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THE EFFECT OF VERTICAL INTEGRATION IN THE QUALITY OF DISPOSABLE PRODUCTS[1]

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ABSTRACT

This paper studies a single-product distribution channel where a manufacturer produces goods, some of which are defective, and a retailer, detecting only a subset of the defective goods, passes the rest along to customers, who end up discarding them. Conjecturing the structure of the demand and cost functions that assume customers to have a decreasing marginal aversion to bad quality while both the supplier and the retailer make marginally increasing efforts to avoid bad quality, we deduce several implicit parameters, including quality cost, based on observable data, such as the share of the channel margin. Once all the parameters of the model are available, we analyze the result of vertical integration. We not only confirm the well-known fact that vertical integration improves the quality perceived by the customer, but also characterize the attitude of the supplier, who may or may not provide a better service, in terms of the sum and the difference of logarithms of the margins.

Keywords: Supply Chain, Quality, Vertical Integration, Nash Equilibrium
JEL Classification: C72, L14, L15.

RESUMEN

Este artículo estudia una cadena de distribución en la que un proveedor fabrica artículos de un cierto tipo, algunos de los cuales tienen fallas, los que son comercializados por un distribuidor que sólo detecta una parte de los artículos defectuosos, traspasándole el resto al consumidor final. Se conjetura una cierta estructura de las funciones de demanda y de costo, que suponen que los consumidores tienen una aversión marginalmente decreciente a la mala calidad y que tanto el proveedor como el distribuidor requieren de esfuerzos marginalmente

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crecientes para evitar productos defectuosos. Con ello se deducen algunos parámetros implícitos del modelo a partir de datos observables, tales como la participación en el margen de la cadena de distribución. Una vez que están disponibles los parámetros, se analiza el resultado de una integración vertical. No sólo se confirma que dicha integración mejora la calidad percibida por el cliente final, sino además se caracteriza el comportamiento del proveedor, el que puede o no entregar una mejor calidad, dependiendo de la suma y la diferencia del logaritmo de los márgenes.

This paper studies a single-product distribution channel where a manufacturer produces goods, some of which are defective, and a retailer, detecting only a subset of the defective goods, passes the rest along to customers, who end up discarding them. The supplier and the retailer simultaneously decide on their effort to reduce errors as a function of the demand and the cost of different quality strategies. Regarding final demand, we assume that the supplier and the retailer have forged some kind of alliance, as described by Amaldoss *et al.* (2000). This alliance, although allows firms to access a greater base of resources, carries the risk of firms forgoing their ability to control the destiny in the marketplace. In our case, this interrelation is translated into the demand that both agents face, which the customer defines as a function of the combined performance of the supply chain. In other words, we assume that the supplier cannot go looking for other firms who could market his products and that the retailer cannot find a different supplier, so the supplier and the retailer end up competing as a team against other alliances, mainly in terms of the quality they are able to provide.

Banker *et al.* (1998), who study the relationship between quality and the intensity of competition in a given industry, define quality as the degree to which a product is attractive to consumers. It can be characterized as either "design quality" or "conformance quality". The former is based on such characteristics as performance, reliability, durability and serviceability, while the latter refers to the degree to which product specifications are met. Our model makes the usual assumptions that firms view quality in the same way their customers do, and that quality can be aggregated along a single dimension of interest. However, we focus on conformance quality, which can range from 100 per cent of the product design to a percentage so low it drives firms out of business. Thus, we are concerned about errors that degrade the product's appeal to customers and therefore reduce the demand for it.

Regarding the cost of quality, we focus on those systems where variable costs and benefits for both the supplier and the retailer are generated only by items that are finally accepted by the customer, not by those rejected by either the retailer or the customer. We refer to such cost function as related to disposable items, meaning that agents incur an insignificant cost to dispose of a defective item. This is usually the case in capital-intensive industries, such as microelectronics, where raw materials are insignificant with respect to fixed expenses, so the marginal cost of manufacturing and delivering is minimal. As long as there are no active capacity constraints, any defective item will not cause a substantial damage other than the loss of demand. The extreme case of this situation is a service industry that uses no raw materials at all, such as the telecom industry, where the variable cost of malfunctioning calls is negligible.

Even if raw materials are important, in some cases there is a systematic surplus that is discarded anyway. A good example of this is the fast food industry that instantly trashes any item that does not comply with the standard, fearing that bad quality will frighten customers away. Another example is provided by retailing channels that can liquidate defective items at zero profit. The clothing industry, for example, can mark down defective garments to such a low price that it makes little difference if the products are sent back to the suppliers or sold in the marketplace.

According to Starbird (2001), defining contracts and compliance-related incentives has been a main issue in the scientific literature concerned with quality, since they influence market structure, product quality and other operational aspects. Because in our model agents are autonomous and know that their payoff is a result of the strategy that they both choose, we adopt a non-cooperative game theoretical perspective, as do Reyniers and Tapiero (1995), who describe a model where the supplier decides how much effort to invest in quality, while the producer (the retailer in our case) resolves whether or not to inspect incoming materials. These authors study the effect of contract design on equilibrium behavior and identify conditions that will result in a mutually advantageous solution. Donoso and Singer (2002) focus on the penalties applied in the transportation business when products are returned to the plant due to bad quality. They model this process as a tournament game and measure the effect of the accuracy of the systems that monitor who is responsible for the bad quality. Lim (2001) derives the optimal contract between a manufacturer and his supplier when there is asymmetric information because the quality of the latter is private information to him. Using the revelation principle for games with incomplete information, he finds the proper combination of rebate prices and end customer warranties so that opportunistic behavior by any supplier is discouraged while incentives to trade are preserved. Although Cachon and Zipkin (1999) study the inventory management problem, their work is related to ours in terms of the structure of the game they propose. They show that the difference between the optimal policy and the equilibrium depends on how inventories are measured, so they derive a linear transfer scheme that aligns agents closer to the optimum.

