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A congested and dwell time dependent transit corridor assignment model

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SUMMARY

This research proposes an equilibrium assignment model for congested public transport corridors in urban areas. In this model, journey times incorporate the effect of bus queuing on travel times and boarding and alighting passengers on dwell times at stops. The model also considers limited bus capacity leading to longer waiting times and more uncomfortable journeys. The proposed model is applied to an example network, and the results are compared with those obtained in a recent study. This is followed by the analysis and discussion of a real case application in Santiago de Chile. Finally, different boarding and alighting times and different vehicle types are evaluated. In all cases, demand on express services tends to be underestimated by using constant dwell time assignment models, leading to potential planning errors for these lines. The results demonstrate the importance of considering demand dependent dwell times in the assignment process, especially at high demand levels when the capacity constraint should also be considered.

KEY WORDS: transit corridor; express services; transit assignment; dwell time; capacity constraint

1. INTRODUCTION

The challenge of offering a service which can compete with the private car has always been present during the design, planning and operational stages of running urban public transport systems. High standards of service should be offered during all the stages of a journey: access and egress time, waiting time and the in-vehicle journey time represented by the commercial speed of the service. This latter point has been the motivation behind investment in public transport corridors with additional priority systems, and, more recently, the design of express services which increase commercial speeds by reducing the number of stops along the corridor. The advantages of these express services have already been widely researched and demonstrated over the last 20 years, and a key element during their design is the demand these new services will attract, which makes up the main input for the frequency and fleet optimization methodologies. Also, the demand they attract strongly depends on the frequencies these services offer.

When demand is high yet not high enough to activate the vehicle capacity constraint, these models assume practically constant journey times. This fact turns out to be particularly questionable on public transport corridors where the lines have common itineraries and are differentiated mainly by the stops they serve. For a bus line, the time saved because of not stopping in several stops becomes its competitive advantage over others.

Also, as public transport corridors move thousands of travelers per hour and direction, and because different frequencies may induce different sets of attractive lines for their trips, dwell times affected by boarding and alighting times can no longer be considered constant either [1]. Traffic assignment should consider all these effects as also the impact in waiting times and reduced comfort if bus capacity

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is limited. For these reasons, we propose an assignment model, which considers both passenger boarding and alighting times as well as the impact of limited capacity on waiting times and comfort. Furthermore, congestion at bus stops, because of bus queuing leading to bus delays, is modeled by adding a penalty function for each vehicle at each stop.

The following section summarizes the literature review on transit assignment models, concentrating on those modeling dwell times. This is followed by a presentation of the hypothesis and formulation of the proposed model. In section 4, we describe the solution algorithm, which is initially applied to a test network in section 5, where the results are compared with those obtained applying the method proposed by [2] over the same network. An application with the data from the Pajaritos Corridor in Santiago de Chile is provided in section 6 along with a sensitivity analysis. Finally, the main conclusions drawn from this work are presented in section 7.

2. LITERATURE REVIEW

For decades now public transport assignment models have been one of the most analyzed and studied topics within the field of transport. These models consider that users plan their journeys in such a way as to minimize their perceived waiting and in-vehicle journey times. Many models assume that waiting times may be influenced by the journey times of the most popular lines [3–5], by the demand itself and by vehicle capacity limits [6–9] whereas in-vehicle journey times are assumed to be independent of demand. Many models incorporate the effect of vehicle capacity limits with a journey time penalty along public transport links depending on demand [4, 6, 10–12].

All the above mentioned works have used two basic methodologies: frequency-based models or schedule-based models. Within this distinction, each model typology has evolved as new research has introduced ever more complexity and realism into the modeling itself or in how the public transport network is presented, triggering debates about which type of model should be preferred. For example, [13, 14] proposed headways of between 10 and 15 min to distinguish between high and low frequency services and, therefore, the type of model required. A visual description of the evolution of transit assignment models can be found in [15].

[14] performed an interesting comparison between the two approaches and highlighted that low computing costs were one advantage of frequency-based models. They also found that they required less detail for input and network coding and were applicable to different passenger regimes other than FIFO at bus stops. Note the debate and the open question in[16] about the potential future application of schedule-based models to large networks with a reasonable computing time.

As far as the authors are aware [17, 18] are the first and only works which use a combined frequency-based and scheduled-based approach to transit assignment. The proposed model uses aggregate line frequencies to parameterize bus headways and a micro-simulator to enforce capacity constraints on individual vehicles, comparing the results with those obtained by [6] using the same network example.

[19] proposed an alternative approach which tried to replicate the schedule-based approach using a frequency-based model. They introduced a factor called “fail-to-board” as a way of simulating the fact that some passengers are unable to board the first bus arriving and have to wait for the next, representing on board and bus stop congestion. This allowed the authors to use the advantages of frequency-based modeling while they adapted the model to simulate effects that at the time could only be considered using schedule based methodologies, but without the disadvantages already mentioned above. Although the assignment process is dynamic, route choice is “partially dynamic” because it is performed based on the initial situation of each period and does not respond to the situation in successive periods.

Later improvements to this model were proposed by [20] including another parameter called “fail-to-seat” to represent the availability of seating, differentiating between seated and standing passengers when calculating perceived journey times. See also the work developed by [21].

The dynamic approach has been used in other work which has generally applied schedule-based modeling as it has been shown to be better at adapting to real changes in user behavior with respect to route choice because of current information systems [22–25].

In spite of a great richness in approaches and models (clearly there are many more than can be mentioned in this brief analysis), little work has been found to include dwell time as a variable in the assignment model.

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[11] proposed a model in which the route cycle time was dependent on the dwell time at each stop and where dwell times depended on the demand. This approach also considered effective frequencies being affected by the demand itself while the in-vehicle travel time, as perceived by users, remained constant and fixed.

The first work found to consider the boarding and alighting times at intermediate bus stops as part of the in-vehicle journey time perceived by the user was [2]. They assumed that in-vehicle journey time was a function of the demand at the successive bus stops. They proposed an algorithm based on incremental assignment, achieving a new user equilibrium at each demand increment without taking capacity constraints into consideration.

More recently, [26] proposed an assignment model in which dwell times are modeled as the maximum of the boarding and alighting time (as if they happened simultaneously through different doors). They applied the transit equilibrium model proposed by [9] and used a revised optimal strategy algorithm and a modified MSA, called Method of Successive Weighted Averages (MSWA) [24] to solve the problem. However, the authors did not consider the effects of limited capacity in their assignment model.

This paper contributes to this issue and proposes a frequency-based transit assignment model based on [6] but considering variable dwell times as well as the impact of limited capacity on waiting times, passenger comfort and delays at bus stops because of bus queuing as part of in-vehicle travel time. A sensitivity analysis has been carried out to study the influence of dwell times and demand on user decision making, and, therefore, on the final assignment. Furthermore, we propose an alternative solution to the [2] incremental assignment by using the Method of Successive Averages (MSA) algorithm. The results of these models and algorithms are compared on the same example network.

