PONTIFICIA UNIVERSIDAD CATOLICA DE CHILE
INSTITUTO DE ECONOMIA
DOCTORADO EN ECONOMIA

TESIS DE GRADO
DOCTORADO EN ECONOMIA

Guadalupi, Carla

Junio, 2015

PONTIFICIA UNIVERSIDAD CATOLICA DE CHILE
INSTITUTO DE ECONOMIA
DOCTORADO EN ECONOMIA

Prices as Signals of Quality: Experimentation and Information Acquisition

Carla Guadalupi

Comisión

Nicolás Figueroa, Juan Pablo Montero, Joaquín Poblete y Carlos Ponce

Santiago, Junio de 2015

PONTIFICIA UNIVERSIDAD CATOLICA DE CHILE
INSTITUTO DE ECONOMIA
DOCTORADO EN ECONOMIA

Prices as Signals of Quality: Experimentation and Information Acquisition

ACKNOWLEDGEMENTS: I would like to express special thanks and appreciation to my advisor Nicolás Figueroa, who has been a tremendous mentor, both through encouraging my research and for allowing me to grow as a research scientist. I would also like to thank the committee members, professor Joaquín Poblete, professor Juan Pablo Montero, professor Carlos Ponce and professor Gonzalo Cisternas for their time and effort. In large part due to their contributions, my defense was an enjoyable moment, and thanks to their brilliant comments and suggestions, I am happy with the resulting work. Special thanks are given to Conicyt for the financial support. A special thanks to my family and friends who supported and encouraged me to strive towards completing my goal. Finally, I would like to express appreciation to my husband Paul Wander for his unconditional support.

Index

Strategic Experimentation and Price Signaling: Low Prices Signal High Quality

Carla Guadalupi

Abstract

This paper examines the optimal pricing strategy for newly introduced experience goods in a two-period monopoly market with experimentation and private information about quality. Consumers learn about quality through price signaling and experimentation, and communicate their findings to other buyers via word of mouth. We show the existence of a unique separating equilibrium that satisfies the intuitive criterion. In this equilibrium, a high-quality seller signals high quality through a low introductory price that rises in the next period (after experimentation has occurred), while a low-quality one charges a high introductory price, which declines over time because the revealed information is likely to be bad. This result helps explain recent empirical evidence and case studies on the introductory pricing strategies of firms entering foreign product markets.

1 Introduction

Extensive literature deals with the optimal pricing strategy for new products of initially uncertain quality (Dean 1969; Krishnan, Bass and Jain 1999; Noble and Gruca 1999). On the one hand, high introductory prices may signal high quality, according to the folk wisdom "you get what you pay for", while low-quality products are always cheaper (Bagwell and Riordan 1991). The logic behind this adage takes many forms, but high prices generally signal high quality because increased production costs are seen to imply both higher prices and higher quality. On the other hand, the experimentation literature suggests that since every purchase yields additional (valuable) information about quality, firms prefer consumers to have more quality information rather than less, so that they often penetrate the market with a low introductory price to encourage use and build reputation (Schlee 2001). A good-quality product benefits from consumer experimentation, while a low-quality product will likely see fewer repeat customers.

In the case of newly introduced experience goods, a firm often has a precise idea of the quality of the product it provides (its "type"), but consumers are unable to observe quality prior to purchase. Consider for example, the market for premium wines, in which the producer knows considerably more about growing-season climate, the production process, and the final characteristics of a bottle of wine than consumers. Given this imbalance, consumers can draw inference from the price (signaling), but they can also learn about quality through repeat purchases and word of mouth (Arndt 1967; Chen and Xie 2008; Zhu and Zhang 2010) (experimentation). These mechanisms do not necessarily operate in isolation and their interaction can lead to new implications for the introductory pricing strategy. For the wine industry, some authors find that high prices reflect high quality (Rosen 1974; Landon and Smith 1998; Storchman and Schnabel 2010), while others show that the wine price-quality relationship is hard to predict, especially for lesser-known wines (Oczkowoski 2001; Lecocq and Visser 2006; Miller et al. 2007). In the case of newly introduced premium wines, reputation and country of origin are found to have a higher impact on price than true quality (Roberts and Reagan 2007; Berríos and Saens 2012; Rodriguez and Felzensztein

2013), and this "country brand trap" may explain the aggressive entry strategy of wines from the New World (Australia, New Zealand, Argentina, Chile, South Africa, Canada and Israel) into international wine markets, such as the US and the UK. Macchiavello (2010) found empirical evidence for low introductory prices, that increase over time, for premium Chilean wines entering the UK market. The empirical and anecdotal evidence suggests that in many cases firms encourage experimentation using penetration prices rather than relying on high prices as a signal of quality.

We approach this issue by setting up a dynamic monopoly model with experimentation and asymmetric information regarding quality, in which a long-lived monopolist faces a sequence of short-lived consumers. The monopolist can be good or bad, and is aware of its type (here the probability of producing high quality). Consumers are initially uninformed about the firm's type, but they have two learning mechanisms: price signaling and/or experimentation. The monopolist's pricing strategy provides a signal from which buyers can infer its type, even as the quantity sold determines the diffusion of information regarding product quality. More sales lead to higher product exposure in the subsequent period, which amplifies the good (or bad) news generated by the product, so that prices become an instrument through which the firm may encourage or discourage experimentation. We look for separating and pooling (pure-strategy) equilibria and apply the intuitive criterion refinement (Cho and Kreps 1987) to eliminate implausible off-equilibrium path beliefs. First we characterize the separating and pooling equilibrium when both types have zero marginal cost and consumers only learn through experimentation. We then study how this equilibrium changes when price signaling operates as an additional intertemporal information diffusion mechanism, i.e. when consumers learn through both signaling and experimentation. Finally we consider the case in which high quality is more costly to produce.

For the first scenario, we show the existence of a unique (first-period) separating equilibrium in which low prices signal high quality. The good monopolist signals high quality through a low introductory price, which rises in the next period, after experimentation has occurred, while a low-quality one charges a high introductory price, which declines over time because the revealed information is likely to be bad. We show that when experimentation and signaling interact the (low price) experimentation result dominates. Moreover, the effect of signaling may switch directions as low prices act as a signaling tool for high quality. The intuition is that low prices are costly, and will only be used by firms confident enough that increased experimentation will yield good news and therefore increased future demand. We also characterize the set of pooling equilibria and show that no pooling equilibrium satisfies the intuitive criterion. Our second result is that the (low) separating price charged by the good monopolist in equilibrium decreases as the impact of signaling on consumers' beliefs increases: a good firm must work harder to discourage the bad one from mimicking its behavior if consumers pay less attention to experimentation. The separating equilibrium disappears altogether if the weight given to signaling is sufficiently high. Finally, we obtain a new separating equilibrium in which the good monopolist signals high quality through high prices, when allowing for correlation between quality and cost.

We contribute to the existing literature by developing a dynamic monopoly model with experimentation and asymmetric information about quality. Most strategic experimentation models consider a setting of incomplete but symmetric information, in which the firm has the same (lack of) information as the buyers. We extend this branch of the literature by assuming that consumers are alone in facing uncertainty regarding product quality. Moreover, only few signaling models allow for experimentation to be used as a learning mechanism, and most assume correlation between cost and quality. Finally, very little has been said about the optimal introductory price when both experimentation and signaling interact. We find that low prices signal high quality, which low-quality firms cannot mimic because of learning effects. The price charged by the high-quality firm in equilibrium is even lower than in the absence of asymmetric information, as in the experimentation literature. Our results help explain the findings of several empirical and case studies. In fact, it is consistent with the initially low introductory prices that rise over time observed for New World premium wines entering the USA and UK markets (Macchiavello 2010), and for high-quality antidepressant drugs in the US market (Chen and Rizzo 2012).

Related Literature. Generally, the logic behind the concepts of strategic experimentation and signaling suggest different optimal pricing strategies for new experience goods. A low introductory price may dominate in an environment defined by experimentation, where the pricing strategy and subsequent purchase decisions help both the firm and the consumers observe quality. In a duopoly with price competition, the equilibrium price is lower than the myopic price, and the amount of experimentation in equilibrium is efficient because experimentation costs are equally distributed between buyers and sellers (Bergemann and Valimaki, 1996). In a similar model, in which an established firm and a new entrant compete in prices, the equilibrium price is associated with excessive early sales by the new firm, that decrease over time as buyers and sellers become informed about the quality of the new product (Bergemann and Valimaki, 1997). The same result is obtained in the monopoly case (Schlee, 2001). If the monopolist has private information, the decision to trade or not becomes strategic and buyers learn by observing ex-post outcomes and the monopolist´s decision to trade or not. In equilibrium, good firms never stop selling, while bad firms still sell with positive probability (Bar-Isaac, 2003).

The signaling literature, however, suggests that a high introductory price may lead consumers to infer that the product is of high quality. In general, this result is due to cost effects. Signaling through high prices generates lower sales, which are less damaging to a higher-cost producer. In a dynamic monopoly model, in which both the introductory price and advertising may be used to signal quality, advertising as "money-burning" indicates that the product is of high-quality. Price signaling can also occur, but the extent to which each instrument is used depends on the difference in costs across qualities (Milgrom-Roberts, 1986). For example, in a static market for durable goods, in which quality is correlated with costs, high prices signal high quality (Bagwell and Riordan, 1991). Moreover, as consumers gain knowledge about product quality, prices decline over time. The result is confirmed when advertising is allowed: the monopolist efficiently signals high quality via high prices and dissipative advertising (Linnemer, 2002). Finally, even if quality and costs are uncorrelated, high prices signal high quality when the buyers and the seller possess some amount of private information (Judd and Riordan, 1994).

2 The model

We consider a two-period model, in which a firm produces a new experience good of unobservable quality to consumers. The quality of the product may be either high or low, $q \in \{H, L\}$, and depends on firm type θ, which is private information and can be either good or bad, $\theta \in \{g, b\}$. A type-θ firm produces high quality with probability θ and low quality with probability $(1-\theta)$, with $0 < b < g < 1$. The production technology is common knowledge and both types have zero marginal cost[1]. We relax this assumption in section 3.4, allowing a correlation between costs and quality. The firm's decision variable is the price charged in each period and we assume no discounting.

Each period a continuum of consumers of mass 1, with unit demand, enters the market[2]. Consumers have common reservation values for the low-quality product $x_i(L) = 0$, for every i. Moreover, they have heterogeneous reservation values for the high-quality product, which are uniformly[3] distributed on the unit interval, $x_i(H) \sim U[0,1]$. Therefore, given a belief μ about firm type, consumer i will buy the product if $x_i(H)[\mu g + (1-\mu) b] - P \geq 0$, which leads to the aggregate demand:

$$Q(P, \mu) = 1 - \frac{P}{\mu g + (1-\mu) b}.$$

Consumers have two possible learning mechanisms: they can interpret the price as a signal of quality (price signaling) and experiment in order to ascertain quality (experimentation)[4]. At the beginning of period 1 consumers believe the firm is good with probability μ_0 and this prior belief is common knowledge. After observing first-period prices P_1, they update beliefs

[1]Product quality is treated as exogenous and is unrelated to marginal costs. The quality of wine produced by a given vineyard in a given year, for example, depends on the length of growing season, rainfall, humidity and many other climate-related factors determined before the firm chooses how to introduce the product. Similarly, we can think of quality as determined by R&D prior to production and sale.

[2]Assuming short-lived consumers is standard in the literature (see Bergemann and Valimaki (2000), and Bagwell and Riordan (1991)). This assumptions is also made for convenience as having long-lived buyers would require a value function for consumers too, complicating the analysis.

[3]The uniform distribution is convenient because it generates a linear demand for a high-quality product.

[4] The sales level, itself a function of consumers' purchase decisions and monopolist's pricing strategy, determines the diffusion of product quality information.

to $\mu_1 = Pr(\theta = g \mid P_1)$ and make their purchase decisions[5]. Note that pooling prices do not provide any information and the posterior belief will be the same as the prior ($\mu_1 = \mu_0$), whereas separating prices induce the belief $\mu_1 = 1$ if $P_1 = P_1^g$ and $\mu_1 = 0$ if $P_1 = P_1^b$, where P_1^g and P_1^b denote the separating equilibrium prices charged by the good and the bad firm, respectively.

At the beginning of period 2, new consumers arrive with beliefs $\mu_{2,q}$. In order to capture signaling and experimentation effects on consumers belief formation, we think of $\mu_{2,q}$ as a weighted average of μ_1, the belief derived from (indirect) observation of first-period prices, and the belief derived from (indirect) experience[6] with the product:

$$\mu_{2,q} = \alpha\mu_1 + (1-\alpha)\left[\mu_q Q_1 + \mu_0 (1 - Q_1)\right], \tag{1}$$

where Q_1 is the quantity sold in the first period, which influences the speed of information diffusion, $q \in \{H, L\}$, and μ_q is the Bayesian update after observing the realized quality:

$$\mu_H = Pr(\theta = g \mid q = q_H) = \frac{\mu_0 g}{\mu_0 g + (1-\mu_0) b}$$

$$\mu_L = Pr(\theta = g \mid q = q_L) = \frac{\mu_0 (1-g)}{\mu_0 (1-g) + (1-\mu_0)(1-b)}.$$

This construction of $\mu_{2,q}$ corresponds to the following intuition: second-period buyers receive two different signals, both noisy, about quality. Previous period prices and previous users observed quality are weighed when forming beliefs. An alternative interpretation for $\mu_{2,q}$ is that second-period consumers interact with previous users with probability α, and communicate about price. Nevertheless, with probability $(1-\alpha)$ they do not interact with them, and consumer reports or expert recommendations are the only information source available. Generally, when $\alpha = 0$ we think of second-period consumers as learning only through experimentation, while with $\alpha = 1$ we assume that they only learn through prices. The beliefs equation 1 represents a reduced form and a formal argument for it can be found

[5]It is worth noting that period 1 consumers only learn through prices, given that, for an experience good, product characteristics and quality can be ascertained only upon consumption.

[6]We say that experimentation and signaling are indirect because they only occur through any sort of communication between different period consumers. It is not the case that second-period consumers directly observe prices nor experience the good by buying it. Therefore, online reviews, consumer reports and word of mouth are an important example of information source for newly arrived consumers.

in the appendix.

Given this belief structure, the expected profits of a type-θ firm, setting an introductory price P_1 and believed to be good with probability μ_1, are given by:

$$\Pi^{\theta}(P_1, \mu_1) = P_1 Q_1^{\theta} + \theta \pi_2(\mu_{2,H}) + (1-\theta)\pi_2(\mu_{2,L})$$

where $\pi_2(\mu)$ denote second-period profits (given beliefs μ), expressed as:

$$\pi_2(\mu) = \max_P \; P\left[1 - \frac{P}{\mu g + (1-\mu) b}\right] = \frac{1}{4}\left[\mu g + (1-\mu) b\right].$$

The timing of the game is as follows: Nature determines firm's type $\theta \in \{g, b\}$ at the outset. At the beginning of period 1 the firm sets the introductory price P_1; consumers, after observing P_1, update beliefs to μ_1 and make their purchase decisions. Quality is then realized. At the beginning of period 2, new consumers start the stage game with beliefs $\mu_{2,q}$ and the firm sets P_2 as a function of the updated beliefs. Buyers make their purchase decisions and second-period profits are realized.

3 Equilibrium

We now look for separating and pooling (pure-strategy) equilibria and apply the intuitive criterion refinement (Cho and Kreps, 1987) to eliminate implausible off-equilibrium path beliefs. First we consider that both types have zero marginal costs and second-period beliefs only depend on experimentation, i.e. $\alpha = 0$. We then analyze the case with $\alpha > 0$, exploring the effect of price signaling as an intertemporal information diffusion mechanism. Finally we consider the case in which high quality is more costly to produce.

3.1 Separating Equilibrium with $\alpha = 0$

We analyze conditions for the existence of a separating equilibrium with $\alpha = 0$[7]. The beliefs

[7] With $\alpha = 0$ second-period consumers cannot observe previous period prices and only learn through experimentation. In this case P_1 influences second-period beliefs only through quantity, but not through the informational content of the separating strategy.

equation 1 reduces to:

$$\mu_{2,q} = \mu_q Q_1 + \mu_0 (1 - Q_1).$$

We solve the game by backward induction. In $t = 2$ there is no reason for firms to differentiate themselves[8] and the second-period pooling price is given by $P_2^* (\mu_{2,q}) = \frac{[\mu_{2,q} g + (1-\mu_{2,q}) b]}{2}$ with profits $\pi_2 (\mu_{2,q}) = \frac{1}{4} [\mu_{2,q} g + (1 - \mu_{2,q}) b]$.

A separating equilibrium in $t = 1$ is a sequential equilibrium at which consumers can distinguish good and bad firms by the different pricing choices they made. Separating prices induce beliefs $\mu_1 = 1$ if $P_1 = P_1^g$ and $\mu_1 = 0$ if $P_1 = P_1^b$. Moreover, off-equilibrium prices $P_1 \neq \left\{P_1^g, P_1^b\right\}$ are assumed to induce pessimistic beliefs $\mu_1 = 0$. Firms' expected profits are:

$$\Pi^g (P_1^g, 1) = P_1^g Q_1^g + g [\pi_2 (\mu_{2,H})] + (1 - g) [\pi_2 (\mu_{2,L})]$$

$$\Pi^b \left(P_1^b, 0\right) = P_1^b Q_1^b + b [\pi_2 (\mu_{2,H})] + (1 - b) [\pi_2 (\mu_{2,L})]$$

where $Q_1^g = 1 - \frac{P_1^g}{g}$ and $Q_1^b = 1 - \frac{P_1^b}{b}$. Moreover $\pi_2 (\mu_{2,H})$ and $\pi_2 (\mu_{2,L})$ depend on P_1^g and P_1^b through the quantity sold in the first period.

Definition 3.1. *(Separating Equilibrium)* A separating equilibrium in $t = 1$ is a pair $\left(P_1^b, P_1^g\right)$ such that three conditions hold:

- **C1.** $\Pi^b \left(P_1^b, 0\right) \geq \Pi^b (P_1, 0)$, for every $P_1 \neq P_1^g$.
- **C2.** $\Pi^b \left(P_1^b, 0\right) \geq \Pi^b (P_1^g, 1)$, and
- **C3.** $\Pi^g (P_1^g, 1) \geq \Pi^g (P_1, 0)$, for every $P_1 \neq P_1^g$.

For the bad firm, P_1^b must dominate any price $P_1 \neq P_1^g$ under beliefs $\mu_1 = 0$ (C1). Moreover, the bad firm should not have incentives to mimic the good one, even if this implies optimistic

[8] It can be easily shown that the only second-period equilibrium is given by pooling prices.

beliefs, $\mu_1 = 1$ (C2). For the good firm, P_1^g must dominate any other price P_1 that induce beliefs $\mu_1 = 0$ (C3).

Lemma 3.2. *In any separating equilibrium $P_1^b = P_1^{b*}$, where P_1^{b*} is the maximizer of $P_1\left(1 - \frac{P_1}{b}\right) + b\pi_2\left(\mu_{2,H}\right) + (1-b)\,\pi_2\left(\mu_{2,L}\right)$*[9].

Proof. This is a necessary condition for C1 to be satisfied. ■

Note that $P_1^{b*} > \frac{b}{2}$, the static monopoly price under beliefs $\mu_1 = 0$. The bad firm charges a higher price than in a static environment because the gains of increased sales in the first period are outweighed by the existence of a second period sub-game with improved information. Moreover C2 implies that $\Pi^b\left(P_1^{b*}, 0\right) \geq \Pi^b\left(P_1^g, 1\right)$, which rules out the possibility that the bad firm would mimic the good one. Analogously for the good firm the following lemma holds:

Lemma 3.3. *For the good firm, it is sufficient to check that $\Pi^g\left(P_1^g, 1\right) \geq \Pi^g\left(P_1^{g*}, 0\right)$, where P_1^{g*} is the maximizer of $P_1\left(1 - \frac{P_1}{b}\right) + g\pi_2\left(\mu_{2,H}\right) + (1-g)\,\pi_2\left(\mu_{2,L}\right)$.*

Proof. The price P_1^g is a separating equilibrium if it dominates any other price P_1 that induce beliefs $\mu_1 = 0$ (C3). Thus it is sufficient to control for the best deviation, which occurs at the price that maximizes profits over both periods under the worst belief.■

We now examine in more detail the prices available to the good firm in a separating equilibrium. Lemma 3.2 allows us to restrict attention to prices P_1^g such that $\Pi^b\left(P_1^{b*}, 0\right) - \Pi^b\left(P_1^g, 1\right) \geq 0$. This inequality asserts that a bad monopolist would choose its maximizing price, under the most pessimistic beliefs, rather than mimicking the good monopolist in order to be perceived as good. The bad firm has no incentive to mimic prices that are either "too low" or "too high": $P_1^g \leq \underline{P}_b$ or $P_1^g \geq \overline{P}_b$, where $\underline{P}_b$ and $\overline{P}_b$ are the roots of the quadratic inequality $\Pi^b\left(P_1^{b*}, 0\right) - \Pi^b\left(P_1^g, 1\right) \geq 0$. For prices below $\underline{P}_b$, the gain of increased sales is outweighed by the loss due to improved information. For prices above $\overline{P}_b$, the loss of overall sales outweighs the gain from keeping the low-quality product relatively unknown. The gains

[9] It is worth noting that $\mu_1 = 0$, and $\mu_{2,q}$ depends on P_1 through Q_1, as in (1).

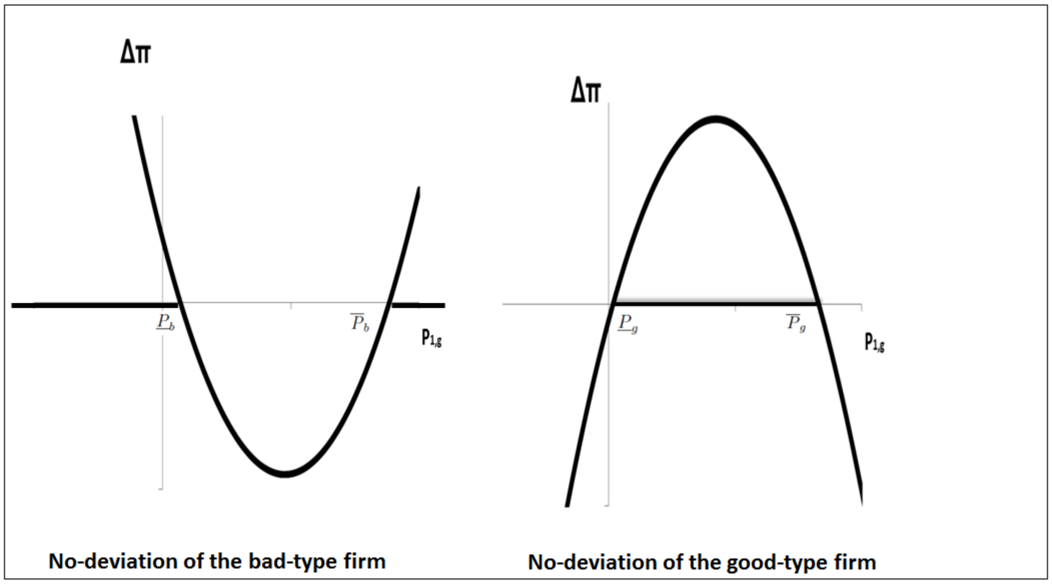

Figure 1:

outweigh the losses inside the parabola, where the bad monopolist has incentives to imitate the good one as shown in figure 1. On the other hand, separation can occur only if the good monopolist chooses not to monopolize the market, being perceived as a bad firm. Lemma 3.3 allows us to consider only prices P_1^g such that $\Pi^g\left(P_1^g,1\right)-\Pi^g\left(P_1^{g*},0\right)\geq 0$, which implies that a good monopolist would rather choose the equilibrium price P_1^g and be perceived as good than be perceived as a bad firm and optimize accordingly. The good firm has no incentive to deviate when the equilibrium price P_1^g is such that $\underline{P}_g \leq P_1^g \leq \overline{P}_g$, where $\underline{P}_g$ and $\overline{P}_g$ are the roots of the quadratic inequality $\Pi^g\left(P_1^g,1\right)-\Pi^g\left(P_1^{g*},0\right)\geq 0$. When the separating equilibrium price is either "too high" or "too low", the cost of selling at extremely low or high prices outweighs the benefit of inducing the belief $\mu_1 = 1$. These conditions are shown in figure 1 and summarized in the following Corollary[10].

