PONTIFICIA UNIVERSIDAD CATÓLICA DE CHILE
ESCUELA DE INGENIERÍA

ROBUST OPTIMIZATION MODEL FOR LASER-BEAM MACHINING EQUIPMENT INVESTMENT UNDER DEMAND UNCERTAINTY

JUAN JOSÉ FELLER GOUDIE

Thesis submitted to the Office of Research and Graduate Studies
in partial fulfillment of the requirements for the degree of
Master in Science in Engineering

Advisor:
ALEJANDRO FRANCISCO MAC CAWLEY VERGARA
JORGE RAMOS GREZ

Santiago de Chile, January 2019

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ACKNOWLEDGEMENTS

I would like to express my gratitude to both of my advisors, Alejandro Mac Cawley and Jorge Ramos. Alejandro gave me his support and guided me during those moment in which I felt lost and that I would never finish my thesis. His guidance allowed me to grow as a student, as a professional and as a person. On the other hand, I would like to thank Jorge who helped me with the research and the modeling of the laser cutting process.

In addition, I want to thank all my friends, especially Andres Perez, who over the years has always encouraged me and helped to accomplish my goals. I would also like to express my gratitude to my family, my parents, my siblings and my nephews and nieces, for their unconditional love and support during my whole life, especially during my university years. I am sure that they are the reason behind each one of my life's achievements.

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ABSTRACT

Productive industries such as the manufacturing industry are constantly facing dynamic, changing and unpredictable situations. Robust capacity and production planning protects companies against the effects of uncertain inputs and processes variability. Unpredictable coverage can be achieved through a robust optimization approach, which accounts for uncertainty when determining an optimal solution.

Laser cutting technology is a manufacturing technique developed during the 1970s, able to process diverse types of materials (metallic and non-metallic) with different thicknesses with high precision (0.02 mm) and speed. Companies using this technology access several productive benefits, such as rapid prototyping and mass production.

In this thesis, we develop a robust model that optimizes equipment investing decisions and finds the optimal production plan based on the available machinery. This model assumes that demand is uncertain and subject to variability. The research questions are: Is it possible to lineally integrate laser cutting models to determine an optimal investment and production plan? Does a robust reformulation of the problem provide a solution that considers capacity coverage against demand variability? Is there a robust optimal plan that increases profitability and operational efficiency while providing capacity coverage?

The model is used to determine the investment and productive plan for a company which offers Laser-Beam Machining (LBM) services. The case study validates the effectiveness of the proposed model and proves the robustness of the solution. Results show that unlike the optimal linear plan, the optimal robust plan considers the investment in extra machines, increasing the company's ability to fill the uncertain demand. Results show that the optimal robust solution can increase the expected profits of the company in 6.39%.

Keywords: Robust Optimization, Manufacture Industry, Laser-beam Machining, Uncertainty, Optimization.

RESUMEN

Las industrias productivas tales como la manufacturera, se ven enfrentadas constantemente a situaciones dinámicas, cambiantes e impredecibles. La generación de planes de inversión y producción robustos pueden proteger a las empresas de los efectos variables e inciertos de los procesos. Se puede generar coberturas a través de la optimización robusta, la cual permite considerar incertidumbre en la determinación de una solución óptima.

La tecnología de corte por láser es un proceso de fabricación desarrollada durante la década de 1970, la cual permite procesar diversos tipos de materiales, con diferentes espesores, alta precisión (0,02 mm) y velocidad. Con esta tecnología las empresas pueden fabricar prototipos rápidamente y producir en masa.

En esta tesis, desarrollaremos un modelo de optimización robusto para la inversión y producción basado en la maquinaria disponible. Este modelo asume que la demanda está sujeta a incertidumbre. Las preguntas de investigación se centran en: ¿Es posible integrar modelos lineales de corte láser para determinar un plan de inversión y producción óptimo? ¿Una reformulación robusta del problema proporcionará una solución que cubra la capacidad frente a la demanda variable? ¿Existe un plan robusto óptimo que aumente las ganancias y la eficiencia operativa con cobertura productiva?

El modelo se utiliza para determinar el plan de inversión y de producción para una empresa que ofrece servicios de mecanizado por rayo láser (LBM). El caso de estudio valida la efectividad del modelo propuesto y demuestra la solidez de la solución. Los resultados muestran que, a diferencia del plan lineal óptimo, el plan robusto óptimo considera la inversión en más maquinaria, lo que aumenta su capacidad productiva. Los resultados muestran que la solución óptima robusta óptima puede aumentar las ganancias en 6.39 %.

Palabaras Clave: Optimización Robusta, Industria Manufacturera, LBM, Incertidumbre, Optimización.

1. INTRODUCTION

1.1. Background

1.1.1. Manufacturing Industry Background

The manufacturing industry refers to those companies which manufacture or process items for the creation of products. Companies take raw materials and elaborate semi-elaborated products or finished goods (INE, 2012). Raw material are obtained from different sources and transformed into new products that could be either sold or further processed for the creation of finished goods products. (SOFOFA, 2015) stats that the manufacturing industry can be divided in seven sub-sectors: Food, Beverages and Tobacco; Cellulose and Paper; Wood and furniture; Nonmetallic minerals; Chemistry, Petrol, Rubber and Plastic; Metal products, Machinery and Equipment; Textiles and Leather. The seven sub-sectors which can be observed in Figure 1.1.

In 2015, this industry represented 10.9% of the Chilean GDP, which corresponds to the fourth most important production sector. The other three sectors are Company Services (13.4%), Mining (13.0%), and Personal Services (11.9%). Manufacturing industries represent the third sector with more employees in the country, with 11.1% of the workforce of the country (Banco Central de Chile, 2018).

Food, Beverage, and Tobacco are the most relevant sector of the manufacturing industry, with a 40% of the GDP for this industry. On the other hand, Textiles and Leathers represent the smallest share of the Manufacturing Industry's GDP. In this thesis, we are going to work with Metal products, Machinery and Equipment. This sub-sector has the third position in contribution to GDP for the manufacturing industry, corresponding to 16%.

The Metallic Products, Machinery, Equipment and Others subsector is responsible for the manufacture of metal and machinery products. This sub-sector has the highest concentration of companies and employees. Some examples of metallic products are: metal

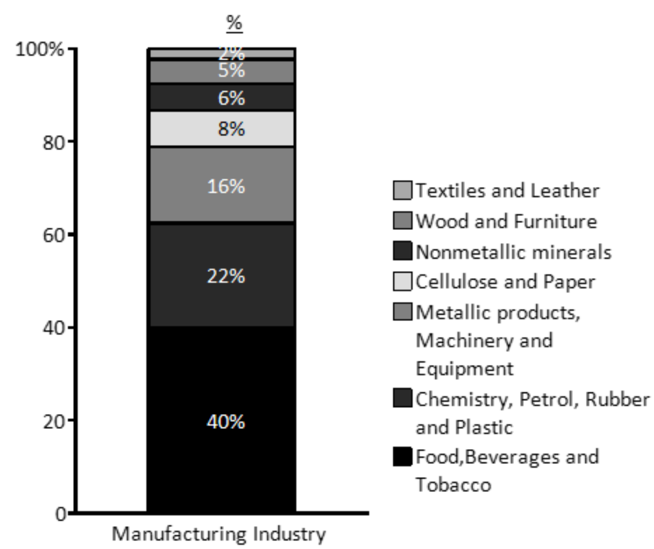

Figure 1.1. Relevance of each sub-sector in Manufacturing GDP industry in 2015

containers, structures, water tanks, heaters ,among others. On the other hand, examples of machinery and equipment are: manufacture of fixed and mobile devices, manufacture of parts to produce and distribute energy, among others (Observatorio Laboral Chile, 2016; SOFOFA, 2015).

1.1.2. Laser Technology

LASER is an acronym termed Light Amplification by the Stimulated Emission of Radiation. Laser technology is based on light; it is generated from a light source that has been amplified in a way similar to how a microphone is used to amplifies sound (Gould, 1959). This amplification is known as the optical amplification. The beam of light is used to create an excitation in the atoms that are present in the lasing medium, which could be solid (e.g. Fiber), gas (e.g. CO_2) or liquid (e.g. hydrogen fluoride).

A laser basically consists of three parts: an amplifying or gain medium, an optical resonator and a pumping system. The gain medium consists of atoms, molecules, ions or electrons that are used to produce the exciting laser light, by the absorption of energy, and its subsequent release in the form of photons of light. The pumping system is responsible to set up the conditions for the light amplification by supplying the necessary energy to excite and keep exciting atoms during the lasing process. The optical resonator is used to achieve the needed magnification in the laser medium. It is contained of a pair of reflecting mirrors (plane, curved, or mixture).

The laser light is characterized by three properties: monochromaticity, directionality, and coherency. The first one is related to the single wavelength and single color of the light beam (also known as monochromaticity). This important property of laser is responsible for the high intensity energy achievable in the laser. This is possible as the energy is concentrated at this single wavelength (Mahamood, 2018).

Directionality is related with low divergences of the light beam. The light emitted by a laser is confined to a rather narrow cone. When the beam propagates outwards, it slowly diverges or fans out. According to the encyclopedia of laser physics and technology, the beam divergence of a laser beam is a measure for how fast the beam expands from the beam waist.

Finally, coherency is the ability of the light to exhibit the effect of interference. A light field is called coherent when there is a fixed phase relationship between the electric field values at different locations or at different times. Spatial coherence means a strong correlation between the electric field at different locations across the beam profile. Temporal coherence means a strong correlation between the electric field at one location but different times (Dr. Rüdiger Paschotta, n.d.).

1.1.3. Laser Types

Mahamood (2018) defines the three most common laser types: Solid state laser, Gaseous state laser and Liquid state laser. The main difference among them is the state of the lasing medium utilized.

1. Solid state laser are lasers in which the gain medium is solid at room temperature. This laser uses crystalline solids doped with ions. It is usually optically pumped using a flash tube or another laser with a shorter wavelength than the lasing medium wavelength. The electrons in the lasing medium are first excited to higher energy states through the absorption of the pumped photons. The excited electrons lose photons in order to leave their excited state. This laser type is capable of producing high power in the infrared light spectrum at a wavelength of 1064 nm. It is commonly used for the cutting of metal and the welding of metals. The main limitation of the solid state laser is the high temperature in the lasing medium which is produced from the excess pump power that heats up the medium and reduces the quantum efficiency.

