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A Model of Final Offer Arbitration in Regulation*

JUAN-PABLO MONTERO
Catholic University of Chile and MIT
Department of Economics, Vicuna Mackena 4860, Santiago, Chile
E-mail: jmontero@faceapuc.cl

Abstract
I study a regulatory process in which both the regulator and the regulated firm propose prices that, in case of disagreement, are settled through final-offer arbitration (FOA)—a practice currently used in Chile for setting prices in the water sector. Rather than submitting a single offer, each party simultaneously submits an offer for each of the firm's cost units (e.g., cost of raw water, capital cost). While a multiple-offers scheme allow the arbitrator to better approximate her ideal settlement, it may induce parties to submit widely divergent offers. This divergence, however, does not affect the arbitrator's ability to learn from the offers.

Key words: final-offer arbitration, price regulation, Nash equilibrium

JEL Classifications: L50, L90

1. Introduction

Departing from the more traditional rate-of-return and price-cap regulations, prices of public utilities in Chile are set using a particular form of yardstick regulation in which the benchmarking is based on a hypothetical efficient firm.[1] Under this price-setting process—introduced first in the electricity sector in the early 1980s—both the regulator and the regulated firm have a very explicit interaction. Based on their own estimation of the long-term costs of this hypothetical efficient firm, both parties propose the price to be charged by the regulated firm

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1 See Vogelsang (2002) for an overview of the different regulatory approaches practiced over the last 20 years.

for the duration of the review period (4–5 years).[2] If the parties cannot agree on the price, the disagreement is settled through an arbitration process.

Since 1999 this arbitration process has taken a distinct form in the water sector. In order to prevent parties' offers from significantly diverging, as has occurred in the other regulated sectors, the legislation that governs the water sector considers a final-offer arbitration mechanism in which the arbitrator is constrained to choose one of the parties' offers as a settlement.[3] But because parties do not submit a single offer for the entire firm but rather submit an offer for each of the cost units into which the firm has been divided,[4] the actual arbitration mechanism looks more like a hybrid between final-offer arbitration and conventional arbitration.[5]

While division of the regulated firm into various units was aimed at introducing greater transparency into the regulatory process and avoiding subsidization across cost units, evidence from the first round of applying this price-setting process for the different water utilities in the country has not been uncontroversial. As shown in Table 1, in most cases the regulator's offer for the entire firm, p^r, diverged significantly from the firm's offer, p^f (to facilitate the exposition p^r has been normalized to 100).[6] In five cases, the parties failed to negotiate the final price, p^s, and instead resorted to final-offer arbitration (FOA).

This great divergence in parties' offers has raised important questions. Some observers have challenged the advantages of the current regulatory mechanism over more conventional mechanisms, particularly price caps as practiced in the UK, while others have questioned the privatization process itself, arguing that the increase in information asymmetries have more than offset any productivity gains.[7] Rather than introducing radical changes in both the privatization program and the regulatory scheme, the authority is exploring ways in which the actual divergence in parties' offers could be diminished. In particular, it is proposing to substantially reduce the multiplicity of offers, i.e., the number of units into which the regulated firm is divided. Reducing the number of offers seems reasonable since one would think that at the limiting case in which parties are required to submit a single offer,

2 In reality, each party constructs an efficient firm and announces the long-term total cost that such a firm would incur by providing the service during the review period. In this construction, the parties may differ not only in unit costs but also in their projections of future demand.

3 The use of final-offer arbitration is commonly seen in settlements of labor disputes (baseball serving as a classic example) but I am not aware of its explicit use elsewhere in a regulatory context.

4 There are approximately 200 units including, among others, cost of raw water, cost of capital, and cost of replacing pavement (Sánchez and Coria 2003).

5 In conventional arbitration, the arbitrator is not constrained to any particular settlement. So, as the number of units becomes increasingly large, final-offer arbitration would seem to approach conventional arbitration since the arbitrator is able to choose almost any settlement by using some combination of parties' offers.

6 The numbers shown are based on parties' announcements of long-term total costs.

7 See Gomez-Lobo and Vargas (2002) for further discussion of the shortcomings of the current regulatory scheme.

Table 1. Firms' Characteristics, Parties' Offers, and Settlements

Firm	Location	Size*	Ownership	p^r	p^f	p^s	FOA
ESSAT	I	3.3	state	100	148	118	yes
ESSAN	II	3.3	state	100	110	106	no
EMSSAT	III	1.9	state	100	112	102	no
ESSCO	IV	4.1	state	100	128	108	no
ESVAL	V	12.9	private	100	184	141	yes
SMAPA	MR	4.7	state	100	125	107	no
Aguas Cordillera	MR	2.7	private	100	156	113	no
Aguas Andinas	MR	37.2	private	100	256	139	yes
ESSEL	VI	4.3	private	100	137	109	no
ESSAM	VII	4.7	state	100	131	113	yes
ESSBIO	VIII	10.8	private	100	115	104	no
ESSAR	IX	4.4	state	100	127	112	no
ESSAL	X	3.9	private	100	146	117	yes
EMSSA	XI	0.6	state	100	137	108	no
ESMAG	XII	1.2	state	100	119	109	no

Source: Superintendencia de Servicios Sanitarios (Agency of Water Services).
*Size is the percentage of consumers served.

they would have little incentive to submit a distant offer that has a low probability of being chosen by the arbitrator.

To better understand agents' behavior in this price-setting process, in this paper I extend the one-dimensional final-offer arbitration models of Farber (1980) and Gibbons (1988) to the case in which parties simultaneously submit offers for each of the units that are part of the item in dispute. Under this multiple-offers scheme, the arbitrator is limited to choosing one party's offer or the other for each unit, so that in principle, the arbitrator is free to fashion a compromise by awarding some offers to one party and the rest to the second party. Despite the fact that this multi-dimensional variant of final-offer arbitration was already recognized by Farber in his article as "issue by issue" final-offer arbitration, the literature contains no formal analysis of such a problem.[8]

Understanding the equilibrium properties of this arbitration game is not only relevant for the price-setting process that motivated this paper,[9] but more generally for any final-offer arbitration in which more than one issue is in dispute

8 A seemingly related problem in the literature is the analysis of split-award auctions where it is possible for a buyer to split a production award between two or more suppliers (Anton and Yao 1989, 1992). These problems have little in common, however. In split-award auctions, bidding parties seek to coordinate in high prices that would report positive profits for both, while in final-offer arbitration, parties have no incentive to coordinate in any particular outcome since they have opposing preferences.

9 This arbitration scheme has also been proposed in place of the current mechanisms used to settle disputes over regulated prices in the electricity and telecommunication sectors in Chile.

(e.g., a government and a contractor renegotiating a multipart contract). The model of the paper is standard in that it is based on a one-period game that considers two parties (i.e., the firm and the regulator) with opposing preferences that simultaneously submit offers to an arbitrator whose ideal settlement is imperfectly known by both parties (recall that parties' uncertainty regarding the arbitrator's preferences is what leads to the divergence of their offers).[10] In the spirit of the legislation, the arbitrator wants to choose efficient prices, i.e., prices that are closest to the long-term cost of the hypothetical efficient firm. But since the parties are much better informed about the cost of this efficient firm than the arbitrator (in part because they conduct detailed studies before submitting their offers), I follow Gibbons (1988) in that the arbitrator may eventually learn a great deal from the parties' (equilibrium) offers about the true cost of this efficient firm. Learning from parties' offers is particularly relevant in this price-setting process because much of the information is case-specific, so it is difficult for arbitrators to acquire additional, reliable information beyond that provided by the two parties.[11]

The results of this paper can be grouped and presented as answers to three basic questions that I tackle in different sections of the paper. The first question (addressed in Section 3) is to what extent the introduction of multiple offers (two or more) affects the divergence between the parties' overall offers (I refer to a party's overall offer as the offer for the entire firm, which is constructed from the party's individual offers for each cost unit). I show that when parties have perfect knowledge about the arbitrator's ideal settlement, parties' offers exhibit, as in the single-offer game, perfect convergence. When parties are uncertain about the arbitrator's preferences, as is usually the case, division of the firm into just two cost units results in multiple equilibria with a divergence between parties' offers that can be arbitrarily large but never smaller than that in the single-offer game, i.e., the case in which parties are required to make just one offer for the entire firm.