This paper is intended to unravel the strategic behavior regarding quality within a supplier-retailer partnership in a disposable product industry. We are concerned with questions such as, what is the effect of the different cost and demand parameters? In what circumstances can non-observable parameters be inferred? How do observable parameters influence the quality performance of a vertical integration? To answer these questions, Section I proposes a static game with complete information that matches the usual assumptions in the literature on demand and cost functions. Section II shows our analysis of Nash equilibrium that allows us to deduce from observable data some parameters that are not easy to verify otherwise. Section III studies the effect of vertical integration and derives the conditions that define the global optimum. Finally, Section IV presents our main conclusions.

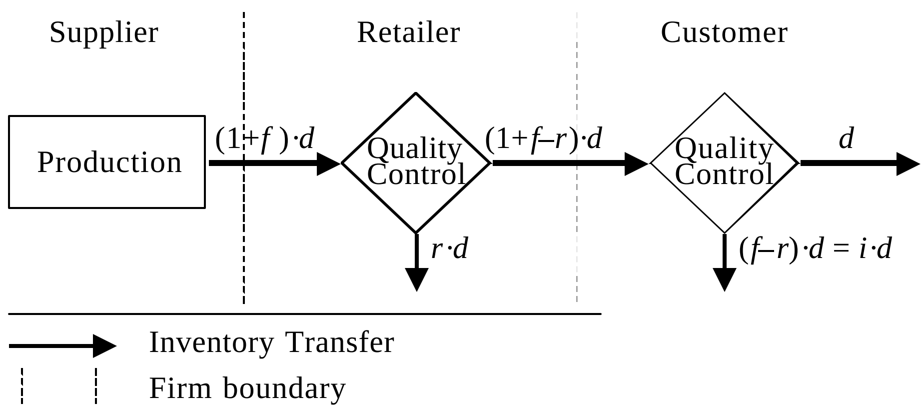
I. A QUALITY MODEL FOR DISPOSABLE PRODUCT SUPPLY CHAINS

We consider the supply chain with two echelons, the supplier and the retailer, whose demand is defined by a customer as follows:

d Demand for non defective products by the end customer. It can be observed since it defines the income of both the supplier and the retailer.

Production is not performed with total quality, so both the retailer and the customer perform quality control, eventually rejecting defective products as illustrated by Figure 1. As in the double marginalization problem presented by Tirole (1988 pp. 174-175), the final output the customer receives is a concatenation of the decisions of non-cooperative players, although such problem is related to price, not the focus of this paper.

FIGURE 1
SUPPLY CHAIN FROM SUPPLIER TO CUSTOMER



In order to model this situation as a static game in the normal form, players, strategies and payoffs must be defined. We consider only two players, namely the supplier and the retailer, who simultaneously decide their quality strategies, that are characterized as follows:

f Fraction of faulty products that are manufactured by the supplier in addition to d, so its total output is $(1 + f)\cdot d$. Usually it cannot be directly observed, since otherwise the defective items would be immediately discarded, although it can be deduced as explained below.

r Fraction of d of defective products that are detected by the retailer, so its output is a flow $(1 + f - r)\cdot d$ that is passed on to the customer and a flow $r\cdot d$ that is rejected. This value can be easily observed, since returns are usually well registered for contractual reasons.

i Fraction of imperfect products that the customer receives, equal to $f - r \geq 0$. We assume that he discards an item if and only if it is defective. Many companies keep a record of customers' satisfaction, so we assume that i can be observed.

We assume that $f > 0$ and $r > 0$, so manufacturing errors and quality controls exist. Errors are generated only by the supplier, so the retailer does not damage items due to careless handling or other reasons. When performing the quality control, the retailer only makes Type I mistakes, i.e. accepting an item that is defective, but does not make Type II errors, which correspond to rejecting an item in good conditions, so it holds that $r \leq f$. These percentages are defined as the excess over the demand d, so $0 \leq f, r \leq \propto$. For instance, if $f = 4$ and $r = 3$, then 4 out of 5 items produced are defective, and then 3 are rejected by the retailer, so half the items the customer receives are defective. Sousa and Voss (2002) report empirical evidence supporting the positive correlation between quality and market share, so final demand is determined, among other aspects, by variable i. As deduced by the Bayesian model of customer choice behavior by Gans (2002), the demand function is not only increasing but also convex on the quality level, so when quality is good the customer becomes very sensitive to errors, while as the percentage of imperfect products i grows, the demand asymptotically approaches zero. From the above considerations we conjecture the following demand function, which is consistent with the exponential model by Vörös (2002):

$$d = D \cdot exp(-E \cdot i). \tag{1}$$

Parameters D and E are positive constants that have the following meaning:

D Maximum demand if the customer received 100% quality, that is, if $i = 0$. Although marketing departments may have a rough estimation of this quantity, it is not verifiable unless the customer indeed gets perfect quality. Therefore, we deduce this quantity from empirical data as explained in Section II.

E Convexity of the demand curve with respect to i, so $E = -(\partial^2 d/\partial i^2)/(\partial d/\partial i)$, meaning that the higher the value of E the more sensitive the customer is to quality. The demand elasticity with respect to i is $-E \cdot i$, so it can be obtained from historical data by correlating sales d of non-defective items with the quality perceived by the customer in terms of i.

We define the income functions in terms of the following parameters related to the variable profit of each agent:

M Margin or net income for the supplier from each item free of defects that is sold to the customer. It includes the revenue from the price minus the variable cost of production, without considering the quality cost nor any other sunk cost. We assume it can be accurately calculated by the company.

N Margin or net income for the retailer from each item free of defects that is sold to the customer, subject to the conditions explained above for M.

