PHYSICAL REVIEW D **79,** 013005 (2009)

# Neutrino masses in split supersymmetry

Marco Aurelio Díaz,[1] Pavel Fileviez Pérez,[2,3] and Clemencia Mora[4]

[1]*Departamento de Física, Universidad Católica de Chile, Avenida Vicuña Mackenna 4860, Santiago, Chile*
[2]*Department of Physics, University of Wisconsin-Madison, 1150 University Avenue, Madison, Wisconsin 53706, USA*
[3]*CFTP, Instituto Superior Tecnico, Avenue Rovisco Pais 1, 1049-001, Lisbon, Portugal*
[4]*Département de Physique Nucléaire et Corpusculaire, Université de Genève, 24 Quai Ernest-Ansermet, 1211 Genève 4, Suisse*
(Received 1 March 2007; published 15 January 2009)

We investigate the possibility of generating neutrino masses in the context of split supersymmetric scenarios where all sfermions are very heavy. All relevant contributions coming from the R-parity violating terms to the neutrino mass matrix up to one-loop level are computed showing the importance of the Higgs bosons one-loop corrections. We conclude that it is not possible to generate all neutrino masses and mixings in split SUSY with bilinear R-parity violating interactions. In the case of partial split SUSY, the one-loop Higgs bosons contributions are enough to generate the neutrino masses and mixings in agreement with the experiment. In the context of minimal SUSY SU(5), we find new contributions that help us to generate neutrino masses in the case of split SUSY.

DOI: 10.1103/PhysRevD.79.013005 PACS numbers: 14.60.Pq, 12.15.Ff, 12.15.Lk, 12.60.Jv

## I. INTRODUCTION

Supersymmetric extensions of the standard model (SM) have been considered as one of the most appealing candidates for physics beyond the SM. Recently, different supersymmetric scenarios have been studied extensively. We mention low-energy SUSY [1], where the supersymmetric scale is around TeV, and split SUSY where all the scalars, except for one Higgs doublet, are very heavy [2]. In both supersymmetric scenarios mentioned above it is possible to achieve unification of the gauge interactions at high scale and the lightest supersymmetric particle could be a natural candidate to describe the cold dark matter of the Universe once the so-called R-parity is imposed as an exact symmetry of the theory. In split- SUSY scenarios, by ignoring the hierarchy problem, most of the unpleasant aspects of low-energy SUSY, such as excessive flavor and *CP* violation, and very fast dimension-five proton decay, are eliminated.

It is very-well known that in general interactions that break the lepton or baryon number (or R-parity) are present in any SUSY extension of the SM. Therefore, we have the possibility of generating the neutrino masses and mixing [3], and we have to understand the predictions for proton stability [4]. For several phenomenological aspects of R-parity violating interactions see Ref. [5]. The possibility of describing the neutrino properties with R-parity violating interactions in the context of the minimal supersymmetricstandard model has been studied in detail for several groups in the context of low-energy supersymmetry (See, for example, Refs. [6,7]). In the context of split SUSY the possibility of describing the masses and mixing of neutrinos has been studied in Ref. [8], where the authors concluded that it is not possible to use the R-parity bilinear terms alone to describe the neutrino properties.

In this work we re-examine the possibility of describing the properties of neutrinos using the R-parity violating interactions in the context of split supersymmetric scenarios. We agree with the results presented in Ref. [8] that in split supersymmetry, where only one Higgs doublet remains at the weak scale, it is not possible to generate the neutrino masses in agreement with the experiments and explain the reasons in detail. We study an alternative split SUSY scenario, where only the sfermions are very heavy, while all Higgs can be light. We refer to this scenario as "partial split SUSY". Notice that in this scenario we can keep the nice features of split SUSY such as the suppression of proton decay due to R-parity violation and unification of gauge couplings at high scale. In this SUSY scenario, we show that it is possible to generate neutrino masses using all relevant interactions once the heavy sfermions are integrated out. Computing all contributions up to one-loop level, we find an example solution, where it is shown that all constraints coming from neutrino experiments on the R-parity violating interactions are satisfied. In this scenario even if R parity is broken one could have the gravitino as a possible cold dark matter candidate.

We conclude that in partial split SUSY (PSS) it is possible to generate all neutrino masses and mixing in agreement with the experiments using the bilinear terms alone and the trilinear R-parity violating couplings are essentially irrelevant. The key element is that the symmetry of the neutrino mass matrix at tree level is broken by the Higgs bosons loops together with neutralinos and charginos. The terms that break the symmetry of neutrino mass matrix vanish in the decoupling limit, making the description of the neutrino masses in the "standard" split SUSY scenario impossible. We study the same issue in the context of the minimal supersymmetric SU(5), where one finds new contributions that help us to generate neutrino masses in agreement with the experiments in the case of split SUSY.

## II. R-PARITY VIOLATION AND NEUTRINO MASSES IN SPLIT SUSY

As we know in any supersymmetric extension of the standard model there are interactions terms that break the so-called R-parity. The R-parity is defined as $R = (-1)^{3(B-L)+2S}$, where $L$, $B$, and $S$ are the lepton and baryon number, and the spin, respectively. Usually this symmetry is considered as an exact symmetry of the minimal supersymmetric extension of the standard model (MSSM) in order to avoid the dimension four contributions to proton decay, and at the same time there is a possibility of havng the lightest supersymmetric particle as a good candidate for the cold dark matter of the Universe.

In the context of the MSSM, the so-called R-parity violating terms are given by

$$\mathcal{W}_{\mathrm{NR}} = \alpha_{ijk}\hat{Q}_i\hat{L}_j\hat{D}^C_k + \beta_{ijk}\hat{U}^C_i\hat{D}^C_j\hat{D}^C_k + \gamma_{ijk}\hat{L}_i\hat{L}_j\hat{E}^C_k + \epsilon_i\hat{L}_i\hat{H}_u, \tag{1}$$

where $\beta_{ijk} = -\beta_{ikj}$ and $\gamma_{ijk} = -\gamma_{jik}$. As it is well known due to the presence of the first and second terms in the above equation one has the so-called the dimension-four contributions to the decay of the proton. In this case, in order to satisfy the experimental bounds on the proton decay lifetime, one has to assume that the multiplication of the couplings $\alpha_{ijk}$ and $\beta_{ijk}$ is of the order $10^{-21}$ when the SUSY scale is at electroweak scale. In order to avoid these very small couplings in the theory, one imposes by hand the R-parity symmetry. There is a second way to avoid these small couplings if the SUSY breaking scale is large,; this is the case of split SUSY. Since in this case there is no need to impose any symmetry by hand, we stick to this possibility and study the generation of neutrino masses in this context.

Let us discuss how to generate neutrino masses through this mechanism in three different scenarios:

(i) *MSSM with split SUSY:* In this supersymmetric scenario called split SUSY, all scalars are very heavy, except for one Higgs doublet. Integrating out the heavy scalars all possible R-parity conserving interactions in split supersymmetric scenarios are given by [2]

$$\begin{aligned}\mathcal{L}^{\mathrm{split}}_{\mathrm{susy}} = {}& \mathcal{L}^{\mathrm{split}}_{\mathrm{kinetic}} + m^2 H^\dagger H - \frac{\lambda}{2}(H^\dagger H)^2 \\ & - \Big[ Y_u \bar{q}_L u_R i\sigma_2 H^* + Y_d \bar{q}_L d_R H + Y_e \bar{l}_L e_R H \\ & + \frac{M_3}{2}\tilde{G}\tilde{G} + \frac{M_2}{2}\tilde{W}\tilde{W} + \frac{M_1}{2}\tilde{B}\tilde{B} + \mu \tilde{H}^T_u i\sigma_2 \tilde{H}_d \\ & + \frac{1}{\sqrt{2}} H^\dagger(\tilde{g}_u \sigma\tilde{W} + \tilde{g}'_u\tilde{B})\tilde{H}_u + \frac{1}{\sqrt{2}} H^T i\sigma_2 \\ & \times(-\tilde{g}_d\sigma\tilde{W} + \tilde{g}'_d\tilde{B})\tilde{H}_d + \mathrm{H.c.}\Big],\end{aligned} \tag{2}$$

where

$$H = \begin{pmatrix} H^+ \\ \frac{1}{\sqrt{2}}(v + \phi^0 + i\varphi^0) \end{pmatrix} \tag{3}$$

is the SM Higgs. In the above equations, we have the SM fields $q_L$, $u_R$, $d_R$, $l_L$, $e_R$, and the superpartners of the Higgs and gauge bosons present in the MSSM. Following our notation, $\tilde{G}$, $\tilde{W}$, and $\tilde{B}$ are the gauginos associated to the SU(3), SU(2), and U(1) gauge groups, respectively. While $\tilde{H}_u$ and $\tilde{H}_d$ correspond to the up and down Higgsinos. The parameters in Eq. (2) are the following: $m$ is the Higgs mass parameter, $\lambda$ is the Higgs quartic self-coupling; $Y_u$, $Y_d$, and $Y_e$ are the Yukawa couplings; $M_3$, $M_2$, and $M_1$ are the gaugino masses, $\mu$ the Higgsino mass, and $\tilde{g}_u$, $\tilde{g}'_u$, $\tilde{g}_d$, and $\tilde{g}'_d$ are trilinear couplings between the Higgs boson, gauginos, and Higgsinos.

The Higgs-gaugino-Higgsino couplings in Eq. (2) satisfy matching conditions at the scale $\tilde{m}$. Above this scale, the theory is supersymmetric and the squarks, sleptons, and heavy Higgs doublet have a mass assumed to be nearly degenerate equal to $\tilde{m}$. The supersymmetric Lagrangian includes the terms

$$\mathcal{L}_{\mathrm{susy}} \ni -\mu\tilde{H}^T_u i\sigma_2\tilde{H}_d - \frac{H^\dagger_u}{\sqrt{2}}(g\sigma\tilde{W} + g'\tilde{B})\tilde{H}_u - \frac{H^\dagger_d}{\sqrt{2}}(g\sigma\tilde{W} - g'\tilde{B})\tilde{H}_d, \tag{4}$$

which implies the following boundary conditions at $\tilde{m}$:

$$\begin{aligned}\tilde{g}_u(\tilde{m}) &= g(\tilde{m})\sin\beta(\tilde{m}), \\ \tilde{g}_d(\tilde{m}) &= g(\tilde{m})\cos\beta(\tilde{m}) \\ \tilde{g}'_u(\tilde{m}) &= g'(\tilde{m})\sin\beta(\tilde{m}), \quad \text{and} \\ \tilde{g}'_d(\tilde{m}) &= g'(\tilde{m})\cos\beta(\tilde{m}),\end{aligned} \tag{5}$$

where $g(\tilde{m})$ and $g'(\tilde{m})$ are the gauge coupling constants evaluated at the scale $\tilde{m}$. At the same time the angle $\beta$ is the mixing angle between the two Higgs doublets $H_d$ and $H_u$ of the supersymmetric model. In order to set our notation the two doublets are given by

$$\begin{aligned}H_d &= \begin{pmatrix} \frac{1}{\sqrt{2}}(v_d + \phi^0_d + i\varphi^0_d) \\ H^-_d \end{pmatrix}, \\ H_u &= \begin{pmatrix} H^+_u \\ \frac{1}{\sqrt{2}}(v_u + \phi^0_u + i\varphi^0_u) \end{pmatrix},\end{aligned} \tag{6}$$

and $\tan\beta = v_u/v_d$. In terms of these two Higgs doublets of the MSSM, the light fine-tuned Higgs doublet $H$ in the low-energy effective model is $H = -i\sigma_2 H^*_d \cos\beta(\tilde{m}) + H_u \sin\beta(\tilde{m})$.

As we mentioned before in split SUSY scenarios at low energy we have the SM fields, the charginos, and neutralinos. Using the above notation the chargino mass matrix is given by

$$\mathbf{M}_{\chi^+}^{\mathrm{SS}} = \begin{bmatrix} M_2 & \frac{1}{\sqrt{2}}\tilde{g}_u v \\ \frac{1}{\sqrt{2}}\tilde{g}_d v & \mu \end{bmatrix}, \tag{7}$$

while the neutralino mass matrix reads as

$$\mathbf{M}_{\chi^0}^{\mathrm{SS}} = \begin{bmatrix} M_1 & 0 & -\frac{1}{2}\tilde{g}'_d v & \frac{1}{2}\tilde{g}'_u v \\ 0 & M_2 & \frac{1}{2}\tilde{g}_d v & -\frac{1}{2}\tilde{g}_u v \\ -\frac{1}{2}\tilde{g}'_d v & \frac{1}{2}\tilde{g}_d v & 0 & -\mu \\ \frac{1}{2}\tilde{g}'_u v & -\frac{1}{2}\tilde{g}_u v & -\mu & 0 \end{bmatrix}. \tag{8}$$