Contrary to the single-offer game, in which parties' equilibrium offers are unique (Farber 1980), the multiplicity of equilibria raises this second question: to what extent the arbitrator's ability to learn from the parties' offers is either lost or severely reduced by the introduction of multiple offers (Section 4). As in the single-offer game, in which the arbitrator perfectly recovers parties' cost information from the average of the parties' offers (Gibbons 1988), I find that the introduction of multiple offers does not affect the arbitrator's ability to learn from the parties' offers. This is because in (separating Bayesian) equilibrium the arbitrator does not

10 As in Farber (1980) and the literature that has followed, I do not include a previous stage in which parties bargain over the final price before going to arbitration, so I do not intend to explain what makes parties more likely to reach an agreement rather than end in arbitration. For more, see Farber and Bazerman (1989).

11 More generally, empirical studies of arbitrator behavior indicate that arbitrators do use parties' offers to compute their ideal settlement (e.g., Ashenfelter and Bloom 1984; Farber and Bazerman 1986).

learn from the absolute value of the individual or overall offers submitted by the parties but from the relationship that these offers exhibit in equilibrium, a relationship that remains regardless of the divergence that parties' offers actually exhibit in equilibrium.

If the introduction of multiple offers does not affect learning, despite the fact that parties' offers can exhibit substantial divergence, the remaining question deals with welfare gains or losses from introducing multiple offers (Section 5). Intuitively, one could argue that multiple offers provide the arbitrator with more flexibility to put together a settlement closer to her ideal settlement (i.e., the cost of the efficient firm) by combining offers from both parties. Although one can construct examples where the arbitrator is further away from her ideal choice, I show that in equilibrium, the parties' offers are structured in such a way that it is always possible for the arbitrator to choose a final price (which combines offers from both parties) that is expected to be closer to her ideal settlement than in the single-offer case.

The model developed in this paper provides results that have important implications for the design of final-offer arbitration mechanisms. In particular, they indicate that the introduction of multiple offers is likely to enhance welfare, despite the increase in the divergence between parties' offers. Before proceeding, however, I should emphasize that this paper is by no means an attempt to discuss the merits of the regulatory approach under study over alternative approaches such as price caps, but rather is an attempt to better understand the effect of regulatory design on parties' behavior. With that objective in mind, the rest of the paper is organized as follows. In Section 2, I introduce the model using the single-offer game. In Sections 3–5, I extend the model to two offers and use it to address, respectively, the three questions raised above. Concluding remarks are provided in Section 6.

2. Single-offer Arbitration

Let us start with the single-offer arbitration game. In this case the two parties (i.e., the regulator and the firm) are asked to submit a single offer for the entire firm (i.e., p^r and p^f, respectively) and the arbitrator is constrained to choose one of the parties' offers as a settlement.

2.1. Preferences and Information

The arbitrator is characterized by the parameter z, which describes the arbitrator's preferred settlement. If the actual settlement is p, the arbitrator's utility is $v_a(p, z) = -(p - z)^2$. Since the spirit of the legislation is to charge (efficient) prices to consumers that just cover the long-term costs of a hypothetical efficient firm, we assume that the arbitrator's ideal price settlement is directly related to the cost of this efficient firm, which we denote by c. In particular, I assume that $z(c) = c$. This assumption is also consistent with the idea that the arbitrator wishes to be rehired (Ashenfelter and Bloom 1984).

Unlike the arbitrator, the firm and the regulator are assumed to be risk-neutral.[12] As in Farber (1980) and Gibbons (1988), both parties are assumed to have strictly opposed preferences: the firm seeks to maximize the arbitrator's expected settlement, while the regulator seeks to minimize it. It may seem odd that these preferences are totally disconnected from the cost of the hypothetical efficient firm. While this may be little problematic for a firm that faces an inelastic demand,[13] it is unlikely that the regulator would care only about consumer surplus and put no weight on firm's profits. As shown in Montero (2003), however, the results do not qualitatively change if the regulator puts some weight on firm's profits because the parties' incentives work basically the same as long as their preferences are not perfectly aligned. Accordingly, I maintain the assumption that parties have strictly opposed preferences in order to keep the analysis simple.

Neither the arbitrator nor the parties have perfect information about the true cost of the hypothetical efficient firm (which is not necessarily the same as the actual firm), but they do not necessarily share the same perceptions about this cost. Following Gibbons' (1988) information structure (I also follow Gibbons' notation very closely), let the arbitrator's perception about the true cost c be summarized by the noisy signal

$$c^a = c + \varepsilon^a, \tag{1}$$

where c is normally distributed with mean m and precision h, and ε^a is normally distributed with zero mean and precision h^a. The parameters m and h are common knowledge and can be interpreted as the publicly observable facts relevant for the regulation of the firm. Note that as h^a grows infinitely large (i.e., variance of ε^a goes to zero), the arbitrator can perfectly infer the cost c.

Similarly, let the parties' knowledge about the true cost c be summarized by the noisy signal

$$c^p = c + \varepsilon^p, \tag{2}$$

where ε^p is normally distributed with zero mean and precision h^p. It is important to emphasize that this information structure assumes that the parties—the firm and the regulator—share the same perception about the true cost c. While letting $h^p > h^a$, this information structure captures the idea that both parties are considerably better informed than the arbitrator; it is not so clear that both parties should share the same perception about c. Since c is not the cost of the firm that is currently providing the service (although is related), the firm is likely to be better informed about site specificities while the regulator, making use of information collected from all the other regulated water firms, may be better informed

12 The introduction of risk aversion complicates the algebra without producing a qualitative change in the results. See Montero (2003).

13 A sufficiently low price elasticity ensures that, in equilibrium, the firm will never submit a price offer above its monopoly price.

about some parameters that are common across firms (e.g., labor productivity).[14] It would certainly add more realism to the analysis the introduction of asymmetric information via different random shocks with different levels of precision, but that has not been done for the single-offer case—much less so for the case of multiple offers. I return to this point in the concluding section of the paper.

The information structure can be summarized as follows: the arbitrator observes c^a, the parties both observe c^p, no one observes c, and m, h, h^p, and h^a are common knowledge. In addition, the three random variables c, ε^a, and ε^p are assumed to be independent of each other, which facilitates the computation of the Bayesian updating following the arrival of new information (e.g., signals, parties' offers). For example, the conditional distribution of c given c^j, where $j = a, p$, is normal with mean $M^j(c^j)$ and precision H^j, where

$$M^j(c^j) = \frac{hm + h^j c^j}{h + h^j} \tag{3}$$

and

$$H^j = h + h^j. \tag{4}$$

Similarly, the conditional distribution of c given c^a and c^p is normal with mean $M^{ap}(c^a, c^p)$ and precision H^{ap}, where

$$M^{ap}(c^a, c^p) = \frac{hm + h^a c^a + h^p c^p}{h + h^a + h^p} \tag{5}$$

and

$$H^{ap} = h + h^a + h^p. \tag{6}$$

I will make use of these definitions of beliefs updating in the models that follow.

2.2. Arbitration without Learning

Let us consider first the case in which the arbitrator only pays attention to the noisy signal c^a in constructing her ideal settlement. Acknowledging that the arbitrator ignores their offers, the parties will form the common belief that the arbitrator's ideal settlement z is randomly distributed according to some cumulative distribution (to be determined below) function $F(z)$, with density $f(z)$. Since the arbitrator is constrained to choose one of the parties' offers as the settlement, she will choose the offer that is closer to her ideal settlement z. Assuming for the moment that in equilibrium the regulator's offer, p^r, does not exceed the firm's offer, p^f, the arbitrator will choose the regulator's offer if and only if $z < \overline{p}$, where $\overline{p} = (p^r + p^f)/2$; hence, the probability that p^r is picked by the arbitrator is $F(\overline{p})$.