3. PROPOSED MODEL

3.1. Hypotheses and notations

The proposed model follows a frequency-based approach, developed by [6]. As in [2], we consider a unidirectional public transport corridor with n bus stops, ordered in the same sequence, where stops p are served by the buses travelling along the corridor ($p = 1,\ldots,n$). This corridor is used by a set L of bus lines which each offer a given frequency and visit a subset of the corridor stops. A line could also be defined by its set of consecutive arcs in which each arc starts in one of its stops and ends in the following one. We also assume a fixed and known journey demand, defined by a "stop to stop" O–D matrix with the elements denoted as T_w.

The model assumes that the users reach their destinations without transfers. Each user chooses a subset of the lines connecting his/her origin and destination that minimizes the total travel time (in-vehicle travel time + waiting time). This subset of bus lines is grouped together in what is called a *route section*, defined as a fictitious link with the origin bus stop as the origin node and the destination bus stop as the destination node of the link and the subset of lines (attractive lines) which service it. Thus, a route section is understood as a set of lines. As [2] shows, when dwell times depend on boarding and alighting, the problem not only cannot be separated by O/D pair, but also the equilibrium assignment we are seeking may require users making the same trip to select a different set of attractive lines.

Thus, our model must consider several route sections representing different subsets of lines serving a given pair of nodes, in accordance with [6]. The set of route sections serving O/D pair w will be denoted as S_w while the origin and the destination of w will be denoted as $o(w)$ and $d(w)$, respectively. One element of this set S_w will be the full set of lines serving the pair w denoted as L_w. We will also assume that once vehicles approach their capacity their associated waiting times and comfort become affected.

3.2. Cost function with capacity constraint

The expected travel time on route section s for the pair w (ETT_s^w) is considered to be [3]:

$$ETT_s^w = WT_s^w + TT_s^w s\hat{I}S_w,\ w\hat{I}W \tag{1}$$

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where:WT_s^w is the waiting time of route section s for the pair w. $s \in S_w$, $w \in WTT_s^w$ is the (in vehicle) travel time of route section s for the pair w. $s \in S_w$, $w \in WW$ is the set of origin destination pairs w.

Waiting time in the route section can be expressed in the following way, in which we assume Poisson bus arrivals processes [3]:

$$WT_s^w = 1/\sum_{l \in s} f_l \rightarrow WT_s^w = 1/f_s s\hat{I}S_w,\ w\hat{I}W \tag{2}$$

where:f_l = frequency of line l.f_s = aggregated frequency of all lines in route section s.

We extend the model proposed by [2] to consider limited vehicle capacity and its effect on waiting times. As demand increases, the effective capacity available for a passenger waiting for a line drops because there will be less room inside the bus to accommodate additional passengers. The closer this number gets to the physical capacity, the more likely that a passenger will not be able to board the bus. Thus, our model must recognize not only that demand affects dwell times but also that limited capacity has a direct impact on passenger waiting times and perceived in-vehicle travel times.

Because the set of attractive lines for a certain trip depends on the level of demand, we must recognize that as congestion grows, all lines connecting a pair of nodes are "potentially" attractive. Under no congestion, an initial group can be defined as the attractive set containing the "quickest" lines corresponding to the set of common routes defined by [3]. However, as passenger demand grows, these routes become increasingly congested, causing slower routes to become more attractive as travel times on the set of "fastest" routes increase because of longer dwell times and waiting times.

This model assumes a similar approach to [6] in which the perceived frequency of a line at a stop p drops with its degree of saturation defined as the ratio between the load of the line arriving at stop p minus those passengers alighting at p plus the passengers boarding at p and the capacity of the line. This relationship between perceived frequency and the degree of saturation is certainly non-linear because for low and intermediate demand levels, the perceived frequency should not change by much.

Thus, the perceived frequency of a line l at stop p, $f^*_{l,p}$ will always be less than or equal to the real frequency. To establish a relationship between this perceived frequency and the average waiting time experienced by users we formulated the following model [6]:

$$f^*_{l,p} = \left(1/wt_{l,p}\right), l\hat{I}L,\ p\epsilon\{1 \,....n\} \tag{3}$$

where:$wt_{l,p}$ is the *representative* waiting time of line l at bus stop p that will be modeled as:

$$wt_{l,p} = (1/f_l)\cdot\left(1 + \left(x_{l,p}\right)^{\varphi}\right), p\hat{I}\{1 \,....n\} \qquad , l\hat{I}L \tag{4}$$

where ϕ is a parameter to be estimated that should be larger than 1, while $x_{l,p}$ corresponds to the degree of saturation of line l at bus stop p and is given by:

$$x_{l,p} = \frac{\sum\limits_{w \in W, o(w)=p,} \sum\limits_{s \in S_w l \in s} \left(y_s^w \dfrac{f^*_{l,p}}{\sum\limits_{j \in s} f^*_{j,p}} \right) + V_{l,p}}{(C_l \cdot f_l)}, p\hat{I}\{1 \ldots n\}, l, j\hat{I}L \tag{5}$$

where:C_l is the vehicle capacity of line l.y_s^w is the flow of route section s for pair $w$$V_{l,p}$ is the passing flow on line l: passengers with origin (o) upstream from p and destination (d) downstream of p given by:

$$V_{l,p} = \sum_{o=1}^{p-1} \sum_{d=p+1}^{n} \sum_{s \in S_{(o,d)}} \left(y_s^{(o,d)} \frac{f^*_{l,o}}{\sum\limits_{j \in s} f^*_{j,o}} \right), p\hat{I}\{1 \ldots n\}, l, j\hat{I}L. \tag{6}$$

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So, the waiting time of a route section s of a pair w will be modeled as:

$$WT_s^w = 1/\sum_{l \in s} f_{l,o(w)}^* \rightarrow 1/f_s^*, s\hat{I}S_w,\ w\hat{I}W \tag{7}$$

where f_s^* is the effective frequency of route section s obtained by:

$$\mathrm{f}_\mathrm{s}^* = \sum_{\mathrm{l\hat{I}s}} \mathrm{f}_\mathrm{l}^*, \mathrm{s\hat{I}S_w},\ \mathrm{l\hat{I}L}\ . \tag{8}$$

Once a passenger boards a vehicle at a bus stop, the travel time to reach its destination corresponds to the running travel time plus all the dwell times experienced by the vehicle.

The travel time on route section s for the pair w (TT_s^w) is considered to be [3]:

$$TT_s^w = \sum_{l \in s} t_l^w(V) \cdot f_l^* / \sum_{l \in s} f_l^* \rightarrow TT_s^w = \sum_{l \in s} t_l^w(V) \cdot f_l^* / f_s^* s\hat{I}S_w,\ w\hat{I}W \tag{9}$$

where t_l^w is the travel time on line l from stop to stop (origin-destination pair w). This travel time includes the time spent by passengers boarding and alighting and additional delays because of bus queuing at intermediate bus stops; t_l^w will be affected by passengers from all o–d pairs whose origin or destination is visited by line l on its journey to join pair w. Therefore, t_l^w depends on the network passenger flow, represented by the vector $V = \{v_l^w\}, \forall l, w$.

