, because each school used a unique set of tests to calculate the mathematics GPA for each year. While we accounted for this variation by including classroom fixed effects, other studies may account for this variation within classroom to increase the precision of the treatment effect estimates. Fifth, this study only explored the effects of the treatment on math anxiety, and it cannot assert the reasons why this variation may have occurred. Sixth, we found a significant treatment effect on language performance for students with high math anxiety (see above, Table 3—5) and it may be the case that in our sample math anxious students were also highly anxious in other types of anxiety (e.g. language anxiety). Future studies may assess whether the effect of completing non-academic activities is moderated by or affects other types of anxiety considering that, for instance, non-familial environmental risk factors associated with general anxiety influence the students' math anxiety levels (Wang et al., 2014).

4. FOSTERING PARENT-STUDENT RELATIONSHIP THROUGH MATH ACTIVITIES

4.1. Introduction

The relationships we establish with others through the use of language (Hausfather, 1996; Vygotsky, 1978) are shaped by our context (Bronfenbrenner, 1986). This is especially true of family relationships. When it comes to school, early adolescence can lead to an increase in parent-student conflicts (Brković, Keresteš, & Puklek Levpušcˇek, 2014).There is a significant body of literature on the importance of cultivating quality relationships between parents and adolescents (Branstetter & Furman, 2013; Qu, Fuligni, Galvan, & Telzer, 2015). However, schools have failed to play a leading role in fostering the development of adolescents by positively mediating said relationship. An adolescent's experience at school should meet their needs as a young adult (Burke & Weir, 1979; Eccles et al., 1993). The school must therefore provide a suitable setting that allows the parent-adolescent relationship to develop. This becomes increasingly necessary with at-risk students, where the parent-adolescent relationship is often highly negative (Hagan, Roubinov, Adler, Boyce, & Bush, 2016; Suldo, 2009).

4.1.1. Parent-Adolescent Relationship

Various studies have shown that closeness, communication and time spent with parents decreases as a child grows older (Dubas & Gerris, 2002; Larson, Richards, Moneta, Holmbeck, & Duckett, 1996; Larson & Verma, 1999; McGue, Elkins, Walden, & Iacono, 2005; Meeus, 2016). This change in the relationship is linked to the conflict that arises from an adolescent's search for independence (Bandura, 1994). During this phase, adolescents face a series of challenges and losses, such as the end of parental omnipotence and the imminent development of personal independence (Comín, 2014; Roche et al., 2014). By doing so, they break the bonds that linked them so closely to their parents during childhood, leading to an increase in their emotional autonomy (Salguero, Palomera, & Fernández-Berrocal, 2012). As a result of this process, adolescents start to question, rebel against and detach themselves from their parents (Comín, 2014).

Conflict provides an opportunity to readjust the roles held by adolescents and their parents. Such conflict can therefore either have a positive or negative effect on their relationship (Laursen & Collins, 2009). A gradual improvement in the way in which parents and adolescents resolve their conflicts can foster a more horizontal relationship between the two of them (Van Doorn, Branje, & Meeus, 2011). However, when the conflict between parent and child is negative, this can lead to the adolescent becoming socially isolated (Bañez, 2017; Espada, Botvin, Griffin, & Méndez, 2003) and cutting communication with the people that care for them. This is because parents often avoid situations and topics of conversation that may cause tension (Fingerman, 1995; Hagestad, 1981, 1987).

When it comes to school, one of the main points of conflict between a parent and an adolescent is homework (Bernedo, Fuentes, & Fernández, 2005; Bosma et al., 1996; Del Valle, 1994). School-related conflicts become more and more frequent in early adolescence (Brković et al., 2014), a period in which school work becomes increasingly complex (Lyons, 2006). One of the main drivers of this tension is that parents spend considerable amounts of time and effort completing homework with their children, despite not feeling qualified to do so (Biscoglio & Langer, 2011; Solomon, Warin, & Lewis, 2002). Homework that is more academic, or geared towards academic achievement, is therefore often viewed in a negative light by adolescents (Leone & Richards, 1989). Furthermore, poor academic results are also another source of conflict between parents and adolescents(Brković et al., 2014; Dotterer, Hoffman, Crouter, & McHale, 2008). In this sense, a parent's concern for their child's future leads to added pressure for the adolescent, negatively affecting their relationship (Biscoglio & Langer, 2011; Solomon et al., 2002).

A positive (or negative) experience between a parent and their adolescent child can change the perception they have of their relationship. For example, many parents feel that (in retrospect) doing sport or going on vacation with their child brought them closer together, especially in the case of fathers (Golish, 2000). This change in how parents view the closeness of their relationship with their child can be understood from Mezirow's (Mezirow, 1996) perspective. In this sense, the author defines adults' transformative learning as the "process of using a prior interpretation to construe a new or revised interpretation of the meaning of one's experience." He also suggests that people can reformulate their beliefs and assumptions through a process of reflection, even when these beliefs and assumptions have been acquired through personal experience and provide the foundations of our view of ourselves and the world (Dirkx, 1998).

Therefore, based on the literature, a lot of the conflict between parents and adolescents stems from issues relating to school and homework. This can sometimes lead to a negative view of their relationship and of school itself. It is therefore important for schools to provide a space that promotes a different kind of interaction between parents and adolescents, without the tension that comes with grades and academic achievement. Such interaction would allow for a change in at least one aspect of their relationship. Furthermore, if schools can provide a positive parent-adolescent experience, parents may reconsider their view of the school. It is therefore also important to understand the parent-school relationship.

4.1.2. Parent-School Relationship

The parent-school relationship is often characterized by a sense of mutual ignorance, lack of communication and disagreement (Finger-Elam, 2014; Romagnoli & Cortese, 2007). This fosters the definition of a relationship based on the idea of “disengaged parents and indifferent teachers”, with a sense of mutual disdain and blame when it comes to a student’s poor performance or behavior (Finger-Elam, 2014; Romagnoli & Cortese, 2007). Children and their parents share specific codes and knowledge, often as a result of their culture. However, these codes and knowledge are not necessarily shared by the school. Performance at school therefore improves if the relationship between parents and teachers is strengthened, leading to shared codes and knowledge (Gubbins, 2016; Ichou & Oberti, 2014).

Improving a parent’s commitment to their child’s school not only fosters their child’s emotional development, it also fosters their learning process. A parent’s behavior and attitude towards learning is one of the most important factors when it comes to their child’s performance at school (Romagnoli & Cortese, 2007). In this sense, the parent’s level of commitment and involvement is key to their child’s education. By becoming more involved in their child’s work and showing an interest in the educational process, parents can show their children that they believe in the importance of school. This can then lead to a positive effect on the child's level of commitment, motivation and academic performance (Zhang, Haddad, Torres, & Chen, 2011).

Given the importance of a parent's behavior and attitude towards school, and how this can be shaped by the parent-adolescent or parent-school relationship, our first research question asks: "In terms of the relationship with their child and the school, how do parents perceive the experience of working with their child on non-academic math assignments?"

4.1.3. Potential Benefits of Parents and Adolescents Spending Time Together

The benefits of social interactions between parents and students are usually associated with the stage of the child's life between birth and the first years of school (Hirsh-Pasek et al., 2015). However, a series of studies in recent decades have led us to believe that the interaction between an adolescent and their parents also provides an important opportunity for a young person to learn and develop. When analyzing the available evidence, we can distinguish between the benefits associated with parent-adolescent interactions both in and outside of a school setting, as well as the benefits of spending time together.

Firstly, parent-student interactions that take place within a school setting have been linked to positive outcomes for adolescent students. For example, the frequency with which parents and adolescents communicate about school-related topics has been positively associated with general academic performance (Jeynes, 2005, 2007; Laursen & Collins, 2004; Sui-Chu & Willms, 1996). Furthermore, the frequency with which parents get involved in school activities (e.g. doing homework or talking about school) is an important predictor of the positive feelings their child will have towards school (school bonding), as well as their overall grades (Perkins et al., 2016). However, the correlational nature of this results did not rule out bidirectional effects.

Secondly, parent-student interactions that take place in a non-school setting have also been shown to have a positive effect on adolescents. The results from the PISA test reveal that students from countries where parents discuss social or political issues with their children perform better at reading (OECD, 2012). Based on these results, the OECD suggested that adults can contribute to their child's development by talking to them about different issues, without requiring a huge amount of time or specialist knowledge. However this results are also correlational, so a causal inference is not warranteed.

Thirdly, adolescents who spend more time in the company of an adult (such as their mother or father) have more positive qualities when it comes to childhood development, such as feeling satisfied with their life (Ma & Shek, 2014). The frequency with which an adolescent spends time with their nuclear family, such as at dinner, is positively associated with their general wellbeing and mental health (Elgar, Craig, & Trites, 2013; Meier & Musick, 2014). However, spending time together on non-school activities, such as playing sports or having fun, is not a predictor of school bonding nor on adolescent's overall grades (Perkins et al., 2016).

The available evidence suggests that parent-adolescent interactions are associated with a range of positive indicators. However, given the correlational nature of the available studies, it is not possible to establish a causal relationship between this interaction and the potential benefits. Similarly, Utter et al. (2017) suggest that interventions should be designed with the aim of increasing the interaction between adolescents and their families so as to see whether or not this interaction alone is beneficial for adolescents.

Given the above, it seems there is a space to explore the effect of parental involvement in non-academic homework tasks and how this complements the teacher's work in the classroom. Furthermore, there is also little evidence regarding the sort of behavior that this kind of intervention will foster among parents, as well as how their relationship with their child and school may change. Our second and third research questions therefore ask: "What sort of behavior is fostered among parents by school-driven, non-academic activities?" and "Given this kind of activity, what are the conditions that can promote changes in the parent-adolescent relationship?

4.2.Methodology

4.2.1. Present Study

The aim of this paper is to study the parents' perception of working with their children on homework assignments in math, without requiring any formal knowledge of the subject. In particular, the paper will focus on the parents' view of the relationship with their child and the school. In this sense, the study looks to build theory rather than reflect on or test it (Twining, Heller, Nussbaum, & Tsai, 2016). The aim of these homework activities was to allow the students to approach math through experiences from their everyday life. By doing so, the student can incorporate a non-academic experience from their personal life and link it to a mathematical concept in the classroom. This concept should also form part of the student's prior knowledge. Students often have a negative perception of mathematics. This is because the concepts are not related to the student's daily life, or because they have had a negative experience with the subject in the past. This kind of activity therefore looks to change this perception by making mathematics part of their life and by relating it to their personal experience.

These activities do not require the parents or students to have any specialist knowledge of the subject. The time required to complete the activities was also kept to a minimum. An activity was sent each week via Short Message Service (SMS) for a period of 12 weeks. The teacher, after the activity, connected it to the learning objective for the class using a one-page handout. An example of the SMS and class material and the teachers' supporting material is included in Appendix. The homework activity was sent every Friday afternoon, with the goals of the activity reinforced in a separate message sent on Saturday morning. The teacher was expected to use the class material at any stage during the following week. A complete list of the messages that were sent to the parents, corresponding to one classroom, can found in Table Appendix 3—A.

Math teachers were included in this study for two main reasons. Firstly, parents lose contact with subject teachers and become less involved with school when their child leaves elementary school (Eccles & Harold, 1993). This drop in the level of involvement has been linked to a lack of a clear point of contact between school and parents as the number of teachers and subjects increases (N. E. Hill & Tyson, 2009b;

Simons-Morton & Crump, 2003). Secondly, the complex nature of the subject may lead to parents having a negative perception of mathematics. It may also lead to increased conflict between parents and students. This conflict may then be worsened by the lack of clarity that parents

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4.2.2. Procedure

Parents were contacted by telephone in order to understand their perception of the experience of working with their child on non-academic homework assignments. The interview also allowed the researchers to understand the sort of behavior that was fostered by these activities. Telephone interviews have been shown to maximize response rates (Sturges & Hanrahan, 2004), as well as gathering similar evidence to face-to-face interviews on specific topics (Galan, Rodríguez-Artalejo, & Zorrilla, 2004; Irvine, Drew, & Sainsbury, 2013), such as the interviewee’s perception of a particular experience (Sturges & Hanrahan, 2004). This process involved three tentative calls at different times (morning, afternoon & evening) and on different days of the week. If the parent failed to answer after three calls, no further attempt was made to contact them.

Semi-structured interviews were conducted, split into three different sections. The first section focused on the parents' general perception and behavior during the experience. During this section, the parents were asked if they could remember the content of the messages, whether they had done the activities, and whether they could describe the activities they did with their child. If they had not done the activities, they were asked why not. Furthermore, the parents were asked whether or not they enjoyed the experience and whether they thought it had been a pleasant experience for their child. In both cases, follow-up questions were asked regarding their reasons for liking or disliking the activities. The second section aimed for the parents to compare the non-academic activities (which did not require any formal knowledge of mathematics) with traditional homework assignments. The parents were asked directly which type of activity they preferred and why. In the third section, the parents had to say whether or not the experience had had any impact on their relationship with their child. If the parents had not talked about this spontaneously in the previous sections, they were asked directly whether they felt their relationship with their child had changed in any way, and why.

The telephone interviews were recorded and transcribed. Based on the transcripts, fragments of text were grouped together so as to identify certain themes that could be used to build content categories. Following this, the researchers then looked to find a common denominator for a group of text fragments, i.e., using open coding (Böhm, 2004; Osses Bustingorry, Sánchez Tapia, & Ibáñez Mansilla, 2006). Given the objective and design of the intervention, communication and closeness were considered expected theoretical categories as they are both key elements of a relationship between two people (Ge, Natsuaki, Neiderhiser, & Reiss, 2009; Withers, McWey, & Lucier-Greer, 2016). Other categories that arose unexpectedly during the interview process were considered as emerging categories. In this sense, the interviews produced emerging categories that had not been included in the original interview script, with topics that had not been considered initially but were clearly important to the interviewees (McMillan & Schumacher, 2014; Osses Bustingorry et al., 2006). As recommended by Twining, Heller, Nussbaum, & Tsai (2016), the open coding was done by a social sciences professional and a second researcher. The codes were then triangulated and collapsed into fewer categories.

A comparative case study was performed in order to understand the conditions that can promote changes in the parent-adolescent relationship. In this sense, a case is considered an interview completed by a parent (unit of analysis). The comparative case study was framed within the interpretive paradigm of Grounded Theory (Corbin & Strauss, 1990). This methodology is particularly suitable for gaining access to subjective processes from the perspective of the subjects themselves, which in this case are the parents of the adolescent children involved in the study.

Critical case sampling (Patton, 1990) was used to understand in which cases parents from low socioeconomic status schools perceived a change in the relationship with their adolescent child. A critical case is the one that can act as a frame of reference for the rest of the population, or part of the population, with regards to the subject of the study. This method allows logical generalizations to made based on the weight of the evidence that can be produced by even just a single case (Alaminos & Castejón, 2006; García-Izquierdo & Conde, 2012). In order to select the critical cases, three cases were chosen at random in which the parents mentioned an improvement in some aspect of the relationship with their child. Three cases in which the parents explicitly stated that the relationship with their child had not changed were also chosen at random.

Each case was analyzed following the coding paradigm described by Strauss (Böhm, 2004). To analyze the data, each parent was taken as a separate case, determining a central phenomenon and a network of underlying relationships for each one. According to the model described by Strauss, the discourse focuses on a topic that develops into the central phenomenon of the subject's experience. This central phenomenon is then used to identify the origin, context, actions and consequences of said experience. The origin helps understand the appearance or development of the phenomenon. The context represents the circumstances currently surrounding the phenomenon. The actions and interactions are both processes and are often goal-oriented. The consequences are the effects that the phenomenon has on the school's current situation. Following this, the context, origin, actions and consequences were described for each of the cases that was identified. Finally, a comparative analysis was conducted based on the parents' experiences. This analysis compared the experience of parents who identified an improvement in the relationship with their child with that of parents who failed to identify any change.

4.2.3. Sample

From a total of 134 parents of 9th and 10th grade students, 59 (44%) completed the telephone interview. The final sample therefore consists of these 59 parents. Two parents (1.5%) refused to do the interview, 6 (4.5%) said they were not the student's parents (even though they were registered as parents at the school), while the remaining parents (50%) could not be contacted. The parents who were interviewed (men = 7 [11.9%], women = 52 [88.1%]) were parents of students from 11 different classes at 8 schools in at-risk areas of Santiago, Chile. Of the parents that were contacted, 44 (74.6%) said they had done at least one of the activities and 31 of them (70.5%) were able to remember and describe 2 or more of the activities during the interview. Thirty of the interviewees (50.8%) filled out a questionnaire during a (previous) parent meeting. This questionnaire included information that was used as a proxy for characterizing the sample. Twenty-five mothers (83.3%), 2 fathers (6.7%), 2 aunts (6.7%) and 1 grandmother (3.3%) completed the questionnaire; the estimated age of the parents and guardians that were interviewed ranged between 29 and 70 (average of 44.1).

To answer the first two research questions, relating to the parents' behavior and appraisal of the experiment, all of the interviewed parents were included in the analysis, regardless of whether or not they were able to remember and describe at least two activities. To answer the third research question, relating to changes in the parent-adolescent relationship, 6 cases were included where the parents talked about changes in communication and closeness. No changes in the other dimensions (autonomy and conflict) were mentioned by the parents during the interview.

4.3. Results

4.3.1. In terms of the Relationship with their Child and the School, how do Parents Perceive the Experience of Working with their Child on Non-Academic Math Assignments?

The following results were found for the parents' perception of the intervention with regards to their relationship with their child and the school. Thirty parents (50.8%) found the activities were an opportunity to spend time with their child, an opportunity that was valued from two different perspectives.

Firstly, the parents found that it strengthened their relationship with their child. Twenty-three (39%) of the parents valued an improvement in at least one aspect of their relationship, the forming or strengthening of a bond with their child, or the opportunity to spend time together. Thirteen parents (22%) felt that it was an opportunity to communicate more with their child, to get to know them better, to be closer to them or empathize with them.

Four parents (6.8%) acknowledged that the activities gave them the opportunity to talk about issues related to the activity itself. Talking about the activity allowed the parents to learn about the student's interests and preferences. Given the general lack of connection that parents can feel with their adolescent children, this opportunity to bond really stood out. One mother who received instructions for an activity that led her and her child to talk about the music they both like said the following:

> Her personality's like that, she's really reserved and doesn't talk about her life much, but I always ask her. But the topic came up during that activity. (…) She asked me about, you know, about songs, I remember that one because she was like

> ‘But which songs did you use to listen to? And why did you like them?... But she listens to another kind [of music], so (…) she [the student] wanted to learn more about me, and I’d ask her ‘What about you? Why do you listen to that? Why do you like that?’ We’d ask each other questions. (Participant 1)

Six parents (10.2%) said that they felt these conversations were an opportunity to get to know their child better. One mother talked about the activities she did with her 16-year-old child and said the following:

> I realized that L (the student) has trouble with things that I hadn’t realized before, because he has good grades in general, but he’s missing loads. I hadn’t noticed from his reports [because he has good grades], but when you sit down and talk it’s like: ‘ahhhhh’, there are a few gaps that need filling. And just by talking to him, something so simple, I realized his problem isn’t math, it’s language, the lack of vocabulary, he’s got a terrible lack of vocabulary. (Participant 4)

Six parents (10.2%) suggested that the activities allowed them to get closer to their child. One parent suggested that they liked the experience “because you interact and start talking to your son more and remember what it’s like to be their age”. Adolescence was characterized by 16 parents (27.1%) as a stage in which the child is absent, in their own world, always busy, always tired, stuck to the screen, always alone or always with their friends. In this sense, one mother said the following:

> It’s that [the student] is at that age when they don’t want to do anything, when everything tires them out, they’re always tired, so I’d get her out of bed and say ‘A_s (student), let’s do the… activity’, (with a tired voice), ‘alright mum, let’s do it’, like she was forced to do it (…) with some of the activities, but with others she’d say ‘alright mum, let’s do it’ straight away, but it was a really nice experience, like I

said, it brought me a little closer to her , so in that sense it was good. (Participant 3)

Three parents (5%) said that the activities were an opportunity to empathize with their child. This empathy mainly came from the parents putting themselves in their child's position based on their own experience. One father said:

> Yeah, I thought it was really fun because you always look for ways to get through to your children and… I liked it. (…) and with this activity, you know, I remembered how it was for me, like how I used to study, dad too, and that led to us telling different stories and we thought it was fun. (Participant 6)

Secondly, the parents found that it helped them become more involved in their child's school life. Eight parents (13.6%) suggested that the activities helped them to be more aware of their child's school work. When the parent and student did the activities at home, topics of conversation came up such as: (i) talking about the activities the teacher did in class, (ii) talking about how the student got on with their teachers, and (iii) how the student was doing at school. One mother said that she felt motivated to ask her son more about school when asked if she preferred these activities or traditional math homework:

> Look, honestly, I'm more used to the old school system, where they'd set 20 tons of homework. I'm over the shock, I think it's different. I can't say if it's better or worse, just different. I think that (these activities) are useful though. You start off with something so silly (the activities), and then it was like 'What are you up to? What are you studying? What did they teach you?' And then we'd go through his notebook and of course I was on top of it a lot more. (Participant 27)

Furthermore, another mother said that she'd had a negative view of math at school and that this was an opportunity for her daughter to have a different view of the subject:

> Yeah, it was worth it because when I was young I hated math, the teacher would come in to the classroom and I'd hate her, because she'd always be angry and shout at us (she laughs), so I reckon that doing math like this, I reckon she sees it like that, that the teacher comes in and they all think about how silly the activity was, like 'my mum made this face', 'she told my brother…' that's how I imagined it was, and I reckon they had to have talked about it, too! At the end of the day that's what it's all about, bringing the family together and stop thinking about math as something so lame and boring. (Participant 10)

Furthermore, one mother was grateful because her daughter knew that she was in contact with her teacher. Three parents (5%) spontaneously mentioned that they valued the fact that the teacher cared about their students.