14 See Teeples and Glyer (1987) for a discussion of differences in production efficiency across water utilities.

The timing of the FOA game is as follows. First, the regulator and the firm simultaneously submit their offers to the arbitrator.[15] Second, the arbitrator chooses the offer that maximizes her utility function $v_a(p, z)$ as the settlement. The parties' Nash equilibrium offers (p^f and p^r) maximize their expected payoffs, so they are found by simultaneously solving

$$\max_{p^f} p^r F(\overline{p}) + p^f[1 - F(\overline{p})] \tag{7}$$

and

$$\min_{p^r} p^r F(\overline{p}) + p^f[1 - F(\overline{p})]. \tag{8}$$

The first-order conditions for this optimization problem are[16]

$$1 - F(\overline{p}) = (p^f - p^r) f(\overline{p})/2 \tag{9}$$

and

$$F(\overline{p}) = (p^f - p^r) f(\overline{p})/2 \tag{10}$$

that, rearranged, yields

$$F(\overline{p}) = 1/2 \tag{11}$$

and

$$p^f - p^r = 1/f(\overline{p}). \tag{12}$$

Equations (11) and (12) summarize Farber's (1980) Nash equilibrium: the parties' offers are centered around the median of the parties' belief about the arbitrator's ideal settlement and the distance between the equilibrium offers decreases as this belief becomes more precise (i.e., higher $f(\cdot)$). Notice that in equilibrium $p^f > p^r$, as previously assumed. In deciding about its offer, each party must consider a trade-off between presenting a more aggressive offer and reducing the probability that the offer will be chosen by the arbitrator. In the limit, when there is no uncertainty about the arbitrator's preferences (h infinitely large), both parties submit the arbitrator's ideal settlement, that is, $p^r = p^f = z$.

The equilibrium values of p^r and p^f depend on $F(z)$. The parties know from (3) that the arbitrator's ideal settlement (in the absence of learning) would be

$$z(c^a) = M^a(c^a) = \frac{hm + h^a c^a}{h + h^a}. \tag{13}$$

15 As in Farber (1980) and subsequent papers I do not explicitly model a first stage, where parties can bargain before going to arbitration. We can think of p^r and p^f as the last offers during the bargaining period.

16 Note that the convexity of the arbitrator's utility function assures the existence of equilibrium.

Given c^p, the parties know that $F(z(c^a))$ is a normal distribution with mean m' and precision h', where

$$m' = \frac{hm + h^a M^p(c^p)}{h + h^a} \tag{14}$$

and

$$h' = \frac{(h + h^p)(h + h^a)}{h^a(h + h^a + h^p)}, \tag{15}$$

which imply that the equilibrium offers reduce to

$$p^f = m' + \sqrt{\frac{\pi}{2h'}} \tag{16}$$

and

$$p^r = m' - \sqrt{\frac{\pi}{2h'}} \tag{17}$$

Note that c^p has an effect on the parties' equilibrium offers not because it improves their knowledge about c but because it affects the parties' belief about the arbitrator's ideal settlement.

2.3. Learning

As explained by Gibbons (1988), it is not sequentially rational for the arbitrator to ignore parties' offers because she can learn from them. In fact, the average of the offers is m', so from (3) and (14), the arbitrator can obtain a point estimate of c^p, that is $c^p(m')$. Sequential rationality then requires that the arbitrator's ideal settlement be not $M^a(c^a)$ but $M^{ap}(c^a, c^p(m'))$, which, from (5), is given by

$$z(c^a, p^f, p^r) = \frac{hm + h^a c^a + h^p c^p(m')}{h + h^a + h^p}. \tag{18}$$

In this way, the parties' offers help the arbitrator to have a more precise estimate of c. Knowing that the arbitrator may learn from their offers, each party now takes into account also the effect that his or her offer could have on the arbitrator's inference about the ideal settlement. Gibbons (1988) demonstrates that there exists a separating perfect Bayesian equilibrium in which the arbitrator perfectly infers c^p from the average of the parties' offers. Despite parties consider the gain from misleading the arbitrator when choosing their offers, in equilibrium parties find it optimal not to do so. To conserve space, I leave the development of the learning equilibrium for the multiple-offers case (Section 4).

3. Multiple Offers without Learning

An important difference between Farber's and Gibbon's models and the regulatory scheme studied in this paper is that parties do not submit a single offer but multiple offers. Consider then the case in which the regulated firm is divided into two units or production centers, 1 and 2 (e.g., water production and water distribution).[17] Note that the possibility of submitting multiple offers affects only parties' strategy space, not the actual operation of the water utility, so both parties and the arbitrator only care about the overall offer $p = p_1 + p_2$ (i.e., the final price to be paid by consumers), not the offer chosen by the arbitrator for each individual unit.

I retain the information structure from the single-offer case in that $c = c_1 + c_2$, $\varepsilon^a = \varepsilon_1^a + \varepsilon_2^a$, and $\varepsilon^p = \varepsilon_1^p + \varepsilon_2^p$ are independent random variables with mean and precision as before. I do not impose, however, any particular correlation between c_1 and c_2 and between ε_1^j and ε_2^j, where $j = a, p$.

In this multiple-offers game, the regulator and the regulated firm submit simultaneously price offers for each of the two units. The regulator's offer is denoted by the pair $\mathbf{p}^r = \{p_1^r, p_2^r\}$ and the firm's offer by the pair $\mathbf{p}^f = \{p_1^r, p_2^r\}$. Thus, the regulator's overall offer is $p^r = p_1^r + p_2^r$ and the firm's overall offer is $p^f = p_1^f + p_2^f$. The arbitrator's task is to choose a price offer for each unit following a FOA procedure. The arbitrator will choose prices p_1 and p_2 that maximize her utility $v_a(p_1, p_2, z) = -(p_1 + p_2 - z)^2$. Then, there will be four possible offer combinations for the arbitrator to choose from: $\{p_1^r, p_2^r\}$, $\{p_1^f, p_2^r\}$, $\{p_1^r, p_2^f\}$, and $\{p_1^f, p_2^f\}$. In this section I study the case of no learning and leave for the next section the case in which the arbitrator uses the parties' offers to obtain a better estimate of c.

3.1. Certainty about the Arbitrator's Preferences

I start by studying the game in which both parties know the arbitrator's ideal settlement (i.e., $\varepsilon_k^a = \varepsilon_k^p = 0$, where $k = 1, 2$) because it helps to illustrate equilibrium properties that carry over to the case in which the parties are uncertain about the arbitrator's ideal settlement. The parties' action space and the arbitrator's ideal settlement z are depicted in Figure 1. More specifically, the parties' offers for units 1 and 2 are in the horizontal and vertical axes, respectively. For example, point A represents a regulator's offer consisting of ${}^A p_1^r$ for the first unit and ${}^A p_2^r$ for the second unit. The line z, on the other hand, contains those combinations of p_1 and p_2 that add up to z. The arbitrator is indifferent between any two combinations that lie on this line.

As in the one-offer case, an obvious equilibrium of the game is for each party $i = r, f$ to submit a pair $\{p_1^i, p_2^i\}$, where $p^i = p_1^i + p_2^i = z$. We know that if party i submits an overall offer of $p^i = z$, party j's best response is not constrained to any offer because the arbitrator would pick p^i regardless of party j's offer. But for $p^i = z$ to be a best response to party j's offer, we must necessarily have $p^j \equiv p_1^j + p_2^j = z$.