As shown by [2], the user equilibrium assignment of this problem should be structured at the route section level, for which the flow vector $Y = \{y_s^w\}$ in route section s for O–D pair w is defined. As [6] show the route section flows (y_s^w) determine the flows on each line l, for an O–D pair w in the following way:

$$v_l^w = \sum_{s \in S_w} \left(y_s^w \frac{f_{l,o(w)}^*}{f_s^*} \right), w\hat{I}W,\ l\hat{I}L_w. \tag{10}$$

Now we can define the flow of line l on link a, which can be expressed as:

$$v_{l,a} = \sum_{w \in W} v_l^w \cdot \delta_{l,a}^w a\hat{I}A_l, l\hat{I}L \tag{11}$$

where:$v_{l,a}$ is the flow of line l on link a.$\delta_{l,a}^w$ = binary variable taking a value of 1 if arc $a \in A_{l,w}$ and 0 otherwise

Delays at traffic lights and junctions are assumed to be included in the link travel time. So the in-vehicle travel time t_l^w from equation (9) can be defined as:

$$t_l^w = rt_l^w + \sum_p dt_{l,p} \cdot \delta_{l,p}^w \tag{12}$$

where:rt_l^w = running time of line l for the pair w given by:

$$rt_l^w = \sum_{a \in A_{l,w}} t_{l,a} + \sum_p \vartheta_p \cdot \delta_{l,p}^w\ l\hat{I}L,\ w\hat{I}W \tag{13}$$

where:$t_{l,a}$ = running time of line l on arc a assuming no delays to reach the stop at the head of arc $aA_{l,w}$ = set of arcs of line l needed to travel to join pair $w \vartheta_p$ is an additional delay time because of bus queuing, which can be modeled by a volume-delay function depending on the degree of saturation of the bus stops [27]:

$$\vartheta_p = \chi \cdot \exp\left(\varsigma \frac{\sum_{l \in L} f_l \cdot \delta_{l,p}}{BSC_p}\right) \tag{14}$$

where ς and χ are parameters to be estimated and BSC_p is the bus stop capacity of stop p according to the Transit Capacity and Quality of Service Manual TCQSM [28].

$\delta_{l,p}$ = binary variable taking a value of 1 if stop p is served by line l and 0 otherwise.$dt_{l,p}$ = the dwell time; the time a bus on line l spends at stop p for passenger boarding/alighting, which can be expressed, following TCQSM [28]:

$$dt_{l,p} = t_{oc} + \left(B_{l,p} \cdot t_b + A_{l,p} \cdot t_a\right)/f_l p\hat{I}\{1 \ldots. n\}, l\hat{I}L \tag{15}$$

where:t_{oc} = door opening and closing time (h).t_b = average boarding time per passenger (h).t_a = average alighting time per passenger (h).$B_{l,p}$ = trips boarding line l at stop p, which can be obtained as:

$$B_{l,p} = \sum_{w \in W, o(w)=p} \sum_{s \in S_w, l \in s} \left(y_s^w \frac{f_{l,p}^*}{f_s^*}\right) p\hat{I}\{1 \ldots. n\}, l\hat{I}L \tag{16}$$

$A_{l,p}$ = trips alighting line l at stop p, which can be obtained as:

$$A_{l,p} = \sum_{w \in W, d(w)=p} \sum_{s \in S_w, l \in s} \left(y_s^w \frac{f_{l,p}^*}{f_s^*}\right) p\hat{I}\{1 \ldots. n\}, l\hat{I}L. \tag{17}$$

Expression (15) assumes that boarding and alighting happen sequentially at every stop. Under different boarding and alighting conditions (i.e. simultaneous processes) expression (15) should be adjusted accordingly [26]. However, behavior in transit corridors equipped with off-board fare payment stations is very often similar to metro systems, where alighting users precede boarding.

Furthermore, a term of discomfort [4] can be introduced to reflect the crowding effect. This term can be included to redefine the running time of line l for the pair w (rt_l^w) of equation (13) as:

$$rt_l^w = \sum_{a \in A_{l,w}} \left(\left(t_{l,a} + \sum_p \vartheta_p \cdot \delta_{a,p}\right) \cdot \left(1 + \alpha \cdot \left(\frac{v_{l,a}}{C_l \cdot f_l}\right)^\beta\right)\right), w\hat{I}W,\ l\hat{I}L \tag{18}$$

where α and β are parameters to be estimated and $\delta_{a,p}$ is a binary variable taking a value of 1 if the head of arc a is p and 0 otherwise. Notice that bus queueing time at bus stops has been included as part of the running time. By doing this, it is considered that users still suffer the crowding conditions at these additional times between stops.

Based on the described time structure, each user making a trip in pair w chooses the route section that minimizes his/her total travel time (in-vehicle travel time + waiting time), given by:

$$ETT_s^w = \left(\frac{1}{f_s^*} + \frac{\sum_{l \in s} \left(rt_l^w \cdot f_l^*\right)}{f_s^*} + \frac{\sum_{l \in s} \left(\sum_p dt_{l,p} \cdot \delta_{l,p}^w\right) \cdot f_l^*}{f_s^*}\right), \qquad w\hat{I}W,\ s\hat{I}S_w. \tag{19}$$

3.3. Equilibrium formulation

In accordance with Wardrop's first principle, the problem can be formulated with a variational inequality of the following type:

$$\overline{ETT}\left(\overline{Y}^*\right) \cdot \left(\overline{Y}^* - \overline{Y}\right) \le 0, \qquad \forall \overline{Y} \in \Omega \tag{20}$$

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where $\overline{Y}$ is any feasible flow vector in each route section for each pair $w\ \{y_s^w\}$, $\overline{Y}*$ represents the equilibrium flow solution, Ω is the group of feasible flow vectors and $\overline{ETT}$ is the route section cost vector.

4. SOLUTION ALGORITHM

4.1. Description of the algorithm

It can be seen that the problem being considered has a non-separable cost function: the cost function for each arc is not separable as a function of its own flow but rather depends on the arcs with which it shares one or more services. So, the Jacobian matrix is non diagonal and asymmetric; hence, no equivalent optimization formulation of the Beckman type exists for the variational problem.

One of the simplest and most robust schemes for solving equilibrium problems is the well-known MSA which has been extensively used in transport applications, and its particular usefulness has been widely demonstrated for solving stochastic equilibrium assignment problems. Its application to congested public transport assignment can be found in a wide number of research ([9, 11, 14, 19] cited in this paper) and can be applied in problems with non-separable cost functions [29].

Given the linear nature of public transport corridors, the transit assignment problem with capacity constraints can be considered as a sequential assignment with respect to the first stop on the route. So, travelers between successive nodes will experience increased waiting times depending on the degree of saturation along the line caused by the trips starting at the previous nodes.

A sequential transit assignment is solved first at each iteration of the MSA algorithm; then, the MSA algorithm updates the route section flows by taking the average value of route section flows from the previous iterations.

The above description means that the proposed algorithm can be described in the following steps:

Step 1 *Initialization of the algorithm*: establish $it = 0$, compute the initial (free flow) travel time on each line for each pair w: $(t_l^w)^{it}$.