Both M and N are required to be positive, otherwise in a non-cooperative game theoretical framework the agent has no incentives to generate any output, so without loss of generality $M, N \geq 1$. We assume that the aggregated unitary margin $M + N$ is equal to a constant C, independent of d, since the final price is fixed by some type of regulation, or the market share of the channel is low so it has little impact on the equilibrium price, while the unit cost is constant. Therefore, in our model quality enhancements generate benefit to the supply channel by increasing sales without sacrificing prices, one of the two "market routes" for improving business performance that are described by Sousa and Voss (2002). The values of M and N are defined exogenously as explained by Lim (2001), most likely as a result of the negotiating power of each agent, which according to Kadiyali *et al.* (2000) is related to demand factors, cost factors and the nature of channel interactions. One of the aims of this paper is to show how such definition affects quality when agents are autonomous, and what is the expected result if they become vertically integrated.

An alternative interpretation of the income function of supplier $M \cdot (D \cdot \exp(-E(f - r)))$, where the unitary net margin M is fixed while the demand d

decreases according to expression $D \cdot \exp(-E(f - r))$, is to assume that the function is of the form $D \cdot (M \cdot \exp(-E(f - r)))$, so the demand is constant but the margins decrease when the quality is low. More generally, the expression $(M \cdot D) \cdot \exp(-E(f - r))$ suggests that the combination of demand times the unitary income of the supplier is penalized by defective items. The same explanation is valid for the retailer, although for clarity reasons, in what follows we consider only the interpretation relating to the demand function.

We define the cost of quality to be increasing and convex in the level of quality offered and with no economies of scale, which is consistent with the conclusions by Li and Rajagopalan (1998). Convexity means that for the supplier, avoiding one mistake when f = 10% is easier than when f = 5%, so quality becomes more difficult as performance approaches to zero errors. Analogously for the retailer, detecting an additional percentage point of defective products when r = 5% is easier than when r = 10%, so catching all the errors generated by the supplier becomes progressively difficult. That there are no economies of scale means that the supplier must pay a given cost for keeping faulty items down to a certain percentage, regardless of the final demand d, which is a function of both f and r. Although this may be unlikely for any value d may have, in reality the demand does not change by large but rather by limited percentages, so the cost of a quality improvement program may have a constant value.

We conjecture that the quality cost functions for the supplier and the retailer are $F\exp(-f)$ and $R(\exp(r) - 1)$ respectively, as opposed to Banker *et al.* (1998) who assume functions to be quadratic on the level of quality. Parameters F and R have the following meaning:

- F Cost of achieving total quality, that is, producing zero faulty items. This could be obtained by quoting the cost of Total Quality Management (TQM) programs, although there is no guarantee that both the investment and the results will match the estimates. As an alternative we deduce it from the observed equilibrium, as explained in Section II.
- R Cost factor that measures how rapidly the retailer's cost of detecting an additional percentage point of defective items grows. This can be difficult to obtain without an Activity Based Costing (ABC) system. As an alternative we also deduce it from the observed equilibrium, as explained in Section II.

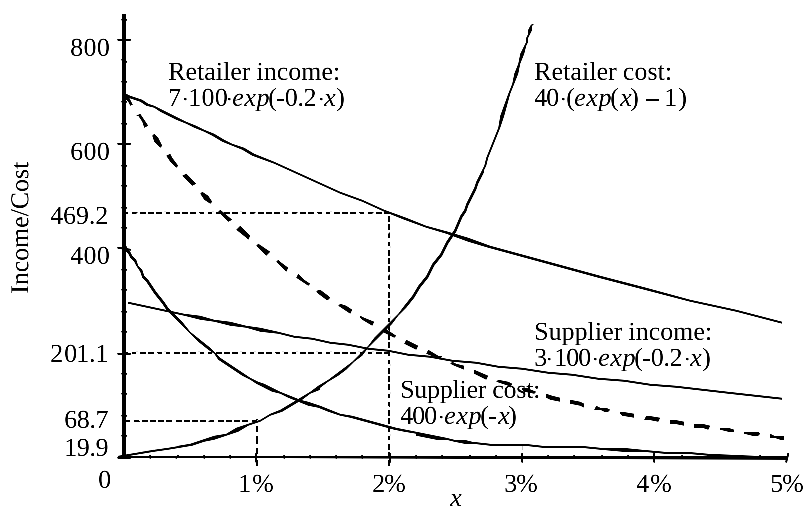
Therefore, the payoff functions for the supplier and for the retailer are the following:

$$\pi_S(f, r) = M \cdot D \cdot exp(-E(f - r)) - F \cdot exp(-f) \tag{2}$$

$$\pi_R(f, r) = N \cdot D \cdot exp(-E(f - r)) - R \cdot (exp(r) - 1) \tag{3}$$

In order to clarify ideas, if D = 100 and E = 0.2, then when the percentage of errors i = 1%, the total demand d is equal to 81.87. If the percentage of errors is incremented by 0.01, that is from 1 to 1.01 percent, then the demand function drops to 81.71, which is approximately $-E \cdot i$ = -0.2·1% = - 0.2%. We restrict E < 0.5 because when E = 0.5 a demand equal to 100 with zero errors drops to 0.08 with 5% of errors, a quality sensitivity that seems acute enough. Suppose also that F = 400, R = 40, M = 3 and N = 7. The income and the cost of each are depicted in Figure 2 as functions of their independent variable. If for some reason the supplier decides to do f = 3% while the retailer chooses r = 1%, their corresponding costs are 19.9 and 68.7, while the percentage of imperfect products is i = 2% so the 81.9 units that are demanded bring an income of 201.1 and 469.2. The dotted line shows the income that the retailer would collect if E were equal to 0.5, which drastically decreases with bad quality.