Let us explore now whether a pair of offers equally distant from the line z, such as A and B in Figure 1 ($\overline{OA} = \overline{OB}$), could also constitute an equilibrium of

17 The case with three or more offers yields the same results (Montero 2003).

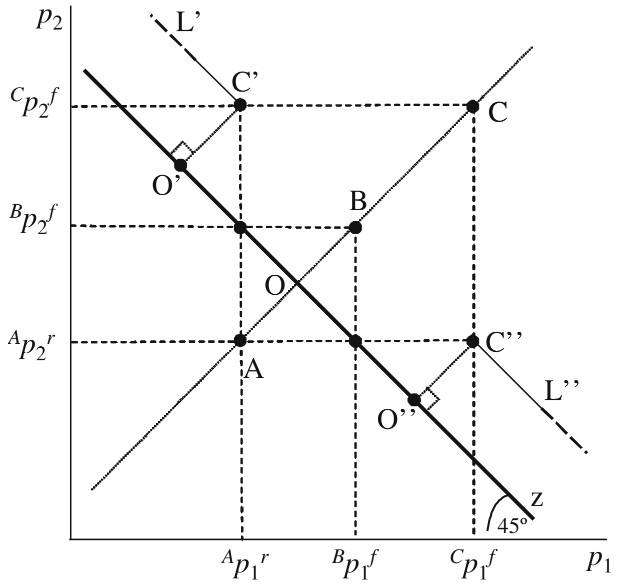

Figure 1. Two-offers game under certainty

the game. If this were the case, we could observe offers divergence in equilibrium but with the same settlement outcome as above. In fact, the arbitrator would be indifferent between the pairs $\{^A p_1^r, ^B p_2^f\}$ and $\{^B p_1^f, ^A p_2^r\}$ because both yield z, her ideal settlement. However, this cannot be an equilibrium. If the regulator plays A, the firm's best response is not to play B but to play either the pair C or a pair along line L' or L'', where $\overline{O'C'} = \overline{O''C''} = \overline{OA} - \epsilon$ and ϵ is a very small positive number. This play leaves the arbitrator indifferent between any pair along L' and L'', including pairs $C' = \{^A p_1^r, ^C p_2^f\}$ and $C'' = \{^C p_1^f, ^A p_2^r\}$. In either case, the overall price settlement is $z + \overline{AO} - \epsilon > z$.[18] Following the same logic, A cannot be the best response to either C or any pair along L' or L''. Therefore, there is no best-response correspondence off the z-line. To summarize

Proposition 1: *If both parties know the arbitrator's preference z, the Nash equilibria of the two-offers game are $p^i = p_1^i + p_2^i = z$ for $i = r, f$.*

Proposition 1 indicates that the introduction of multiple offers (as many as the number of units into which the firm has been divided) does not affect the perfect convergence of the parties' overall offers when there is certainty about the arbitrator's preferences. The logic behind this result is simple. If, say, the firm starts

18 If for any reason the regulator's offer is to the northeast of line z, the firm's best response is to play any pair equally or further distant from z in the northeast direction.

by submitting an (overall) offer that is strictly above the arbitrator's ideal settlement, z, the regulator can secure a settlement that is below z (and a bit closer to z in absolute terms) by slightly undercutting the firm's offer. The firm, in turn, has incentives to undercut the new regulator's offer by making an offer that moves the settlement even closer (in absolute terms) to z from above. This iterative process ends when the parties have no incentives to undercut each other, which occurs only when both parties submit (overall) offers exactly equal to z. Although it has only been formally shown for the two-offers case, it should be clear that Proposition 2 extends to the case of three or more offers simply because a party's overall offer different from z can always be profitably undercut by the other party.[19]

This perfect convergence is an interesting result because one would think that as the number of offers increases, the arbitration process would approach conventional arbitration in the sense that the arbitrator can impose almost any settlement she wishes by choosing the right combination of parties' offers. But since in conventional arbitration we expect to observe any pair of offers in equilibrium (as in any cheap-talk game) or, alternatively, maximum differentiation if the arbitrator is believed to split differences, Proposition 1 indicates that the outcome of a final-offer arbitration scheme with a large number of offers does not converge to the outcome of a conventional arbitration scheme.

3.2. Uncertainty about the Arbitrator's Preferences

Let us now turn to the more realistic case in which the parties are uncertain about the arbitrator's preferences, but let us maintain the assumption, for now, that the arbitrator ignores parties' offers in constructing her ideal settlement. To estimate the probability that the arbitrator chooses a particular offers combination, we need first to discuss some regularities that will prevail in equilibrium. We know that the regulator's overall offer cannot exceed the firm's overall offer in equilibrium (i.e., $p^f = p_1^f + p_2^f \geq p^r = p_1^r + p_2^r$), but there are in principle two types of equilibrium configurations consistent with that. One type of configuration is that in which each of the firm's individual offers is equal or greater than the regulator's (i.e., $p_k^f \geq p_k^r$ for $k = 1, 2$). A second type is that in which one of the firm's individual offers is smaller than the regulator's and the other individual offer is large enough that $p^f \geq p^r$. While in theory both configurations are possible, empirical evidence from the price-setting process in the Chilean water sector is largely consistent with the first type of configuration.[20] Accordingly, in the analysis that

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19 Consider, for example, a three-offers game in which the arbitrator's ideal settlement is $z = \$10$. If the regulator submits the offer $p^r = \{1, 2, 3\}$, which is \$4 off the z-plane, the firm's best response is not to play a symmetrically distant offer such as $p_a^f = \{3, 5, 6\}$ but to play $p_b^f = \{8.99, 9.99, 10.99\}$, where 0.01 is the smallest possible number, say, a penny. By submitting the latter the firm assures itself a settlement of 13.99. Since p^r is, by the same arguments, not the regulator's best response to p_b^f, we cannot have an equilibrium with parties' offers located off the z-plane.

20 With very few exceptions, associated mostly with minor cost units, the firm's offer for each of the cost units was always above the regulator's offer, and particularly so in those pricing processes that ended up in arbitration (personal communication with Ronaldo Bruna from the Agency for Water Services, April, 2004).

follows I focus on the equilibrium solution under the first type of configuration (but I do mention the differences with the equilibrium solution under the second type of configuration).

Since p_1 and p_2 are perfect substitutes, we can adopt the convention that in equilibrium $p_2^i \geq p_1^i$ for $i = r, f$, which implies that $p_1^r + p_2^f \geq p_1^f + p_2^r$. The probabilities can then be found by dividing the z space into four different regions, each supporting the election of one particular offers combination. Depending on the parties' offers there will be values $z_1 < z_2 < z_3$ such that if z falls in the region $(-\infty, z_1)$, the arbitrator will choose $\{p_1^r, p_2^r\}$, if z falls in the region $[z_1, z_2)$ the arbitrator will choose $\{p_1^f, p_2^r\}$, if z falls in the region $[z_2, z_3)$ the arbitrator will choose $\{p_1^r, p_2^f\}$, and if z falls in the region $[z_3, +\infty)$ the arbitrator will choose $\{p_1^f, p_2^f\}$.

As before, the parties' Nash equilibrium offers maximize their expected payoffs so are found by simultaneously solving

$$\max_{p_1^f, p_2^f} (p_1^r + p_2^r)F(z_1) + (p_1^f + p_2^r)[F(z_2) - F(z_1)] + (p_1^r + p_2^f)[F(z_3) - F(z_2)] + (p_1^f + p_2^f)[1 - F(z_3)], \tag{19}$$

$$\min_{p_1^r, p_2^r} (p_1^r + p_2^r)F(z_1) + (p_1^f + p_2^r)[F(z_2) - F(z_1)] + (p_1^r + p_2^f)[F(z_3) - F(z_2)] + (p_1^f + p_2^f)[1 - F(z_3)], \tag{20}$$

where

$$z_1 = (p_1^r + 2p_2^r + p_1^f)/2, \tag{21}$$

$$z_2 = (p_1^r + p_2^r + p_1^f + p_2^f)/2, \tag{22}$$

$$z_3 = (p_1^r + p_1^f + 2p_2^f)/2 \tag{23}$$

and $F(z)$ is a cumulative normal distribution with mean and precision given, respectively, by (14) and (15).