Step 2 Sequential assignment to public transport:

2.1 Set p = 1

2.2 *Transit Assignment*: The times $(t_l^w)^{it}$ are used to calculate the attractive lines in all route sections starting from node p determining $min_s\ ETT_s^w$, according to (19). All journeys starting from node p, are assigned to the identified route section.

2.3 *Update* (wt_l^w): the flows obtained from the route sections and line sections ($y_s^w; v_l^w$) are used to update the waiting times ($wt_{l,p}$) perceived by the travelers starting their trip in $p+1$, according to (4). Based on these perceived frequencies, we compute f_s^* for all route sections starting in p + 1, according to (8).

2.4 p ← p + 1

2.5 *G*o to *2.2.*until $p = n$

Step 3 *Updating*: with the obtained flows per route section $Y^n = (y_s^w)^{it}$, and travelers per line $V^{it} = (v_l^w)^{it}$ update the expected travel time times: $(ETT_s^w)^{it}$ as a function of V^{it}.

Step 4 : *Search for the direction of movement*: with the new expected travel times from *Step 3*, carry out another *sequential assignment*, obtaining an auxiliary flow vector for each route section and pair w: $(z_s^w)^{it}$

Step 5 *Movement*: Obtain the new journey flow pattern per line according to:

$$\left(y_s^w\right)^{it+1} = \left(y_s^w\right)^{it} + (1/it)\left(\left(z_s^w\right)^{it} - \left(y_s^w\right)^{it}\right) \tag{21}$$

Step 6 *Convergence Criteria* (Δ): If convergence is found, stop; if not, perform $it \leftarrow it + 1$ and return to *Step 3*.

5. NETWORK TEST APPLICATION

The proposed model has been checked using the example network proposed by[2]. The network consists of a corridor of four nodes and three lines with the characteristics shown in Figure 1.

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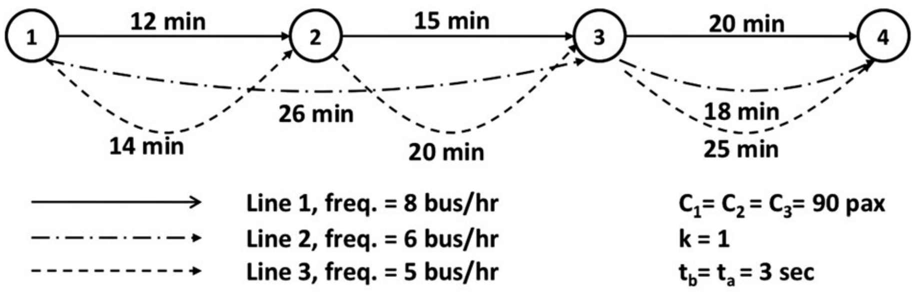

w	1-2	1-3	1-4	2-3	2-4	3-4
Tw	300	600	200	200	400	300

Figure 1. Network test configuration.

The algorithm is first used to identify the assignment for the uncapacitated case with and without taking into account dwell times (only boarding and alighting times are computed), arriving at the same results reported by [2] and displayed in Table I. This table shows that in this equilibrium the 3–4 pair is the only O/D pair served by two route sections. For low levels of demand, only route section S4 (L1, L2) will be attractive, but once demand grows then section S7 (L1, L2, L3) also becomes attractive.

However, if passenger load is analyzed by line segment, the load on L1 reaches 826.37 between nodes 2 and 3, exceeding its capacity. Thus, the assignment is carried out again taking into account the capacity constraint and discomfort penalty (bus queuing effect in bus delays is avoided here because of low frequencies).

The equilibrium assignment when vehicle capacities are considered (with parameter values $\phi = 40$, $\alpha = 0.5$ *and* $\beta = 10$) is displayed in Table II, in which OD pairs 1–3, 1–4 and 3–4 are now served by two route sections with identical journey times. This reassignment of flows prevents L1 from becoming saturated in arc 2–3.

This can be seen more clearly in Table III which shows the passenger load and the degree of occupancy of each line segment on the network. We have also added in the table the case in which dwell times are assumed independent of the flows. In the new assignment, the segment 2–3 of line *1* is no

Table I. Application of the algorithm on the uncapacitated network: equilibrium journey times and route section flows.

Equilibrium ETTs							
		W					
Section	Lines	1–2	1–3	1–4	2–3	2–4	3–4
S1	L1	24.66	44.88	69.11	27.72	51.95	31.73
S2	L2		41.00	61.69			30.69
S3	L3	28.31	51.38	78.38	35.08	62.08	39.00
S4	L1,L2		38.93	61.65			27.00
S5	L1,L3	21.45	42.77	68.07	25.93	51.23	29.91
S6	L2,L3		40.27	63.83			29.01
S7	L1,L2,L3		39.05	62.89			27.00
Equilibrium route section flows							
S1	L1	0.00	0.00	0.00	0.00	0.00	0.00
S2	L2		0.00	0.00			0.00
S3	L3	0.00	0.00	0.00	0.00	0.00	0.00
S4	L1,L2		600.00	200.00			212.31
S5	L1,L3	300.00	0.00	0.00	200.00	400.00	0.00
S6	L2,L3		0.00	0.00			0.00
S7	L1,L2,L3		0.00	0.00			87.69

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Table II. Application of the algorithm on the capacitated network: equilibrium journey times and route section flows.

Section	Lines	Equilibrium ETTs					
		1–2	1–3	1–4	2–3	2–4	3–4
S1	L1	23.857	47.800	71.395	32.048	55.643	31.095
S2	L2		42.292	**63.895**			31.603
S3	L3	29.080	53.153	80.180	36.073	63.100	39.027
S4	L1,L2		**41.153**	**63.895**			**27.027**
S5	L1,L3	**21.251**	45.243	70.158	28.833	53.811	29.531
S6	L2,L3		41.774	65.843			29.523
S7	L1,L2,L3		**41.153**	65.023			**27.027**
Equilibrium route section flows							
S1	L1	0.00	0.00	0.00	0.00	0.00	0.00
S2	L2		0.00	**185.86**			0.00
S3	L3	0.00	0.00	0.00	0.00	0.00	0.00
S4	L1,L2		**306.94**	**14.14**			**207.38**
S5	L1,L3	**300.00**	0.00	0.00	**200.00**	**400.00**	0.00
S6	L2,L3		0.00	0.00			0.00
S7	L1,L2,L3		**293.06**	0.00			**92.62**

Bold text: route sections with minimum ETT under equilibrium conditions.

Table III. Application of the algorithm: comparison of travellers and degree of occupation per line segment for each assignment.