However, getting involved in school life through this intervention was uncomfortable for 4 of the parents (6.8%). One mother said that it was awkward taking their child out of their environment. One father said that it was awkward being told to get closer to his son, in reference to one of the Saturday messages about the parent getting closer to their child.

> One day was like weird because [the teacher] said [in the message] like… we had to be closer to our child, interact with them, like, you know… like to trust them. And I was like 'Does Teacher A know that we trust each other or is it the same for everyone? (Participant 8)

As well as seeing the activities as an opportunity to interact with their child, the parents also talked about the design of the intervention and the activities. Twenty-seven parents (45.8%) gave a positive appraisal of the design of the activities, suggesting that they were enjoyable, fun, novel and a good opportunity to spend time with their child. They also said that they were easy, that they connected math with everyday life and that it provided a challenge every week. Three parents (5%) valued the opportunity to break with routine and do something different. One mother said the following about why she liked the experience: "It was fun, it was a challenge every week, even if we couldn't do it, you know? It was a chance to do something a little different, to get away from the day-to-day stuff" (Participant 18).

The aspect that stood out was the non-academic nature of the activity (4 parents, 6.8%). This mainly came from the fact that the activities did not require any specialist knowledge and the parents did not feel the pressure of needing to have academic knowledge of the subject. For example, one student's grandmother said: "I'm an old woman, nearly 80 years old (…) How long ago was 3rd grade? (…), I can't be at her (the student's) level" (Participant 44).

Furthermore, the parents also valued how the activities did not require them to play a supervisory role, which is often the case with traditional homework assignments. One mother stated: "[Here] we work together, whereas with [traditional] math homework you have to be threatening them" (Participant 58).

Three parents (5%) said they had a negative view of traditional homework assignments or the traditional school dynamic, when compared with non-academic activities:

> Look, I prefer this kind of activity, it's great. The others [traditional homework assignments] are a drag, like when you were a kid and went to school: they'd send a load of homework, $%#" all day, stuck at school and then to go home with homework, I mean it's quite lame." (Participant 33)

Although the majority of feedback regarding the design of the non-academic activities was positive, 2 parents (3.4%) said that some of the activities were not fun. In those cases, the parents suggested the activities were a little childish and that adolescents are not that easy to motivate.

4.3.2. What Sort of Behavior is Fostered among Parents by School-Driven, Non-Academic Activities?

With regards to the question about parent behavior and to what extent this was shaped by the experience, we were able to identify 3 groups of behavioral patterns based on the parent's level of commitment to the intervention, the level of communication between the parent and their child, and the time they spent together (degree of encounter).

Level of commitment to the intervention

Three levels of commitment were identified: (i) not doing any of the activities, (ii) not doing all of the activities, and (iii) creating strategies to involve the student. Fifteen parents (25.4%) said that they did not do any of the activities. This was due to reasons such as a lack of time (3 parents, 5%), a lack of interest from the student (1 parent, 1.7%), or because the weekend is for resting (2 parents, 3.4%). Fifteen parents (25.4%) said that they did not do all of the activities because (i) they do not spend much time with their child or could not find a time when they were both available (9 parents, 15.3%), (ii) the child sometimes did not want to do them (2 parents, 3.4%), or (iii) they put off doing them and then forgot (1 parent, 1.7%). Seven parents (11.9%) said that they developed strategies to motivate their child and involve them in the activities.

The strategies reported by the parents include: telling their child that it was an obligation (1 parent, 1.7%), showing them how quick and easy the activities were (1 parent, 1.7%), talking about the goal of the intervention (1 parent, 1.7%), and showing them how fun each activity could be (4 parents, 6.8%). With regards to this last strategy, one mother talked about what she did to motivate her son:

> Look, we were counting pencils and I said let's bet, separating them into colors, who would win? I'd say 'Who'll win?', 'Who'll get more of this color?', 'Who'll finish first?', 'Who'll come first?', so for example let's say my mum had like 100, more than 500 pencils, so we'd throw a load on the bed, then I'd say 'Who'll win?', 'Who'll get more reds?', 'I'll get blues', 'Who's going to win?' So it was fun. (Participant 56)

Level of communication between parent and student.

There were three levels: (i) three parents (5%) said that they had not done any of the activities but had at least talked about them with their child. Two parents (3.4%) said that they had talked about some of the activities over the phone with their child, as they were not able to find the time to see each other, (ii) two parents (3.4%) had conversations that were limited to the topic of the activity, and (iii) ten parents (16.9%) had conversations that went beyond the activity itself.

Degree of encounter by the parent and child.

Three levels were identified: (i) twelve parents (20.3%) said that they were with their child when doing the activity, (ii) three parents (5%) said that they went out with their child when doing the activity, and (iii) fifteen parents (25.4%) said that they laughed with their child when doing the activity

4.3.3. Given this Kind of Activity, What are the Conditions that Can Promote Changes in the Parent-Adolescent Relationship?

In the first of these cases, the mother said that although the teacher explained the objective of the activities to the parents, she felt that her child did not understand the point of them. The mother saw the activities as an opportunity to share her opinion with her son and to be closer to him. She mentioned that going to the park to do the activities was a good way to spend time with her child in a fun environment. Furthermore, the mother also felt that her son was more interested in being on his own or being with his friends. She therefore described her child as being somewhat absent. She expressed a desire to be able to “get closer to her child” and was therefore keen to motivate and share her experience with her son during the activities. Following the intervention, the mother said that the student talked more with both parents and that she felt closer to him.

In the second case, the mother felt that she had a good relationship with her daughter, although she was a little “reserved” and “dry”, as she did not talk much about her private life. In this case, the mother said that she knew very little about mathematics and liked that the goal of this experience was to increase communication. When doing the activity, they asked each other questions, which the mother saw as a sign of interest from her daughter. After the intervention, the mother felt that the student was more willing to talk about school-related issues.

In the third case, the mother felt that the activities had two objectives: to spend time with her child, while at the same time learning. The mother had a negative view of mathematics as she felt she constantly had to be telling her son off, whereas the activities in this intervention were not academic obligations. This mother explained how she laughed a lot with her son about the situations that came up during the activities. After the intervention, she felt she was closer to her son and that he was more willing to talk about school-related issues. In all three cases, the mothers could clearly remember at least two of the activities.

Three cases were also identified in which the parents did not perceive any change in the relationship with their child. In the first of these cases, the mother felt that she had a good relationship with her child, despite them spending little time together. Time was also identified as one of the reasons why they did not do the activities. The mother felt that the activities were useful as they helped her daughter to learn about mathematics. During her interview, she suggested that the activities were a good opportunity to spend time together and have fun. She also commented that the conversation after an activity was limited to talking about the activity itself. After the intervention, the mother felt that the communication with her daughter had not improved.

In a second case, the mother described her son as being very reserved and that her schedule at work meant that they did not get to see each other too often. Although they did some of the activities, this was only when the child wanted to. When he did not want to, they did not do them. She felt that her relationship with her son had not changed and that they did not speak any more after the intervention than before it.

In the third case, the mother felt that communication with her daughter before the intervention was good. She explained that they talked to each other quite a lot during the activities, that they talked about the activities a lot and that she thought they were fun. However, she did not see any reason for the activities to change the nature of their relationship. In all three of these cases, although the mothers claimed to have done some of the activities, they were not able to clearly describe any of them.

4.4. Discussion

When studying the parents' perception of their experience of participating in a non-academic school intervention, we found that a significant number of parents value the opportunity to communicate with and be closer to their adolescent child. This complements the work by Withers et al. (2016), who found that communication, closeness, autonomy and conflict are components that act independently of one another within the parent-adolescent relationship in high-risk settings. The kind of assignments described in the present study therefore create opportunities for communication and closeness that would not otherwise exist naturally. Using such activities, the mathematics teacher has the potential to foster and strengthen the parent-adolescent relationship. This can be achieved through two of the aforementioned components (communication and closeness), by sending the parents non-academic assignments that do not require formal knowledge of the topics that are studied in math class.

Furthermore, our study also reveals that these opportunities to communicate help the parents get to know their child better and empathize with them. This relationship between communication, getting to know each other and empathizing is in line with the findings by (Philippe & Seiler, 2006), who suggest that communication works as a vehicle to develop areas of mutual interest and for two people to get to know each other better. Therefore, from the perspective of bioecological theory, a system in which two people interact (Bronfenbrenner, 1979, 1987), is a good source of social bonding, where complementarity provides the substance of the relationship between the two individuals. In this sense, such a system may allow for the possibility of mutual development and learning (Balswick, King, & Reimer, 2016; Solla, 2011).

The present study also reveals that parents appreciate the opportunity to get more involved in school life through instances such as this. Non-academic activities that are suggested by the school provide an opportunity for parents and their child to talk about school-related issues. These issues come up in conversations, even when the activities did not explicitly suggest any topics of conversation, beyond the topic of the activity itself (e.g. ice cream). This may explain why Perkins et al. (2016) found that positive feelings towards the school are related only to the frequency with which the parents get involved in activities that rise from the school context, and not with activities outside this context. This hypothesis, regarding the importance of the context in which an opportunity to interact arises, really stands out given how unexplored this area is. In this sense, there have been very few studies involving interventions that look to increase the number of opportunities that parents and adolescents have to interact, particularly ones based on non-academic activities set by the school.

Although being together hypothetically gives the adolescent the opportunity to talk about emotional and social issues (Eisenberg, Neumark-Sztainer, Fulkerson, & Story, 2008; Elgar et al., 2013), the comparative analysis of the critical cases in this study suggest that spending time together is not enough to generate a discussion between parent and child. When comparing the cases in which the parents describe a change in the relationship with those in which there was no change (Figure 1), we can see that both the parent and the child (or at least the parent) show an interest in getting to know one another. This includes trying to motivate the child to take part in the activities or asking questions that go beyond the minimum requirements of the activity. This interaction interest, that comes in a context of poor communication between parent and child, partly explains why it may be beneficial for an adolescent to spend time with their parents (such as at dinner), even when they have a poor relationship with their parents or it is not easy to talk to them (Elgar, Craig, & Trites, 2013; Meier & Musick, 2014).

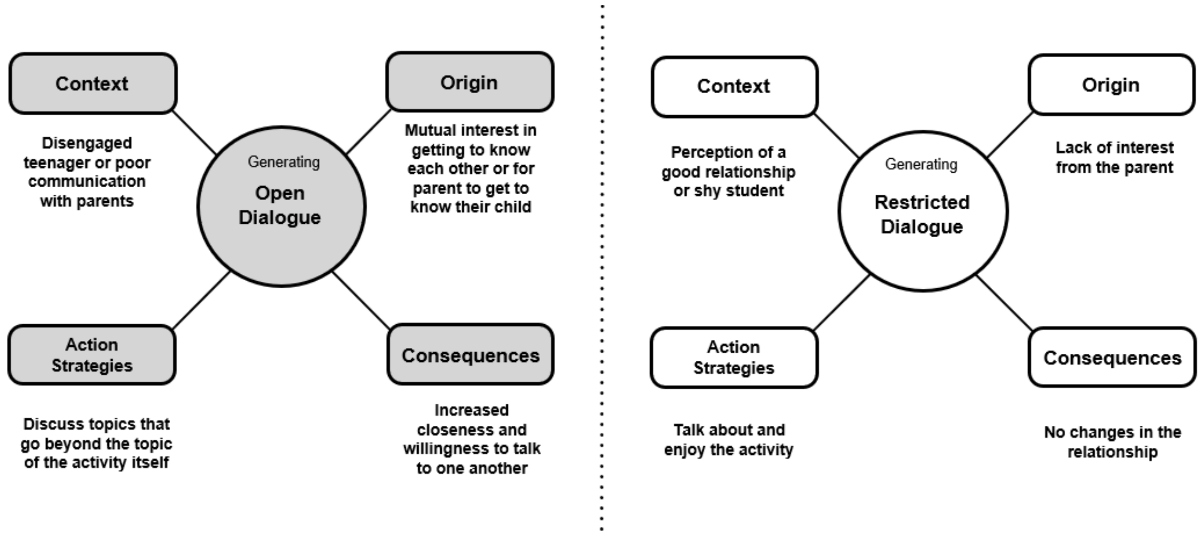

Figure 1. Cases where the mothers identified a change in the relationship with their child (left) and where no changes were reported (right), analyzed using Strauss' model.

The parent behavior that was observed, such as the level of commitment, level of communication and time spent with their child, may provide solid indicators with which to model the potential impact of non-academic interventions. Time spent together in a non-academic school setting, where the parents do not require any formal knowledge of the math topics studied in class, allow the parents and child to spend time together, go out and laugh with one another, in a setting that is designed to avoid any negative connotations.

From the perspective of bioecological theory, proximal processes are "processes of progressively more complex reciprocal interaction between an active, evolving biopsychological human organism and the persons, objects, and symbols in its immediate external environment. To be effective, the interaction must occur on a fairly regular basis over extended periods of time" (Bronfenbrenner & Morris, 1998). From this perspective, "human development" involves lasting changes in the way in which a person perceives and relates to their environment. In other words, there is a process of psychological mediation between the stimulus (environment) and the subject's response, shaping the way in which a person interprets and experiences a situation (Esteban-Guitart, 2016; Gifre & Guitart, 2013).

Given the above, creating opportunities for interaction or "proximal processes" in a family setting based on non-academic activities may lead to changes in the way in which an individual interprets and experiences a given situation, in this case the parent-child relationship. The focus is on the way in which the subject perceives the situation and this requires somewhat lasting changes in behavior. Considering the "interrelationships among two or more settings in which the person actively participates" (Bronfenbrenner, 1987), the school and family provide the potential for developing an educational setting, especially when considering the number of supporting links between these two settings and the other contexts in which the adolescent and their parents or guardians find themselves. In line with our findings, such interrelationships may define the nature of the shared activities. This is particularly important given the potential of such interaction in an adolescent's development.

This study addresses the well-known generational gap between an adolescent and their parents, using opportunities for interaction that are otherwise normally fraught with conflict. Conflicts, at this stage of an adolescent's development, can be regulated by the level of intimacy between a parent and their child, as well as the affective environment in which the development takes place. Promoting opportunities for non-academic interaction provides a space for conversation in a non-conflictive affective environment.

Our research reveals that in order to create attractive non-academic activities for parents and adolescents they must strike a balance between being childish and fun. Adolescence is a period that is characterized by a redefinition of the bonds between a child and their parents. It involves the affective nature of the child's process of becoming more autonomous, leading to a feeling of becoming separated from their parents and abandoning their childhood bonds. Emotional autonomy refers to the degree to which an adolescent has managed to let go of the bonds that were tying them to their childhood (Salguero, Palomera, & Fernández-Berrocal, 2012). Within this context, it is understandable that parents could mention that they found the activities to be too childish for an adolescent. Furthermore, the students are used to a school setting where they are faced with a difficult and decontextualized content. The teacher is therefore faced with the challenge of motivating their students using these activities, while at the same time providing activities in class that break this paradigm. In this sense, the teachers need to play an active role in the process of involving their students.

4.5. Conclusions

Our first research question explores the parents' perception of participating in school activities that do not require any formal knowledge of the math topics studied in class. We found that a significant number of parents value the opportunity to communicate with and be closer to their child. A school-driven intervention can therefore indirectly reinforce the parents' perception of the school and subsequently the relationship that the parent has with the school. In order to do so, opportunities must be provided for the parent and child to interact, share their mutual knowledge and be closer to one another. By giving non-academic homework assignments, we can break down the barriers in the parent-child relationship that traditional homework assignments often erect. Therefore, by sending non-academic activities that do not require any formal knowledge of the subject, the mathematics teacher is able to foster and strengthen the parent-adolescent relationship. Furthermore, our study reveals that these opportunities for interaction allow the parents to get to know their child better and empathize with them. Additionally, this study also reveals that parents value the possibility of getting more involved in their child's school life as they have the opportunity to talk with them about school-related issues. It is therefore possible for a school-driven intervention to help build a relationship between a parent and their adolescent child. This adds to the body of knowledge regarding the potential effects of interventions that looks to increase the level of interaction between parents and adolescents. Our second research question looks to analyze the parents' behavior when participating in non-academic school activities. Different levels of commitment were observed, specifically when it comes to the level of communication between the parents and their child and the time that they spent together. This is then reflected in the quality of their interactions, which ranged from just being together to actually having fun. This study also reveals that activities related to everyday life can either lead to conversations limited specifically to the topic of the activity itself, or to conversations that go beyond the minimum requirements of the activity. In other words, the activities can generate an open or restricted dialogue (Figure 1). Its relevance lies that it provides the foundations for assessing the quality of the parent-adolescent encounter space.

Finally, our third research question looked at the conditions that are needed in order to promote a change in the nature of the relationship. In this sense, we were able to see that this happens when both the parent and the child (or at least the parent) show an interest in getting to know one another. This includes trying to motivate the child to participate in the activities or asking questions that go beyond the minimum requirements of the activity. In this sense, it is important to note that simply building a bridge between the two is not enough. This therefore determines the conditions that should be taken into account when designing interventions that look to provide opportunities for parents and adolescents to interact.

Furthermore, the intervention described in this paper was shown to foster empathy among some parents to their students. This has been linked to prosocial behavior by students in the classroom, such as helping their peers, trusting and being friendly (Richaud de Minzi, Lemos, & Mesurado, 2011; Vicenta, Mesurado, Tur-Porcar, Samper García, & Richaud, 2014).

4.6. Limitations and future work

While it was possible to reinterpret the relationship between parents and adolescents, this intervention was not able to generate opportunities for interaction in cases where the relationship was poor or completely broken, or where the parents were not motivated to try to involve their child. However, the continuity and contiguity of this type of activities with a larger sample of parents and over a longer period of time may lead to changes in this relationship.

This intervention also provides the opportunity to build quality relationships between a parent and their adolescent child, because a healthy relationship needs the adolescent to feel a connection to their parents (Bowlby, 2008; Resnick, Harris, & Blum, 1993; Siegel, 2015) and to feel that their parents support them in general (Baumrind, Larzelere, & Owens, 2010; Christian, Perryman, & Portrie-Bethke, 2017). Therefore, future work should look to understand the adolescents' perceptions and behavior. It would also be interesting to look at results in different academic settings, not just high-risk settings, to see how much of an impact the context has on the described results.