Now, since we are interested in the possibility of describing the neutrino masses in split SUSY, we write all relevant R-parity violating interactions as

$$\mathcal{L}_{\mathrm{RpV}}^{\mathrm{split}} = \epsilon_i \tilde{H}_u^T i\sigma_2 L_i - \frac{1}{\sqrt{2}} a_i H^T i\sigma_2(-\tilde{g}_d \sigma \tilde{W} + \tilde{g}'_d \tilde{B}) L_i + \mathrm{H.c.}, \tag{9}$$

where $\epsilon_i$ are the parameters that mix Higgsinos with leptons, and $a_i$ are dimensionless parameters that mix gauginos with leptons. Notice that the first term is the usual bilinear term, while the last two terms are obtained once we integrate out the sleptons using the bilinear soft terms ($\tilde{L}_i H_u$), which break explicitly R parity. As it is well known, we can also write the usual R-parity violating trilinear terms ($\hat{Q}\hat{D}^C\hat{L}$, $\hat{L}\hat{L}\hat{E}^C$). However, since the sfermions are very heavy in split SUSY and the contributions to the neutrino mass matrix coming from those terms are at one-loop level, those interactions cannot play any important role. Using Eq. (9), after the Higgs acquires a vacuum expectation value (vev), we find the relevant terms for neutrino masses

$$\mathcal{L}_{\mathrm{RpV}}^{\mathrm{split}} = -[\epsilon_i \tilde{H}_u^0 + \tfrac{1}{2} a_i v(\tilde{g} c_\beta \tilde{W}_3 - \tilde{g}' c'_\beta \tilde{B})]\nu_i + \mathrm{H.c.} + \ldots, \tag{10}$$

where $v$ is the vacuum expectation value of the SM-like Higgs field $H$. Knowing all R-parity violating interactions, we can write the neutralino/neutrino mass matrix as

$$\mathcal{M}_N^{\mathrm{SS}} = \begin{bmatrix} \mathrm{M}_{\chi^0}^{\mathrm{SS}} & (m^{\mathrm{SS}})^T \\ m^{\mathrm{SS}} & 0 \end{bmatrix}, \tag{11}$$

where $\mathrm{M}_{\chi^0}^{\mathrm{SS}}$ is given by Eq. (7) and $m$ reads as

$$m^{\mathrm{SS}} = \begin{bmatrix} -\frac{1}{2}\tilde{g}'_d a_1 v & \frac{1}{2}\tilde{g}_d a_1 v & 0 & \epsilon_1 \\ -\frac{1}{2}\tilde{g}'_d a_2 v & \frac{1}{2}\tilde{g}_d a_2 v & 0 & \epsilon_2 \\ -\frac{1}{2}\tilde{g}'_d a_3 v & \frac{1}{2}\tilde{g}_d a_3 v & 0 & \epsilon_3 \end{bmatrix}. \tag{12}$$

We define the parameters $\lambda_i \equiv a_i \mu + \epsilon_i$, which are related to the traditional bilinear R-parity violation (BRpV) parameters $\Lambda_i$ [9] by $\Lambda_i = \lambda_i v_d$. Integrating out the neutralinos, we find that the neutrino mass matrix is given by

$$\begin{aligned} \mathbf{M}_\nu^{\mathrm{eff}} &= -m^{\mathrm{SS}}(\mathrm{M}_{\chi^0}^{\mathrm{SS}})^{-1}(m^{\mathrm{SS}})^T \\ &= \frac{v^2}{4\det M_{\chi^0}^{\mathrm{SS}}}(M_1\tilde{g}_d^2 + M_2\tilde{g}_d'^2) \\ &\quad \times \begin{bmatrix} \lambda_1^2 & \lambda_1\lambda_2 & \lambda_1\lambda_3 \\ \lambda_2\lambda_1 & \lambda_2^2 & \lambda_2\lambda_3 \\ \lambda_3\lambda_1 & \lambda_3\lambda_2 & \lambda_3^2 \end{bmatrix}, \end{aligned} \tag{13}$$

where the determinant of the neutralino mass matrix is

$$\det M_{\chi^0}^{\mathrm{SS}} = -\mu^2 M_1 M_2 + \tfrac{1}{2}v^2\mu(M_1\tilde{g}_u\tilde{g}_d + M_2\tilde{g}'_u\tilde{g}'_d) + \tfrac{1}{16}v^4(\tilde{g}'_u\tilde{g}_d - \tilde{g}_u\tilde{g}'_d)^2. \tag{14}$$

Notice that the effective neutrino mass matrix $\mathbf{M}_\nu^{\mathrm{eff}}$ has only one eigenvalue different from zero. As in the case of R-parity violation in the MSSM with bilinear terms, at tree level only one neutrino is massive. Therefore, we have to investigate all possible one-loop contributions to the neutrino mass matrix, which help us to generate the atmospheric and solar neutrino masses. It has been argued in the literature [8] that using the bilinear terms it is not possible to explain the neutrino masses and mixing. We study this issue in detail, and as we will show in the next section, once we include the one-loop contributions to the neutrino mass matrix it is not possible to generate all neutrino masses in agreement with the experiment.

(ii) *MSSM with partial split SUSY* Let us study the same issue of how to generate neutrino masses through R-parity violating interactions in partial split SUSY, where only the sfermions are very heavy, while the Higgs can be light. Notice that in this case proton decay can be suppressed, and the unification of the gauge interactions at high scale is possible as well. We will show that in this scenario the contributions from the light Higgs bosons is enough to generate the neutrino masses at one loop, and study the decoupling limit in order to have a better understanding of the results presented in the previous section.

We integrate out the heavy squarks and sleptons and find that the R-parity conserving (RpC) interactions below the scale $\tilde{m}$ are given by

$$\begin{aligned} \mathcal{L}_{\mathrm{PSS}}^{\mathrm{RpC}} \ni -\Big[ & m_1^2 H_d^\dagger H_d + m_2^2 H_u^\dagger H_u - m_{12}^2(H_d^T \epsilon H_u + \mathrm{H.c.}) + \frac{1}{2}\lambda_1(H_d^\dagger H_d)^2 + \frac{1}{2}\lambda_2(H_u^\dagger H_u)^2 \\ & + \lambda_3(H_d^\dagger H_d)(H_u^\dagger H_u) + \lambda_4|H_d^T\epsilon H_u|^2\Big] \\ & + h_u \bar{\mu}_R H_u^T \epsilon q_L - h_d \bar{d}_R H_d^T \epsilon q_L - h_e \bar{e}_R H_d^T \epsilon l_L \\ & - \frac{1}{\sqrt{2}} H_u^\dagger(\tilde{g}_u \sigma \tilde{W} + \tilde{g}'_u \tilde{B})\tilde{H}_u - \frac{1}{\sqrt{2}} H_d^\dagger(\tilde{g}_d \sigma \tilde{W} - \tilde{g}'_d \tilde{B})\tilde{H}_d + \mathrm{H.c.} \end{aligned} \tag{15}$$

In the above equations, the two Higgs doublets that survive at the weak scale are $H_d$ and $H_u$. The parameters in Eq. (15) not defined before are the following: $m_1^2$, $m_2^2$, and $m_{12}^2$ are the Higgs mass parameters, $\lambda_i$, $i = 1, 2, 3, 4$ are the Higgs quartic self-couplings; and $h_u$, $h_d$, and $h_e$ are the Yukawa couplings. The Higgs-gaugino-Higgsino, gauge, and Yukawa couplings in Eq. (15) satisfy matching conditions at the scale $\tilde{m}$. Above this scale, the theory is supersymmetric, and the squarks and sleptons have a mass assumed to be nearly degenerate to $\tilde{m}$. The supersymmetric Lagrangian above $\tilde{m}$ includes the terms

$$\begin{aligned}\mathcal{L}_{\rm susy}^{\rm RpC} \ni -\Big[ & m_1^2 H_d^\dagger H_d + m_2^2 H_u^\dagger H_u - m_{12}^2 (H_d^T \epsilon H_u \\ & + \text{H.c.}) + \frac{1}{8}(g^2 + g'^2)(H_d^\dagger H_d)^2 \\ & + \frac{1}{8}(g^2 + g'^2)(H_u^\dagger H_u)^2 + \frac{1}{4}(g^2 - g'^2) \\ & \times (H_d^\dagger H_d)(H_u^\dagger H_u) - \frac{1}{2} g^2 |H_d^T \epsilon H_u|^2 \Big] \\ & + \lambda_u \bar{u}_R H_u^T \epsilon q_L - \lambda_d \bar{d}_R H_d^T \epsilon q_L - \lambda_e \bar{e}_R H_d^T \epsilon l_L \\ & - \frac{1}{\sqrt{2}} H_u^\dagger (g \sigma \tilde{W} + g' \tilde{B}) \tilde{H}_u - \frac{1}{\sqrt{2}} H_d^\dagger (g \sigma \tilde{W} \\ & - g' \tilde{B}) \tilde{H}_d + \text{H.c.} \end{aligned} \tag{16}$$

Consequently, at the scale $\tilde{m}$ we have the following boundary conditions for the Higgs couplings:

$$\begin{aligned} \lambda_1 &= \lambda_2 = \tfrac{1}{4}(g^2 + g'^2), \\ \lambda_3 &= \tfrac{1}{4}(g^2 - g'^2), \\ \lambda_4 &= -\tfrac{1}{2} g^2 \end{aligned} \tag{17}$$

for the Yukawa couplings $h_u = \lambda_u$, $h_d = \lambda_d$, $h_e = \lambda_e$, and for the Higgsino-gaugino Yukawa couplings, $\tilde{g}_u = \tilde{g}_d = g$, $\tilde{g}'_u = \tilde{g}'_d = g'$. All of them are evaluated at the scale $\tilde{m}$. Note the difference between these boundary conditions and the corresponding ones in the original split supersymmetric model: the former do not involve the angle $\beta$. At the weak scale, the minimization of the Higgs potential leads to a vacuum expectation value for both Higgs doublets, which satisfy $v_d^2 + v_u^2 = v^2$, such that $m_W^2 = 1/2 g^2 v^2$ and $m_Z^2 = 1/2(g^2 + g'^2)v^2$, as usual for a two Higgs doublet model.

As we mentioned before, in split SUSY scenarios, the charginos and neutralinos survive at low energies. Using the above notation the chargino mass matrix is given by

$$\mathbf{M}_{\chi^+}^{\rm PSS} = \begin{bmatrix} M_2 & \frac{1}{\sqrt{2}} v \tilde{g}_u s_\beta \\ \frac{1}{\sqrt{2}} v \tilde{g}_d c_\beta & \mu \end{bmatrix}, \tag{18}$$

while the neutralino mass matrix reads as

$$\mathbf{M}_{\chi^0}^{\rm PSS} = \begin{bmatrix} M_1 & 0 & -\frac{1}{2}\tilde{g}'_d c_\beta v & \frac{1}{2}\tilde{g}'_u s_\beta v \\ 0 & M_2 & \frac{1}{2}\tilde{g}_d c_\beta v & -\frac{1}{2}\tilde{g}_u s_\beta v \\ -\frac{1}{2}\tilde{g}'_d c_\beta v & \frac{1}{2}\tilde{g}_d c_\beta v & 0 & -\mu \\ \frac{1}{2}\tilde{g}'_u s_\beta v & -\frac{1}{2}\tilde{g}_u s_\beta v & -\mu & 0 \end{bmatrix}. \tag{19}$$

The difference with the split supersymmetric case in Eqs. (7) and (8) is in the mixings between Higgsinos and gauginos. Now, with the neutrino masses in mind, we write all relevant R-parity violating interactions in partial split SUSY

$$\mathcal{L}_{\rm PSS}^{\rm RpV} = -\epsilon_i \tilde{H}_u^T \epsilon L_i - \frac{1}{\sqrt{2}} b_i H_u^T \epsilon (\tilde{g}_d \sigma \tilde{W} - \tilde{g}'_d \tilde{B}) L_i + \text{H.c.}, \tag{20}$$

with $b_i$ dimensionless parameters. Using Eq. (20), after the Higgs acquires a vev, we find the relevant terms for neutrino masses

$$\mathcal{L}_{\rm PSS}^{\rm RpV} = -[\epsilon_i \tilde{H}_u^0 + \tfrac{1}{2} b_i v_u (\tilde{g}_d \tilde{W}_3 - \tilde{g}'_d \tilde{B})]\nu_i + \text{H.c.} + \ldots, \tag{21}$$

where $v_d = v c_\beta$ and $v_u = v s_\beta$ are the vev of the two Higgs doublets. The neutralino/neutrino mass matrix still has the form given in Eq. (11), but in this scenario the matrix $m$ reads as

$$m^{\rm PSS} = \begin{bmatrix} -\frac{1}{2}\tilde{g}'_d b_1 v_u & \frac{1}{2}\tilde{g}_d b_1 v_u & 0 & \epsilon_1 \\ -\frac{1}{2}\tilde{g}'_d b_2 v_u & \frac{1}{2}\tilde{g}_d b_2 v_u & 0 & \epsilon_2 \\ -\frac{1}{2}\tilde{g}'_d b_3 v_u & \frac{1}{2}\tilde{g}_d b_3 v_u & 0 & \epsilon_3 \end{bmatrix}. \tag{22}$$

The effective neutrino mass matrix obtained after diagonalizing by blocks is

$$\begin{aligned}\mathbf{M}_\nu^{\rm eff} &= -m^{\rm PSS} (\mathrm{M}_{\chi^0}^{\rm PSS})^{-1} (m^{\rm PSS})^T \\ &= \frac{M_1 \tilde{g}_d^2 + M_2 \tilde{g}'^2_d}{4 \det M_{\chi^0}^{\rm PSS}} \begin{bmatrix} \Lambda_1^2 & \Lambda_1\Lambda_2 & \Lambda_1\Lambda_3 \\ \Lambda_2\Lambda_1 & \Lambda_2^2 & \Lambda_2\Lambda_3 \\ \Lambda_3\Lambda_1 & \Lambda_3\Lambda_2 & \Lambda_3^2 \end{bmatrix}, \end{aligned} \tag{23}$$

with $\Lambda_i = \mu b_i v_u + \epsilon_i v_d$, and with the determinant of the neutralino submatrix equal to

$$\det M_{\chi^0}^{\rm PSS} = -\mu^2 M_1 M_2 + \tfrac{1}{2} v_u v_d \mu (M_1 \tilde{g}_u \tilde{g}_d + M_2 \tilde{g}'_u \tilde{g}'_d), \tag{24}$$

which is analogous to Eq. (14).