2. Gaseous state laser corresponds to laser generated in a gas lasing medium. This laser generates stimulated emission from the low-energy transitions between vibration and rotation states of the gas's molecular bonds. The main advantage of gas lasers is that they are relatively cheaper than other types of lasers. Gas lasers are also produced from vaporized metal ion to generate deep ultraviolet wavelengths.

3. Liquid state laser is a type of optically pumped laser in which the lasing medium is liquid at room temperature. This laser allows a wide selection of the emission wavelength and polarization from the lasing medium. The spectrum spans from the near ultraviolet to near infrared radiation depends on the type of dye that is utilized. This dye is doped into the liquid crystal to produce a continuous spectrum of lasing. The main advantages of the liquid state are the higher efficiency, and its adjustability various frequencies, which make them ideal for scientific and medical applications. The main drawbacks of this laser are the liquid instability as a results of high heat intensity, and the change of the active substance's refractive index due to the heating.

1.1.4. Laser Cutting Process

Laser cutting is mainly a thermal process in which a high energy density laser beam is focused on the material that will be processed. As the material absorbs thermal energy, the heat transforms it into a molten, vaporized or chemically changed state. A high pressure assist gas removes the molten material and creates a kerf. Typically this gas (inert) is nitrogen or oxygen as they do not react exothermically with the molten material, and thus do not contribute to the energy input.

The effectiveness of this process depends on thermal and the optical properties of the laser rather than the mechanical properties of the material to be processed. Therefore, materials that exhibit a high degree of brittleness, or hardness, and have favorable thermal properties, such as low thermal diffusivity and conductivity, are particularly well suited for laser machining (Dubey & Yadava, 2008a). This process does not generates cutting forces between the machine and the material. The cutting process only occurs through irradiation.

A continuous cut is produced by moving the laser beam under CNC (Computer numerical control). This method controls the movements of a laser machinery directly through coded instructions in the form of numerical data. The coding language is named G. The laser cutting process can be automated utilizing offline CAD/CAM controlling systems. Figure 1.2 shows an schema of a laser cutting system (Dubey & Yadava, 2008a).

Laser cutting is a powerful machining method for cutting complex profiles and drilling holes in a wide range of materials. This process is suitable for precise processing of micro-parts. It is ideal to drill micro-holes with diameters up to 5 mm with an accuracy of 0.02 mm. Laser cutting produces finished parts that do not need to be further processed to be used, due to its great precision and flexibility.

Among the variety of the materials that Laser cutting technology can process are: steel (Carbon, T1, PAS 500, Corten, XAR PLUS, WELDOX, HARDOX, etc), stainless steel (AISI 302, 304, 310, 316, 420, 430, etc), aluminum, copper, bronze, brass, plastics, acrylics, wood, among others.

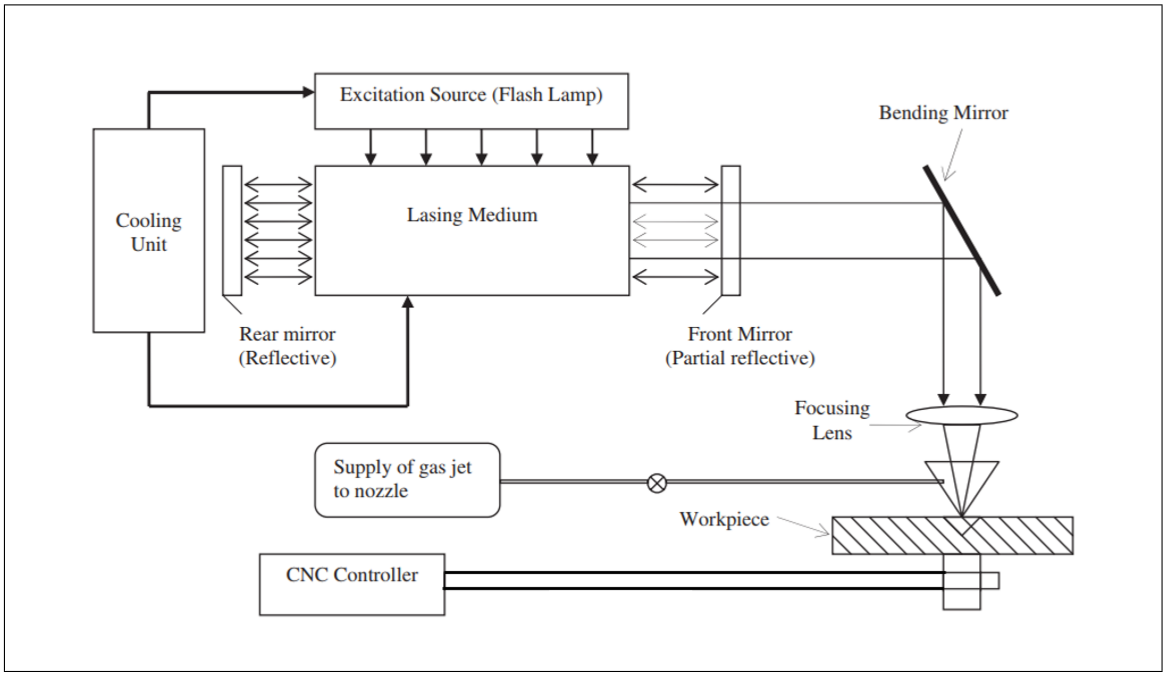

Figure 1.2. Schematic of a laser beam cutting system

1.1.5. Laser Cutting Machine Market

According to Grand View Research (2015), the global laser cutting machine market was valued at US$ 3.02 billion in 2015. The growing trend of automation in the manufacturing sector and the increasing demand for the end-use industry is expected to augment the demand for these machines in the future. Laser cutting machines precisely process parts with complex patterns at high speeds with consistent results. Manufacturers are investing in automation of laser cutting to reduce the downtime, human error and increase efficiency in the energy utilization.

The key players of this industry are focusing on reducing the price of these machines due to the intense competition among vendors. The presence of a vast number of manufacturers (laser cutting companies) has led to a price reduction and the increase of the market share of this technology. However, the high implementation cost of these devices, the lack of technical expertise and the high power consumption are expected to challenge the industry growth.

North America is expected to be a significant region for the development of the laser cutting industry. This region accounted for over 31% of the revenue share of the laser cutting market in 2015, and is estimated to reach over US$ 1.97 billion by 2024. Asia Pacific is projected to be the fastest-growing region due to the presence of robust economies such as China, India, and Japan. The increasing adoption of laser systems in the manufacturing sector is boosting the regional growth. Moreover, the strong growth in industries that use laser cutting to produce semiconductors, consumer electronics, and automotive components is expected to drive demand.

The intense competition is enabling market participants to develop innovative products that are efficient and able to provide faster cycle times for cutting and engraving processes. Strategic investments and partnerships are anticipated to be the most efficient way to gain quick access to the emerging markets.

1.2. Objectives

The main objective of this research is to develop a robust optimization approach for investment and production planning, in environments where demand is exposed to uncertainty. The proposed model is used to optimize the investment decisions and production plan for a company that offers Laser-Beam Machining (LBM) services. For a defined timeframe, it determines the right moment to invest and increase the production capacity. Based on the results of the robust optimization model and assuming that demand follows a normal distribution, the model calculates the optimal production plan. In order to achieve this main objective, we defined the following secondary objectives:

(i) Study the available mathematical representations of the laser cutting process and integrate them into a linear optimization model able to determine the optimal investment and production plan for a manufacturing company.

(ii) Reformulate the optimization problem into a robust optimization approach to account for demand uncertainty when determining the optimal investment and production plan, so the company can be protected against the demand variability.

(iii) Simulate the operation of a company that offers Laser-Beam Machining (LBM) services under uncertain demand, to determine if there is an optimal investment and production plan that provides capacity coverage that avoid unfilled demand due to sales variability.

1.3. Hypothesis

In order to achieve the proposed objectives we propose the following research hypothesis:

(i) It is possible to determine an optimal investment and production plan for manufacturing companies based on the linear integration of mathematical models that represent the laser cutting process.

(ii) The inclusion of uncertainty in the optimization model through a robust optimization approach allows the company to have the optimal extra capacity to be covered against demand variability and increase its profitability.

(iii) There is an optimal investment and production plan for the case study company that provides capacity coverage against demand uncertainty and that leads to operational and economical benefits.

1.4. Thesis Outline

This thesis is based on the presentation of a paper that shows the main findings of the research. The thesis is organized as follows. Chapter 1 is an introductory section that presents the context of the manufacturing industry, the laser beam technology, the laser beam market and the main objectives of this work. Chapter 2 corresponds to the journal article written from this work. Finally, Chapter 3 contains the main conclusions of this work and suggests some further research on this topic.

2. ROBUST OPTIMIZATION MODEL FOR LASER-BEAM MACHINING EQUIPMENT INVESTMENT UNDER DEMAND UNCERTAINTY

The acquisition of Advanced Manufacturing Technologies generally involves high levels of investment, its payback period is usually longer and there is a moderate-to-high risk involved in adopting these technologies. Production planners work towards robust solutions where demand can be filled despite its variation. In this work, we develop a robust model that optimizes equipment investing decisions and finds the optimal production plan based on the available machinery. This model assumes that demand is uncertain and subject to variability. First, we propose a linear investment model based on demand historical information. Then, it is transformed into a robust optimization model which considers demand uncertainty. Second, we determine the optimal production plan based on the results of the robust optimization model and assuming that demand follows a normal distribution. The model is used to determine the investment and productive plan for a company which offers Laser-Beam Machining (LBM) services. The case study validates the effectiveness of the proposed model and proves the robustness of the solution. For this specific application of the model, results show that the optimal robust solution can increase the expected profits of the company in 6.39%.

