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Economic modelling of water storage, irrigation and crop choice under water scarcity

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General Introduction

A vast amount of literature has been developed on the climate change effects on water resources. Changes in means and variances and its spatial distribution are expected in the future (IPCC, 2008). In parallel, economic development is increasing the competition between the different uses like human consumption, agriculture and energy generation (OECD, 2011). Historically water management strategies have been based on past hydrological records but in this context, those records are not a good guide to future conditions and so the current practices may not be robust (Draper & Lund, 2004; IPCC, 2008).

Furthermore, there are other challenges than the ones related to the climate since the physical impacts will influence the human behavior (Rehana and Mujumdar 2014; Olmstead, Fisher-Vanden and Rimsaite 2016; Olmstead 2010). So, human adaptation is also observable as a rational response from water users like farmers. To completely understand the impacts of climate change in an agricultural context, these rational decisions and their dynamics must be considered.

In this context of a changing climate and increasing resource pressure, it is imperative to develop efficient mechanisms that help secure the water availability and allow for its allocation and reallocation. To do so, dynamic economic modelling appears as one alternative to help understand the interactions between physical inputs and farmers or water managers. Human behavior is integrated by optimizing benefits on the long run in an uncertain context.

So, the general objective of the present thesis is to study the dynamics of water decisions for agriculture in the presence of climate change. To do so, three adaptation strategies are analyzed and each poses a specific question to address:

1. Changing the irrigation policy: when permanent crops are already in production and water is not enough, what are the drivers for an optimal decision?
2. Water storage policies: when storage is an alternative and future water inputs are uncertain, do dams mitigate the effects of climate change?
3. Changing the crop mix: when a crop mix is affected in a region that faces water scarcity, does water become the limiting input instead of land?

The flowing chapters develop each analysis in detail.

Economic modelling of perennial crops under water shortage

Contents

Abstract

How farmers respond to water availability uncertainty is still a question under development. Special attention is required when permanent crops are grown. Climate change projections consider the increase of extreme events such as droughts and floods. Due to the nature of perennials, they are more exposed to uncertainty than annual crops. One of the adaptation alternatives that farmers have is the irrigation strategy but it has intertemporal effects that must be analyzed. We develop a dynamic theoretical economic model to assess the optimal irrigation decisions of a farmer who has a productive permanent crop. Both water dose and irrigated area are considered variable inputs. We build on previous literature by including three intertemporal effects: crop area survival between periods, irrigation impacts on future production, and probability of water restriction contingent on crop area. Our results are the same as previous research when enough water is available but when water is restricted the tradeoff between water dose and irrigated area becomes relevant. We calculate a threshold for water availability that determines which of the extensive or intensive irrigation strategies is optimal. Permanent crop dynamics also determine which irrigation practice should be applied. Ignoring these considerations may lead to suboptimal decisions with profit and water efficiency losses.

1. Introduction

Perennial crops have largely expanded due to their high value returns. Between 2000-2020 the area dedicated increased 31% reaching 10% of global cropland (FAO, 2020). A key characteristic of perennials is that they are long-term investments as opposed to annuals that have a one-year cycle. Its production starts after a few years of growth, so payoff is highly dependent on the expected availability of inputs. Due to this climate change poses a large challenge for growers who need to keep the risk at the minimum. Future projections on water resources include a risk increase of floods and droughts (Bates, Kundzewicz, & Wu, 2008). So, to enhance adaptation and efficiency, farmers, water administrators and policy makers need to understand the economic dynamics of permanent crops.

We study how permanent crops production can adapt to stochastic water shortages. Focusing on water as an uncertain resource, we model a farmer's irrigation decisions for an already productive orchard. Managing irrigation is part of the strategies that can be implemented when water becomes scarce (Fereres, Goldhamer, & Sadras, 2012). Previous literature has accounted for irrigation decisions as an adaptation practice when less water is available, but the dynamics of perennials have been ignored. Also, crop land is usually taken as a fixed input, inducing less flexibility in the decisions. We found that this restriction induces suboptimal decisions resulting in loss of profits.