			No dwell time dependent			*Dwell time dependent without capacity constraint*		*Dwell time dependent with capacity constraint*	
line	*Node A*	*Node B*	*Cap*	*Pax*	*Degree of sat (x)*	*Pax*	*Degree of sat (x)*	*Pax*	*Degree of sat (x)*
1	1	2	720	641.76	0.89	641.75	0.89	491.48	0.68
	2	3	720	980.22	1.36	826.37	1.15	676.10	0.94
	3	4	720	685.71	0.95	518.68	0.72	411.73	0.57
2	1	3	540	342.86	0.63	342.86	0.63	416.01	0.77
	3	4	540	214.29	0.40	204.39	0.38	416.01	0.77
3	1	2	450	115.38	0.26	115.38	0.26	310.05	0.57
	2	3	450	76.92	0.17	230.77	0.51	192.51	0.43
	3	4	450	0.00	0.00	176.92	0.39	307.89	0.68

longer saturated, whereas it was clearly overloaded with both unrestricted assignments which did not consider vehicle capacity.

The three processes described above lead to quite different passenger assignments to line segments. Table IV shows trip assignments per line obtained by the three assignment models. When capacity constraint is considered line L3 significantly increases its demand in spite of its journey times at first appearing uncompetitive.

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Table IV. Application of the algorithm to the example network: comparison of travellers per line.

	Trips per line (pax/h)		
	L1	L2	L3
No *dwell time dependent*	1336.26	471.43	192.31
Dwell time dependent without capacity constraint	1169.23	461.54	369.23
Dwell time dependent with capacity constraint	1023.99	532.51	443.51

6. APPLICATION TO A REAL NETWORK: PAJARITOS CORRIDOR (SANTIAGO DE CHILE)

Once the proposed model has been checked, a larger network is needed in order to check the travel time variability because of dwell times, for this reason the model has been used with data from the segregated bus lane corridor that operates along Pajaritos Avenue in Santiago, Chile. This corridor is 3.3 km long and there are 10 bus stops in each direction. Operational parameters (lines, frequencies and bus sizes) have been taken from[30] and are shown in Figure 2. This corridor is served by 294 buses per hour divided into four lines. The demand is shown in Table V and has been obtained from [30] and [31].

All possible cases have been analyzed in this application: (i) transit assignment with constant dwell times and no capacity constraint; (ii) transit assignment with demand dependent dwell times and no capacity constraint; (iii) transit assignment with constant dwell times and capacity constraint and (iv) transit assignment with demand dependent dwell times and capacity constraint. We will assume a constant dwell time equal to 20 s for (i) and (iii). This value has been obtained as an aggregate of those reported by [1, 32, 33].

Table VI shows the values taken for all parameters in each case. Convergence criteria have been the gap between two successive iterations, defined as:

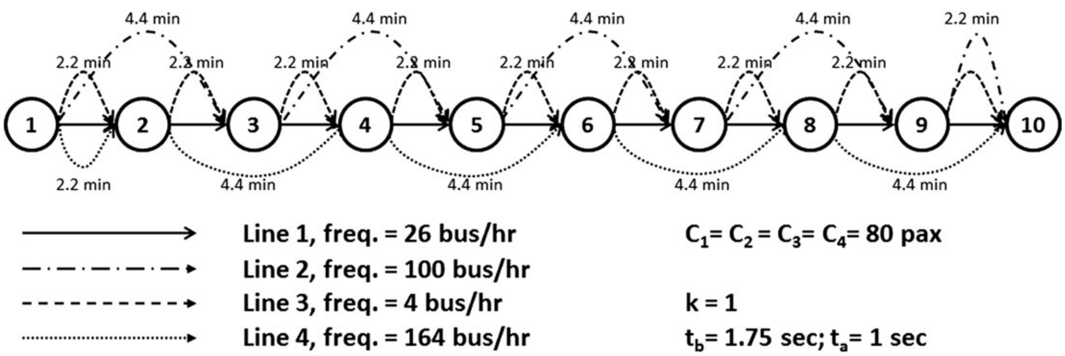

Figure 2. Pajaritos corridor scheme.

Table V. Pajaritos corridor OD matrix (pax/h).

T_w	1	2	3	4	5	6	7	8	9	10	Boarding
1		4453	2898	2422	1494	1261	733	430	373	2481	16546
2			312	271	173	159	79	46	40	267	1346
3				310	197	121	91	54	48	319	1140
4					0	0	140	93	84	617	934
5						219	107	65	57	384	832
6							134	75	67	453	729
7								86	73	508	667
8									87	559	646
9										626	626
10											0
Alighting	0	4453	3210	3003	1864	1760	1284	849	828	6213	23465

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Table VI. Model parameters.

	Case i	Case ii	Case iii	Case iv
α	n/a	n/a	0,5	0,5
β	n/a	n/a	10	10
ϕ	n/a	n/a	40	40
Dwell time	20 s	Variable	20 s	Variable
t_b	n/a	1.75	n/a	1.75
t_a	n/a	1	n/a	1
t_{oc}	n/a	3	n/a	3
Bus size	n/a	n/a	90	90
ς	0.9152	0.9152	0.9152	0.9152
χ	7.7609	7.7609	7.7609	7.7609
BSC	210	210	210	210

$$\Delta \rightarrow \sum_{s \in S} abs\left(V_s^{it+1} - V_s^{it}\right) < 0.01. \tag{22}$$

The results obtained from all the transit assignments show how there is enough capacity to satisfy these higher demand rates in spite of the high demand at the beginning of the corridor.

However, in spite of not reaching the capacity constraint, it is important to note that the demand varies significantly when the dwell time variable is considered. In fact, the aggregated results are quite similar whether the capacity constraint has been considered or not.

Table VII shows how the express lines become more relevant in these cases because their travel times are more attractive because of the fact that such lines do not serve every bus stop; the demand on L2 increases by more than 300 passengers and more than 500 on L4, representing an increase of 4.5% over the initial demand. This data becomes clearer for the all-stop lines, which lose the same number of passengers, but in this case representing more than 20% of the demand.

If the results are already significant for the number of passengers per line, the analysis per section along the corridor reveals even more variations. As can be seen in Figure 3, the first sections, with the highest demand, show the greatest variations in passenger numbers, with variations of 40% in demand for all-stop lines and 6.9% on express lines.

These values should not be underestimated as they represent variations of up to 400 passengers/h in each direction along a single section and could lead to potential planning errors for these lines if they were not taken into account.

Therefore, as can be seen in Figure 3, the assignment, and thus, the demand of each line are highly conditioned by dwell times. This fact is clearly reflected by express lines (L2 and L4) where travel times become even more competitive.

An analysis of the route sections chosen by the users for their journeys shows OD pairs where the users continue to choose more than one route section. In the case of the 1–3 pair, the users are shared between sections S2, served by L2, and S11 served by L1, L2 and L3; and the 8–10 pair, where journeys are made on sections S4, corresponding to L4 and S13, served by lines L1, L3 and L4.

Furthermore, a sensitivity analysis has been carried out considering different levels of demand and different boarding and alighting times. Thus, the OD matrix shown in Table V has been modified by applying two coefficients equal to 0.5 and 1.5 in order to decrease and to increase the demand, respectively.

The results show that congestion (discomfort; waiting times) does not influence travel times when the demand is much lower than the operational capacity of the corridor. The loads of each line remain

Table VII. Pajaritos corridor application: passengers per line.