5. CONCLUSIONS AND DISCUSSION

Research has strongly pointed out the importance of parental engagement in their child developmental process. However, it is often hard for parents to participate in the adolescent school life. The parents' limitations of technical knowledge can make parents anxious and, so the student, or make parents engage in undesired ways. To foster fruitful parent-student interactions, non-technical parental involvement is promoted using non-academic activities. Evidence in this thesis does not allow to accept the hypothesis that the intervention on average significantly improve the students' academic achievement in math (H1). However, results in this work lends support to the idea that non-technical parental involvement in mathematics using non-academic activities on average at least does not negatively affect the student's GPA in mathematics. Both field experiments, described in Chapters 3 and 4, showed the effect of being assigned to the treatment, had a positive magnitude, however, only in the first study the effect of the treatment was significant (roughly .40 SD, $p<0.05$, see Table 2—3). In the second study the effect was not significant (roughly 0.1 SD, $p=0.324$, see Table 3—3), i.e. we cannot assure that it is different from zero, but its magnitude kept the same direction of the first field experiment (positive in magnitude).

Regarding the second hypothesis of this thesis (H2), chapters 2, 3 and 4 together provide strong evidence that the effect of the intervention on the student outcomes depends of the student's math anxiety at the outset. Chapter 3 showed that math anxiety moderated the relationship between the student's mathematics performance at the end of the year and being assigned to the intervention. The field experiment showed that the more anxious the student was at the outset of the intervention, the higher their improvement in their mathematic achievement due to the intervention, because the intervention decreased the student's anxiety about math for students who suffered of elevated levels of this anxiety. In Chapter 2, the student's math anxiety was not analyzed, but it was mentioned that the teacher perceived the students as highly anxious about math. Later, we analyzed these students' math anxiety and confirmed that these students were often anxious at the outset (see section 9.5.1 below, Appendix 5), which may explain the differences in the magnitudes of the effect sizes detected in these two chapters (2 and 3). Moreover, In Chapter 2 and 4, parents described they had fun with the student which may drive the decrease in the student's math anxiety found in Chapter 3. Therefore, non-academic math activities that are fun for parents and children alike help diminish students' math anxiety by weakening the association between math and difficulty. This support Dowker et al.'s (2016) hypothesis that the way adults frame the subject can decrease math anxiety for the students.

Regarding our third hypothesis (H3), parents did have several perceptions and behaviors when facing the non-academic activities. While many of them valued the activities, highlighting aspects related to their relationship with their child, it is important to note that simply building a bridge between the two, parent and student, is not enough. As described in Chapter 4, to potentially change the nature of the parent-student relationship, we saw that it is necessary that both the parent and the child (or at least the parent) show an interest in getting to know one another.

Finally, the intervention was designed to help the math teacher connect the learning objective to daily life; there are several learning objectives and areas in the mathematics curriculum where this can be addressed. Therefore, it is interesting to know in which learning objectives teachers can benefit most when inviting parents to participate with their children in simple activities. For this purpose, we studied the relative relevance of several factors that explain mathematics performance in the 8^{th} grade SIMCE test. We found that the two most salient predictors were student performance in Numbers Skills in 4^{th} grade and the student's school socioeconomic background (see Table Appendix 1—A1, Appendix 1—A). These explanatory variables were similar in effect size to explain future performance, and the magnitude of their effect was bigger than students' achievement in 4^{th} grade in other performance areas (language, algebra, geometry, data analysis) and gender. This study was reported in a paper that was sent to the Elementary School Journal (see section 9.1 below, Appendix 1—A).

6. THESIS CONTRIBUTION

This thesis contributes to the growing body of literature suggesting that low-touch, text message-enabled interventions, can have a meaningful impact on parental engagement and student learning. In our particular case, the study focused on low-income Chilean students from highly disadvantaged backgrounds. It shows that it is possible to work with parents to reduce the anxiety that the math subject promotes due to its complexity, and to improve mathematics performance for those students who suffer from elevated levels of math anxiety through parent-teacher collaboration. Moreover, it shows that simple forms of ICT can play a key role in the family-student-teacher ecosystem. It provides a way in which the school can bridge the communication between parent and their adolescent child, by providing moments in which they can share and talk about school issues and have fun. These interactions can build the parent-student relationship, from the parents' perspective.

7. LIMITATIONS AND FUTURE WORK

Parents perceive activities as positive and describe them as an opportunity to activate the sleeping dimensions of their relationship with their child (e.g. communication), but it does not necessarily mean that the quality of the relationship will change due to the completion of the series of activities. Future studies could address whether school can promote some type of non-technical parental involvement using non-academic activities able to create a relationship for those students whose relationship with their parents is mostly or totally broken. Parents perceive the intervention as positive for their relationship and for maintaining knowledge about school, but it does not necessarily mean that it is perceived as positive by the students. The aim of the qualitative research was not to test whether this intervention would work as an affective source for students, but to explore the quality of the bridge provided by the teacher for parent-student communication.

The results of this study do not allow to assert whether the intervention benefit students with low levels of math anxiety. It may have no effect, as it might be hard for students who suffer from low anxiety levels, to decrease their levels of anxiety. This thesis provides evidence of the magnitude of effect size and future research can explore the effects of the intervention in a larger population that allows to detect a minimum effect size of roughly 0.1.

As shown in Appendix 5 (see section 9.5.2 below), female students suffered consistently of higher levels of math anxiety when compared to male students in the samples analyzed in this thesis, which is aligned with what prior literature has shown (e.g. Dowker et al., (2016)). However, no differential treatment effect on students' math achievement was found when comparing female students with male students (see section 9.5.3 below, Appendix 5). Future studies may analyze the role of both parent and student gender when promoting non-technical parental involvement using non-academic activities in math, considering that prior studies have shown differences in the relationship between different levels of parent's math anxiety and students' outcomes, depending on whether parent-student are same-gender dyads or mixed-gender dyads (Casad, Hale, & Wachs, 2015). Moreover, this thesis considers students' levels of math anxiety, but does not measure parents' math anxiety. Research shows that these two are related (e.g. Maloney et al., 2015), therefore, future studies encouraging parents to participate in activities with the student may include this measure.

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9. APPENDIXES

9.1. Appendix 1. Curricular Content Areas Study

How Prioritizing Number Skills Can Act as a Mediator for Socioeconomic Inequality within a National Math Compulsory Curriculum

9.1.1. Introduction

The discussion regarding the quality of education takes into consideration aspects such as which skills, knowledge and values should be encouraged in order for individuals to become successful members of society. Within an educational system, the curriculum framework fulfills this role and is developed by discussing cultural, political, social and economic aspects (UNESCO, 2005). In addition to what a curriculum may specify, identifying which topics have a greater impact on overall performance in mathematics is essential for teachers to be able to prioritize (Schumm, Vaughn, & Leavell, 1994). This is especially relevant in developing countries, where many students do not achieve a minimum level of proficiency in mathematics at any stage of their education (Agencia de Calidad de la Educación, 2015; Bos, Elías, Vegas, & Zoido, 2016; Glewwe & Kremer, 2006).

Content prioritization by teachers is becoming increasingly important due to a series of conditions that can be observed in the 21st century classroom. Firstly, there is an increasing trend away from content and towards higher-order thinking skills and different literacies. This includes reading, writing, mathematics and ICT, as well as creativity, critical thinking, problem solving and decision making, among others (Apino & Retnawati, 2017; Binkley et al., 2012; Schleicher, 2012; UNESCO, 2005). Secondly, teachers must consider the different pace at which individual students learn (Pritchett & Beatty, 2012). As a consequence, many educators fail to cover the contents and skills that are set out in the annual curriculum framework (Ramírez, 2006).

The present study aims to explore which are the most critical math content to be prioritized and, therefore, the ones to be primarily and effectively covered.

Effective content coverage phenomenon is understood to be a combination of instructional delivery and the teacher's instructional effectiveness (Schmidt, Cogan, Houang, & McKnight, 2011). However, the effect of such coverage may vary depending on the school's socioeconomic context. Students at low-income schools may have less opportunity to learn. This is often due to educational disadvantages, such as a lack of teaching resources. Consequently, covering some content areas might have a relative bigger impact in future achievement, depending on the educational context in which they are taught. The results from PISA 2012 showed that familiarity with topics such as Algebra and Geometry, as well as how often students are faced with formal mathematical problems, explains roughly a third of the socioeconomic achievement gap. This gap is related to the students' socioeconomic level and their academic performance in mathematics (Schmidt, Burroughs, Zoido, & Houang, 2015). However, this study did not include how being exposed to the different content areas contribute to future mathematics performance and, thus, to achievement inequality.

Understanding what topics on the curriculum should be effectively covered is increasingly important, as lower levels of effective content coverage may lead to poorer future educational outcomes (Levin, 2006). In the following section, with this aim, we summarize the literature on some key predictors of future mathematics performance, and define the research questions that drive this work.

9.1.2. Literature Review

9.1.2.1. Early Numeracy Skills and Performance in Mathematics. The level of knowledge acquired by students in the early school years can affect the level of knowledge they acquire in subsequent years. Therefore, weak (or strong) knowledge in the early years can increase (or decrease) inequality overtime (Bodovski & Farkas, 2007). Indeed, several studies have found associations between numeracy in early childhood and subsequent performance in mathematics in elementary school (Geary, 2011; Jordan, Glutting, & Ramineni, 2010; Passolunghi & Lanfranchi, 2012) and high school (Watts, Duncan, Siegler, & Davis-Kean, 2014). However, most of studies have relied on overall measures of math knowledge (Rittle-Johnson, Fyfe, Hofer, & Farran, 2017), i.e. only a few studies have explored the associations between specific areas of knowledge and future proficiency in elementary school mathematics.

For example, Nelson, Parker & Zaslofsky (2016) explored the Minnesota Math Corps database while also assessing the importance of improving math fact fluency. This study involved 1,493 students from 4^{th} to 8^{th} grade who had been identified as being at risk of failing to meet proficiency standards. Other researchers have also studied the importance of the math skills that are developed during elementary years and their effect on overall performance in mathematics in the future. With a sample of 134 Swedish students aged between 10 and 13, Träff (2013) studied the contribution of different arithmetic skills (i.e. arithmetic fact retrieval, arithmetic calculation and word problem solving) to general cognitive tasks. He found that the students’ proficiency in problem solving was underpinned by their number skills and general cognitive ability, while math fact retrieval was linked only to their number skills. Furthermore, Siegler et al. (2012) explored the relative contribution of the students’ math skills at age ten, such as addition, subtraction, multiplication, division and fractions, in predicting their overall achievement in early high-school math. This was done by analyzing 3,677 students in the UK and 699 students in the US. Across both samples, knowledge of fractions and division in 4^{th} grade improved overall achievement in mathematics 5 or 6 years later. This was especially true with algebra.

In regards to the content knowledge, in algebra, topics such as understanding concepts as equality have been shown to impact the student future performance in late elementary grades (e.g. understanding of the equal sign, Knuth, Stephens, McNeil, & Alibali, 2006) . Regarding geometry content area, there is evidence available on the effect of spatial skills and knowledge of shapes on math future performance (e.g. Logan & Lowrie, 2017a; Rittle-Johnson et al., 2017). While how to teach the abilities related with data analysis content area, such as making sense of graphs, have been widely studied (e.g., Friel, Curcio, & Bright, 2001), there is little documentation about the effect of skills related to data analysis content area on the students' future mathematics performance. Evidence available indicates that data analysis in preschool significantly predicts fifth graders' mathematics performance and the magnitude of its association does not differ from geometry's magnitude (Nguyen et al., 2016).

To the best of our knowledge, there is no evidence on the relative contribution of the main math content areas in late first level of elementary school (Numbers, Algebra, Geometry and Data Analysis) as predictors of future mathematics performance in late elementary school (4^{th} grade to 8^{th} grade). The evidence available corresponds to preschool or early years of elementary school (e.g. Kieran, 2018). It indicates that content areas separately contribute to future performance and that numbers skills have the strongest association with future performance (Nguyen et al., 2016; Rittle-Johnson, Zippert, & Boice, 2018). Due to the lack of evidence for late elementary school, numbers is often assumed to be the most important, but nobody has actually tested it. Otherwise, people usually assume that every previous knowledge is equally important for future performance. The relative contribution of the content areas is a critical topic to be explored for developing policy regarding math curricula and guiding teachers' content prioritization.

As shown previously, research in this field has so far come from developed countries and has been based on small- and medium-scale studies. However, the reality faced by developing countries is somewhat different and can lead to cultural differences (e.g. Chinn & Fairlie, 2010). Furthermore, national data covering a country's entire population can provide a much wider view. Large scale studies in this area of work are hard to find, because it is usually difficult for authors to access the data set required for their research. Given this, our first research question asks: "When it comes to prior knowledge, which areas of mathematics are the best predictors of future performance in this subject for an entire cohort of students in Chile?"

9.1.2.2. Other Factors Associated to Mathematics Performance. In order to fully understand the specific contribution made by prior knowledge of the main areas of mathematics, other relevant variables for explaining student performance in mathematics must also be taken into account. It has been found that knowledge of reading and writing is fundamental to other academic areas, such as Mathematics and Science (Mullis, Martin, & Foy, 2013; Norris & Phillips, 2003; Savolainen, Ahonen, Aro, Tolvanen, & Holopainen, 2008; Vilenius-Tuohimaa, Aunola, & Nurmi, 2008). Specifically regarding performance in Mathematics, evidence reveals that students with poorer performance in reading consistently demonstrate poorer performance in Mathematics (Mullis et al., 2013). Regardless of the available evidence on the importance of language skills for the mathematics subject (Gersten, Jordan, & Flojo, 2005; Powell, Driver, Roberts, & Fall, 2017; Vilenius-Tuohimaa, Aunola, & Nurmi, 2008), people usually assume that mathematics knowledge, such as numbers, algebra, geometry or data analysis will better predict future mathematics performance better than language skills, without testing it. We acknowledge the importance of this set of skills and, thus, we include them as part of the analysis conducted in this study to confirm that the results of our models correspond with what was found previously in the literature (details are described in the Methods section).

9.1.2.3. Socioeconomic Status (SES) and Performance in Mathematics. Ample evidence has been found regarding the importance of socioeconomic status to general academic performance (Bradley & Corwyn, 2002; Buchmann, 2002; Dahl Lochner, 2005; OECD, 2012; PISA, 2012; Sirin, 2005; Sullivan, 2007); specifically mathematics (Jordan, Kaplan, Nabors Oláh, & Locuniak, 2006; Sirin, 2005b). Several studies have estimated that an individual's socioeconomic status has a mean effect size (r) of roughly 0.40 on their general academic achievement. This effect size is larger (roughly 0.6) when socioeconomic status is measured on an aggregated level, such as school or school district level (Sirin, 2005b). The effect on performance in mathematics is even greater (Sirin, 2005). Furthermore, there is also evidence to suggest there is a link between socioeconomic status in early years and subsequent performance in mathematics (Krajewski & Schneider, 2009).

Given the above, it is important to understand which topics within the mathematics curriculum should be prioritized. Doing so, we improve the students' future performance in mathematics, which may in turn help reduce socioeconomic inequality. Therefore, our second research question asks: "What is the relationship between students' prior knowledge in mathematics, students' achievement in the future and the socioeconomic context where they were taught?"

9.1.3. Chilean context

This study analyzes the case of Chile, a middle-income country (GDP per capita of US$ 13,792) (The World Bank, 2017). The national curriculum for mathematics in Chile includes the learning objectives that the Chilean government has defined as being compulsory (Valverde, 2004). However, the curriculum framework does not define which of these topics are the most important, nor which should be mandatorily attained by students. It also fails to define what percentage of these learning goals must be achieved in each grade (Cox, 2003).

Elementary education is compulsory in Chile and is composed of two educational levels: 1st to 4th grade and 5th to 8th grade. This corresponds to International Standard Classification of Education levels 1 and 2, respectively [ISCED 1 and 2] (OECD, 2015). The elementary curriculum for mathematics has been revised three times (Ministry of Education, 2002, 2009, 2012). However, the framework structure and contents have not changed significantly.

During the first level of elementary school (1st to 4th grade), more time (50% in each of the four grades) is spent teaching number skills and arithmetic operations with natural numbers than geometry, fractions and spatial skills (Ministry of Education, 2004). During the second level, this percentage increases for number skills. An analysis of the results from the 2003 Trends in International Mathematics and Science Study (TIMSS) revealed that 72% of 8th graders in Chile were taught with an emphasis on number skills (i.e. whole numbers, fractions, decimals, percentages) (Mullis et al., 2000). The time spent by teachers on each topic was not shown to vary significantly among schools, regardless of their socioeconomic status (Ministry of Education, 2004).

The students' performance in mathematics is measured nationwide using the SIMCE test (*System for Measuring the Quality of Education),* a national standardized test that is applied universally in Chile in different grade levels (4th and 8th grade) to align with the state curriculum (Agencia de la Calidad de la Educación, 2018). The majority of elementary schools in Chile do not separate students by ability (Marcel & Raczynski, 2009). Therefore, two students in fourth-grade with different levels of prior knowledge in mathematics will still be in the same class.

In Chile, there are essentially two kinds of schools: private schools, accounting for approximately 5% of the student population, and state-subsidized schools, accounting for the rest of the population (95%) (Ministry of Education, 2014). The private system is mainly accessed by the richest section of the population. This is partly because there is huge income inequality within the country. For instance, while 89% of 15-year-olds with high SES attend high SES schools, 78% of low SES students attend low SES schools (Schmidt et al., 2015). Furthermore, in 2006 the richest 10% of the country accounted for 45% of national household income (Solimano & Torche, 2008), and this inequality has shown to be highly persistent over time (Chacón Espejo & Paredes Araya, 2015). As a consequence, the household income of students attending private school is four times bigger than that of students attending state-subsidized school. Additionally, private schools also have at least five times more resources per student than state-subsidized schools (Cabezas, Paredes, Bogolasky, Rivero, & Zarhi, 2017).

9.1.4. Methodology

9.1.4.1. Data

For this descriptive correlational study, four national, student-level databases were used to construct a longitudinal dataset. All databases were provided by the Chilean Ministry of Education. These datasets include: (i) 4th grade individual answer sheets for mathematics (disaggregated data for the different areas of mathematics) (SIMCE 2007), (ii) 4th grade individual test scores for language arts (SIMCE 2007), (iii) parent responses to 4th grade questionnaire (SIMCE 2007), (iv) 8th grade individual test scores for mathematics (SIMCE 2011) and (v) 2nd and 3rd graders annual and individual academic performance in 2005 and 2006, respectively.

The longitudinal data set of this study comprises all students who sat both the 8th grade SIMCE test in 2011 as well as the 4th grade SIMCE test in 2007. Our data set of children consists of all individuals who can be identified both in the datasets of 4th and 8th grade, have no missing data on their mathematics 4th grade 2007 answer sheet and have a language score for the 4th grade 2007 SIMCE test. We didn't impose any requirement regarding students or parents' place of birth or current citizenship. In total, the effective sample includes 158,818 students, which represents 67% of the universal set of 238,776 students who were registered to sit the 4th grade SIMCE test in 2007 (see Table A1, Appendix A for details). The main reasons for this loss of students include the cumulative effect of students repeating a grade during those four years (Ministry of Education, 2014), random absences on the day of the test (González, Cuesta, & Larroulet, 2017), and the annual drop-out rate (Ministry of Education, 2008, 2014). The effective sample corresponds to students from 7539 (95%) of the 7,956 schools in Chile. The socioeconomic characteristics of the schools in both samples (i.e. universe and effective sample) are similar, although students in the follow-up sample have higher scores on both the 4th grade math and language arts SIMCE tests than students in the 4th grade population. For descriptive statistics of both samples, the universe of 4th grade students and the final follow-up sample see Table A1, Appendix A.

In order to ensure that the sample was representative in terms of geography (rural/urban) and school type (private and state-subsidized), sampling weights were calculated and included in the estimations (Education and Technology Center- ENLACES, 2011).