2. Literature review

How agriculture will deal with climate change events is still an open question (Chavas, Chambers, & Pope, 2010). One way to analyze the farmers' response to changes in water resources is using crop-water production functions. García-Vila and Fereres (2012) explore the decisions of crop choice and water dose using a farm-scale model that maximizes farm income. They include a crop-water production function for annual crops. Impacts in the income were analyzed for different external effects such as water restrictions, prices, and policies. Similar results have been found for perennial crops: Berbel and Mateos (2014) show that lowering water doses may be a strategy for farmers to adapt to water scarcity in olive orchards. Their findings remark on the importance of the crop-water production function since it determines farmers' income. Exposito and Berbel (2020) developed the microeconomics of the water allocation of the previous model applied in almonds. Crop-water functions are well documented for annual crops but perennials are still under development due to the complexity of long term responses and intertemporal effects (Fereres et al., 2012).

Another related literature for this research is the analysis of uncertain water availability when perennial crops are grown. Feinerman and Tsur (2014) analyzed the economics of an uncertain cycle duration induced by stochastic water supply regimes. They develop a crop's drought vulnerability index based on drought hazard to capture the limit between the positive and negative returns. Their model also accounts for the intertemporal effects of low irrigation, considering two years of crop survival in dry seasons. Their results demonstrate how a perennial crop may have negative expected profits if there exists a probability of drought. Arellano-González and Moore (2020) extended Feinerman and Tsur framework by including the possibility of water storage. Their results show that the adoption of perennials increases when water banks are available.

In previous literature, water and crop land are key inputs in all cases, but most studies consider the second as a fixed input. Restricting the farmer options to change only the water dose reduces the adaptation alternatives. To understand the optimal decisions for permanent crops, both inputs should be considered as flexible. In addition, the irrigation of one year affects the growth and production of subsequent years (Fereres et al., 2012). This intertemporal effect is one of the big differences between annuals and perennials and forces any analysis to be dynamic. Finally, previous work literature has focused on scarce water resources in stationary conditions (Chai et al., 2016; Expósito & Berbel, 2020) but how to incorporate uncertain shocks is still under development.

We develop a dynamic theoretical economic model to assess the optimal irrigation decisions of a farmer that has a productive perennial crop. Our work extends the model of Expósito and Berbel (2020) by including the dynamics and intertemporal effects of irrigation on perennial production. Also, we consider a yield-water function based on the recommendation of previous research (Fereres et al., 2012). Finally, uncertainty on water availability is explored, similar to the development of Feinerman and Tsur (2014).

The organization of the present paper remains as follows. In the next section, the model and theory are developed. A general approach is first presented, followed by specific scenarios to illustrate the problem. The fourth section considers a simulation of the model using a numerical example. We finish with a discussion of the findings and conclusions.

3. Profit dynamics of permanent crops in the context of water shortage

3.1 Overview

We model the decisions of a farmer that maximizes the expected benefits of harvesting a permanent crop and faces water shortage. The farmer decides the amount of crop area to irrigate and the water dose to apply. Water shortage occurs when the water available for irrigation is not enough to apply an optimal water dose to the whole crop area. The permanent crop has reached its productive age. These years are of special interest since it's the period that allows the farmer to recover the investment made in the trees and their growing stage. The production is highly sensitive to irrigation, so a lower water dose will impact the farmer profits.

One period $\boldsymbol{t}$ refers to a year that includes the irrigation and the harvesting season. At the beginning of the analysis, the crop area is $\boldsymbol{L}$. Since the crop is perennial, crop area cannot increase from one period to the next, but the farmer can choose to reduce it. For each period $\boldsymbol{t}$, the water available for irrigation W_t is known and the farmer decides the crop area to irrigate given by $\boldsymbol{s_t}$ $(\leq \boldsymbol{L})$ and the water dose to apply, $\boldsymbol{w_t}$. Thus, total water applied in each period is less or equal to the total amount of water available in that period, $w_t s_t \leq W_t$. We assume that irrigation is homogenous for all area $\boldsymbol{s_t}$ and water storage is not possible. We also assume that the trees die if they are not irrigated in one period. The farmer does not consider any other use of the land.

In each period, the profit obtained is $\boldsymbol{\pi_t} = p \cdot \vartheta(w_{t-A}, w_t, s_t), where$ $\boldsymbol{p}$ is the output net price and $\boldsymbol{\vartheta}(w_{t-A}, w_t, s_t)$ is the production function. The production is the multiplication of the yield function and the irrigated area: $[\delta(w_{t-A})y(w_t)] \cdot s_t$.