(iii) *SUSY SU(5) with split SUSY:* Now, let us discuss how one can find the R-parity violating couplings in the context of the simplest UV completion of the MSSM, the minimal SUSY SU(5). In this context the relevant superpotential is given by

$$\mathcal{W}_{\rm NR}^{\rm SU(5)} = \eta_i \hat{\bar{5}}_i \hat{5}_H + c_i \hat{\bar{5}}_i \hat{24}_H \hat{5}_H + \Lambda_{ijk} \hat{10}_i \hat{\bar{5}}_j \hat{\bar{5}}_k, \tag{25}$$

where our notation is $\bar{5}^T = (\hat{D}^C, -\hat{L}^T i\sigma_2)$, $10 = (\hat{Q}, \hat{U}^C, \hat{E}^C)$, $5_H^T = (\hat{T}, \hat{H}_u)$, and $\hat{24}_H = (\hat{\Sigma}_8, \hat{\Sigma}_3, \hat{\Sigma}_{(3,2)}, \hat{\Sigma}_{(\bar{3},2)}, \hat{\Sigma}_{24})$. Since all trilinear terms are coming from the same term in SU(5) one finds

$$\alpha_{ijk}/2 = \beta_{ijk} = \gamma_{ijk} = \Lambda_{ijk} = -\Lambda_{ikj}, \tag{26}$$

and the relevant interactions for the generation of neutrinos masses are given by

$$\mathcal{L}_{\rm RpV} = -a_i \nu_i \tilde{H}_u^0 + \frac{1}{2} c_i \nu_i \tilde{\Sigma}_3^0 H_u^0 + \frac{3c_i}{2\sqrt{15}} \nu_i \tilde{\Sigma}_{24} H_u^0 + \text{H.c.}, \tag{27}$$

where at the renormalizable level $M_{\Sigma_3} = 5M_{\Sigma_{24}} = M_\Sigma$. Therefore, in this case one has the usual contribution from the bilinear term plus an extra contribution for the neutrino masses once we integrate out the neutral component of $\Sigma_3$ and $\Sigma_{24}$. It is important to mention that $a_i = \eta_i - 3\langle\Sigma_{24}\rangle c_i/2\sqrt{15}$. Now, integrating out the fields $\Sigma_3$ and $\Sigma_{24}$ one finds that the mass matrix for neutrinos is given by

$$M_{ij}^{\rm SU(5)} = M_{ij}^{\rm SS} + \frac{v_u^2}{M_\Sigma} c_i c_j, \tag{28}$$

where one can have $M_\Sigma \approx 10^{14-15}$ GeV in agreement with the unification constraints [10].

## III. ONE-LOOP CORRECTIONS TO THE NEUTRINO MASS MATRIX

The one-loop corrections are crucial for the correct characterization of neutrino phenomena. In the MSSM usually the most important one-loop contributions to the neutrino mass matrix are the bottom squarks, charginos, and neutralinos contributions.

### A. Split SUSY case

In split SUSY all scalars, except for one light Higgs boson, are superheavy. Therefore, in this case the only potentially important contributions are charginos and neutralinos together with $W$, $Z$, and light Higgs inside the loop. We show in Appendix A that $Z$ and $W$ loops are just a small renormalization of the tree-level contribution. The Higgs boson loop together with neutralinos has the same property in the decoupling limit. We discuss those contributions in detail in this section.

In general, the one-loop contributions to the neutrino mass matrix can be written as [6]

$$\Delta M_\nu^{ij} = \Pi_{ij}(0) = -\frac{1}{16\pi^2} \sum_{f,b} G_{ijfb} m_f B_0(0; m_f^2, m_b^2), \tag{29}$$

where the sum is over the fermions $(f)$ and the bosons $(b)$ inside the loop, $m_f$ is the fermion mass, and $G_{ijfb}$ is defined by the couplings between the neutrinos and the fermions and bosons inside the loop. Once the smallness of the $\epsilon_i$ and $\lambda_i$ parameters is taken into account, each contribution can be expressed in the form

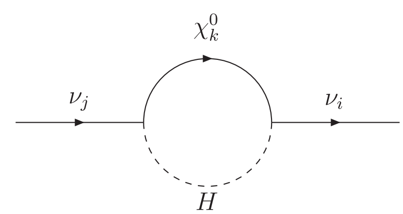
$$\Delta\Pi_{ij} = A^{(1)}\lambda_i\lambda_j + B^{(1)}(\epsilon_i\lambda_j + \epsilon_j\lambda_i) + C^{(1)}\epsilon_i\epsilon_j, \tag{30}$$

with $A^{(1)}$, $B^{(1)}$, and $C^{(1)}$ parameters independent of $\epsilon_i$ and $\lambda_i$, but dependent on the other SUSY parameters. The superindex (1) refers to the one-loop contribution. The tree-level neutrino mass matrix in Eq. (13) has the form $\mathbf{M}_{\nu ij}^{\rm eff} = A^{(0)}\lambda_i\lambda_j$ with

$$A^{(0)} = \frac{v^2}{4\det M_{\chi^0}^{\rm SS}} (M_1 \tilde{g}_d^2 + M_2 \tilde{g}_d'^2), \tag{31}$$

and we define the one-loop corrected parameters $A = A^{(0)} + A^{(1)}$, $B = B^{(1)}$, and $C = C^{(1)}$.

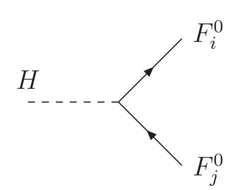
In the MSSM with BRpV the neutral Higgs bosons mix with the sneutrinos forming two sets of 5 scalars and 5 pseudoscalars. Nevertheless, in split SUSY, all the sneutrinos are extremely heavy and decouple from the light Higgs boson $H$. In addition, the heavy Higgs boson also has a very large mass, leaving the light Higgs as the only neutral scalar able to contribute to the neutrino masses. This contribution is represented by the following Feynman graph,




which is proportional to the neutralino mass $m_{\chi_k^0}$. Here, $\chi_k^0$ and $H$ are the neutralino and Higgs mass eigenstates, but the graph is calculated in the basis where $\nu_i$ are not mass eigenstates. The fields $\nu_i$ are the neutrino fields associated to the effective mass matrix given in Eq. (13). This contribution to Eq. (29) proceeds with the coupling [6]

$$G_{ijk}^h = \tfrac{1}{2}(O_{Ljk}^{nnh} O_{Lki}^{nnh} + O_{Rjk}^{nnh} O_{Rki}^{nnh}), \tag{32}$$

where the relevant vertex is




$$= i\left[O_{Lij}^{nnh}\frac{(1-\gamma_5)}{2} + O_{Rij}^{nnh}\frac{(1+\gamma_5)}{2}\right]$$

Here, $F_i^0$ are the seven eigenvectors linear combination of the Higgsinos, gauginos, and neutrinos. The $O_L$ and $O_R$ couplings satisfy $O_{Lij}^{nnh} = (O_{Rji}^{nnh})^*$ and above the scale $\tilde{m}$ we have

$$O_{Rij}^{nnh} = \tfrac{1}{2}\{\mathcal{N}_{i4}(gs_\beta\mathcal{N}_{j2} - g's_\beta\mathcal{N}_{j1}) - \mathcal{N}_{i3}(gc_\beta\mathcal{N}_{j2} - g'c_\beta\mathcal{N}_{j1}) + \mathcal{N}_{i\ell+4}(gs_\ell\mathcal{N}_{j2} - g's_\ell\mathcal{N}_{j1}) + (i \leftrightarrow j)\}, \tag{33}$$

where we have an implicit sum over $\ell = 1, 2, 3$. We allow the matrix elements of the matrix $\mathcal{N}$ to be imaginary when one of the eigenvalues is negative, such that we do not need to include explicitly the sign called $\eta_i$ in Ref. [6]. The difference with the MSSM couplings given in Ref. [11] lies in the fact that in our case $\mathcal{N}$ is a $7 \times 7$ matrix, and the Higgs mixing angle has been replaced by $\alpha = \beta - \pi/2$, valid in the decoupling limit [12]. In addition, the third term is not present in the MSSM and comes from the second term in the supersymmetric Lagrangian of Eq. (9).

Comparing the Lagrangian below the scale $\tilde{m}$ in Eq. (9) with the relevant term of the supersymmetric Lagrangian above $\tilde{m}$ given by

$$\mathcal{L}_{\text{SUSY}} \ni -\frac{1}{\sqrt{2}}\tilde{L}_i^\dagger(g\sigma^a\tilde{W}^a - g'\tilde{B})L_i \tag{34}$$

and considering the mixing between sleptons and Higgs bosons above that scale, a correspondence is found when the replacement $\tilde{L}_i^* \to -s_i i\sigma_2 H$ is made. The relevant matching condition at $\tilde{m}$ is

$$a_i(\tilde{m}) = \frac{s_i(\tilde{m})}{\cos\beta(\tilde{m})}, \tag{35}$$

where the parameters $s_i(\tilde{m})$ represent the amount of slepton $\tilde{L}_i$ in the low-energy Higgs $H$, and related to the sneutrino vev present above the scale $\tilde{m}$,

$$\tilde{L}_i = \begin{pmatrix} \frac{1}{\sqrt{2}}(v_i + \tilde{\ell}_{si}^0 + i\tilde{\ell}_{pi}^0) \\ \tilde{\ell}_{Li}^- \end{pmatrix}, \tag{36}$$

as explained in Appendix B. Using the approximation for the matrix $\mathcal{N}$ from Appendix A, we obtain for the coupling below the scale $\tilde{m}$

$$O_{Rik}^{\nu\chi h} = \tfrac{1}{2}\{-(\tilde{g}s_\beta N_{k2} - \tilde{g}'s'_\beta N_{k1})\xi_{i4} - N_{k4}(\tilde{g}s_\beta\xi_{i2} - \tilde{g}'s'_\beta\xi_{i1}) + (\tilde{g}c_\beta N_{k2} - \tilde{g}'c'_\beta N_{k1})(\xi_{i3} - a_i) + N_{k3}(\tilde{g}c_\beta\xi_{i2} - \tilde{g}'c'_\beta\xi_{i1})\}, \tag{37}$$

Notice that there is no term proportional to $\epsilon_i$ since there is a cancellation in $\xi_{i3} - a_i$. It can be checked using Eqs. (A4) and the definition of $\lambda_i = a_i\mu + \epsilon_i$. This implies that the contribution of the light Higgs boson has the form

$$\Delta\Pi_{ij}^h = A^h\lambda_i\lambda_j, \tag{38}$$

which does not break the symmetry of the neutrino mass matrix at tree level. The detailed expression is given by

$$\Delta\Pi_{ij}^h = -\frac{1}{16\pi^2}\sum_{k=1}^{4}(\tilde{O}_k^{\nu\chi h})^2\lambda_i\lambda_j m_{\chi_k^0}B_0(0; m_{\chi_k^0}^2, m_h^2), \tag{39}$$

with

$$\tilde{O}_k^{\nu\chi h} = \tfrac{1}{2}\{-(\tilde{g}s_\beta N_{k2} - \tilde{g}'s'_\beta N_{k1})\xi_4 - N_{k4}(\tilde{g}s_\beta\xi_2 - \tilde{g}'s'_\beta\xi_1) + (\tilde{g}c_\beta N_{k2} - \tilde{g}'c'_\beta N_{k1})(\xi_3 - 1/\mu) + N_{k3}(\tilde{g}c_\beta\xi_2 - \tilde{g}'c'_\beta\xi_1)\}. \tag{40}$$

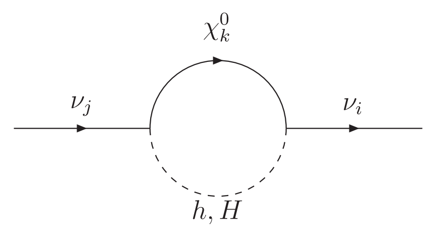
Since the gauge and Goldstone boson contribute to the neutrino mass matrix in the same form, as can be checked in the Appendix, we conclude that it is not possible to generate the neutrino masses in split supersymmetry with bilinear R-parity violating interactions alone. This conclusion is in agreement with the results presented in Ref. [8], and in Ref. [13], where the contribution from the Higgs boson can be inferred taking the decoupling limit [17].

## B. Partial split SUSY case

In this scenario, the five physical Higgs states, $h$, $H$, $A$, $H^\pm$ are light and contribute to the neutrino mass matrix. In the following subsections, we divide them in $CP$-even, $CP$-odd, and charged Higgs contributions.

### *1. CP-even neutral Higgs bosons*

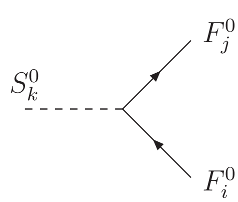
The two $CP$-even neutral Higgs bosons contribute to the neutrino mass matrix through the following graphs,




where the $G$ factor in Eq. (29) is,

$$G_{ijkr}^s = \tfrac{1}{2}(O_{Ljkr}^{nns}O_{Lkir}^{nns} + O_{Rjkr}^{nns}O_{Rkir}^{nns}). \tag{41}$$

The relevant coupling above the scale $\tilde{m}$ is the $CP$-even neutral scalar couplings to two neutral fermions, given by,




$$= i\left[O_{Lijk}^{nns}\frac{(1-\gamma_5)}{2} + O_{Rijk}^{nns}\frac{(1+\gamma_5)}{2}\right],$$

where

$$O_{Lijk}^{nns} = \tfrac{1}{2}[(-R_{k1}^0\mathcal{N}_{j3}^* + R_{k2}^0\mathcal{N}_{j4}^* - R_{k\ell+2}^0\mathcal{N}_{j\ell+4}^*) \times (g\mathcal{N}_{i2}^* - g'\mathcal{N}_{i1}^*) + (i \leftrightarrow j)] \tag{42}$$

and $O_{Rijk}^{nns} = (O_{Lijk}^{nns})^*$. The fields $S_k^0$ are linear combinations of $CP$-even Higgs and sneutrinos whose mass matrix in the basis $(\phi_d^0, \phi_u^0, \tilde{\ell}_{si}^0)$ is given in Appendix B. In the PSS, the mass matrix can be diagonalized by

$$\begin{pmatrix} h \\ H \\ \tilde{\nu}_s^i \end{pmatrix} = \begin{pmatrix} -s_\alpha & c_\alpha & -s_s^j \\ c_\alpha & s_\alpha & -t_s^j \\ -s_\alpha s_s^i + c_\alpha t_s^i & c_\alpha s_s^i + s_\alpha t_s^i & \delta_{ij} \end{pmatrix} \begin{pmatrix} \phi_d^0 \\ \phi_u^0 \\ \tilde{\ell}_{sj}^0 \end{pmatrix}, \tag{43}$$

where the angle $\alpha$ is analogous to the $CP$-even neutral Higgs bosons mixing angle of he MSSM. An expression for the mixing angles $s_s^i$ and $t_s^i$ above the scale $\tilde{m}$ can be found in Appendix B. Comparing the supersymmetric Lagrangian above the scale $\tilde{m}$ in Eq. (34) with the terms of the psSUSY Lagrangian in Eq. (20) we find the following matching conditions:

$$s_s^i(\tilde{m}) = -b_i(\tilde{m})c_\alpha; \qquad t_s^i(\tilde{m}) = -b_i(\tilde{m})s_\alpha, \tag{44}$$

where $s_s^i(\tilde{m})$ represents the amount of slepton $\tilde{L}_i$ present in the low-energy light Higgs $h$, and analogously with $t_s^i(\tilde{m})$ for the low-energy heavy Higgs $H$. In the limit where the sleptonic fields have a very large mass, they satisfy

$$s_s^i \longrightarrow -c_\alpha \frac{v_i}{v_u}, \qquad t_s^i \longrightarrow -s_\alpha \frac{v_i}{v_u}, \tag{45}$$

which tells us that the parameter $b_i$, defined below $\tilde{m}$, is directly proportional to the sneutrino vacuum expectation value $v_i$, defined above the scale $\tilde{m}$.