FIGURE 2
EXAMPLE OF INCOME AND COST WITH f = 3% AND r = 1%



As expected, both cost functions tend to zero as quality is relaxed: $F \cdot \exp(-\infty) = 0$ and $R \cdot (\exp(0) - 1) = 0$. We have implicitly assumed that the elasticity of the supplier's cost functions is $-f$, which does not lose generality since the magnitude by which the errors are measured can be adjusted. However, the elasticity of the retailer's cost functions is r, so we are imposing a relationship between the elasticities of the cost of quality for both agents. Notice that in reality the conformance cost $R(\exp(r) - 1)$ is a function not only of r but also of f, since rejecting r = 1% of defective items when f = 20% is cheaper than when f = 2%. For the sake of simplicity we assume independence, which is a realistic simplification when the value range of f is relatively limited. Also, we are disregarding the variable cost of quality, which is realistic if the demand d is subject to limited changes, as is our case.

II. EQUILIBRIUM FOR THE SUPPLIER-RETAILER ALLIANCE

The alliance forged by the supplier and the retailer can be modelled by either a cooperative or a non-cooperative game. In the first case, the players may negotiate a binding contract according to the so-called Nash bargaining solution which, among other features, is a Pareto optimum that depends on the utility that each agent attains if they do not reach an agreement. In the case of non-cooperative games, players should converge to a Nash equilibrium outcome where neither of them has incentives to unilaterally move away from it. For the problem at hand we select the second approach since, as suggested by Cachon and Zipkin (1999), a co-operative framework should specify penalties for deviations, which are hard to impose in practice. On the contrary, Nash equilibrium are self enforcing and therefore more realistic.

Convergence to the Nash equilibrium vector (f_e, r_e) is justified by the fact that each player chooses the quality level as his best response to the actions of the other, which is an application of the rational choice theory that conceptualizes that agents select among their feasible actions the ones that maximize their expected utility. Such capability is challenged by Marini (1992) who explains that cognitive psychology realizes that managers select their strategies limited by oversimplification of complex problems, selective perception, use of stereotypes and wishful thinking, weaknesses that are usually named "bounded rationality." In spite of this, Satz and Ferejohn

(1994) assert that even if players show cognitive downsides, the rational choice theory is an accurate conceptual framework for anticipating what average players will choose, especially if they are subject to defiant constraints. In competitive supply chains, the constraint of survival is critical, so one can assume that agents reach equilibrium either by strategic analysis or by trial and error, since otherwise they will perish sooner or later.

Accepting that the rational choice theory is a proper framework for describing the supplier-retailer relationship, we find the Nash equilibrium of the above game by intersecting the best-response functions of both players. In the case of the supplier, such function $f(r)$ represents the optimal level for f assuming that the retailer chooses r.

Proposition 1: Recalling that the feasible strategies impose that 0 ≤ r ≤ f, the best-response function of the supplier is:

$$f(r) = \begin{cases} \text{i)} \quad \dfrac{-Er + \ln\left(F/(MDE)\right)}{1-E} & \Leftrightarrow \quad r \le \ln \dfrac{F}{MDE} \\ \text{ii)} \quad r & \Leftrightarrow \quad \ln \dfrac{F}{MDE} \le r \end{cases} \tag{4}$$

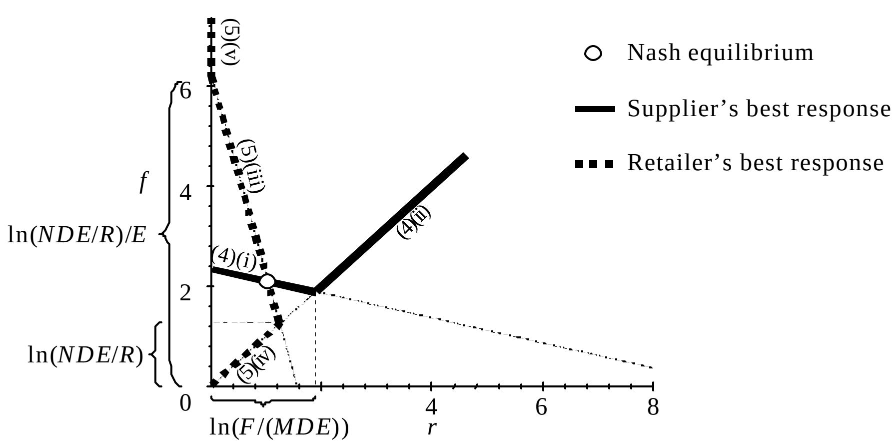
Proposition 2: Recalling that the feasible strategies impose that $0 \le r \le f$, the best-response function of the retailer is:

$$r(f) = \begin{cases} \text{iii)} \quad \dfrac{-Ef + \ln\left(NDE/R\right)}{1-E} & \Leftrightarrow \quad \ln \dfrac{NDE}{R} \le f \le \dfrac{1}{E} \ln \dfrac{NDE}{R} \\ \text{iv)} \quad f & \Leftrightarrow \quad f \le \ln \dfrac{NDE}{R} \\ \text{v)} \quad 0 & \Leftrightarrow \quad \dfrac{1}{E} \ln \dfrac{NDE}{R} \le f \end{cases} \tag{5}$$

The proofs for the above propositions are omitted because they follow a standard procedure, based on the property that any continuous and differentiable function must have its maximum value either at stationary point or at an extreme point of its convex domain. For **Proposition 1** the domain is $[r, L]$ where L is a large enough constant, while for **Proposition 2** it is $[0, f]$. In both cases, the extreme points either coincide with the stationary points, or their derivative discards them as maximums since they are positive at the lower bound of the domain or negative at the upper bound.