Corollary 3.4. *A separating equilibrium in* $t = 1$ *is a pair* $\left(P_1^{b*},P_1^g\right)$ *such that* $P_1^g \in \left(\underline{P}_b,\overline{P}_b\right)^c \cap \left[\underline{P}_g,\overline{P}_g\right]$.

[10]See the appendix for the expressions of the quadratic inequalities.

In a separating equilibrium, a good firm signals high quality through low prices, while a bad one charges a higher price (the profit maximizing price). The intuition is simple: a good monopolist suffers less from separating through low introductory prices, because they result in more sales, increased knowledge of product quality and finally better second-period profits. On the other hand, a bad monopolist benefits more from high introductory prices, which induce low sales in the first period, limiting the diffusion of information regarding quality and allowing for second period sales, even though the realized quality was poor. In Spence's famous job-market signaling model, good-type workers signal high-quality via increased education, which at some critical point bad types are unable to replicate because education is assumed to be more costly for them. Here, the logic of the signaling mechanism does not depend on cost differences across types, instead copycat behavior is prevented by repeat purchases due to the existence of a second-period with improved information. Thus, when allowing for experimentation, signaling generates an inverse relationship between price and quality. The situation in which a separating equilibrium exists is depicted in figure 2. Moreover, this equilibrium exhibits the following price dynamics: the price of high (low) quality products increases (decreases) over time, as consumers learn about product quality. Since the second period pooling price is linear in beliefs, it will be higher (lower) if good (bad) news were revealed in the first period when the product was introduced.

Proposition 3.5. *There is always a separating equilibrium with* $P_1^g \leq \underline{P}_b$*, but there is no separating equilibrium with* $P_1^g > \underline{P}_b$ *if* $\overline{P}_b > \frac{g}{b} P_1^{b*}$*. Since* $\underline{P}_b < P_1^{g*}$*, in a separating equilibrium, the good monopolist always charges a price* P_1^g *which is below the profit maximizing price* P_1^{g*}*.*

Proof. See Appendix.■

Most of these separating equilibria involve beliefs that are implausible, since any deviation is interpreted as coming from a bad firm. We now restrict attention to equilibria that satisfy the "intuitive criterion" of Cho and Kreps. To understand the Cho and Kreps refinement in

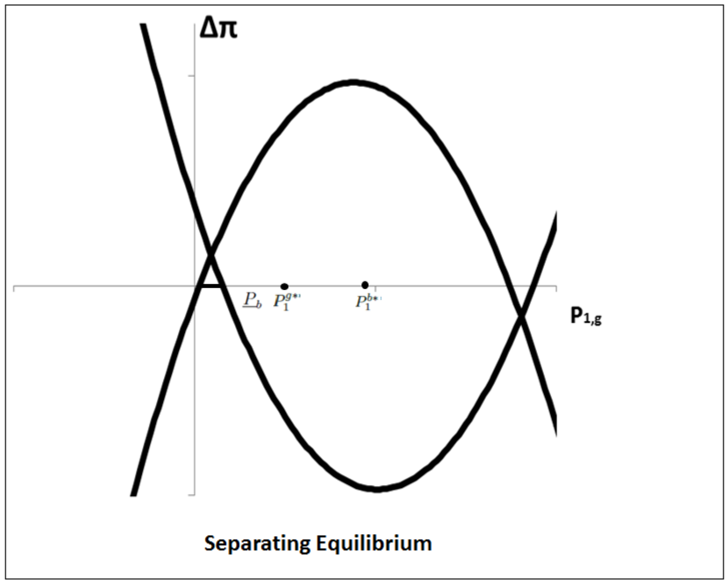

Figure 2:

this context, consider an equilibrium in which the good monopolist's profits are $\Pi^g\left(P_1^g,1\right)$ while the bad firm earns profits $\Pi^b\left(P_1^{b*},0\right)$. The equilibrium fails the intuitive criterion if there exists a price P' such that: a) $\Pi^g\left(P^{'},1\right)\geq\Pi^g\left(P_1^g,1\right)$ and b) $\Pi^b\left(P^{'},1\right)<\Pi^b\left(P_1^{b*},0\right)$. That is, if there exists a price $P^{'}$ such that the good firm is better off by deviating and the bad and makes more profits following the equilibrium price, even if consumers have the most optimistic beliefs. Intuitively, if such a price $P^{'}$ exists, consumers should interpret such a deviation as coming from a good firm, making the equilibrium fail.

Proposition 3.6. *The pair* $\left(P_1^{b*},\underline{P}_b\right)$ *is the only separating equilibrium that satisfies the intuitive criterion.*

Proof. The proof consists of two steps. We first show that there is no equilibrium price $P<\underline{P}_b$ that satisfies the intuitive criterion. Consider the price $P<\underline{P}_b$ such that $\left(P_1^{b*},\,P\right)$ is a separating equilibrium. Define $P^{'}=P+\varepsilon$. Then it is easy to see that a) $\Pi^g\left(P^{'},1\right)\geq\Pi^g\left(P,1\right)$ and b) $\Pi^b\left(P^{'},1\right)<\Pi^b\left(P_1^{b*},0\right)$. Let P_1^{g**} be the price that maximizes the good firm's profits under the most optimistic belief $\mu_1=1$. Noting that $P_1^{g**}>P_1^{g*}$and $P^{'}<\underline{P}_b<P_1^{g*}$, we get that $P<P^{'}<P_1^{g**}$. Therefore $\Pi^g\left(P^{'},1\right)\geq\Pi^g\left(P,1\right)$. By Corollary 1 we know that $\Pi^b\left(P,1\right)<\Pi^b\left(P_1^{b*},0\right)$, and by continuity $\Pi^b\left(P^{'},1\right)<\Pi^b\left(P_1^{b*},0\right)$. Thus for any price $P < \underline{P}_b$ condition a) is not satisfied, violating the intuitive criterion. We now show that $\left(P_1^{b*}, \underline{P}_b\right)$ satisfies the intuitive criterion, that is there is no equilibrium price P' such that a) $\Pi^g\left(P', 1\right) \geq \Pi^g\left(\underline{P}_b, 1\right)$ and b) $\Pi^b\left(P', 1\right) < \Pi^b\left(P_1^{b*}, 0\right)$. If $P' < \underline{P}$ condition a) is not satisfied. Moreover, $P' \geq \overline{P}_b$ otherwise b) is not satisfied. By Corollary 1 we have that, for $P' > \overline{P}_b$, $\Pi^g\left(P', 1\right) < \Pi^g\left(P_1^{g*}, 0\right)$. Then $\left(P_1^{b*}, \underline{P}_b\right)$ is the only separating equilibrium that satisfies the intuitive criterion. ■

3.2 Pooling Equilibrium with $\alpha = 0$

We now analyze conditions for the existence of a pooling equilibrium[11] with $\alpha = 0$ and show that there is no pooling equilibrium ($P_1^g = P_1^b = P_1$) that satisfies the intuitive criterion. Under pooling equilibria prices are uninformative and only reflect consumers' prior expectation of quality.

We solve the game by backward induction. We know the second-period pooling price is given by $P_2^*\left(\mu_{2,q}\right) = \frac{\left[\mu_{2,q}g + (1-\mu_{2,q})b\right]}{2}$ with associated profits $\pi_2\left(\mu_{2,q}\right) = \frac{1}{4}\left[\mu_{2,q}g + \left(1 - \mu_{2,q}\right)b\right].$ In $t = 1$ the posterior belief $\mu_1 = \mu_0$. As before we assume that off-equilibrium prices $\widetilde{P}_1 \neq P_1$ induce $\mu_1 = 0$. Firms' expected profits are:

$$\Pi^\theta\left(P_1, \mu_0\right) = P_1 Q_1 + \theta\pi_2\left(\mu_{2,H}\right) + \left(1 - \theta\right)\pi_2\left(\mu_{2,L}\right),$$

with $Q_1 = 1 - \frac{P_1}{\mu_0 g + (1-\mu_0)b}$, and $\theta \in \{g, b\}$.

Definition 3.7. *(Pooling Equilibrium)* A pooling equilibrium in $t = 1$ is a price P_1 that satisfies $\Pi^\theta\left(P_1, \mu_0\right) \geq \Pi^\theta\left(\widetilde{P}_1, 0\right)$ for every $\widetilde{P}_1 \neq P_1$ and for $\theta \in \{g, b\}$.

Lemma 3.8. *For both types, it is sufficient to check that:* $\Pi^\theta\left(P_1, \mu_0\right) \geq \Pi^\theta\left(P_1^{\theta*}, 0\right)$, *where* $P_1^{\theta*}$ *is the maximizer of* $P_1\left(1 - \frac{P_1}{b}\right) + \theta\pi_2\left(\mu_{2,H}\right) + \left(1 - \theta\right)\pi_2\left(\mu_{2,L}\right)$, *for* $\theta \in \{g, b\}$.

Proof. The definition of *Pooling Equilibrium* requires that P_1 must dominate any other price $\widetilde{P}_1$ that induce beliefs $\mu_1 = 0$. Thus it is sufficient to control for the best deviation, which

[11] We do not study the implications or details of the pooling equilibria here, but such equilibria may be worthy of further study given the fact that Chilean wines entering the Chinese market have been shown to use pooling prices.

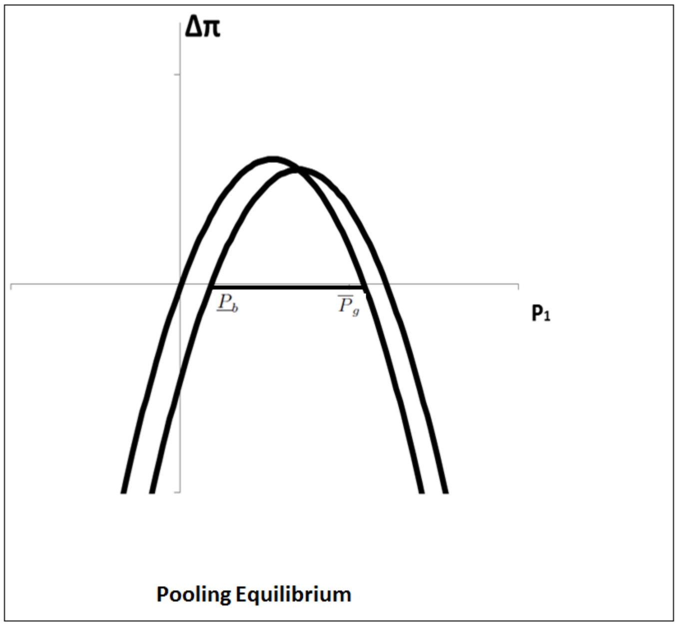

Figure 3:

occurs, for both types, at the price that maximizes profits over both periods under the worst beliefs.■

Lemma 3.8 states that pooling can occur only if both types prefer not to monopolize the market, which allow us to consider only prices P_1 that satisfy the following inequalities: $\Pi^b(P_1, \mu_0) - \Pi^b\left(P_1^{b*}, 0\right) \geq 0$ and $\Pi^g(P_1, \mu_0) - \Pi^g\left(P_1^{g*}, 0\right) \geq 0$. These inequalities assert that both types would rather choose P_1 and be perceived as average quality than be perceived as bad firms and optimize accordingly. The first-period pooling equilibrium prices P_1 are such that $\overline{P}_b \leq P_1 \leq \overline{P}_g$, where $\overline{P}_b$ and $\overline{P}_g$ are the roots of the previous quadratic inequalities. Moreover, no pooling equilibrium satisfies the intuitive criterion. The situation in which a pooling equilibrium exists is depicted in figure 3.

Proposition 3.9. *There is always a pooling equilibrium with $P_1 \leq \overline{P}_g$, but no pooling equilibrium satisfies the intuitive criterion.*

Proof. The proof consists of two steps. The first step is presented in the appendix and show the existence of pooling equilibria. We now show that no pooling equilibrium satisfies the intuitive criterion. Fix some pooling equilibrium P_1. At this equilibrium price, a type-θ firm earns profits $\Pi^{\theta}(P_1, \mu_0) = P_1 Q_1 + \theta \pi_2(\mu_{2,H}) + (1-\theta)\pi_2(\mu_{2,L})$. We show that there exists a price $P < P_1$ that the good firm would strictly prefer to deviate to if by doing so it would be seen as good; whereas the bad firm would rather adhere to the equilibrium price, even if deviating leads to be thought of as a good monopolist. Thus if consumers observe the price P, they would automatically believe that the firm is good. In other words, we show the existence of a price $P < P_1$ that violates the intuitive criterion by satisfying the following conditions:

a) $\Pi^g(P_1, \mu_0) < \Pi^g(P, 1)$ and

b) $\Pi^b(P_1, \mu_0) \geq \Pi^b(P, 1)$.

Let consider a price $P < P_1$ such that $\Pi^g(P_1, \mu_0) = \Pi^g(P, 1)$. It is easy to show that the bad firm is strictly worse off by charging P ($\Pi^b(P_1, \mu_0) > \Pi^b(P, 1)$). We denote $U^2(Q_1, \theta)$ the second-period profits of a type-θ firm who sold the quantity Q_1 in the first period. We can rewrite the expected profits of a type-θ firm, that charges P_1 inducing beliefs μ_1 in the first period, as $\Pi^{\theta}(P_1, \mu_1) = P_1 Q_1 + U^2(Q_1, \theta)$. Then

$$\Pi^b(P, 1) - \Pi^b(P_1, \mu_0)$$

$$= P\left(1 - \frac{P}{g}\right) + U^2\left(1 - \frac{P}{g}, b\right) - P_1\left(1 - \frac{P_1}{\mu_0 g + (1-\mu_0) b}\right) - U^2\left(1 - \frac{P_1}{\mu_0 g + (1-\mu_0) b}, b\right)$$

$$< P\left(1 - \frac{P}{g}\right) + U^2\left(1 - \frac{P}{g}, g\right) - P_1\left(1 - \frac{P_1}{\mu_0 g + (1-\mu_0) b}\right) - U^2\left(1 - \frac{P_1}{\mu_0 g + (1-\mu_0) b}, g\right) = 0,$$

where the first inequality comes from the fact that $g > b$, and the second equality comes from the assumption about P such that $\Pi^g(P_1, \mu_0) = \Pi^g(P, 1)$. Thus there exists a price P such that $\Pi^g(P_1, \mu_0) = \Pi^g(P, 1)$ and $\Pi^b(P_1, \mu_0) > \Pi^b(P, 1)$. Therefore, $P' = P - \varepsilon$ violates the intuitive criterion for any sufficiently small ε .■

3.3 Intertemporal Price Signaling: $\alpha > 0$

We now turn to the general case with $\alpha > 0$, in order to capture simultaneously signaling and experimentation effects on second-period consumers beliefs. When forming second-period beliefs, consumers now give weight to both experimentation and first-period prices, which act as an intertemporal information mechanism[12].

The main result here is that a higher impact of first-period prices on second-period beliefs (higher α) decreases the separating price charged by the good firm in $t = 1$. A good signal in the first period ($P_1 = P_1^g$) carries more positive information to second-period buyers, making it more attractive. To discourage the bad firm from mimicking, a lower price must be charged by the good monopolist. For sufficiently high α, the separating equilibrium might disappear: even at $P_1^g = 0$ incentives for the bad firm to mimic the good one might be too strong. In the limiting case with $\alpha = 1$ second period consumers only learn through prices. Thus the only existing equilibrium is a pooling one.

Proposition 3.10. *The separating equilibrium price P_1^g is decreasing in α. Moreover, if $b \geq \frac{g}{2}$ a separating equilibrium always exists if $\alpha < 1$; if $b < \frac{g}{2}$, there exists an $\overline{\alpha} < 1$ such that a separating equilibrium exists if and only if $\alpha \leq \overline{\alpha}$.*

Proof. From section 3.1 we know that a separating equilibrium is such that $P_1^g \leq \underline{P}_b$ and the only pair that satisfy the intuitive criterion is $\left(P_1^{b*}, \underline{P}_b\right)$. It is therefore enough to prove that: 1) $\underline{P}_b$ is monotonically decreasing in α and 2) $\underline{P}_b$ becomes negative, for sufficiently high α and $b < \frac{g}{2}$.

1) When $P_1^g = \underline{P}_b$, the bad firm is indifferent between following the equilibrium strategy $\left(P_1^{b*}\right)$ and mimicking the good-type firm. We rewrite the condition $\Pi^b\left(P_1^{b*}, 0\right) = \Pi^b\left(\underline{P}_b, 1\right)$ as $H(\alpha) = \Pi^b\left(\underline{P}(\alpha), 1\right)$, where

$$H(\alpha) = \max_{P_1} \; P_1 Q_1 + b\pi_2\left(\mu_{2,H}\right) + (1-b)\,\pi_2\left(\mu_{2,L}\right).$$

[12] Second-period beliefs are now given by equation 1.

Using the Envelope Theorem we can calculate:

$$\frac{\partial H(\alpha)}{\partial \alpha} = \frac{(g-b)}{4}\left[b\frac{\partial \mu_{2,H}}{\partial \alpha} + (1-b)\frac{\partial \mu_{2,L}}{\partial \alpha}\right] < 0.$$

Moreover, $\frac{\partial \Pi(\underline{P}(\alpha),1)}{\partial \alpha} > 0$. Thus the bad firm payoffs are decreasing in α when it follows the separating equilibrium strategy P_1^{b*}, but they also are increasing in α when it mimics the good firm by charging P_1^g. Therefore, the condition $\Pi^b\left(P_1^{b*},0\right) \geq \Pi^b\left(P_1^g,1\right)$ weakens as the parameter α increases, which implies that $\underline{P}_b$ is decreasing in α.

2) We show that if $b \geq \frac{g}{2}$ a separating equilibrium always exists if $\alpha < 1$, whereas if $b < \frac{g}{2}$, there exists an $\overline{\alpha} < 1$ such that a separating equilibrium exists if and only if $\alpha \leq \overline{\alpha}$. In particular, we consider the extreme case with $\alpha = 1$ and we calculate the smaller root of the quadratic inequality $\Pi^b\left(P_1^{b*},0\right) - \Pi^b\left(P_1^g,1\right) \geq 0$, $\underline{P}_b$. When $\alpha = 1$, second-period buyers only consider first-period prices as signals when forming second-period beliefs, consequently $\mu_{2,q} = \mu_1$. Given $\alpha = 1$, the price that maximizes the bad-type firm profits is $P_1^{b*} = \frac{b}{2}$, the myopic monopoly price. We can easily calculate the negative root of the quadratic inequality $\underline{P}_b = \frac{1-\sqrt{2\left(1-\frac{b}{g}\right)}}{\frac{2}{g}}$, which is negative for $b < \frac{g}{2}$. We conclude that for sufficiently high α and $b < \frac{g}{2}$, $\underline{P}_b < 0$ and it is to be too costly for the good firm to distinguish itself from the bad one, which makes the separating equilibrium collapse. In the other case, $b \geq \frac{g}{2}$, for every $\alpha > 0$, the good-type firm separating equilibrium price will be $P_1^g \leq \underline{P}_b$.$\blacksquare$

3.4 Separating Equilibrium with Cost Heterogeneity

The result presented above is contingent on the assumption that quality and cost are not correlated. In this setting low prices signal high quality when experimentation is possible. Bagwell and Riordan (1991) analyze a monopoly market for durable goods in which high quality goods are more costly to produce. In their static setting, (same period) price is the only signaling instrument available. Experimentation does not come in to play and high prices signal high quality. We arrive at a similar result when a marginal cost $c > 0$ is introduced for the good monopolist, even we allow experimentation.

Following Bagwell and Riordan, since no second-period pooling equilibrium satisfies the intuitive criterion (our chosen refinement tool), we only focus on separating equilibria. In $t = 2$, the sole separating equilibrium that satisfies the intuitive criterion is analogous to the Bagwell and Riordan result: a low-quality, low-cost firm does not mimic the high-quality, high-cost firm because its monopoly output is well above the separating level, given its low cost of production, and the distortion costs of mimicry outweigh the gains. The second-period separating equilibrium eliminates the incentive for the good monopolist to utilize the experimentation channel to provide information because second-period consumers can infer quality directly from second-period prices. The setting is therefore reduced to a static game, traditional price signals dominate (even when $\alpha = 0$) and the only separating equilibrium that satisfies the intuitive criterion is one in which a good monopolist signals high quality through high prices while a bad monopolist charges a lower price.

4 Conclusion

This paper studies the optimal pricing strategy for new experience goods in a dynamic monopoly model with experimentation and private information about product quality. The monopolist's product may be of high or low quality, and the probability of producing high quality (the monopolist's type) is private information. The interaction of two different learning mechanisms (signaling and experimentation) makes low introductory prices a powerful tool to signal high quality. Our main result is that low prices signal high quality. The intuition is that low prices are costly (they imply a short-term revenue loss), and will only be used by firms confident enough that more introductory sales will lead to higher future profits (through higher consumer experimentation). This equilibrium exhibits the following price dynamics: the introductory price of high (low) quality products increases (decreases) over time, as consumers learn about product quality.

Our prediction of low but increasing prices as signal of high quality helps explain recent empirical and case studies on introductory pricing strategies adopted by firms entering foreign markets or launching new products. In fact, it is consistent with the initially low introductory prices that rise over time observed for New World premium wines entering the USA and UK markets (Macchiavello 2010), and for high-quality antidepressant drugs in the US market (Chen and Rizzo 2012). Moreover, our findings have important managerial implications for firms considering new product introductions and designing word-of-mouth advertising campaigns.

Many interesting extensions can be derived from this two-period framework. For example, the firm can separate and communicate quality through other instruments, such as future discounts for repeat consumers, return policies or advertising. Adding these instruments can test the model's robustness. Advertising, for example, can reveal the importance of the price instrument when word of mouth is the main learning force. As signaling becomes more important in belief formation, the importance of advertising as a signaling tool should increase. On the other hand, when experimentation is the only learning mechanism, the use of low introductory price does more to improve future sales than advertising. Furthermore, relaxing the assumption of short-lived consumers or allowing for mixed strategies could allow us to endogenize experimentation. We could also compare welfare results between the separating and the pooling equilibria and analyze price convergence to the static monopoly price by extending the model to an infinite time horizon.

References

[1] ARNDT, J. (1967): "Role of Product-Related Conversations in the Discussion of a New Product", *Journal of Marketing*, Vol. 4, pp. 291-295.

[2] BAGWELL, K. and M. RIORDAN (1991): "High and Declining Prices Signal Product Quality", *The American Economic Review*, Vol. 81, No. 1, pp. 224-239.

[3] BAR-ISAAC, H. (2003): "Reputation and Survival: Learning in a Dynamic Signalling Model", *Review of Economic Studies*, Vol. 70, pp. 231-251.

[4] BAR-ISAAC, H. and S. TADELIS (2008): "Seller Reputation", *Foundations and Trends in Microeconomics*, Vol. 4, No. 4, pp. 273-351.

[5] BASS, F. (1969): "A New Product Growth for Model Consumer Durables", *Management Science*, Vol. 15, No. 5, pp. 215-227.

[6] BERGEMANN, J. and J. VALIMAKI (1996): "Learning and Strategic Pricing", *Econometrica*, Vol. 64, pp. 1125-49.