The first-order conditions for this optimization problem are

$$[p_1^f]: 1 - F(z_1) + F(z_2) - F(z_3) + (p_1^r - p_1^f)[f(z_1) - f(z_2) + f(z_3)]/2 + (p_2^r - p_2^f)f(z_2)/2 = 0, \tag{24}$$

$$[p_2^f]: 1 - F(z_2) + (p_1^r - p_1^f)[-f(z_2)/2 + f(z_3)] + (p_2^r - p_2^f)f(z_2)/2 = 0, \tag{25}$$

$$[p_1^r]: F(z_1) - F(z_2) + F(z_3) + (p_1^r - p_1^f)[f(z_1) - f(z_2) + f(z_3)]/2 + (p_2^r - p_2^f)f(z_2)/2 = 0, \tag{26}$$

$$[p_2^r]: F(z_2) + (p_1^r - p_1^f)[f(z_1) - f(z_2)/2] + (p_2^r - p_2^f)f(z_2)/2 = 0. \tag{27}$$

Although the solution involves multiple equilibria as in the certainty case (any of the four equations is a linear combination of the other three; in particular, $[p_1^f]+[p_1^r]=[p_2^f]+[p_2^r]$, where $[p_k^i]$ denotes the first-order condition for p_k^i), they all must satisfy the conditions above that, when rearranged, leads to

Proposition 2: *The parties' overall offers, i.e.,* $p^r=p_1^r+p_2^r$ *and* $p^f=p_1^f+p_2^f$, *are centered around the median of the parties' belief about the arbitrator's ideal settlement, i.e.,* $F(\overline{p})=1/2$, *and the distance between them is not unique and never smaller than the distance in the single-offer case.*

Proof: Let us first prove that $F(z_2=\overline{p})=1/2$. Combine (24) with (26) and (25) with (27) to obtain, respectively,

$$F(z_2)=F(z_1)+F(z_3)-1/2, \tag{28}$$

$$F(z_2)=1/2+(p_1^f-p_1^r)[f(z_1)-f(z_3)]/2. \tag{29}$$

In addition, we know from (21) to (23) that

$$z_3-z_2=z_2-z_1. \tag{30}$$

Given the perfect colinearity between first-order conditions (which implies that we have three equations for four unknowns), we can make an unrestricted selection for one of the four offers, or alternatively, for $\Delta\equiv p_1^f-p_1^r\geq 0$. Furthermore, any particular value of Δ leads to a unique equilibrium given the parties' objective functions (including the arbitrator's) that we are considering here.[21] And since $f(z_1)=f(z_3)$ and $F(z_2)=1/2$ is an equilibrium candidate in that it solves the system (28)–(30) for any $\Delta\geq 0$ and a symmetric density function such as the normal distribution, uniqueness implies that $F(z_2)=1/2$. On the other hand, to find an expression for the distance between parties' offers, add (24) and (26) and rearrange to obtain

$$p^f-p^r=\frac{1}{f(z_2)}-(p_1^f-p_1^r)\left[\frac{f(z_3)+f(z_1)-2f(z_2)}{f(z_2)}\right], \tag{31}$$

where $p^f=p_1^f+p_2^f$ and $p^r=p_1^r+p_2^r$. Replacing $f(z_3)=f(z_1)$ and $z_2=\overline{p}$, equation (31) can be rewritten as

$$p^f-p^r=\frac{1}{f(\overline{p})}+2(p_1^f-p_1^r)\left[\frac{f(\overline{p})-f(z_1)}{f(\overline{p})}\right] \tag{32}$$

21 Uniqueness (for the first type of equilibrium configuration) can be easily proved using the results from the certainty case. If the regulator's offer is, say, the pair A of Figure 1, the firm's best response for a given value of z is unique and equal to the pair C of Figure 1 (if for some value of z the pair A falls to the northeast of the z-line, the firm's best response is A). And since the firm's best response is a non-decreasing function of z (strictly increasing if A is to the southwest of the z-line), the firm's best response to A is unique when z distributes according to $F(z)$.

Since $\Delta \equiv p_1^f - p_1^r \geq 0$ and $f(\overline{p}) \geq f(z_1)$, the distance between offers cannot be smaller than in the single-offer case.[22] ■

Provided that in the absence of learning, $F(\cdot)$ is a normal distribution with mean m' and precision h', the parties' (overall) equilibrium strategies satisfy[23]

$$p^f = m' + \sqrt{\frac{\pi}{2h'}} + \gamma \tag{33}$$

and

$$p^r = m' - \sqrt{\frac{\pi}{2h'}} - \gamma, \tag{34}$$

where $p^f = p_1^f + p_2^f$, $p^r = p_1^r + p_2^r$, $\gamma = \Delta \cdot (1 - \exp[-(p^f - p^r - \Delta)^2 h'/8]) \geq 0$, and $\Delta \equiv p_1^f - p_1^r$ is some arbitrary non-negative value.[24]

Unlike in the single-offer game where parties' offers approach one another as the uncertainty about the arbitrator's preferences disappears, Proposition 2 establishes that the equilibrium in the multiple-offers game does not necessarily follow such a pattern. In fact, when parties are fully certain about the arbitrator's ideal settlement, the equilibrium of the multiple-offers game shows perfect convergence, but when parties are just a bit uncertain, divergence between the parties' offers can be substantial.

To understand this result, consider an example in which parties have a good idea about the arbitrator's ideal settlement (i.e., high h'), yet parties' offers are quite apart in equilibrium.[25] The probability that the arbitrator will pick either the regulator's offer, $\mathbf{p}^r = \{p_1^r, p_2^r\}$, or the firm's offer, $\mathbf{p}^f = \{p_1^f, p_2^f\}$, as the settlement is virtually zero. Despite the little uncertainty about the arbitrator's preferences,

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22 Under the second type of equilibrium configuration (i.e., $p_1^f \leq p_1^r$, $p_2^f \geq p_2^r$, and $p^f \geq p^r$), the distance between parties' (overall) offers becomes

$$p^f - p^r = \frac{1}{f(\overline{p})} + 2(p_1^f - p_1^r)\frac{f(z_1)}{f(\overline{p})},$$

where $\overline{p}$ and z_1 are as before. Since $p_1^f - p_1^r$ is some arbitrary non-positive number (although limited by the fact that $p^f \geq p^r$), one could in principle observe perfect convergence in equilibrium or, at least, more convergence than in the single-offer case. This type of equilibrium configuration, however, is not consistent with what we observe in practice (see footnote 20). For example, we do not observe arbitrators choosing the entire offer of one particular party as the settlement, as this type of configuration would predict, but rather constructing the settlement with offers from both parties.

23 Note that $z_1 = (p^f + p^r - (p^f - p^r) + p_1^f - p_1^r)/2$.

24 The value of Δ cannot be arbitrarily large but is bounded by the set of feasible solutions of the problem (for more, see Section 5). Note also that because γ depends on p^f and p^r, equilibrium values of p^f and p^r are to be found numerically.

25 In Montero (2003), I develop an example in which $F(\cdot)$ is a uniform distribution over the interval $[a, b]$ and show how to construct equilibrium offers where $p^r = p_1^r + p_2^r \ll a$ and $p^f = p_1^f + p_2^f \gg b$.

however, it is not clear for the parties whether the arbitrator will pick the combination $\{p_1^f, p_2^r\}$ or the higher combination $\{p_1^r, p_2^f\}$ as the settlement (recall that $p_1^f + p_2^r < p_1^r + p_2^f$). The fact that the parties are not sure about the final settlement is what allows for an equilibrium with such distant offers (in the absence of uncertainty this cannot occur because parties can perfectly anticipate the arbitrator's choice). In this equilibrium, the regulator, for example, would not have incentives to marginally increase (decrease) p_2^r because the gains (losses) from increasing (decreasing) the probability that the arbitrator picks the lower combination (i.e., $\{p_1^f, p_2^r\}$) are exactly offset by the losses (gains) from introducing a higher (lower) combination. Neither would the regulator have incentives to change p_1^r. Similar arguments apply for the firm.

The large distance between parties' offers that may eventually prevail in equilibrium raises the question as to whether the use of multiple offers prevents the arbitrator from improving her knowledge about the cost c. It may be hard to believe that the arbitrator can learn the same about c regardless of whether parties' offers are close to each other or far apart. I turn to this issue in the following section.