In the coupling in Eq. (42), we take the first neutral fermion as a neutrino and the second as a neutralino, obtaining the following couplings for both Higgs bosons $h$ and $H$,

$$\begin{aligned} O_{Lik}^{\nu\chi h} &= \tfrac{1}{2}[(s_\alpha N_{k3}^* + c_\alpha N_{k4}^*)(-g\xi_{i2} + g'\xi_{i1}) \\ &\quad + (-s_\alpha \xi_{i3} - c_\alpha \xi_{i4} + s_s^i)(gN_{k2}^* - g'N_{k1}^*)] \\ O_{Lik}^{\nu\chi H} &= \tfrac{1}{2}[(-c_\alpha N_{k3}^* + s_\alpha N_{k4}^*)(-g\xi_{i2} + g'\xi_{i1}) \\ &\quad + (c_\alpha \xi_{i3} - s_\alpha \xi_{i4} + t_s^i)(gN_{k2}^* - g'N_{k1}^*)]. \end{aligned} \tag{46}$$

After isolating the terms proportional to $\epsilon_i$ in the couplings, and using Eq. (44), we find the following expressions valid below $\tilde{m}$:

$$\begin{aligned} O_{Lik}^{\nu\chi h} &= \tilde{O}_{Lk}^{\nu\chi h}\Lambda_i + \frac{1}{2\mu s_\beta}\cos(\alpha-\beta)(gN_{k2}^* - g'N_{k1}^*)\epsilon_i \\ O_{Lik}^{\nu\chi H} &= \tilde{O}_{Lk}^{\nu\chi H}\Lambda_i + \frac{1}{2\mu s_\beta}\sin(\alpha-\beta)(gN_{k2}^* - g'N_{k1}^*)\epsilon_i, \end{aligned} \tag{47}$$

with the term proportional to $\Lambda_i$ given by

$$\begin{aligned} \tilde{O}_{Lk}^{\nu\chi h} &= -\frac{1}{2}\Big[(s_\alpha N_{k3}^* + c_\alpha N_{k4}^*)(g\xi_2 - g'\xi_1) \\ &\quad + \Big(s_\alpha\xi_3 + c_\alpha\xi_4 + \frac{c_\alpha}{\mu v_u}\Big)(gN_{k2}^* - g'N_{k1}^*)\Big] \\ \tilde{O}_{Lk}^{\nu\chi H} &= \frac{1}{2}\Big[(c_\alpha N_{k3}^* - s_\alpha N_{k4}^*)(g\xi_2 - g'\xi_1) \\ &\quad + \Big(c_\alpha\xi_3 - s_\alpha\xi_4 - \frac{s_\alpha}{\mu v_u}\Big)(gN_{k2}^* - g'N_{k1}^*)\Big]. \end{aligned} \tag{48}$$

Notice that the presence of the term proportional to $\epsilon_i$ in Eq. (47) implies that the contribution of the $CP$-even Higgs bosons has the form

$$\Delta\Pi_{ij} = A\Lambda_i\Lambda_j + B(\Lambda_i\epsilon_j + \Lambda_j\epsilon_i) + C\epsilon_i\epsilon_j \tag{49}$$

breaking the symmetry of the neutrino mass matrix at tree level, and generating a solar mass. Explicitly, this contribution is

$$\begin{aligned} \Delta\Pi_{ij}^{hH} &= -\frac{1}{16\pi^2}\sum_{k=1}^{4}\sum_{n=1}^{2}(E_k^n\Lambda_i + F_k^n\epsilon_i)(E_k^n\Lambda_j \\ &\quad + F_k^n\epsilon_j)m_{\chi_k^0}B_0(0; m_{\chi_k^0}^2, m_{H_n}^2), \end{aligned} \tag{50}$$

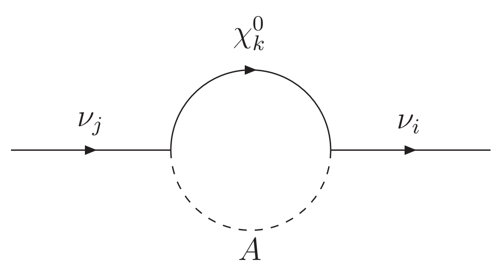
with

$$\begin{aligned} E_k^1 &= \tilde{O}_{Lk}^{\nu\chi h}, \qquad F_k^1 = \frac{\cos(\alpha-\beta)}{2\mu s_\beta}(gN_{k2}^* - g'N_{k1}^*) \\ E_k^2 &= \tilde{O}_{Lk}^{\nu\chi H}, \qquad F_k^2 = \frac{\sin(\alpha-\beta)}{2\mu s_\beta}(gN_{k2}^* - g'N_{k1}^*), \end{aligned} \tag{51}$$

where we work in the Feynman gauge.

### 2. *CP-odd neutral Higgs bosons*

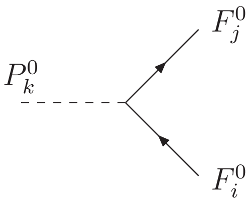
Loops including the $CP$-odd Higgs boson $A$ must be added through the graph,




where the $G$ factor in Eq. (29) is

$$G_{ijkr}^p = -\tfrac{1}{2}(O_{Ljkr}^{nnp}O_{Lkir}^{nnp} + O_{Rjkr}^{nnp}O_{Rkir}^{nnp}). \tag{52}$$

The relevant coupling above the scale $\tilde{m}$ is the $CP$-odd neutral scalar couplings to two neutral fermions, given by




$$= \left[ O^{nnp}_{Lijk} \frac{(1-\gamma_5)}{2} + O^{nnp}_{Rijk} \frac{(1+\gamma_5)}{2} \right]$$

where

$$\begin{aligned} O^{nnp}_{Lijk} &= -\tfrac{1}{2}[(-R^p_{k1}\mathcal{N}^*_{j3} + R^p_{k2}\mathcal{N}^*_{j4} - R^p_{k\ell+2}\mathcal{N}^*_{j\ell+4}) \\ &\quad \times (g\mathcal{N}^*_{i2} - g'\mathcal{N}^*_{i1}) + (i \leftrightarrow j)] \end{aligned} \tag{53}$$

and $O^{nnp}_{Rijk} = -(O^{nnp}_{Ljik})^*$. The fields $P^0_k$ are linear combinations of $CP$-odd Higgs and sneutrinos whose mass matrix in the basis $(\varphi^0_d, \varphi^0_u, \tilde{\ell}^0_{pi})$ is given in Appendix B. In the psSUSY, the mass matrix can be diagonalized by

$$\begin{pmatrix} G \\ A \\ \tilde{\nu}^i_p \end{pmatrix} = \begin{pmatrix} -c_\beta & s_\beta & -s^j_p \\ s_\beta & c_\beta & -t^j_p \\ -c_\beta s^i_p + s_\beta t^i_p & s_\beta s^i_p + c_\beta t^i_p & \delta_{ij} \end{pmatrix} \begin{pmatrix} \varphi^0_d \\ \varphi^0_u \\ \tilde{\ell}^0_{pj} \end{pmatrix}. \tag{54}$$

An expression for the mixing angles $s^i_p$ and $t^i_p$ above the scale $\tilde{m}$ can be found in Appendix B. Comparing the supersymmetric Lagrangian above the scale $\tilde{m}$ in Eq. (34) with the terms of the psSUSY Lagrangian in Eq. (20) we find the following matching conditions:

$$s^i_p(\tilde{m}) = b_i(\tilde{m})s_\beta; \qquad t^i_p(\tilde{m}) = b_i(\tilde{m})c_\beta, \tag{55}$$

where $s^i_p(\tilde{m})$ represents the amount of slepton $\tilde{L}_i$ present in the Goldstone boson $G$, and analogously with $t^i_p(\tilde{m})$ for the low-energy $CP$-odd Higgs $A$. In the limit where the sleptonic fields have a very large mass,

$$s^i_p \to s_\beta \frac{v_i}{v_u}, \qquad t^i_p \to c_\beta \frac{v_i}{v_u}, \tag{56}$$

which indicates $b_i = v_i/v_u$ in agreement with the $CP$-even case.

If we take the coupling in Eq. (53) and expand on small R-parity violating parameters we find for the $CP$-odd Higgs bosons couplings

$$\begin{aligned} O^{\nu\chi a}_{Lik} &= -\tfrac{1}{2}[(-s_\beta N^*_{k3} + c_\beta N^*_{k4})(-g\xi_{i2} + g'\xi_{i1}) \\ &\quad + (s_\beta\xi_{i3} - c_\beta\xi_{i4} + t^i_p)(gN^*_{k2} - g'N^*_{k1})]. \end{aligned} \tag{57}$$

If we isolate the terms proportional to $\epsilon_i$, using Eq. (55), we find

$$O^{\nu\chi a}_{Lik} = \tilde{O}^{\nu\chi a}_{Lk}\Lambda_i + \frac{1}{2\mu s_\beta}(gN^*_{k2} - g'N^*_{k1})\epsilon_i. \tag{58}$$

It is shown in Appendix A that the Goldstone boson contribution completely cancels out when gauge dependent terms from gauge couplings and tadpoles are included. The $\tilde{O}$ coupling is defined by

$$\begin{aligned} \tilde{O}^{\nu\chi a}_{Lk} &= -\frac{1}{2}\bigg[(s_\beta N^*_{k3} - c_\beta N^*_{k4})(g\xi_2 - g'\xi_1) \\ &\quad + \left(s_\beta\xi_3 - c_\beta\xi_4 + \frac{c_\beta}{\mu v_u}\right)(gN^*_{k2} - g'N^*_{k1})\bigg]. \end{aligned} \tag{59}$$

In this way, the $CP$-odd contribution is

$$\begin{aligned} \Delta\Pi^A_{ij} &= \frac{1}{16\pi^2}\sum_{k=1}^{4}(E^3_k\Lambda_i + F^3_k\epsilon_i)(E^3_k\Lambda_j \\ &\quad + F^3_k\epsilon_j)m_{\chi^0_k}B_0(0; m^2_{\chi^0_k}, m^2_A), \end{aligned} \tag{60}$$

with

$$E^3_k = \tilde{O}^{\nu\chi a}_{Lk}, \qquad F^3_k = \frac{1}{2\mu s_\beta}(gN^*_{k2} - g'N^*_{k1}). \tag{61}$$

Note that the $CP$-odd contribution in Eq. (61) has the opposite sign of the $CP$-even contribution. In addition, the $\epsilon_i\epsilon_j$ terms in the limit of equal neutral Higgs masses. This is because the $CP$-even terms are proportional to $\cos^2(\alpha - \beta)B_0(0; m^2_{\chi^0_k}, m^2_h)$ and $\sin^2(\alpha - \beta)B_0 \times (0; m^2_{\chi^0_k}, m^2_H)$, while the $CP$-odd term is proportional to $-B_0(0; m^2_{\chi^0_k}, m^2_A)$.

## IV. NUMERICAL RESULTS

### A. Partial split SUSY

As seen in the previous chapters, partial split supersymmetry is determined by the following supersymmetric parameters: the supersymmetric Higgs mass $\mu$, the gaugino masses $M_1$ and $M_2$, the mass of the lightest $CP$-even Higgs $m_h$, the $CP$-odd Higgs mass $m_A$, and the tangent of the $CP$-odd Higgs mixing angle $\tan\beta$. As a working scenario we choose the numerical values given in Table I. In this scenario the four neutralino masses are $m_\chi = 147$, 282, 455, 476 GeV, with the lightest neutralino the lightest supersymmetric particle. In the Higgs sector, the charged Higgs mass is $m^+_H = 1003.2$ GeV, the heavy neutral $CP$-even Higgs mass is $m_H = 1000.2$ GeV, and the $CP$-even Higgs mixing angle is given by $\sin\alpha = 0.101$.