[7] BERGEMANN, J. and J. VALIMAKI (1997): "Market Diffusion with Two-Sided Learning", *The RAND Journal of Economics*, Vol. 28, No. 4, pp. 773-795.

[8] BERGEMANN, J. and J. VALIMAKI (2005): "Monopoly Pricing of Experience Goods", Cowles Foundation Discussion Paper No. 1463 .

[9] BERRIOS, R. and R. SAENS (2012): "The Country Brand Trap", Cepal Review No. 106.

[10] CHEN, Y. and J. XIE (2008): "Word-of-Mouth as a New Element of Marketing Communication Mix", *Managment Science,* Vol. 54, No. 3, pp. 477 - 491.

[11] CHEN, J. and J.A. RIZZO (2012): "Pricing Dynamics and Product Quality: the case of antidepressant drugs", *Empirical Economics,* Vol. 42, No. 1, pp. 279-300.

[12] CHEVALIER, J. and D. MAYZLIN (2006): "The Effect of Word of Mouth on Sales", *Journal of Marketing Research*, Vol. 43, No. 3, pp. 345-354.

[13] CHO, I. and D. KREPS (1987): "Signaling Games and Stable Equilibria", *The Quarterly Journal of Economics*, Vol. 102, No. 2, pp. 179-221.

[14] DAWAR, N. and M. SARVARY (1997): "The Signaling Impact of Low Introductory Price on Perceived Quality and Trial", *Marketing Letters*, Vol. 8, No. 3, pp. 251-259.

[15] DEAN, J. (1969): "Pricing Pioneering Products", *The Journal of Industrial Economics,* Vol. 17, No. 3, pp. 165-179.

[16] FELZENSZTEIN, C. and C. RODRIGUEZ (2014): "New world Wines in the UK Market: re-thinking the right strategies for 2020", Emerald Emerging Markets Case Studies, Vol. 3, No. 4, pp. 1-10.

[17] JUDD, K. and M. RIORDAN (1994): “Price and Quality in a New Product Monopoly”, *The Review of Economic Studies,* Vol. 61, No. 4, pp. 773-789.

[18] KALISH, S. (1985): “A New Product Adoption Model with Price, Advertising and Uncertainty”, *Managment Science,* Vol. 31, No. 12, pp. 1569-1585.

[19] KRISHNAM, T.V., BASS, F.M. and D.C. JAIN (1999): “Optimal Pricing Strategy for New Products”, *Managment Science,* Vol. 45, No. 12, pp. 1650-1663.

[20] LANDON, S. and C.E. SMITH (1998): “Quality Expectations, Reputation, and Price”, *Southern Economic Journal*, Vol. 64, No. 3, pp. 628–647.

[21] LECOCQ, S. and M. VISSER (2006): “What Determines Wine Prices: Objective vs. Sensory Characteristics”, *Journal of Wine Economics*, Vol. 1, pp. 42–56.

[22] LINNEMER, L. (2002): “Price and Advertising as Signals of Quality when Some Consumers are Informed”, *International Journal of Industrial Organization,* Vol. 20, pp. 931-947.

[23] MACCHIAVELLO, R. (2010): “Development Uncorked: Reputation Acquisition in the New Market for Chilean Wines in the UK”, CEPR Discussion Papers No. 7698.

[24] MILGROM, P. and J. ROBERTS (1986): “Price and Advertising Signals of Product Quality”, *Journal of Political Economy,* Vol. 94, No. 4, pp. 796-821.

[25] MILLER, J.R., GENC, I., and A. DRISCOLL (2007): “Wine Price and Quality: In Search of a Signaling Equilibrium in 2001 California Cabernet Sauvignon”, *Journal of Wine Research*, Vol. 18, No. 1, pp. 35-46.

[26] NOBLE, P.M. and T.S. GRUCA (1999): “Industrial Pricing: Theory and Managerial Practices”, *Marketing Science,* Vol. 18, No. 3, pp. 435-454.

[27] OCZKOWSKI, E. (2001): “Hedonic Wine Price Functions and Measurement Error”, *Economic Record,* Vol. 77, pp.374–382.

[28] ROBERTS, P.W. and R. REAGANS (2007): "Critical Exposure and Price-Quality Relationships for New World Wines in the U.S. Market", *Journal of Wine Economics*, Vol. 2, No. 1, pp. 56-69.

[29] ROSEN, S. (1974): "Hedonic Prices and Implicit Markets: Product Differentiation in Pure Competition", *Journal of Political Economy*, Vol. 82, No. 1, pp. 34–55.

[30] SCHLEE, E. (2001): "Buyer Experimentation and Introductory Pricing", *Journal of Economic Behavior and Organization,* Vol. 44, pp. 347-362.

[31] SHAPIRO, C. (1983): "Optimal Pricing of Experience Goods", *The Bell Journal of Economics,* Vol. 14, No. 2, pp. 497-507.

[32] SCHNABEL, H. and K. STORCHMANN (2010), "Prices as Quality Signals: Evidence from the Wine Market", *Journal of Agricultural and Food Industrial Organization*, Vol. 8, No. 1, pp. 1-23.

[33] ZHU, F. and X. ZANG (2010): "Impact of Online Consumer Reviews on Sales: The Moderating Role of Product and Consumer Characteristics", *Journal of Marketing,* Vol. 74, No. 2, pp. 133-148.

Appendix

1. Second-period beliefs

In order to capture signaling and experimentation effects, we construct $\mu_{2,q}$ as a weighted average of three noisy signals: μ_0(the prior belief), μ_1 (the belief derived from (indirect) observation of P_1), and μ_q (the belief derived from (indirect) experience with the product):

$$\mu_{2,q} = \alpha\mu_1 + (1-\alpha) Q_1 \mu_q + (1-\alpha)(1-Q_1)\mu_0,$$

where $\alpha \in (0,1)$ and Q_1 is the quantity sold by the monopolist in the first period, which influences the speed of information diffusion. We assume that μ_0, μ_1 and $\mu_{2,q}$ are normally and independently distributed. Specifically we assume that $\mu_0 \equiv \mu + \varepsilon_0$, where ε_0 is a normally (i.i.d) distributed error with mean 0 and precision s, ($\varepsilon_0 \sim N(0,s)$), $\mu_1 \equiv \mu + \varepsilon_1$, with $\varepsilon_1 \sim N(0, s_1(Q))$ and $\mu_q \equiv \mu + \varepsilon_2$, with $\varepsilon_2 \sim N(0, s_2(Q))$. Normality has the pleasent feature of implying linear updating rules for consumers, who learn about μ through the observations of the μ_0, μ_1 and μ_q. This learning process is well-known given the normality and independence assumptions about the signals:

$$\mu_{2,q} = \frac{s_1(Q)}{s + s_1(Q) + s_2(Q)}\mu_1 + \frac{s_2(Q)}{s + s_1(Q) + s_2(Q)}\mu_q + \frac{s}{s + s_1(Q) + s_2(Q)}\mu_0.$$

Since we have one degree of freedom, for arbitrary s, we solve for $s_1(Q)$ and $s_2(Q)$:

$$s_1^*(Q) = \frac{s}{(1-\alpha)\left[\frac{1}{\alpha} - \frac{(1-\alpha)Q}{\alpha} - Q\right]}$$

$$s_2^*(Q) = \frac{(1-\alpha)s}{\alpha(1-\alpha)\left[\frac{1}{\alpha} - \frac{(1-\alpha)Q}{\alpha} - Q\right]} - s,$$

with $\frac{\partial s_1^*(Q)}{\partial Q} > 0$ and $\frac{\partial s_2^*(Q)}{\partial Q} > 0$.

Note that the precisions s_1^* and s_2^* depend positively on the quantity sold, which means that more sales are associated with more (precise) information diffusion. Any functional form for beliefs $\mu_{2,q}$ such that $\frac{\partial^2 \mu_{2,q}}{\partial \mu_q \partial Q} > 0$ allows for signaling and experimentation effects. We choose the functional form presented in section 2 for simplicity.

2. Separating Equilibrium with $\alpha = 0$: Quadratic Inequalities

Condition 2 of Definition 1 rules out the possibility that the bad firm would mimic the good one, and allows us to restrict attention to prices P_1^g such that $\Pi^b\left(P_1^{b*}, 0\right) - \Pi^b\left(P_1^g, 1\right) \geq 0$. The bad firm has no incentive to mimic prices that are either "too low" or "too high": $P_1^g \leq \underline{P}_b$ or $P_1^g \geq \overline{P}_b$, where $\underline{P}_b$ and $\overline{P}_b$ are the roots of the quadratic inequality depicted in Figure 1:

$$\Pi^b\left(P_1^{b*}, 0\right) - \Pi^b\left(P_1^g, 1\right) = P_1^{g^2} - P_1^g X_b + Y_b \geq 0$$

where

$$X_b = \left\{ g + \frac{\mu_0 (g-b)}{4} - \frac{\mu_0 (1-b)(1-g)(g-b)}{4\left[\mu_0 (1-g) + (1-\mu_0)(1-b)\right]} - \frac{\mu_0 b g (g-b)}{4\left[\mu_0 g + (1-\mu_0) b\right]} \right\},$$

$$Y_b = \left\{ -\frac{g}{b} P_1^{b*^2} + P_1^{b*} \left[g + \frac{\mu_0 g (g-b)}{4b} - \frac{\mu_0 g (1-b)(1-g)(g-b)}{4b\left[\mu_0 (1-g) + (1-\mu_0)(1-b)\right]} - \frac{\mu_0 b g^2 (g-b)}{4b\left[\mu_0 g + (1-\mu_0) b\right]} \right] \right\}$$

$$P_1^{b*} = \left\{ b - \frac{\mu_0 b g (g-b)}{4\left[\mu_0 g + (1-\mu_0) b\right]} - \frac{\mu_0 (1-b)(1-g)(g-b)}{4\left[\mu_0 (1-g) + (1-\mu_0)(1-b)\right]} + \frac{\mu_0 (g-b)}{4} \right\}.$$

At the same time, Condition 3 of Definition 1 states that separation can occur only if the good monopolist chooses not to monopolize consumers and be perceived as a bad firm. Thus the good-type firm has no incentive to deviate and monopolize the entire market when the equilibrium price P_1^g is such that $\underline{P}_g \leq P_1^g \leq \overline{P}_g$, where $\underline{P}_g$ and $\overline{P}_g$ are the roots of the quadratic inequality depicted in Figure 1:

$$\Pi^g\left(P_1^g, 1\right) - \Pi^g\left(P_1^{g*}, 0\right) = -P_1^{g^2} + P_1^g X_g + Y_g \geq 0$$

where

$$X_g = \left\{ g + \frac{\mu_0 (g-b)}{4} - \frac{\mu_0 (1-g)^2 (g-b)}{4 [\mu_0 (1-g) + (1-\mu_0)(1-b)]} - \frac{\mu_0 g^2 (g-b)}{4 [\mu_0 g + (1-\mu_0) b]} \right\},$$

$$Y_g = \left\{ \frac{g}{b} P_1^{g*^2} - P_1^{g*} \left[g + \frac{\mu_0 g (g-b)}{4b} - \frac{\mu_0 g (1-g)^2 (g-b)}{4b [\mu_0 (1-g) + (1-\mu_0)(1-b)]} - \frac{\mu_0 g^3 (g-b)}{4b [\mu_0 g + (1-\mu_0) b]} \right] \right\}$$

$$P_1^{g*} = \left\{ b - \frac{\mu_0 g^2 (g-b)}{4 [\mu_0 g + (1-\mu_0) b]} - \frac{\mu_0 (1-g)^2 (g-b)}{4 [\mu_0 (1-g) + (1-\mu_0)(1-b)]} + \frac{\mu_0 (g-b)}{4} \right\}$$

3. Pooling Equilibrium with $\alpha = 0$: Quadratic Inequalities

Lemma 8 states that we can restrict attention to pooling prices P_1 such that two quadratic inequalities simultaneously hold: $\Pi^b (P_1, \mu_0) - \Pi^b \left(P_1^{b*}, 0\right) \geq 0$ and $\Pi^g (P_1, \mu_0) - \Pi^g (P_1^{g*}, 0) \geq 0$. We shown that the first-period pooling equilibrium is such that $\overline{P}_b \leq P_1 \leq \overline{P}_g$, where $\overline{P}_b$ and $\overline{P}_g$ are the upper roots of the quadratic inequalities $\Pi^b (P_1, \mu_0) - \Pi^b \left(P_1^{b*}, 0\right) \geq 0$ and $\Pi^g (P_1, \mu_0) - \Pi (P_1^{g*}, 0) \geq 0$, respectively. We can write the former inequalities as: $\Pi^b (P_1, \mu_0) \geq \Pi^b \left(P_1^{b*}, 0\right) = -P_1^2 + P_1 X_b + Y_b \geq 0$, with

$$X_b = \left\{ \frac{4(\mu_0 g + (1-\mu_0) b) + (g-b) \left[\mu_0 - \frac{\mu_0 b g}{\mu_0 g + (1-\mu_0) b} - \frac{\mu_0 (1-b)(1-g)}{\mu_0 (1-g) + (1-\mu_0)(1-b)} \right]}{4} \right\}$$

$$Y_b = \left\{ \frac{(\mu_0 g + (1-\mu_0) b)}{b} P_1^{b*^2} - P_1^{b*} (\mu_0 g + (1-\mu_0) b) \left[\frac{4b + (g-b) \left(\mu_0 - \frac{\mu_0 b g}{\mu_0 g + (1-\mu_0) b} - \frac{\mu_0 (1-b)(1-g)}{\mu_0 (1-g) + (1-\mu_0)(1-b)} \right)}{4b} \right] \right\}$$

$$P_1^{b*} = \left\{ b - \frac{\mu_0 b g (g-b)}{4 [\mu_0 g + (1-\mu_0) b]} - \frac{\mu_0 (1-b)(1-g)(g-b)}{4 [\mu_0 (1-g) + (1-\mu_0)(1-b)]} + \frac{\mu_0 (g-b)}{4} \right\}.$$

and $\Pi^g (P_1, \mu_0) \geq \Pi^g (P_1^{g*}, 0) = -P_1^2 + P_1 X_g + Y_g \geq 0$, with

$$X_g = \left\{ \frac{4(\mu_0 g + (1-\mu_0) b) + (g-b)\left[\mu_0 - \frac{\mu_0 g^2}{\mu_0 g + (1-\mu_0) b} - \frac{\mu_0 (1-g)^2}{\mu_0 (1-g) + (1-\mu_0)(1-b)}\right]}{4} \right\},$$

$$Y_g = \left\{ \frac{(\mu_0 g + (1-\mu_0) b)}{b} P_1^{g*^2} - P_1^{g*} (\mu_0 g + (1-\mu_0) b) \left[\frac{4b + (g-b)\left(\mu_0 - \frac{\mu_0 g^2}{\mu_0 g + (1-\mu_0) b} - \frac{\mu_0 (1-g)^2}{\mu_0 (1-g) + (1-\mu_0)(1-b)}\right)}{4b} \right] \right\}.$$

$$P_1^{g*} = \left\{ b - \frac{\mu_0 g^2 (g-b)}{4[\mu_0 g + (1-\mu_0) b]} - \frac{\mu_0 (1-g)^2 (g-b)}{4[\mu_0 (1-g) + (1-\mu_0)(1-b)]} + \frac{\mu_0 (g-b)}{4} \right\}.$$

4. Proof of Proposition 5

The proof consists of two steps. STEP 1: We show that there is always a separating equilibrium for $P_1^g \leq \underline{P}_b$. It is sufficient to show that $\Pi^g(\underline{P}_b, 1) > \Pi^g(P_1^{g*}, 0)$, i.e. the good firm is better off by charging the equilibrium price $P_1^g = \underline{P}_b$ rather than deviating to the profit-maximizing price P_1^{g*}. Note that the equilibrium price $P_1^g = \underline{P}_b$ is such that the bad firm is indifferent between following the equilibrium strategy and mimicking the good monopolist $\left(\Pi^b(\underline{P}_b, 1) = \Pi^b\left(P_1^{b*}, 0\right)\right)$.

We denote $U^2\left(Q_1^\theta, \theta\right)$ the second-period profits of a type-θ firm who sold the quantity Q_1^θ in the first period. So we can rewrite the expected profits of a type-θ firm, that charges P_1 inducing beliefs μ_1 in the first period, as: $\Pi^\theta(P_1, \mu_1) = P_1 Q_1^\theta + U^2\left(Q_1^\theta, \theta\right)$. Now we can write: $\Pi^g(\underline{P}_b, 1) - \Pi^g(P_1^{g*}, 0)$

$$= \underline{P}_b\left(1 - \tfrac{\underline{P}_b}{g}\right) + U^2\left(1 - \tfrac{\underline{P}_b}{g}, g\right) - P_1^{g*}\left(1 - \tfrac{P_1^{g*}}{b}\right) - U^2\left(1 - \tfrac{P_1^{g*}}{b}, g\right)$$

$$= P_1^{b*}\left(1 - \tfrac{P_1^{b*}}{b}\right) + U^2\left(1 - \tfrac{P_1^{b*}}{b}, b\right) - U^2\left(1 - \tfrac{\underline{P}_b}{g}, b\right) + U^2\left(1 - \tfrac{\underline{P}_b}{g}, g\right) - P_1^{g*}\left(1 - \tfrac{P_1^{g*}}{b}\right) - U^2\left(1 - \tfrac{P_1^{g*}}{b}, g\right)$$

$$\geq P_1^{g*}\left(1 - \tfrac{P_1^{g*}}{b}\right) + U^2\left(1 - \tfrac{P_1^{g*}}{b}, b\right) - U^2\left(1 - \tfrac{\underline{P}_b}{g}, b\right) + U^2\left(1 - \tfrac{\underline{P}_b}{g}, g\right) - P_1^{g*}\left(1 - \tfrac{P_1^{g*}}{b}\right) - U^2\left(1 - \tfrac{P_1^{g*}}{b}, g\right)$$

$$= U^2\left(1 - \tfrac{P_1^{g*}}{b}, b\right) - U^2\left(1 - \tfrac{\underline{P}_b}{g}, b\right) + U^2\left(1 - \tfrac{\underline{P}_b}{g}, g\right) - U^2\left(1 - \tfrac{P_1^{g*}}{b}, g\right)$$

$$\geq U^2\left(1 - \tfrac{P_1^{g*}}{b}, b\right) - U^2\left(1 - \tfrac{\underline{P}_b}{b}, b\right) + U^2\left(1 - \tfrac{\underline{P}_b}{b}, g\right) - U^2\left(1 - \tfrac{P_1^{g*}}{b}, g\right),$$

where the first equality uses the fact that $\underline{P}_b$ is such that the bad firm is indifferent between following the equilibrium strategy and mimicking the good monopolist $\left(\Pi^b\left(\underline{P}_b,1\right)=\Pi^b\left(P_1^{b*},0\right)\right)$. The first inequality comes from the fact that the bad-type monopolist will be better off charging her maximizing price (P_1^{b*}) rather than charging the good firm maximizing price (P_1^{g*}), and the last inequality comes from the fact that a bad firm will earn more profits in the second period if it is perceived to be good with smaller probability.

Finally, a simple computation yields that

$$U^2\left(1-\frac{P_1^{g*}}{b},b\right)-U^2\left(1-\frac{\underline{P}_b}{b},b\right)+U^2\left(1-\frac{\underline{P}_b}{b},g\right)-U^2\left(1-\frac{P_1^{g*}}{b},g\right)\geq 0$$

iff $\left(1-\frac{\underline{P}_b}{b}\right)\geq\left(1-\frac{P_1^{g*}}{b}\right)$, or equivalently $\underline{P}_b\leq P_1^{g*}$. To check that effectively $\underline{P}_b\leq P_1^{g*}$, it is enough to check that $\Pi^b\left(P_1^{b*},0\right)-\Pi^b\left(P_1^{g*},1\right)<0$, i.e. the bad firm strictly prefers to mimic the good one when the separating equilibrium price is $P_1^g=P_1^{g*}$. We then evaluate the quadratic inequality $\Pi^b\left(P_1^{b*},0\right)-\Pi^b\left(P_1^g,1\right)=P_1^{g^2}-P_1^gX_b+Y_b$ at $P_1^g=P_1^{g*}$ and show that it is negative. Simple computation yields that:

$$P_1^{g*}\left[(g-b)-\frac{\mu_0 g\,(g-b)^2}{4\left[\mu_0 g+(1-\mu_0)\,b\right]}+\frac{\mu_0\,(1-g)\,(g-b)^2}{4\left[\mu_0\,(1-g)+(1-\mu_0)\,(1-b)\right]}\right]<0.$$

STEP 2: We now show that there is no separating equilibrium with $P_1^g>\underline{P}_b$ if $\overline{P}_b>\frac{g}{b}P_1^{b*}$. It is sufficient to show that $\Pi^g\left(\overline{P}_b,1\right)<\Pi^g\left(P_1^{g*},0\right)$, i.e. the good firm is better off by deviating to the profit-maximizing price P_1^{g*}, rather than charging the equilibrium price $P_1^g=\overline{P}_b$. Note that the equilibrium price $P_1^g=\overline{P}_b$ is such that the bad firm is indifferent between following the equilibrium strategy and mimicking the good monopolist $\left(\Pi^b\left(\overline{P}_b,1\right)=\Pi^b\left(P_1^{b*},0\right)\right)$. We denote $U^2\left(Q_1^\theta,\theta\right)$ the second-period profits of a type-θ firm who sold the quantity Q_1^θ in the first period. So we can rewrite the expected profits of a type-θ firm, that charges P_1 inducing beliefs μ_1 in the first period, as $\Pi^\theta\left(P_1,\mu_1\right)=P_1Q_1^\theta+U^2\left(Q_1^\theta,\theta\right)$. Now we can write $\Pi^g\left(\overline{P}_b,1\right)-\Pi^g\left(P_1^{g*},0\right)$

$$=\overline{P}_b\left(1-\tfrac{\overline{P}_b}{g}\right)+U^2\left(1-\tfrac{\overline{P}_b}{g},g\right)-P_1^{g*}\left(1-\tfrac{P_1^{g*}}{b}\right)-U^2\left(1-\tfrac{P_1^{g*}}{b},g\right)$$

$$= P_1^{b*}\left(1-\tfrac{P_1^{b*}}{b}\right)+U^2\left(1-\tfrac{P_1^{b*}}{b},b\right)-U^2\left(1-\tfrac{\overline{P}_b}{g},b\right)+U^2\left(1-\tfrac{\overline{P}_b}{g},g\right)-P_1^{g*}\left(1-\tfrac{P_1^{g*}}{b}\right)-U^2\left(1-\tfrac{P_1^{g*}}{b},g\right)$$

$$< P_1^{b*}\left(1-\tfrac{P_1^{b*}}{b}\right)+U^2\left(1-\tfrac{P_1^{b*}}{b},b\right)-U^2\left(1-\tfrac{\overline{P}_b}{g},b\right)+U^2\left(1-\tfrac{\overline{P}_b}{g},g\right)-P_1^{b*}\left(1-\tfrac{P_1^{b*}}{b}\right)-U^2\left(1-\tfrac{P_1^{b*}}{b},g\right)$$

$$= U^2\left(1-\tfrac{P_1^{b*}}{b},b\right)-U^2\left(1-\tfrac{\overline{P}_b}{g},b\right)+U^2\left(1-\tfrac{\overline{P}_b}{g},g\right)-U^2\left(1-\tfrac{P_1^{b*}}{b},g\right),$$

where the first equality uses the fact that $\overline{P}_b$ is such that $\Pi\left(\overline{P}_b,1\right)=\Pi^b\left(P_1^{b*},0\right)$, and the first inequality comes from the fact that the good monopolist is better off by charging her maximizing price (P_1^{g*}) rather than charging the bad firm maximizing price (P_1^{b*}).