9.1.4.2. Measures and Covariates

Our dependent variable corresponds to the overall score on the 8th grade (2011) SIMCE test in mathematics. The IRT-scaled scores for the 8th grade SIMCE test were provided by the Ministry of Education**.** There are four different formats for this test. Based on a set of items that are included on each of the formats, an equating procedure is carried out by the Chilean Ministry of Education. This is done in order to obtain a single score for each student that is comparable across the formats at a school level. The areas of the curriculum that were evaluated by this test were: Number Skills (roughly 40% of the item for each format), Algebra (12%), Geometry (27%) and Statistics and Probability (21%). In our sample, the score ranges between 135.35 and 395.66 points, with a mean of 264.78 points and a standard deviation of 48.63 points. In order to help interpret the regression coefficients, this outcome was standardized to have a mean of 0 and standard deviation of 1.

Our main predictors are the scores in the four areas assessed by the SIMCE test in mathematics sat by 4th grade students in 2007. As the individual answer sheets were available, we were able to estimate scores for each area. Sub-scales for the different curricular areas were obtained using the Barlett method (DiStefano, Zhu, & Mindrila, 2009). As a result, exploratory factor analysis was used to obtain the students' scores for each area. This was done based on tetrachoric correlations for each of the binary items. For each scale, one factor was retained using two joint criteria: (i) an eigenvalue greater than one (for every scale there was only one factor greater than 1) and (ii) looking for the breaking point using the scree test (Osborne & Costello, 2009).

Each SIMCE test is administered using four different formats within each school (Format A, B, C and D). This rotating design allows the whole curriculum to be assessed during the allotted time (Ministry of Education, 2003; Mullis, Martin, Gonzalez, & Chrostowski, 2004). The score for each area of the curriculum was therefore estimated for each sub-sample (i.e. each format). It is worth noting that using an equating procedure across the different formats of the test was ruled out. This is because the different formats only shared a small number of items for each area.

The following list is a broad description of the content areas, that separately include an integrated assessment of problem-solving (Ministry of Education, 2007):

- Number Skills: The score obtained on the 2007 4th grade mathematics SIMCE for this area of the curriculum. This area mainly assessed the students' ability to read and write whole numbers and establish relationships between them. This included ordering the numbers, as well as looking for simple patterns. It also included comprehension of fractions based on identifying them in picture form or identifying their relationship to the parts of a whole. This area also included the students' ability to add, subtract, multiply and divide whole numbers in different situations, as well as the use of conventional algorithms for carrying out operations.
- Algebra: The score obtained on the 2007 4th grade mathematics SIMCE for this area of the curriculum. This area mainly involved solving equations with an unknown in different parts of the equation.
- Geometry: The score obtained on the 2007 4th grade mathematics SIMCE for this area of the curriculum. This area assessed spatial awareness, the ability to describe and interpret trajectories and locations on a chart, and the ability to visualize different viewpoints of shapes. This area also assessed the students' ability to recognize, classify and compare geometric shapes and figures.
- Data Analysis: The score obtained on the 2007 4th grade mathematics SIMCE for this area of the curriculum. This area assessed the students' use of numbers in order to read, interpret and organize information in tables and graphs.

Table 9—1 shows the descriptive statistics (mean and standard deviations), the eigenvalue for the retained factor, the measures of reliability and the number of items for each scale. It is worth mentioning that, in general, the estimated scores have acceptable to good internal consistency, with alphas varying from 0.61 to 0.84 (Table 9—1). These levels of internal consistency were not achieved in just three cases. This means that the findings for algebra in sub-samples A (a=0.59), C (a=0.55) and D (a=0.54) should be interpreted with caution. Finally, in order to subsequently compare the results, the scores were standardized by dividing the variable that was generated for each format by two standard deviations. This was done independently for each of the four sub-samples of the test administered in 2007 (i.e. the four different formats).

Table 9—1. Descriptive Statistics of the Standardized Co-Variables per Sub-Sample

	Sub-sample A (n=41630)							Sub-sample B (n=40417)							Sub-sample C (n=39348)							Sub-sample D (n=37423)					
Variable	Nq	Alpha	Ev	Mean	SD	Min	Max	Nq	Alpha	Ev	Mean	SD	Min	Max	Nq	Alpha	Ev	Mean	SD.	Min	Max	Nq	Alpha	Ev	Mean	SD.	Min
Numbers	17	0.82	6.24	1.5	0.5	0	2.1	16	0.77	4.98	1.6	0.5	0	2.3	17	0.84	6.82	1.5	0.5	0	2.1	15	0.82	6.0	1.6	0.5	0
Algebra	5	0.59	1.86	1.3	0.5	0	1.9	5	0.61	1.90	1.3	0.5	0	1.8	4	0.55	1.49	1.2	0.5	0	1.7	4	0.54	1.50	1.3	0.5	0
Geometry	9	0.65	2.82	1.7	0.5	0	2.3	11	0.63	2.94	1.8	0.5	0	2.4	10	0.71	3.57	1.8	0.5	0	2.4	10	0.67	3.14	1.9	0.5	0
Data Analysis	5	0.63	2.27	1.6	0.5	0	2	5	0.62	2.34	1.7	0.5	0	2.1	5	0.62	2.19	1.4	0.5	0	1.9	6	0.66	2.55	1.4	0.5	0
Language	-	-	-	2.7	0.5	1.1	3.8	-	-	-	2.6	0.5	1.2	3.8	-	-	-	2.6	0.5	1.2	3.7	-	-	-	2.6	0.5	1.2
School type (reference: private non-subsidized)																											
State subsidized School	-	-	-	0.93	-	-	-	-	-	-	0.93	-	-	-	-	-	-	0.93	-	-	-	-	-	-	0.19	-	-
Female	-	-	-	0.52	-	-	-	-	-	-	0.52	-	-	-	-	-	-	0.52	-	-	-	-	-	-	0.52	-	-

Note. Nq = Number of question per area in each format of the test; Ev = Eigenvalue retained for score calculation

Additionally, the following predictors where included in the analysis: (1) 4th grade Language SIMCE score, which corresponds to the IRT-scaled score obtained on the 2007 4th grade SIMCE in language arts. This test assesses reading comprehension by using tasks that involve locating, interpreting, relating to and reflecting on information contained in literary and non-literary texts (Ministry of Education, 2007). The scores range between 112.32 and 379.35, with a mean of 265.23 (calculated for the total sample). The score is standardized by dividing by two standard deviations for this variable (50.23) (2) Female, which is a self-declaration obtained from responses to the SIMCE questionnaire (binary variable), (3) Type of school, which was coded as 1 if the student attends a state-subsidized school (either a public school or a voucher school) and 0 if they attend a private school and (4) Students' overall GPA in 3rd and 4th grade, which ranged from 3.5 to 7 (M=6.3, SD= 0.48) and from 4.1 to 7 (M=6.1, SD=0.50), respectively.

Model

Given our first research question and the results from the literature review, as well as the kind of information that was available on an individual level, the following regression model was considered:

$$MS_{ij} = \beta_0 + \beta_1 NUM_{ij} + \beta_2 ALG_{ij} + \beta_3 GEOM_{ij} + \beta_4 STAT_{ij}$$

$$+\beta_5 LANG_{ij} + \beta_6 GEN_{ij} + \beta_7 Y_{ij} + (\varepsilon_{ij} + \mu_{ij}) \quad (1)$$

where MS_{ij} is the IRT-scaled score on the 2011 SIMCE for student i at school j; NUM, ALG, GEOM, STAT are the standardized scores for each specific area for student i at school j; and LANG is the IRT-scaled score on the 2007 SIMCE in language arts, standardized by dividing by two standard deviations. Furthermore, GEN ij is the gender of the student, while Y_{ij} is the type of school attended by the student, ε_{ij} is an individual-level error term, and μ_{ij} is the school-level error term.

In order to fulfil the objectives of this study, disaggregated data for the different areas of mathematics was needed. We therefore analyzed the four different formats of the 4th grade SIMCE test separately, i.e. we separated our original sample into four sub-samples (i.e. Sub-sample 1: Format A of the test, Sub-sample 2: Format B, etc.). The hierarchical linear model was therefore estimated for each of the four formats, using the sub-sample for each area as an independent variable. Several Wald tests were conducted in order to compare the regression coefficients and identify whether the differences between them are statistically significant. This is a parametric statistical test for testing hypotheses.

After running this procedure, we assessed whether Numbers Skills had a larger association with the students' future achievement because its scale was comprised of a larger number of items (compared to the other content areas). To this aim, we replaced the original Numbers scale for a new scale composed of a smaller number of items and conducted the same models, for each sub-sample. The new scale was calculated by averaging 8 to 9 random items, depending on the test format. This quantity is half the number of items in the original scale and, it is less or equal to the number of items for Geometry content area. While the estimated Numbers skill coefficients were found to be smaller than the ones presented in the main results, main conclusions remained the same (see Table Appendix 1--D1, Appendix 1--D).

To answer our second research question, we carried out two complementary analytical approaches. In the first one, we compared the magnitude of the regression coefficients of our variables of interest (area of mathematics and school type), which were estimated using the same model that was used to answer our first research question, i.e. aforementioned equation 1. This is similar to the strategy used by Schlicht, Stadelmann-Steffen, & Freitag (2010) and Nguyen et al. (2016), who also compared regression coefficients to answer to their questions. Given the relationship between school type and SES highlighted previously, the coefficient associated with school type is used as a proxy for the socioeconomic achievement gap. Following the method proposed by Gelman (2008), the scores for the continuous variables, were centered and standardized by dividing by two standard deviations (for details, see Appendix E). In this case, the continuous variables correspond to the four areas of mathematics (Number Skills, Algebra, Geometry and Data Analysis) and the score on 4th grade SIMCE test in language arts. The aim of this was to make these coefficients comparable with the coefficients for school type (binary variable). Binary inputs were used as benchmarks for rescaling. In the second analytical approach, we separated the sample by school type (i.e. private or state-subsidized) and then estimated the same regression model (Equation 1) for each sub-sample. Doing so allowed us to analyze whether there was any variation in the effect size for each area of mathematics depending on the student's school type, i.e. the socioeconomic context where they are taught.

Due to the nested structure of the data, where students are grouped by schools, Hierarchical Linear Models (HLM) (Goldstein, 1995) were estimated using sampling weights. Using an HLM model allows for the introduction of a non-observable variable or school effect, which controls for the effects of the individual covariates.

This study is correlational and descriptive in nature. The estimations for the regression models in this article do not require a causal interpretation, at least not in the way suggested by recent discussions of causal inference (Imbens & Rubin, 2015). Our findings provide information about associations between outcomes and a given independent variable, controlled by the other covariates used in the regression models. Future research must examine the role of unobserved confounders.

All of the analyses were carried out using Stata 12.

9.1.4.3. Sensitivity Analysis

A random effects model was adjusted and is presented in the results section. This type of model is used when the aim is to test the effects of group-level variables (Snijders & Bosker, 2012). However, a fixed effects model with clustered standard errors was also conducted. This is because this kind of model allows non-observed school characteristics to be associated with the other covariates in the model (such as the type of funding received by the school). This assumption is not permitted by a random effects model. In order to check the consistency, both models were adjusted using Equation 1. The results from the fixed effects model are presented in Table B1, Appendix B. As second robustness check, we run an extra model that accounted for prior performance of students' math learning one and two years before our first assessment, i.e. we included 2nd and 3rd grade overall GPA as controls. The results lead to the same conclusions after changing the model specification (see Table B2, Appendix B). Moreover, an individual level SES score was calculated based on the parent responses to 4th grade questionnaire (SIMCE 2007) instead of the type of school and same conclusions were drawn (see Appendix E). Furthermore, Potential problems of multicollinearity between the covariates were checked using a VIF analysis. This analysis returned values between 1.04 and 3.21 and therefore any problems of multicollinearity could be discarded (VIF<4 or VIF<10) (Mandel, 1985; O'brien, 2007).

9.1.5. Results

The results from the hierarchical linear models (HLM) for each sub-sample are included in Table 9—2.

Table 9—2 HLM Models of 8th grade Mathematics Score per Test Format

	4th grade format A		4th grade format B		4th grade format C		4th grade format D	
Parameter	beta	(SE)	beta	(SE)	beta	(SE)	beta	(SE)
Intercept	-2.084***	(0.038)	-1.949***	(0.039)	-2.049***	(0.042)	-2.144***	(0.041)
Level 1 (Student)								
Numbers Skills[a]	0.623***	(0.016)	0.619***	(0.015)	0.656***	(0.016)	0.585***	(0.017)
Algebra[a]	0.328***	(0.013)	0.373***	(0.013)	0.349***	(0.012)	0.321***	(0.013)
Geometry[a]	0.236***	(0.013)	0.238***	(0.014)	0.258***	(0.015)	0.106***	(0.015)
Data Analysis[a]	0.031*	(0.014)	0.028$^{\sim}$	(0.014)	0.125***	(0.014)	0.106***	(0.014)
4th grade Language SIMCE score[a]	0.350***	(0.016)	0.308***	(0.014)	0.234***	(0.015)	0.333***	(0.015)
Female	-0.095***	(0.009)	-0.097***	(0.010)	-0.064***	(0.010)	-0.104***	(0.010)
Level 2 (School)								
School Funding (Reference: Private Non-Subidized Schools)[b]:								
State Subsidized Schools	-0.529***	(0.025)	-0.527***	(0.025)	-0.448***	(0.025)	-0.512***	(0.026)
	Random parameters							
Student level	0.354	(0.005)	0.348	(0.005)	0.347	(0.005)	0.360	(0.005)
95% Conf. Int.	0.344 – 0.363		0.339-0.357		0.338-0.356		0.351-0.370	
School level	0.124	(0.005)	0.130	(0.005)	0.121	(0.005)	0.110	(0.005)
95% Conf. Int.	0.116 - 0.135		0.122 - 0.140		0.111 – 0.130		0.101-0.119	
Intra-class correlation	0.2608		0.252		0.244		0.216	
Nº of students	41630		40417		39348		37423	
Nº of schools	6847		6475		6238		5855	

Note. [a]Language score and mathematics scores were standardized by dividing by two standard deviations (Gelman, 2008). Standard errors in parentheses. The indicated variables are constructed based on different samples corresponding to the four different formats of the test. [b]Private schools coded as 0. *** $p<0.001$, * $p<0.05$, $^{\sim}p<0.1$, two-tailed test.

The results show that the vast majority of variables included in the model (Number Skills, Algebra, Geometry, language arts SIMCE score, gender and school type) have a statistically significant association ($p<0.01$) with performance in 8th grade mathematics. The only exception is the score obtained for Data Analysis, which does not have a statistically significant influence on performance in 8th grade mathematics for one of the sub-samples (Format B). In terms of differences between formats, the coefficients are consistent across the four formats, while the number of students and schools included in each model are also relatively balanced.

In general terms, performance in 4th grade Number Skills and attending a public elite school are both strongly correlated to performance in 8th grade mathematics. An increase of two standard deviations in these two variables is related to an increase of more than half a standard deviation in performance in 8th grade mathematics. This equates to estimate an effect size of roughly 0.3 that can be calculated by dividing the corresponding beta coefficients shown in Table 2, considering the covariates standardization conducted in this study**.** Furthermore, the results show that performance in 4th grade algebra and language arts both have a similar impact on the students' performance on the 8th grade SIMCE test in mathematics. In this case, an increase of two standard deviations in these variables is related to an increase of one-third of a standard deviation in performance in 8th grade mathematics. With the other two areas of mathematics (Geometry and Data Analysis), an increase of two standard deviations is related to an increase of less than one-fourth of a standard deviation in performance in 8th grade mathematics. In particular, Data Analysis effect size is roughly zero (by dividing the corresponding beta coefficients shown in Table 2, is roughly 0.05). Finally, in terms of gender, when controlling for the other covariates, girls is related to a score around 0.09 standard deviations (between 0.064 and 0.104 points) less than boys on the 8th grade SIMCE test in mathematics. All of these associations are statistically significant after controlling for the other covariates and taking into consideration random school effects.

We then estimated the variance explained by these models (Table 2). The intra-class correlation, which ranges between 0.22 and 0.26, shows that up to 25% of the performance on the 8th grade SIMCE test in mathematics can be attributed to the school attended by the student. This reveals the importance of adjusting a hierarchical linear model that accounts for the school effect, as well as showing the degree to which the type of school attended in 4th grade influences the students' performance on the 8th grade SIMCE test in mathematics.

Finally, a comparison of coefficients was conducted (Figure 1) using a 95% confidence interval and chi-squared test (see Table C1, Appendix C for the results). In this case, the results revealed that the marginal effect of the students' performance in Number Skills is significantly greater than the effect of their performance in the other areas and on the language arts test. This is consistent across the four sub-samples. It also revealed that Algebra has a consistently larger predictive power on future performance when compared to Geometry across the different sub-samples (test formats) Furthermore, two of the test formats (Formats A and D) revealed that the score in Algebra and on the language arts SIMCE test are not statistically different. This means that their effect on student performance in 8th grade mathematics is similar. On the other hand, the differences between the two coefficients with Formats B and C are statistically significant (Appendix C). Additionally, the students' performance in language arts appears to be more important than their score in Geometry in three of the sub-samples. More specifically, statistically significant differences (with a 95% confidence interval) can be observed between the two coefficients in each of the sub-samples except for Format C (see Appendix C).

[Insert Figure 1 here]

With regards to the relationship between students' prior knowledge of different areas of mathematics and the context where they were taught, when controlling for the other covariates and the random school effects, the marginal effect of the students' score in Number Skills is similar in size to the marginal effect of the type of school they attend. In terms of SIMCE scores, this could mean that an increase of two standard deviations in number skills would compensate for the achievement gap between private and state-subsidized schools. The results from a confidence interval analysis (Figure 1) and chi-squared test (Appendix C) reveal that the effect of student performance in Number Skills is significantly greater than school type for the four formats of the SIMCE test. In other words, the students' performance in Number Skills may be more important than whether or not they attend a private elite school. The effect size of the remaining areas was significantly smaller than the effect size of the school type (Appendix C).

When assessing whether the effect of the students' prior knowledge of certain areas of mathematics changes depending on the type of school they attend, it is worth noting in Table 9—3 that the coefficients for Number Skills and Geometry increased their magnitude compared to models for the full sample and were greater for students attending private schools than students attending state-subsidized schools. In state-subsidized schools, Algebra's effect size (roughly 0.17) remain stead regardless of the type of school is used. However, the relative importance between Algebra and Geometry changes depending on the type of school that was considered to estimate the regression. In private schools, Algebra have a weaker predictive power than Geometry, in most of the sub-samples.

Table 9—3 HLM Models of 8th grade Mathematics Score per Test Format and per Type of School Funding.