Line	Case (i)	Case (ii)	Case (iii)	Case (iv)
L1	3633	2887	3633	2905
L2	7830	8183	7830	8163
L3	559	444	559	447
L4	11 444	11 951	11 444	11 951

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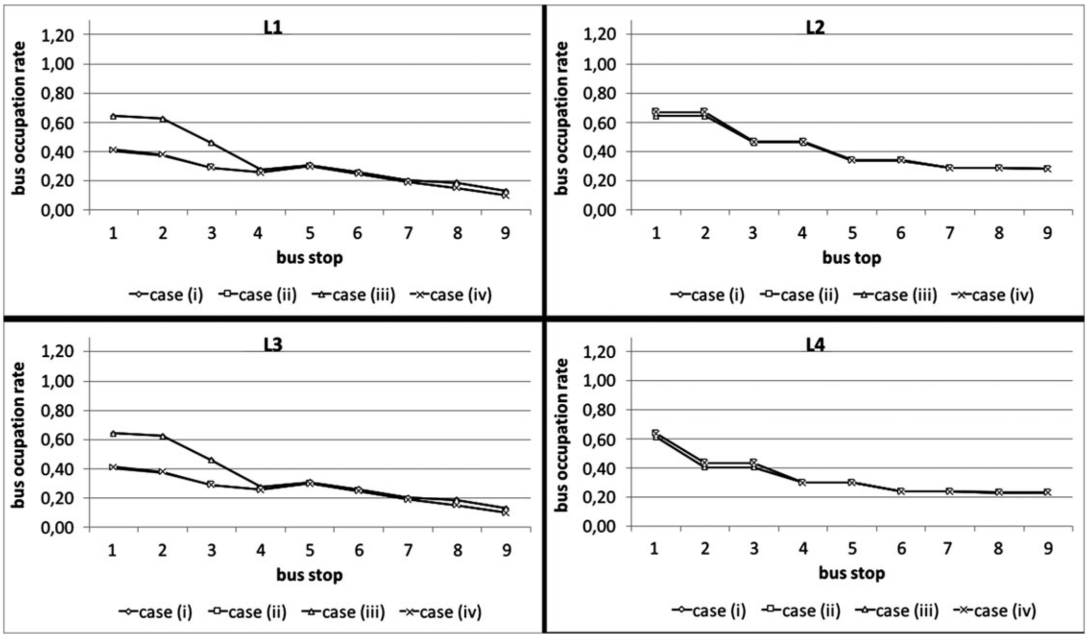

Figure 3. Degree of saturation of each line (base demand).

constant in all the analyzed cases but are highly dependent on dwell times. This effect can be seen in Figure 4, where the passenger loads per line change slightly when dwell time is considered, especially in the initial sections where demand is highest.

However, when demand levels reach operating capacity, travel times are still significantly affected by dwell times, even more than by the capacity constraint as shown in Figure 5. Variations of up to 900 passengers/h are found in each direction on L1 when the capacity constraint is not considered, but 43%, and 400 passengers/h/direction when the capacity constraint is considered. In other words, with high levels of demand, dwell times have a greater influence on the assignment results than the capacity constraint.

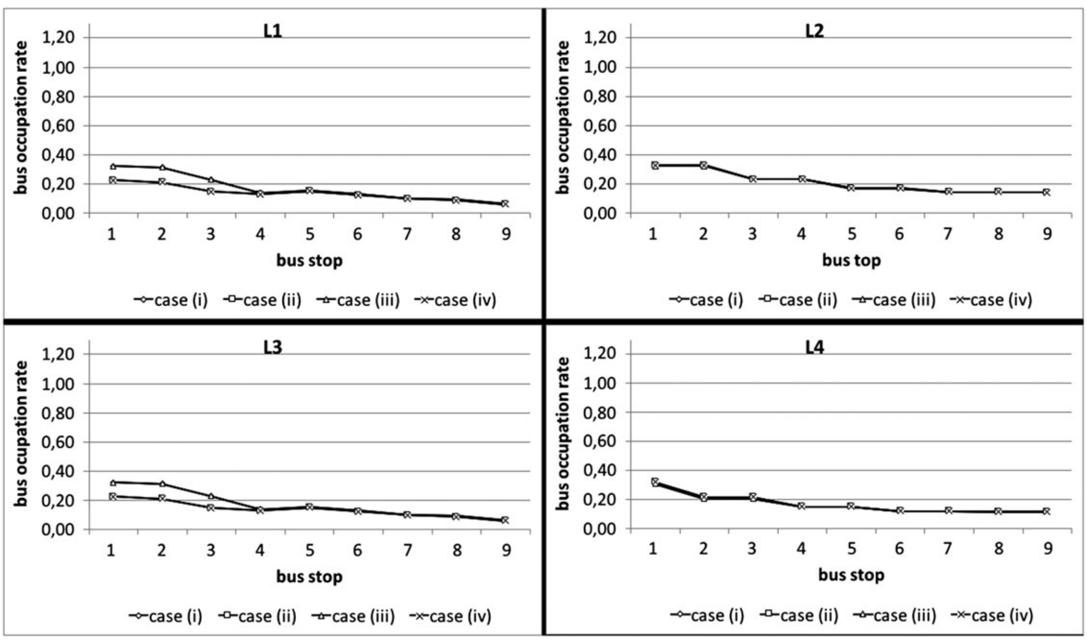

Figure 4. Degree of saturation of each line (demand decreased by 0.5).

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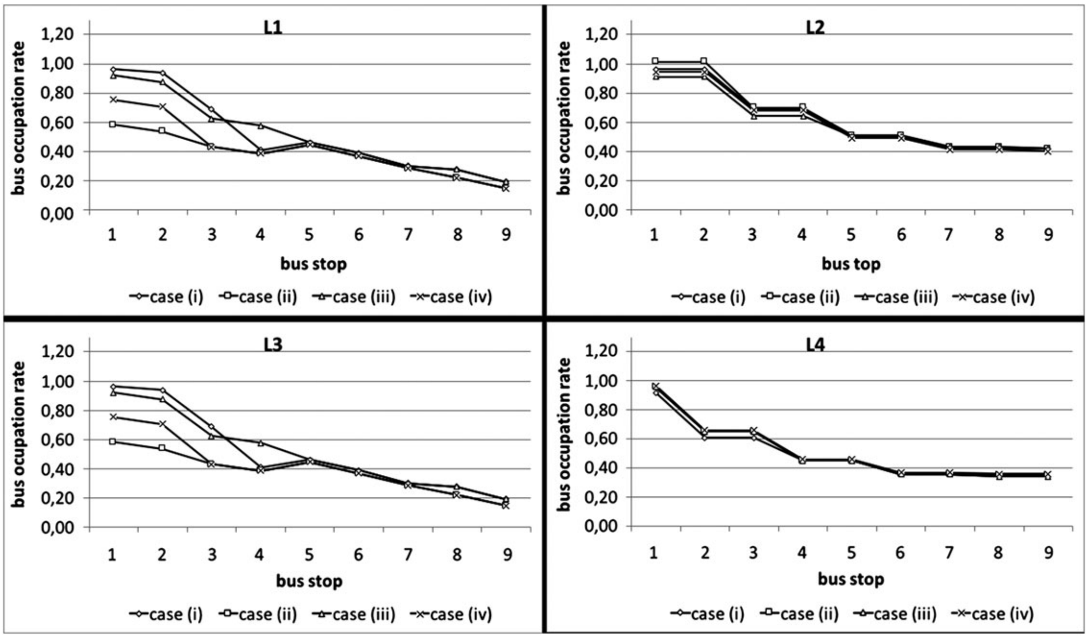

Figure 5. Degree of saturation of each line (demand increased by 1.5).