Finally, a simple computation yields that

$$U^2\left(1-\frac{P_1^{b*}}{b},b\right)-U^2\left(1-\frac{\overline{P}_b}{g},b\right)+U^2\left(1-\frac{\overline{P}_b}{g},g\right)-U^2\left(1-\frac{P_1^{b*}}{b},g\right)<0$$

iff $\left(1-\frac{\overline{P}_b}{g}\right)<\left(1-\frac{P_1^{b*}}{b}\right)$, or equivalently $\overline{P}_b>\frac{g}{b}P_1^{b*}$, which is true by assumption. ■

5. Proof of Proposition 9

STEP 1: We now show that there is always a pooling equilibrium for $P_1\le\overline{P}_g$. It is sufficient to show that $\Pi^b\left(\overline{P}_g,\mu_0\right)>\Pi^b\left(P_1^{b*},0\right)$, i.e. the bad firm is better off by charging the equilibrium price $P_1=\overline{P}_g$ rather than deviating to the profit-maximizing price P_1^{b*}. Note that the pooling equilibrium price $P_1=\overline{P}_g$ is such that the good firm is indifferent between following the equilibrium strategy and charging its maximizing price $\left(\Pi^g\left(\overline{P}_g,\mu_0\right)=\Pi^g\left(P_1^{g*},0\right)\right)$. We denote $U^2\left(Q_1^\theta,\theta\right)$ the second-period profits of a type-θ firm who sold the quantity Q_1^θ in the first period. So we can rewrite the expected profits of a type-θ firm, that charges P_1 inducing beliefs μ_1 in the first period, as: $\Pi^\theta\left(P_1,\mu_1\right)=P_1Q_1^\theta+U^2\left(Q_1^\theta,\theta\right)$. Now we can write: $\Pi^b\left(\overline{P}_g,\mu_0\right)-\Pi^b\left(P_1^{b*},0\right)$

$$=\overline{P}_g\left(1-\tfrac{\overline{P}_g}{\mu_0 g+(1-\mu_0)b}\right)+U^2\left(1-\tfrac{\overline{P}_g}{\mu_0 g+(1-\mu_0)b},b\right)-P_1^{b*}\left(1-\tfrac{P_1^{b*}}{b}\right)-U^2\left(1-\tfrac{P_1^{b*}}{b},b\right)$$

$$=P_1^{g*}\left(1-\tfrac{P_1^{g*}}{b}\right)+U^2\left(1-\tfrac{P_1^{g*}}{b},g\right)-P_1^{b*}\left(1-\tfrac{P_1^{b*}}{b}\right)-U^2\left(1-\tfrac{P_1^{b*}}{b},b\right)+U^2\left(1-\tfrac{\overline{P}_g}{\mu_0 g+(1-\mu_0)b},b\right)-U^2\left(1-\tfrac{\overline{P}_g}{\mu_0 g+(1-\mu_0)b},g\right)$$

$$\ge P_1^{b*}\left(1-\tfrac{P_1^{b*}}{b}\right)+U^2\left(1-\tfrac{P_1^{b*}}{b},g\right)-P_1^{b*}\left(1-\tfrac{P_1^{b*}}{b}\right)-U^2\left(1-\tfrac{P_1^{b*}}{b},b\right)+U^2\left(1-\tfrac{\overline{P}_g}{\mu_0 g+(1-\mu_0)b},b\right)-U^2\left(1-\tfrac{\overline{P}_g}{\mu_0 g+(1-\mu_0)b},g\right)$$

$$= U^2\left(1-\frac{P_1^{b*}}{b}, g\right) - U^2\left(1-\frac{P_1^{b*}}{b}, b\right) + U^2\left(1-\frac{\overline{P}_g}{\mu_0 g+(1-\mu_0)b}, b\right) - U^2\left(1-\frac{\overline{P}_g}{\mu_0 g+(1-\mu_0)b}, g\right)$$

$$\geq U^2\left(1-\frac{P_1^{b*}}{\mu_0 g+(1-\mu_0)b}, g\right) - U^2\left(1-\frac{P_1^{b*}}{\mu_0 g+(1-\mu_0)b}, b\right) + U^2\left(1-\frac{\overline{P}_g}{\mu_0 g+(1-\mu_0)b}, b\right) - U^2\left(1-\frac{\overline{P}_g}{\mu_0 g+(1-\mu_0)b}, g\right),$$

where the first equality uses the fact that $\overline{P}_g$ is such that the good firm is indifferent between following the equilibrium strategy and charging her maximizing price $\left(\Pi^g\left(\overline{P}_g, \mu_0\right) = \Pi^g\left(P_1^{g*}, 0\right)\right)$. The first inequality comes from the fact that the good-type monopolist will be better off charging her maximizing price (P_1^{g*}) rather than charging the bad firm maximizing price (P_1^{b*}), and the last inequality comes from the fact that a bad firm will earn more profits in the second period if it sells a smaller quantity in the first period, whereas for a good firm the contrary is true. Finally

$$U^2\left(1-\frac{P_1^{b*}}{\mu_0 g+(1-\mu_0)b}, g\right) - U^2\left(1-\frac{P_1^{b*}}{\mu_0 g+(1-\mu_0)b}, b\right) + U^2\left(1-\frac{\overline{P}_g}{\mu_0 g+(1-\mu_0)b}, b\right) - U^2\left(1-\frac{\overline{P}_g}{\mu_0 g+(1-\mu_0)b}, g\right) \geq 0$$

iff $\left(1-\frac{P_1^{b*}}{\mu_0 g+(1-\mu_0)b}\right) \geq \left(1-\frac{\overline{P}_g}{\mu_0 g+(1-\mu_0)b}\right)$, or equivalently $P_1^{b*} \leq \overline{P}_g$. To check that effectively $P_1^{b*} \leq \overline{P}_g$, it is enough to check that $\Pi^g\left(P_1^{g*}, 0\right) - \Pi^g\left(P_1^{b*}, \mu_0\right) < 0$, i.e. the good firm strictly prefers to follow the pooling equilibrium $(P_1 = P_1^{b*})$ rather than monopolize the market. Simple computation shows that when evaluating the quadratic inequality $\Pi^g\left(P_1^{g*}, 0\right) - \Pi^g\left(P_1, \mu_0\right)$, at $P_1 = P_1^{b*}$, it is indeed negative. ■

Price Signaling with Information Acquisition

Abstract

We study a market in which the buyer has no information about product quality, while the seller has private probabilistic information about it. Buyers observe price and can procure an inspection, which provides valuable information about the good for sale. With costless inspections, there is no separating equilibrium. We then show that when information acquisition is costly, there is a separating equilibrium that satisfies the intuitive criterion, in which high prices signal high quality and furthermore, the dynamic separating equilibrium showing higher separating prices than the static one. Finally we discuss the implications of time-on-the-market on separating equilibria. Specifically, when there is only one asset on sale over both periods (therefore both price and time-on-the-market can signal quality) there is no separating equilibrium even if single-crossing is satisfied. The key to this result is that the second-period buyer cannot observe why the asset did not sell in the first period. Notably, the failure to sell can be attributed to overpricing or an unfavorable inspection outcome. Therefore the copycat behavior is more attractive to the poor-quality seller because he benefits more from an increase in buyer beliefs than his high-quality counterpart. Allowing only the first-period buyer to acquire information on quality, we show the existence of a separating equilibrium in which high prices and time-on-the-market signal high quality.

1 Introduction

Information acquisition is a common activity before making a purchase. Since consumers often cannot ascertain the quality of the product for sale, they are willing to invest time and money on inspecting the good before purchasing it. Inspections are pervasive in the real estate, car and art markets, where prospective buyers spend significant resources trying to avoid costly mistakes. Since inspections are costly and imperfect, agents are strategic in their use. The price of an asset as well as the prior assessment of its quality influence the willingness to pay for information acquisition. High-priced items are more susceptible to be inspected, but also pessimistic beliefs about the item's quality encourage the gathering of information. When the seller is better informed about product quality than the buyers, prices might be used as a signaling device by good-quality sellers. This is so because high prices encourage information acquisition, and inspections are more costly for low-quality sellers (it is more often than their products are found wanting). We characterize the optimal pricing strategy for high-quality assets when information acquisition prior to purchase is possible.

We set up a model with asymmetric information regarding quality and information acquisition, in which the buyer has no information about the asset quality, while the seller has private information about the probability of owning a high-quality asset. Buyers observe prices and can procure an inspection, which provides valuable information about the asset for sale. The seller's pricing strategy provides a signal from which buyers can infer its type, even as it determines the precision of the information regarding product quality they can acquire. Higher prices lead to higher product exposure via inspections, so that prices become an instrument through which the seller may encourage or discourage information acquisition. We look for separating (pure-strategy) equilibria and apply the intuitive criterion refinement (Cho and Kreps 1987) to eliminate implausible off-equilibrium path beliefs.

We first consider, as a benchmark, the case with costless inspections, and show there is no separating equilibrium. Any strategy that is profitable for the high-quality seller can be imitated by the low-quality one, who just sells less often. We then consider the case in which the buyer might increase his signal precision, through costly information acquisition. Here, we show there is a separating equilibrium in which high prices signal high quality. The intuition is that high prices induce more information acquisition, which is incentivized by sellers confident enough that inspections will yield good news and therefore will not decrease demand substantially.

We finally discuss the implications of time-on-the-market on separating equilibria, when the selling season is composed of two periods and there is only one asset on sale over both periods. Then history becomes important since it carries significant information. When facing a decision in the second period, the buyer knows that the first-period one chooses not to buy for one of two reasons. Either the common component signal was bad (which is very relevant for him) or the private component was low (which is irrelevant). Higher prices put a higher weight on the overpricing explanation, decreasing information transmission. We show that the sufficient conditions for the existence of a separating equilibrium do not always hold. The key to this result is that the good seller sells more often in the first period, ending the game, which in turn makes the poor-quality seller more interested in the reputational effects induced by a no-sale in the first period. Since this is achieved through high prices, the possibilities of mimicking the good type are enhanced, making separation often impossible. Finally, we show the existence of a separating equilibrium in which high prices and time-on-the-market signal high quality, when only first-period buyer is allowed to acquire information. In this case, the second-period buyer learns about quality from observing the price history as well as the past purchase decisions.

Literature Review. The model developed here is related to the literature on signaling (high) quality through (high) prices. Bagwell and Riordan (1991) show that high (and declining) prices signal high quality, in a monopoly market for durable goods, in which quality is correlated with costs. Judd and Riordan (1994) reach the same result by examining a two-period signal-extraction model with learning. Even though no correlation between quality and costs is assumed, private information on both sides of the market allows the seller to signal high quality through high prices. Neither costly information acquisition nor time-time-on-the-market play a role in achieving an effective high-price signal. In both models, imperfectly informed consumers may interpret signals to effectively improve information, but inspections or other forms of information acquisition are not available. This paper is also closely related to the literature on signaling with information acquisition. For example, Bester and Ritzberger (2001) consider a static model in which an informed monopolist chooses the optimal pricing strategy for an asset of unknown quality to consumers. Consumers can infer quality from the price or pay for access to an external source of information. For small costs of information acquisition, there is no separating equilibrium in pure strategies, which confirms the Grossman-Stiglitz paradox. Prices cannot be informative, because if they were, no one will pay for information and a low-quality seller would mimic the high-quality one. Furthermore, they show there is a unique partial pooling equilibrium, that resolves the paradox and satisfies the intuitive criterion, which involves mixed strategies and sufficiently small costs of information acquisition. The model presented here achieves a separating equilibrium without the use of mixed strategies or exogenous informational sources. Gertz (2014) examines a monopolistic market with quality uncertainty and information acquisition, a setting very similar to the one presented in Bester and Ritzberger (2001). Nevertheless information acquisition is now endogenously determined, by allowing the buyer to optimally choose the search effort. He characterizes all possible market equilibria and focuses on consumer's behavior and welfare. The main result is that consumer's welfare is maximized at a pooling equilibrium with no search. If the buyer is given the possibility of information acquisition, he can use this search ability as a threat (even if the search proves fruitless), which forces down the equilibrium price. This paper is the closest to the model presented here, even as the focus is on consumer behavior and welfare rather than the strategic actions of firms. Furthermore, time-on-the-market is not considered. Mezzetti and Tsoulouhas (2000) analyze a principal-agent model where the principal is privately informed about his type and the agent could gather information about the principal's type, at a monetary cost, before engaging in a relationship. They find that, if uncertainty is high and the precontractual investigation is not too costly, there exists a separating equilibrium in which a favorable principal is able to separate himself from his unfavorable counterpart. Separation can occur due to the renegotiation option offered by the principal in the worst case scenario that the investigation results in an unfavorable outcome. The idea of signaling with costly information acquisition is present here, but applied to the context of a principal-agent optimal contract problem. Finally we extend the literature on time-on-the-market as sign of quality, initiated by Taylor (1999). Taylor (1999) explores the effect of time-on-the-market on pricing in a two-period model with asymmetric information and a single object for sale (a house). The parametric assumption made about the quality of the item allows him to rule out separating equilibria and focus attention to consumer learning. The main result is obtained without considering information acquisition and involves a pooling equilibrium, in which the low-quality seller mimic his high-quality counterpart. Depending on the information structure, the seller may post a higher or a lower price in the first-period.

2 The Model

We consider a model with asymmetric information regarding quality and information acquisition, in which the buyer has no information about product quality, while the seller has private probabilistic information about it. The quality of the asset may be either high or low, $q \in \{0, 1\}$, and the seller is aware of his type θ, the probability of owning a high quality asset, that can be either good or bad, $\theta \in \{g, b\}$, with $0 < b < g < 1$. The ex post valuation of the buyer is qv, where q represents the common valuation (objective quality) and v is the buyer individual valuation or "taste" for the product, drawn from a distribution $G(v)$ continuously differentiable with $G'(v) = g(v) > 0$, for all $v \in [0, 1]$. The seller's valuation of the asset and his production cost are zero and there is no discounting.

Each period, after observing the price and before making his purchase decision, the buyer procures an inspection on quality. The outcome of the inspection may be either favorable or unfavorable, $s \in \{F, NF\}$ and it is characterized by the following conditional probabilities:

	0	1
F	$1-\sigma$	1
NF	σ	0

A high-quality asset always results in a favorable outcome, whereas a low-quality one generates it with probability $(1-\sigma)$. Therefore a favorable outcome does not guarantee high quality, whereas an unfavorable outcome can be thought of as discovering a flaw in the asset, fully revealing low quality. Note that no buyer will buy the asset if the inspection outcome was unfavorable. Here, $\sigma \in [0, \overline{\sigma}]$, can be interpreted as a measure of the signal precision[13]. Let $C(\sigma)$ be the cost of acquiring information about the asset quality through by procuring an inspection, with $C'(\sigma) > 0$ and $C''(\sigma) > 0$, for $\sigma > 0$; $C(0) = C'(0) = 0$ and $C'(\overline{\sigma}) > 1$.

The timing of the game is the following: the seller's type is drawn by Nature at the outset; the seller learns his type and chooses a pricing strategy. After observing the price, the

[13]The results obtained in this paper are robust to more general signal structures that satisfies the following assumptions: $Pr(F \mid q = 1) > Pr(F \mid q = 0)$ and $Pr(NF \mid q = 1) < Pr(NF \mid q = 0)$.

buyer updates beliefs on the asset quality and procures the inspection on quality. The inspection's outcome is realized, then the buyer updates beliefs accordingly and makes a purchase decision.

The buyer starts the game with a prior belief on the asset being of high quality, $\mu_0 = Pr(q=1)$, and updates beliefs according to Bayes rule, after observing the price. Note that separating prices $\left(P^b, P^g\right)$ induce beliefs $\mu = b$ if $P = P^b$ and $\mu = g$ if $P = P^g$, whereas pooling prices do not provide any information and the posterior will be the same as the prior, $\mu = \mu_0$. After observing the price, the buyer procures the inspection on quality. Given a price P and a belief μ, the buyer solves

$$\max_{\sigma \in [0,\overline{\sigma}]} \mu \left(v - P\right) - P\left(1-\mu\right)\left(1-\sigma\right) - C\left(\sigma\right).$$

The assumptions on $C\left(\cdot\right)$ ensure the existence of a unique solution to this problem, $\sigma^* = \sigma\left(P,\mu\right) < \overline{\sigma}$, defined by $C'\left(\sigma^*\right) = P\left(1-\mu\right)$. Note that $\sigma^* = \sigma\left(P,\mu\right)$ is increasing in the price and decreasing in the buyer's assessment of quality. The buyer then receives the signal and makes a purchase if his expected utility is positive, which leads to the demand

$$\overline{D}\left(P,\mu\right) = 1 - G\left(\frac{\mu P + P\left(1-\mu\right)\left(1-\sigma^*\right) + C\left(\sigma^*\right)}{\mu}\right).$$

We denote by $\pi\left(\theta,P,\mu\right)$ the one-period profits of a type-θ seller who sets the price P, inducing beliefs μ:

$$\pi\left(\theta,P,\mu\right) = PD\left(\theta,P,\mu\right)$$

$$D\left(\theta,P,\mu\right) = \overline{D}\left(P,\mu\right)\left[\theta + \left(1-\theta\right)\left(1-\sigma^*\right)\right].$$

We define and analyze conditions for the existence of separating equilibria in pure strategies. First we characterize the benchmark case, in which the cost of procuring the inspection on quality is zero and σ is a fixed parameter (the buyer always chooses the maximum amount of information), and show that there is no separating equilibrium. We then show that when allowing the buyer to choose the precision of the signal, by costly acquire information, sellers are able to separate themselves in equilibrium. We finally discuss the implications of time-on-the-market on separating equilibria, in the case that the selling season is composed of two periods and there is one asset on sale over both periods.

3 Separating Equilibrium

A separating equilibrium is a sequential equilibrium at which the buyer can distinguish the good and the bad seller by the different pricing choices they made. Note that separating prices allow the buyer to infer the seller's type, but not the true quality of the asset. Separating prices $\left(P^b, P^g\right)$ induce beliefs $\mu = b$ if $P = P^b$ and $\mu = g$ if $P = P^g$, whereas pooling prices do not provide any information and the posterior will be the same as the prior, $\mu = \mu_0$. Moreover, off-equilibrium prices $P \neq \left\{P^b, P^g\right\}$ are assumed to induce pessimistic beliefs $\mu = b$, to make the existence of the equilibrium easier.

Definition 4.1. *(Separating Equilibrium)* A separating equilibrium is a pair $\left(P^b, P^g\right)$ such that three conditions hold:

C1. $\pi\left(b, P^b, \mu = b\right) \geq \pi\left(b, P, \mu = b\right)$, for every $P \neq P^g$.

C2. $\pi\left(b, P^b, \mu = b\right) \geq \pi\left(b, P^g, \mu = g\right)$, and

C3. $\pi\left(g, P^g, \mu = g\right) \geq \pi\left(g, P, \mu = b\right)$, for every $P \neq P^g$.

For the bad seller, P^b must dominate any price $P \neq P^g$ under pessimistic beliefs (C1). Moreover, the bad seller should not have incentives to mimic the good one, even if this implies optimistic beliefs (C2). For the good seller, P^g must dominate any other price P that induce pessimistic beliefs (C3).

Lemma 4.2. In any separating equilibrium $P^b = P^{b*}$, where P^{b*} is the maximizer of $\pi(b, P, \mu = b)$. Moreover, for the good seller, it is sufficient to check that $\pi(g, P^g, \mu = g) \geq \pi(g, P^{g*}, \mu = b)$, where P^{g*} is the maximizer of $\pi(g, P, \mu = b)$.

Proof. $P^b = P^{b*}$ is a necessary condition for C1 to be satisfied. Moreover C3 requires that the good seller should not have any incentive to deviate from the equilibrium price, with such deviation implying pessimistic beliefs. Then it is sufficient to control for best deviation which occurs at P^{g*}, the maximizer of $\pi(g, P, \mu = b)$. ■

3.1 Benchmark case: costless inspections

In subsequent sections we assume that the buyer chooses the precision of the signal by acquiring information at rising cost. Prior to study this problem, it is useful to have a benchmark case against which to compare the effect of information acquisition on the existence and characterization of a separating equilibrium. Hence in this section the cost of procuring the inspection on quality is zero and $\sigma = \overline{\sigma}$ is a fixed parameter (the buyer always chooses the maximum amount of information). We denote by μ^s the updated belief on $q = 1$, after observing the price and the inspection outcome. Note that the buyer will anticipate procuring a favorable inspection when updating beliefs, so that we can restrict attention to beliefs $\mu^F(\mu)$ since no buyer will buy the asset at any price if $s = NF$. Then $\mu^F(\mu) = \frac{\mu}{\mu + (1-\mu)(1-\overline{\sigma})}$, and $\mu = b$ if $P = P^b$ and $\mu = g$ if $P = P^g$. Conditional on a favorable inspection outcome, the buyer will buy the good if $\mu^F(\mu)v - P \geq 0$, which leads to the demand $\overline{D}\left(P, \mu^F(\mu)\right) = 1 - G\left(\frac{P}{\mu^F(\mu)}\right)$ and associated profits

$$\pi\left(\theta, P, \mu^F(\mu)\right) = PD\left(\theta, P, \mu^F(\mu)\right),$$

with $D\left(\theta, P, \mu^F(\mu)\right) = \overline{D}\left(P, \mu^F(\mu)\right)[\theta + (1-\theta)(1-\overline{\sigma})]$.

Lemma 4.3. With costless inspections, there is no separating equilibrium.

Proof. The pair $\left(P^{b*}, P^{g}\right)$ is a separating equilibrium if two conditions simultaneously hold:

1. $\pi\left(b, P^{b*}, \mu^F(b)\right) \geq \pi\left(b, P^{g}, \mu^F(g)\right)$, and

2. $\pi\left(g, P^{g}, \mu^F(g)\right) \geq \pi\left(g, P^{g*}, \mu^F(b)\right)$.

Consider the equilibrium price $P^g = \overline{P} > P^{b*}$ such that $\pi\left(b, P^{b*}, \mu^F(b)\right) = \pi\left(b, \overline{P}, \mu^F(g)\right)$. At this price the good seller should not have any incentive to deviate to his "monopoly" price, i.e. $\pi\left(g, \overline{P}, \mu^F(g)\right) > \pi\left(g, P^{g*}, \mu^F(b)\right)$, which is equivalent to require:

$$\pi\left(g, P^{b*}, \mu^F(b)\right) > \pi\left(g, P^{g*}, \mu^F(b)\right)$$

given that we define $\overline{P}$ such that $\pi\left(b, P^{b*}, \mu^F(b)\right) = \pi\left(b, \overline{P}, \mu^F(g)\right)$. This cannot be true since $P^{g*} = P^{b*} = \frac{\mu^F(b)\left(1-G\left(\frac{P}{\mu^F(b)}\right)\right)}{g\left(\frac{P}{\mu^F(b)}\right)}$. Therefore, the only existing equilibrium of this game is a pooling one. ■

If inspections are free, there is no separating equilibrium mainly because the consumer will always fully inspect both types ($\sigma = \overline{\sigma}$) and the bad type is unable to inhibit information acquisition by lowering the price (as is shown in the previous section, σ is inversely related to price, and if a product is cheap, it is better to buy directly rather than pay for an inspection). Mimicking the good type is therefore the only recourse for the bad type. When inspections have a cost, on the other hand, a separating equilibrium can be achieved because σ varies positively with price (and higher prices imply higher inspection intensity), therefore the bad type has incentive to charge less in order to avoid inspections and revelation of his type.

4 Costly information acquisition

We now consider costly information acquisition. It is useful to note, again, the amount of information acquired by the consumer depends positively on prices and negatively on beliefs. In this context, the price of the asset serves as a learning mechanism for quality via two channels: first, directly as a standard price signal and secondly, as a factor which can encourage or discourage inspections. Moreover, both channels operate in the same direction: high prices signal high quality both via the standard signaling mechanism, but also because a high price is essentially an invitation to inspect.