4. Multiple Offers with Learning

We now turn to the central model of the paper. Since we have already seen that it is not sequentially rational for the arbitrator to ignore parties' offers, the objective of this section is to show, as in Gibbons' (1988) single-offer game, that there exists a separating perfect Bayesian equilibrium in this multiple-offers arbitration game.[26] Suppose that the arbitrator believes that $\overline{p}$, the average of the parties' overall offers, perfectly reveals c^p, both on and off the equilibrium path. This means that for any pair of multiple offers, $\mathbf{p}^f = \{p_1^f, p_2^f\}$ and $\mathbf{p}^r = \{p_1^r, p_2^r\}$, the arbitrator computes the point estimate $c^p = c^p(\overline{p} = (p_1^f + p_2^f + p_1^r + p_2^r)/2)$.[27] From (5), the arbitrator's ideal settlement is then

$$z(c^a, \mathbf{p}^f, \mathbf{p}^r) = \frac{hm + h^a c^a + h^p c^p(\overline{p})}{h + h^a + h^p}. \tag{35}$$

26 Gibbons (1988) explains there is also a continuum of pooling equilibrium in which the arbitrator learns nothing from the parties' offers in equilibrium. Gibbons further argues, however, that the empirical evidence (e.g., Ahsenfelter and Bloom 1984; Farber and Bazerman 1986) is largely consistent with the notion that arbitrators learn a great deal from parties' offers. In our case, learning seems to be particularly important because much of the information the arbitrator uses in constructing her ideal settlement is firm-specific and not readily available to the arbitrator other than from the parties' offers.

27 Gibbons (1988) mentions that other separating equilibria may exist in which a different function of $\mathbf{p}^f$ and $\mathbf{p}^r$ reveals c^p to the arbitrator. I see this as a reasonable possibility in the multiple-offers case with random variables that are not normally distributed because in such a case the parties' equilibrium offers are no longer centered around the median of the parties' beliefs about the arbitrator's ideal settlement but they can be centered above or below it, i.e., $F(\overline{p}) \neq 1/2$ (Montero 2003).

As in the no-learning case, depending on the parties' offers there will be cut-off values $z_1 < z_2 < z_3$ such that if $z(c^a, \mathbf{p}^f, \mathbf{p}^r)$ falls in the region $(-\infty, z_1)$, the arbitrator will choose $\{p_1^r, p_2^r\}$ as the settlement, if $z(c^a, \mathbf{p}^f, \mathbf{p}^r)$ falls in the region $[z_1, z_2)$, she will choose $\{p_1^f, p_2^r\}$, if $z(c^a, \mathbf{p}^f, \mathbf{p}^r)$ falls in the region $[z_2, z_3)$ she will choose $\{p_1^r, p_2^f\}$, and if $z(c^a, \mathbf{p}^f, \mathbf{p}^r)$ falls in the region $[z_3, +\infty)$, she will choose $\{p_1^f, p_2^f\}$, where z_1, z_2, and z_3 are given by (21), (22), and (23), respectively.

Using (35), we can then express the event that the arbitrator chooses $\{p_1^r, p_2^r\}$ as $c^a < C_1(z_1, z_2)$, that she chooses $\{p_1^f, p_2^r\}$ as $C_1(z_1, z_2) \le c^a < C_2(z_2)$, that she chooses $\{p_1^r, p_2^f\}$ as $C_2(z_2) \le c^a < C_3(z_2, z_3)$, and that she chooses $\{p_1^f, p_2^f\}$ as $C_3(z_2, z_3) < c^a$, where (recall that $z_2 = \overline{p}$)

$$C_1(z_1, z_2) = \frac{h^a z_1 + h(z_1 - m) + h^p(z_1 - c^p(z_2))}{h^a}, \tag{36}$$

$$C_2(z_2) = \frac{h^a z_2 + h(z_2 - m) + h^p(z_2 - c^p(z_2))}{h^a}, \tag{37}$$

$$C_3(z_2, z_3) = \frac{h^a z_3 + h(z_3 - m) + h^p(z_3 - c^p(z_2))}{h^a}. \tag{38}$$

Given the probability that the parties assign to each of these four events occurring, a derivation analogous to that leading to the first-order conditions (24)–(27) results in the following equilibrium conditions

$$\begin{aligned}[p_1^f]&: 1 - F(C_1(z_1, z_2)|c^p) + F(C_2(z_2)|c^p) - F(C_3(z_2, z_3)|c^p) \\ &+ (p_2^r - p_2^f) f(C_2|c^p) \frac{\partial C_2}{\partial p_1^f} \\ &+ (p_1^r - p_1^f)\left[f(C_1|c^p)\frac{\partial C_1}{\partial p_1^f} - f(C_2|c^p)\frac{\partial C_2}{\partial p_1^f} + f(C_3|c^p)\frac{\partial C_3}{\partial p_1^f}\right] = 0,\end{aligned} \tag{39}$$

$$\begin{aligned}[p_2^f]&: 1 - F(C_2|c^p) + (p_2^r - p_2^f) f(C_2|c^p) \frac{\partial C_2}{\partial p_2^f} \\ &+ (p_1^r - p_1^f)\left[f(C_1|c^p)\frac{\partial C_1}{\partial p_2^f} - f(C_2|c^p)\frac{\partial C_2}{\partial p_2^f} + f(C_3|c^p)\frac{\partial C_3}{\partial p_2^f}\right] = 0,\end{aligned} \tag{40}$$

$$\begin{aligned}[p_1^r]&: F(C_1|c^p) - F(C_2|c^p) + F(C_3|c^p) + (p_2^r - p_2^f) f(C_2|c^p) \frac{\partial C_2}{\partial p_1^r} \\ &+ (p_1^r - p_1^f)\left[f(C_1|c^p)\frac{\partial C_1}{\partial p_1^r} - f(C_2|c^p)\frac{\partial C_2}{\partial p_1^r} + f(C_3|c^p)\frac{\partial C_3}{\partial p_1^r}\right] = 0,\end{aligned} \tag{41}$$

$$\begin{aligned}[p_2^r]&: F(C_2|c^p) + (p_2^r - p_2^f) f(C_2|c^p) \frac{\partial C_2}{\partial p_2^r} \\ &+ (p_1^r - p_1^f)\left[f(C_1|c^p)\frac{\partial C_1}{\partial p_2^r} - f(C_2|c^p)\frac{\partial C_2}{\partial p_2^r} + f(C_3|c^p)\frac{\partial C_3}{\partial p_2^r}\right] = 0.\end{aligned} \tag{42}$$

where $F(\cdot)$ is now the distribution of c^a conditional on c^p, which is normal with mean $M^p(c^p)$ given by (3) and precision

$$H' = \frac{(h+h^p)h^a}{h+h^a+h^p}. \tag{43}$$

As in the no-learning case, the first-order conditions (39)–(42) do not lead to a unique equilibrium because any of the four conditions is a linear combination of the other three. A derivation analogous to that leading to (28) and (29) then yields the equilibrium conditions

$$F(C_2(z_2)|c^p) = F(C_1(z_1,z_2)|c^p) + F(C_3(z_2,z_3)|c^p) - 1/2, \tag{44}$$

$$F(C_2(z_2)|c^p) = \frac{1}{2} + \frac{(p_1^f - p_1^r)(f(C_1(z_1,z_2)|c^p) - f(C_3(z_2,z_3)|c^p))}{2} \cdot \left(\frac{\partial C_3}{\partial p_2^f} - \frac{\partial C_1}{\partial p_2^f}\right). \tag{45}$$

Since $\partial C_3/\partial p_2^f > \partial C_1/\partial p_2^f$, from the arguments leading to Proposition 2 we know that these two conditions imply that $C_2(z_2) = M^p(c^p)$.

To compute $C_2(z_2)$ (and also $C_1(z_1,z_3)$ and $C_3(z_2,z_3)$), we make use of the properties that $c^p(z_2)$, the rule the arbitrator uses to infer the value of c^p from the parties' offers, must satisfy in equilibrium. If the equilibrium value of $\overline{p}$ $(=z_2)$ is to reveal c^p, it must hold that $c^p(z_2) = c^p$ in equilibrium, so substituting $c^p(z_2)$ for c^p in $M^p(c^p)$ and using $C_2(z_2) = M^p(c^p)$ yields

$$c^p(\overline{p} = z_2) = \frac{(h+h^p)\overline{p} - hm}{h^p}. \tag{46}$$

Replacing (46) in (36)–(38) yields $C_1(z_1,z_2) = z_1 - (z_2 - z_1)(h+h^p)/h^a$, $C_2(z_2) = z_2$, and $C_3(z_2,z_3) = z_3 + (z_3 - z_2)(h+h^p)/h^a$. These results imply both that parties' offers are centered around the median of the parties' belief about the arbitrator's ideal settlement $(\overline{p} = M^p(c^p))$ and that, by arguments analogous to those leading to (32), the distance between the parties' offers is given by

$$p^f - p^r = \frac{1}{f(\overline{p})} + 2(p_1^f - p_1^r)\left[\frac{f(\overline{p}) - f(C_1(z_1,\overline{p}))}{f(\overline{p})}\right], \tag{47}$$

where $f(\overline{p}) \geq f(C_1(z_1,\overline{p}))$ and $\Delta \equiv p_1^f - p_1^r \geq 0$ is, as before, the arbitrary choice that defines a particular equilibrium.