TABLE I. PSS and neutrino mass matrix parameters.

| Parameter | Solution | Units |
|---|---|---|
| $\tan\beta$ | 10 | — |
| $\mu$ | 450 | GeV |
| $M_2$ | 300 | GeV |
| $M_1$ | 150 | GeV |
| $m_h$ | 120 | GeV |
| $m_A$ | 1000 | GeV |
| $Q$ | 830 | GeV |
| $A$ | −2.7 | eV/GeV$^4$ |
| $B$ | −0.0005 | eV/GeV$^3$ |
| $C$ | 0.315 | eV/GeV$^2$ |

TABLE II. BRpV parameters and neutrino observables.

| Parameter | Solution | Units |
|---|---|---|
| $\epsilon_1$ | 0.0346 | GeV |
| $\epsilon_2$ | 0.265 | GeV |
| $\epsilon_3$ | 0.322 | GeV |
| $\Lambda_1$ | $-0.0269$ | GeV$^2$ |
| $\Lambda_2$ | $-0.00\,113$ | GeV$^2$ |
| $\Lambda_3$ | 0.0693 | GeV$^2$ |
| $\Delta m^2_{\rm atm}$ | $2.34 \times 10^{-3}$ | eV$^2$ |
| $\Delta m^2_{\rm sol}$ | $8.16 \times 10^{-5}$ | eV$^2$ |
| $\tan^2\theta_{\rm atm}$ | 1.04 | — |
| $\tan^2\theta_{\rm sol}$ | 0.455 | — |
| $\tan^2\theta_{13}$ | 0.0247 | — |
| $m_{ee}$ | 0.00 394 | eV |

The one-loop corrected parameters $A$, $B$, and $C$ introduced in Eq. (49) are calculated with the results in Eq. (50) for the neutral $CP$-even Higgs bosons, in Eq. (60) for the neutral $CP$-odd Higgs boson, and in Eq. (A24) for the charged Higgs boson. These contributions give rise to a set of parameters $A$, $B$, and $C$ given in Table I. The value of $A = -2.7\ \mathrm{eV/GeV^2}$ is mainly due to the tree-level contribution, and $C = 0.315\ \mathrm{eV/GeV^4}$ is completely generated by radiative corrections.

The parameter $C$ is subtraction scale independent, while the parameters $A$ and $B$ depend on the subtraction scale $Q$. As a way of fixing this scale, we have chosen $Q$ such that it minimizes the parameter $B$, making the solar mass completely scale independent. For the scenario in Table I, we find that $Q = 830$ GeV gives rise to $B = -0.0005\ \mathrm{eV/GeV^3}$, which is already negligible.

We notice that in the decoupling limit scenario the light $CP$-even Higgs $h$ contribution to the solar mass (or equivalently, to the parameter $C$) is negligible, since it is proportional to $\cos(\alpha - \beta) \to 0$. Therefore, it can be said properly that the solar mass comes exclusively from the contributions of the heavy Higgs bosons $H$ and $A$. Furthermore, as indicated by Eqs. (51) and (61) the contributions from $H$ and $A$ have opposite signs and tend to cancel each other in the decoupling limit, where $\sin(\alpha - \beta) \to 1$ and $m_H \to m_A$. In our scenario, $\cos(\alpha - \beta) = 0.0016$ and $m_H - m_A = 0.2$ GeV, and the cancellation between $H$ and $A$ contributions to $C$ is at the 0.07% level.

Within the scenario in Table I,we look for a solution to the neutrino observables varying $\vec{\epsilon}$ and $\vec{\Lambda}$. An example solution is given in Table II. This solution satisfies $\epsilon_1 \ll \epsilon_2, \epsilon_3$, and $|\Lambda_2| \ll |\Lambda_1|, \Lambda_3$. The sign of these parameters has a very small influence. Also in Table II we list the neutrino observables. The atmospheric mass $\Delta m^2_{\rm atm} = 2.34 \times 10^{-3}\ \mathrm{eV}^2$ and the solar mass $\Delta m^2_{\rm sol} = 8.16 \times 10^{-5}\ \mathrm{eV}^2$ are practically at the center of the experimentally allowed regions. The atmospheric angle $\tan^2\theta_{\rm atm} = 1.04$ is slightly deviated from maximal mixing, while the solar angle $\tan^2\theta_{\rm sol} = 0.455$ is nonmaximal with a value centered on the experimentally allowed region. The other two parameters, the reactor angle $\tan^2\theta_{13} = 0.0247$ and the neutrinoless double beta decay mass $m_{ee} = 0.00394$ eV, have not been experimentally measured, and the predictions of our model are well below the experimental upper bounds.

In order to study the dependence of the neutrino physics solutions on different parameters we have implemented the following $\chi^2$:

$$\chi^2 = \left(\frac{10^3\Delta m^2_{\rm atm} - 2.35}{0.95}\right)^2 + \left(\frac{10^5\Delta m^2_{\rm sol} - 8.15}{0.95}\right)^2 + \left(\frac{\sin^2\theta_{\rm atm} - 0.51}{0.17}\right)^2 + \left(\frac{\sin^2\theta_{\rm sol} - 0.305}{0.075}\right)^2. \quad (62)$$

In each of these terms, we evaluated how many standard deviations the prediction is from the measured experimental central values [15]. In Fig. 1, we have $\chi^2$ in the vertical axis as a function of $A$ and $C$, in perspective in the left frame and level contours in the right frame. The preferred solution of Table I appears at the center of the graphs. Neutrino observables are very sensitive to the parameters $A$ and $C$ as shown by contours, where the darkest ellipsoid (blue) corresponds to $\chi^2 \lesssim 10$, while the white center

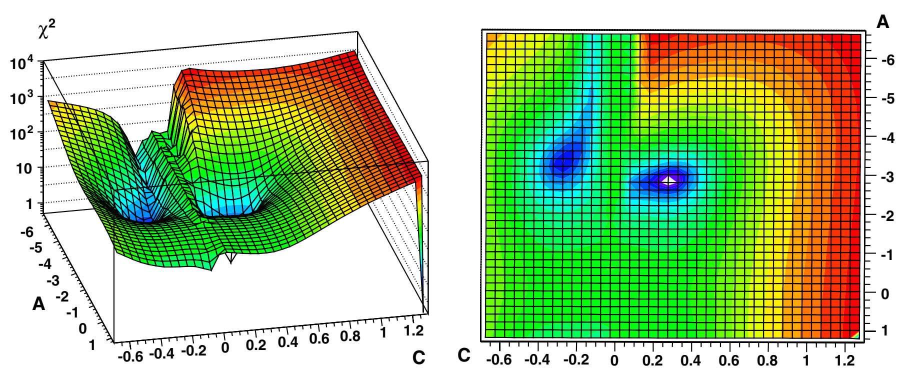



FIG. 1 (color online). Neutrino physics $\chi^2$ as a function of the neutrino mass matrix parameters A and C, keeping $\vec{\epsilon}$ and $\vec{\Lambda}$ as indicated in Table I.

corresponds to $\chi^2 \lesssim 1$. There is a second minima, but it does not reach values near unity.

A good approximation for the neutrino masses in this scenario is the following:

$$m_3 = C|\vec{\epsilon}|^2 + A\frac{(\vec{\epsilon}\cdot\vec{\Lambda})^2}{|\vec{\epsilon}|^2} \qquad m_2 = A\frac{|\vec{\epsilon}\times(\vec{\Lambda}\times\vec{\epsilon})|^2}{|\vec{\epsilon}|^4} \tag{63}$$

with the third neutrino massless [14]. Despite the fact that $C$ is one-loop generated and $A$ receives contributions at tree level, the first term in $m_3$ is dominant, and thus more important for the atmospheric mass scale. The $A$ term is the only one contributing to the solar mass, as indicated in Eq. (63).

In Fig. 2, we plot $\chi^2$ as a function of $\epsilon_2$ and $\epsilon_3$ in two frames as described for the previous figure. The rest of the BRpV parameters are fixed to the values in Table I, while the values of $A$ and $C$ are calculated from the loop contributions. In our scenario, an approximated expression can be found when $\epsilon_1$ and $\Lambda_2$ are neglected. It turns out that the atmospheric angle and mass squared difference depend strongly on $\epsilon_2$ and $\epsilon_3$. They are given by

$$\Delta m^2_{\rm atm} \approx C^2(\epsilon_2^2 + \epsilon_3^2)^2 \tan^2\theta_{\rm atm} \approx \left(\frac{\epsilon_2}{\epsilon_3}\right)^2. \tag{64}$$

Notice that it is the atmospheric mass that receives the main contribution from loop corrections, with $C$ generated entirely at one loop. Equal values for the atmospheric mass correspond to circles around the origin in the $\epsilon_2$-$\epsilon_3$ plane, while equal values for the atmospheric angle are represented by straight lines passing through the origin. This geometry can be visualized in Fig. 2.

In Fig. 3, we plot $\chi^2$ as a function of $\Lambda_1$ and $\Lambda_3$ with the other parameters as indicated in Table I. The solar mass squared difference and angle depend strongly on $\Lambda_1$ and $\Lambda_3$ as indicated by the following approximations:

$$\begin{aligned}\Delta m^2_{\rm sol} &\approx A^2\left[\Lambda_1^2 + \frac{\Lambda_3^2}{1+(\epsilon_3/\epsilon_2)^2}\right]^2 \tan^2\theta_{\rm sol}\\ &\approx \frac{\Lambda_1^2}{\Lambda_3^2}\left[1+\left(\frac{\epsilon_3}{\epsilon_2}\right)^2\right].\end{aligned} \tag{65}$$

When the $\epsilon$ parameters are kept constant, equal values for the solar mass are represented by ellipses, while constant values for the solar angle are represented by straight lines passing through the origin. As with the previous figure, this geometry can be visualized also in Fig. 3.

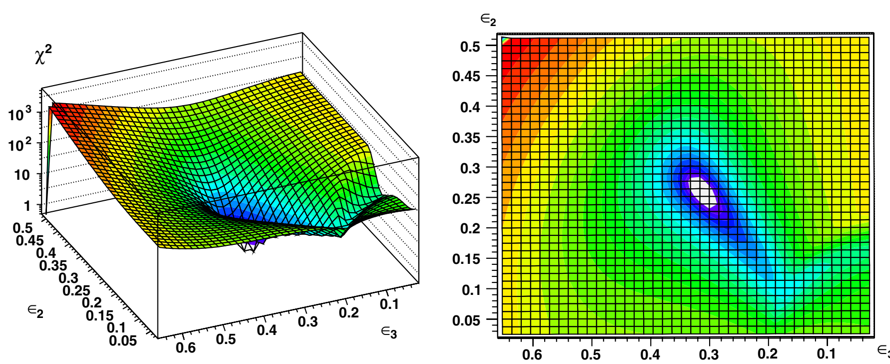



FIG. 2 (color online). Neutrino physics $\chi^2$ as a function of the BRpV parameters $\epsilon_2$ and $\epsilon_3$, keeping the rest of the parameters as indicated in Table I.

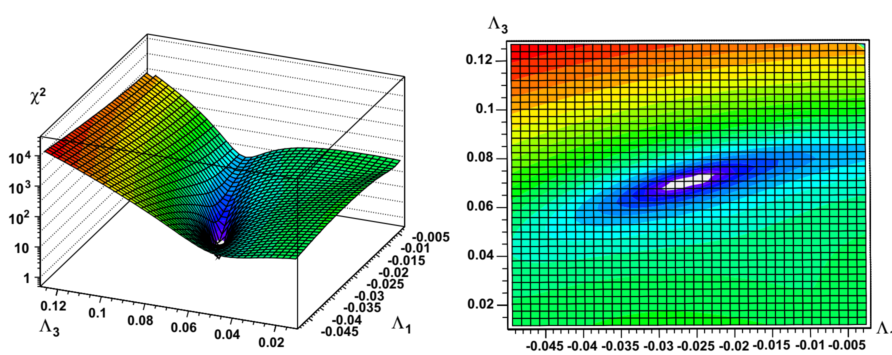



FIG. 3 (color online). Neutrino physics $\chi^2$ as a function of the BRpV parameters $\Lambda_1$ and $\Lambda_3$, keeping the rest of the parameters as indicated in Table I.

TABLE III. SU(5) split SUSY and neutrino mass matrix parameters.

| Parameter | Solution I | Solution II | Units |
|---|---|---|---|
| $\tan\beta$ | 10 | 10 | — |
| $\mu$ | 450 | 450 | GeV |
| $M_2$ | 300 | 300 | GeV |
| $M_1$ | 150 | 150 | GeV |
| $M_\Sigma$ | $9\times10^{15}$ | $5\times10^{15}$ | GeV |
| $A$ | $-1.7\times10^3$ | $-1.7\times10^3$ | eV/GeV$^2$ |
| $C$ | $6.7\times10^{-3}$ | $1.2\times10^{-2}$ | eV |

TABLE IV. SU(5) BRpV parameters and neutrino observables.

| Parameter | Solution I | Solution II | Units |
|---|---|---|---|
| $c_1$ | 0.62 | 0.51 | — |
| $c_2$ | $-0.52$ | $-1.49$ | — |
| $c_3$ | 0.85 | 1.38 | — |
| $\lambda_1$ | 0.0008 | 0.0015 | GeV |
| $\lambda_2$ | $-0.0037$ | $-0.0016$ | GeV |
| $\lambda_3$ | $-0.0038$ | $-0.0011$ | GeV |
| $\Delta m^2_{\rm atm}$ | $2.4\times10^{-3}$ | $2.6\times10^{-3}$ | eV$^2$ |
| $\Delta m^2_{\rm sol}$ | $8.2\times10^{-5}$ | $8.3\times10^{-5}$ | eV$^2$ |
| $\tan^2\theta_{\rm atm}$ | 1.02 | 1.00 | — |
| $\tan^2\theta_{\rm sol}$ | 0.45 | 0.50 | — |
| $\tan^2\theta_{13}$ | 0.026 | 0.049 | — |
| $m_{ee}$ | 0.004 | 0.005 | eV |

### B. SUSY SU(5)

As expressed in Eq. (28), the tree-level contribution from BRpV to the neutrino mass matrix is complemented in our SU(5) supersymmetric model by a contribution suppressed by one power of the $M_\Sigma$ mass scale. The dimensionless coefficients $c_i$ are expected to be of order unity, but different from each other due to renormalization group equations effects. Despite that in split supersymmetric scenarios the light Higgs cannot contribute at one loop to the neutrino mass matrix, this extra SU(5) term is capable of generating a solar mass.

Keeping the low-energy supersymmetric parameters equal to their values in the examples shown for partial split supersymmetry in the previous section, we look for solutions in the case of SU(5) split SUSY with BRpV. In Table III we show two solutions for two different values of the scale $M_\Sigma$. The resulting neutrino mass coefficients $A$ and $C$ are also shown in the same table. The $A$ coefficient is independent of the mass scale $M_\Sigma$, but $C$ is inversely proportional to it.