Yield $\delta(w_{t-A})y(w_t)$ has two components. As discussed in the previous section, there is an important effect in yields of permanent crops when a poor water dose is applied in one period. It does not only affect the present production but also the output obtained in subsequent seasons. To capture this intertemporal effect, we propose a penalty function $\delta(w_{t-A})$. $\boldsymbol{A}$ are the number of future seasons that are influenced by the water dose. This penalty takes values between $[0,1]$.

The second term $y(w_t)$ accounts for the crop-yield-water function, very common in the agricultural literature (Fereres et al., 2012). We consider a stationary function for all periods since our analysis involves only the productive age of the tree (Tregeagle & Zilberman, 2016). We assume $y(w_t)$ is increasing, continuous and twice-continuous differentiable and strictly concave (Feinerman & Tsur, 2014). Keeping other inputs constant, yield increases with water dose up to a maximum, y_{max}, and then decreases. This upper level is attained when a water dose of w^o is applied.

We assume risk neutrality and a fixed output price so the problem to solve is:

$$Max_{\{w_1, w_2, \ldots, w_T, s_1, s_2, \ldots, s_T\}} \qquad \sum_{t=1}^{T} p \cdot \beta^{t-1} \cdot E[\boldsymbol{\vartheta}(w_{t-A}, w_t, s_t)\,]$$

$$\begin{aligned} s.t. \qquad & s_1 \leq L \\ & s_t \leq s_{t-1}\,, \forall\, t = 2\ \ldots T \\ & w_t \cdot s_t \leq W_t \end{aligned}$$

w_{t-A} are known for $t \leq A$

The expectation refers to the level of the water availability W_t that is uncertain for $t \geq 2$. We are interested in analyzing choices in a water shortage context. For the first period, water shortage occurs when $W_1 < W_O := w^o \cdot L$ and for $t \geq 2$ we have $W_t < w^o \cdot s_{t-1}$. Water shortage implies a relation of the water dose and area to irrigate as follows: $w_t \cdot s_t = W_t$. The farmer has only one choice to make, the crop area to irrigate (the water dose) since the other variable will be determined by the previous relationship. In presence of water shortage, the problem is to find the optimal combination of (w_t, s_t) that satisfies this water restriction and maximizes the profit. The problem is presented graphically in Figure2.