Suppose now that the buyer can acquire information by choosing the precision of the signal, $\sigma \in [0, \overline{\sigma}]$ at a cost $C(\sigma)$. We analyze conditions for the existence of a separating equilibrium in the one-shot game, in which σ is endogenously determined. Consider a buyer, whose "taste" for the asset is given by v, with beliefs μ about $q = 1$ prior to procuring the inspection. The problem facing such a buyer is:

$$\max_{\sigma \in [0,\overline{\sigma}]} \mu (v - P) - P (1 - \mu) (1 - \sigma) - C (\sigma) .$$

The assumptions on $C(\cdot)$ ensure the existence of a unique solution to this problem, $\sigma^* = \sigma(P, \mu) < \overline{\sigma}$, defined by $C'(\sigma^*) = P(1 - \mu)$. Note that $\sigma^* = \sigma(P, \mu)$ is increasing in the price and decreasing in his assessment of quality. Then the buyer will drop out of the market once the price reaches P defined by

$$\mu (v - P) - P (1 - \mu) (1 - \sigma^*) - C (\sigma^*) = 0$$

which leads to a demand

$$\overline{D} (P, \mu) = 1 - G \left(\frac{\mu P + P (1 - \mu) (1 - \sigma^*) + C (\sigma^*)}{\mu} \right)$$

and associated profits

$$\pi (\theta, P, \mu) = P D (\theta, P, \mu)$$

$$D(\theta, P, \mu) = \overline{D}(P, \mu)\left[\theta + (1-\theta)(1-\sigma^*)\right].$$

A separating equilibrium in defined according to Definition 1.

Lemma 4.4. In any separating equilibrium, $P^b = P^{b*}$, where P^{b*} is the maximizer of $\pi(b, P, \mu = b)$. For the good seller, it is sufficient to check that $\pi(g, P^g, \mu = g) \geq \pi(g, P^{g*}, \mu = b)$, where P^{g*} is the maximizer of $\pi(g, P, \mu = b)$. Moreover $P^{g*} > P^{b*}$ if $\frac{\sigma_P}{\sigma} \geq -\frac{\overline{D}_P}{\overline{D}}$, where $\frac{\sigma_P}{\sigma}$ represents the price-elasticity of information acquisition and $-\frac{\overline{D}_P}{\overline{D}}$ represents the price-elasticity of demand.

Proof. $P^b = P^{b*}$is a necessary condition for C1 in Definition 1 to be satisfied. The price P^g is a separating equilibrium if it dominates any other price P that induce beliefs $\mu = b$ (C3). Thus it is sufficient to control for the best deviation, which occurs at P^{g*},the price that maximizes profits under the worst belief. We show now that $P^{g*} > P^{b*}$, which is implied by $\frac{\partial^2\pi(\theta,P,\mu)}{\partial P\partial\theta} > 0:$

$$\frac{\partial^2\pi(\theta, P, \mu)}{\partial P\partial\theta} = \frac{\partial}{\partial\theta}\left\{PD_P(\theta, P, \mu) + D(\theta, P, \mu)\right\}$$

$$= PD_{P\theta}(\theta, P, \mu) + D_\theta(\theta, P, \mu),$$

which is positive if $\frac{\sigma_P}{\sigma} \geq -\frac{\overline{D}_P}{\overline{D}}$. Note that $D_\theta(\theta, P, \mu) = \overline{D}(P, \mu)\,\sigma(P, \mu) > 0$ and $D_{P\theta}(\theta, P, \mu) = \overline{D}_P(P, \mu)\,\sigma(P, \mu) + \overline{D}(P, \mu)\,\sigma_P(P, \mu)$ is positive if $\frac{\sigma_P}{\sigma} \geq -\frac{\overline{D}_P}{\overline{D}}$.■

Proposition 4.5. There is always a separating equilibrium $\left(P^{b*}, P^g\right)$ with $P^g > P^{g*}$ if $-\frac{\sigma_\mu}{\sigma} \leq \frac{D_\mu}{D}$, where $\frac{\sigma_\mu}{\sigma}$ represents the beliefs-elasticity of information acquisition and $\frac{D_\mu}{D}$ the beliefs-elasticity of demand.

Proof. Consider the price $\overline{P} > P^{b*}$ such that $\pi\left(b, P^{b*}, \mu = b\right) = \pi\left(b, \overline{P}, \mu = g\right)$. If $\overline{P} \leq P^{g*}$, then, by the Envelope Theorem, it is straightforward to show that $\pi\left(g, \overline{P}, \mu = g\right) \geq \pi\left(g, P^{g*}, \mu = b\right)$, and $\left(P^{b*}, P^{g*}\right)$ is a separating equilibrium.

Thus we can restrict attention to the case $\overline{P} > P^{g*}$. At this price the good seller should not have any incentive to deviate to his "monopoly" price, i.e. $\pi\left(g, \overline{P}, \mu = g\right) \geq \pi\left(g, P^{g*}, \mu = b\right)$, which is equivalent to:

$$\pi\left(g, \overline{P}, \mu = g\right) - \pi\left(g, P^{g*}, \mu = b\right) \geq \pi\left(b, \overline{P}, \mu = g\right) - \pi\left(b, P^{b*}, \mu = b\right) = 0.$$

We can rewrite the left and right hand sides of this inequality as

$$\left[\pi\left(g, \overline{P}, \mu = g\right) - \pi\left(g, \overline{P}, \mu = b\right)\right] + \left[\pi\left(g, \overline{P}, \mu = b\right) - \pi\left(g, P^{g*}, \mu = b\right)\right] \geq$$

$$\left[\pi\left(b, \overline{P}, \mu = g\right) - \pi\left(b, \overline{P}, \mu = b\right)\right] + \left[\pi\left(b, \overline{P}, \mu = b\right) - \pi\left(b, P^{b*}, \mu = b\right)\right].$$

Therefore it is enough to show that

1. $\left[\pi\left(g, \overline{P}, \mu = g\right) - \pi\left(g, \overline{P}, \mu = b\right)\right] \geq \left[\pi\left(b, \overline{P}, \mu = g\right) - \pi\left(b, \overline{P}, \mu = b\right)\right]$ and

2. $\left[\pi\left(g, \overline{P}, \mu = b\right) - \pi\left(g, P^{g*}, \mu = b\right)\right] \geq \left[\pi\left(b, \overline{P}, \mu = b\right) - \pi\left(b, P^{b*}, \mu = b\right)\right]$.

Condition 1 is implied by $\frac{\partial^2 \pi(\theta, P, \mu)}{\partial\theta\partial\mu} > 0$:

$$\frac{\partial^2 \pi\left(\theta, P, \mu\right)}{\partial\theta\partial\mu} = \frac{\partial}{\partial\mu}\left\{P D_\theta\left(\theta, P, \mu\right)\right\}$$

$$= \overline{D}_\mu\left(P, \mu\right)\sigma\left(P, \mu\right) + \overline{D}\left(P, \mu\right)\sigma_\mu\left(P, \mu\right)$$

which is positive if $-\frac{\sigma_\mu}{\sigma} \leq \frac{D_\mu}{D}$.

Since $P^{g*} > P^{b*}$condition 2 is implied by $\frac{\partial^2 \pi(\theta,P,\mu)}{\partial P \partial \theta} > 0$, which is true if $\frac{\sigma_P}{\sigma} \geq -\frac{\overline{D}_P}{\overline{D}}$, as shown in Lemma 4.■

Two conditions therefore guarantee the existence of a separating equibrium in which high prices signal high quality: $\frac{\partial^2 \pi(\theta,P,\mu)}{\partial P \partial \theta} > 0$ and $\frac{\partial^2 \pi(\theta,P,\mu)}{\partial \theta \partial \mu} > 0$. The first condition is standard single-crossing: the cost of signaling through high prices is lower for the good type. The second condition requires that the shift from pessimistic to optimistic beliefs is more attractive to the good type than to his bad-type counterpart. In our setup, signaling requires profits to be marginally more sensitive to type for both prices P and beliefs μ. Note that this is in contrast to Spence´s job-market signaling model where the worker´s utility is quasilinear in beliefs, therefore the second condition is automatically satisfied (with equality). Figure 1 illustrates the role of the above-mentioned conditions for the proof.

The separating equilibrium for the two-period extension of the model can be found in the appendix.

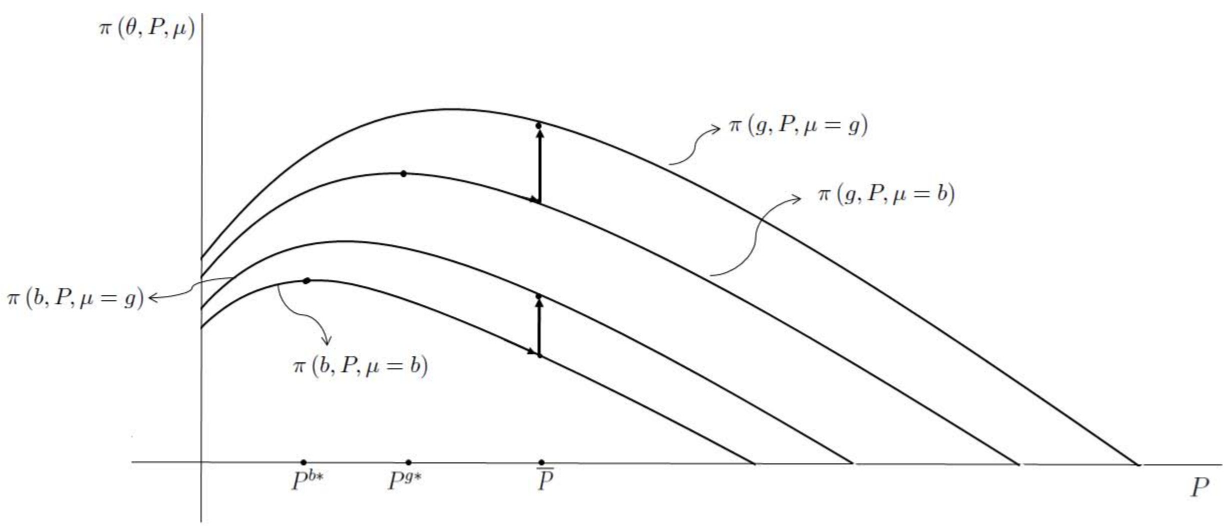

Figure 1: Sketch of the Proof.

Most of these separating equilibria involve beliefs that are implausible, since any deviation is interpreted as coming from a bad seller. We now restrict attention to equilibria that satisfy the "intuitive criterion" of Cho and Kreps. To understand the Cho and Kreps refinement in this context, consider an equilibrium in which the good monopolist's profits are $\pi(g, P^g, \mu = g)$ while the bad firm earns profits $\pi\left(b, P^{b*}, \mu = b\right)$. The equilibrium fails the intuitive criterion if there exists a price $P^{'}$ such that: a) $\pi\left(g, P^{'}, \mu = g\right) \geq \pi(g, P^g, \mu = g)$ and b) $\pi\left(b, P^{'}, \mu = g\right) < \pi\left(b, P^{b*}, \mu = b\right)$. That is, if there exists a price $P^{'}$ such that the good seller is better off by deviating and the bad one makes more profits following the equilibrium price, even if the deviation would have generated optimistic beliefs. Intuitively, if such a price $P^{'}$ exists, consumers should interpret such a deviation as coming from a good seller, making the equilibrium fail.

Proposition 4.6. The pair $\left(P^{b*}, \overline{P}\right)$ is the only separating equilibrium that satisfies the intuitive criterion.

Proof. The proof consists of two steps. We first show that there is no equilibrium price $P > \overline{P}$ that satisfies the intuitive criterion. Consider the price $P > \overline{P}$ such that $\left(P^{b*}, \overline{P}\right)$ is a separating equilibrium. Define $P^{'} = P - \varepsilon$. Then it is easy to see that a) $\pi\left(g, P^{'}, \mu = g\right) \geq \pi(g, P^g, \mu = g)$ and b) $\pi\left(b, P^{'}, \mu = g\right) < \pi\left(b, P^{b*}, \mu = b\right)$. Let P^{g**} be the price that maximizes the good firm's profits under the most optimistic beliefs $\mu = g$, $P^{g**} = argmax_P\, \pi(g, P, \mu = g)$. Noting that $P^{g*} < P^{g**}$ and $P^{g*} < \overline{P} < P^{'}$, we get that $P^{g**} < P^{'} < P$. Therefore $\pi\left(g, P^{'}, \mu = g\right) \geq \pi(g, P^g, \mu = g)$. By Proposition 5 we know that $\pi\left(b, P^{'}, \mu = g\right) < \pi\left(b, P^{b*}, \mu = b\right)$ to ensure that the bad seller would not deviate, then by continuity $\pi(b, P, \mu = g) < \pi\left(b, P^{b*}, \mu = b\right)$. Thus for any price $P > \overline{P}$ condition a) is not satisfied, violating the intuitive criterion. We now show that $\left(P^{b*}, \overline{P}\right)$ is the only separating equilibrium that satisfies the intuitive criterion, that is there is no equilibrium price $P^{'}$ such that a) $\pi\left(g, P^{'}, \mu = g\right) \geq \pi(g, P^g, \mu = g)$ and b) $\pi\left(b, P^{'}, \mu = g\right) < \pi\left(b, P^{b*}, \mu = b\right)$. If $P^{'} > \overline{P}$ condition a) is not satisfied. Then, $P^{'} \leq \overline{P}$. But if $P^{'} < \overline{P}$, there is no separating equilibrium, as shown in Proposition 5. Then it must be $P^{'} = \overline{P}$, and $\left(P^{b*}, \overline{P}\right)$ is the only separating equilibrium that satisfies the intuitive criterion. ■

5 Time-on-the-Market

5.1 Costly information acquisition and time-on-the-market

We now consider a selling season composed of two periods with only one asset on sale over both periods. Note that now both price and time-on-the-market may signal quality. In this case the game proceeds as follows: at the beginning of the first period the seller posts a separating price P. After observing the price, the buyer updates beliefs on the asset quality and procures an inspection on quality, then decides whether to buy or not. If no sale occurs in the first period, then we get to the second stage of the game, where a new buyer enters the market. The second-period buyer takes into account the fact that the asset did not sell in the first period at price P (time-on-the-market) when forming beliefs. He then procures an inspection on quality, and makes a purchase decision. Note that he cannot observe the outcome of the first-period inspection or even if one was procured, therefore he did not know why the asset did not sell in the first period. Specifically, the buyer cannot distinguish between two possible reasons: (i) the asset was overpriced with respect to first-period buyer valuation, or (ii) the inspection's outcome was unfavorable revealing low quality, even if the first buyer was ready to buy it. Therefore, time-on-the-market enters as a variable at the time of updating beliefs on the seller's type and the quality of the asset.

We solve the game by backward induction. The problem facing the buyer in $t = 2$ is

$$\max_{\sigma_2 \in [0,\overline{\sigma}]} \mu_2 \left(v - P_2\right) - P_2 \left(1 - \mu_2\right) \left(1 - \sigma_2\right) - C\left(\sigma_2\right),$$

which leads to $\sigma_2^* = \sigma\left(P_2, \mu_2\right)$ such that $C^{'}\left(\sigma_2^*\right) = P_2\left(1 - \mu_2\right)$, where μ_2 denotes the beliefs about the asset being of high quality, taking into account time-on-the-market:

$$\mu_2\left(P, \mu\right) = \frac{\mu G\left(f\left(P, \mu\right)\right)}{G\left(f\left(P, \mu\right)\right) + \left[1 - G\left(f\left(P, \mu\right)\right)\right] \sigma\left(P, \mu\right)\left(1 - \mu\right)},$$

where $G(f(P,\mu))$ denotes the probability that second-period buyer assigns to reason (i):

$$G(f(P,\mu)) = G\left(\frac{\mu P + (1-\mu)(1-\sigma^*(P,\mu))P + C(\sigma^*(P,\mu))}{\mu}\right)$$

and $[1-G(f(P,\mu))]\,\sigma(P,\mu)(1-\mu)$ denotes the probability that a flaw was discovered at the inspection (reason (ii)).

Note that, when time-on-the-market comes into play, not only the buyer updates beliefs accordingly, but also the seller. We denote by $\widetilde{\theta}$ the posterior probability assigned by the seller to high quality given that asset did not sell in the previous period:

$$\widetilde{\theta}(\theta,P,\mu) = \frac{\theta G(f(P,\mu))}{G(f(P,\mu)) + [1-G(f(P,\mu))]\,\sigma(P,\mu)(1-\theta)}.$$

Hence, a buyer with valuation v for the asset will drop out of the market once the price reached P_2 defined by

$$\mu_2(v-P_2) - P_2(1-\mu_2)(1-\sigma_2^*) - C(\sigma_2^*) = 0$$

which leads to second-period demand

$$\overline{D}_2(P_2,\mu_2) = 1 - G\left(\frac{\mu_2 P_2 + P_2(1-\mu_2)(1-\sigma_2^*) + C(\sigma_2^*)}{\mu_2}\right),$$

and associated profits

$$\pi\left(\widetilde{\theta},P_2,\mu_2\right) = P_2 D_2\left(\widetilde{\theta},P_2,\mu_2\right) = P_2\overline{D}_2(P_2,\mu_2)\left[\widetilde{\theta} + \left(1-\widetilde{\theta}\right)(1-\sigma_2^*)\right].$$

Note that the inspection procured by the buyer in the second period is more intense than the one procured by first-period "potential" buyer, $\sigma_2^* \geq \sigma_1^*$. As $\mu_2 < \mu$, the result follows from $\frac{\partial\sigma^*}{\partial\mu} = -\frac{P}{C''(\sigma^*)} < 0$. This is because second-period buyer worries that time-on-the-market was due to an unfavorable inspection, therefore detection of low quality, in the first period and as a result procures a more intense inspection in the second period.

For any history of separation $\left(P^{b*}, P^{g}\right)$, second-period prices are given by $\left(P_2^{b*}, P_2^{g**}\right)$, where P_2^{b*} is the maximizer of $\pi\left(\widetilde{b}, P_2, \mu_2\left(P^{b*}, \mu = b\right)\right)$ and P_2^{g**} is the maximizer of $\pi\left(\widetilde{g}, P_2, \mu_2\left(P^{g}, \mu = g\right)\right)$. We denote by $\Pi\left(\theta, P, \mu\right)$ the profits over both periods of a type-θ seller who sets the price P in the first period, inducing beliefs μ, and look for separating equilibria as well.

When there is only one asset on sale (therefore both price and time-on-the-market can signal quality) there might be no separating equilibrium even if single-crossing is satisfied. The key to this result is that second-period buyer cannot observe why the asset did not sell in the first period. Notably, the failure to sell can be attributed to (i) overpricing or (ii) an unfavorable inspection outcome. Therefore the good seller can influence buyer beliefs by choosing a high price, in order to make reason (i) seem more plausible. Failure to sell in the first period conveys a much weaker assessment of quality when the price is high than when it is low. The problem, however, is that the bad seller has more incentive to hide behind a high initial price because he benefits more from an increase in buyer beliefs than his high-quality counterpart, given his lower probability of sale in the first period. It follows that the bad type will always prefer to imitate a high quality seller using a high initial price because the change in consumer beliefs that results from a first-period sale inflict more damage than a non-sale, especially if the latter can be easily justified by a high price.

Fact 4.7. With information acquisition and time-on-the-market on of the sufficient condition for the existence of a separating equilibrium, $\frac{\partial^2 \Pi(\theta,P,\mu)}{\partial\theta\partial\mu} > 0$ (1) is not satisfied.

Even if we can assure single-crossing, condition (1) cannot be satisfied, since the shift from pessimistic to optimistic beliefs is more attractive to the bad seller, making the equilibrium fail:

$$\frac{\partial^2 \Pi\left(\theta, P, \mu\right)}{\partial\theta\partial\mu} = \frac{\partial^2}{\partial\theta\partial\mu}\left\{\left[P - \pi\left(\widetilde{\theta}, P_2, \mu_2\right)\right] D\left(\theta, P, \mu\right) + \pi\left(\widetilde{\theta}, P_2, \mu_2\right)\right\}$$

$$= \frac{\partial}{\partial \mu} \left\{ \left[P - \pi \left(\widetilde{\theta}, P_2, \mu_2 \right) \right] D_\theta \left(\theta, P, \mu \right) + \left[1 - D \left(\theta, P, \mu \right) \right] \frac{\partial \pi \left(\widetilde{\theta}, P_2, \mu_2 \right)}{\partial \theta} \right\}$$

$$= \left[P - \pi \left(\widetilde{\theta}, P_2, \mu_2 \right) \right] D_{\theta\mu} \left(\theta, P, \mu \right) - \frac{\partial \pi \left(\widetilde{\theta}, P_2, \mu_2 \right)}{\partial \mu} D_\theta \left(\theta, P, \mu \right)$$

$$+ \left[1 - D \left(\theta, P, \mu \right) \right] \frac{\partial \pi \left(\widetilde{\theta}, P_2, \mu_2 \right)}{\partial \theta \partial \mu} - \frac{\partial \pi \left(\widetilde{\theta}, P_2, \mu_2 \right)}{\partial \theta} D_\mu \left(\theta, P, \mu \right).$$

There are two terms in the previous expression which are unequivocally negative, and possibly quite important. The first is $\left[-\frac{\partial \pi \left(\widetilde{\theta}, P_2, \mu_2 \right)}{\partial \mu} D_\theta \left(\theta, P, \mu \right) \right]$. This reflects the fact that inducing better beliefs at $t = 2$ (higher μ_2 through optimistic μ) is more valuable for the bad type, since he is more likely to have an unsold asset at that time. The second term, $\left[-\frac{\partial \pi \left(\widetilde{\theta}, P_2, \mu_2 \right)}{\partial \theta} D_\mu \left(\theta, P, \mu \right) \right]$, shows that inducing higher first-period beliefs is more appealing to the bad type than the good, since the outside option of waiting until the next period is worse for the former.

5.2 Costly information acquisition, time-on-the-market and separating equilibrium

We now consider the case in which only first-period buyer is allowed to acquire information to infer the common value of the object, whereas the second-period one learns from his predecessor purchase decision and makes his choice accordingly. We show there is a separating equilibrium in which high prices (and time-on-the-market) signal high quality. Note that now the second-period buyer updates beliefs about the asset quality only observing that no sale occurred in the first period at price P.

We solve the game by backward induction. Since no inspection on quality occurs in the second period, given beliefs μ_2 , the buyer will buy if $\mu_2 v - P_2 \geq 0$, which leads to second-period demand

$$\overline{D}_2\left(P_2, \mu_2\right) = 1 - G\left(\frac{P_2}{\mu_2}\right),$$

and associated profits

$$\pi\left(P_2, \mu_2\right) = P_2 \overline{D}_2\left(P_2, \mu_2\right),$$

where μ_2 is defined as before and denotes the beliefs about the asset being of high quality, taking into account time-on-the-market:

$$\mu_2\left(P, \mu\right) = \frac{\mu G\left(f\left(P, \mu\right)\right)}{G\left(f\left(P, \mu\right)\right) + \left(1 - G\left(f\left(P, \mu\right)\right)\right)\left(\sigma\left(P, \mu\right)\left(1 - \mu\right)\right)}.$$

For any history of separation $\left(P^{b*}, P^{g}\right)$, second-period prices are given by $\left(P_2^{b*}, P_2^{g**}\right)$, where P_2^{b*} is the maximizer of $\pi\left(P_2, \mu_2\left(P^{b*}, \mu = b\right)\right)$ and P_2^{g**} is the maximizer of $\pi\left(P_2, \mu_2\left(P^{g}, \mu = g\right)\right)$. We denote by $\Pi\left(\theta, P, \mu\right)$ the profits over both periods of a type-θ seller who sets the price P in the first period, inducing beliefs μ, and look for separating equilibria as well.