The interpretation of **Proposition 1** and **Proposition 2** is depicted by Figure 3, which shows the best-response functions using the data from Section I: $D = 100$, $E = 0.2$, $F = 400$, $R = 40$, $M = 3$ and $N = 7$. According to (4), the supplier's strategy f reaches the highest quality, that is, the minimum percentage of faulty products, when his counterpart chooses a strategy $r = \ln(F/(MDE))$. Below that threshold of r the supplier relaxes his quality standard to avoid being taken advantage of, and above $\ln(F/(MDE))$ the supplier can benefit from the retailer's willingness to detect a relatively large number of defective items. According to (5), the retailer's strategy r reaches a maximum level of concern for quality, that is, the maximum percentage of rejection, when his counterpart chooses a strategy $f = \ln(NDE/R)$. If the errors generated by the supplier are below such threshold, then r decreases since customers are receiving total quality so $f = r$. Otherwise, the retailer relaxes his quality-control effort to avoid being taken advantage of, up to a point of no-concern $\ln(NDE/R)/E$ where he stops rejecting items given the poor quality of the supplier. This last situation, (5)(v), is not covered by our analysis, which focuses on supply chains where the retailer does perform quality control.

FIGURE 3
BEST-RESPONSE FUNCTIONS AND NASH EQUILIBRIUM



Notice that $\ln(F/(MDE))/(1 - E)$ is the constant term in (4)(i), so a higher quality cost parameter F and a lower margin M for the supplier will shift its best response to higher percentages of faulty products. Analogously, $\ln(NDE/R)/(1 - E)$ is the constant term in (5)(iii), so a lower quality cost parameter R and a higher margin N for the retailer will shift his best response to higher percentages of rejection of defective products.

Proposition 3: If there is a Nash equilibrium such that r_e and i_e are positive, then it is unique and $\ln(F/(MDE)) > \ln(NDE/R)$.

Proof: If $r_e > 0$ and $i_e = f_e - r_e > 0$ then the equilibrium must lay in (4) (i) and (5) (iii), a unique intersection since their slopes are $E/(E - 1)$ and $(E - 1)/E$ respectively, and they are different when $E < 0.5$. Therefore:

$$f_e = \frac{E\ ln\dfrac{R}{NDE} + (1-E)ln\dfrac{F}{MDE}}{1-2E},\ r_e = \frac{(1-E)\,ln\dfrac{NDE}{R} + E\ ln\dfrac{MDE}{F}}{1-2E},\ i_e = \frac{ln\dfrac{FR}{D^2E^2MN}}{1-2E} \tag{6}$$

If the percentage of imperfect products i_e is positive, then $\ln(F/(MDE)) > \ln(NDE/R)$.

The interpretation of the results above are consistent with most anecdotic, empirical and theoretical results about quality that are reported by Li and Rajagopalan (1998): Lower quality cost parameters F and R will reduce manufacturing errors, will increment quality control that will ultimately improve quality for the end customer.

Proposition 4: If the Nash equilibrium is such that $i_e = 0$ and it is unique, then the equilibrium must also lie in (4) (i) and (5) (iii).

Proof: If $i_e = 0$ then the equilibrium must also be found in (4) (ii) and (5) (iv), although to be unique it must hold that $\ln(F/(MDE)) = \ln(NDE/R)$, and therefore $f_e = r_e = \ln(F/(MDE))$, which satisfy both in (4) (i) and (5) (iii)

As explained in Section I, it is usually the case that variables r_e, i_e, d_e as well as parameters E, M and N can be observed from the steady-state operation of the supply chain. If the conditions of **Proposition 3** or **Propo-**

sition 4 hold, then variable f and parameters D, F and R can be deduced from the above, since the equilibrium is unique. For instance, if it is observed that $r_e = 1.04\%$, $i_e = 1.07\%$ and $d_e = 80.67$, then $f_e = r_e + i_e = 2.11\%$ and all the parameters described in Figure 3 can be derived from the system of equations (1) and (6), which yields:

$$D = d_e/(\exp(-E(f_e - r_e))); F = M \cdot d_e \cdot E \cdot \exp(f_e);\ R = N \cdot d_e \cdot E \cdot \exp(r_e) \quad (7)$$

Replacing expression (7) in (2) and (3) we obtain the payoff functions for the supplier and the retailer:

$$\pi_S = M \cdot d \cdot (1 - E);\ \ \pi_R = N \cdot d \cdot (1 - E) + N \cdot d \cdot E \cdot \exp(-r_e) \quad (8)$$

Notice that the conditions of **Proposition 3** can be easily verified: r_e and i_e are positive. However, the conditions of **Proposition 4** cannot be verified, since when $i_e = 0$ there is no guarantee that the equilibrium is unique, because it can lie within the interval $[\ln(NDE/R), \ln(F/(MDE))]$.

III. OPTIMALITY UNDER VERTICAL INTEGRATION

Under some circumstances, the supplier and the retailer can behave in a cooperative manner by selecting f and r in order to maximize the aggregated profit of the supply chain and then decide how to split it according to their negotiating power and other considerations. This situation may be possible due to binding contracts that require a one-shot investment that commits each agent to a quality level from where they cannot deviate, or because there is a dominant supplier who imposes “vertical restraints” to the retailer as described by Tirole (1988 pp. 170-172). For instance, the retailer may pay a “franchise fee” that consists of a fixed charge plus a variable amount that depends on the quantity sold. Finally, agents may develop a trustful relationship since, as described by Kumar (1996 pp. 95), they realize that “by working together as partners, retailers and manufacturers can provide the greatest value to customers at the lowest possible cost.” In this section we are concerned with the agents’ optimal selection of strategies but not with the manner they share profits, which may be solved using the Nash bargaining solution or any other "fairness" criterion. This is consistent with a scenario of vertical integration whereby players become one single company so profits are not split at all, or with a first-stage negotiation process where representatives from both companies are trying to find the Pareto optimum that maximizes the "pie size" that will be divided later on. In what follows we label this cooperative setup as vertical integration, although its conclusions hold for the other circumstances described above. Formally, suppose that a central planner wants to maximize $\pi = \pi_S + \pi_R$, which is obtained by adding the expressions (2) and (3), that is:

$$\pi(f, r) = (M + N) \cdot D \cdot \exp(-E(f - r)) - F \cdot \exp(-f) - R \cdot (\exp(r) - 1) \quad (9)$$