2.1. Introducction

The decision to acquire Advanced Manufacturing Technologies (AMT), such as laser-beam machining equipment, generally involves a high level of investment, with a moderate-to high level of risk in the adoption of the technology and payback periods which extend longer than those that business enterprises are used (Chan, Chan, Lau, & Ip, 2001). Also over the last decades, the usage and development of AMT has grown rapidly, impacting the daily operation of all manufacturing companies (Olesen, 1990). In order to remain competitive, companies must improve their productivity, reducing fix and variable cost; which can be achieved by the investment in new manufacturing technologies (Vranakis & Chatzoglou, 2011). However, decision-makers need to ponder a number of complex factors at the moment of deciding the adequate machinery and the optimal moment to invest (Hofmann & Orr, 2005). The factors that need to be taken into consideration are: product demand, quality, cost, production efficiency, capital expenditure, among some of the factors needed to be taken into account. An important factor which needs to be taken into account, and sometimes neglected is equipment flexibility, which determines the ability of the equipment to perform cost-effectively and rapidly a set of different tasks (Gustavsson, 1984; Lloréns, Molina, & Verdú, 2005; Kaschel & Bernal, 2006; Sethi & Sethi, 1990). Slack (Slack, 1983) states that flexibility is an important concept that must be taking into account in the productive industries due to the the lack of stability and predictability of the environment. Thus, manufacturing equipment must be flexible enough to allow mass production efficiencies (Avittathur & Swamidass, 2007; Swamidass & Newell, 1987; Sethi & Sethi, 1990). Hence, making the right equipment type acquisition decision, at the right time, will provide considerable operational and competitive benefits for the enterprise(Orr, 1999).

In the AMT literature, the use of optimization has been mostly focused in modelling the process parameters and their interactions, which generally involve a large number, and the selection of the exact parameters settings which considerably affect its performance (See Rao & Kalyankar for an in depth review (Rao & Kalyankar, 2014)). In the equipment selection methods, most of the research in AMT has been focused in multi-criteria decision-making methods (Hodgett, 2016; Yazdani-Chamzini, 2014; Hadi-Vencheh & Mohamadghasemi, 2015) or fuzzy techniques (Chan, Chan, Chan, & Humphreys, 2006; Evans, Lohse, & Summers, 2013; Sadeghi, Ahmady, & Ahmady, 2012). Optimization approaches to equipment selection has been previously used in the mining context (C. Burt, Caccetta, Fouché, & Welgama, 2016; C. N. Burt & Caccetta, 2014) and in the print production environments (Rai, Gross, & Ettam, 2015). However none of them has account for demand uncertainty into their decision making process.

Our objective is to develop and solve a linear robust optimization model, taking into account the demand variability, to determine the optimal equipment selection and production planning for a AMT laser machining company. Our contribution is the use of robust optimization approach which accounts the parameters ambiguity and stochastic uncertainty (Gabrel, Murat, & Thiele, 2014). The model produces an optimal investment plan for a defined period of time and the adequate equipment required to prevents gaps in supply due to demand fluctuations. In addition, we determine the equipment required for a robust production plan, and the key productivity components and variables. We compare the results generated by a Linear Optimization model with a Robust Optimization one, in order to understand the effects of demand uncertainty in the equipment selection and production planning process. To validate our findings we present the case study of a company which offers Laser-Beam Machining (LBM) services.

The rest of this paper is structured as follows: in section 2 we present a bibliographic review of robust optimization models with different degrees of uncertainty applied to manufacture industries. In section 3 we describe the model and its parameters, and show the mathematical formulation for the optimization problem. In section 4 we explain the Robust Optimization methodology based on the approach of (Bertsimas & Sim, 2004). Section 5 presents a case study where we apply the proposed model to a company which offers laser cutting services. In section 6, we analyze the results of the model, discussing the level of robustness and the implications of them. Finally, we conclude the research and present potential further research that could be continued on this topic.

2.2. Literature Review

Investment in new technologies, such as AMT, has become a requirement for companies to stay competitive. Benefits from investing in AMT arise from: reduced inventory, less floor space, improved return on equity, reduction in unit production cost and cycle time, increased flexibility, improved product quality, increased productivity, quick response to customer demand, increased ease of operation, and improved employee relations (Gustavsson, 1984; Cheng et al., 2018; Dıaz, Machuca, & Álvarez-Gil, 2003; Jonsson, 2000; Chung, 1991). A growing global market competition along with a diversity of customers, adds uncertainty into the investment decisions; and enterprises are paying increasing attention to the agile, networked, service-oriented, green, social, and other manufacturing characteristics (Tao, Cheng, Zhang, & Nee, 2017). For companies to remain competitive, is crucial to determine the optimal investment, the durability of the equipment and the correct timing for replacement or reinvestment (Smith, 1961). On the other hand, capacity planning in arguably the most critical factor for long term success in the manufacturing industry (Wu, Erkoc, & Karabuk, 2005). Therefore, investment and production decisions must be made jointly. Mula et al. (Mula et al., 2006) reviews various production planning models that account for uncertainty in their mathematical formulation. Some of the reviewed models are used for: aggregate planning, material requirement planning, capacity planning, among others.

Investing in AMT generally involves a high level of investment, with a moderate-to high level of risk in the adoption of the technology and payback periods which extend longer than those that business enterprises are used (Chan et al., 2001). Dixiti et al. (Dixit & Pindyck, 1993) states that the investment decision has three main characteristics. First, investment decision are partially or completely irreversible. Second, there is a level of uncertainty over the futures rewards of the investment. Finally, postponing the investment allows the decision-maker to have more information and less uncertainty; however, it cannot be completely eliminated. (van Mieghem, 2003) states that the inclusion of uncertainty can typically improve results' accuracy while optimizing a capacity plan. Authors have proposed different approaches to obtain an optimal capacity plan. (Paraskevopoulos et al., 1991) developed a robust approach for capacity planning that accounts for demand uncertainty. Alternatively, (Eppen et al., 1989) developed a long term capacity planning model for an automobile manufacturer using different demand scenarios.

In the AMT literature the decision to invest in a given technology has been approached using multi-criteria decision-making methods (Hodgett, 2016; Yazdani-Chamzini, 2014;

Hadi-Vencheh & Mohamadghasemi, 2015) or fuzzy techniques (Chan et al., 2006; Evans et al., 2013; Sadeghi et al., 2012). The use of optimization approaches has been studied in the mining context (C. Burt et al., 2016; C. N. Burt & Caccetta, 2014) and in the print production environments (Rai et al., 2015). However none of them has accounted for demand uncertainty into their decision making process.

Optimization models are frequently used across all companies in the manufacturing industry. Although models are exposed to different sources of uncertainty, some companies decide to optimize production assuming that all information is known; this is not adequate for real problems (Alvarez & Vera, 2014; Bertsimas, Brown, & Caramanis, 2011). The literature shows that there are two main approaches to incorporate data uncertainty in a single and in multi-period decision-making process, which are Stochastic Programming (SP) and Robust Optimization (RO) (Varas, Maturana, Pascual, Vargas, & Vera, 2014; Bertsimas & Thiele, 2006; Gorissen, Yanıkoğlu, & den Hertog, 2015; Maggioni, Potra, & Bertocchi, 2014).

Stochastic Programming (SP) is a rigorous approach that assumes that the probability distributions of the uncertain parameters are known and can be accurately estimated. This approach requires an accurate probabilistic description of the variables and data, which in some cases is very difficult to obtain (Birge & Louveaux, 2011; Maggioni et al., 2014). Stochastic Programming is the most complex and hard method for managers to understand (Varas et al., 2014). Its use has been widely reported in the literature (Birge & Louveaux, 2011) in fields as system capacity expansion, portfolio selection, scheduling maintenance personnel, investment decision, among others.

On the other hand, Robust Optimization (RO) is a relatively new methodology which includes uncertainty into the problem model. Its goal is to obtain solutions that remain feasible for any uncertain outcome of a given set of parameters (Bertsimas et al., 2011; Bertsimas & Sim, 2004; Bertsimas & Thiele, 2006; Ben-Tal & Nemirovski, 1999; Beyer & Sendhoff, 2007; Ben-Tal, Goryashko, Guslitzer, & Nemirovski, 2004). This approach accounts for variations in the optimal results caused by fluctuations on the parameters

(Marijit, 2009). Thus, it delivers solutions that are less sensitive to the variations of the parameters (Deb & Gupta, 2006). The RO methodology generates solutions that are progressively less sensitive to data uncertainty; it does not need detailed probabilistic knowledge nor specific distributions of the uncertain parameters (Gorissen et al., 2015; Mulvey, Vanderbei, & Zenios, 1995). Unlike stochastic programming, RO has been more extensively used as it requires less information and still accounts for uncertainty. Nonetheless, it is important to mention that RO is more complex and requires more computational power than other methodologies (Mulvey et al., 1995).

The RO methodology was first developed by (Soyster, 1973). He proposes a deterministic linear optimization model which finds solutions that will always be feasible for all the data points in a convex set. So, in a RO approach the decision maker needs to determine the level of robustness or level in which the problem will continue to be feasible under an uncertain outcome of a given set of parameters (Bertsimas & Sim, 2004; Bertsimas et al., 2011; Varas et al., 2014). The RO approach requires additional inputs called uncertainty budgets, which should be based on the decision-maker risk aversion since these inputs determine the conservatism of the solution. The impact of uncertainty budgets variations on the problem solution are not obvious and the results' analysis tends to be more complicated. Several authors describe the structure that uncertainty sets must have for the RO outputs to be accurate and reliable. In addition, literature defines different types of reformulations for constraints that include uncertain variables or parameters. (Alvarez & Vera, 2014) proposes a robust optimization with a polyhedral uncertainty set for a Sawmill planning problem where the uncertainty lies in the yield parameter for the cutting process. The same approach is used by (Varas et al., 2014) who presents a robust optimization model for a Sawmill production scheduling with uncertainty in product demand and raw material supply. On both examples, the authors evaluate the robustness and conservatism level of the solution, providing several managerial insights that could help production schedulers to choose the appropriate level of conservatism based on the uncertainty they are facing. (Sungur et al., 2008) compare three uncertainty sets (Box, Ellipsoidal, and Convex Hull) for a capacitated vehicle routing problem with uncertain demand. The conclusions are that box uncertainty sets lead to the most unfilled demand and highest cost; convex hull uncertainty sets result in the lowest value of the objective function; and ellipsoidal uncertainty sets generate an objective function value greater than the other two approaches. In (Sungur et al., 2008)'s model all uncertainty behave with similar trends, and the differences are caused by the chosen uncertainty set.