	4th grade format A		4th grade format B		4th grade format C		4th grade format D	
	State Subsidized Schools	Private Schools	State Subsidized Schools	Private Schools	State Subsidized Schools	Private Schools	State Subsidized Schools	Private Schools
Parameter	beta (SE)	beta (SE)	beta (SE)	beta (SE)	beta (SE)	beta (SE)	beta (SE)	beta (SE)
Numbers	0.613***	0.877***	0.614***	0.748***	0.644***	0.971***	0.574***	0.903***
	(0.016)	(0.089)	(0.015)	(0.086)	(0.017)	(0.096)	(0.017)	(0.115)
Algebra	0.327***	0.364***	0.374***	0.357***	0.348***	0.406***	0.320***	0.371***
	(0.013)	(0.067)	(0.013)	(0.090)	(0.012)	(0.092)	(0.013)	(0.073)
Geometry	0.231***	0.392***	0.226***	0.568***	0.261***	0.271*	0.245***	0.427***
	(0.014)	(0.076)	(0.013)	(0.098)	(0.015)	(0.113)	(0.015)	(0.087)
Data Analysis	0.037**	0.038	0.031*	0.052	0.129***	0.094	0.114***	0.052
	(0.014)	(0.108)	(0.014)	(0.120)	(0.014)	(0.096)	(0.015)	(0.097)
Female	-0.095***	-0.097**	-0.099***	-0.072	-0.067***	-0.020	-0.102***	-0.149***
	(0.009)	(0.036)	(0.010)	(0.046)	(0.010)	(0.050)	(0.010)	(0.043)
Language	0.341***	0.401***	0.307***	0.259***	0.221***	0.318***	0.321***	0.396***
	(0.016)	(0.070)	(0.014)	(0.066)	(0.015)	(0.057)	(0.016)	(0.053)
Intercept	-2.576***	-2.999***	-2.454***	-2.665***	-2.456***	-2.879***	-2.615***	-3.197***
	(0.027)	(0.149)	(0.028)	(0.220)	(0.027)	(0.223)	(0.029)	(0.166)
ICC	0.261	0.249	0.275	0.245	0.254	0.278	0.229	0.267
N (students)	38210	3420	37149	3268	36186	3162	34377	3046

Note. Language score and mathematics scores were standardized by dividing by two standard deviations (Gelman, 2008). Standard errors in parentheses. The indicated variables are constructed based on different samples corresponding to the four different formats of the test. Random parameters omitted for reasons of space. *** p<0.001, ** p<0.01, * p<0.05, ~p<0.1, two-tailed test

9.1.6. Discussion

Using a nationally-representative sample of students in Chile, this study explored what are the most relevant mathematics content areas to be taught and what is their role in the mathematics socioeconomic achievement inequality. Our results show that Number Skills are key to the students' future performance in mathematics (see Table 9—2). The effect size for this area is ~0.30, which is slightly bigger than the mean effect size found by Hill, Bloom, Black, & Lipsey (2008) for elementary school studies (0.23). This result is relevant considering that the representation of Numbers Skills items on the 8th grade state test is lower than in 4th grade one, i.e. the prevalence of items goes from almost 50% to roughly 40%. As mentioned in the methods section, the estimates for Numbers Skills were found to be smaller when considering a smaller number of items on its scale. However, they were larger or at least equal to Algebra content area's coefficients across all sub-samples and the different sets of questions considered in the sensitivity analysis (see Table D1, Appendix D). The current finding confirms the general belief about the importance of Numbers content area. This evidence also generalizes the importance of the specific skills related to this content area, such as arithmetic skills, that was previously found in small- and medium-scale studies conducted in developed countries. When faced with low levels of curricular implementation and the need to prioritize, this result is particularly relevant. It provides policymakers with correlational evidence regarding the importance of focusing on Number Skills in the early years in order to improve the students' future performance in mathematics.

The regression results show considerable differences in the relationship between the remaining 4th grade-areas of math with future performance. As shown in Table 9—2, Algebra relationship with future performance is stronger than Geometry's and it at least doubles the effect size on future performance when compared to Data Analysis. This finding is complementary to previous studies that highlight the importance of managing Algebra at late elementary school (Liang, Heckman, & Abedi, 2018; Spielhagen, 2006). It was particularly unexpected considering that Algebra have a smaller number of items in the 8th-grade state test than Geometry or than Statistics and Probability (12% compared to 27% and 21%, respectively). This evidence is against the common intuition that a larger number of items, i.e. a broader range of learning objectives would better predict future performance. Additionally, literature showed that knowledge of shapes before entering school is not a predictor for future performance in fifth grade (Rittle-Johnson et al., 2017). However, knowledge of shapes (Geometry) in fourth grade turned to be a strong predictor for eight grade performance (see Table 9—2). Furthermore, the small effect size (<0.1) of Data Analysis content area stress the importance of exploring whether this content area should have as much attention as the algebraic concepts during the very first elementary school years. The Chilean 4th grade test includes same amount of questions in both areas (Geometry and Data Analysis) suggesting equal relevance for the curriculum, however, it is necessary to acknowledge that the number of items does not necessarily represent the effective coverage each of the areas receive.

The results in Table 9—2 also reveal the importance of the students' performance in 4th grade language arts and gender to their overall performance in mathematics. With regards to the first predictor, having a good score on the SIMCE test in language arts turned out to be almost as important as Algebra, and more important than Geometry or Data analysis. This finding is consistent with previous research, which has revealed the association between performance in language arts and performance in mathematics (Mullis et al., 2013; Norris & Phillips, 2003; Savolainen et al., 2008; Vilenius-Tuohimaa et al., 2008). In addition to this, it also provides further evidence regarding the importance of reinforcing or ensuring basic literacy in areas such as numeracy, reading and writing. With regards to gender, the disadvantage found for female students is consistent with previous findings for Chile based on results from the 2003 and 2011 TIMSS (Mullis, Martin, Foy, & Arora, 2012; OECD, 2012). Additionally, gender disadvantages showed to be more important than data analysis to the students' future performance in mathematics, but less than the remaining content areas (see Table 9—2).

Regarding the relationship between students' prior knowledge in mathematics, students' achievement in the future and the socioeconomic context where they were taught, three major insights should be considered. Firstly, the importance of Numbers Skills is further reinforced by the fact that the effect of a student's performance in this area is the same as the effect of their socioeconomic status, reflected by the type of school they attend (see Table 9—2). Secondly, the remaining areas relative importance change depending on the type of school where they were measured (see Table 9—3). Thirdly, the magnitude of the effect size of Numbers Skills and Geometry increased when measured in private elite schools in comparison with public schools, reflecting that advantages and disadvantages in prior knowledge are more beneficial or detrimental for the student's future achievement (see Table 9—3). The basic literacies related to Numbers Skills, such as counting up to 50, on average are managed by the students earlier in the year in private schools (Ministry of Education, 2004), which could potentially free space for developing a better understanding and manipulations of tens (Ministry of Education, 2004). However, our results suggest that whether students in private elite schools who do not manage the prior knowledge in Numbers Skills, they will be more likely to fall behind and may acquire the crucial abilities later than required for their math class. In the same line, in the case of Geometry, teachers working in higher socioeconomic contexts indicate that they cover a larger number of geometrical shapes (Ministry of Education, 2004), neglecting perhaps the management of the most basic figures. Fourthly, the estimated effect size for Algebra remains quite steady independently of the type of school where the content area was measured. This means that managing Algebra is equally fruitful for students' future achievement, regardless of the type of school it is taught.

9.1.7. Conclusions, Future Work and Limitations

This study looks to answer two research questions. Firstly, which areas of mathematics are the best predictors of a student's future performance in this subject? The result of our regressions analysis showed that Numbers Skills is the content area most strongly related to future performance and its association with future achievement is even stronger than the socioeconomic context of the school (see Table 9—2). The relative importance of the remaining areas varies depending on the type of school where they are measured (see Table 3). Secondly, what is the relationship between the students' prior knowledge of certain areas of mathematics, their overall future performance in the subject and the socioeconomic context? As discussed previously, the predictive relevance of Number Skills and Geometry is stronger for private elite schools in comparison with public schools. These differences might be driven by the depth and complexity of the teaching practices, widening the gap for students who are exposed to more learning opportunities, but are not prepared to receive the new knowledge. This interpretation highlight the importance of studying further what are the most important areas of mathematics to prioritize and what is the effect of exposing students to more complex knowledge before acquiring the minimum and basic skills. It also underlines the need of examining the role that this content areas can play as a moderator for socioeconomic inequality. This is particularly relevant in countries such as Chile, where socioeconomic and cultural differences among students are some of the greatest among OECD nations, as shown by the PISA results (OECD, 2013a).

We warn the readers not to interpret these results as: “we should expose (or deprive) all students to harder and more complex context”. Students who are exposed to more learning opportunities, may achieve more in math, but might also develop more negative emotions around the subject (Brunyé et al., 2013; Hannula, 2002) . On the contrary, these results should encourage researchers to explore the different aspects of the specific content areas in order to improve the way they are taught and to spend time on how to ensure the most crucial skills that could be leaving students' behind.

As mentioned previously, language skills are also key to the students' future performance in mathematics (see Table 9—2 and Discussion Section above). Considering the importance of language skills, it would be important to study whether these skills are particularly relevant for the case for mathematics, or whether it is also true for other subjects such as science. Gender association with future performance was weaker when compared to most of the prior knowledge areas (Numbers Skills, Algebra, Geometry and Language), but it was stronger than the association between Data Analysis and forthcoming achievement. As this is a descriptive study, we cannot assert causal explanations for relationship between gender and future performance. This difference may be due to the differences in how male and female students process many special tasks (Logan & Lowrie, 2017). However, it may also not actually reflect a difference in ability but instead be related to other factors, such as what girls are told and their beliefs about gender differences in mathematics (Hill, Corbett, & St Rose, 2010). Alternatively, it may also be due to differences in the way students cope with the testing experience (Dweck, 2006).

This study has certain limitations that should be acknowledged. Firstly, a test that is originally intended to collect information on a unidimensional ability (overall performance in 4th grade mathematics) was used to estimate area-specific scores. This is also a limitation in terms of the number of items per area per format, which was not always the same. This could be improved if area-specific tests were designed and applied with 4th grade students in order to analyze their association with future performance in mathematics. Secondly, some people might argue that Numbers Skills is the most important one because it could be the dimension that is less correlated to specific knowledge acquired through studying, or because it could be related with non-cognitive dimensions and not solely to cognitive ones. In other words, it could be assessing more of a general ability (or certain abilities) rather than content management. While this study does not have information to assert these hypothesis, future research could address these questions by also including students math-specific abilities and, attitudinal and motivational covariates. Thirdly, this study does not account for prior contributions to the differing areas of math learning before 4th grade, however, robustness check showed that the magnitude of the coefficients remained quite stable when adding students' performance in second and third grade (Table B2, Appendix B). This result suggests a weak relationship between the relative importance of the different content areas on students' future mathematics performance and their prior performance. Future studies could explore further this relation. Furthermore, as highlighted previously, our analyses are descriptive in nature. From descriptive work a policy cannot directly be prescribed. However, based on these findings researchers can spend their resources on analyzing what are the best policies, for example, for school score-based accountability. In order to advance towards a causal interpretation of our findings, future research should include causal designs and evaluate how sensitive our findings are to confounding.

9.2. Appendix 2. Supplementary Material in "Having Fun Doing. Math…"

9.2.1 Appendix 2—A Balance Check

Table Appendix 2—A1

Balance check between treatment and control conditions among students with available long term outcome (sample used in Models 4, 5, 6, and 7 of Table2)

	Control			Treatment			Sig. diff.
Variable	N	Mean	SD	N	Mean	SD	p
Grade	25			26			
8th grade	25	.40		26	.46		0.48
9th grade	25	.32		26	.29		0.49
10th grade	25	.28		26	.25		0.93
Male	25	.48		26	.50		0.23
Previous academic achievement							
2013 Math GPA	22	4.59	0.96	26	4.83	0.80	0.08†
2013 Overall GPA	22	5.50	0.54	26	5.60	0.56	0.45
2014 Math GPA	22	4.83	0.91	26	4.78	1.07	0.65
2014 Overall GPA	22	5.55	0.63	26	5.56	0.65	0.70
2015 Math GPA	22	4.87	1.05	26	4.80	1.06	0.99
2015 Overall GPA	22	5.41	0.57	26	5.40	0.65	0.95
Fall 2016 Math GPA	25	5.09	1.38	26	4.96	1.09	0.68

Note. The p-value is estimated using fixed effects. † p<0.1. GPA means reported here correspond to the GPA for students from 8th, 9th and 10th grade together. Sample used in this table includes all students who participated in the study for whom there is information on the average Math GPA of Fall 2017 (N=51). Thus, 2 students from the treatment condition and 3 students from the control condition are left out of this sample.

9.2.2. Appendix 2—B Text Messages Treatment Group

Test messages for the treatment group, by week

During the first two weeks of the intervention, the parents were informed that they would receive text messages with activities or academic information on their child (whose name was no more than 12 characters long), based on their randomly-assigned group (treatment or control). Table B1 shows the messages that were received by all of the parents in the treatment group during the first two weeks. These messages were personalized to include the name of the student, shown here as [Name 12]. However, from week 3 each grade (8^{th}, 9^{th} and 10^{th} grade) received a set of messages with a differentiated activity based on the corresponding objective taken from the math curriculum (see Table B2).

Table Appendix 2—B1. Text messages with an invitation to the treatment group

Week	Day	Grade(s)	Message
1	Friday	8^{th}, 9^{th} and 10^{th}	Hi, it's Miss Cami. Every Friday you will receive a message with an activity to do with [Name 12]. They will be short and simple!
2	Friday	8^{th}, 9^{th} and 10^{th}	We hope that you help [Name 12] by doing the activity and answering the question you receive via text message (or WhatsApp).

Table Appendix 2—B2. Differentiated text messages for the treatment group by week and by grade

Week	Day (Category)	Grade(s)	Description of Message
3	Friday (The "soccer players activity")	8^{th}	Hi, it's Miss Cami. With [Name 12] you should choose a position in soccer, for example central defender. Find 5 soccer players who play in that position.
3	Friday (The "soccer teams activity")	9^{th}	Hi, it's Miss Cami. With [Name 12] you should think of a professional footballer who you both like. Research and name which teams this player has played for.
3	Friday (The "ground activity")	10^{th}	Hi, it's Miss Cami. Using any kind of object (clothes, spoons, etc.), make two figures that are the same shape but different sizes. Take a photo of them.
4	Monday (Reminder - Teacher Contact)	8^{th}, 9^{th} and 10^{th}	Hi, it's Miss Cami. Were you able to do the activity? What did you think of it?
4	Wednesday (Teacher Suggestion)	8^{th}, 9^{th} and 10^{th}	During this weekend's activity, we recommend asking your child how they feel in math class. Listen to how they feel and don't criticize them.
4	Friday (The "habits activity")	8^{th}	Hi, it's Miss Cami. You should talk about the activities you like to do during the week and each of you should choose one. In general, how many times do you do this activity each week?
4	Friday (The "poem activity")	9^{th}	Hi, it's Miss Cami. Think of a poem. Choose a song that you both like and change the lyrics for the words in the poem. Record yourselves singing it (audio or video).
4	Friday (The "container activity")	10^{th}	Hi, it's Miss Cami. Talk about the largest container, pot or jar that you have ever filled with water or any other liquid. How much liquid did it hold?
5	Monday (Reminder - Teacher Contact)	8^{th}, 9^{th} and 10^{th}	Hi, it's Miss Cami! Were you able to talk with [Name 12]?
5	Wednesday (Teacher Suggestion)	8^{th}, 9^{th} and 10^{th}	During this weekend's activity, we recommend telling your child that you are proud of her when she tries to get better at math and that you value her effort.

Table Appendix 2—B2. Differentiated text messages for the treatment group by week and by grade (continued…)

Week	Day (Category)	Grade(s)	Description of Message
5	Friday (The "hands and feet activity")	8th	Hi, it's Miss Cami. With [Name 12] you should measure the front of your house using your feet and note it down. Later, measure it again, but this time using your hands.
5	Friday (The "sounds activity")	9th	Hi, it's Miss Cami. With [Name 12] go outside. Look for and listen to 5 outdoor sounds (such as a barking dog, a car or a bird) and try to copy them.
5	Friday (The "mapping activity")	10th	Hi, it's Miss Cami. With ##name## draw a detailed map of your house on a blank sheet of paper. Include the walls, doors and furniture, etc.
6	Monday (Reminder - Teacher Contact)	8th, 9th and 10th	Hi, it's Miss Cami. What did you think of this week's activity?
6	Wednesday (Teacher Suggestion)	8th, 9th and 10th	During this weekend's activity, we recommend telling your child that you are proud of him/her when he/she works with you on his/her homework.
6	Friday (The "bus stop activity")	8th	Hi, it's Miss Cami. With [Name 12] you should go to a bus stop. Each of you should choose a color. For 10 minutes, how many cars of that color passed by?
6	Friday (The "favorite food activity")	9th	Hi, it's Miss Cami. With [Name 12] go outside and ask 5 people what their favorite food is. Write down their answers.
6	Friday (The "nearby place activity")	10th	Hi, it's Miss Cami. With [Name 12] think of somewhere close to your house. Do you think you know how far it is from your house? Try to measure it.
7	Monday (Reminder - Teacher Contact)	8th, 9th and 10th	Were you able to do the activity?
7	Wednesday (Teacher Suggestion)	8th, 9th and 10th	[Name 12] can do anything if he/she puts his/her mind to. This weekend, tell him/her that you believe this.
7	Friday (The "bugs activity")	8th	Hi, it's Miss Cami. With [Name 12] you should go outside and take a glass with you. Look for some bugs and put them in the glass. How many bugs would it take to fill the glass?
7	Friday (The "relatives' ages activity")	9th	Hi, it's Miss Cami. With [Name 12] talk about the age of your relatives and the people who live in your house. If you can, write them down.
7	Friday (The "rock-paper-scissors activity")	10th	Hi, it's Miss Cami. Make yourselves comfortable and play rock, paper, scissors 20 times. In a notebook, write down how many times each person wins.

9.2.3. Appendix 2—C Text Messages Control Group

Text messages for the control group, by week, grade and group

Table Appendix 2—C1 shows the messages that were sent every Friday to the parents in the control group, depending on the student's grade level.

Table Appendix 2—C1. Text messages per grade for the control group

<table>
<tr><th colspan="4">Control Group: Informative Messages</th></tr>
<tr><td>Week</td><td colspan="3">Messages for all grade levels.</td></tr>
<tr><td>1</td><td colspan="3">Hi, it's Miss Cami. This term I will send a weekly reminder of the student's math tests, quizzes and homework.</td></tr>
<tr><td>2</td><td colspan="3">Hi, it's Miss Cami. This term I will send a weekly reminder of the student's math tests, quizzes and homework.</td></tr>
<tr><td>Week</td><td>8th Grade</td><td>9th Grade</td><td>10th Grade</td></tr>
<tr><td>3</td><td>This Wednesday there is a math quiz. Regards.</td><td>This Monday there is a math test. Regards.</td><td>There's a quiz this Wednesday. Regards.</td></tr>
<tr><td>4</td><td>This Tuesday they should complete the exercises and take them to school. Regards.</td><td>This Wednesday there is an open-book quiz. Regards.</td><td>This Tuesday they should complete the exercises and take them to school. Regards.</td></tr>
<tr><td>5</td><td>This Wednesday there is a math test. Regards</td><td>This Tuesday they should complete the exercises and take them to school. Regards.</td><td>This Wednesday there is an open-book quiz. Regards.</td></tr>
<tr><td>6</td><td>This Wednesday there is an open-book quiz. Regards.</td><td>This Monday there is an open-book quiz. Regards.</td><td>This Wednesday there is an open-book quiz. Regards.</td></tr>
<tr><td>7</td><td>This Tuesday they should complete the exercises and take them to school. Regards.</td><td>This Monday there is a math test. Regards.</td><td>This Tuesday they should complete the exercises and take them to school. Regards.</td></tr>
</table>

9.2.4. Appendix 2—D Tables

Table Appendix 2—*D1*
Correlations between annual math GPA and overall GPA per year and grade.

	8th grade	9th grade	10th grade
2013	0.76	0.28	0.75
2014	0.65	0.54	0.50
2015	0.50	0.20	0.82

Note. Correlations between annual math GPA and overall GPA for the entire sample (8th, 9th and 10th grade) are 0.83, 0.83 and 0.85 for 2013, 2014 and 2015, respectively. **The correlations between yearly math GPAs across years are 0.76 (2013 and 2014), 0.85 (2014 and 2015) and 0.75 (2013 and 2015). The correlations between yearly overall GPAs across years are 0.87 (2013 and 2014), 0.88 (2014 and 2015) and 0.90 (2013 and 2015).**

Table Appendix 2—*D2*
Effects of Non-Academic Tasks on Academic Achievement in Math for Spring 2016 and Fall 2017 (three composite covariates for previous academic performance)

	Model 1 No controls, only group dummies.		Model 2, controlling for previous achievement in math		Model 3, controlling for previous achievement in math and overall		Model 4, Model 3 using sample of Models 5,6,7.		Model 5, No controls, only group dummies.		Model 6, controlling for previous achievement in math		Model 7, controlling for all previous achievement	
	Effect on Math GPA, Spring 2016[a]								**Effect on Math GPA, Fall 2017[b]**					
	β	SE	β	SE	β	SE	β	SE	β	SE	β	SE	β	SE
Treatment	0.416	0.260	0.388 †	0.221	0.467*	0.226	0.490*	0.233	0.339	0.253	0.415*	0.177	0.392*	0.192
N	56		56		56		51		51		51		51	
R^2	0.11		0.38		0.44		0.55		0.25		0.64		0.65	
Controls														
Fall 2016 Math GPA			X		X		X				X		X	
Male					X		X						X	
Annual Math and Overall GPA[c] (years 2015, 2014, 2013)					X		X						X	
Group dummies	X		X		X		X		X		X		X	

Note: †$p<0.10$; * $p<0.05$. Outcome variables and continuous covariates are standardized within sample (M=0; SD=1).