As an example, Figure 6 shows the variations in the passengers who board at the first stop along the corridor where there is the greatest demand according to the Origin Destination matrix. The results were normalized by representing the percentile variations compared with the base scenario considered in case (i), in other words, without congestion and with constant dwell times. This graph highlights the important drop in passengers boarding lines L1 and L3, and corroborates that the variations are more significant in cases (ii) and (iv), when variable dwell times are considered, than when the capacity constraint is activated.

Because in this case dwell times seem to have a strong impact in the results, the passenger boarding and alighting times at bus stops have also been varied. Lower values have been used, corresponding to

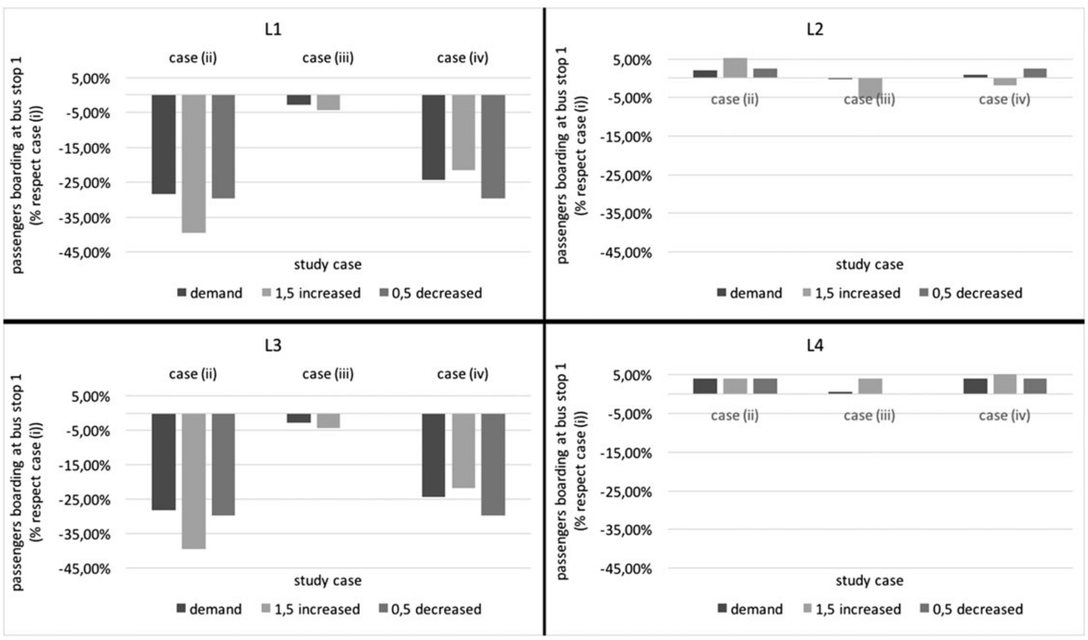

Figure 6. Demand relative variation for each case at bus stop #1.

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0.5 s/pass and 0.3 s/pass for boarding and alighting, respectively for the observed demand on the corridor (Table V) and assuming a 50% growth factor. The influence of more extreme values was also analyzed by notably increasing these times up to 2 s/pass and 1.2 s/pass, respectively. Intermediate times of 1 s/pass and 0.75 s/pass were also tested according to [28, 34, 35]. These values may fit the conditions of different vehicle types (normal and articulated buses with three and four doors) and fare payment systems. The comparison between the different levels of occupancy per line along the corridor is presented in Figures 7 and 8. It can be seen that, for all the analyzed cases, the difference in occupancy level grow as dwell times depend more heavily on the boarding and alighting process. Differences of around 40% were found for the base demand case and of 27% for the amplified demand, with

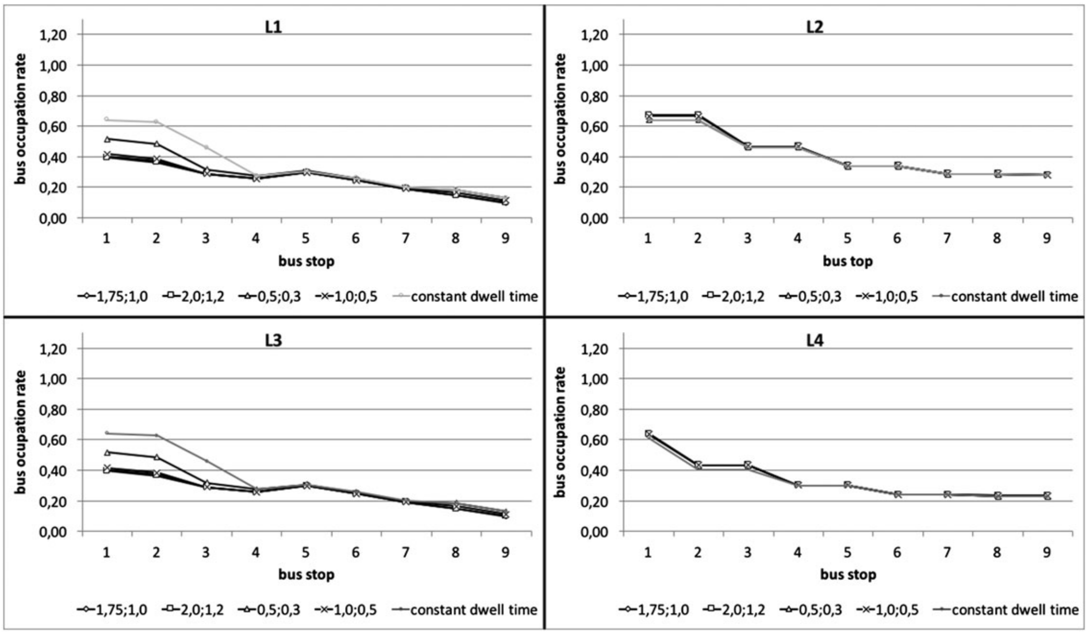

Figure 7. Degree of saturation of each line with boarding/alighting times variation (base demand).