In the case of an specific AMT technology, such as Laser-Beam Machining (LBM). The optimization literature has mostly focused on optimizing the machine's parameters to maximize effectiveness and efficiency (Rao & Kalyankar, 2014). For more details of Laser cutting process we recommend to review (Kalpakjian & Schmid, 2013; Powell, 1993; Ion, 2005; Steen, 2010). The main differences between laser technologies are conduction, machine power and the materials that can be processed. CO_2 laser is an electrically pumped gas laser that radiates at a wavelength of 10.6μm (Brecher, Emonts, Rosen, & Hermani, 2011). It is used for fine cutting metals sheets at high speeds due to its high average beam power, improved efficiency, and beam quality. Nd: Yag laser is an optically pumped solid state laser that operates at a wavelength of 1.06μm (Gadallah & Abdu, 2015). Due to its shorter wavelength, this laser is more suitable for processing reflective metals, in addition to the common ones. The fiber laser is especially suited for efficient material treatment due to its high beam quality and high power efficiency of up to 40%. In this type of laser, the radiation of the light is guided by the motor spindle via a light conducting cable (Brecher et al., 2011). (Gadallah & Abdu, 2015) develop a model to optimize the main parameters for laser cutting machines, as cutting speed, power and gas pressure.

2.3. Equipment Investment and Production Planning Model

2.3.1. Description of the model

To determine the optimal equipment selection and production planning for a AMT laser machining company, we have divided the problem into two integrated decision models: first, an investment and second, a production planning model. The investment model tries to determine which is the best selection LBM technology and the moment to make the investment decision, in order to increase the production capacity of a company which manufactures (p) different types of products. Based on the machine configuration generated by the first model, the second model determines best production strategy which maximize the economic benefits of the company.

For our case study, there are (m) different types of machinery each with different features. Each machine was modelled using two parameters that directly affect the production process. The first parameter is the production rate (cutting velocity) (ν_p^m) which represent how many products can be processed in a certain period of time. This parameter varies depending on the products and on the processing machine. The second one is the production cost (G_p^m) per period of time, which represent the amount of money that the company has to pay for a machine to produce a specific product on a period of time. (α_p) represents the price of each minute of production for a specific item.

On each time period, the company faces a certain demand ($D_{p,t}$) for each product. Demand must be filled to avoid a monetary penalization of (ρ_p) for not filling it (penalization is different for each product). The company can invest only in one machinery per period of time, which will be operative at the beginning of the next period. Each machine has a value of (γ^m), and can only process a certain set of products.

We will now describe the technical parameters which govern de LBM process and the proceed to develop the investment and production model.

2.3.2. Laser Cutting machinery parameters: cutting speed and gas consumption

The performance of laser beam machining mainly depends on laser parameters (e.g. laser power, wavelength, mode of operation), material parameters (e.g. type, thickness) and process parameters (e.g. feed rate, focal plane position, frequency, energy, pulse duration, assist gas type and pressure) (Dubey & Yadava, 2008b). We will focus on 2-D cutting laser machines and the two main parameters which affect the technology choice and operational costs are: the cutting speed and the gas consumption.

2.3.2.1. Laser Cutting speed

Laser Cutting Speed is one of the main parameters for this advanced production systems. It controls the heat transfer of the laser to the material, and it requires to be adjusted according to the material to be cut and its thickness. There is an inverse relationship between cutting velocity and material thickness; hence thick materials need lower cutting speed while thinner ones can have higher cutting speeds. On Appendix A we developed a simple model for understanding the physical process.

Table 2.1. Components of equation for laser cutting

P_c:	Laser Power, W
w:	kerf, m
t:	thickness, m
$\overrightarrow{v_c}$:	cutting speed, m/s
ρ:	density of the processing material, kg/m3
η_c:	efficiency of laser beam

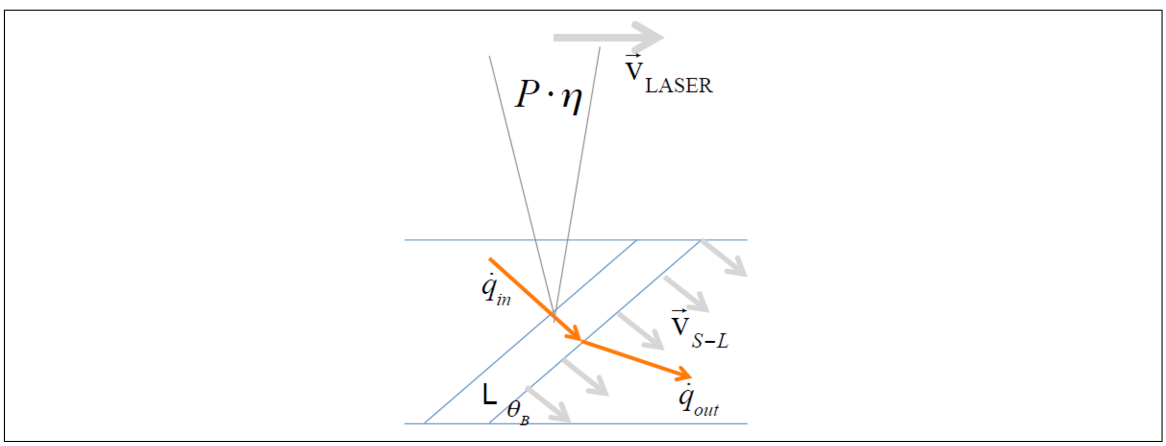

Figure 2.1. Representation of laser cutting dynamics process in a material

Table 2.1 shows the parameter needed to determine the laser cutting speed. The laser cutting process is represented in Figure 2.1. We model two different laser technologies, fiber laser and CO_2 laser. To obtain the cutting velocity we used different laser power values for both technologies. In Figure 2.2 we can observe the behaviour of the laser cutting velocity for both technologies, and for different laser power and thickness of an specific material. In equation 2.1 we present the mathematical relation between the parameters and the laser cutting speed for different materials.

$$|\vec{v}_{S-L}| = \frac{\frac{P\eta}{A\rho}}{\Delta H_{S-L}} = \frac{P \cdot \eta \cdot \cos\theta_B}{t \cdot w \cdot \rho \cdot \Delta H_{S-L}} \tag{2.1}$$

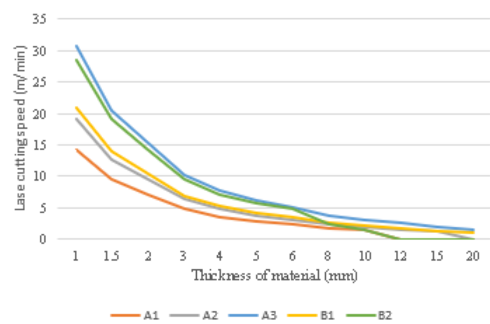

Figure 2.2. Relationship between laser cutting speed vs thickness for different Laser types and power, for steel material.

2.3.2.2. Gas Consuming Model

A second important parameter for deciding which laser cutting machines technology to acquire is the amount of gas that machines consume during the cutting process, since gas consumption is one of the most relevant costs of the laser cutting operation. Figure 2.3 shows a simple representation of the process. There is a gas cylinder, a gas regulator (1) and the nozzle (2), from where the laser is beamed out. In Appendix 2 we present a more extensive explanation of the process. Equations 2.2 and 2.3 represents the relation between the gas velocity through the nozzle and the flow. It is important to take into consideration that each material requires a specific nozzle diameter.

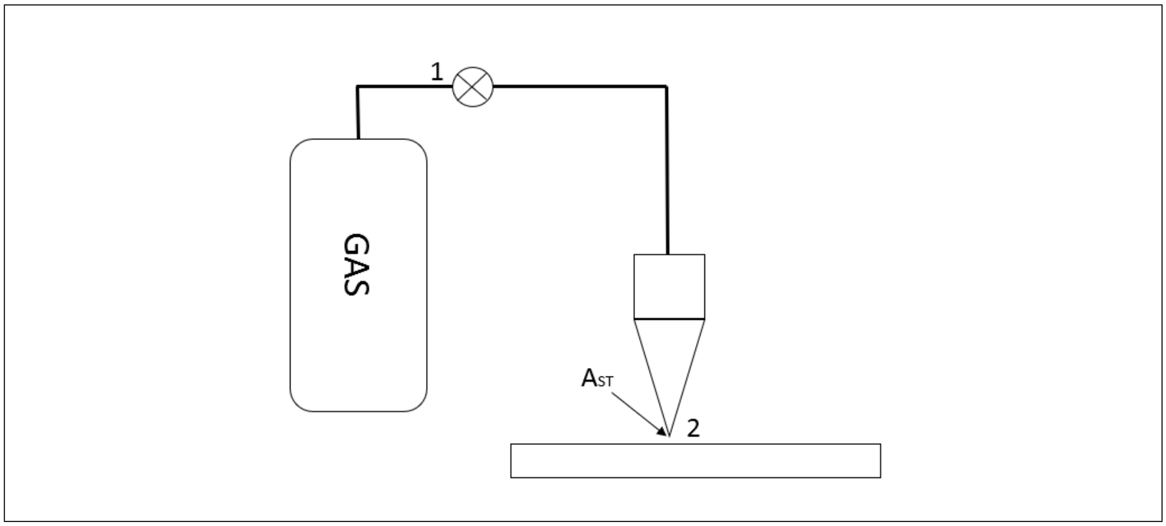

Figure 2.3. Representation of gas transportation in laser cutting process

$$\vec{v}_2 = \sqrt{2\left(\frac{P_1}{\rho_1} - \frac{P_2}{\rho_2}\right)} \tag{2.2}$$

$$Q = \vec{v}_2 \cdot A_{nozzle} \tag{2.3}$$

2.3.3. Deterministic optimization model formulation

Using the previous developed equations of laser speed and gas consumption for each type of technology we will proceed now to present the cost optimization model to determine the optimal equipment selection and production planning for a AMT laser machining company. We will first present the model indices, variables and parameters and the present the formulation of the deterministic optimization model.