[a] Short term outcome is the grade point average between October and December, the third academic term in 2016.

[b] Long term outcome is the grade point average between March and May 2017, the first academic term in 2017.

[c] Three covariates: average between math GPA and overall GPA at end of each academic year (March and December) for 2013, 2014 and 2015. Imputed for 6 missing cases.

Table Appendix 2—*D3*
Effects of Non-Academic Tasks on Academic Achievement in Math for Spring 2016 and Fall 2017 (one composite covariate for previous academic performance)

	Model 1 No controls, only group dummies.		Model 2, controlling for previous achievement in math		Model 3, controlling for previous achievement in math and overall		Model 4, Model 3 using sample of Models 5,6,7.		Model 5, No controls, only group dummies.		Model 6, controlling for previous achievement in math		Model 7, controlling for all previous achievement	
	Effect on Math GPA, Spring 2016[a]								**Effect on Math GPA, Fall 2017[b]**					
	β	SE	β	SE	β	SE	β	SE	β	SE	β	SE	β	SE
Treatment	0.416	0.260	0.388 †	0.221	0.424†	0.225	0.450*	0.218	0.339	0.253	0.415*	0.177	0.387*	0.183
N	56		56		56		51		51		51		51	
R^2	0.11		0.38		0.40		0.50		0.25		0.64		0.65	
Controls														
Fall 2016 Math GPA			X		X		X				X		X	
Male					X		X						X	
Previous achievement composite (years 2015, 2014, 2013)[c]					X		X						X	
Group dummies	X		X		X		X		X		X		X	

Note: †$p<0.10$; * $p<0.05$. Outcome variables and continuous covariates are standardized within sample (M=0; SD=1).

[a] Short term outcome is the grade point average between October and December, the third academic term in 2016.

[b] Long term outcome is the grade point average between March and May 2017, the first academic term in 2017.

[c] One composite: average between the 2013-2015 covariates, corresponding to math GPA and overall GPA at end of each academic year (March and December). Imputed for 6 missing cases.

Table Appendix 2—*D4 Model 4, full specifications.*

Outcome	Spring 2016 Math GPA	Fall 2017 Math GPA
Treatment	0.488	0.413
	(0.230)*	(0.194)*
Fall 2016 Math Grade	0.588	0.822
	(0.184)**	(0.155)***
2013 Math GPA	0.083	0.149
	(0.240)	(0.203)
2014 Math GPA	-0.028	0.064
	(0.274)	(0.232)
2015 Math GPA	0.158	-0.467
	(0.381)	(0.322)
2013 Overall GPA	-0.468	-0.378
	(0.321)	(0.271)
2014 Overall GPA	-0.016	-0.096
	(0.338)	(0.285)
2015 Overall GPA	0.597	0.549
	(0.396)	(0.334)
Male	0.205	-0.167
	(0.246)	(0.208)
Constant	-0.285	-0.204
	(0.187)	(0.158)
R^2	0.55	0.68
Adjusted R2	0.43	0.59
N	51	51
F	4.24	6.18

Notes: * p<0.05, ***p<0.001

Table Appendix 2—*D5*
Effects of Non-Academic Tasks on Academic Achievement in Math for Spring 2016 and Fall 2017 (raw outcomes)

	Model 1 No controls, only group dummies.		Model 2, controlling for previous achievement in math		Model 3, controlling for previous achievement in math and overall		Model 4, Model 3 using sample of Models 5,6,7.		Model 5, No controls, only group dummies.		Model 6, controlling for previous achievement in math		Model 7, controlling for all previous achievement	
	Effect on Math GPA, Spring 2016[a]								**Effect on Math GPA, Fall 2017[b]**					
	β	SE	β	SE	β	SE	β	SE	β	SE	β	SE	β	SE
Treatment	0.343	0.215	0.317†	0.182	0.373†	0.191	0.420*	0.185	0.384	0.286	0.470*	0.198	0.459*	0.217
N	56		56		56		51		51		51		51	
R^2	0.11		0.38		0.44		0.53		0.25		0.65		0.66	
Controls														
Fall 2016 Math GPA			X		X		X				X		X	
Male					X		X						X	
Annual Math and Overall GPA[c] (years 2015, 2014, 2013)					X		X						X	
Group dummies	X		X		X		X		X		X		X	
Outcome mean				5.602								4.941		
Outcome SD				0.825								1.131		

Note: †p<0.10; * p<0.05. Outcome variables are standardized (M=0; SD=1). Continuous covariates were standardized within class to have a mean of 0 and standard deviation of 1 in each class.

[a] Short term outcome is the grade point average between October and December, the third academic term in 2016.

[b] Long term outcome is the grade point average between March and May 2017, the first academic term in 2017.

[c] Average between math GPA and overall GPA at end of each academic year (March and December) for 2013, 2014 and 2015. Imputed for 6 missing cases.

9.2.5. Appendix 2—E Figures

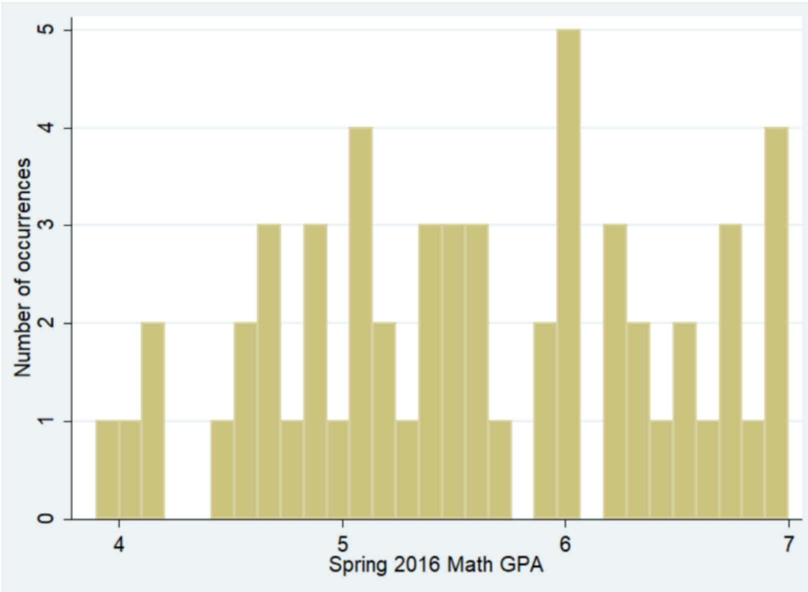

Figure E1. Number of occurrences of Spring 2016 Math GPA outcome

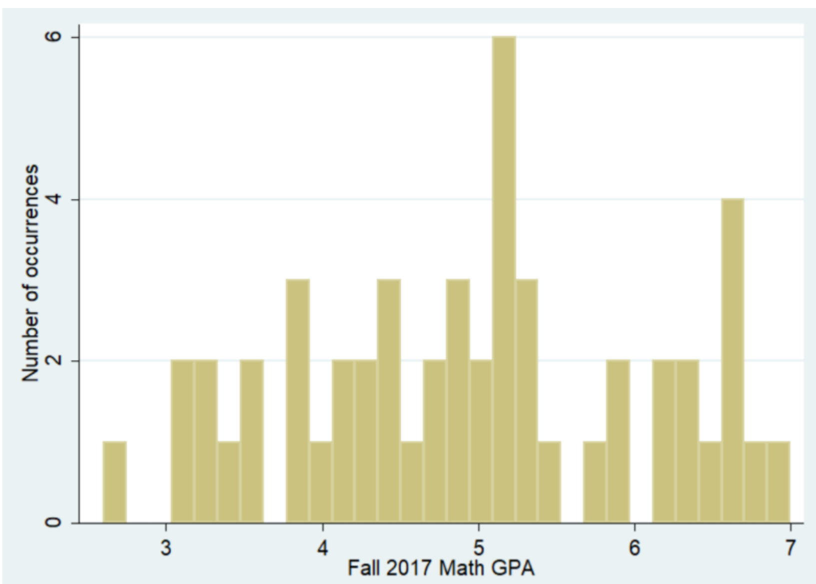

Figure E2. Number of occurrences of Fall 2017 Math GPA outcome

9.3. Appendix 3. Supplementary Material in "Let's Spend Time Together …"

9.3.1. Appendix 3—A Messages by teacher and grade

Table Appendix 3—A1. Fridays' messages: Activities sent to parents of 9th grade students

		Teacher 3, 4 and 5	Teacher 6	Teacher 9	Teacher 11
Week 1	Message	Hello, it's [Teacher's name]. Look for a piece of paper with [Student's name] and cut it in half. See how many times you can keep cutting it in half.	Hello, it's [Teacher's name]. Look for a piece of paper with [Student's name] and cut it in half. See how many times you can keep cutting it in half.	Hello, it's [Teacher's name]. Have a race with [Student's name], giving them a 10 second head-start before trying to catch them.	Hello, it's [Teacher's name]. Go somewhere with [Student's name] that isn't built-up and look at the mountains. Try to find two mountains that are similar shapes.
Week 2		Hello, it's [Teacher's name]. Go outside with [Student's name] and listen for five different sounds (such as a dog barking, a car or a bird). Try to imitate them.	Hello, it's [Teacher's name]. Work with [Student's name] by putting different items of fruit on a shelf and grouping them by color. Then, try to identify other things that each group has in common.	Hello, it's [Teacher's name]. With [Student's name], take a bunch of pens and pencils and count how many different colors there are. Then count how many there are of each color.	Hello, it's [Teacher's name]. Look for some clothes hangers or other triangular objects with [Student's name] and describe what happens when you hang them (do they move?).
Week 3		Hello, it's [Teacher's name]. Work with [Student's name] by putting different items of fruit on a shelf and grouping them by color. Then, try to identify other things that each group has in common.	Hello, it's [Teacher's name]. With [Student's name], think of a car or vehicle that you both like. Draw all of the different parts of the car/vehicle that you know (for example, the wheels).	Hello, it's [Teacher's name]. With [Student's name], think of a professional footballer who you both like. Research and write down a list of all the teams that player has played for.	Hello, it's [Teacher's name]. With [Student's name], decide which shape is larger: a circle, a star or a square. There isn't just one right answer.
Week 4		Hello, it's [Teacher's name]. With [Student's name], imagine that you are going to eat a pizza. Who else would you invite? How many pizzas would you buy? How many slices would each person eat?	Hello, it's [Teacher's name]. Talk with [Student's name] about a poem or part of a song that you know by memory.	Hello, it's [Teacher's name]. Talk with [Student's name] about a poem or part of a song that you know by memory.	Hello, it's [Teacher's name]. With [Student's name], imagine that you are going to eat a pizza. Who else would you invite? How many pizzas would you buy? How many slices would each person eat?
Week 5		Hello, it's [Teacher's name]. Talk with [Student's name] about a poem or part of a song that you know by memory.	Hello, it's [Teacher's name]. Go outside with [Student's name] and listen for five different sounds (such as a dog barking, a car or a bird). Try to imitate them.	Hello, it's [Teacher's name]. Go for a walk around the block with [Student's name]. Have a conversation and see how long it takes you to go around the block.	Hello, it's [Teacher's name]. Talk with [Student's name] and think about a product that has recently gone up in price.

Week 6		Hello, it's [Teacher's name]. With [Student's name], think of a professional footballer who you both like. Research and write down a list of all the teams that player has played for.	Hello, it's [Teacher's name]. With [Student's name], think of a professional footballer who you both like. Research and write down a list of all the teams that player has played for.	Hello, it's [Teacher's name]. Survey at least 7 people who live on your block or near your house. Ask them how old they are and how many people live in their house.	Hello, it's [Teacher's name]. Look for a piece of paper with [Student's name] and cut it in half. See how many times you can keep cutting it in half.
Week 7		Hello, it's [Teacher's name]. Talk with [Student's name] about the summer and your favorite ice cream. Count how many ice creams you could buy with 2,000 pesos.	Hello, it's [Teacher's name]. Have a race with [Student's name], giving them a 10 second head-start before trying to catch them.	Hello, it's [Teacher's name]. Think about famous people with [Student's name]. For every 2 famous people that [Student's name] can name, you should name 1.	Hello, it's [Teacher's name]. With [Student's name], take some pieces of fruit (the same fruit) and see how many you need in order for it to weigh as much as a kilo of rice.
Week 8		Hello, it's [Teacher's name]. Work with [Student's name] to name 10 famous people who appear on TV. Write down how many each of you came up with	Hello, it's [Teacher's name]. Talk with [Student's name] about the summer and your favorite ice cream. Count how many ice creams you could buy with 2,000 pesos.	Hello, it's [Teacher's name]. Talk about your favorite ice creams with [Student's name] and say whether you prefer ice lollies or ice creams in a cone.	Hello, it's [Teacher's name]. Play heads or tails with [Student's name]. Toss the coin 5 times and see who guesses heads or tails correctly more times. Enjoy!
Week 9		Hello, it's [Teacher's name]. Have a race with [Student's name], giving them a 10 second head-start before trying to catch them.	Hello, it's [Teacher's name]. With [Student's name], think about which socks you'd like to use on each day of the week. Then, count how many of each color there are, as well as choosing your favorites.	Hello, it's [Teacher's name]. Find a wheel with [Student's name] (for example, on a car, truck, bicycle or motorbike) and look at all of the measurements you can think of. Write them down.	Hello, it's [Teacher's name]. With [Student's name], talk about things that happen in nature that you think are important.
Week 10		Hello, it's [Teacher's name]. With [Student's name], take a bunch of pens and pencils and count how many different colors there are. Then count how many there are of each color.	Hello, it's [Teacher's name]. Think about famous people with [Student's name]. For every 2 famous people that [Student's name] can name, you should name 1.	Hello, it's [Teacher's name]. Take a walk with [Student's name] along a street with trees or houses that are similar to one another. Look at the trees or houses and see how they are similar.	Hello, it's [Teacher's name]. With [Student's name], think of a car or vehicle that you both like. Draw all of the different parts of the car/vehicle that you know (for example, the wheels).
Week 11		Hello, it's [Teacher's name]. With [Student's name], choose two different kinds of fruit. How many pieces of each fruit do you think you would need in order for it to weigh as much as a kilo of rice?	Hello, it's [Teacher's name]. Go for a walk around the block with [Student's name]. Have a conversation and see how long it takes you to go around the block.	Hello, it's [Teacher's name]. Find a sunny spot and use steps to measure how tall [Student's name] is and how long their shadow is. Do the same with your height and the length of your shadow.	Hello, it's [Teacher's name]. With [Student's name], find a coin, some paper and a pen. Draw around the coin in two different parts of the paper. Measure the distance between the two.

Week 12		Hello, it's [Teacher's name]. Think about famous people with [Student's name]. For every 2 famous people that [Student's name] can name, you should name 1.	Hello, it's [Teacher's name]. With [Student's name], take a bunch of pens and pencils and count how many different colors there are. Then count how many there are of each color.	Hello, it's [Teacher's name]. Go outside with [Student's name] and look for two buildings. Count how many floors there are in each building and use steps to measure how long their shadows are.	Hello, it's [Teacher's name]. Think about famous people with [Student's name]. For every 2 famous people that [Student's name] can name, you should name 1.

Note. The messages shown in tables A1, A2 and A3 were personalized to include the name of the student, shown here as [Student's name] and the name of the teacher, show here as [Teacher's name]. Colors are used to indicate the content area the activity is related to: blue represents numbers and arithmetic, green represent geometry, pink algebra and yellow data analysis.

Table Appendix 3—A2. Fridays' messages: Activities sent to parents of 10th grade students

	Teacher 1	Teacher 2	Teacher 7	Teacher 8	Teacher 10
Week 1	Hello, it's [Teacher's name]. Have you ever played chess or checkers? Talk with [Student's name] about how many squares you think there are on the board.	Hello, it's [Teacher's name]. Have you ever played chess or checkers? Talk with [Student's name] about how many squares you think there are on the board.	Hello, it's [Teacher's name]. With [Student's name], draw a square (with all four sides the same length). Draw a line between two opposite corners. Try to measure the lines in your drawing.	Hello, it's [Teacher's name]. With [Student's name], write a different number of 3 pieces of paper. Then choose two of them without looking. Write down the numbers that were drawn. Repeat 5 times.	Hello, it's [Teacher's name]. Look for a piece of paper with [Student's name] and cut it in half. See how many times you can keep cutting it in half.
Week 2	Hello, it's [Teacher's name]. If you can, go to the park with [Student's name] and feed the pigeons. Count how many there are at the beginning and how many by the end.	Hello, it's [Teacher's name]. If you can, go to the park with [Student's name] and feed the pigeons. Count how many there are at the beginning and how many by the end.	Hello, it's [Teacher's name]. Have you ever played chess or checkers? Talk with [Student's name] about how many squares you think there are on the board.	Hello, it's [Teacher's name]. Talk with [Student's name] about how tall different members of the family are. If possible, write down their heights on a piece of paper.	Hello, it's [Teacher's name]. With [Student's name], take 4 steps forward. Repeat 3 times. Now count how many steps you need to go back to where you were.
Week 3	Hello, it's [Teacher's name]. Find a wheel with [Student's name] (for example, on a car, truck, bicycle or motorbike) and look at all of the measurements you can think of. Write them down.	Hello, it's [Teacher's name]. Talk with [Student's name] about the largest container that you've ever filled with water or juice. How much liquid was there?	Hello, it's [Teacher's name]. Talk with [Student's name] about the largest container that you've ever filled with water or juice. How much liquid was there?	Hello, it's [Teacher's name]. With [Student's name], draw a square (ideally with sides of 5cm). Draw a line between two of its corners. Try to measure this line.	Hello, it's [Teacher's name]. Work with [Student's name] to split 20 stones into two bags. Look for stones in the first bag that are the same as a stone in the second bag.
Week 4	Hello, it's [Teacher's name]. With [Student's name], draw two figures in the ground that are the same shape but different sizes. If you can, take a photo of them.	Hello, it's [Teacher's name]. With [Student's name], draw two figures in the ground that are the same shape but different sizes. If you can, take a photo of them.	Hello, it's [Teacher's name]. With [Student's name], take 4 steps forward. Repeat 3 times. Now count how many steps you need to go back to where you were.	Hello, it's [Teacher's name]. With [Student's name], take 5 steps forward and then 12 steps to the right. Count how many steps there are from the point where you started to the point where you ended up and write it down	Hello, it's [Teacher's name]. Try cooking something with [Student's name]. If possible, check whether the ingredients are out of date
Week 5	Hello, it's [Teacher's name]. With [Student's name], imagine that you are walking around the inside of the Great Pyramids of Egypt. How do you imagine they'd be from the inside?	Hello, it's [Teacher's name]. Look for a piece of paper with [Student's name] and cut it in half. See how many times you can keep cutting it in half.	Hello, it's [Teacher's name]. With [Student's name], think of or find at least 2 different ways you could call a football.	Hello, it's [Teacher's name]. With [Student's name], look for a card (travel card, credit card etc.). Between the two of you, design and draw a new card.	Hola soy **nombre**. Junto a ##nombre## vayan al parque y miren a lo lejos. Compare que tan lejos puede ver cada uno. Aprovechen de divertirse en el parque

Week 6	Hello, it’s [Teacher’s name]. Go somewhere with [Student’s name] that isn’t built-up and look at the mountains. Try to find two mountains that are similar shapes.	Hello, it’s [Teacher’s name]. Talk with [Student’s name] about things that are useless on their own but useful when joined together.	Hello, it’s [Teacher’s name]. Talk with [Student’s name] about things that are useless on their own but useful when joined together.	Hello, it’s [Teacher’s name]. Find a sunny spot and use steps to measure how tall [Student’s name] is and how long their shadow is. Do the same with your height and the length of your shadow.	Hello, it’s [Teacher’s name]. Talk with [Student’s name] about things that are useless on their own but useful when joined together.
Week 7	Hello, it’s [Teacher’s name]. Find a sunny spot and use steps to measure how tall [Student’s name] is and how long their shadow is. Do the same with your height and the length of your shadow.	Hello, it’s [Teacher’s name]. With [Student’s name], draw two figures in the ground that are the same shape but different sizes. If you can, take a photo of them.	Hello, it’s [Teacher’s name]. Work with [Student’s name] to split 20 stones into two bags. Look for stones in the first bag that are the same as a stone in the second bag.	Hola soy **nombre**. Imaginen que visitan una piramide en Mexico. Junto a ##nombre## conversen acerca de como creen que seria la sombra de la piramide.	Actividad de esta semana: Junto a ##nombre## busquen algunas frutas y luego agrupenlas por color, determinen alguna caracteristica comun en los grupos.
Week 8	Hello, it’s [Teacher’s name]. Go outside with [Student’s name] and look for two buildings. Count how many floors there are in each building and use steps to measure how long their shadows are.	Hello, it’s [Teacher’s name]. Go somewhere with [Student’s name] that isn’t built-up and look at the mountains. Try to find two mountains that are similar shapes.	Hello, it’s [Teacher’s name]. With [Student’s name], try different drinks and see which are more sour and which are less.	Hello, it’s [Teacher’s name]. Look for a piece of paper with [Student’s name] and cut it in half. See how many times you can keep cutting it in half.	Hello, it’s [Teacher’s name]. With [Student’s name], think of a car or vehicle that you both like. Draw all of the different parts of the car/vehicle that you know (for example, the wheels).
Week 9	Hello, it’s [Teacher’s name]. Talk about things that are typical of September 18th (national holidays). Draw a kite and write down any measurements you can think of.	Hello, it’s [Teacher’s name]. Go outside with [Student’s name] to look at the stars and identify different triangles. If it’s not a clear night, you can look at stars on the internet instead.	Hello, it’s [Teacher’s name]. With [Student’s name], drink some fruit juice (such as orange or banana) and drink some water. See which you like the most and which is more sour.	Hola soy **nombre**. Midan la altura de una puerta usando las manos de ##nombre##. Midan tambien la altura de la mitad de la puerta.	Hello, it’s [Teacher’s name]. Talk with [Student’s name] about a poem or part of a song that you know by memory.
Week 10	Hola soy **nombre**. Imaginen que visitan una piramide en Mexico. Junto a ##nombre## conversen acerca de como creen que seria la sombra de la piramide.	Hello, it’s [Teacher’s name]. Talk about things that are typical of September 18th (national holidays). Draw a kite and write down any measurements you can think of.	Hello, it’s [Teacher’s name]. Measure the height of a door using [Student’s name]’s hands. Measure the height of half the door, too.	Hello, it’s [Teacher’s name]. Talk with [Student’s name] about things that are useless on their own but useful when joined together.	Hello, it’s [Teacher’s name]. With [Student’s name], think of a professional footballer who you both like. Research and write down a list of all the teams that player has played for.