In Solution I, with a high value for $M_\Sigma = 9\times10^{15}$ GeV, the $A$ term in the neutrino mass matrix dominates over the $C$ term, such that the atmospheric mass comes mainly from $A\lambda_i\lambda_j$, and the smallness of the reactor angle is achieved with a small value for $\lambda_1$. The solar mass is generated with the $Cc_ic_j$ term, with the $c_i$ of order unity.

In Solution II, we lower the value for $M_\Sigma = 5\times10^{15}$ GeV, reversing the situation. Now the $Cc_ic_j$ term dominates generating the atmospheric mass. Since we look for solutions with $c_i$ of order one (we accept $0.5 < c_i < 1.5$), the value of $\tan^2\theta_{13}$ grows to values close to its experimental upper bound. In this way, lower values of $M_\Sigma$ are severely restricted. In this solution, the solar mass is generated by the $A\lambda_i\lambda_j$ term. See Table IV.

## V. SUMMARY

We have studied in detail the possibility of describing the neutrino masses and mixing angles in the context of split supersymmetric scenarios, where the sfermions and/or Higgses are very heavy. We have considered all relevant contributions to the neutrino mass matrix up to one-loop level coming from the R-parity violating interactions, showing the importance of the Higgs one-loop corrections in the case of partial split SUSY, where only the sfermions are very heavy. We have found new contributions in the context of the minimal supersymmetric SU(5), which can help us to generate the neutrino masses in agreement with the experiments in the split SUSY scenario.

## ACKNOWLEDGMENTS

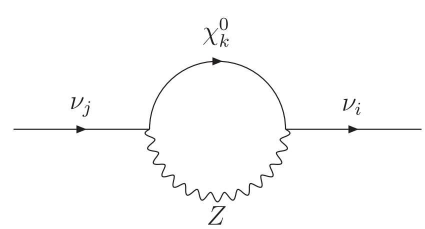
The work of M. A. D. was partly founded by Conicyt Grant No. 1060629 and Conicyt and Banco Mundial Grant “Anillo Centro de Estudios Subatómicos.” The work of P. F. P. has been supported by *Fundação para a Ciência e a Tecnologia* (FCT, Portugal) through the project CFTP, under Contract No. POCTI-SFA-2-777. Also P. F. P. has been supported in part by the U.S. Department of Energy Contract No. DE-FG02-08ER41531 and in part by the Wisconsin Alumni Research Foundation. We would like to thank E. J. Chun and G. Senjanović for a discussion. P. F. P thanks the Instituto de Fisica Corpuscular (IFIC) in Valencia for hospitality. The work by C. M. was supported by ALFA-EC under the framework of the HELEN program. C. M. would like to thank CERN for their hospitality.

## APPENDIX A: GAUGE AND GOLDSTONE BOSON LOOPS IN SPLIT SUSY

In this appendix we show the properties of the gauge boson one-loop contributions to the neutrino mass matrix.

### 1. $Z$ and neutral Goldstone boson loops

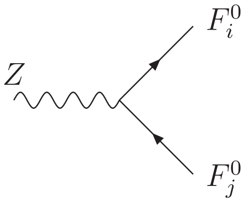
In $Z$ loops the fermionic sum in Eq. (29) is over neutral fermions $F^0_k$, of which only the neutralinos are relevant. There is no bosonic sum since only $Z$ contributes.



The coupling $G^Z_{ijk}$ is equal to

$$G^Z_{ijk} = -2(O^{nnz}_{Ljk}O^{nnz}_{Rki} + O^{nnz}_{Rjk}O^{nnz}_{Lki}), \tag{A1}$$

where the coupling of a $Z$ boson to two neutral fermions is [16]




$$= i\left[O^{nnz}_{Lij}\frac{(1-\gamma_5)}{2} + O^{nnz}_{Rij}\frac{(1+\gamma_5)}{2}\right]$$

with

$$\begin{aligned} O^{nnz}_{Lij} &= -(O^{nnz}_{Rij})^*, \\ O^{nnz}_{Rij} &= -\frac{g}{2c_W}\Big(\mathcal{N}^*_{i4}\mathcal{N}_{j4} - \mathcal{N}^*_{i3}\mathcal{N}_{j3} \\ &\quad - \sum_{a=1}^{3}\mathcal{N}^*_{ia+4}\mathcal{N}_{ja+4}\Big). \end{aligned} \tag{A2}$$

The matrix $\mathcal{N}$ diagonalizes the $7\times 7$ neutrino/neutralino mass matrix, giving non-negative eigenvalues. Without including the final rotation on the neutrino sector, it can be approximated in the following way [6]:

$$\mathcal{N} \approx \begin{bmatrix} N & N\xi^T \\ -\xi & 1 \end{bmatrix}, \tag{A3}$$

where $N$ diagonalizes the $4\times 4$ neutralino mass submatrix. The parameters $\xi$ are defined by

$$\begin{aligned} \xi_{i1} &= \frac{\tilde{g}'_d \mu M_2}{2\,\mathrm{det}M_{\chi^0}}\Lambda_i \qquad \xi_{i2} = -\frac{\tilde{g}_d \mu M_1}{2\,\mathrm{det}M_{\chi^0}}\Lambda_i \\ \xi_{i3} &= \frac{v_u}{4\,\mathrm{det}M_{\chi^0}}(M_1\tilde{g}_u\tilde{g}_d + M_2\tilde{g}'_u\tilde{g}'_d)\Lambda_i - \frac{\epsilon_i}{\mu} \\ \xi_{i4} &= -\frac{v_d}{4\,\mathrm{det}M_{\chi^0}}(M_1\tilde{g}_d^2 + M_2\tilde{g}_d'^2)\Lambda_i. \end{aligned} \tag{A4}$$

For notational brevity we define the $\xi_i$ parameters as $\lambda_i\xi_1 = \xi_{i1}$, $\lambda_i\xi_2 = \xi_{i2}$, $\lambda_i\xi_3 - \epsilon_i/\mu = \xi_{i3}$, and $\lambda_i\xi_4 = \xi_{i4}$. The couplings in Eq. (A2) can be approximated with the help of Eq. (A3) to

$$O^{\nu\chi z}_{Rik} \approx \frac{g}{2c_W}(2N_{k4}\xi_{i4} + N_{k1}\xi_{i1} + N_{k2}\xi_{i2}), \tag{A5}$$

where $i$ labels the three neutrinos and $k$ labels the four neutralinos. Considering Eq. (A4) we conclude that

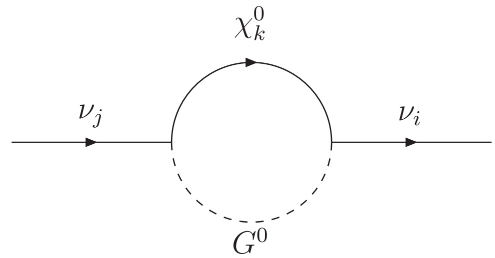
$$\Delta\Pi^Z_{ij} = A^Z\lambda_i\lambda_j, \tag{A6}$$

with

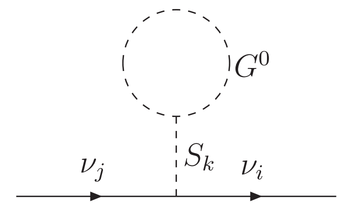
$$\begin{aligned} A^Z = -\frac{g^2}{16\pi^2 c_W^2}\sum_{k=1}^{4}&(2N_{k4}\xi_4 + N_{k1}\xi_1 \\ &+ N_{k2}\xi_2)^2 m_{\chi^0_k}B_0(0; m^2_{\chi^0_k}, m_Z^2). \end{aligned} \tag{A7}$$

This contribution is only a renormalization of the tree-level mass matrix, which does not break its symmetry, i.e., it does not generate mass to all neutrinos.

There is an extra contribution to $A^Z$ dependent on the gauge parameter $\xi$. This is canceled by the following loops involving the neutral Goldstone boson,



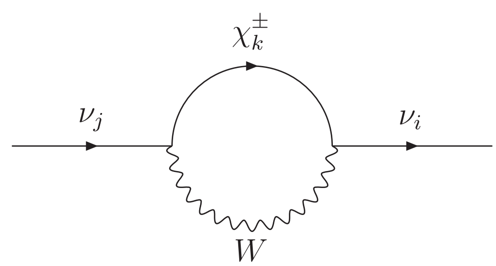

+




as demonstrated in Ref. [6].

### 2. *W* and chargedGoldstone boson loops

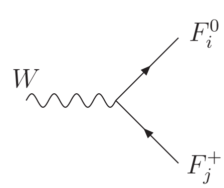
In $W$ loops the fermionic sum in Eq. (29) is over charged fermions $F_k^+$, of which only the charginos are relevant. There is no bosonic sum since only $W$ contributes.




The coupling $G^W_{ijk}$ is equal to

$$G^W_{ijk} = -4(O^{ncw}_{Ljk}O^{ncw}_{Rik} + O^{ncw}_{Rjk}O^{ncw}_{Lik}), \tag{A8}$$

where the coupling of a $W$ boson to two fermions is




$$= i\left[O^{ncw}_{Lij}\frac{(1-\gamma_5)}{2} + O^{ncw}_{Rij}\frac{(1+\gamma_5)}{2}\right]$$

with

$$O^{ncw}_{Lij} = -g\left(\mathcal{N}^*_{i2}\mathcal{U}_{j1} + \frac{1}{\sqrt{2}}\mathcal{N}^*_{i3}\mathcal{U}_{j2} + \frac{1}{\sqrt{2}} \times \sum_{a=1}^{3}\mathcal{N}^*_{ia+4}\mathcal{U}_{ja+2}\right) \tag{A9}$$

$$O^{ncw}_{Rij} = -g\left(\mathcal{N}_{i2}\mathcal{V}^*_{j1} - \frac{1}{\sqrt{2}}\mathcal{N}_{i4}\mathcal{V}^*_{j2}\right).$$

The $\mathcal{U}$ and $\mathcal{V}$ matrices diagonalize the $5\times 5$ chargino/charged lepton mass matrix, and can be approximated to [6]

$$\mathcal{U} \approx \begin{bmatrix} U & U\xi_L^T \\ -\xi_L & 1 \end{bmatrix}, \qquad \mathcal{V} \approx \begin{bmatrix} V & 0 \\ 0 & 1 \end{bmatrix}, \tag{A10}$$

where $U$ and $V$ diagonalize the $2\times 2$ chargino submatrix. The parameters $\xi_L$ are

$$\xi_L^{i1} = \frac{\tilde{g}_d}{\sqrt{2}\,\det M_{\chi^+}}\Lambda_i, \qquad \xi_L^{i2} = -\frac{\tilde{g}_u\tilde{g}_d v_u}{2\mu\,\det M_{\chi^+}}\Lambda_i - \frac{\epsilon_i}{\mu}, \tag{A11}$$

with

$$\det M_{\chi^+} = \mu M_2 - \tfrac{1}{2}\tilde{g}_u\tilde{g}_d v_u v_{d,} \tag{A12}$$

and similar to what we did in the previous subsection, we define the parameters $\xi_j^L$, $j = 1, 2$, with the relations $\xi_L^{i1} = \xi_1^L\lambda_i$ and $\xi_L^{i2} = \xi_2^L\lambda_i - \epsilon_i/\mu$. The couplings in Eq. (A9) can be approximated to

$$O^{\nu\chi w}_{Rij} \approx g\left(V^*_{j1}\xi_{i2} - \frac{1}{\sqrt{2}}V^*_{j2}\xi_{i4}\right)$$

$$O^{\nu\chi w}_{Lij} \approx g\left(U_{j1}\xi_{i2} - \frac{1}{\sqrt{2}}U_{j2}[\xi_L^{i2} - \xi_{i3}] - \frac{1}{\sqrt{2}}U_{j1}\xi_L^{i1}\right), \tag{A13}$$

where $i$ labels the three neutrinos and $j$ labels the two charginos. Similar to what happened with the $Z$ contributions, the $W$ contribution depends only on the $\lambda_i$

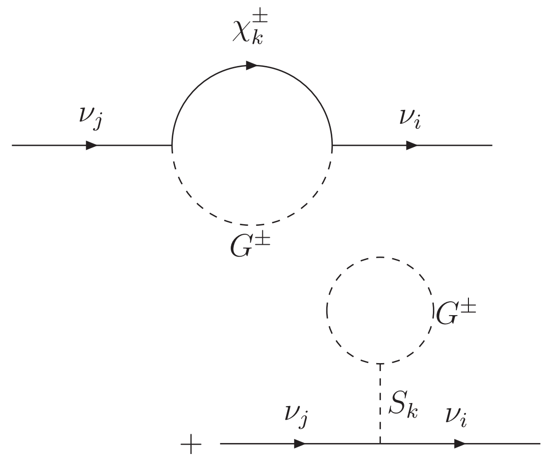
$$\Delta\Pi^W_{ij} = A^W\lambda_i\lambda_j, \tag{A14}$$

with

$$A^W = \frac{g^2}{2\pi^2}\sum_{k=1}^{2}\left[U_{k1}\xi_2 - \frac{U_{k2}}{\sqrt{2}}(\xi_2^L - \xi_3) + \frac{U_{k1}}{\sqrt{2}}\xi_1^L\right] \times \left(V_{k1}\xi_2 - \frac{V_{k2}}{\sqrt{2}}\xi_4\right)m_{\chi_k^+}B_0(0; m^2_{\chi_k^+}, m_W^2), \tag{A15}$$

adding to the tree-level contribution without changing the symmetry. Therefore, the $W$ and $Z$ loops do not help us to generate mass to all neutrinos.

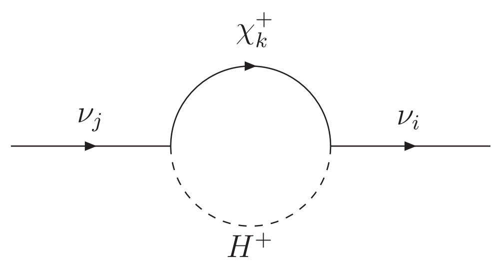
As for the case of $A^Z$, there is an extra contribution to $A^W$ dependent on the gauge parameter $\xi$. This is canceled by loops involving the charged Goldstone boson,




. The rest of the tadpoles form a gauge invariant set, and renormalize the vacuum expectation values [6].