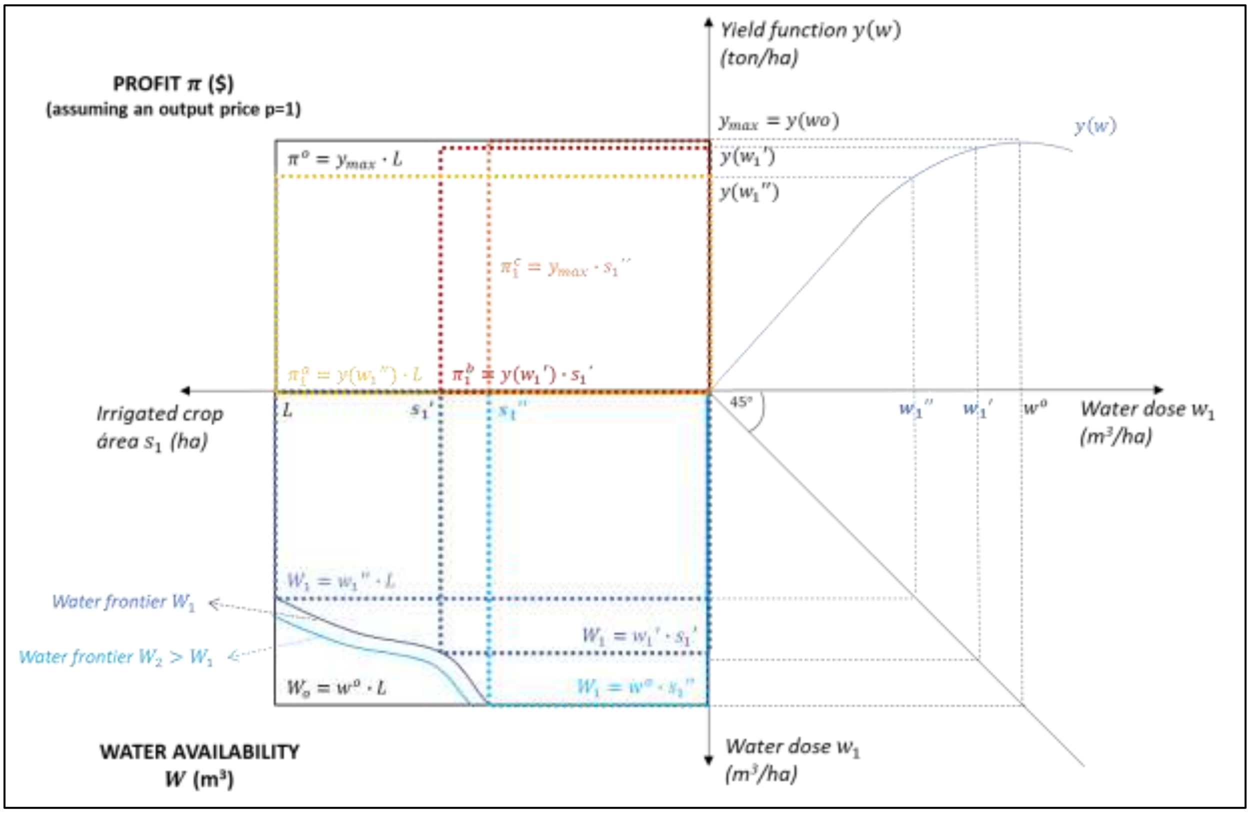

Figure 1: Yield function, profits and water availability for different combinations of (w_t, s_t).

The yield function $y(w)$ is plotted in the first quadrant. The right horizontal axis is the water dose w. Different levels of water dose are shown with its respective yield: w^o, w_1' and $w_1{}''$ with $y(w^o) = y_{max}$, $y(w_1{}')$ and $y(w_1'')$. For values $w \leq w^o$ the yield function is increasing. Values over w^o are out of the analysis since the focus is water shortage.

We assume that the penalty function $\delta(w_{t-A}) = 1$ and an output price $p = 1$, so the profit is given by $\pi = y(w) \cdot s$. It is shown in the second quadrant with the left horizontal axis as the irrigated area s. Different combinations of (w, s) allows for different profit levels, with a maximum of $\pi^o = y_{max} \cdot L$.

The third quadrant shows water availability $W = w \cdot s$. The higher water available is given by $W_O = w^o \cdot L$. Given $W_1 \leq W_O$, different irrigation strategies may be used. An extensive one implies the irrigation of all the crop area that is available and adjusting the water dose. This scenario is given by $W_1 = w_1'' \cdot L$. On the other hand, an intensive policy maintains the water dose that maximizes the yield and adjusts the irrigated area. This case is $W_1 = w^o \cdot s_1{}''$.

If we connect all combinations of (w, s) that satisfy $W = w \cdot s$ for a given W we form a frontier, like an isoquant. A higher water availability, $W_2 > W_1$, will implicate a new level of this frontier. The optimal solution to the problem is the combinations of (w, s) that is part of a water frontier that results on the higher profit.

In the following sections we will develop in detail the conditions for an optimum in two periods. This time horizon is enough to illustrate the relevance of considering both irrigated area and water dose as input variables. Also, the impact of the intertemporal effects can be explained. The extension to more periods is evident.

3.2 Optimal choices in two periods

We assume water shortage will be only present in the first period, so $W_1 < W_O$. Water dose and irrigated area are jointly chosen following the relation $w_1(s_1) = \frac{W_1}{s_1}$. The second period has no restriction, i.e. $W_2 \geq w^o \cdot s_1$, so both variables w_2 and s_2 are chosen. The problem becomes then:

$$
\begin{aligned}
&Max_{\{s_1, s_2, w_2\}} \quad p \cdot y(w_1(s_1)) \cdot s_1 + p \cdot \beta \cdot \delta(w_1(s_1)) \cdot y(w_2) \cdot s_2 \\
&s.t. \quad s_1 \leq L \\
&\quad\quad\; s_2 \leq s_1
\end{aligned}
$$

Optimal choices for the second period are predictable since no water restriction is present. Our interest is in the decisions of the first period, which are determined by the MNB of the irrigated area in t=1, s_1:

$$p \cdot \left[y\big(w_1(s_1)\big) - \frac{W_1}{s_1} y'\big(w_1(s_1)\big) \right] - p \cdot \beta \cdot \delta'\big(w_1(s_1)\big) \cdot \frac{W_1}{s_1} \cdot \frac{s_2}{s_1} \cdot y(w_2) - \lambda_1 + \lambda_2 = 0$$

λ_1 and λ_1 are the shadow values of the irrigated area in $t = 1$ and $t = 2$ respectively.