Week 11	Hello, it's [Teacher's name]. With [Student's name], take a napkin and fold it by joining two corners to form a triangle. Measure its three sides.	Hello, it's [Teacher's name]. With [Student's name], take a napkin and fold it by joining two corners to form a triangle. Measure its three sides.	Hello, it's [Teacher's name]. Talk with [Student's name] about things that are useless on their own but useful when joined together.	Hola soy **nombre**. Junto a ##nombre## tomen algun jugo de fruta (como de naranja o de platano) y tomen agua. Vean cual les gusta mas y cual es mas acido.	Hello, it's [Teacher's name]. Think about famous people with [Student's name]. For every 2 famous people that [Student's name] can name, you should name 1.
Week 12	Hello, it's [Teacher's name]. Work with [Student's name] to design a watch or a clock. Talk about any watch or clock you know and choose your favorite.	Hello, it's [Teacher's name]. Go to the park with [Student's name] and look into the distance. Compare how far each of you can see. Take advantage of the moment and have fun in the park!	Hello, it's [Teacher's name]. With [Student's name], imagine that you are walking around the inside of the Great Pyramids of Egypt. How do you imagine they'd be from the inside?	Hello, it's [Teacher's name]. If you can, go to the park with [Student's name] and feed the pigeons. Count how many there are at the beginning and how many by the end.	Hello, it's [Teacher's name]. Go for a walk around the block with [Student's name]. Have a conversation and see how long it takes you to go around the block.

Table Appendix 3—A3. Saturdays' messages: Information about the activities and the intervention per treatment arm.

	Activities track (A)		**Script Treatment (B)**		**Differences in messages between treatment tracks (C)**
Week	**Message objective**	**Text message**	**Message objective**	**Text message**	
1	Motivation for completing the activity and clarify that it is not a compulsory activity	La actividad que recibio en su celular es para apoyar a ##nombre## en matemática. No es obligatoria. Al hacerla no hay respuestas correctas ni incorrectas.	Motivation, anxiety and not compulsory activity	La actividad que recibio en su celular es para apoyar a ##nombre## en matemática. No es obligatoria. Al hacerla no hay respuestas correctas ni incorrectas.	Texts are identical for both treatment arms, i.e. Motives Treatment and Script Treatment are reminded that the activity is to support the student, is not compulsory and there are no correct answers.
2	Motivation and not graded activity	La actividad que recibio por mensaje de texto se utilizara y servira para la clase de matematica. Sin embargo, no tiene nota y no es obligatoria.	Motivation and not graded activity	La actividad que recibio por mensaje de texto se utilizara y servira para la clase de matematica. Sin embargo, no tiene nota y no es obligatoria.	Texts are identical for both treatment arms, i.e. Motives Treatment and Script Treatment are reminded about the class usage of the activity and that the activity is not graded and not compulsory.

3	Motivation and not graded activity	Les recuerdo: la actividad que recibio por mensaje de texto se usara y servira para la clase de matematica. Sin embargo, no tiene nota y no es obligatoria.	Motivation and not graded activity	Les recuerdo: la actividad que recibio por mensaje de texto se usara y servira para la clase de matematica. Sin embargo, no tiene nota y no es obligatoria.	Texts are identical for both treatment arms, i.e. same message for Motives Treatment and for Script Treatment
4	Reminder + Reinforcement of good moment together	Hola soy **nombre**. Recuerde hacer la actividad con ##nombre##. El unico requisito es hacerla juntos y aprovechar de compartir un buen momento.	Reminder + Reinforcement of good moment together	Hola soy **nombre**. Recuerde hacer la actividad con ##nombre##. El unico requisito es hacerla juntos y aprovechar de compartir un buen momento.	Texts are identical for both treatment arms, i.e. same message for Motives Treatment and for Script Treatment
5	Objective Boosting	Hola soy **nombre**. La actividad siempre sera simple. Cuando la realizas con ##nombre##, el ejemplo que usamos en clases es mas real y tiene mas sentido	Script: The brain is like a muscle – Growth Mindset	Hola soy **#nombre**. Cuando estes con ##nombre##, aprovecha y dile que el cerebro es como un musculo. El cerebro necesita trabajar para hacerse mas fuerte.	Completely different content for each treatment arm. Motives Treatment is reinforcing the class usage of the activity. Script Treatment is prompting parents to say to their kid about growth mindset.
6	Objective Boosting	Hola soy **nombre**. No es necesario que anoten resultados de la actividad. El ejemplo que ##nombre## ve en clases estara relacionado con lo que hicieron	Script: Effort Value	Hola soy **nombre**. Le recomiendo expresar que used se siente orgulloso cuando ##nombre## se esfuerza por mejorar en matematica. Quien se esfuerza, mejora.	Completely different content for each treatment arm. Motives Treatment is reinforcing the class usage of the activity. Script Treatment is prompting parents to say to their kid. that abilities are growable
7	Not my phone	Hola soy **nombre**. Recuerde que le escribo desde un telefono que no es mio, por lo que si me responde no lo podre ver altiro.	Expectativas and not my phone	Soy **nombre**. Dile a ##nombre## que crees en sus capacidades. Recuerde: le escribo desde un telefono que no es mio y si me responde, no lo podre ver altiro	Both treatment arm are reminded that the phone does not belong to the teacher. In addition, Script Treatment prompt parents to remind the student that they believe en their capabilities.

8	Motivation and not compulsory activity	La actividad que recibio en su celular es para apoyar a ##nombre## en matematica. No es obligatoria. Al hacerla no hay respuestas correctas ni incorrectas.	Script: Effort Value	Soy **nombre**. Aprovecha de decirle a ##nombre## que quien pone suficiente tiempo, esfuerzo y estudio puede desarrollar cualquier habilidad.	Completely different content for each treatment arm. Motives Treatment is encouraging parents to support the student in a non-compulsory acitvity. . Script Treatment is prompting parents to say to their kid about the value of effort and that the skills are growable.
9	Objective Boosting	Hola soy **nombre**. Aproveche las actividades para estar un momento juntos con ##nombre##.	Objective Boosting Script: Effort Value	Soy **nombre**. Aproveche la actividad para estar un momento juntos. Recuerdele a ##nombre## que puede superar cualquier desafio si trabaja y se esfuerza	Both treatment arm are reminded that the activity can be used to share a good parent-student moment together. In addition, Script Treatment prompt parents to remind the student about the value of effort and that that skills are growable.
10	Objective Boosting	Hola soy **nombre**. Si ##nombre## no quiere hacer la actividad con usted, no se enoje. Digale que quiere que esten juntos.	Objective Boosting and parent-student expectation	Hola soy **nombre**. Si ##nombre## no quiere hacer la actividad con usted, no se enoje. Digale que cree mucho en sus capacidades y quiere que esten juntos.	Both treatment arm are reminded that the activity should not be a reason to have an arguement and parents are encourage to communicate that they want to share a good moment with the student. In addition, Script Treatment prompt parents to say they believe in the student's capabilities.
11	First ending	Estamos terminando con la serie de actividades que queria que hicieran. Aunque no las sigan recibiendo, siempre intenten tener buenos momentos juntos.	Fist ending	Estamos terminando con la serie de actividades que queria que hicieran. Aunque no las sigan recibiendo, siempre intente decirle a ##nombre## es muy capaz	Both treatment arm are reminded that the intervention is ending. Motives treatment prompt parents to continue sharing moments with the student. Script treatment prompt parents to continue saying the student that he/she is very capable.

12	End and Trascend	Ayer les envie la ultima actividad que tenia preparada para ustedes. Espero puedan seguir creando espacios para estar juntos y fortalecer su relacion.	End and Trascend	Ayer les envie la ultima actividad que tenia preparada para ustedes. Espero puedan seguir creando espacios para estar juntos y fortalecer su relacion.	Texts are identical for both tratment arms, i.e. Motives Treatment and Script Treatment are informed that the activities are over. Both treatment arms are encouraged to continue sharing moments and to continue building their relationship.

Table Appendix 3—A4. Control group messages by teacher.

Teacher 1
Hi, I'm [Teacher's name]. Next week [Student's name] is having a quiz in mathematics.
Hi, I'm [Teacher's name]. [Student's name] has an exam next Wednesday.
Hi, I'm [Teacher's name]. Next Friday, [Student's name] is having a quiz in mathematics.
Teacher 2
Hi, I'm [Teacher's name]. On Thursday, [Student's name] has a workshop in mathematics
Hi, I'm [Teacher's name]. Next Monday, [Student's name] is having a quiz in mathematics.
Hi, I'm [Teacher's name]. [Student's name] has an mathematics exam next Monday.
Hi, I'm [Teacher's name]. [Student's name] is having a quiz in mathematics this Thursday.
Teacher 3
Hi, I'm [Teacher's name]. [Student's name] is having a quiz in mathematics on Monday.
Hi, I'm [Teacher's name]. [Student's name] is having a mathematics exam next Thursday
Teacher 4
No messages were sent to the control group
Teacher 5
Hi, I'm [Teacher's name]. [Student's name] has a mathematics exam next Wednesday.
Hi, I'm [Teacher's name]. [Student's name] has the mathematics final exam of the semestar on [date].
[Student's name] has a mathematics exam next Thurday
Teacher 6
No messages were sent to the control group
Teacher 7
Hi, I'm [Teacher's name]. [Student's name] has a mathematics exam next [date]. Do not forget to take pencil and eraser.
Hi, I'm [Teacher's name]. [Student's name] has a mathematics exam this Tuesday
Hi, I'm [Teacher's name]. [Student's name] has a mathematics exam next Tuesday.
Teacher 8
No messages were sent to the control group
Teacher 9
Hi, I'm [Teacher's name]. Next Monday, there is a mathematics quiz.
This Tuesday, [Student's name] has the mathematics final exam for this quarter.
On Monday, [Student's name] has a mathematics quiz next Monday.
Next Monday, we are having a mathematics quiz.
Next week there will be no quiz and no exam in mathematics.
Teacher 10
Hi, I'm [Teacher's name]. Hay prueba de matematica mañana lunes.
Hi, I'm [Teacher's name]. Constanza tiene prueba parcial de matematica la proxima semana.
Hi, I'm [Teacher's name]. Luis tiene control de matematica el proximo viernes.
Hi, I'm [Teacher's name]. Anthony tiene control de matematica el proximo viernes.
On Friday, [Student's name] is having a mathematics quiz.
Teacher 11
Hi, I'm [Teacher's name]. Mañana hay control de matematica.
This Tuesday, [Student's name] has the mathematics final exam for this quarter.
El lunes [Student's name] tiene control de matematica.
On Monday, we are having a mathematics quiz.
Next week there will be no quiz and no exam in mathematics.
Teacher 12
[Student's name] is having a mathematics quiz this Tuesday.
[Student's name] is having a mathematics quiz this Monday.
On Monday, we are having a mathematics quiz.
Next week there will be no quiz and no exam in mathematics.

9.3.2. Appendix 3—B Students' and Teachers' Handout

Students' Handout. The first section of the students' handout asked the adolescent to think of the same situation proposed in the non-academic activity, with the aim of equating the teacher emphasis on the topic for every student, independent if they did or not the activity with their parent. The second section was a math activity that serves as a scaffold for the student to connect the non-academic topic with the mathematics learning objective. The third section included an academic activity with the aim of applying the mathematics learning objective to the non-academic activity. For an example translated to English see below, Figure Appendix **3—**B1.

Teachers' Handout. Each teacher received a guide on how to use the activity in their classrooms. In the first section, there was a motivation for the teacher to use the material in class. This message was always the same, every week and for every teacher. In the second section, a description of what the connection was between the non-academic activity and the math learning objective. In the third section, we included some suggestions for encouraging students' participation. Some extra exercises were included occasionally for the teacher to reinforce the aimed ability in the classroom. For an example translated to English see below, Figure Appendix **3—**B2.

I) **Is there a soccer player who you like the most? If there is no player that you like, work with a partner who likes soccer. Write the name of your favorite player and write down at least two teams in which he has played.**

Player: ____________________

Team 1: ____________________
Team 2: ____________________
Team 3: ____________________
Team 4: ____________________

II) **In the following two pictures, there are two soccer teams.**

Can you see whether there is any player that appears in the three pictures? Is there any player who appears in only two of the three pictures?

Respuesta:

III) **In the following expression: 3ab + ac + 3ab, what are the "common factors" that you can use to factor by the "common factor method"? Which are not "common factors"?**

Figure Appendix **3—**B1. Students' Handout

Teacher's guide

[VOLUNTARY work for the teacher]

Connecting the activity received by parents with the curricular aim is crucial because whether the teacher does not connect it to the curricular aim, they may feel that teacher is asking them to do things that are useless.

1. **What is the connection between the activity and the curricular objective that was thought by the person who created the text message?**

   The algebraic terms are like soccer teams, they are made up of different players. To conduct the factoring procedure, you need to choose the players who are in all the teams at the same time.

2. **Suggested questions to encourage the students' participation in the classroom**

   Are there players that does not appear in some of the pictures but not in all of them?

   Is it easy to find who are the players that appear in all the pictures?

   What are the similarities between an algebraic term and a soccer team?

   When conducting the factoring procedure, we use an every-day skill called "to identify". To find what are the "common factors", I need to be able to identify what is the factor that appear in every algebraic term, just as I identify a player in a football team.

3. **Practical tasks to write on the whiteboard and to explore further the connection between the activity and the curricular objective.**

   Have the students solving factoring exercises in which the original expression has two or more algebraic terms. Ideally: Start working with factors whose exponent is not greater than 1, so as not to generate confusion for the student.

Figure Appendix **3—**B2. Teachers' Handout

9.3.3. Appendix 3—C Tables

Table Appendix **3**—C1. Differential Attrition by covariate

Variables	Is Math GPA Missing?	Is Language GPA Missing?	Is post-intervention Math Anxiety Missing?
Treatment x Student's age	-0.074**	-0.077**	-0.072†
	(0.03)	(0.03)	(0.04)
Treatment x Student is female	-0.034	-0.021	0.022
	(0.07)	(0.05)	(0.07)
Treatment x Overall GPA 2016	0.032	0.039†	-0.003
	(0.04)	(0.02)	(0.04)
Treatment x Mathematics GPA 2016	0.013	0.01	-0.013
	(0.04)	(0.02)	(0.04)
Treatment x Number of people living at home	0.037*	0.040*	0.081**
	(0.02)	(0.02)	(0.03)
Treatment x Math Anxiety (pre)	-0.039	-0.051	-0.047
	(0.03)	(0.04)	(0.05)
Treatment x Quality of Family Relation (pre)	0.056	0.060†	-0.036
	(0.03)	(0.04)	(0.06)
Treatment x Intrinsic motivation (pre)	0.018	0.017	0.029
	(0.03)	(0.03)	(0.05)
Treatment x Self-efficacy (pre)	0.05	0.055	0.081
	(0.03)	(0.03)	(0.05)
Treatment x Self-determination (pre)	0.024	0.019	0.053
	(0.03)	(0.03)	(0.05)
Treatment x Parental motivation (pre)	0.009	-0.005	0.002
	(0.03)	(0.03)	(0.05)
Treatment x Personal relevance (pre)	0.019	0.007	-0.022
	(0.03)	(0.04)	(0.05)
Treatment x Grade motivation (pre)	0.012	0.011	0.027
	(0.03)	(0.03)	(0.05)
Treatment x Career motivation (pre)	0.034	0.018	0.019
	(0.03)	(0.03)	(0.05)
Treatment x Parent is female (pre)	0.122	0.064	0.089
	(0.07)	(0.05)	(0.07)
Treatment x Authoritarian score (pre)	0.066	0.002	0.001
	(0.06)	(0.02)	(0.05)
Treatment x Authoritative score (pre)	0.115*	0.021	-0.008
	(0.06)	(0.02)	(0.05)

Standard errors in parentheses

** p<0.01, * p<0.05, † p<0.1

Table Appendix **3**—C2. Randomization checks (treatment vs control)

The Effect of Treatment Status on Pre-Treatment Covariates by the Randomization Sample and Student Learning Outcomes Sub-Samples

Panel A: Regressions of Students Characteristics on Treatment Status (Control group is the omitted category in all regressions)

	Randomized Sample	N	Sample A: Has Math GPA Outcome	N	Sample B: Has Language GPA Outcome	N	Sample C: Math Anxiety Outcome	N
Administrative Data								
Age (in years)	0.145†	365	0.193*	341	0.159*	322	0.152†	283
	(0.080)		(0.079)		(0.080)		(0.087)	
Female	0.014	422	0.028	352	0.043	328	0.030	290
	(0.051)		(0.053)		(0.054)		(0.058)	
Overall GPA 2016[a,]	0.034	367	0.000	345	0.002	322	0.068	287
	(0.094)		(0.095)		(0.098)		(0.104)	
Missing Mathematics GPA 2016	-0.044	366	-0.056	344	-0.032	321	0.005	286
	(0.099)		(0.100)		(0.102)		(0.110)	
Pre-intervention Students Survey Data								
Number of people living at home	-0.152	233	-0.226†	218	-0.330*	199	-0.321*	176
	(0.131)		(0.134)		(0.136)		(0.146)	
Math Anxiety and Motivation [c]								
Math Anxiety (MA)	0.095	238	0.125	222	0.133	203	0.098	179
	(0.124)		(0.129)		(0.133)		(0.145)	
Math Motivation								
Intrinsic Motivation (IM)	0.001	239	-0.024	223	-0.005	204	-0.079	180
	(0.129)		(0.133)		(0.135)		(0.150)	
Self Efficacy (SE)	-0.006	238	-0.053	222	-0.052	203	-0.111	179
	(0.128)		(0.134)		(0.134)		(0.151)	
Self Determination (SD)	-0.036	238	-0.063	222	-0.102	203	-0.116	179
	(0.126)		(0.130)		(0.134)		(0.151)	
Parental Motivation (PM)	-0.109	238	-0.098	222	-0.118	203	-0.164	179
	(0.128)		(0.133)		(0.134)		(0.153)	

Personal Relevance (PR)	-0.051	238	-0.072	222	-0.074	203	-0.103	179
	(0.127)		(0.132)		(0.131)		(0.149)	
Grade Motivation (GM)	0.027	238	0.021	222	0.005	203	-0.033	179
	(0.125)		(0.132)		(0.132)		(0.150)	
Career Motivation (CM)	-0.196	238	-0.228†	222	-0.209	203	-0.245	179
	(0.130)		(0.135)		(0.134)		(0.149)	
Missing pre-survey data	-0.018	373	-0.014	349	-0.020	325	-0.011	290
	(0.027)		(0.026)		(0.025)		(0.030)	
Pre-intervention Parents Survey Data								
Age (in years)[c]	0.237	144	0.225	139	0.216	133	0.231	114
	(0.173)		(0.175)		(0.178)		(0.190)	
Parenting Style [c]								
Authoritative	-0.161	172	0.022	166	-0.239	158	-0.255	134
	(0.156)		(0.045)		(0.164)		(0.179)	
Authoritarian	-0.026	172	-0.199	166	-0.041	158	0.012	134
	(0.149)		(0.159)		(0.152)		(0.166)	

Notes: Each row represents a separate regression model (only the coefficients of the treatments status are reported). All regressions include classroom fixed effects. The omitted reference group in all regressions is the control group. Sample size for each regression varies according to missing cases for each covariate. Motivational scores, family relations scores and GPAs variables are in standard deviation units. Statistical significance levels: †p<0.10; *p<0.05; **p<0.01. The model conducted is the following: $X_{ics} = \alpha + \beta_1 \cdot T_{ics} + \gamma_c + \varepsilon_{is}$ where, X_{ics} is a vector of the listed pre-treatment covariates. The binary variable T_{ics} stands for whether the parent of student i was assigned to receive activities during year t, regardless of the treatment arm they belonged . γ_c is classroom fixed effect and ε_{is} is a student-level error-term.