Proposition 4.8. With first-period information acquisition and time-on-the-market there is a separating equilibrium $\left(P^{b*}, P^{g}\right)$ with $P^{g} > P^{g*}$, if the following two conditions hold:

1. $\overline{g} \leq \frac{G(P)^2}{\overline{\sigma}}$,

2. $\frac{\partial D_\theta}{\partial \mu}\frac{\mu}{D_\theta} \geq -\frac{\partial\left[P - \pi\left(P_2, \mu_2\right)\right]}{\partial \mu}\frac{\mu}{\left[P - \pi\left(P_2, \mu_2\right)\right]}$.

Proof. We can write $\Pi\left(\theta, P, \mu\right)$ as

$$\Pi\left(\theta, P, \mu\right) = \left[P - \pi\left(P_2, \mu_2\right)\right] D\left(\theta, P, \mu\right) + \pi\left(P_2, \mu_2\right),$$

with

$$D(\theta, P, \mu) = \overline{D}(P, \mu)\left[\theta + (1-\theta)(1-\sigma(P, \mu))\right].$$

As we know, we need to check that:

1) $\frac{\partial^2 \Pi(\theta,P,\mu)}{\partial P \partial \theta} > 0$ and

2) $\frac{\partial^2 \Pi(\theta,P,\mu)}{\partial \theta \partial \mu} > 0$.

Condition (1) is equal to:

$$\frac{\partial^2 \Pi(\theta, P, \mu)}{\partial P \partial \theta} = \frac{\partial}{\partial \theta}\left\{\left[P - \pi(P_2, \mu_2)\right] D_P(\theta, P, \mu) + \left[1 - \frac{\partial \pi(P_2, \mu_2)}{\partial P}\right] D(\theta, P, \mu) + \frac{\partial \pi(P_2, \mu_2)}{\partial P}\right\}$$

$$= \left[P - \pi(P_2, \mu_2)\right] D_{P\theta}(\theta, P, \mu) + \left[1 - \frac{\partial \pi(P_2, \mu_2)}{\partial P}\right] D_\theta(\theta, P, \mu),$$

Note that we are looking for separation through high prices, so that we can assume $P > \pi(P_2, \mu_2)$. Moreover $D_{P\theta}(\theta, P, \mu) = \overline{D}_P(P, \mu)\sigma(P, \mu) + \overline{D}(P, \mu)\sigma_P(P, \mu)$ is positive if $\frac{\sigma_P}{\sigma} > -\frac{\overline{D}_P}{\overline{D}}$ (as shown in Lemma 4) and $D_\theta(\theta, P, \mu) = \overline{D}(P, \mu)\sigma(P, \mu) > 0$. So that it is enough to show that $\frac{\partial \pi(P_2, \mu_2)}{\partial P} < 1$:

$$\frac{\partial \pi(P_2, \mu_2)}{\partial P} = \frac{P_2^{*2}}{\mu_2^2} g\left(\frac{P_2^*}{\mu_2}\right)\frac{\partial \mu_2}{\partial P} = \left[1 - G\left(\frac{P_2^*}{\mu_2}\right)\right]\frac{1 - G\left(\frac{P_2^*}{\mu_2}\right)}{g\left(\frac{P_2^*}{\mu_2}\right)}\frac{\partial \mu_2}{\partial P}$$

using the fact that at the optimum $\frac{P_2^*}{\mu_2} = \frac{1-G\left(\frac{P_2^*}{\mu_2}\right)}{g\left(\frac{P_2^*}{\mu_2}\right)}$. So that it is enough to show that $\frac{\partial \mu_2}{\partial P} < 1$:

$$\frac{\partial \mu_2}{\partial P} = \frac{\partial}{\partial P}\left\{\frac{\mu G(f(P, \mu))}{G(f(P, \mu)) + (1 - G(f(P, \mu)))(\sigma(P, \mu)(1-\mu))}\right\}$$

$$= \frac{\mu\left(1-\mu\right)\left[g\left(f\right)\sigma^{*}\frac{\partial f(P,\mu)}{\partial P}-\sigma_{P}^{*}G\left(f\right)\left(1-G\left(f\right)\right)\right]}{\left[G\left(f\right)+\left(1-G\left(f\right)\right)\left(\sigma^{*}\left(1-\mu\right)\right)\right]^{2}}$$

$$= \frac{\mu\left(1-\mu\right)\left[g\left(f\right)\sigma^{*}\left(\frac{1-\sigma^{*}(1-\mu)}{\mu}\right)-\sigma_{P}^{*}G\left(f\right)\left(1-G\left(f\right)\right)\right]}{\left[G\left(f\right)+\left(1-G\left(f\right)\right)\left(\sigma^{*}\left(1-\mu\right)\right)\right]^{2}}$$

using the fact that

$$f\left(P,\mu\right)=\frac{\mu P+\left(1-\mu\right)\left(1-\sigma^{*}\right)P+C\left(\sigma^{*}\right)}{\mu}$$

and

$$\frac{\partial f\left(P,\mu\right)}{\partial P}=\frac{\mu+\left(1-\mu\right)\left(1-\sigma^{*}\right)}{\mu}.$$

Now if

$$\left[g\left(f\right)\sigma^{*}\left(\frac{1-\sigma^{*}\left(1-\mu\right)}{\mu}\right)-\sigma_{P}^{*}G\left(f\right)\left(1-G\left(f\right)\right)\right]\leq 0$$

the proof is completed. From now on we consider the case in which

$$\left[g\left(f\right)\sigma^{*}\left(\frac{1-\sigma^{*}\left(1-\mu\right)}{\mu}\right)-\sigma_{P}^{*}G\left(f\right)\left(1-G\left(f\right)\right)\right]>0.$$

$$\frac{\partial\mu_{2}}{\partial P}=\frac{\mu\left(1-\mu\right)\left[g\left(f\right)\sigma^{*}\left(\frac{1-\sigma^{*}(1-\mu)}{\mu}\right)-\sigma_{P}^{*}G\left(f\right)\left(1-G\left(f\right)\right)\right]}{\left[G\left(f\right)+\left(1-G\left(f\right)\right)\left(\sigma^{*}\left(1-\mu\right)\right)\right]^{2}}<1$$

$$\left(1-\mu\right)g\left(f\right)\sigma^{*}-\left(1-\mu\right)^{2}g\left(f\right)\sigma^{*^{2}}-\sigma_{P}^{*}\mu\left(1-\mu\right)G\left(f\right)\left(1-G\left(f\right)\right)$$

$$<G\left(f\right)^{2}+\left(1-G\left(f\right)\right)^{2}\left(\sigma^{*}\left(1-\mu\right)\right)^{2}+2G\left(f\right)\left(1-G\left(f\right)\right)\left(\sigma^{*}\left(1-\mu\right)\right).$$

Then it is enough to show that

$$(1-\mu)\,g\,(f)\,\sigma^{*} < G\,(f)^{2}\,,$$

moreover as $(1-\mu)\,g\,(f)\,\sigma^{*} \leq (1-\mu)\,g\,(f)\,\overline{\sigma}$, and $f\,(P,\mu) = \frac{\mu P+(1-\mu)(1-\sigma^{*})P+C(\sigma^{*})}{\mu} > P$ implies $G\,(f) > G(P)$, it is enough that

$$(1-\mu)\,g\,(f)\,\overline{\sigma} < G(P)^{2}.$$

Therefore $\frac{\partial\pi(P_2,\mu_2)}{\partial P} < 1$ is always true if $\overline{g} \leq \frac{G(P)^2}{\overline{\sigma}}$.

At the same time we require $\frac{\partial^2\Pi(\theta,P,\mu)}{\partial\theta\partial\mu} > 0$:

$$\frac{\partial^{2}\Pi\,(\theta,P,\mu)}{\partial\theta\partial\mu} = \frac{\partial}{\partial\mu}\left\{\left[P-\pi\,(P_2,\mu_2)\right]D_{\theta}\,(\theta,P,\mu)\right\} =$$

$$\left[P-\pi\,(P_2,\mu_2)\right]D_{\theta\mu}\,(\theta,P,\mu) - \frac{\partial\pi\,(P_2,\mu_2)}{\partial\mu}D_{\theta}\,(\theta,P,\mu) > 0$$

if

$$\frac{\partial D_{\theta}}{\partial\mu}\frac{\mu}{D_{\theta}} \geq -\frac{\partial\left[P-\pi\,(P_2,\mu_2)\right]}{\partial\mu}\frac{\mu}{\left[P-\pi\,(P_2,\mu_2)\right]}.\blacksquare$$

6 Conclusion

This paper examines the optimal pricing strategy in a monopoly market with asymmetric information about product quality and information acquisition. The seller has private information about the probability of owning a high-quality asset, whereas the buyer is initially uninformed about quality. The buyer has two learning mechanisms: price signaling and information acquisition prior to purchase. Information acquisition is costly - in terms of time and money - and imprecise. Moreover, it is increasing in prices and decreasing in the prior assessment of quality. We show existence of a unique separating equilibrium that satisfies the intuitive criterion in which high prices signal high quality because a high price is essentially an invitation to inspect.

We then discuss the implications of time-on-the-market on the separating equilibrium, when the selling season is composed of two periods and there is only one object for sale. The two conditions that guarantee the existence of a separating equilibrium might not hold in this case, since a failure to sell in the first period conveys a much weaker assessment of quality when the price is high than when it is low. Hence the low-quality seller has more incentive to hide behind a high initial price and will always prefer to imitate a high quality seller. We solve the problem by allowing only first-period buyers to acquire information. When the second-period buyer learns from the price history and his predecessors purchase decision, a separating equilibrium exists in which high prices (and time-on-the-market) signal high quality.

Our model has many applications and can help explain price dynamics in real estate, auto, arts and clothes markets, for example. Consumers of high-end products usually spend more time researching product attributes. In particular, real estate and auto purchase decisions are usually made after inspections that range from casual to professional, and have corresponding costs. We explain the price path observable in such situations.

Many interesting extensions can be derived from this analysis. For example, a finite or infinite horizon may be used to study price dynamics. Furthermore, a multi-period framework may illuminate the unresolved conclusion about the existence of a separating equilibrium when both prices and time-on-the-market signal quality. That open question may also be approached by discounting the future in different ways for the high-quality and low-quality seller, respectively. Finally, the analysis of different market structures, where strategic interaction among firms also comes into play, might yield different conclusions about the optimal pricing strategy.

References

[1] AKERLOF, G. A. (1970): “The Market for "Lemons": Quality Uncertainty and the Market Mechanism”, *The Quarterly Journal of Economics*, Vol. 84, No. 3., pp. 488-500.

[2] BAGWELL, K. and M. RIORDAN (1991): “High and Declining Prices Signal Product Quality”, *The American Economic Review*, Vol. 81, No. 1, pp. 224-239.

[3] BERGEMANN, J. and J. VALIMAKI (1996): “Learning and Strategic Pricing”, *Econometrica*, Vol. 64, pp. 1125-49.

[4] BERGEMANN, J. and J. VALIMAKI (1997): “Market Diffusion with Two-Sided Learning”, *The RAND Journal of Economics*, Vol. 28, No. 4, pp. 773-795.

[5] BERGEMANN, J. and J. VALIMAKI (2005): “Monopoly Pricing of Experience Goods”, Cowles Foundation Discussion Paper No. 1463 .

[6] BESTER, H. and K. RITZBERGER (2001): “Strategic pricing, signalling, and costly information acquisition”, *International Journal of Industrial Organization,* Vol. 19, pp. 1347-1361.

[7] CHADE, H. and V. VERA de SERIO (2002): “Pricing, learning, and strategic behavior in a single-sale model”, *Economict Theory,* Vol. 19, pp. 333-353.

[8] CHAN, Y. and H. LELAND (1982): “Prices and Qualities in Markets with Costly Information”, *The Review of Economic Studies*, Vol. 49, No. 4, pp. 499-516.

[9] CHO, I. and D. KREPS (1987): “Signaling Games and Stable Equilibria”, *The Quarterly Journal of Economics*, Vol. 102, No. 2, pp. 179-221.

[10] GERTZ, C. (2014): “Quality Uncertainty with Imperfect Information Acquisition”, Institute of Mathematical Economics, Working paper 487.

[11] JUDD, K. and M. RIORDAN (1994): “Price and Quality in a New Product Monopoly”, *The Review of Economic Studies,* Vol. 61, No. 4, pp. 773-789.

[12] KIHLSTROM, R. (1974): "A Bayesian Model of Demand for Information about Product Quality", *International Economic Review,* Vol. 15, No. 1, pp. 99-118..

[13] LINNEMER, L. (2002): "Price and Advertising as Signals of Quality when Some Consumers are Informed", *International Journal of Industrial Organization,* Vol. 20, pp. 931-947.

[14] MEZZETTI, C. and T. TSOULOUHAS (2000): "Gathering information before signing a contract with a privately informed principal", *International Journal of Industrial Organization,* Vol. 18, pp. 667-689.

[15] MILGROM, P. (1981): "Rational Expectations, Information Acquisition, and Competitive Bidding", *Econometrica,* Vol. 49, No. 4, pp. 921-943.

[16] MILGROM, P. and J. ROBERTS (1986): "Price and Advertising Signals of Product Quality", *Journal of Political Economy,* Vol. 94, No. 4, pp. 796-821.

[17] SCHLEE, E. (2001): "Buyer Experimentation and Introductory Pricing", *Journal of Economic Behavior and Organization,* Vol. 44, pp. 347-362.

[18] TAYLOR, C. (1999): "Time-on-the-Market as a Sign of Quality", *The Review of Economic Studies,* Vol. 66, pp. 555-578.

[19] VOORNEVELD, M. and J.W. WEIBULL (2011): "A Scent of Lemon—Seller Meets Buyer with a Noisy Quality Observation", *Games*, Vol. 2, pp. 163-186.

[20] WOLINSKY, A. (1983): "Prices as Signals of Product Quality", *The Review of Economic Studies*, Vol. 50, No. 4, pp. 647-658.

Appendix

Costless inspections and two-period game

We now consider the selling season is composed of two periods and there is an object for sale each period. We denote by μ^s the updated belief on $q = 1$, after observing the price and the inspection outcome. We restrict attention to beliefs $\mu^F(\mu)$ since no buyer will buy the asset at any price if $s = NF$. Then $\mu^F(\mu) = \frac{\mu}{\mu+(1-\mu)(1-\bar{\sigma})}$, and $\mu = b$ if $P = P^b$ and $\mu = g$ if $P = P^g$ [14]. We solve the game by backward induction. Following a history of separation $\left(P^{b*}, P^g\right)$, second-period prices are given by $\left(P_2^{b*}, P_2^{g**}\right)$[15], where $P_1^{b*} = P_2^{b*} = P^{b*}$ is the maximizer of $\pi\left(b, P, \mu^F(b).\right)$ and P_2^{g**} is the maximizer of $\pi\left(g, P, \mu^F(g).\right)$. We denote by $\Pi\left(\theta, P, \mu^F(\mu)\right)$ the profits over both periods of a type-θ seller who sets the price P in the first period, inducing beliefs $\mu^F(\mu)$, conditional on a favorable inspection outcome.Thus equilibrium profits are given by

$$\Pi\left(b, P^{b*}, \mu^F(b)\right) = 2P^{b*}D\left(b, P^{b*}, \mu^F(b)\right)$$

$$\Pi\left(g, P^g, \mu^F(g)\right) = P^g D\left(g, P^g, \mu^F(g)\right) + P_2^{g**}D\left(g, P_2^{g**}, \mu^F(g)\right).$$

Proposition 4.9. In the dynamic game, with costless inspections, there is no separating equilibrium.

Proof. A separating equilibrium in $t = 1$ is a pair $\left(P^{b*}, P^g\right)$ such that two conditions simultaneously hold:

C1. $\Pi\left(b, P^{b*}, \mu^F(b)\right) \geq \Pi\left(b, P_1^g, \mu^F(g)\right)$, and

[14] Note that $\mu_1^F = \mu_2^F = \mu^F(\mu)$.

[15] Second-period equlibrium prices are calculated by maximizing profits, since sellers' private information was fully revealed in the first period, where a separating equilibrium was played.

C2. $\Pi\left(g, P^g, \mu^F(g)\right) \geq \Pi\left(g, P^{g*}, \mu^F(b)\right)$.

Separation can occur if the bad seller chooses its maximizing price rather than mimicking the good one, even if this implies optimistic beliefs (C1), and the good seller chooses not to monopolize the market by charging his maximizing price P^{g*} in the first period, being perceived as a bad seller (C2). The proof follows the same reasoning of Lemma 3 proof. Consider the equilibrium price $P^g = \overline{P} > P^{b*}$ such that $\Pi\left(b, P^{b*}, \mu^F(b)\right) = \Pi\left(b, \overline{P}, \mu^F(g)\right)$ (1). Condition (1) is equivalent to

$$2\left[P^{b*}\overline{D}\left(P^{b*}, \mu^F(b)\right)(b + (1-b)(1-\overline{\sigma}))\right] =$$

$$\overline{PD}\left(\overline{P}, \mu^F(g)\right)(b + (1-b)(1-\overline{\sigma})) + P_2^{g**}\overline{D}\left(P_2^{g**}, \mu^F(g)\right)(b + (1-b)(1-\sigma))$$

At this price the good seller should not have any incentive to deviate to his "monopoly" price, i.e. $\Pi\left(g, P_1^g, \mu^F(g)\right) > \Pi\left(g, P_1^{g*}, \mu^F(b)\right)$ (2):

$$2\left[P^{g*}\overline{D}\left(P^{g*}, \mu^F(b)\right)(g + (1-g)(1-\overline{\sigma}))\right] \leq$$

$$\overline{PD}\left(\overline{P}, \mu^F(g)\right)(g + (1-g)(1-\overline{\sigma})) + P_2^{g**}\overline{D}\left(P_2^{g**}, \mu^F(g)\right)(g + (1-g)(1-\overline{\sigma})).$$

By equality (1) this is equivalent to

$$\overline{PD}\left(\overline{P}, \mu^F(g)\right)(g + (1-g)(1-\overline{\sigma})) + P_2^{g**}\overline{D}\left(P_2^{g**}, \mu^F(g)\right)(g + (1-g)(1-\overline{\sigma})) <$$

$\overline{PD}\left(\overline{P}, \mu^F(g)\right)(g + (1-g)(1-\overline{\sigma})) + P_2^{g**}\overline{D}\left(P_2^{g**}, \mu^F(g)\right)(g + (1-g)(1-\overline{\sigma}))$, which cannot be true since $P^{b*} = P^{g*}$.

Thus there is no separating equilibrium satisfying both (1) and (2) at the same time. ■

Costly inspections and two-period game

We now consider the case in which the selling season is composed of two periods and there is an object on sale each period. When information acquisition is costly, the dynamic separating equilibrium shows higher separating prices than the static one.

Proposition 4.10. There is always a separating equilibrium $\left(P^{b*}, P_1^g\right)$ with $P_1^g \geq P^{g*}$ if $\frac{\sigma_P}{\sigma} > -\frac{\overline{D}_P}{\overline{D}}$ and $-\frac{\sigma_\mu}{\sigma} < \frac{\overline{D}_\mu}{\overline{D}}$, where $\frac{\sigma_P}{\sigma}$ $\left(\frac{\sigma_\mu}{\sigma}\right)$ represents the elasticity of the information precision to the price (beliefs) and $\frac{\overline{D}_P}{\overline{D}}$ $\left(\frac{\overline{D}_\mu}{\overline{D}}\right)$ the elasticity of demand to price (beliefs). Moreover the dynamic separating equilibrium shows higher separating prices than the static one, $P_1^g > P^g$.

Proof. The proof follows the same steps as for the static case (see Proof of Proposition 5). We now show that the dynamic separating equilibrium shows higher separating prices than the static one $P_1^g > P^g$. We defined $\overline{P}$ as the price at which the bad seller was indifferent between following the equilibrium strategy and mimicking the good one in the static game, $\pi\left(b, P^{b*}, \mu = b\right) = \pi\left(b, \overline{P}, \mu = g\right)$ (1). Now define $\widetilde{P}$ as its equivalent for the two-period game, the price such that $\Pi\left(b, P^{b*}, \mu = b\right) = \Pi\left(b, \widetilde{P}, \mu = g\right)$ (2). Expressions (1) and (2) can be written as

$$P^{b*}\overline{D}\left(P^{b*}, \mu = b\right)\left(b + (1-b)\left(1 - \sigma\left(P^{b*}, \mu = b\right)\right)\right) = \overline{P}\overline{D}\left(\overline{P}, \mu = g\right)\left(b + (1-b)\left(1 - \sigma\left(\overline{P}, \mu = g\right)\right)\right)$$
(1)

$$2\left[P^{b*}\overline{D}\left(P^{b*}, \mu = b\right)\left(b + (1-b)\left(1 - \sigma\left(P^{b*}, \mu = b\right)\right)\right)\right] =$$
$$\widetilde{P}\overline{D}\left(\widetilde{P}, \mu = g\right)\left(b + (1-b)\left(1 - \sigma\left(\widetilde{P}, \mu = g\right)\right)\right) + P_2^{g**}\overline{D}\left(P_2^{g**}, \mu = g\right)\left(b + (1-b)\left(1 - \sigma\left(P_2^{g**}, \mu = g\right)\right)\right)$$
(2)

Suppose that $\widetilde{P} = \overline{P}$. If this is the case, then condition (2) reduces to

$$P^{b*}\overline{D}\left(P^{b*}, \mu = b\right)\left(b + (1-b)\left(1 - \sigma\left(P^{b*}, \mu = b\right)\right)\right) =$$
$$P_2^{g**}\overline{D}\left(P_2^{g**}, \mu = g\right)\left(b + (1-b)\left(1 - \sigma\left(P_2^{g**}, \mu = g\right)\right)\right),$$

which cannot be true since $\pi\left(b, P_2^{g**}, \mu = g\right) > \pi\left(b, P^{b*}, \mu = b\right)$. Then, to mantain the equality in condition (2) it must be that $\widetilde{P} > \overline{P}$.■

Price Signaling and Herding

1 Introduction

In many markets consumer decisions are influenced by the choices of their peers. Facing uncertainty about the characteristics of a new product, consumers try to infer its quality from previous buyers' purchase decisions. Imitation, the influence of opinion leaders, word-of-mouth communication, as well as past experience are often the leitmotiv of consumer choice. Such behavior is quite common in the technology industry: the potential adopters of new technologies, such as computer software, new medical equipment or a new type of vehicle, typically look to the sequence of past purchases for information on quality. We study the optimal pricing strategy for new products in a dynamic monopoly market with asymmetric information about product quality, in which buyers learn from each other's purchases.

We set up a two-period model, in which a long-lived monopolist faces short-lived buyers each period. The buyer has no information about product quality, while the seller has private information about the probability of producing a high-quality product. Buyers have a common valuation for the product, but also an individual valuation or "taste" for it, independently and identically distributed. The first-period buyer observes prices and obtains a private signal regarding the common value of the product, before deciding whether or not to make the purchase. While the inspection outcome is private information to the buyer, the purchase decision becomes public. Based on the previous buyer's purchase decisions and price history, the second-period buyer makes his choice. Each period, the monopolist posts a price without observing the signal recieved by the buyer as a result of inspection, but knowing his own type. Note that, in this context, the monopolist uses the pricing strategy not only to signal his type, but also to manipulate the buyers' learning process. Beliefs are higher after a first-period sale at relatively high prices. If a firm is able to sell at a very high price, after being inspected, it must be good! We look for separating (pure-strategy) equilibria and show that high prices signal high quality, under different assumptions about the signal precision and the objects for sale.