The first parenthesis accounts for the current marginal effect of s_1 in the first period MNB. It is the trade-off of increasing s_1 that includes a positive impact since more area is productive. But also a second negative impact is present. It captures the effect of s_1 in the water dose w_1. When more irrigated area is chosen, the water dose decreases reducing the MNB.

Changes in s_1 will also impact on the MNB of the second period. Two effects are identified. The first is the effect on future production, through the penalty function $\delta(w_1(s_1))$. We expect this impact to be negative since an increase in s_1 will lower the water dose $w_1$which in turn decreases the potential production in $t = 2$. The other term is λ_2. Irrigating more crop land in $t = 1$ allows for additional land in $t = 2$ generating a future marginal benefit.

To find the optimal solution for $t = 1$ we look at the shadow value of irrigated area, λ_1:

$$\Rightarrow \lambda_1 = p \cdot \left[y\big(w_1(s_1)\big) - \frac{W_1}{s_1} y'\big(w_1(s_1)\big) \right] - p \cdot \beta \cdot \delta'\big(w_1(s_1)\big) \cdot \frac{W_1}{s_1} \cdot \frac{s_2}{s_1} \cdot y(w_2) + \lambda_2 \geq 0$$

λ_2 is positive and already known, so $\lambda_1 \geq 0$ will imply two alternatives:

1) $\lambda_1 > 0$, the MNB of s_1 is positive, so it will be optimal to irrigate all crop land available. Following an extensive strategy, the optimal decision will be ${s_1}^* = L$ and ${w_1}^* = \frac{W_1}{L}$.

2) $\lambda_1 = 0$, the optimal solution is $({w_1}^*, {s_1}^*)$ that results on a MNB of s_1 equal to zero. In this case ${s_1}^* < L$ and ${w_1}^* = \frac{W_1}{{s_1}^*} > \frac{W_1}{L}$, so an intensive strategy is present.

So, extensive versus intensive solutions will depend on the level of the water available. The optimal choice is not only determined by the level of W_1 but also due to the intertemporal effects. This connects with the previous description of the isoquants of Figure 1. There is a threshold for W_1 that will determinate the irrigation strategy. This limit is related to the functional form of $y(w_1)$. Higher W_1 allows values for the water dose w_1 in the neighborhood of w^o. Equivalently, $y'(w_1)$ and the indirect effect of s_1, are near zero. This situation is comparable to the problem without water shortage described in the first section of this chapter. When W_1 is below this threshold, the indirect effect of $s_1$is relevant so water shortage becomes a problem. This case is where our analysis diverges from previous research. By allowing both area and water dose to vary, the profits may be higher than in the case where land is fixed.

4. Simulation

We simulate the previous model to explore the optimal decisions of a farmer that grows a permanent crop and faces a water shortage. We explore different time horizons and water availability scenarios. The analysis is developed in the same order as the previous chapter. Intensive versus extensive irrigations practices are identified. Also, intertemporal effects are present when water is restricted.

At the beginning of the analysis, we assume that the farmer has $L = 15$ ha of a permanent crop. For the production function, we follow agronomic literature and consider a third order polynomial for the yield response to water function (Goldhamer & Fereres, 2017). Given the previous assumptions we have $w^o = 662\, m^3/ha$ so $W_O = w^o \cdot L = 9.924\, m^3$. The intertemporal effect of a low water dose is assumed to extend for only one season so $\boldsymbol{A} = 1$. The penalty function is defined as $\delta(w) := \left(\frac{w}{w^o}\right)^\alpha$ with $\alpha = 0.7$, for $w \in [0, w^o]$. For a water dose over w^o, $\delta(w) = 1$. Figures 3 and 4 illustrates both components of the production function $\vartheta(w_{t-1}, w_t, s_t) = \delta(w_{t-1})y(w_t)$:

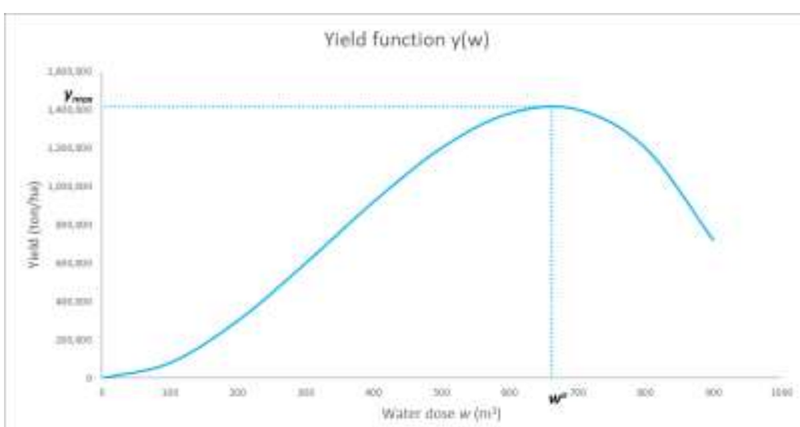

Figure 2: Yield function as a 3rd order polynomial[1].

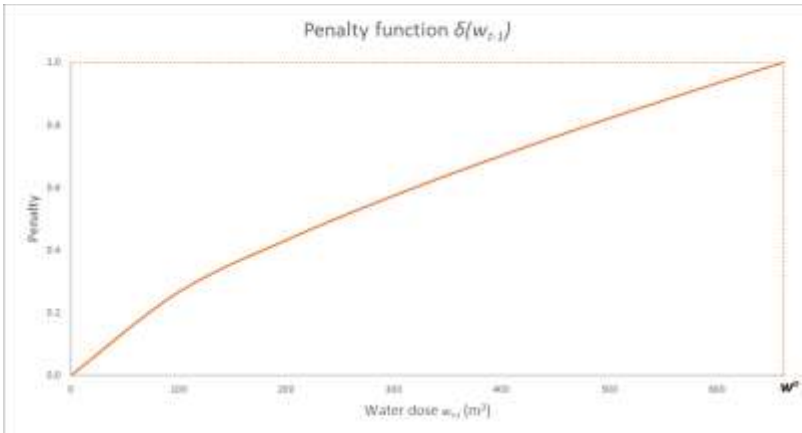

Figure 3: Yield penalty function.

We first analyze what happens in one period. As explained before, the water availability determines if the best solution will be an intensive or extensive irrigation strategy. Without water shortage, the optimum will be $w_1^* = w^o$ and $s_1^* = L$ (extensive irrigation). But when $W_1 < W_O$ and the water restriction becomes active, the trade-off between irrigated area and water dose appears. As explained in the previous chapter, water shortage implies a double effect of $s_1$in the MNB (Figure 4). The first one is a positive one given by $p \cdot\ y\big(w_1(s_1)\big)$. The second one is a negative one since the water dose will decrease with a marginal increment of s_1. This effect is $p \cdot w_1(s_1) \cdot y'\big(w_1(s_1)\big)$. When both effects are equal, we have $\lambda_1 = 0$, so $s_1^* < L$. We see this happening for $w_1^* = 500\ m^3/ha$. This is the lower limit for w_1.

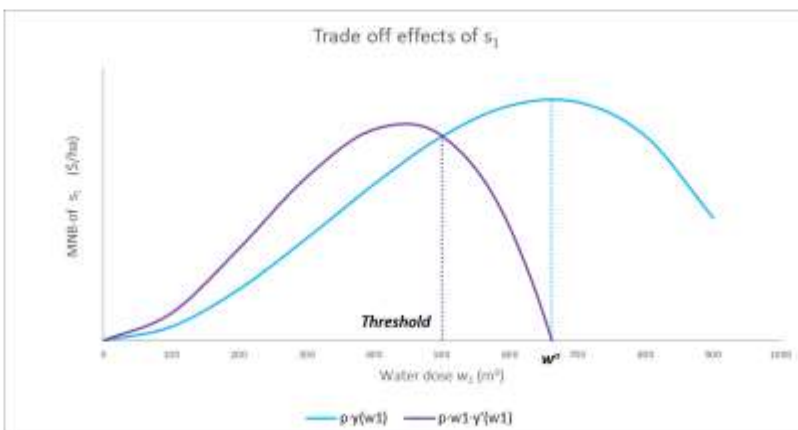

Figure 4: Trade off effects in the MNB of s_1.