Since $\pi(f, r)$ can be differentiated at any point, given that it is the sum of exponential functions that are differentiable at their domain, the optimum must be either at a stationary point (f_s, r_s) or at an extreme of its domain. When solving $\partial\pi(f, r)/\partial f = \partial\pi(f, r)/\partial r = 0$ there is only one stationary point:

$$f_s = \frac{E \ln\frac{R}{F} + \ln\frac{F}{(M+N)DE}}{1-2E}, r_s = \frac{(1-E)\ln\frac{F}{R} + \ln\frac{(M+N)DE}{F}}{1-2E}, i_s = \frac{\ln\frac{FR}{D^2E^2(M+N)^2}}{1-2E}$$

If the conditions of **Proposition 3** or **Proposition 4** hold, replacing in the above expression the parameters D, F and R as calculated by (7), it follows that:

$$f_s = f_e - \frac{\ln\left(\left(\frac{M}{N}\right)^E \frac{M+N}{M}\right)}{1-2E}; r_s = r_e - \frac{\ln\left(\left(\frac{M}{N}\right)^E \frac{N}{M+N}\right)}{1-2E}; i_s = (f_e - r_e) + \frac{\ln\left(\frac{MN}{(M+N)^2}\right)}{1-2E} \quad (10)$$

Proposition 5: The percentage of faulty items f_s at the stationary point (f_s, r_s) is strictly lower than the percentage of faulty items f_e in equilibrium.

Proof: Given that $E < 0.5$, if $(M/N)^E(M+N)/M > 1$ then $f_s < f_e$. Replacing

M by $C - N$ the function $((C - N)/N)^E \cdot C/(C - N)$ is convex when $1 \le N \le C$, so applying the first order condition the minimum is at $N = EC$ so the function is equal to $(1 - E)^{E-1}E^{-E}$ which is strictly greater than 1 for any $E > 0$.

Proposition 6: The stationary point always generates fewer errors to the end customer.

Proof: Given that $E < 0.5$ and that $M \cdot N < (M + N)^2$ so for $M, N \ge 1$, expression (10) states that i_s has a value strictly lower that i_e.

The interpretation of **Proposition 5** and **Proposition 6** is that if the stationary point is feasible, the vertical integration has a positive effect on quality in two aspects: The supplier reduces the number of errors and simultaneously the end customer receives fewer defective items.

Proposition 7: The percentage of returns r_s at the stationary point (f_s, r_s) must be positive.

Proof: Assume by contradiction that r_s is smaller than or equal to zero, so expression (10) is equivalent to imposing $E \cdot \ln(M) + (1 - E) \cdot \ln(N) \ge \ln(M + N) + (1 - 2E) \cdot r_e$. Since $(1 - 2E) \cdot r_e \ge 0$ it must hold that $E \cdot \ln(M) + (1 - E) \cdot \ln(N) \ge \ln(M + N)$, which is equivalent to $M^E \cdot N^{(1-E)} \ge M + N$. On the other hand, if $E \le 1$ and $M, N \ge 1$ it holds that $M + N > E \cdot M + (1 - E) \cdot N$ which is greater than or equal to $M^E \cdot N^{(1-E)}$ due to the weighted average inequality.

From **Proposition 5** it holds that the stationary point (f_s, r_s) must lie below the horizontal $f = f_e$ in Figure 4; from **Proposition 6** it must lie to the right of the 45-degree line $i_s = i_e$ that crosses the Nash equilibrium (f_e, r_e); and from **Proposition 7** it must lie to the right of the vertical line $r = 0$.

Proposition 8: If $\ln M + \ln N \geq 2\ln C - (1-2E)(f_e - r_e)$ then the stationary point (f_s, r_s) is the global optimal operation of the supply chain. If there is strict inequality, then $f_s > r_s$ so the customer receives errors.

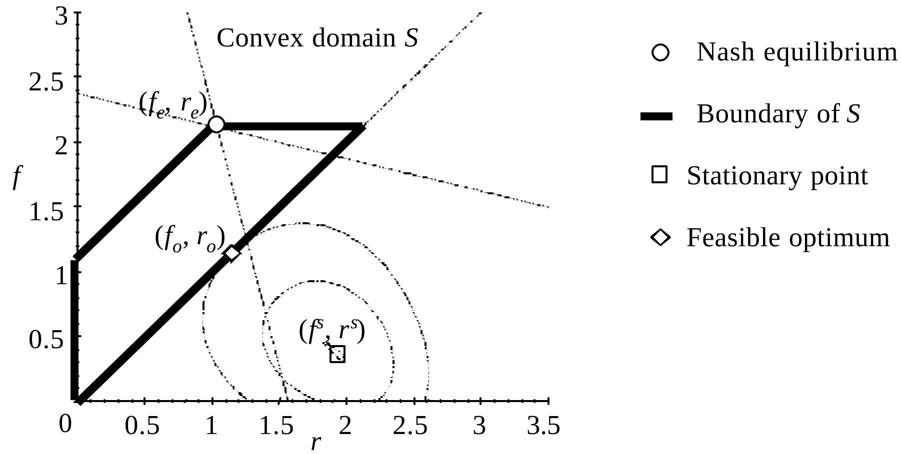
Proof: In the Appendix.

Corollary: If $\ln M + \ln N \geq 2\ln C - (1-2E)(f_e - r_e)$ then the equilibrium is never optimal, since the stationary point, which is the optimum in this case, is different from the equilibrium according to **Proposition 5**.

Proposition 9: If $\ln M + \ln N \leq 2\ln C - (1-2E)(f_e - r_e)$ then the optimal operation (f_o, r_o) of the supply chain is $(0, 0)$ if $\ln M - \ln N \leq -(f_e + r_e)$ or $f_o = r_o = ½(f_e + r_e + \ln(M/N))$ otherwise.