Table 2.2. Indices

$\mathcal{P}$:	Set of Products (indexed by p)
$\mathcal{M}$:	Set of Machines (indexed by m)
$\mathcal{T}$:	Set of Periods (indexed by t, j)

Table 2.3. Variables

$x_{p,t}^{m,j}$:	Cutting distance of product p, in machine m,j in period t
$w_t^{m,j}$:	1 if a machine m,j is available on period t, 0 if not
$y_t^{m,j}$:	1 if an investment decision on machine m,j is made on period t, 0 if not

Table 2.4. Parameters

α_p:	sale price for each cutting minute of product p
ν_p^m:	cutting speed for each product p in machine m
G_p^m:	sale cost for each minute of gas consumed by machine m in product p
E_p^m:	sale cost for each minute of electricity consumed by machine m in product p
$D_{p,t}$:	Demand of product p in period t
ρ_p:	penalty cost for each minute of incomplete demand of product p
γ^m:	Price of machine m
Ω:	Available time for each period
r:	Discount rate
Mat^m:	set of products that machine m can process
Maq^m:	set of machines that can not be acquired

(Q1) Max $z = \sum_t \left[\sum_p \sum_m \sum_j \frac{x_{p,t}^{m,j}}{\nu_p^m}(\alpha_p - (G_p^m + E_p^m)) - \sum_p \rho_p (D_{p,t} - \sum_m \sum_j \frac{x_{p,t}^{m,j}}{\nu_p^m}) - \sum_m \sum_j y_t^{m,j} \gamma^m \right] \frac{1}{(1+r)^t}$

subject to:

$$\sum^{M} \sum^{J} \frac{x_{p,t}^{m,j}}{\nu_p^m} \leq D_{p,t} \qquad \forall p, t \tag{2.4}$$

$$\sum^{M} y_t^{m,j} \leq 1 \qquad \forall t \tag{2.5}$$

$$\sum^{P} \sum^{M} \sum^{J} \frac{x_{p,t}^{m,j}}{\nu_p^m} \leq \Omega \sum^{M} \sum^{J} w_t^{m,j} \qquad \forall t \tag{2.6}$$

$$x_{p,t}^{m,j} \leq M w_t^{m,j} \qquad \forall p, m, j, t \tag{2.7}$$

$$y_{t-1}^{m,j} = w_t^{m,j} - w_{t-1}^{m,j} \qquad \forall m, j, t \tag{2.8}$$

$$x_{p,t}^{m,j} = 0 \qquad \forall i, j, t \notin Mat^m \tag{2.9}$$

$$w_0^{m,j} = 1 \qquad \forall j \notin Maq^m \tag{2.10}$$

$$y^{m,t}, w^{m,t} \in (0, 1) \qquad |\forall m, t \tag{2.11}$$

$$x_{p,t}^{m,j} \geq 0 \qquad |\forall p, t, m, j \tag{2.12}$$

The equipment selection and production planning problem (Q1) can be modelled as a linear programming optimization model. At his stage the model is deterministic, so we do not consider uncertainty at this stage. Later we will present how add uncertainty into the model by applying robust optimization..

The objective function (Q1) maximizes the company profit by obtaining the operational income of each type of technology and subtracting both the investment cost and the cost of lost sales. We determine the operational cost for each product and machinery combination, by determining the equivalent cutting speed (calculated in section 4.2.1) and relate it to the time required to cut and finally obtaining the gas and electricity consumption during the process. The cost of not filling the demand was estimated as 30% of the selling price of each product. The first constraint 2.4 ensures that on each period of time, production time sold for a specific item is never higher than the demand. Constraint 2.5 guarantees that the company can only buy one machine per period of time, due to the considerable cost of each piece of equipment. Constraint 2.6 limits the use of the machinery to the available operational time of each period. We define an operational time as 145,800 minutes per machine per period. Constraint 2.7 represents the relation between two decision variables, where a machine is unavailable for production if it has not been purchased. Constraint 2.8 shows that each machine can only process specific products. Constraints 2.9 and 2.10 represent the initial condition of the problem. Finally, constraints 2.11 and 2.12 correspond to the nature of each decision variable.

2.4. Robust reformulation

In this section, we will describe how we added uncertainty into the previous formulation using a RO approach proposed by Bertsimas and Sim (2004). First, we will describe how the RO methodology works and the apply it to model the demand uncertainty.

2.4.1. Robust Optimization methodology

Our first assumption is that uncertainty in the parameters affects only the elements in the right hand of the constraint matrix. We will start with a general representation of the deterministic model, given by:

$$
\begin{aligned}
\text{minimize} \quad & c^T x \\
\text{subject to:} \quad & a_i^T \leq b_i \quad \forall i = 1, ..., m \\
& l \leq x \leq u
\end{aligned}
\tag{2.13}
$$

To model uncertainty into the constraints (**A**), each uncertain coefficient $a_{i,j}$ can take values based on a symmetric distribution with mean equal to the nominal value $a_{i,j}$ in the interval $[\overline{a}_{i,j} - \hat{a}_{i,j}, \overline{a}_{i,j} + \hat{a}_{i,j}]$. The exact value of the deviation component $\hat{a}_{i,j}$ is unknown, but can be estimated. As is unlikely that all coefficients will be equal to their nominal value, it is also unlikely that they will be equal to their worst-case value.

The objective now is to adjust the level of conservativeness of the solution, in order to achieve a reasonable trade-off between robustness and performance. On literature, authors defined the scaled deviation of a parameter $a_{i,j}$ from its nominal values as $z_{i,j} = (a_{i,j} - \overline{a}_{i,j})/\hat{a}_{i,j}$. This variable can only take values between [-1,1]. For each constraint that is subject to uncertainty, a threshold (not necessarily integer) is introduced to bound the total variations of uncertain parameters, as follows:

$$\sum_{(i,j)\in J} |z_{i,j}| \leq \Gamma$$

where J is the set of indices of uncertain parameters. When $\Gamma = 0$ we obtain a average case, and when it takes $\Gamma = |J|$ we get the worst case. Bertsimas & Sim Bertsimas and Sim (2004) states that this allows greater flexibility to build a robust model without excessively affecting the optimum of the optimization problem. This decision will be defined according to the risk aversion of the decision maker. To build the robust counterpart of the nominal problem, we will follow the uncertainty set proposed by Bertsimas and Sim (2004). It is constructed by maximizing the left-hand side of the constraints over the set of admissible scaled deviations. This leads to the following problem:

$$\begin{aligned} \text{maximize} \quad & c^T x \\ \text{subject to:} \quad & \overline{a}_i^T x + \beta_i(x, \Gamma_i) \leq b_i, \quad \forall i = 1, ..., m \\ & x \geq 0 \end{aligned} \tag{2.14}$$

where $\overline{a}_i^T$ represents the nominal data for row i and the protection function for each constraint $i = 1, ..., m$ is defined as a new optimization problem, which objective function consist in: $\beta_i(x, \Gamma_i) =$ maximize $\sum\limits_{j\in J_i} |x_j|\hat{a}_{i,j} z_{i,j}$ subject to $\sum\limits_{j\in J_i} \leq \Gamma_i$ and $\leq z_{i,j} \leq 1, \forall j \in J_i$. By the application of strong duality we can reformulate the problem as equivalent to:

$$\begin{aligned} \text{maximize} \quad & c^T x \\ \text{subject to:} \quad & \overline{a}_i^T x + z_i \Gamma_i + \sum_{j\in J_i} p_{i,j} \leq b_i, \quad && \forall i \\ & z_i + p_{i,j} \geq \hat{a}_{i,j} y_j, && \forall i, j \in J_i \\ & -y_j \leq x_j \leq y_j, && \forall j \\ & y_j, z_i, x_j \geq 0, && \forall j \end{aligned}$$

$$p_{i,j} \geq 0, \qquad \forall i, j \in J_i \tag{2.15}$$

where variables z_i and $p_{i,j}$ are dual variables of the problem 2.15. The problem is linear, so there is no difficulty on solving suing standard linear methods.

2.4.2. Uncertainty in Demand

We will consider uncertainty only on the demand parameter $D_{p,t}$. Using a RO approach we will find solutions, which up to certain level, are still feasible up to certain degree of demand variability.

To simplify the modelling of the robust counterpart of the model (Q1), we will substitute $D_{p,t}\rho_p$ with $\Pi_{p,t}$ in the objective function, and add the following constraint to the model.

$$D_{p,t}\rho_p \geq \Pi_{p,t} \qquad |\forall p, t \tag{2.16}$$

This change of variable allows us to remove $D_{p,t}\rho_p$ from objective function without affecting the problem's solution. To re-formulate (Q1) as a robust model, we will rewrite constraints 2.4 and 2.16 which contain an uncertain parameter and add $\Gamma_{p,t}$ as the uncertainty budget for the new constraints.

$$\begin{aligned}
\beta_{p,t}(\hat{D}_{p,t};\Gamma_{p,t}) = \text{Max } &\{\sum_{l}^{t} \rho_p z_{p,l} \hat{D}_{p,l}\} \\
\text{subject to: } \sum_{l}^{t} z_{p,l} &\leq \Gamma_{p,t} \qquad \forall p, l \leq t \\
0 \leq z_{p,l} &\leq 1 \qquad \forall p, l \leq t
\end{aligned} \tag{2.17}$$

Using duality we can transform 2.17 into the following problem:

$$\text{(Q2) min } \{v_{p,t}\Gamma_{p,t} + \sum_{l}^{t} s_{l,p,t}\}$$

$$\text{subject to: } v_{p,t} + s_{l,p,t} \geq \hat{D}_{l,k} \qquad |\forall p, t, l \leq t \tag{2.18}$$

$$v_{p,t}, s_{l,p,t} \geq 0 \qquad |\forall l \leq t \tag{2.19}$$

We can write another protection function for constraint 2.4 of the deterministic model. If we replace the original deterministic model constraints with the robust ones, by the previous obtained protection functions, we obtain:

$$\sum_{j}^{T} \sum_{m}^{M} \frac{x_{p,t}^{m,j}}{\nu_p^m} \leq \overline{D}_{p,t} - \Gamma_{p,t} q_{p,t} - \sum_{k}^{t} u_{k,p,t} \qquad |\forall p, t, k \leq t$$

$$\Pi_{p,t} \geq \overline{D}_{p,t} + \Gamma_{p,t} v_{p,t} + \sum_{l}^{t} s_{l,p,t} \qquad |\forall p, t, l \leq t$$

Additionally, we add two more constraints to the original problem to account for the relation between dual variables, the uncertainty budget $v_{p,t}$, $q_{p,t}$, $s_{l,p,t}$, $u_{k,p,t}$ and uncertainty variable (constraints 2.18 and 2.23). We also add constraints corresponding to the nature of the new variables (2.31).