We simulate different values of W_1 as a percentage of W_O. The optimal choices (w_1^*, s_1^*) will follow a pattern divided in two sections (Figure 5). Without water shortage, the solution will be up at the right. When W_1 less but close to W_O, all crop area will keep irrigated so $s_1^* = L$. Therefore, the water dose will

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[1] $y(w_t) = a_1 \cdot w_t^3 + a_2 \cdot w_t^2 + a_3 \cdot w_t$ with $a_1 = -0.01,\ a_2 = 10\ and\ a_3 = -100$.

diminish to absorb the effect of having less water availability, following an extensive approach. This strategy will be maintained for a decreasing W_1. When w_1 reaches its lower limit, the intensive approach is preferred and will be kept for more severe shortages. $W_1 = 76\% W_O$ is the water availability threshold that divides the extensive and intensive sections.

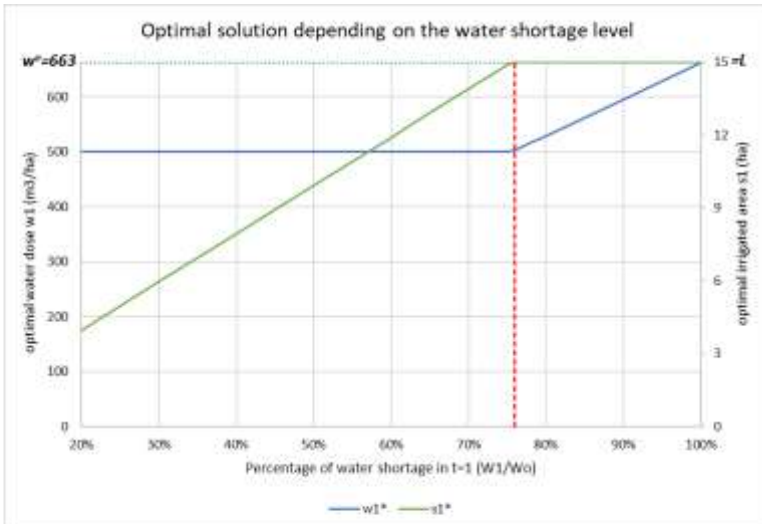

Figure 5: Optimal irrigation strategies depending on the water availability.

We extend the results by including a second period. Results are presented in Table 1. The first column has the variables and results of the profits, penalty and probability functions. The next two columns consider the one period analysis. The last three consider two periods, two that are deterministic and one with uncertainty. Water shortage is the same for the one period analysis that we detailed before and is assumed to be 90% for $t = 2$ ($W_2 = 90\% \cdot w^o \cdot s_1$). The third scenario considers W_2 as a stochastic variable with a function probability defined as $g(s_1) = G\ \frac{\ln{(s_1)}}{\ln{(L)}}$ with $G = 0.7$.

	One period		Two periods, deterministic		Two periods, stochastic
	$W_1 \geq W_o$	W_1= 50%W_o	W_1= 50%W_o and $W_2 \geq W_1$	W_1= 50%W_o and W_2=90%·w^o·s_1	W_1= 50%W_o and W2=90%·w^o·s_1 with probability $g(s_1)$
Optimum using intensive or extensive strategies					
s_1	15	10	11	12	12
w_1	662	500	433	419	424
Profits in t=1	21,225,991	11,909,315	11,683,950	11,582,828	11,623,430
s_2			11	12	12
w_2			662	595	595
$\delta(w_1)$			0.7	0.7	0.7
$g(s_1)$					0.6
Discounted profits in t=2			9,644,589	9,461,536	9,526,937
Total profits	**21,225,991**	**11,909,315**	**21,328,539**	**21,044,364**	**21,150,367**
Mantaining an extensive strategy (s1=s2=L)					
s_1	15	15	15	15	15
w_1	662	331	331	331	331
Profits in t=1	21,225,991	10,488,940	10,488,940	10,488,940	10,488,940
s_2			15	15	15
w_2			662	595	595
$\delta(w_1)$			0.6	0.6	0.6
$g(s_1)$					0.7
Discounted profits in t=2			10,452,904	10,155,824	10,244,948
Total profits	**21,225,991**	**10,488,940**	**20,941,844**	**20,644,764**	**20,733,888**
Losses	0	-1,420,374	-386,695	-399,600	-416,479
% Losses	**0%**	**-12%**	**-2%**	**-2%**	**-2%**

Table 1: Simulation results for one and two periods, considering different scenarios of water shortage.