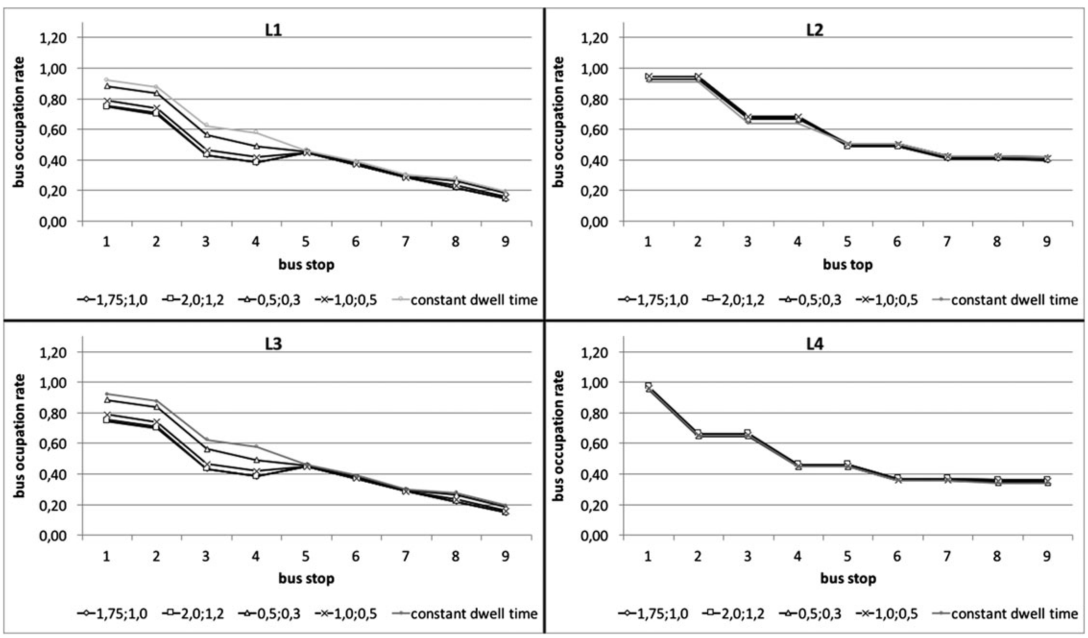

Figure 8. Degree of saturation of each line with boarding/alighting times variation (demand increased by 1.5).

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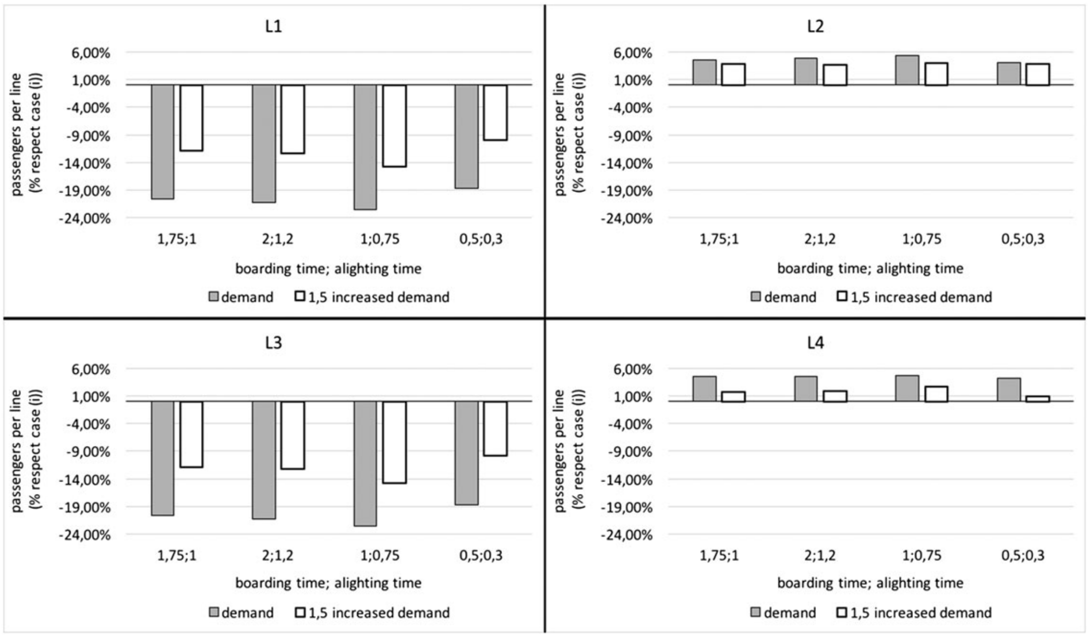

Figure 9. Demand relative variation of each line with boarding/alighting times variation and different levels of demand.

variations of 550 and 400 passengers, respectively, on certain line sections when compared to the constant dwell times case.

The influence of boarding/alighting times on passenger assignment has been found to be quite similar for intermediate values: 1–2 s/pass and 0.5–1.2 s/pass for boarding and alighting times, respectively. These times are comparable with observed service times for three door standard buses. Lower passenger service times (0.5 and 0.3 s/pass) replicating four door articulated buses lead to slightly different results. However, differences of around 20% were found in passengers per line for the actual demand case and of 17% for the factored demand.

The results obtained in this analysis highlight the importance of considering the impact of variable dwell times during the assignment process because the route section flows (therefore line flows) may vary greatly with respect to conventional transit assignment models. These differences become more apparent at the individual line level, as can be seen in Figure 9. This figure shows the relative passenger variations for each line with respect to the assignment with capacity constraint and constant dwell times. The "all-stop" lines (L1 and L3) show flow drops of over 20% in most of the cases analysed for the base demand case, and of more than 12% when the demand is amplified. The express lines (L2 and L4) show increments of around 5% in the base demand case and over 3.5% when demand is amplified. As in previous cases, the assignment in the scenario with the fastest boarding/alighting times is the closest to the assignment with constant dwell times. However, even in this scenario the lowest increase is found on Line 4 for the factored demand, representing about 200 passengers/h in a scenario where the capacity constraint is reached. This may lead to problems associated with bad planning for the lines along the corridor.

Finally, the assignment model, both with and without capacity constraints have shown that some origin–destination pairs are served by more than one route section. As [2] found, identical users faced with the same trip must be allowed to take different decisions for an equilibrium assignment to exist.

7. CONCLUSIONS

This article has proposed a new assignment model for public transport corridors. In this model limited vehicle capacity has been considered influencing waiting time at bus stops, and "discomfort" during a journey when occupancy at buses approach capacity. Dwell times because of passenger boarding and alighting times at bus stops have also been included as part of the in-vehicle journey times, as well as an additional delay inside the bus because of bus queuing at stops.

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The MSA algorithm with a sequential assignment has been used to solve it. The application of the algorithm to the uncapacitated text network reported by [2] has provided identical results.

The application shows significant variations in passenger assignment per line segment compared to traditional assignment models, implying the importance of taking into account the dwell time, even during high levels of demand where dwell times have a more significant influence than the capacity constraint on the assignment results. Variations of up to 40% passengers/h in each direction have been found along a single section in most cases.

Different boarding and alighting times have been analyzed in order to mimic different vehicle types: normal and articulated buses with three and four doors. Lower passenger service times (0.5 and 0.3 s/pass) replicating four door articulated buses lead to different results from those obtained with intermediate times and with constant dwell times. However, differences of around 20% were found for the base demand case and of 17% for the amplified demand.

In all cases, constant dwell time assignment models tend to underestimate the demand of express services (and therefore, demand on all-stop services is overestimated). This could lead to planning errors undersupplying these services if variable dwell times were not considered.

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