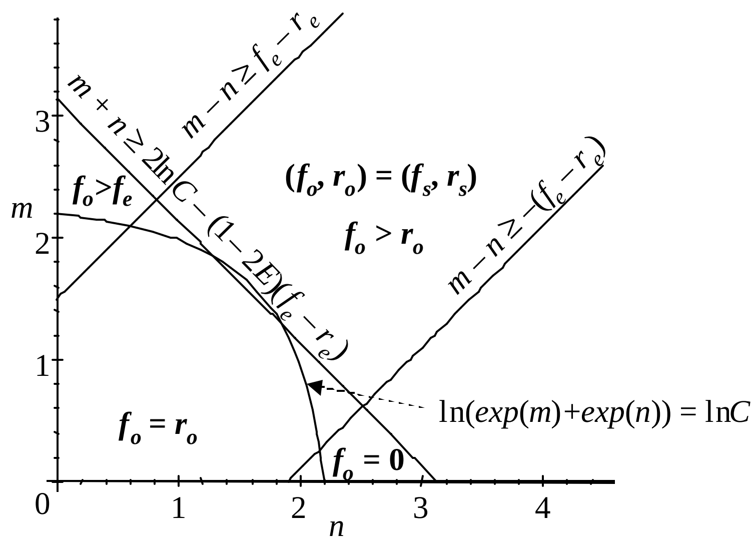
Proof: If $\ln M + \ln N = 2\ln C - (1-2E)(f_e - r_e)$ then according to (10) $i_s = 0$ so $f_s = r_s$. Otherwise $i_s < 0$ so the stationary point is outside the polyhedron S defined by inequalities $f \leq f_e$, $r \geq 0$, $f - r \leq f_e - r_e$ and $f \geq r$ depicted by Figure 4. Consider the half space $f - r \geq 0$, which is a convex domain. The optimum (f_o, r_o) for $\pi(f, r)$ is not within this space, since it includes neither a stationary nor a non-differentiable point, so (f_o, r_o) must be within its boundary $f = r$. Substituting this equation in expression (9) yields $\pi_{f=r}(g) = (M + N)\cdot D - F\cdot exp(-g) - R\cdot(exp(g) - 1)$ with $g \geq 0$, which is concave, since $\partial\pi_{f=r}(g)/\partial g = -(F\cdot exp(-2g)+R)\cdot exp(g) < 0$. From the first-order condition, it holds that the optimum is $g = ½\ln(F/R)$ that combined with (7) is equal to $g = ½(f_e + r_e + \ln(M/N))$. If $g \geq 0$ then it is feasible so $f_o = r_o = g$; otherwise the optimum must lie on an extreme of the domain so $f_o = r_o = 0$.

FIGURE 4
GLOBAL FEASIBLE AND INFEASIBLE OPTIMA



From the above propositions, it is possible to draw the constraints defining the value of the margins M and N, such that the optimal operation (f_o, r_o) of the vertically integrated supply chain has different features. Figure 5 shows such constraints in the space of $m = \ln M$ and $n = \ln N$ for $E = 0.01$, $M + N = C = 10$, $f_e = 1.7$, $r_e = 0.2$, where most of them have a linear shape but $\ln(exp(m)+exp(n)) = \ln C = \ln(M + N)$. If $\ln M + \ln N > 2\ln C - (1- 2E)(f_e - r_e)$, represented by the upper right semi plane, then $(f_o, r_o) = (f_s, r_s)$ so the customer receives a positive number of errors, although fewer than before. If $\ln M + \ln N \leq 2\ln C - (1- 2E)(f_e - r_e)$ then $f_o = r_o$ so the customer receives no errors. In this case, if $\ln M - \ln N \leq -(f_e - r_e)$ then $f_o = r_o = 0$, meaning that the supplier stops generating defective products. On the contrary, if $\ln M - \ln N > f_e - r_e$ then $f_o > f_e$, so the supplier generates more defective products than before, imposing a higher burden to the retailer to catch them all. Although this latter result does not contradict the literature in terms that the integration does in fact improve final quality, it is surprising because it shows that the supplier's attitude towards quality degenerates, providing a service to the retailer that is worse than before.

FIGURE 5
VERTICAL INTEGRATION IN THE SPACE OF m = lnM and n = lnN



The above results are consistent with the anecdotic evidence provided by Kumar (1996) and with the empirical research by Stanley and Wisner (2001) who find strong positive relationships between the implementation of cooperative purchasing/supplier relationships, internal service quality, and the quality of services and products provided to external customers. This bears a resemblance with the double marginalization problem where a chain of monopolies makes less profit than the integrated industry, since the autonomous agents misalign their decisions. In our case, final quality, and therefore total profit, is also jeopardized by the non-cooperative behavior of the players. Also, our model concurs with the idea of “optimal quality” by Juran (1988) that there is an optimum level of conformance quality above which it is not advantageous to improve. The novelty of our work is that it partially responds to the challenge posted by Sousa and Voss (2002): To identify under which conditions quality may not be free. We conclude that the quality effort in cooperative or integrated supply chains is determined by the agents' initial share of the margin. If the margins were similar, then both agents would be already eager to reduce the number of defective products, so integration improves quality, although not by much. If margins were unequal, then integration would be especially effective since the agent that was undermined would show a great advance on quality. For the case of the supplier, if his margin were below a given threshold, then in the integrated chain he would generate no errors, while if such margin were above another level then he would free-ride the supplier, who would significantly increment his effort to detect defective items.

IV. CONCLUSIONS

We have studied a distribution channel where the supplier and the retailer determine how much effort to invest in improving quality as a function of an end customer's demand and the cost of the different strategies. We conjecture the structure of the demand and cost functions based on assumptions that are common to the scientific literature, with two exceptions: we are assuming that products are disposable and that the cost elasticities of both agents are related.