We first consider the case in which the private signal is perfectly informative, and there is one object on sale each period. We then analyze how the result changes when time-on-the-market comes into play and the second period is reached only if the first-period buyer decides not to buy the product. We then analyze conditions for the existence of a separating equilibrium for the more general case with imperfectly informative signals, with both two and one objects for sale over both periods.

We show that there exists a separating equilibrium in which high prices signal high quality, even as the equilibrium with time-on-the-market requires more restrictive conditions. As first period prices can be used to manipulate information transmission, sellers can take advantage of this in order to increase second period profits (unless second period profits are linear in beliefs). Such an opportunity is more valuable for good sellers, since they sell more often. However, with time on the market, a force in the opposite direction plays a role, since bad type sellers are on the second period market more often.

Related Literature. The model developed here is closely related to the herding and observational learning literature, initiated by Banerjee (1992) and Bikhchandani, Hirshleifer and Welch (1992). They show that, when prices are fixed, and each agent observes both a private signal and past purchase decisions, a pathological phenomenon may arise. At some point all agents will ignore their own signals and base their decisions only on the observed behavior of the previous agents, which will prevent further learning and may lead to informational cascades. An extension of the herding literature studies strategic pricing and experimentation, in a context of incomplete but symmetric information. Firms use their pricing policies to manipulate the consumers' learning process. Specifically firms optimally choose low prices in order to incentivize experimentation and information diffusion, therefore encouraging herding (see, for example, Bergemann and Valimaki (1996), Caminal and Vives (1996), Vettas (1997), Schlee (2001)). Bose et al. (2006, 2008) study the optimal dynamic pricing in a monopoly market with observational learning and private information on the consumer side. They find, in contrast to the above mentioned literature, that a low introductory price induces little information transmission. The monopolist here uses the price as a screening device, therefore choosing between a low initial price, that conveys very little information, and a high initial price that allows differentiation between consumers vis-a-vis the private information recieved.

This paper is also closely related to the literature on signaling (high) quality through (high) prices. Bagwell and Riordan (1991) show that high (and declining) prices signal high quality, in a monopoly market for durable goods, in which quality is correlated with costs. Judd and Riordan (1994) reach the same result by examining a two-period signal-extraction model with learning. Even though no correlation between quality and costs is assumed, private information on both sides of the market allows the seller to signal high quality through high prices. In both models learning from others is not considered. Bar-Isaac (2003) studies a dynamic learning model with a privately informed monopolist. In his model, signaling comes from the monopolist's strategic decision to sell or not, which in turn affects consumers' learning by observing ex-post outcomes. In equilibrium, good firms never stop selling, while bad firms still sell with positive probability.

Finally we extend the literature on time-on-the-market as sign of quality, initiated by Taylor (1999). Taylor (1999) explores the effect of time-on-the-market on pricing in a two-period model with asymmetric information and a single object for sale (a house). The parametric assumption made about the quality of the item allows him to rule out separating equilibria and focus attention to consumer learning. The main result involves a pooling equilibrium, in which the low-quality seller mimic his high-quality counterpart. Depending on the information structure, the seller may post a higher or a lower price in the first-period.

2 The Model

We consider a model in which a single seller faces a potential buyer each period. The quality of the product may be either high or low, $q \in \{0, 1\}$, which is unknown to both the seller and the buyer. The seller has private information about his type θ, the probability of producing high quality, that can be either good or bad, $\theta \in \{g, b\}$, with $0 < b < g < 1$. The ex post valuation of the buyer is $q + v$[16], where q represents the common value (objective quality) and v is the buyer individual valuation or "taste" for the product, drawn from a distribution $G(v)$ continously differentiable with $G'(v) = g(v) > 0$, for all $v \in [0,1]$. The seller's decision variable is the selling price and there is no discounting.

The timing of the game is the following: the seller's type is drawn by Nature at the outset; the seller learns his type and chooses a pricing strategy. The buyer observes the price and procures an inspection on quality, the inspection's outcome is realized, then the buyer updates beliefs according to Bayes rule and makes a purchase decision. At the beginning of the second period a new buyer arrives, who, after observing previous user's purchase decision, updates beliefs on the product quality and decides whether to buy or not. Second-period profits are then realized and the game ends.

The first-period buyer, after observing the price and before making his purchasing decision, procures an inspection on quality. The outcome of the inspection may be either favorable or unfavorable, $s \in \{F, NF\}$ and it is characterized by the following conditional probabilities:

	0	1
F	$1-\sigma$	1
NF	σ	0

A high-quality product always results in a favorable outcome, whereas a low-quality one generates it with probability $(1-\sigma)$. Therefore a favorable outcome does not garantee high quality, whereas an unfavorable one can be thought of as discovering a flaw in the product, fully revealing low quality. Here, $\sigma \in [0, \overline{\sigma}]$, can be interpreted as a measure of the signal precision[17]. The second-period buyer observes the price history and the previous user's purchase decision, $p \in \{S, NS\}$, where S stands for "sale" and NS for "no-sale" in the previous period.

[16]The results are easily generalizable to any function $f(q, v)$.

[17]The results obtained in this paper are robust to more general signal structures that satisfies the following assumptions: $Pr(F \mid q = 1) > Pr(F \mid q = 0)$ and $Pr(NF \mid q = 1) < Pr(NF \mid q = 0)$.

The first-period buyer starts the game with a prior $\mu_0 = Pr(q = 1)$, and updates beliefs to $\mu^s(\mu)$, after observing the price and the inspection outcome. Note that separating prices $\left(P^b, P^g\right)$ induce beliefs $\mu = b$ if $P = P^b$ and $\mu = g$ if $P = P^g$, whereas pooling prices do not provide any information and the posterior will be the same as the prior, $\mu = \mu_0$. Then

$$\mu^F = \frac{\mu}{\mu + (1-\mu)(1-\sigma)}$$

$$\mu^{NF} = 0.$$

Conditional on the inspection outcome, the buyer will buy the good if $\mu^s(\mu) + v - P \geq 0$, which leads to the demand $\overline{D}(P, \mu) = 1 - G(P - \mu^s(\mu))$ and associated profits

$$\pi(\theta, P, \mu) = PD(\theta, P, \mu)$$

$$D(\theta, P, \mu) = \overline{D}\left(P, \mu^F(\mu)\right)[\theta + (1-\theta)(1-\sigma)] + \overline{D}(P, 0)(1-\theta)\sigma.$$

At the beginning of the second-period a new buyer arrives with a belief $\mu^p(P, \mu)$ about the product quality, where $\mu^p(P, \mu)$ the probability of the product being of high quality, after observing previous period prices and purchase decision. Conditional on history, the second-period buyer will buy the product if $\mu^p(P, \mu) + v - P_2 \geq 0$, which leads to second-period demand $\overline{D}(P_2, P, \mu) = 1 - G(P_2 - \mu^p(P, \mu))$ and associated profits

$$\pi(P_2, P, \mu) = max_{P_2} P_2 \overline{D}(P_2, P, \mu).$$

Therefore the expected profits over both periods of a type-θ seller, who sets a price P in the first period, inducing beliefs μ are

$$\Pi(\theta, P, \mu) = PD(\theta, P, \mu) + E_{(P,\mu)}[\pi(P_2, P, \mu)]$$

$$= PD(\theta, P, \mu) + D(\theta, P, \mu)\left[\pi\left(P^*_{2,S}, P, \mu\right)\right]$$

$$+ (1 - D(\theta, P, \mu))\left[\pi\left(P^*_{2,NS}, P, \mu\right)\right],$$

where $D(\theta, P, \mu)$ denotes the probability of a sale in the first period, and $P^*_{2,S}$ and $P^*_{2,NS}$ are the maximizers of $\pi(P_{2,S}, P, \mu)$ and $\pi(P_{2,NS}, P, \mu)$, respectively, following any history of separation in the first period. Note that second-period profits can be either linear or convex in beliefs, depending on the assumptions made about $G(v)$. We will discuss the implications of linearity and convexity on the separating equilibrium in the subsequent sections.

We define and analyze conditions for the existence of separating equilibria in pure strategies. We characterize separating equilibria for the benchmark case, in which the signal is perfectly informative. We then analyze the general case with an imperfect signal structure. For both cases, we consider the scenario with an object for sale each period and one object for sale over both periods, discussing the implications of time-on-the-market on separating equilibria.

3 Separating Equilibrium

A (first-period) separating equilibrium is a sequential equilibrium at which buyers can distinguish the good and the bad seller by the different pricing choices they made. Note that separating prices allow the buyer to infer the seller's type, but not the true quality of the product. Separating prices $\left(P^b, P^g\right)$ induce beliefs $\mu = b$ if $P = P^b$ and $\mu = g$ if $P = P^g$, whereas pooling prices do not provide any information and the posterior will be the same as the prior, $\mu = \mu_0$. Moreover, off-equilibrium prices $P \neq \left\{P^b, P^g\right\}$ are assumed to induce pessimistic beliefs $\mu = b$, to make the existence of the equilibrium easier.

Definition 4.11. *(Separating Equilibrium)* A separating equilibrium is a pair $\left(P^b, P^g\right)$ such that three conditions hold:

C1. $\Pi\left(b, P^b, \mu = b\right) \geq \Pi\left(b, P, \mu = b\right)$, for every $P \neq P^g$.

C2. $\Pi\left(b, P^b, \mu = b\right) \geq \Pi\left(b, P^g, \mu = g\right)$, and

C3. $\Pi\left(g, P^g, \mu = g\right) \geq \Pi\left(g, P, \mu = b\right)$, for every $P \neq P^g$.

For the bad seller, P^b must dominate any price $P \neq P^g$ under pessimistic beliefs (C1). Moreover, the bad seller should not have incentives to mimic the good one, even if this implies optimistic beliefs (C2). For the good seller, P^g must dominate any other price P that induce pessimistic beliefs (C3).

Lemma 4.12. In any separating equilibrium $P^b = P^{b*}$, where P^{b*} is the maximizer of $\Pi\left(b, P, \mu = b\right)$. Moreover, for the good seller, it is sufficient to check that $\Pi\left(g, P^g, \mu = g\right) \geq \Pi\left(g, P^{g*}, \mu = b\right)$, where P^{g*} is the maximizer of $\Pi\left(g, P, \mu = b\right)$.

Proof. $P^b = P^{b*}$ is a necessary condition for C1 to be satisfied. Moreover C3 requires that the good seller should not have any incentive to deviate from the equilibrium price, with such deviation implying pessimistic beliefs. Then it is sufficient to control for best deviation which occurs at P^{g*}, the maximizer of $\Pi\left(g, P, \mu = b\right)$. ■

4 Benchmark case: perfectly informative signals

4.1 Two objects for sale

In subsequent sections it is assumed first-period buyer receives a noisy signal when procuring the inspection on quality. Prior to study this scenario, it is useful to have a benchmark case, in which the inspection provides first-period buyer with a perfectly informative signal about the product quality, $\sigma = 1$, and, therefore, prices do not carry out any signaling role (for the first-period buyer).

We solve the game by backward induction. Second-period buyer, after observing previous user's purchase decision $p \in \{S, NS\}$, updates beliefs to $\mu^p(P, \mu)$:

$$\mu^{NS} = 0$$

$$\mu^S = \frac{\mu}{\mu + (1-\mu)[1 - G(P)]},$$

where $\mu = b$ if $P = P^b$ and $\mu = g$ if $P = P^g$. Conditional on a "no-sale" history, second-period demand is given by $\overline{D}(P_{2,NS}) = 1 - G(P_{2,NS})$, whereas, conditional on a "sale" history, it is given by $\overline{D}(P_{2,S}, P, \mu) = 1 - G\left(P_{2,S} - \mu^S(P, \mu)\right)$, with associated profits $\pi(P_{2,NS}) = P_{2,NS}\overline{D}(P_{2,NS})$ and $\pi(P_{2,S}, P, \mu) = P_{2,S}\overline{D}(P_{2,S}, P, \mu)$, respectively. Then, for any history of separation $\left(P^{b*}, P^g\right)$, second-period prices are given by $P^*_{2,NS}$ and $P^*_{2,S}$, the maximizers of $\pi(P_{2,NS})$ and $\pi(P_{2,S}, P, \mu)$, respectively.

With perfectly informative signals, first-period buyer beliefs are independent of the pricing strategies. Then $\mu^F = 1$, and $\mu^{NF} = 0$. Note that, conditional on a favorable inspection outcome, the buyer will always buy the product, since we are assuming $P < 1$. Then first-period profits are independent of the beliefs generated by the separating strategy and given by $\pi(\theta, P) = PD(\theta, P)$, with

$$D(\theta, P) = [\theta + [1 - G(P)](1 - \theta)].$$

Therefore the expected profits of a type-θ seller, who sets a price P in the first period, inducing beliefs μ are

$$\Pi(\theta, P, \mu) = PD(\theta, P) + D(\theta, P)\pi\left(P^*_{2,S}, P, \mu\right) + [1 - D(\theta, P)]\pi\left(P^*_{2,NS}\right).$$

A separating equilibrium is defined according to Definition 1. We now show that there is always a separating equilibrium in which high prices signal high quality. The intuition behind this result is the following: on the one hand, a good seller has a higher probability of sale in the first period at any price; on the other hand, and more importantly, the increase in second-period profits, conditional on a first-period sale and high price, is higher for the good seller. Secon-period beliefs are responsible for such a result: if a firm is able to sell at a very high price, after being inspected, then it must be good! Note that the intensity of this result is due to the functional form of second-period profits. In particular the convexity of second-period profits amplifies the effect of beliefs, then requiring lower prices - therefore a lower cost of signaling - to the good seller to achieve separation.

Proposition 4.13. There is always a separating equilibrium $\left(P^{b^*}, P^g\right)$ with $P^g > P^{g*}$.

Proof. Consider the price $\overline{P} > P^{b*}$ such that $\Pi\left(b, P^{b*}, \mu = b\right) = \Pi\left(b, \overline{P}, \mu = g\right)$. If $\overline{P} \leq P^{g*}$, then, by the Envelope Theorem, it is straightforward to show that $\Pi\left(g, \overline{P}, \mu = g\right) \geq \Pi\left(g, P^{g*}, \mu = b\right)$, and $\left(P^{b*}, P^{g**}\right)$ is a separating equilibrium, where P^{g**} is the price that maximizes the good seller profits under optimistic beliefs, $\Pi\left(g, P, \mu = g\right)$.

Thus we can restrict attention to the case $\overline{P} > P^{g*}$. At this price the good seller should not have any incentive to deviate to his "monopoly" price, i.e. $\Pi\left(g, \overline{P}, \mu = g\right) \geq \Pi\left(g, P^{g*}, \mu = b\right)$, which is equivalent to:

$$\Pi\left(g, \overline{P}, \mu = g\right) - \Pi\left(g, P^{g*}, \mu = b\right) \geq \Pi\left(b, \overline{P}, \mu = g\right) - \Pi\left(b, P^{b*}, \mu = b\right) = 0.$$

We can rewrite the left and right hand sides of this inequality as

$$\left[\Pi\left(g, \overline{P}, \mu = g\right) - \Pi\left(g, \overline{P}, \mu = b\right)\right] + \left[\Pi\left(g, \overline{P}, \mu = b\right) - \Pi\left(g, P^{g*}, \mu = b\right)\right] \geq$$

$$\left[\Pi\left(b, \overline{P}, \mu = g\right) - \Pi\left(b, \overline{P}, \mu = b\right)\right] + \left[\Pi\left(b, \overline{P}, \mu = b\right) - \Pi\left(b, P^{b*}, \mu = b\right)\right].$$

Therefore it is enough to show that

1. $\left[\Pi\left(g,\overline{P},\mu=g\right)-\Pi\left(g,\overline{P},\mu=b\right)\right]\geq\left[\Pi\left(b,\overline{P},\mu=g\right)-\Pi\left(b,\overline{P},\mu=b\right)\right]$ and

2. $\left[\Pi\left(g,\overline{P},\mu=b\right)-\Pi\left(g,P^{g*},\mu=b\right)\right]\geq\left[\Pi\left(b,\overline{P},\mu=b\right)-\Pi\left(b,P^{b*},\mu=b\right)\right]$.

Conditions 1 and 2 are implied by: $\frac{\partial^2\Pi(\theta,P,\mu)}{\partial P\partial\theta}>0$ and $\frac{\partial^2\Pi(\theta,P,\mu)}{\partial\theta\partial\mu}>0$. The first condition is standard single-crossing: the cost of signaling through high prices is less detrimental to the good type than to the bad one. On the other hand, the second condition requires that the shift from pessimistic to optimistic beliefs is more attractive to the good type than to the bad one.

1. $\frac{\partial^2\Pi(\theta,P,\mu)}{\partial P\partial\theta}\geq 0$

$$\frac{\partial^2\Pi\left(\theta,P,\mu\right)}{\partial P\partial\theta}=\frac{\partial}{\partial\theta}\{[PD_P\left(\theta,P\right)+D\left(\theta,P\right)]+D_P\left(\theta,P\right)\pi\left(P_{2,S}^*,P,\mu\right)$$

$$+D\left(\theta,P\right)\frac{\partial\pi\left(P_{2,S}^*,P,\mu\right)}{\partial P}-D_P\left(\theta,P\right)\pi\left(P_{2,NS}^*\right)\}$$

$$=\{PD_{P\theta}\left(\theta,P\right)+D_\theta\left(\theta,P\right)+D_{P\theta}\left(\theta,P\right)\pi\left(P_{2,S}^*,P,\mu\right)$$

$$+D_\theta\left(\theta,P\right)\frac{\partial\pi\left(P_{2,S}^*,P,\mu\right)}{\partial P}-D_{P\theta}\left(\theta,P\right)\pi\left(P_{2,NS}^*\right)$$

$$=D_\theta\left(\theta,P\right)\left[1+\frac{\partial\pi\left(P_{2,S}^*,P,\mu\right)}{\partial P}\right]+D_{P\theta}\left(\theta,P\right)\left[P+\pi\left(P_{2,S}^*,P,\mu\right)-\pi\left(P_{2,NS}^*\right)\right]$$

which is always positive since $D_\theta\left(\theta,P\right)=G(P)$, $\frac{\partial\pi\left(P_{2,S}^*,P,\mu\right)}{\partial P}=P_{2,S}^*g(P_{2,S}^*-\mu^S)\frac{\mu(1-\mu)}{[\mu+(1-\mu)(1-G(P))]^2}>0$, $D_{P\theta}\left(\theta,P\right)=g(P)>0$ and $\pi\left(P_{2,S}^*,P,\mu\right)>\pi\left(P_{2,NS}^*\right)$.

We show now that $\frac{\partial^2\Pi(\theta,P,\mu)}{\partial\theta\partial\mu} \geq 0$:

$$\frac{\partial^2\Pi(\theta,P,\mu)}{\partial\theta\partial\mu} = \frac{\partial}{\partial\theta}\left\{D(\theta,P)\left[\frac{\partial\pi\left(P^*_{2,S},P,\mu\right)}{\partial\mu}\right]\right\}$$

$$= D_\theta(\theta,P)\left[\frac{\partial\pi\left(P^*_{2,S},P,\mu\right)}{\partial\mu}\right]$$

which is positive since $\frac{\partial\pi\left(P^*_{2,S},P,\mu\right)}{\partial\mu} = P^*_{2,S}g(P^*_{2,S}-\mu^S)\ \frac{1-G(P)}{[\mu+(1-\mu)(1-G(P))]^2} > 0.$■

4.2 Time-on-the-Market

We now consider the case in which there is only one asset on sale over both periods (therefore both price and time-on-the-market can signal quality) and look for separating equilibria as well. At the beginning of the first period the seller posts a separating price P. After observing the price and the inspection's outcome, the buyer makes his purchase decision. If no sale occurs in the first period, then we get to the second stage of the game, where the buyer update beliefs on the seller's type, taking into account the fact that the asset did not sell in the first period (time-on-the-market), and $\mu^N = 0$ as before. Note that in this case the expected profits of a type-θ seller who sets the price P in the first period, inducing beliefs μ are

$$\Pi(\theta,P,\mu) = PD(\theta,P) + [1-D(\theta,P)]\,\pi\left(P^*_{2,NS}\right)$$

where $[1-D(\theta,P)]$ represents the probability of a "no-sale" in the first period and $\pi\left(P^*_{2,NS}\right)$ are the profits associated with that history. We show that there is always a separating equilibrium in which high prices (and time-on-the-market) signals quality.

Proposition 4.14. There is always a separating equilibrium $\left(P^{b^*},P^g\right)$ with $P^g > P^{g*}$.

As we know, we need to check that:

1) $\frac{\partial^2 \Pi(\theta,P,\mu)}{\partial P \partial \theta} > 0$ and

2) $\frac{\partial^2 \Pi(\theta,P,\mu)}{\partial \theta \partial \mu} > 0$.

$$\frac{\partial^2 \Pi\left(\theta, P, \mu\right)}{\partial P \partial \theta} = \frac{\partial}{\partial \theta}\left\{\left[P D_P\left(\theta, P\right) + D\left(\theta, P\right)\right] - D_P\left(\theta, P\right) \pi\left(P^*_{2,NS}\right)\right\}$$

$$= \left\{P D_{P\theta}\left(\theta, P\right) + D_\theta\left(\theta, P\right) - D_{P\theta}\left(\theta, P\right) \pi\left(P^*_{2,NS}\right)\right\}$$

$$= \left\{D_\theta\left(\theta, P\right) + D_{P\theta}\left(\theta, P\right)\left[P - \pi\left(P^*_{2,NS}\right)\right]\right\}$$

which is positive if $P > \pi\left(P^*_{2,NS}\right)$. Since $P_{2,NS} = argmax_{P_2} P_2\left[1 - G\left(P\right)\right]$, and $P^{g*} = argmax_P\left[\theta + \left[1 - G\left(P\right)\right]\left(1 - \theta\right)\right] + \pi\left(P^*_{2,NS}\right)$, then $P^{g*} > P^*_{2,NS}$. Finally $P^*_{2,NS} > P^*_{2,NS}\left[1 - G\left(P^*_{2,NS}\right)\right]$. Then we can conclude that $P > \pi\left(P^*_{2,NS}\right)$, for any $P > P^{g*}$.

We show now that $\frac{\partial^2 \Pi(\theta,P,\mu)}{\partial \theta \partial \mu} > 0$:

$$\frac{\partial^2 \Pi\left(\theta, P, \mu\right)}{\partial \theta \partial \mu} = 0 \blacksquare$$

Note that in this case the positive effect of beliefs disappear, as $\frac{\partial^2 \Pi(\theta,P,\mu)}{\partial \theta \partial \mu} = 0$. This is because the game reaches the second stage only after a history of "no-sale", in which case beliefs do not play any role - second-period buyer knows with certainty, after a "no-sale" history, that the inspection's outcome was unfavorable in the previous period. Therefore separation can occur only at a very high cost, that is charging a price $P > \pi\left(P^*_{2,NS}\right)$.

5 Imperfect signals

5.1 Two objects for sale

We now investigate the more general case, in which first-period buyer receives a noisy signal when procuring the inspection on quality, $\sigma \in [0, \overline{\sigma}]$. As before, a high-quality product always yields a favorable outcome, whereas a low-quality one generates it with probability $(1 - \sigma)$.