Provided that $F(\cdot)$ is a normal distribution with mean $M^p(c^p)$ and precision H' given by (43), the parties' (overall) equilibrium strategies satisfy

$$p^f = M^p(c^p) + \sqrt{\frac{\pi}{2H'}} + \Gamma \tag{48}$$

and

$$p^r = M^p(c^p) - \sqrt{\frac{\pi}{2H'}} - \Gamma, \tag{49}$$

where $p^f = p_1^f + p_2^f$, $p^r = p_1^r + p_2^r$, $\Gamma = \Delta \cdot (1 - \exp[-(p^f - p^r - \Delta)^2 (h + h^p)^2/8H']) \geq 0$, and $\Delta \equiv p_1^f - p_1^r$ is some arbitrary non-negative value.[28] These results can be summarized in the following proposition

Proposition 3: *The parties' offers strategies in* (48) *and* (49) *and the arbitrator's decision strategy based on her ideal settlement* (35) *and inference rule* (46) *constitute a separating perfect Bayesian equilibria of the multiple-offers arbitration game. As in Gibbons* (1988)*, in this equilibrium the arbitrator's ideal settlement can be written as* $z = \alpha\overline{p} + (1-\alpha)c^a$*, where* $\alpha = (h + h^p)/(h + h^a + h^p)$.

In this separating equilibrium, the arbitrator infers c^p from the average of the parties' overall offers (not from the absolute value of the offers submitted for each cost unit) according to (46), then uses this value in (35) to compute her "Bayesian-updated" ideal settlement, and finally chooses the combination of individual offers that is closer to this ideal settlement.[29] Anticipating this, parties find it optimal not to mislead the arbitrator and submit offers satisfying (48) and (49). As the precision of the parties' signal about the true cost c increases relative to that of the arbitrator' signal, the arbitrator assigns more weight to the information coming from the parties' offers than to her own signal in constructing her ideal settlement.

The reason the arbitrator is not misled in equilibrium is that parties must balance the opportunity to influence the arbitrator's belief about the true cost of the hypothetical efficient firm against the two considerations that determined the equilibrium in the no-learning case: the gain from having a more favorable settlement and the reduced probability of having that more favorable settlement. This can be explained by extending the example under Proposition 2 to the possibility of learning. In this new equilibrium with learning, the regulator would not have incentives to decrease p_2^r because the losses from decreasing the probability that the arbitrator picks the lower pair (i.e., $\{p_1^f, p_2^r\}$) are exactly offset by both the gains from introducing a lower combination and the gains from influencing the arbitrator's belief that c^p is lower than it actually is. Similarly, the regulator would not have incentives to increase p_2^r because the gains from increasing the probability that the arbitrator picks the lower pair (i.e., $\{p_1^f, p_2^r\}$) are exactly offset by both the losses from introducing a less aggressive (i.e., higher) combination and the losses from influencing the arbitrator's belief that c^p is higher than it actually is.

28 As in Proposition 2, Δ cannot be arbitrarily large.

29 Note that the arbitrator uses the same inference rule (46) regardless of whether parties' offers are on or off the equilibrium path.

One of the main implications of Proposition 3 is that the multiplicity of equilibria does not affect the arbitrator's ability to learn from the parties' offers despite the fact they may exhibit significant divergence in equilibrium. The reason for this is that the regulator does not learn from the absolute value of individual offers but rather from the way offers are related. The arbitrator uses these relationships (equations (44) and (45)) to correctly infer parties' private information from their offers.

5. Flexibility from Multiple Offers

If the introduction of multiple offers does not affect learning, despite parties' offers exhibiting substantial divergence, one could argue that the use of multiple offers is socially desirable as long as it provides the arbitrator with more flexibility to put together a settlement closer to her ideal settlement (i.e., the true cost of the efficient firm) by combining offers from both parties.

To explore such a possibility, let us compare the single-offer scheme and the two-offers scheme using the following example. Assume that in equilibrium the parties' belief about the arbitrator's ideal settlement are represented by the mean $M^p(c^p) = 100$ and precision $H' = 0.0157$ (recall that the introduction of multiple offers does not affect learning in equilibrium). The (unique) equilibrium offers in the single-offer game are $p^r = 90$ and $p^f = 110$. Among the multiplicity of equilibria in the two-offers game, consider the following two equilibria: (i) $\mathbf{p}^r = \{p_1^r = 34.69, p_2^r = 55.31\}$ and $\mathbf{p}^f = \{p_1^f = 34.69, p_2^f = 75.31\}$; and (ii) $\mathbf{p}^r = \{p_1^r = 15.72, p_2^r = 61.54\}$ and $\mathbf{p}^f = \{p_1^f = 31.02, p_2^f = 91.72\}$. Equilibrium (i) is constructed using $\Delta \equiv p_1^f - p_1^r = 0$ and equilibrium (ii) using $\Delta = 15.3$.

We will say that the two-offers scheme is more flexible than the single-offer scheme if the former allows the arbitrator to construct a settlement that is expected to be closer to her ideal settlement. If we compare equilibrium (i) of the two-offers game with the single-offer equilibrium, the two-offers scheme seems to do as well as the single-offer scheme in that both provide the arbitrator with the same set of available settlements, i.e., $\langle 90; 110\rangle$. If, on the other hand, we compare equilibrium (ii) with the single-offer equilibrium, the two-offers scheme expands the set of settlement prices available to the arbitrator from $\langle 90, 110\rangle$ to $\langle 77.26, 92.56, 107.44, 122.74\rangle$. For this expansion to be, in expected terms, attractive for the arbitrator, the probability that the arbitrator's ideal settlement falls closer to any of the prices available under the single-offer set than to any of the prices available under the second-expanded set must be relatively low; more precisely, below 50%. In this example, such probability is only 23.4%.[30]

30 This value is obtained by solving $F((92.56+90)/2) - F((77.26+90)/2) + F((122.74+110)/2) - F((107.44+110)/2) = 0.23$, where $F(\cdot)$ is a normal distribution with mean $M^p(c^p) = 100$ and precision $H' = 0.0157$ (i.e., $\sigma = 7.98$).

Based on these results, the two-offers scheme appears to be more flexible than the single-offer scheme. More generally, however, it can be established that

Proposition 4: *By combining offers from both parties, the two-offers game provides the arbitrator with more flexibility to construct a settlement that is expected to be closer to her ideal settlement than the single-offer game does.*

Proof: Let p_s^i be party i's offer in the single-offer game ($i = r, f$), and recall that for the two-offers game we have that $p^i = p_1^i + p_2^i$ is party i's overall offer, $p^{fr} = p_1^f + p_2^r$, and $p^{rf} = p_1^r + p_2^f$ (with $p^{fr} < p^{rf}$ by construction). The first part of the proof consists of showing that the equilibrium offers in the two-offers game for a given value of $\Delta \equiv p_1^f - p_1^r \geq 0$ are such that (a) $p^r \leq p_s^r \leq p^{fr}$ and (b) $p^{rf} \leq p_s^f \leq p^f$. The second part of the proof consists of showing that given (a) and (b), the probability that the arbitrator's ideal settlement falls closer to any of the two prices of the set $\left\langle p_s^r, p_s^f \right\rangle$ than to any of the four prices of the set $\langle p^r, p^{fr}, p^{rf}, p^f \rangle$ is less than 50%.