### 3. Charged Higgs boson loops

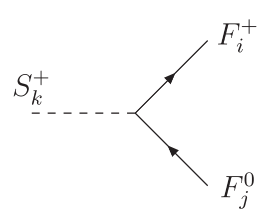
The last loops we consider are the ones that include a charged scalar and a charged fermion. The loop is represented by the following graph,




where the $G$ factor in Eq. (29) is

$$G^{s+}_{ijkr} = (O^{ncs}_{Ljkr}O^{cns}_{Lkir} + O^{ncs}_{Rjkr}O^{cns}_{Rkir}). \tag{A16}$$

The relevant coupling above the scale $\tilde{m}$ is the charged scalar couplings to a charged and a neutral fermion. It is given by




$$= \left[O^{cns}_{Lijk}\frac{(1-\gamma_5)}{2} + O^{cns}_{Rijk}\frac{(1+\gamma_5)}{2}\right]$$

where the $O^{cns}_L$ and $O^{cns}_R$ couplings are

$$O^{cns}_{Lijk} = h_\tau R^{S^+}_{k1}\mathcal{N}^*_{j7}\mathcal{V}^*_{i5} - R^{S^+}_{k2}\left(\frac{g}{\sqrt{2}}\mathcal{N}^*_{j2}\mathcal{V}^*_{i2} + \frac{g'}{\sqrt{2}}\mathcal{N}^*_{j1}\mathcal{V}^*_{i2} + g\mathcal{N}^*_{j4}\mathcal{V}^*_{i1}\right) - h_\tau R^{S^+}_{k5}\mathcal{N}^*_{j3}\mathcal{V}^*_{i5} - \sqrt{2}g' R^{S^+}_{k\ell+5}\mathcal{N}^*_{j1}\mathcal{V}^*_{i\ell+2}$$

$$O^{cns}_{Rijk} = R^{S^+}_{k1}\left(\frac{g}{\sqrt{2}}\mathcal{N}_{j2}\mathcal{U}_{i2} + \frac{g'}{\sqrt{2}}\mathcal{N}_{j1}\mathcal{U}_{i2} - g\mathcal{N}_{j3}\mathcal{U}_{i1}\right) + h_\tau R^{S^+}_{k8}(\mathcal{N}_{j7}\mathcal{U}_{i2} - \mathcal{N}_{j3}\mathcal{U}_{i5})$$
$$+ R^{S^+}_{k\ell+2}\left(\frac{g}{\sqrt{2}}\mathcal{N}_{j2}\mathcal{U}_{i\ell+2} + \frac{g'}{\sqrt{2}}\mathcal{N}_{j1}\mathcal{U}_{i\ell+2} - g\mathcal{N}_{j\ell+4}\mathcal{U}_{i1}\right), \tag{A17}$$

with $O^{cns}_{Lijk} = O^{ncs*}_{Rjik}$ and $O^{cns}_{Rijk} = O^{ncs*}_{Ljik}$. The fields $S^+_k$ are eight linear combinations of charged Higgs bosons and charged sleptons, whose mass matrix in the $(H^+_d, H^+_u, \tilde{\ell}^+_{Lj}, \tilde{\ell}^+_{Rj})$ basis is in Appendix B. This mass matrix is diagonalized in psSUSY by the rotation

$$\begin{pmatrix} G^+ \\ H^+ \\ \tilde{l}^+_{Li} \\ \tilde{l}^+_{Ri} \end{pmatrix} = \begin{pmatrix} c_\beta & s_\beta & -s^j_L & 0 \\ -s_\beta & c_\beta & -t^j_L & 0 \\ c_\beta s^i_L - s_\beta t^i_L & s_\beta s^i_L + c_\beta t^i_L & \delta_{ij} & 0 \\ 0 & 0 & 0 & \delta_{ij} \end{pmatrix} \times \begin{pmatrix} H^+_d \\ H^+_u \\ \tilde{\ell}^+_{Lj} \\ \tilde{\ell}^+_{Rj} \end{pmatrix}. \tag{A18}$$

An expression for the mixing angles $s^i_L$ and $t^i_L$ above the scale $\tilde{m}$ can be found in the Appendix B. Comparing the supersymmetric Lagrangian above the scale $\tilde{m}$ in Eq. (34) with the terms of the psSUSY Lagrangian in Eq. (20), we find the following matching conditions:

$$s^i_L(\tilde{m}) = b_i(\tilde{m})s_\beta; \qquad t^i_L(\tilde{m}) = b_i(\tilde{m})c_\beta, \tag{A19}$$

where $s^i_L(\tilde{m})$ represents the amount of slepton $\tilde{L}_i$ present in the charged Goldstone boson $G^+$, and analogously with $t^i_L(\tilde{m})$ for the low-energy charged Higgs $H^+$. In the limit where the sleptonic fields have a very large mass,

$$s^i_L \longrightarrow s_\beta\frac{v_i}{v_u}, \qquad t^i_L \longrightarrow c_\beta\frac{v_i}{v_u}, \tag{A20}$$

indicating that $b_i = v_i/v_u$ are in agreement with the $CP$-even and $CP$-odd cases. Now we make an expansion of the couplings in Eq. (A17), and we find

$$O^{\nu\chi h+}_{Lik} = c_\beta\left(\frac{g}{\sqrt{2}}\xi_{i2}V^*_{k2} + \frac{g'}{\sqrt{2}}\xi_{i1}V^*_{k2} + g\xi_{i4}V^*_{k1}\right)$$
$$O^{\nu\chi h+}_{Rik} = s_\beta\left(\frac{g}{\sqrt{2}}\xi_{i2}U_{k2} + \frac{g'}{\sqrt{2}}\xi_{i1}U_{k2} + g\xi_{i3}U_{k1}\right) + gt^i_L U_{k1}, \tag{A21}$$

and isolating the terms proportional to $\epsilon_i$, using Eq. (A19), we write

$$O^{\nu\chi h+}_{Lik} = \tilde{O}^{\nu\chi h+}_{Lk}\Lambda_i,$$
$$O^{\nu\chi h+}_{Rik} = \tilde{O}^{\nu\chi h+}_{Rk}\Lambda_i - \frac{1}{\mu s_\beta}gU_{k1}\epsilon_i, \tag{A22}$$

where we have defined

$$\tilde{O}^{\nu\chi h+}_{Lk} = c_\beta\left(\frac{g}{\sqrt{2}}\xi_2 V^*_{k2} + \frac{g'}{\sqrt{2}}\xi_1 V^*_{k2} + g\xi_4 V^*_{k1}\right)$$
$$\tilde{O}^{\nu\chi h+}_{Rk} = s_\beta\left(\frac{g}{\sqrt{2}}\xi_2 U_{k2} + \frac{g'}{\sqrt{2}}\xi_1 U_{k2} + g\xi_3 U_{k1}\right) + gU_{k1}\frac{c_\beta}{\mu v_u}. \tag{A23}$$

Finally, the charged Higgs contribution to the neutrino mass matrix is

$$\Delta\Pi^{h+}_{ij} = -\frac{1}{16\pi^2}\sum_{k=1}^{2}\tilde{O}^{\nu\chi h+}_{Lk}\Big[2\tilde{O}^{\nu\chi h+}_{Rk}\Lambda_i\Lambda_j - \frac{gU_{k1}}{\mu s_\beta}(\Lambda_i\epsilon_j + \Lambda_j\epsilon_i)\Big]m_{\chi^+_k}B_0(0; m^2_{\chi^+_k}, m^2_{H^+}). \tag{A24}$$

Note that there is no $\epsilon_i\epsilon_j$ term.

## APPENDIX B: HIGGS SLEPTON SECTOR

Here, we give details on the Higgs slepton mass matrices and approximations in the case when the slepton masses are much heavier that the Higgs masses.

### *CP*-even Higgs-sneutrino mixing

The $CP$-even Higgs and sneutrino fields mix to form a set of five neutral mass eigenstates $S^0_i$. We organize the mass terms in the Lagrangian in the following way:

$$\mathcal{L} \ni -\frac{1}{2}[\phi^0_d, \phi^0_u, \tilde{\ell}^0_{si}]\mathbf{M}^2_{S^0}\begin{bmatrix} \phi^0_d \\ \phi^0_u \\ \tilde{\ell}^0_{sj} \end{bmatrix}. \tag{B1}$$

The mass matrix is divided into blocks [6]

$$\mathbf{M}^2_{S^0} = \begin{bmatrix} \mathbf{M}^2_{S^0hh} & \mathbf{M}^2_{S^0h\tilde{\nu}} \\ \mathbf{M}^{2T}_{S^0h\tilde{\nu}} & \mathbf{M}^2_{S^0\tilde{\nu}\tilde{\nu}} \end{bmatrix}. \tag{B2}$$

The Higgs $2\times 2$ submatrix is equal to

$$\mathbf{M}^2_{S^0 hh} = \begin{bmatrix} B_0\mu\frac{v_u}{v_d} + \frac{1}{4}g_Z^2 v_d^2 + \mu\vec{\epsilon}\cdot\frac{\vec{v}}{v_d} + \frac{T_d}{v_d} & -B_0\mu - \frac{1}{4}g_Z^2 v_d v_u \\ -B_0\mu - \frac{1}{4}g_Z^2 v_d v_u & B_0\mu\frac{v_d}{v_u} + \frac{1}{4}g_Z^2 v_u^2 - \vec{B}_\epsilon\cdot\frac{\vec{v}}{v_u} + \frac{T_u}{v_u} \end{bmatrix}, \tag{B3}$$

where we call $g_Z^2 = g^2 + g'^2$, and in supergravity models we have $B_\epsilon^i = B_i\epsilon_i$. In this matrix we have eliminated the Higgs soft masses using the minimization conditions of the scalar potential (or tadpole equations) [6]. These Higgs tadpole equations at tree level are

$$\begin{aligned} T_d &= (m_{H_d}^2 + \mu^2)v_d + v_d D - \mu(B_0 v_u + \vec{v}\cdot\vec{\epsilon}) \\ T_u &= -B_0\mu v_d + (m_{H_u}^2 + \mu^2)v_u - v_u D + \vec{v}\cdot\vec{B}_\epsilon \\ &\quad + v_u\vec{\epsilon}^2, \end{aligned} \tag{B4}$$

with $D = 1/8(g^2 + g'^2)(\vec{v}^2 + v_d^2 - v_u^2)$. At tree-level, it is safe to set $T_u = T_d = 0$, and if we take the R-parity conserving limit $\epsilon_i$, $v_i \to 0$, we can recognize the $CP$-even Higgs mass matrix of the MSSM. The $2\times 3$ mixing submatrix is given by

$$\mathbf{M}^2_{S^0 h\tilde{\nu}} = \begin{bmatrix} M^2_{S^0 h_d\tilde{\nu}_i} \\ M^2_{S^0 h_u\tilde{\nu}_i} \end{bmatrix} = \begin{bmatrix} -\mu\epsilon_i + \frac{1}{4}g_Z^2 v_d v_i \\ B_\epsilon^i - \frac{1}{4}g_Z^2 v_u v_i \end{bmatrix}, \tag{B5}$$

which vanishes in the R-parity conserving limit. Finally, the sneutrino submatrix is given by

$$(\mathbf{M}^2_{S^0\tilde{\nu}\tilde{\nu}})_{ij} = (M_{Li}^2 + D)\delta_{ij} + \tfrac{1}{4}g_Z^2 v_i v_j + \epsilon_i\epsilon_{j,} \tag{B6}$$

where we have not yet used the corresponding tadpole equations, and we have assumed that the sneutrino soft mass matrix is diagonal. The sneutrino tadpole equations are given by

$$T_i = v_i D + \epsilon_i(-\mu v_d + \vec{v}\cdot\vec{\epsilon}) + v_u B_\epsilon^i + v_i M_{Li}^2. \tag{B7}$$

It is clear from this equation that if the sneutrino vev's are zero, $\mu\epsilon_i = B_\epsilon^i v_u/v_d$, and therefore, the mixing between the up and down Higgs fields with the sneutrino fields are related by $M^2_{S^0 h_d\tilde{\nu}} = -\tan\beta M^2_{S^0 h_u\tilde{\nu}}$. Of course, this last relation is not valid if the sneutrino vev's are not zero.

In the case of large slepton masses, the mass matrix in Eq. (B2) is diagonalized in two steps by the rotation matrix

$$R_{S^0} = \begin{pmatrix} 1 & 0 & -s_s^j \\ 0 & 1 & -t_s^j \\ s_s^i & t_s^i & \delta_{ij} \end{pmatrix}\begin{pmatrix} -s_\alpha & c_\alpha & 0 \\ c_\alpha & s_\alpha & 0 \\ 0 & 0 & \delta_{ij} \end{pmatrix}, \tag{B8}$$

with the mixing angles at the scale $\tilde{m}$ satisfying

$$\begin{aligned} s_s^i &= \frac{-s_\alpha M^2_{S^0 h_d\tilde{\nu}_i} + c_\alpha M^2_{S^0 h_u\tilde{\nu}_i}}{M^2_{L_i} - m_h^2}, \\ t_s^i &= \frac{c_\alpha M^2_{S^0 h_d\tilde{\nu}_i} + s_\alpha M^2_{S^0 h_u\tilde{\nu}_i}}{M^2_{L_i} - m_H^2}, \end{aligned} \tag{B9}$$

where the Higgs masses can be neglected in front of the slepton masses in this approximation. From Eq. (B5), we find the following limits for large slepton masses:

$$s_s^i \to -c_\alpha\frac{v_i}{v_u}, \qquad t_s^i \to -s_\alpha\frac{v_i}{v_u}, \tag{B10}$$

which links the smallness of the Higgs-sneutrino mixing needed for neutrino physics, with the smallness of the sneutrino vevs.