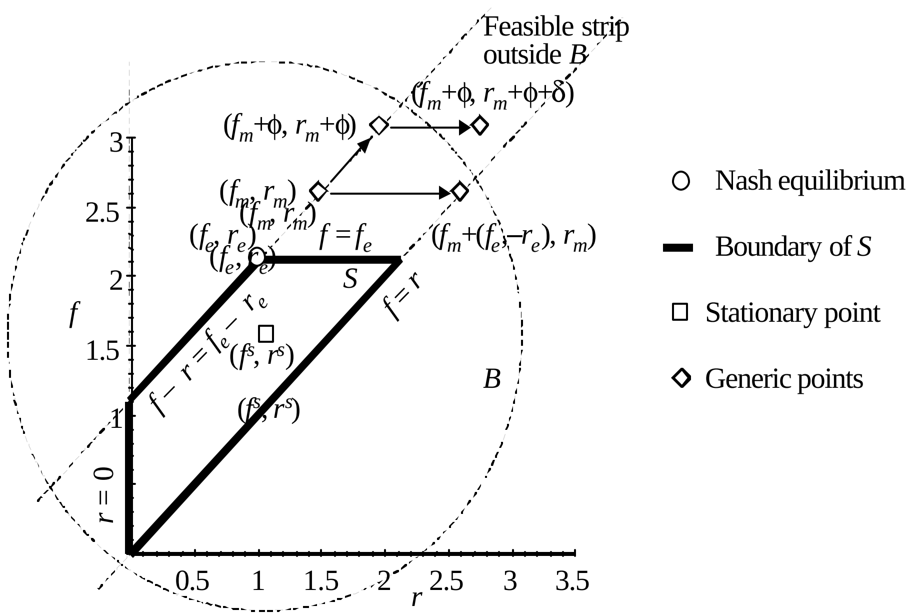
By characterizing the Nash equilibrium, we derive the conditions under which implicit parameters related to quality, namely the total quality cost for the supplier and the compliance cost for the retailer, can be deduced from observable data. With this information, we are able to derive the properties of the global optimum for a vertically integrated supply chain that depends on the margins and the original equilibrium. If the sum of logarithms of the margins is above a given threshold, then the customer receives a positive number of errors, otherwise the customer receives zero errors. In this last case the structure of the solution depends on the difference of logarithms of the margins: if it is below a certain value, then the supplier generates no errors, otherwise he generates more than in the original situation.

The relevance of these findings is that, although it is an accepted fact that vertical integration improves quality for the end customer, we provide additional insight about how this improvement takes place: it may or may not lead to near-zero errors, and it may or may not enhance the supplier's performance. Considering the evidence that vertical integration tends to increase costs, managers should carefully assess whether the loss of efficiency is justified by the expected quality improvement.

APPENDIX

Proof of **Proposition 8**: If $\ln M + \ln N \geq 2\ln C - (1 - 2E)(f_e - r_e)$ is true then $i_s \geq 0$ according to (10), so the stationary point (f_s, r_s) is within the polyhedron S defined by inequalities $f \leq f_e$, $r \geq 0$, $f - r \leq f_e - r_e$ and $f \geq r$ shown by Figure 6.

FIGURE 6
STATIONARY POINT AS GLOBAL OPTIMUM



Consider point (f_m, r_m) such that:

(i) $f_m - r_m = f_e - r_e$, so (f_m, r_m) is in the same 45° line as (f_e, r_e).

(ii) $\pi(f_m, r_m) < \pi(f_s, r_s)$: it can be attained since $\pi(f, r)$ can be arbitrarily low due to the cost of the retailer $R{\cdot}(\exp(r_m) - 1)$ when r_m is large enough.

(iii) $F{\cdot}\exp(-f_m) < R{\cdot}\exp(f_m - (f_e - r_e))$ which holds with f_m large enough.

(iv) $\partial\pi(f, r)/\partial r$ evaluated in $(f_m + (f_e - r_e), r_m) < 0$: it forces $DE{\cdot}\exp(-E(0))(M+N) - F{\cdot}\exp(r_m) < 0$ which is true with r_m large enough.

It holds that $\pi(f_m+\phi,\ r_m+\phi) < \pi(f_m,\ r_m)$ since $\partial((M + N)\cdot D\cdot\exp(-E(f_e - r_e)) - F\cdot\exp(-g) - R\cdot\exp(g - (f_e - r_e)))/\partial g = \partial(- F\cdot\exp(-g) - R\cdot\exp(g - (f_e - r_e)))/\partial g = F\cdot\exp(-f_m) - R\cdot\exp(f_m - (f_e - r_e))$ which is < 0 from (iii). It holds that $\pi(f_m+\phi,\ r_m+\phi+\pi) < \pi(f_m+\phi,\ r_m+\phi)$ since $\partial((M + N)\cdot D\cdot\exp(-E(f_e - (r_e + h))) - F\cdot\exp(-(f_m + \phi)) - R\cdot\exp(r_m + \phi + h))/\partial h = DE\cdot\exp(-E(f_e - (r_e + h))(M+N) - R\cdot\exp(r_m + \phi + h) < 0$ for any $\phi \geq 0$ from (iv). Therefore, any point $(f_m+\phi,\ r_m+\phi+\delta)$ with $\phi,\ \delta \geq 0$ has a lower value than $(f_m,\ r_m)$, which by (ii) has a lower value than $(f_s,\ r_s)$, so the function decreases as f and r grow. Defining a ball B that contains the origin and $(f_m,\ r_m)$, $(f_s,\ r_s)$ is the optimum in B since it is the unique stationary point, while the feasible strip outside B decreases, so $(f_s,\ r_s)$ is the global optimum.

If $\ln M + \ln N > 2\ln C - (1- 2E)(f_e - r_e)$ then when calculating $f_s - r_s$ according to expression (10), it results in a positive number, so the customer receives errors.

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