Let us proceed with the first part of the proof. We know from Proposition 3 that $p^f - p^r \geq p_s^f - p_s^r$ and that these offers are centered around $M^p(c^p)$, so it remains to be demonstrated that $p_s^r \leq p^{fr}$ and $p^{rf} \leq p^{fr}$. From Proposition 3, we know that $(p^{fr} + p^{rf})/2 = \overline{p}$, which implies that p^{fr} and p^{rf} are also centered around $M^p(c^p)$. In addition, rearranging (47) yields

$$p^{rf} - p^{fr} = \frac{1}{f(\overline{p})} - 2(p_1^f - p_1^r)\frac{f(C_1(z_1, \overline{p}))}{f(\overline{p})} < \frac{1}{f(\overline{p})},$$

where $\Delta \equiv p_1^f - p_1^r$ is some arbitrary non-negative value. Thus, we have that when $\Delta = 0$, $p^r = p_s^r = p^{fr}$ and $p^{rf} = p_s^f = p^f$, and when $\Delta > 0$, $p^r < p_s^r < p^{fr}$ and $p^{rf} < p_s^f < p^f$.

For the second part, we need to demonstrate that

$$\begin{aligned}\Psi(\Delta) \equiv\ & F((p_s^r + p^{fr})/2) - F((p_s^r + p^r)/2) \\ & + F((p_s^f + p^f)/2) - F((p_s^f + p^{rf})/2) < 0.5,\end{aligned}$$

where $F(\cdot)$ is a normal distribution with mean $M^p(c^p)$ and precision H'. In the absence of a closed-form approach, I will provide a discussion based on numerical results, which applies to any values of $M^p(c^p)$ and H'. For $\Delta = 0$, it is immediate that $\Psi = 0$ since $p^r = p_s^r = p^{fr}$ and $p^r = p_s^r = p^{fr}$. As Δ increases, $\Psi(\Delta)$ also increases as p^r and p^{fr} (and p^{rf} and p^f) depart from p_s^i in opposite directions. There is, however, a value of Δ after which p^{fr} (p^{rf}) changes direction and starts approaching p_s^r (p_s^f). This change in direction implies that there cannot be an equilibrium where $p^{fr} = p^{rf} = M^p(c^p)$.[31] As Δ continues increasing, the value

31 In the example above, the turning point for p^{rf} (and p^{rf}) occurs, when $\Delta = 10.9$. The equilibrium offers associated with that value lead to $p^r = 82.75$, $p^{fr} = 93.65$, $p^{rf} = 106.35$, and $p^f = 117.25$.

of Ψ does the same until it reaches a maximum value of 0.234 for some critical value of $\overline{\Delta}$ (which does depend on $M^p(c^p)$ and H').[32] Above $\overline{\Delta}$ there is no equilibrium solution to the problem. ■

Some welfare implications of Proposition 4 are possible to establish. One must first notice that the settlement choice may not be very different regardless of whether the arbitrator's objective function is to choose the price that is closest to her belief about the long-term cost of the efficient firm (i.e., c) or to choose the price that maximizes social welfare based on this same belief. In fact, the arbitrator's choice will be the same for a linear demand and constant (or linear) marginal cost in the relevant range.[33] Therefore, unless we believe that arbitrators' beliefs about c are systematically biased in some particular way, the more flexible scheme is also likely to deliver a more socially desirable outcome—more so if the probability that the single-offer scheme provides the arbitrator with the price closest to her ideal settlement is at most 23.4%.[34]

6. Concluding Remarks

Motivated by the price-setting process in the water sector in Chile, I have developed a model of FOA in which the item under dispute (i.e., the long-run cost of a hypothetical efficient firm) has been divided into various dimensions or units (e.g., cost of raw water, capital cost) and the arbitrator is limited to choosing one party's offer or the other for each unit. Rather than discussing the merits of this final-offer arbitration process over alternative regulatory approaches such as price-cap or rate-of-return, this paper has focused on understanding the effect that the introduction of multiple offers (instead of just one single offer for the total cost) has on parties' behavior and on the equilibrium outcome.

I have found that moving from a single-offer arbitration scheme to a multiple-offers scheme with two or more offers can substantially increase the distance between the parties' overall offers in equilibrium. Despite this likely increase in divergence, the use of multiple offers helps the arbitrator to establish a final price closer to her ideal settlement (i.e., the long-run cost of a hypothetical efficient firm) without affecting her ability to learn from the parties' offers about the true cost of the efficient firm.

32 The critical value of Δ in the example above is 15.3, which is the value used to obtain equilibrium (ii).

33 This analysis it is implicitly assumes that if prices are set below c the firm does not shut down but continues operation (perhaps covering only variable costs and, hence, postponing investments).

34 In Montero (2003) I consider the case in which $F(\cdot)$ is a uniform distribution over the interval $[a, b]$ and show that the two-offers scheme is unambiguously superior to the single-offer scheme. The equilibrium offers in the single-offer game are $p^r = a$ and $p^f = b$. The equilibrium offers in the two-offers game produce price sets that can be of two types: either $\langle p^r = a, p^{fr} \geq a, p^{rf} \leq b, p^f = b \rangle$ or $\langle p^r < a, p^{fr} = a, p^{rf} = b, p^f > b \rangle$.

These findings have two important implications. The first is that the authority's proposal calling for a reduction in the number of units into which the firm is divided from something around 200 to 50 offers (or down to two offers, for that matter) would make little difference, if any, in its effort to lower the divergence between parties' offers. The second implication is that it is never optimal to implement a single-offer scheme because that always provides the arbitrator with a set of available settlements that is less attractive than the set from a multiple-offers scheme. Whether the arbitration scheme should be based on two or more offers is open to more empirical analysis because the multiplicity of equilibria associated with any multiple-offers game does not allow us to assure that increasing the number of offers, say, from two to three, will lead to an equilibrium with a more attractive set of prices for the arbitrator to choose from.

Part of the above results depend on the information assumption that parties have symmetric information about the cost of the efficient firm. It is likely, instead, that each party will be better informed about some aspects of the efficient firm than the other party. As mentioned by Gibbons (1988), it is possible that such information asymmetry may influence both the means and the substance of the parties' communication with the arbitrator. The effect can be even larger in multiple-offer arbitration if the arbitrator has a good idea that such party is better informed about that aspect of the efficient firm than the other party. This is an interesting, although difficult, direction for further research.

Another question that deserves future work is why parties came to be in arbitration in the first place. The data summarized in Table 1 provide some insights. Ownership status seems to explain, at least in part, why some parties are more likely to reach agreement than others. In fact, for three of the six privately owned companies,[35] prices were determined through arbitration, while for only two of the nine state-owned companies, prices were determined in such a way. Firm size, which may serve as a proxy for firm's complexity and uncertainty about the arbitrator's preferences, also seems relevant (although the largest two firms also happen to be in private hands). Given the small sample size, however, there is not much else that can be said.

If we believe that negotiated settlements are valuable from a welfare standpoint because they allow parties more discretion in negotiating their own settlement (Farber 1980), it is also relevant to understand whether and how a reduction (or increase) in the number of offers affects the likelihood of parties ending up in arbitration. Empirical and experimental work comparing conventional and (single-offer) final-offer arbitration shows that it is not clear whether dispute rates (i.e., number of negotiations that end in arbitration) and distance between parties' offers are greater in conventional arbitration than in final-offer arbitration (Farber and Bazerman 1986, 1989; Ashenfelter et al. 1992).

35 With the exception of Aguas Cordillera, these companies have gone private only recently: 1–2 years before the price reviews.

Finally, there is the question about the overall optimality of the regulatory approach studied in this paper, relative to alternative approaches. If both the regulator and the firm share the same perception about the efficient firm, as assumed in this paper, the merits of the arbitration scheme seem unclear other than limiting the regulator's discretion in setting prices. To the extent that the firm has information that the regulator does not have, as commonly occurs in practice, there may be important advantages associated with the use of arbitration; yet, those merits need to be better established through more research. Perhaps more useful within the existing arbitration approach is to ask for ways in which the construction of the hypothetical efficient firm could be improved. Following the yardstick regulatory scheme practiced in the water sector in the UK, one possibility is to require, at least partially, the use of actual costs from previous review periods and from other water utilities.

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