### 2. *CP*-odd Higgs-sneutrino mixing

The $CP$-odd Higgs bosons and sneutrinos mix to form a set of five $CP$-odd scalars, whose mass terms in the Lagrangian are

$$\mathcal{L} - \frac{1}{2}[\varphi_d^0, \varphi_u^0, \tilde{\ell}_{pi}^0]\mathbf{M}^2_{P^0}\begin{bmatrix} \varphi_d^0 \\ \varphi_u^0 \\ \tilde{\ell}_{pj}^0 \end{bmatrix}, \tag{B11}$$

where we decompose the $5\times 5$ mass matrix in the following blocks:

$$\mathbf{M}^2_{P^0} = \begin{bmatrix} \mathbf{M}^2_{Phh} & \mathbf{M}^2_{Ph\tilde{\nu}} \\ \mathbf{M}^{2T}_{Ph\tilde{\nu}} & \mathbf{M}^2_{P\tilde{\nu}\tilde{\nu}} \end{bmatrix}. \tag{B12}$$

The Higgs sector is given by the $2\times 2$ mass submatrix

$$\mathbf{M}^2_{Phh} = \begin{bmatrix} B_0\mu\frac{v_u}{v_d} + \mu\vec{\epsilon}\cdot\frac{\vec{v}}{v_d} + \frac{T_d}{v_d} & B_0\mu \\ B_0\mu & B_0\mu\frac{v_d}{v_u} - \vec{B}_\epsilon\cdot\frac{\vec{v}}{v_u} + \frac{T_u}{v_u} \end{bmatrix}, \tag{B13}$$

where the tadpoles $T_u$ and $T_d$ are defined in Eq. (B4). In the R-parity conserving limit we reproduce the $CP$-odd mass matrix in the MSSM. The Higgs-sneutrino mixing is given by the $2\times 3$ matrix

$$\mathbf{M}^2_{Ph\tilde{\nu}} = \begin{bmatrix} M^2_{Ph_d\tilde{\nu}_i} \\ M^2_{Ph_u\tilde{\nu}_i} \end{bmatrix} = \begin{bmatrix} -\mu\epsilon_i \\ -B_\epsilon^i \end{bmatrix}, \tag{B14}$$

which vanishes in the R-parity conserving limit. Finally, the sneutrino $3\times 3$ mass matrix is

$$(\mathbf{M}^2_{P\tilde{\nu}\tilde{\nu}})_{ij} = (M_{Li}^2 + D)\delta_{ij} + \epsilon_i\epsilon_j, \tag{B15}$$

where we have assumed diagonal soft slepton mass parameters.

If slepton masses are very large, the $5\times 5$ mass matrix can be diagonalized with the following rotations:

$$R_{P^0} = \begin{pmatrix} 1 & 0 & -s_p^j \\ 0 & 1 & -t_p^j \\ s_p^i & t_p^i & \delta_{ij} \end{pmatrix}\begin{pmatrix} -c_\beta & s_\beta & 0 \\ s_\beta & c_\beta & 0 \\ 0 & 0 & \delta_{ij} \end{pmatrix}, \tag{B16}$$

with the mixing angles $s_p^i$ and $t_p^i$ satisfying at the scale $\tilde{m}$,

$$s_p^i = \frac{-c_\beta M^2_{Ph_d\tilde{\nu}_i} + s_\beta M^2_{Ph_u\tilde{\nu}_i}}{M^2_{L_i} - m_G^2},$$
$$t_p^i = \frac{s_\beta M^2_{Ph_d\tilde{\nu}_i} + c_\beta M^2_{Ph_u\tilde{\nu}_i}}{M^2_{L_i} - m_A^2}, \tag{B17}$$

and the Higgs masses $m_G^2$ and $m_A^2$ negligible in front of the slepton masses. Using Eqs. (B14) and (B17) we find the following mixing angles in the limit of large slepton masses

$$s_p^i \to s_\beta \frac{v_i}{v_u}, \qquad t_p^i \to c_\beta \frac{v_i}{v_u} \tag{B18}$$

also proportional to the sneutrino vacuum expectation values.

### 3. Charged Higgs slepton mixing

The charged Higgs boson and slepton fields mix to form a set of eight charged eigenstates $S_i^+$, whose mass terms in the Lagrangian are organized according to

$$\mathcal{L} - [H_d^-, H_u^-, \tilde{\ell}_{Li}^-, \tilde{\ell}_{Ri}^-]\mathbf{M}^2_{S^+}\begin{bmatrix} H_d^+ \\ H_u^+ \\ \tilde{\ell}_{Li}^+ \\ \tilde{\ell}_{Ri}^+ \end{bmatrix}. \tag{B19}$$

The $8 \times 8$ mass matrix is written as

$$\mathbf{M}^2_{S^+} = \begin{bmatrix} \mathbf{M}^2_{S^+hh} & \mathbf{M}^2_{S^+h\tilde{\ell}} \\ \mathbf{M}^{2T}_{S^+h\tilde{\ell}} & \mathbf{M}^2_{S^+\tilde{\ell}\tilde{\ell}} \end{bmatrix}, \tag{B20}$$

with the following charged Higgs boson $2 \times 2$ block:

$$\mathbf{M}^2_{S^+hh} = \begin{bmatrix} B_0\mu\frac{v_u}{v_d} + \mu\vec{\epsilon}\cdot\frac{\vec{v}}{v_d} + \frac{1}{4}g^2(v_u^2 - \vec{v}^2) + \frac{1}{2}h^2_{\ell k}v_k^2 + \frac{T_d}{v_d} & B_0\mu + \frac{1}{4}g^2 v_d v_u \\ B_0\mu + \frac{1}{4}g^2 v_d v_u & B_0\mu\frac{v_d}{v_u} - \vec{B}_\epsilon\cdot\frac{\vec{v}}{v_u} + \frac{1}{4}g^2(v_d^2 + \vec{v}^2) + \frac{T_u}{v_u} \end{bmatrix}. \tag{B21}$$

This mass matrix reduces to the charged Higgs mass matrix of the MSSM when the BRpV parameters are taken equal to zero. Mixing between charged Higgs bosons and left- and right-charged sleptons appear through terms in the following $2 \times 6$ block:

$$\mathbf{M}^2_{S^+h\tilde{\ell}} = \begin{bmatrix} M^2_{S^+h_d\tilde{\ell}_{Li}} & M^2_{S^+h_d\tilde{\ell}_{Ri}} \\ M^2_{S^+h_u\tilde{\ell}_{Li}} & M^2_{S^+h_u\tilde{\ell}_{Ri}} \end{bmatrix} = \begin{bmatrix} -\mu\epsilon_i + \frac{1}{4}g^2 v_d v_i - \frac{1}{2}h^2_{\ell i}v_d v_i & -\frac{1}{\sqrt{2}}h^2_{\ell i}v_u\epsilon_i - \frac{1}{\sqrt{2}}A_{\ell i}v_i \\ -B^i_\epsilon + \frac{1}{4}g^2 v_u v_i & -\frac{1}{\sqrt{2}}h_{\ell i}(\mu v_i + \epsilon_i v_d) \end{bmatrix}, \tag{B22}$$

which as expected vanishes in the R-parity conserving limit. The charged slepton submatrix is further divided into left- and right-slepton sectors

$$\mathbf{M}^2_{S^+\tilde{\ell}\tilde{\ell}} = \begin{bmatrix} \mathbf{M}^2_{LL} & \mathbf{M}^2_{LR} \\ \mathbf{M}^{2T}_{LR} & \mathbf{M}^2_{RR} \end{bmatrix}, \tag{B23}$$

which are given by the following expressions:

$$M^2_{LL} = \left[M^2_{L_i} + \frac{1}{8}(g^2 - g'^2)(v_u^2 - v_d^2 - \vec{v}^2) + \frac{1}{2}h^2_{\ell i}v_d^2\right]\delta_{ij} + \frac{1}{4}g^2 v_i v_j + \epsilon_i\epsilon_j$$
$$M^2_{LR} = \frac{1}{\sqrt{2}}(v_d A_{\ell i} - \mu v_u h_{\ell i})\delta_{ij}$$
$$M^2_{RR} = \left[M^2_{R_i} + \frac{1}{4}g'^2(v_u^2 - v_d^2 - \vec{v}^2) + \frac{1}{2}h^2_{\ell i}(v_d^2 + \vec{v}^2)\right]\delta_{ij}. \tag{B24}$$

Slepton soft mass parameters are taken diagonal, and the MSSM expressions are recovered when we make $\epsilon_i = v_i = 0$. As before, if slepton soft masses are large, a diagonalization can be accomplished by the rotations

$$R_{S^+} = \begin{pmatrix} 1 & 0 & -s_L^j & -s_R^j \\ 0 & 1 & -t_L^j & -t_R^j \\ s_L^i & t_L^i & \delta_{ij} & 0 \\ s_R^i & t_R^i & 0 & \delta_{ij} \end{pmatrix}\begin{pmatrix} c_\beta & s_\beta & 0 & 0 \\ -s_\beta & c_\beta & 0 & 0 \\ 0 & 0 & \delta_{ij} & 0 \\ 0 & 0 & 0 & \delta_{ij} \end{pmatrix}, \tag{B25}$$

with the following mixing angles at the scale $\tilde{m}$:

$$s_L^i = \frac{c_\beta M^2_{S^+h_d\tilde{\ell}_{Li}} + s_\beta M^2_{S^+h_u\tilde{\ell}_{Li}}}{M^2_{L_i} - m^2_{H^+}},$$
$$t_L^i = \frac{-s_\beta M^2_{S^+h_d\tilde{\ell}_{Li}} + c_\beta M^2_{S^+h_u\tilde{\ell}_{Li}}}{M^2_{L_i} - m^2_{G^+}}$$
$$s_R^i = \frac{c_\beta M^2_{S^+h_d\tilde{\ell}_{Ri}} + s_\beta M^2_{S^+h_u\tilde{\ell}_{Ri}}}{M^2_{R_i} - m^2_{H^+}},$$
$$t_R^i = \frac{-s_\beta M^2_{S^+h_d\tilde{\ell}_{Ri}} + c_\beta M^2_{S^+h_u\tilde{\ell}_{Ri}}}{M^2_{R_i} - m^2_{G^+}}. \tag{B26}$$

When slepton masses are very large, the right mixing angles vanish while the left mixing angles are proportional to the slepton vevs,

$$s_L^i \to s_\beta\frac{v_i}{v_u}, \qquad t_L^i \to c_\beta\frac{v_i}{v_u}, \qquad s_R^i \to 0, \qquad t_R^i \to 0 \tag{B27}$$

in a similar way as the previous two cases.

[1] For a review see, H. E. Haber and G. L. Kane, Phys. Rep. **117**, 75 (1985).

[2] N. Arkani-Hamed and S. Dimopoulos, J. High Energy Phys. 06 (2005) 073; G. F. Giudice and A. Romanino, Nucl. Phys. **B699**, 65 (2004); **B706**, 65(E) (2005).

[3] For a review see, R. N. Mohapatra and A. Y. Smirnov, Annu. Rev. Nucl. Part. Sci.**56**, 569 (2006); J. Lesgourgues and S. Pastor, Phys. Rep. **429**, 307 (2006).

[4] For a review on proton decay see, P. Nath and P. Fileviez Pérez, Phys. Rep. **441**, 191 (2007).

[5] B. Allanach *et al.* (R-parity Working Group Collaboration), arXiv:hep-ph/9906224; R. Barbier *et al.*, Phys. Rep. **420**, 1 (2005); H. K. Dreiner, arXiv:hep-ph/9707435. For the most general constraints on R-parity violating interactions coming from nucleon decay see, P. Fileviez Pérez, J. Phys. G **31**, 1025 (2005).

[6] M. Hirsch, M. A. Diaz, W. Porod, J. C. Romao, and J. W. F. Valle, Phys. Rev. D **62**, 113008 (2000); **65**, 119901(E) (2002).

[7] R. Hempfling, Nucl. Phys. **B478**, 3 (1996); M. Hirsch, M. A. Diaz, W. Porod, J. C. Romao, and J. W. F. Valle, Phys. Rev. D **62**, 113008 (2000); **65**, 119901(E) (2002); M. A. Diaz, M. Hirsch, W. Porod, J. C. Romao, and J. W. F. Valle, Phys. Rev. D **68**, 013009 (2003); **71**, 059904(E) (2005); M. Drees, S. Pakvasa, X. Tata, and T. ter Veldhuis, Phys. Rev. D **57**, R5335 (1998); F. M. Borzumati, Y. Grossman, E. Nardi, and Y. Nir, Phys. Lett. B **384**, 123 (1996); E. J. Chun, S. K. Kang, C. W. Kim, and U. W. Lee, Nucl. Phys. **B544**, 89 (1999); Y. Grossman and S. Rakshit, Phys. Rev. D **69**, 093002 (2004); S. Davidson and M. Losada, J. High Energy Phys. 05 (2000) 021; A. Dedes, S. Rimmer, and J. Rosiek, J. High Energy Phys. 08, (2006) 005.

[8] E. J. Chun and S. C. Park, J. High Energy Phys. 01 (2005) 009.

[9] M. Nowakowski and A. Pilaftsis, Nucl. Phys. **B461**, 19 (1996).

[10] I. Dorsner, P. Fileviez Perez, and G. Rodrigo, Phys. Lett. B **649**, 197 (2007).

[11] J. F. Gunion, H. E. Haber, G. L. Kane, and S. Dawson, *The Higgs Hunter's Guide* (Perseus Publishing, Cambridge, Massashusetts, 1990).

[12] J. F. Gunion and H. E. Haber, Phys. Rev. D **67**, 075019 (2003).

[13] S. Davidson and M. Losada, Phys. Rev. D **65**, 075025 (2002).

[14] M. A. Diaz, C. Mora, and A. R. Zerwekh, Eur. Phys. J. C **44**, 277 (2005).

[15] M. Maltoni, T. Schwetz, M. A. Tortola, and J. W. F. Valle, New J. Phys. **6**, 122 (2004).

[16] J. Rosiek, arXiv:hep-ph/9511250.

[17] S. Davidson and M. Losada, J. High Energy Phys. 05 (2000) 021.