Table Appendix **3**—C3. Randomization checks (Treatment arms vs control)

The Effect of Treatment Status on Pre-Treatment Covariates by the Randomization Sample and Student Learning Outcomes Sub-Samples

Panel A: Regressions of Students Characteristics on Treatment Arm Status (Control group is the omitted category in all regressions)

	Randomization Sample		Sample A: Has Math GPA Outcome		Sample B: Has Language GPA Outcome		Sample C: Math Anxiety Outcome	
	Treatment Effect Estimates:		Treatment Effect Estimates:		Treatment Effect Estimates:		Treatment Effect Estimates:	
	Activities track	Script track	Activities track	Script track	Activities track	Script track	Activities track	Scrip track
Female	0.079	-0.018	0.070	0.045	0.177	0.086	0.145	0.10
	(0.128)	(0.125)	(0.132)	(0.130)	(0.141)	(0.139)	(0.166)	(0.16
Age (in years)	0.185†	0.098	0.241*	0.137	0.187†	0.140	0.102	0.08
	(0.101)	(0.099)	(0.099)	(0.098)	(0.102)	(0.101)	(0.124)	(0.12
Missing Age	0.005	0.016	0.006	0.016	0.008	0.005	0.022	0.04
	(0.022)	(0.022)	(0.024)	(0.023)	(0.021)	(0.021)	(0.035)	(0.03
Overal GPA 2016	-0.073	0.135	-0.090	0.087	-0.095	0.073	-0.038	0.21
	(0.117)	(0.115)	(0.119)	(0.117)	(0.121)	(0.119)	(0.155)	(0.15
Missing Overall GPA 2016	-0.003	0.002	0.008	0.014	0.010	0.004	0.015	0.01
	(0.016)	(0.016)	(0.014)	(0.014)	(0.014)	(0.014)	(0.019)	(0.01
Math GPA 2016	-0.097	0.007	-0.107	-0.007	-0.084	-0.002	-0.063	0.17
	(0.124)	(0.122)	(0.125)	(0.123)	(0.130)	(0.128)	(0.165)	(0.16
Missing Mathematics GPA 2016	-0.007	-0.003	0.003	0.008	0.004	-0.003	0.006	0.00
	(0.018)	(0.017)	(0.016)	(0.016)	(0.016)	(0.016)	(0.022)	(0.02
Number of people living at home	-0.458	-0.123	-0.504	-0.332	-0.780*	-0.401	-0.758*	-0.69
	(0.304)	(0.287)	(0.310)	(0.293)	(0.322)	(0.302)	(0.378)	(0.37
Pre-Survey								
Math Anxiety (MA)	0.113	0.081	0.085	0.162	0.145	0.164	0.119	0.26
	(0.160)	(0.151)	(0.166)	(0.157)	(0.178)	(0.168)	(0.197)	(0.19
Intrinsic Motivation (IM)	-0.062	0.053	-0.113	0.053	-0.053	0.089	-0.238	0.13

	(0.166)	(0.157)	(0.171)	(0.162)	(0.183)	(0.173)	(0.215)	(0.21
Self Efficacy (SE)	0.048	-0.052	0.003	-0.103	0.018	-0.067	-0.056	-0.13
	(0.165)	(0.156)	(0.173)	(0.164)	(0.179)	(0.169)	(0.213)	(0.21
Self Determination (SD)	0.016	-0.079	-0.028	-0.093	-0.106	-0.108	-0.036	0.08
	(0.162)	(0.152)	(0.166)	(0.158)	(0.174)	(0.165)	(0.201)	(0.20
Parental Motivation (PM)	-0.048	-0.161	-0.003	-0.180	-0.090	-0.111	-0.134	-0.11
	(0.164)	(0.154)	(0.169)	(0.161)	(0.179)	(0.170)	(0.214)	(0.21
Family Relation Quality (FRQ)	-0.033	0.157	-0.071	0.112	-0.053	0.090	0.038	0.21
	(0.174)	(0.165)	(0.182)	(0.177)	(0.200)	(0.193)	(0.217)	(0.22

Notes: Each row represents a separate regression model (only the coefficients of the treatments status are reported). All regressions include classroom fixed effects. The omitted reference group in all regressions is the control group. Sample size for each regression varies according to missing cases fo each covariate. Motivational scores, family relations scores and GPAs variables are in standard deviation units. Statistical significance levels: †p<0.1 *p<0.05; **p<0.01.

Table Appendix **3—**C4. Moderator effect of Math Anxiety

Moderator effect of Math Anxiety on Standardized Students' Mathematics Performance

Panel A: Students Math Performance	Model 9	
Variables	β	SE
Treatment	-0.556*	0.3241677
Math Anxiety (pre-intervention)	-0.239*	0.0891765
Treatment x Math anxiety (pre-intervention)	0.225*	0.1070659
N=	223	
Model Inclusions		
Classroom Fixed Effects	X	
Imbalance pre-treatment covariates	X	
Additional pre-treatment covariates	X	
Extra pre-treatment covariates	X	

†p<0.10; *p<0.05; **p<0.01.

9.3.4. Appendix 3—D Figures

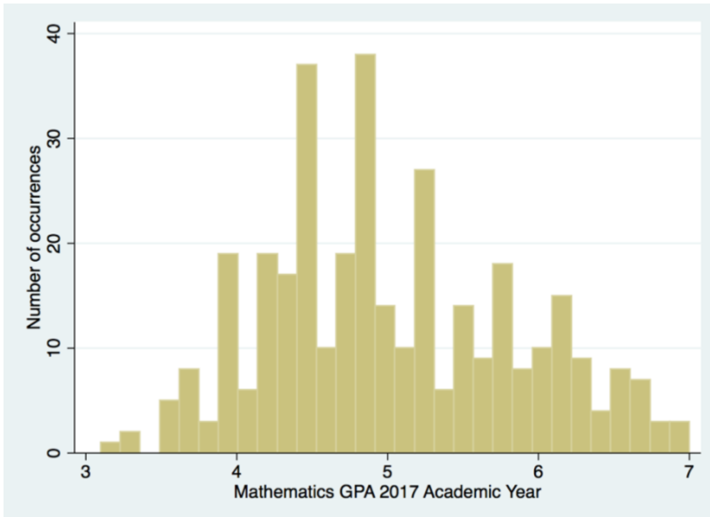

Figure Appendix **3—**D1. Number of occurrences (frequency) of Mathematics GPA 2017 Outcome

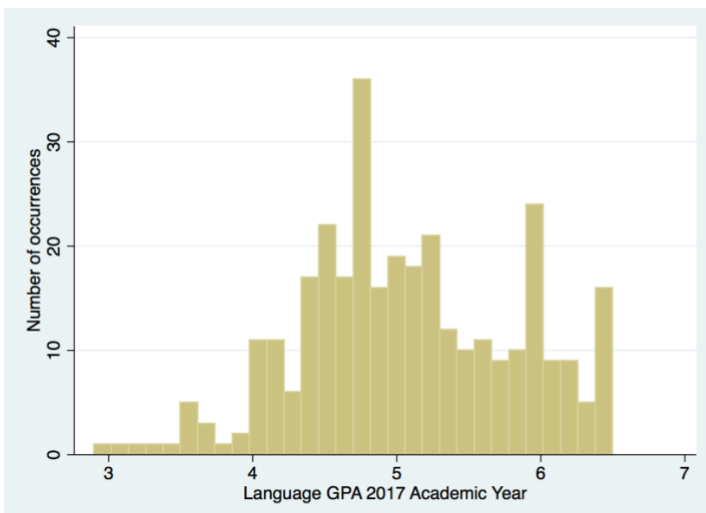

Figure Appendix **3—**D2. Number of occurrences (frequency) of Language GPA 2017 Outcome

9.3.5. Appendix 3—E Methods

Measures and covariates

Math anxiety (Núñez-Peña et al., 2013)

Name of the survey: A Spanish version of the short Mathematics Anxiety Rating Scale (sMARS) (sMARS)
Scale: five-point Likert scale (1 = never, 5 = always)

- Estoy nervioso(a) por cómo me va a ir en las pruebas de matemática
- Me pongo nervioso(a) cuando llega el momento de dar una prueba de matemática
- Me pone nervioso o ansioso tener que resolver una serie sumas o restas
- Me pongo nervioso o ansioso al hacer un control de matemática
- Me pongo nervioso o ansioso al entrar a una clase de matemática
- Me pone nervioso o ansioso tener que resolver multiplicaciones o divisiones

Parent–child relationship quality
Name of the survey: Quality of family relationships
Scale: five-point Likert scale (1 = Totalmente en desacuerdo, 5 = Totalmente de acuerdo)

- ¿Te sientes cercano a tu madre/padre?
- ¿Sientes que tu madre/padre se preocupa por ti?
- La mayoría del tiempo tu madre/padre es cariñosa/o contigo
- Te sientes satisfecho con la forma en la que tú y tu madre se comunican
- En general, te sientes satisfecho con la relación que tienes con tu madre

Global Family Relation Quality

Name of the survey: Quality of family relationships
Scale: five-point Likert scale (1 = *not at all*, 5 = *very much*)

- ¿Sientes que tu familia te entiende?
- ¿Te diviertes con tu familia?
- ¿Sientes que tu familia te presta atención?

Global Family Relation Quality

Name of the survey: Quality of family relationships
Scale: five-point Likert scale (1 = Totalmente en desacuerdo, 5 = Totalmente de acuerdo)

- Do you usually share time with your mother/father?
- Do you talk about life with your mother/father?
- Do you have moments with your mother/father where they can laugh together?

Mathematics Motivation

Note. Previous to this intervention we surveyed 440 students to assess the reliability of the questionnaire, that included the 25 items from Glynn, Brickman, Armstrong, & Taasoobshirazi (2011), 5 items from (Glynn, Taasoobshirazi, & Brickman, 2009) and 5 items that we created to measure Parental Motivation (listed below). We analyzed reliability of the instrument and, consequently, we kept the items described in the main manuscript.

Name of the survey: Parental Motivation
Scale: five-point Likert scale (1 = *not at all*, 5 = *very much*)

- I care to learn to make my parents or family happy
- When I try to improve myself in math, I do it so that my family feels proud
- I consider it important to get the grades that my parents or family expect me to get
- I want to do well in math, so that my family will stop bothering me

Parenting Styles
Name of the survey:

Scale: five-point Likert scale (1 = never, 5 = always)

Authoritative scale

- I know the names of our child's friends.
- I am aware of problems or concerns about our child in school.
- I give praise when our child is good.
- I give comfort and understanding when our child is upset.
- I express affection by hugging, kissing, and holding our child.
- I show sympathy when our child is hurt or frustrated.
- I tell our child that we appreciate what the child tries or accomplishes.
- I am responsive to our child's feelings or needs.
- I encourage our child to talk about the child's troubles.
- I have warm and intimate times together with our child.
- I apologize to our child when making a mistake in parenting.
- I explain the consequences of the child's behavior.
- I give our child reasons why rules should be obeyed.
- I emphasize the reasons for rules.
- I help our child to understand the impact of behavior by encouraging our child to talk about the consequences of his/her own actions.
- I talk it over and reason with our child when the child misbehaves.
- I tell child our expectations regarding behavior before the child engages in an activity.
- I take into account our child's preferences in making plans for the family.
- I allow our child to give input into family rules.
- I take our child's desires into account be fore asking the child to do something.
- I encourage our child to freely express (himself)(herself) even when disagreeing with parents.
- I channel our child's misbehavior into a more acceptable activity.
- I am easy going and relaxed with our child.
- I show patience with our child.
- I joke and play with our child.
- I show respect for our child' s opinions by encouraging our child to express them.

Authoritarian scale

- I show respect for our child' s opinions by encouraging our child to express them.
- I explode in anger towards our child.
- I yell or shout when our child misbehaves.
- I argue with our child.

- I disagree with our child.
- I use physical punishment as a way of disciplining our child.
- I spank when our child is disobedient.
- I slap our child when the child misbehaves.
- I shove our child when the child is disobedient.
- I guide our child by punishment more than by reason.
- I grab our child when he/she is being disobedient.
- I punish by taking privileges away from our child with little if any explanations.
- I punish by putting our child off somewhere alone with little if any explanations .
- I use threats as punishment with little or no justification.
- When two children are fighting, I discipline children first and asks questions later.
- I appear to be more concerned with own feelings than with our child's feelings.
- When our child asks why (he)(she) has to conform, state: because I said so, or I am your parent and I want you to.
- I tell our child what to do.
- I demand that our child does/do things.
- I scold and criticize to make our child improve.
- I withhold scolding and/or criticism even when our child acts contrary to our wishes.

9.4. Appendix 4. Supplementary Material in "Fostering…"

9.4.1. Appendix 4—A Text Messages, Class Material and Teacher Support Material

Example of the SMS and class material

SMS:

Below is an example of an activity used in 10th grade. The placeholders **name** and ##name## represent the name of the teacher and the name of the student, respectively.

> Hi, it's **name**. With ##name##, if you can, go to the park and feed the pigeons. Count how many there are in the beginning and how many there are by the end.

Class material:

I) Suppose that you are with someone feeding the pigeons in the park. Before you got there, there were only a few pigeons. You then started to feed them a few bread crumbs. How many pigeons do you think there were in the beginning? How many pigeons do you think there were by the end? Use your imagination.

Answer:

II) Suppose that every minute the number of pigeons doubles. If in the beginning there was 1 pigeon, how much time would have to pass for there to be 8 pigeons?

Answer:

Teachers' support material

Teacher Guide

[OPTIONAL teacher task]

Connecting the activity that the parents receive with the topics in class is important because, if you don't, the parents might feel that the teacher is asking them to do things that are pointless.

1. How did the person who wrote the message connect it with the learning objective?

The number of pigeons is doubled every minute. You can connect this to exponential growth. The final number of pigeons is the starting number multiplied by 2^n, where n is the number of minutes.

Furthermore, to tell how many minutes have passed to get to a certain number, the brain needs to solve a logarithm: $\log_2(8) = n$

2. Recommended questions in order to involve the students

You can ask certain questions in order to foster pedagogical interaction and connect the activity to the topics in class.

Questions for concrete thinking

If there was 1 pigeon, how many pigeons are there after the first minute?

How many pigeons are there after the second minute?

How many pigeons are there after the third minute?

How much time has passed if there are 4?

How much time has passed if there are 8?

How much time has passed if there are 16?

Questions for making generalizations

With every minute that goes by, what happens to the number of pigeons? [it doubles]

When I double something several times, how can I summarize it? [as a power]

If I have the final number, without knowing already, how can I work out how many times it has been doubled?

$1 * 2^x = 8$

Connecting to logarithms

2 to the power of what is 8? How many times do I have to double the number 2 in order to get to the final number of pigeons?

That question is the logarithm: how many times do I have to raise a number to the power of something in order to get to another number?

9.5. Appendix 5. Exploring further on Math Anxiety

9.5.1. Level of math anxiety of participant students in study "Having Fun Doing Math:…" compared to students in study "Let's Spend Time Together".

In the study "Having Fun Doing Math" (Chapter 2 in this thesis), we measured math anxiety through 2 items (five-point Likert scale; 1 = never, 5 = always; α=.89). We asked students the following prior to the intervention: "Estoy nervioso(a) por cómo me va a ir en las pruebas de matemática" (I'm nervous about how I'm going to do in math tests) and "Me pongo nervioso(a) cuando llega el momento de dar una prueba de matemática" (I'm nervous when I have a math test).

We surveyed 47 out of 56 (84%). Surveyed students did not show significant difference regarding their math GPA prior to the intervention, compared to non-respondent students (4.94 vs 4.99, p=0.95). As shown below in Table Appendix 5—A, surveyed students reported feeling often anxious about math (score: 3.94 out of 5). In our second study, students were classified as "highly anxious" when obtaining a score higher than the observed mean, 2.95, in the math anxiety scale (standing for students feeling sometimes anxious), using a 6-items scale (see above, Measures in Chapter 4 or Measures in Appendix 3—E). Nevertheless, it is important to acknowledge that students in the study "Let's spend time together…" obtained a significantly higher score in the 8-item scale when compared to the two items assessed in the first study (2.95 vs 3.62, p<.001).

Table Appendix 5—A Descriptive Statitsics. Students' Math Anxiety in study "Having Fun Doing Math".

Variable	Obs	Mean	SD	Min	Max
Math Anxiety	47	3.94	1.25	1	5

9.5.2. Gender differences in students' math anxiety

As shown in Table Appendix 5—B1 and Appendix 5—B2, female students in the analyzed samples suffer consistently of higher math anxiety levels.

Table Appendix 5—B1 Gender differentes in students' math anxiety in study "Having Fun Doing Math".

Variable	Mean (female sample)	SD (female sample)	Mean (male sample)	SD (male sample)	t-test	p-value

Math Anxiety	4.27	0.21	3.45	0.30	2.311	p<.05

Table Appendix 5—B2 Gender differentes in students' math anxiety in study "Let's Spend Time Together".

Variable	Mean (female sample)	SD (female sample)	Mean (male sample)	SD (male sample)	t-test	p-value
Math Anxiety (6-item scale)	3.19	0.082	2.74	0.088	2.311	p<.001
Math Anxiety (2-item scale)	3.99	0.109	3.33	0.095	4.477	p<.001

9.5.3. Treatment effect by gender

In both studies, we regress post-intervention mathematics GPA on treatment interacted with gender and included the controls for the full specifications models. No significant interaction was found between treatment and gender. In both studies, the coefficient of interest was close to zero; beta= 0.096 (p=0.804) for "Having Fun Doing Math…" and beta= .0079 (p=0.955) for "Let's Spend Time Together…"