The full robust counterpart (Q4) is the following:

$$\text{(Q4) Max z} = \sum_{t} \left[\sum_{p} \sum_{m} \sum_{j} \frac{x_{p,t}^{m,j}}{\nu_p^m} (\alpha_p + \rho_p - (G_p^m + E_p^m)) - \sum_{p} \rho_p \Pi_{p,t} - \sum_{m} \sum_{j} y_t^{m,j} \gamma^m \right] \frac{1}{(1+r)^t}$$

subject to:

$$\sum_{j}^{T} \sum_{m}^{M} \frac{x_{p,t}^{m,j}}{\nu_p^m} \leq \overline{D}_{p,t} + \Gamma_{p,t} q_{p,t} + \sum_{k}^{t} u_{k,p,t} \qquad |\forall p, t, k \leq t \tag{2.20}$$

$$\Pi_{p,t} \geq \overline{D}_{p,t} + \Gamma_{p,t} v_{p,t} + \sum_{l}^{t} s_{l,p,t} \qquad |\forall p, t, l \leq t \tag{2.21}$$

$$v_{p,t} + s_{l,p,t} \geq \hat{D}_{l,k} \qquad |\forall p, t, l \leq t \tag{2.22}$$

$$q_{p,t} + u_{k,p,t} \geq \hat{D}_{p,k} \qquad |\forall p, t, k \leq t \tag{2.23}$$

$$\sum^{M} y_t^{m,j} \leq 1 \qquad \forall t \tag{2.24}$$

$$\sum^{P}\sum^{M}\sum^{J} \frac{x_{p,t}^{m,j}}{\nu_p^m} \leq \Omega \sum^{M}\sum^{J} w_t^{m,j} \qquad \forall t \tag{2.25}$$

$$x_{p,t}^{m,j} \leq M w_t^{m,j} \qquad \forall p, m, j, t \tag{2.26}$$

$$y_{t-1}^{m,j} = w_t^{m,j} - w_{t-1}^{m,j} \qquad \forall m, j, t \tag{2.27}$$

$$x_{p,t}^{m,j} = 0 \qquad \forall i, j, t \notin Mat^m \tag{2.28}$$

$$w_0^{m,j} = 1 \qquad \forall j \notin Maq^m \tag{2.29}$$

$$y^{m,t}, w^{m,t} \in (0,1) \qquad |\forall m, t \tag{2.30}$$

$$x_{p,t}^{m,j}, q_{p,t}, u_{l,p,t}, v_{p,t}, s_{k,p,t} \geq 0 \qquad |\forall p, t, m, j, k, l \tag{2.31}$$

The objective function of (Q4) is similar to the last one (Q1). The main difference is the term $\Pi_{p,t}$ which represents the uncertain parameter (Demand). Most constraints equal the ones of (Q1) [2.24, 2.25, 2.26, 2.27, 2.28, 2.29, 2.30]. The new demand satisfaction constraint is 2.20, which has the nominal value of the demand, the uncertainty budget and the dual variables of the problem (Q3). Constraint 2.21 allows the removal of the demand from the objective function. This constraint has the nominal value of the uncertain parameter, the uncertainty budget and the dual variables of the problem (Q2). Constraints 2.22 and 2.23 are related with the transformation of the linear problem into the robust problem through the hard duality theorem of optimization.

2.5. Numerical case study

In this section, we will apply our models and perform a numerical case study using the values of a company which offers laser cutting machinery services. The model will determine which should be the best configuration of machinery and production scheme for given level of demand uncertainty. We will determine the technology decision, from a given pool of options available in the market, and amount of machines that should be acquired and the production plan needed to fill out the demand. For this study, we will have available five types of laser beam cutting technologies (M). Table 2.7 presents the information for each type of technology. Additionally, we consider that 11 different types products (P) to be processed. Not all technologies can cut all products, so in Table 2.10 we present which products can be processed on each machine. The model is executed for six periods of time (T) which is equivalent to 5 years. The discount rate is $\sigma = 0.1$. The companies starts with two types of technologies: A2 and B1. We assume a normal distribution for the demand of each product, where the mean and standard deviation are based on historical information

We programmed the optimization models using the IBM ILOG CPLEX Optimization Studio and there were optimized using a laptop with 8 GB of RAM with a 2.4GHz dual-core Intel Core i7.

Laser cutting and gas consumption parameters model were determined for each technology using the previous equations. These numbers are presented in Table 2.5 and Table 2.6. Demand was estimated from the company that offers laser cutting services, using historical information between 2012 and 2017.

Table 2.5. Gas consumption levels for each technology obtained from equation 3.3.2

P_p	Q m^3/hr	$/hr
P1	0.9117	147.6
P2	0.8754	151.2
P3	0.8009	160.2
P4	0.9794	263.4
P5	0.9400	285.0
P6	0.8594	286.8
P7	0.9794	263.4
P8	0.9400	285.0
P9	0.9648	286.8
P10	0.9117	147.6
P11	0.9308	258 .0

Table 2.6. Laser cutting speeds for each technology obtained from equation 3.3.1, in m/min

	Machine type and technology				
P_p	A1	A2	A3	B1	B2
P1	7.15	9.53	15.39	10.48	14.30
P2	2.38	3.18	5.13	3.49	4.77
P3	-	1.59	2.57	1.75	0.04
P4	6.78	9.04	16.68	9.94	13.56
P5	2.26	3.01	3.48	3.31	4.52
P6	1.36	1.81	1.25	1.99	2.71
P7	25.72	34.29	55.39	37.72	51.53
P8	12.86	17.14	27.70	18.86	25.72
P9	8.57	11.43	18.46	12.57	17.14
P10	-	-	-	6.81	9.81
P11	2.38	3.18	5.13	-	-

Table 2.7. Available types of technology

Laser type	Machine type and technology				
	A1	A2	A3	B1	B2
A	X	X	X		
B				X	X

Table 2.8. Ratio between laser cutting speed and the equipment price, in m/min/$

	Machines type and technology				
P_p	A1	A2	A3	B1	B2
P1	1.02E-5	1.06E-5	2.23E-5	1.33E-5	1.43E-5
P2	3.40E-6	3.53E-6	7.43E-6	4.42E-6	4.77E-6
P3	-	1.77E-6	3.72E-6	2.22E-6	4.00E-8
P4	9.69E-6	1.00E-5	2.42E-5	1.26E-5	1.36E-5
P5	3.23E-6	3.34E-6	5.04E-6	4.19E-6	4.52E-6
P6	1.94E-6	2.01E-6	1.81E-6	2.52E-6	2.71E-6
P7	3.87E-5	3.81E-5	8.03E-5	4.77E-5	5.15E-5
P8	1.84E-5	1.90E-5	4.01E-5	2.39E-5	2.57E-5
P9	1.22E-5	1.27E-5	2.68E-5	1.59E-5	1.71E-5
P10	-	-	-	8.62E-6	9.81E-6
P11	3.40E-6	3.53E-6	7.43E-6	-	-
Average	9.02E-2	9.52E-6	1.99E-5	1.23E-5	1.31E-5

Table 2.9. Percentage of demand of special products over the total demand per years, in %

Products	Years					
	0	1	2	3	4	5
P3	23.26	11.82	8.47	6.81	5.80	5.10
P10	0.10	0.17	0.05	0.02	0.06	0.04
P11	0.53	0.26	0.16	0.11	0.09	0.07

Table 2.10. Types of products which can be process on each machine technology

Equipment	P1	P2	P3	P4	P5	P6	P7	P8	P9	P10	P11
A1	X	X		X	X	X	X	X	X		X
A2	X	X	X	X	X	X	X	X	X		X
A3	X	X	X	X	X	X	X	X	X		X
B1	X	X	X	X	X	X	X	X	X	X	
B2	X	X	X	X	X	X	X	X	X	X	

2.6. Result and Discussion

In this section, we present the results obtained from the deterministic optimization model and the RO model. In addition, we study the differences that arises between the models and how does the addition of uncertainty in a RO model can generate economic benefits. Both models have different results as the RO model considers uncertainty on the demand and the LO model which does not consider uncertainty at all. In Figure 2.4 we can observe the results on the number of machines that each model acquires on each period. The deterministic model invest in one machine type A1 in the first period, and never invest again. On the other hand, the robust optimization model shows that the optimal decision is to invest on three machines of type A1 in periods 1,2 and 3. Table 2.11 and Figure 2.5 shows that as we increase the uncertainty budget, the investment plan proposed by the robust model involves purchasing more machines. Also, as the robustness level is increased, the model advances the moment in which the investment is made, until the level of 51-66 robustness, in which the model invests in an additional machine. The machine with technology type A1 seems to be the preferred one.

Table 2.11. Machinery investment during the years

Robustness level	0	1	2	3	4	5	Total Machines
LO	A1						3
0-17	A1						3
18	A1			A1			4
19-28	A1		A1				4
29-50	A1	A1					4
51-66	A1	A1	A1				5

