Cocycles of isometries and denseness of domination

Abstract

We consider the problem of approximating a linear cocycle (or, more generally, a vector bundle automorphism) over a fixed base dynamics by another cocycle admitting a dominated splitting. We prove that the possibility of doing so depends only on the homotopy class of the cocycle, provided that the base dynamics is a minimal diffeomorphism and the fiber dimension is at least 3 . This result is obtained by means of a general theorem on the existence of almost invariant sections for fiberwise isometries of bundles with compact fibers and finite fundamental group. As another consequence of that theorem, we characterize almost coboundaries on compact Lie groups with finite center. The main novelty of the proofs is the use of a quantitative homotopy result due to Calder, Siegel and Williams..