We solve the game by backward induction. Second-period buyer, after observing previous user's purchase decision $p \in \{S, NS\}$, updates beliefs to $\mu^p(P, \mu)$:

$$\mu^{NS} = \frac{\mu G\left(P - \mu^F(\mu)\right)}{\left[1 - \sigma(1-\mu)\right] G\left(P - \mu^F(\mu)\right) + \sigma(1-\mu) G(P)}$$

$$\mu^S = \frac{\mu\left[1 - G\left(P - \mu^F(\mu)\right)\right]}{\left[1 - \sigma(1-\mu)\right]\left[1 - G\left(P - \mu^F(\mu)\right)\right] + \sigma(1-\mu)\left[1 - G(P)\right]},$$

where $[1 - G(\cdot)]$ represents first-period demand, $\mu^F(\mu) = \frac{\mu}{\mu + (1-\mu)(1-\sigma)}$, $\mu = b$ if $P = P^b$ and $\mu = g$ if $P = P^g$. Then, conditional on a "no-sale" history, second-period demand is given by $\overline{D}(P_{2,NS}, P, \mu) = 1 - G\left(P_{2,NS} - \mu^{NS}(P, \mu)\right)$,, whereas, conditional on a "sale" history, it is given by $\overline{D}(P_{2,S}, P, \mu) = 1 - G\left(P_{2,S} - \mu^S(P, \mu)\right)$, with associated profits $\pi(P_{2,NS}, P, \mu) = P_{2,NS}\overline{D}(P_{2,NS}, P, \mu)$, and $\pi(P_{2,S}, P, \mu) = P_{2,S}\overline{D}(P_{2,S}, P, \mu)$, respectively. Then, for any history of separation $\left(P^{b*}, P^g\right)$, second-period prices are given by $P^*_{2,NS}$ and $P^*_{2,S}$, the maximizers of $\pi(P_{2,NS}, P, \mu)$ and $\pi(P_{2,S}, P, \mu)$, respectively.

Note that now first-period buyer takes into account both prices and the inspection's outcome when updating beliefs about product quality. Then $\mu^F(\mu) = \frac{\mu}{\mu + (1-\mu)(1-\sigma)}$ and $\mu^{NF}(\mu) = 0$. Again, conditional on a favorable inspection outcome, the buyer will always buy the product, which leads to the demand $\overline{D}(P, \mu) = 1 - G(P - \mu^s(\mu))$ and associated profits

$$\pi(\theta, P, \mu) = PD(\theta, P, \mu)$$

$$D(\theta, P, \mu) = \overline{D}\left(P, \mu^F(\mu)\right)[\theta + (1-\theta)(1-\sigma)] + \overline{D}(P, 0)(1-\theta)\sigma.$$

Therefore the expected profits of a type-θ seller, who sets a price P in the first period, inducing beliefs μ are

$$\Pi(\theta, P, \mu) = PD(\theta, P, \mu) + D(\theta, P, \mu)\left[\pi\left(P^*_{2,S}, P, \mu\right)\right] + (1 - D(\theta, P, \mu))\left[\pi\left(P^*_{2,NS}, P, \mu\right)\right].$$

Proposition 4.15. If $G\left(P^{g*} - \mu^F(\mu)\right) \geq \frac{1}{2}$ and $g(\cdot) > 0$ is non-decreasing, there is always a separating equilibrium $\left(P^{b^*}, P^g\right)$ with $P^g > P^{g*}$.

Proof. As we know, we need to check that:

1) $\frac{\partial^2 \Pi(\theta,P,\mu)}{\partial P \partial \theta} > 0$ and

2) $\frac{\partial^2 \Pi(\theta,P,\mu)}{\partial \theta \partial \mu} > 0.$

$$\frac{\partial^2 \Pi(\theta, P, \mu)}{\partial P \partial \theta} = \frac{\partial}{\partial \theta}\{[PD_P(\theta, P, \mu) + D(\theta, P, \mu)] + D_P(\theta, P, \mu)\,\pi\left(P^*_{2,Y}, P, \mu\right)$$

$$+D(\theta, P, \mu)\frac{\partial \pi\left(P^*_{2,Y}, P, \mu\right)}{\partial P} - D_P(\theta, P, \mu)\,\pi\left(P^*_{2,N}, P, \mu\right) + (1 - D(\theta, P, \mu))\frac{\partial \pi\left(P^*_{2,N}, P, \mu\right)}{\partial P}\}$$

$$= \{PD_{P\theta}(\theta, P, \mu) + D_\theta(\theta, P, \mu) + D_{P\theta}(\theta, P, \mu)\,\pi\left(P^*_{2,Y}, P, \mu\right)$$

$$+D_\theta(\theta,P,\mu)\frac{\partial\pi\left(P^*_{2,Y},P,\mu\right)}{\partial P}-D_{P\theta}(\theta,P,\mu)\,\pi\left(P^*_{2,N},P,\mu\right)-D_\theta(\theta,P,\mu)\frac{\partial\pi\left(P^*_{2,N},P,\mu\right)}{\partial P}\}$$

$$=D_\theta\left[1+\frac{\partial\pi\left(P^*_{2,Y},P,\mu\right)}{\partial P}-\frac{\partial\pi\left(P^*_{2,N},P,\mu\right)}{\partial P}\right]+D_{P\theta}\left[P+\pi\left(P^*_{2,Y},P,\mu\right)-\pi\left(P^*_{2,N},P,\mu\right)\right],$$

which is positive since $D_\theta(\theta,P)=\sigma\left[G(P)-G(P-\mu^F(\mu))\right]>0$, $D_{P\theta}(\theta,P)=\sigma\left[g(P)-g(P-\mu^F(\mu))\right]$ and $\pi\left(P^*_{2,S},P,\mu\right)>\pi\left(P^*_{2,NS},P,\mu\right)$. To conclude, we now show that $\frac{\partial\pi\left(P^*_{2,S},P,\mu\right)}{\partial P}>\frac{\partial\pi\left(P^*_{2,NS},P,\mu\right)}{\partial P}$ if $G\left(P^{g*}-\mu^F(\mu)\right)\geq\frac{1}{2}$.

We can write $\pi(P_{2,p},P,\mu)$ as

$$\pi(P_{2,p},P,\mu,\eta)=max_{P_{2,p}}P_{2,p}\left[1-G\left(P_{2,p}-\left(\eta\mu^S+(1-\eta)\,\mu^{NS}\right)\right)\right].$$

so that

$$\frac{\partial}{\partial P}\left\{\pi\left(P^*_{2,S},P,\mu\right)-\pi\left(P^*_{2,NS},P,\mu\right)\right\}=\wr_{\eta=0}^{\eta=1}\frac{\partial^2\pi(P_{2,p},P,\mu,\eta)}{\partial P\partial\eta}.$$

Then, it is enough to prove $\frac{\partial^2\pi(P_{2,p},P,\mu,\eta)}{\partial P\partial\eta}\geq 0$.

$$\frac{\partial^2\pi(P_{2,p},P,\mu,\eta)}{\partial P\partial\eta}=\frac{\partial}{\partial P}\left\{\left[1-G\left(P_{2,p}-\left(\eta\mu^S+(1-\eta)\,\mu^{NS}\right)\right)\right]\left(\mu^S(P,\mu)-\mu^{NS}(P,\mu)\right)\right\}$$

$$=\left\{\left[1-G\left(P_{2,p}-\left(\eta\mu^S+(1-\eta)\,\mu^{NS}\right)\right)\right]\frac{\partial}{\partial P}\left(\mu^S(P,\mu)-\mu^{NS}(P,\mu)\right)\right\}$$

$$+\left(\mu^S-\mu^{NS}\right)g\left(P_{2,p}-\left(\eta\mu^S+(1-\eta)\,\mu^{NS}\right)\right)\left[-\frac{\partial P_{2,p}}{\partial P}+\eta\frac{\partial}{\partial P}\left(\mu^S-\mu^{NS}\right)+\frac{\partial\mu^{NS}}{\partial P}\right]$$

$$= \left\{ \left[1 - G\left(P_{2,p} - \left(\eta \mu^S + (1-\eta)\,\mu^{NS}\right)\right)\right] \frac{\partial}{\partial P}\left(\mu^S - \mu^{NS}\right) \right\}$$

$$+ \left(\mu^S - \mu^{NS}\right) g\left(P_{2,p} - \left(\eta \mu^S + (1-\eta)\,\mu^{NS}\right)\right) \left[\left(1 - \frac{H'}{1-H'}\right)\left(\eta \frac{\partial}{\partial P}\left(\mu^S - \mu^{NS}\right) + \frac{\partial \mu^{NS}}{\partial P}\right)\right],$$

where the last equality comes from $\frac{\partial P_{2,p}}{\partial P} = -\frac{H'(\cdot)}{1-H'(\cdot)}\left[\eta \frac{\partial}{\partial P}\left(\mu^S - \mu^{NS}\right) + \frac{\partial \mu^{NS}}{\partial P}\right]$, with $H'(\cdot) = \left(\frac{1-G(\cdot)}{g(\cdot)}\right)' < 0.$

Then a sufficient condition to $\frac{\partial^2 \pi(P_{2,p},P,\mu,\eta)}{\partial P \partial \eta} \geq 0$ is that $\frac{\partial}{\partial P}\left(\mu^S - \mu^{NS}\right) \geq 0.$

$$\frac{\partial}{\partial P}\left(\mu^S - \mu^{NS}\right) = \frac{\mu\sigma(1-\mu)\left\{\left[(1-\sigma(1-\mu))\,G\left(P-\mu^F\right) + \sigma(1-\mu)\,G(P)\right]^2 \left\{g(P)\left[1-G\left(P-\mu^F\right)\right] - g\left(P-\mu^F\right)\left[1-G(P)\right]\right\}\right.}{\left[(1-\sigma(1-\mu))\,G\left(P-\mu^F\right) + \sigma(1-\mu)\,G(P)\right]^2 \left[(1-\sigma(1-\mu))\left(1-G\left(P-\mu^F\right)\right) + \sigma(1-\mu)(1-G(P))\right]^2}$$

$$+ \frac{\left.\left[(1-\sigma(1-\mu))\left(1-G\left(P-\mu^F\right)\right) + \sigma(1-\mu)(1-G(P))\right]^2 \left\{g(P)\,G\left(P-\mu^F\right) - g\left(P-\mu^F\right)G(P)\right\}\right\}}{\left[(1-\sigma(1-\mu))\,G\left(P-\mu^F\right) + \sigma(1-\mu)\,G(P)\right]^2 \left[(1-\sigma(1-\mu))\left(1-G\left(P-\mu^F\right)\right) + \sigma(1-\mu)(1-G(P))\right]^2} \geq 0.$$

We need that

$$\left[(1-\sigma(1-\mu))\,G\left(P-\mu^F\right) + \sigma(1-\mu)\,G(P)\right]^2 \geq$$

$$\left[(1-\sigma(1-\mu))\left(1-G\left(P-\mu^F\right)\right) + \sigma(1-\mu)(1-G(P))\right]^2$$

and $\left\{g(P)\left[1-G\left(P-\mu^F\right)\right] - g\left(P-\mu^F\right)\left[1-G(P)\right]\right\} \geq -\left\{g(P)\,G\left(P-\mu^F\right) - g\left(P-\mu^F\right)G(P)\right\}.$ The first inequality is garanteed by $G\left(P^{g*} - \mu^F(\mu)\right) \geq \frac{1}{2}$, and the second one by $g(\cdot)$ increasing.

We show now that $\frac{\partial^2 \Pi(\theta,P,\mu)}{\partial \theta \partial \mu} \geq 0$:

$$\frac{\partial^2 \Pi(\theta,P,\mu)}{\partial \theta \partial \mu} = \frac{\partial}{\partial \theta}\{P D_\mu + D_\mu \pi\left(P^*_{2,S}, P, \mu\right) + D\left[\frac{\partial \pi\left(P^*_{2,S}, P, \mu\right)}{\partial \mu}\right]$$

$$-D_\mu \pi\left(P_{2,NS}^*, P, \mu\right) + (1-D)\left[\frac{\partial \pi\left(P_{2,NS}^*, P, \mu\right)}{\partial \mu}\right]\}$$

$$= D_{\mu\theta}\left[P + \pi\left(P_{2,S}^*, P, \mu\right) - \pi\left(P_{2,NS}^*, P, \mu\right)\right] + D_\theta\left[\frac{\partial \pi\left(P_{2,S}^*, P, \mu\right)}{\partial \mu} - \frac{\partial \pi\left(P_{2,NS}^*, P, \mu\right)}{\partial \mu}\right].$$

which is positive since $D_\theta(\theta, P) > 0$, $D_{\mu\theta}(\theta, P) = \frac{\sigma(1-\sigma)}{[\mu+(1-\mu)(1-\sigma)]^2}\left[g\left(P - \mu^F(\mu)\right)\right] > 0$, $\pi\left(P_{2,S}^*, P, \mu\right) - \pi\left(P_{2,NS}^*, P, \mu\right) > 0$. We just need to show now that $\frac{\partial \pi\left(P_{2,S}^*, P, \mu\right)}{\partial \mu} > \frac{\partial \pi\left(P_{2,NS}^*, P, \mu\right)}{\partial \mu}$. Again we can write

$$\pi\left(P_{2,p}, P, \mu, \eta\right) = max_{P_{2,p}} P_{2,p}\left[1 - G\left(P_{2,p} - \left(\eta\mu^S + (1-\eta)\mu^{NS}\right)\right)\right],$$

and show that $\frac{\partial^2 \pi(P_{2,p}, P, \mu, \eta)}{\partial \mu \partial \eta} \geq 0$.

$$\frac{\partial^2 \pi\left(P_{2,p}, P, \mu, \eta\right)}{\partial \mu \partial \eta} = \left\{\left[1 - G\left(P_{2,p} - \left(\eta\mu^S + (1-\eta)\mu^{NS}\right)\right)\right]\frac{\partial}{\partial \mu}\left(\mu^S - \mu^{NS}\right)\right\}$$

$$+\left(\mu^S - \mu^{NS}\right) g\left(P_{2,p} - \left(\eta\mu^S + (1-\eta)\mu^{NS}\right)\right)\left[\left(1 - \frac{H'}{1-H'}\right)\left(\eta\frac{\partial}{\partial \mu}\left(\mu^S - \mu^{NS}\right) + \frac{\partial \mu^{NS}}{\partial \mu}\right)\right].$$

A sufficient condition for the former expression to be positive is $\frac{\partial}{\partial \mu}\left(\mu^S - \mu^{NS}\right) \geq 0$.

Since $\frac{\partial}{\partial \mu}\left(\frac{1}{\mu^S} - \frac{1}{\mu^{NS}}\right) = \frac{1}{\mu^{NS^2}}\left[\frac{\partial \mu^{NS}}{\partial \mu} - \frac{\partial \mu^S}{\partial \mu}\left(\frac{\mu^{NS}}{\mu^S}\right)^2\right]$ and $\frac{\mu^{NS}}{\mu^S} < 1$, it is enough to prove that

$$\frac{\partial}{\partial \mu}\left(\frac{1}{\mu^S} - \frac{1}{\mu^{NS}}\right) < 0.$$

Note that $\left(\frac{\partial \mu^{NS}}{\partial \mu} - \frac{\partial \mu^S}{\partial \mu}\right) = \frac{\partial}{\partial \mu}\left\{\sigma(1-\mu)\left[\frac{1-G(P)}{1-G(P-\mu^F)} - \frac{G(P)}{G(P-\mu^F)}\right]\frac{1}{\mu}\right\}$ which is negative.■

5.2 Time-on-the-market

We now consider the case in which there is only one asset on sale over both periods (therefore both price and time-on-the-market can signal quality) and look for separating equilibria as well. At the beginning of the first period the seller posts a separating price P. After observing the price and the inspection's outcome, the buyer makes his purchase decision. If no sale occurs in the first period, then we get to the second stage of the game, where the buyer update beliefs on the seller's type, taking into account the fact that the asset did not sell in the first period (time-on-the-market), and $\mu^N = 0$ as before. Note that in this case the expected profits of a type-θ seller who sets the price P in the first period, inducing beliefs μ are

$$\Pi(\theta, P, \mu) = PD(\theta, P, \mu) + (1 - D(\theta, P, \mu)) \left[\pi\left(P^*_{2,NS}, P, \mu\right)\right].$$

where $[1 - D(\theta, P, \mu)]$ represents the probability of a "no-sale" in the first period and $\pi\left(P^*_{2,NS}, P, \mu\right)$ are second-period profits conditional on that history. We show that there is always a separating equilibrium in which high prices (and time-on-the-market) signals quality.

Proposition 4.16. If $\overline{\sigma} \leq 4\frac{G^2(P^{g*}-1)}{g^2(1)|J|}$, where $\mid J \mid$ is the maximum of $\left(\frac{G(\cdot)}{g(\cdot)}\right)^{\prime}$, there is always a separating equilibrium $\left(P^{b*}, P^g\right)$ with $P^g > P^{g*}$.

Proof. As we know, we need to check that:

1) $\frac{\partial^2\Pi(\theta,P,\mu)}{\partial P \partial \theta} > 0$ and

2) $\frac{\partial^2\Pi(\theta,P,\mu)}{\partial \theta \partial \mu} > 0$.

$$\frac{\partial^2\Pi(\theta, P, \mu)}{\partial P \partial \theta} = \frac{\partial}{\partial \theta}\{[PD_P + D] - D_P\pi\left(P^*_{2,NS}, P, \mu\right) + (1 - D)\frac{\partial \pi\left(P^*_{2,NS}, P, \mu\right)}{\partial P}\}$$

$$= D_\theta \left[1 - \frac{\partial \pi\left(P^*_{2,NS}, P, \mu\right)}{\partial P}\right] + D_{P\theta}\left[P - \pi\left(P^*_{2,NS}, P, \mu\right)\right],$$

which is positive since $P > \pi\left(P^*_{2,NS}, P, \mu\right)$ and $\frac{\partial \pi\left(P^*_{2,NS}, P, \mu\right)}{\partial P} < 1$ if $\overline{\sigma} \leq 4\frac{G^2(P^{g*}-1)}{g^2(1)|J|}$. We now check that $\frac{\partial \pi\left(P^*_{2,NS}, P, \mu\right)}{\partial P} < 1$:

$$\frac{\partial \pi\left(P^*_{2,NS}, P, \mu\right)}{\partial P} = \left[1 - G\left(P^*_{2,NS} - \mu^{NS}\right)\right]\frac{\partial \mu^{NS}}{\partial P}$$

using the fact that at the optimum $P^*_{2,NS} = \frac{1-G\left(P^*_{2,NS}-\mu^{NS}\right)}{g\left(P^*_{2,NS}-\mu^{NS}\right)}$. So that it is enough to show that $\frac{\partial \mu^{NS}}{\partial P} < 1$:

$$\frac{\partial \mu^{NS}}{\partial P} = \frac{\sigma\mu\left(1-\mu\right)\left[G\left(P\right)g\left(P-\mu^F\right) - G\left(P-\mu^F\right)g\left(P\right)\right]}{\left[\left(1-\sigma\left(1-\mu\right)\right)G\left(P-\mu^F\right) + \sigma\left(1-\mu\right)G\left(P\right)\right]^2}$$

$$\leq \frac{\sigma\mu\left(1-\mu\right)\left[G\left(P\right)g\left(P-\mu^F\right) - G\left(P-\mu^F\right)g\left(P\right)\right]}{\left[G\left(P-\mu^F\right)\right]^2}$$

$$\leq \frac{\overline{\sigma}\frac{1}{4}g\left(P-\mu^F\right)g\left(P\right)\left[\frac{G(P)}{g(P)} - \frac{G\left(P-\mu^F\right)}{g\left(P-\mu^F\right)}\right]}{\left[G\left(P-\mu^F\right)\right]^2}.$$

$$\leq \frac{\overline{\sigma}\frac{1}{4}g^2\left(P\right) \mid J \mid \mu^F}{G^2\left(P-\mu^F\right)} \leq \frac{\overline{\sigma}\frac{1}{4}g^2\left(1\right) \mid J \mid \mu^F}{G^2\left(P^{g*}-\mu^F\right)}$$

which is lower than 1 for $\overline{\sigma} \leq 4\frac{G^2(P^{g*}-1)}{g^2(1)|J|}$, where $J \equiv \left(\frac{G(\cdot)}{g(\cdot)}\right)$ and where $\mid J \mid$ is the maximum of $\left(\frac{G(\cdot)}{g(\cdot)}\right)'$. We show now that $\frac{\partial^2 \Pi(\theta,P,\mu)}{\partial\theta\partial\mu} \geq 0$:

$$\frac{\partial^2 \Pi\left(\theta, P, \mu\right)}{\partial\theta\partial\mu} = D_{\mu\theta}\left[P - \pi\left(P^*_{2,NS}, P, \mu\right)\right] - D_\theta\frac{\partial \pi\left(P^*_{2,NS}, P, \mu\right)}{\partial \mu} > 0$$

if

$$\frac{\partial D_\theta}{\partial \mu}\frac{\mu}{D_\theta} \geq -\frac{\partial\left[P - \pi\left(P^*_{2,NS}, P, \mu\right)\right]}{\partial \mu}\frac{\mu}{\left[P - \pi\left(P^*_{2,NS}, P, \mu\right)\right]}. \blacksquare$$

References

[1] BAGWELL, K. and M. RIORDAN (1991): “High and Declining Prices Signal Product Quality”, *The American Economic Review*, Vol. 81, No. 1, pp. 224-239.

[2] BANERJEE, A.V. (1992): “A simple model of herd behavior”, *Quarterly Journal of Economics*, Vol. 107, pp. 797–817.

[3] BAR-ISAAC, H. (2003): “Reputation and Survival: Learning in a Dynamic Signalling Model”, *Review of Economic Studies*, Vol. 70, pp. 231-251.

[4] BECKERG, .S. (1991): “A Note on Restaurant Pricing and Other Examples of Social Influences on Price”, *Journal of Political Economy*, Vol. 99, pp. 1109-1116.

[5] BERGEMANN, J. and J. VALIMAKI (1996): “Learning and Strategic Pricing”, *Econometrica*, Vol. 64, pp. 1125-49.

[6] BIKHCHANDANI, S., HIRSHLEIFER, D. and I. WELCH (1992): “A theory of fads, fashion, custom, and cultural change as informational cascades”, *Journal of Political Economy*, Vol. 100, pp. 992–1026.

[7] BOSE, S., OROSEL, G.O., OTTAVIANI, M., VESTERLUND, L. (2006): “Monopoly pricing in the binary herding model”, *Economic Theory,* Vol. 37, pp. 203-241.

[8] BOSE, S., OROSEL, G.O., OTTAVIANI, M., VESTERLUND, L. (2008): “Dynamic monopoly pricing and herding”, *The Rand Journal of Economics,* Vol. 37, pp. 910–928.

[9] CAMINAL, R. and X. VIVES (1996): Caminal, R., Vives, X.: “Why market shares matter: an information-based theory”, *The Rand Journal of Economics,* Vol. 27, pp. 221– 239.

[10] CHAMLEY, C. (2004): Rational Herds: Economic Models of Social Learning, Cambridge, University Press.

[11] JUDD, K. and M. RIORDAN (1994): “Price and Quality in a New Product Monopoly”, *The Review of Economic Studies,* Vol. 61, No. 4, pp. 773-789.

[12] MILGROM, P. and J. ROBERTS (1986): "Price and Advertising Signals of Product Quality", *Journal of Political Economy,* Vol. 94, No. 4, pp. 796-821.

[13] SCHLEE, E. (2001): "Buyer Experimentation and Introductory Pricing", *Journal of Economic Behavior and Organization,* Vol. 44, pp. 347-362.

[14] SHAPIRO, C. (1983): "Optimal Pricing of Experience Goods", *The Bell Journal of Economics,* Vol. 14, No. 2, pp. 497-507.

[15] TAYLOR, C. (1999): "Time-on-the-Market as a Sign of Quality", *The Review of Economic Studies,* Vol. 66, pp. 555-578.

[16] VETTAS, N. (1997): "On the Informational Role of Quantities: Durable Goods and Consumers' Word- of-Mouth Communication", *International Economic Review,* Vol. 38, No. 4, pp. 915-944.

[17] WOLINSKY, A. (1983): "Prices as Signals of Product Quality", *The Review of Economic Studies*, Vol. 50, No. 4, pp. 647-658.