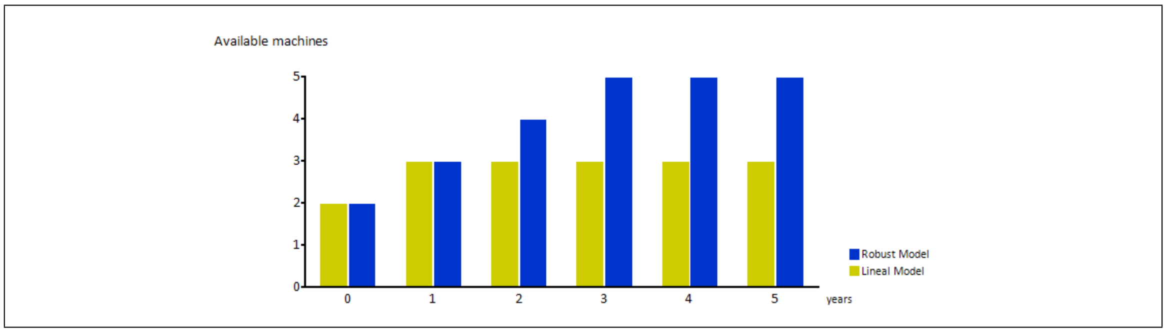

Figure 2.4. Machinery configuration of the proposed models

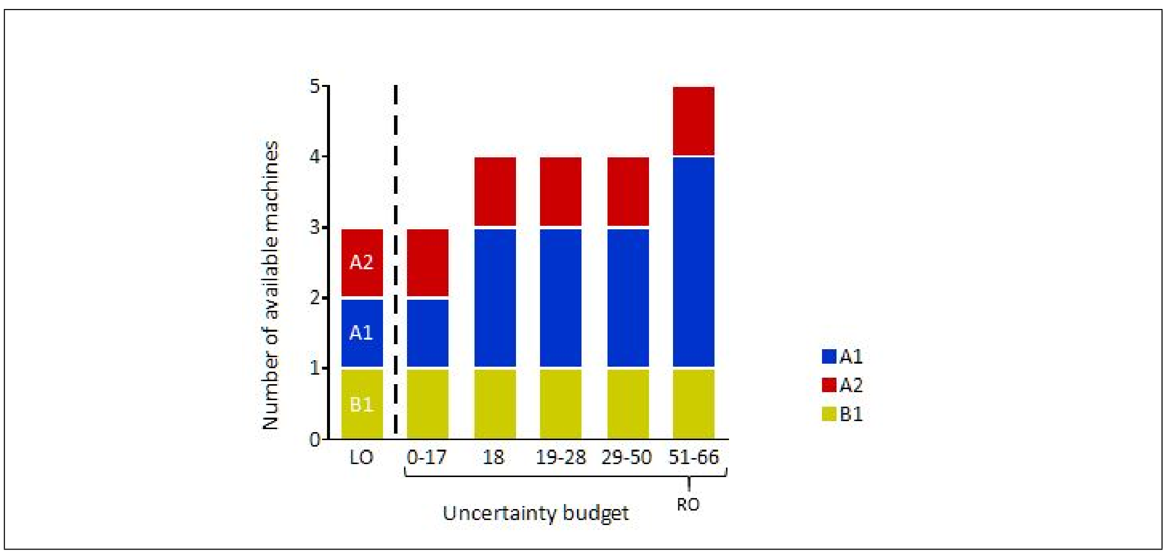

Figure 2.5. Investment plan for each level of uncertainty LO and RO models

Figure 2.6 and Table 2.12 shows the percentage change and values in the objective function for different uncertainty budgets for a given demand variability level. The optimal production plan in the deterministic model does not include uncertainty, so its value remains stable (yellow line). When the uncertainty budget is less than 17, the result of the RO model is slightly smaller than the result of the deterministic one (LO) because the uncertainty budget is too small to take care of the demand variability. As we increase the uncertainty budget, the RO model surpasses the deterministic model because the technology and productive decisions account for the variability on the demand. When the uncertainty budget reaches the 51-66 level, the objective value drops drastically, below the deterministic model, because the decision maker has over-invested in the buffer over the demand uncertainty.

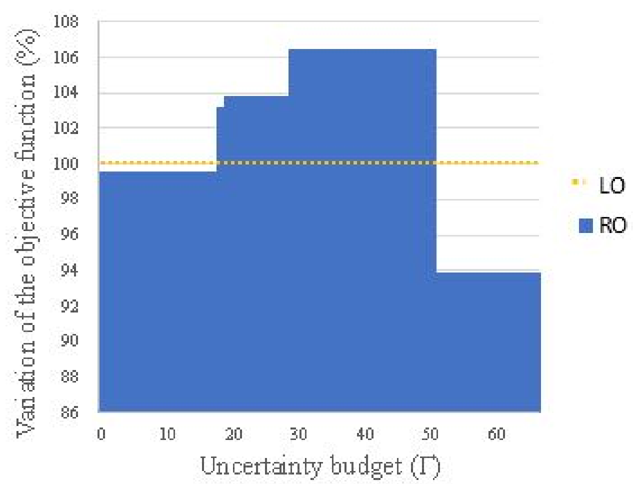

Figure 2.6. Objective function percentage value for different level of uncertainty budgets

Table 2.12. Objective function value and its change under different levels of uncertainty

Lineal	Robust	Uncertainty budget	%
4,142,026	4,120,197	0-17	-0.52%
4,142,026	4,274,466	18	3.19%
4,142,026	4,296,642	19-28	3.73%
4,142,026	4,406,702	29-50	6,39%
4,142,026	3,888,294	51-66	-6.13%

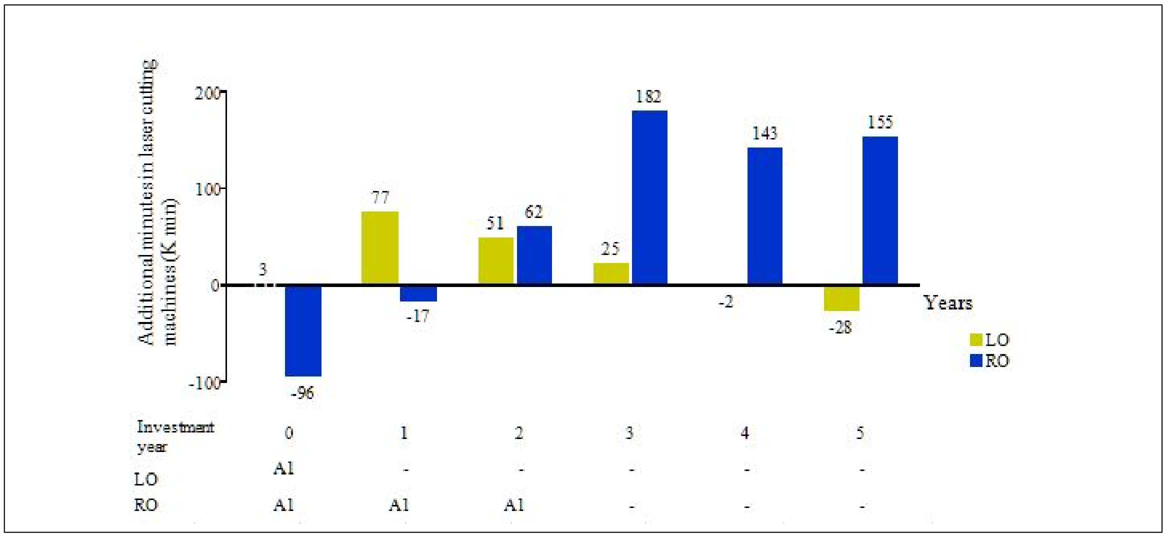

Figure 2.7. Additional minutes for LO, RO productive plan

To analyse the effect that the uncertainty budget has on the technology and the productive plan, we must look at the levels of unfilled demand for each product during the time frame. Figure 2.7 show the minutes consumed by the machines during the years. The deterministic (LO) model (fixed demand) has negative minutes on years 4 and 5 because is unable to fill the demand. On the other hand, the RO model during the first two years the is unable to fill all the required demand, but during the next years, it invests in more technology having enough the capacity to fulfil the required demand.

The objective behind the robustness budget is to add additional capacity into the system to be able to take care of the demand variability. Nevertheless this additional capacity comes with a cost,hence it is important to determine the adequate level of robustness to cope with the demand variability. Figure 2.8 shows the objective value under different levels of demand uncertainty for the deterministic (LO) and RO model. For the LO model we can observe that as the demand uncertainty increases the objective value constantly decreases, indicating that the demand uncertainty negatively affects the deterministic model. On the other side, if we observe the RO model, as the variability in demand is increased the model performance is improved, because the robustness investment is reflected in a given technology acquisition and production plan which allows the company to adjust better to the demand variability. This improvement reaches a maximum, around the 30% variability, after which it decreases indicating that the robustness is not enough to support the increased level of demand variability.

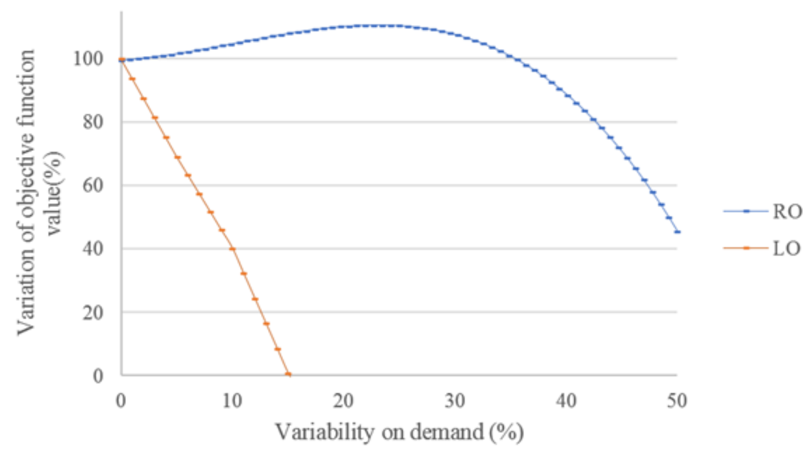

Figure 2.8. Objective function percentage value for different level of demand uncertainty for a given budget

Table 2.13. Variation of the objective function when demand deviation value changes from 0-50%

Demand deviation variation	Objective function %
0	99.5
1	99.7
2	99.9
3	100.2
:	
10	103.6
20	107.9
30	110.6
40	107.4
50	46.8

To analyse how the cost of each technology affects the acquisition decision, we will take the technology of machine A1 which is preferred in by both models, and study the impact of changing its price between 5-20%, without changing the cost of the other technologies. In Figure 2.9 we can observe how the acquisition decision is affected in the RO model under different uncertainty budgets and percentage change of technology A1. As it can be expected, the optimal solution of the RO model changes the machinery configuration, by acquiring more B1 technology instead of A1. Under small changes in the technology price (between 5 % to 10 %), the model still selects at least 1 machine of technology A1, specially under high robustness budgets. But after a 15% in price of technology A1 is too high, so the model decides only for B1 technology. These results are in line to what we can observe in Table 2.8, in here we can see that as the ratio of laser speed versus cost is higher the technology is preferred among others, as long as we can process all products, if this ration is altered due to a change in the cost of the technology, so it will be the acquisition decision.

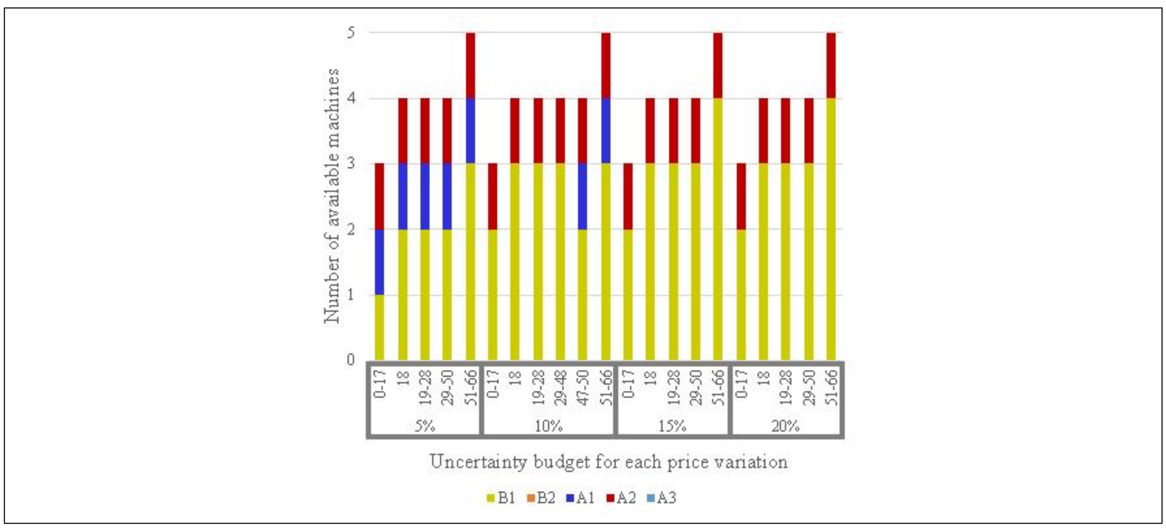

Figure 2.9. Machinery configuration for variation on A1 price from 0-20%

2.7. Conclusion

We have developed and implemented a deterministic and a robust optimization investment and production models. The objective of the models is to determine the optimal machinery configuration and a production plan which maximizes the company's benefits under demand uncertainty. The decision maker has can choose a priori the robustness level which will match their risk aversion. It is important to highlight that a higher robustness level means more protection in case of uncertainty, but it usually has a higher cost.

We implemented the model using real productive data from a laser beam cutting company and performed a sensitivity analysis to find the optimal level of robustness. Results show that the Robust Optimization model renders higher economic benefits than the deterministic linear optimization model under the presence of demand variability. The addition of uncertainty in the model leads to more realistic solutions which can be used to make strategic decisions. The RO can deliver robust solutions which are less sensitive to the parameters' variation. The sensitivity analysis shows that the optimal solution is mainly affected by the level of the uncertain demand and the cost of the laser cutting machinery. Furthermore, changes on those variables highly impact the company's economic benefits.

As the demand variability increases the optimal technology acquisition and production plan of the deterministic (LO) approach does not have enough capacity to process the entire demand. The extra capacity in the RO plan, allows the company to be covered and able to process more orders in the case that demand increases unexpectedly. The higher the uncertainty budget, the higher protection the company has to demand variations. This protection is derived from the extra capacity available, but is not free of expense. As the robustness budget is increased the decision maker may over-invested in the buffer over the demand uncertainty and the profits of the company are decreased.

It is interesting to see that the type of technology which is acquire depends upon two aspects: the range of products which they can process and the cost. As the technology becomes more expensive or the robustness budget is increased, the model selects a different technology portfolio.

Varas et al. Varas et al. (2014) explains that the Robust Optimization has three main advantages over other approaches. First, the RO approach does not require probabilistic distribution knowledge for the uncertain parameters. That information is usually hard to get as it requires to analyze large amounts of data. Second, the RO model maintains the linearity of the problem which is helpful to find the optimal solution in an efficient manner. Finally, based on the company's risk aversion, the decision-maker can determine an uncertainty budget that protects the strategic and production decisions from uncertainty. This uncertainty budget is a critical parameter required to find a robust solution.

In this work we have only assumed only one source of uncertainty: demand. Further research should incorporate other sources of uncertainty, such as cutting process parameters, cost of machinery maintenance, production yield, among others. We have also assumed that the technology costs are fixed during the period, it is common to observe that as technology becomes largely available their cost is reduced. Also we have not added the possibility that new technology and more efficient can become available in the future, rendering the current obsolete, which can change the time and type of technology to acquire.

2.8. APPENDIX

2.8.1. Laser cutting speed parameter

Every cutting process can be modeled by the first law of thermodynamics Cengel (2001) as a Power equation, which relates the volume of the material removed per unit of time and the specific energy of the cutting process. Equation 2.32 represents that relation. If we expand the equation we get:

$$\eta_c \cdot P_c = \dot{V}_c \cdot E_c \tag{2.32}$$

To develop the energy parameter, we use Stefan equation that represents the energy balance from any reaction Ramos Grez (2017).

$$\dot{m}\Delta H_{S-L} = \dot{q}_{in} - \dot{q}_{out} \tag{2.33}$$

where $\dot{m}$ represents the laser flow from the nozzle of the machine, ΔH_{S-L} represents the change in the material as a result of the melting process and $\dot{q}$ corresponds to the heat transfer. Then, we use the enthalpy from Equation 2.33 and decompose the terms of the equation.

$$\vec{v}_{S-L}|\rho A \Delta H_{S-L} = K_L A \frac{dT}{dx}|_L - K_S A \frac{dT}{dx}|_S \tag{2.34}$$

The maximum possible velocity is achieved when: $P\eta = K_L A\frac{dT}{dx}|_L$ and $\frac{dT}{dx}|_S = 0$. When we replace it in Equation 2.34 we obtain:

$$|\vec{v}_{S-L}| = \frac{\frac{P\eta}{A\rho}}{\Delta H_{S-L}} = \frac{P \cdot \eta \cdot \text{cos}\theta_B}{t \cdot w \cdot \rho \cdot \Delta H_{S-L}} \tag{2.35}$$

2.8.2. Gas consumption model

First, we assume that we have an incompressible gas, so we can use the Bernoulli equation. We use it at two points: the gas regulator and the nozzle exit. With this approach, we estimate the speed at which the gas exits the nozzle. Then, we use that information to determine the gas flow during the cutting process.

$$\frac{P_1}{\rho} + \frac{1}{2}\vec{v}_1^2 + z_1 \cdot g = \frac{P_2}{\rho} + \frac{1}{2}\vec{v}_2^2 + z_2 \cdot g \tag{2.36}$$

Where P_i represents the pressure of the gas on each point in the flow line, ρ represents the density of the gas that we are modelling, $\vec{v}_i$ is the speed of the gas on each stage, z_i represent the elevation point over the reference line, and g represents the gravity acceleration. Without losing generality we can assume that $z_1; z_2$ are similar to each other due to the configuration of the system. We can also assume that $\vec{v}_1$ is 0 as at this point the movement of the gas is almost null and $A_1 >> A_2$. Thus, we obtain:

$$\vec{v}_2 = \sqrt{2\left(\frac{P_1}{\rho_1} - \frac{P_2}{\rho_2}\right)} \tag{2.37}$$

To be more specific, P_1 is the pressure of the cylinder and P_2 corresponds to the atmospheric pressure. In the case of a compressible gas, we can assume that the gas behaves as an ideal gas. If we assume an isentropic process, we can apply the first isentropic relation for ideal gases under constant-specific-heat assumptions {Cengel2001.

$$\left(\frac{T_2}{T_1}\right)_{s=const} = \left(\frac{P_2}{P_1}\right)^{\frac{k-1}{k}} \tag{2.38}$$

where s represent the entropy of the gas, and k is the coefficient between the specific heat at constant pressure c_p and the specific heat at constant volume c_v. If we remove T_2 we obtain the following equation

$$T_2 = T_1 \left(\frac{P_2}{P_1} \right)^{\frac{k-1}{k}}$$

With those temperature values we can estimated the velocity of the gas using the kinetic theory of gases which is represented on equation 2.37.

$$v_i = \sqrt{\gamma_i \cdot R_{gas} \cdot T_i} \tag{2.39}$$

When we replace the equation with actual values, we get $\vec{v}_2$, and then we obtain the flow of the gas through the nozzle. It is important to mention that for every material-thickness combinations a special nozzle with a specific area is needed. Furthermore, the gas velocity must not overpass the sound velocity.

$$Q = \vec{v}_2 \cdot A_{nozzle} \tag{2.40}$$

3. GENERAL CONCLUSIONS AND FURTHER RESEARCH

In this work, we developed a robust optimization approach for investment and production planning, in environments where demand is exposed to uncertainty. The proposed model was used to determine the optimal investment decisions and production plan for a company that offers Laser-Beam Machining (LBM) services.

Real problems are constantly exposed to several sources of uncertainty. Companies need to protect themselves against the effects of uncertainty. Robust optimization allows companies to optimize their operations and profitability when exposed to parameters variability. Robustness is defined as the ability to overcome adverse situations and provide protection against deplorable situations. Uncertainty was handled through robustness budgets; its value is directly correlated to uncertainty sources incorporated into the problem. The performed analysis determines the trade off between the robustness budget and the companies' economic benefits.

The robustness of the problem corresponds to the company's' protection against uncertainty. A higher robustness budget has a higher cost, but allows higher flexibility for demand variations. For the case study, an increase of robustness means an increase in the company's productive capacity. Moreover, the robust approach involves the investment in three more machines than the linear model. The studied company produces as products are demanded; they do not build up inventory. Therefore, a versatile configuration of machinery and a high processing capacity is an effective way to be protected against uncertainty.

The cutting speed and the material's gas consumption were calculated using mathematical formulations. The developed model assumes that demand is the only source of uncertainty. To determine the optimal investment plan, demand was estimated using historical information of the company's sales. On the other hand, to calculate the optimal production plan, we assumed that demand is normally distributed.

The analysis of the optimal production plan shows that as the uncertainty budget increases, the optimal value of the objective function can be up to 6.4% higher than the linear case. That value is achieved for an uncertainty budget between 29 and 50. However, if the uncertainty budget exceeds that number, the value of the objective function decreases to 93% of the linear case optimal. On the other hand, a robust model has higher flexibility and coverage to demand variations. The value of the objective function of the linear model presents an abrupt drop for demand variations; a 15% variation generates a negative optimum. For the robust model, a 30% demand variation generates the highest value of the objective function.

The optimal solution of the model involves the acquisition of the same machine on each investment occasion. The model invest in the machine with the lowest cost, which at the same time has the highest production limitations. The initial conditions of the problem consider two machines, with which the company is able to process all 11 product. Therefore, the model invests on the cheapest equipment. In consequence, if the cost of chosen machine increases the model invests in the second cheapest machine.

The proposed model only considers uncertainty in a single parameter (demand). In reality this problem has other sources of uncertainty, such as cutting speeds and gas consumption. Additionally, the model can be expanded to consider the impact of those variables in the problem, leading to more accurate solutions. In addition, the solution of the problem could be analyzed for a different set of initial conditions, such as the initial set of machines. Moreover, further research could be carried out to study the different sets of products, in which materials that cannot be processed by both machines have a higher percentage of the demand. Finally, further analysis could study the impact of uncertainty in the cutting speed and not in the demand. In this case, demand would have a fixed value.

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