Two sets of results are shown. The first one displays the optimum choices for (w_t, s_t) considering that an extensive or intensive irrigation strategy may be applied. The second set maintains extensive practices $(s_t = L)$ so it forces to adapt only the water dose w_t. This second approach is traditionally found in previous literature as discussed in the introduction, thus comparison of the profits of both sets are presented.

Analyzing the first set, intensive irrigation is always preferred for the first period $(s_1 < L = 15)$ in the presence of water shortage. This accounts for the tradeoff between the water dose and irrigated area that has been discussed in the previous sections. Also, the cross effect of s_1 in the production of the second period captured by the penalty function. The results of s_2 indicate an extensive strategy since the all crop area is irrigated. One reason may be that water shortage is drastic for the first period but not so deep in the second.

Water shortage also implies less profits in all cases as expected. In the one period analysis, this loss is 44% when water is restricted compared to the case where water is available. The profits are similar in $t = 1$ in all scenarios where water is restricted. Comparing the two periods with water restriction, profits are higher when shortage is uncertain than when W_2 is deterministic. This result is predictable since there is a probability of not having shortage that elevates the profits in $t = 2$. Both profits are lower than the case where the restriction holds is only for W_1.

If we compare the profits between the two sets previously described, they are higher when the irrigated area is not fixed (see the last line). As we described in the previous sections, intensive management may be optimal when dealing with water shortage. Reducing only the water dose but not the irrigated area has many effects on the future profits. These are balanced when both variables vary. A more flexible irrigation strategy impacts not only in the profits but also on the total water used. For the first period, we have the same amount of water in all cases $(w_1 \cdot s_1 = 50\% W_o = 4.962)$. But when the second period is analyzed, the water used in an intensive strategy is 15% to 22% less than with extensive irrigation.

5. Conclusions

The dynamics of perennial crops are still an open question. They differ from annual crops due to the intertemporal effects that are carried from one period to the next. When water availability is uncertain, these effects become even more relevant.

In the present analysis we have presented the problem of a farmer with a productive perennial crop that faces water shortage. The farmer can adjust not only the irrigation plan through the water dose but also the crop area that is irrigated. We construct an economic model that extends previous literature by considering the dynamics of permanent crops in the presence of relevant water shortages.

Previous research often considers water dose as the only variable input, maintaining crop area as a fixed one. In permanent crops this is usually taken for granted, due to the high initial investment that takes to grow a productive orchard. Our model considers both inputs as flexible allowing for very different irrigation strategies: one as an extensive management when all crop area is irrigated and only the water dose is reduced and at the other hand will be the intensive one, where crop area is reduced and water dose is maintained.

Our first contribution is the finding of a threshold that divides which irrigation strategy is optimal. When the water shortage is small, adapting only the water dose is the best decision to make. But when water shortage becomes higher, crop area should be reduced to minimize losses. So, our model shows the same results as previous research for small water shortages, but when water scarcity is deepened, the tradeoff between water dose and irrigated area needs to be balanced.

A second contribution of our work is the consideration of intertemporal effects in three aspects: crop survival, crop productivity and the probability of having water shortage. The first one is due to the relation of crop area between one period and the next. Only irrigated areas survive from one period to another. This approach is frequent in dynamic problems, where the crop area is the state variable that evolves from one period to the next. The second effect is included in the production function. Agronomic studies indicate that a low water dose will have implications in not only present but also in future production (Fereres et al., 2012). Our work innovates in developing the microeconomics of this impact. A third effect arises due to the probability of water shortage. Whether water available is enough or not for irrigation is contingent on the crop area. We consider a probability function for water shortage that depends on the crop area of the previous period. These intertemporal effects are explicitly developed in the model and our theoretical results show that they are key in the determination of the optimal decisions of water dose and irrigated area.

Given previous results, the analysis of farmer adaptation to water shortage must include both irrigation strategies and the dynamics of perennials. Without these considerations, the decisions modeled may be suboptimal. Our results indicate that including a flexible irrigation strategy avoids profits loss and also enhances water efficiency. This second result is of special attention given the global context of water availability.

We have highlighted the dynamic nature of permanent crops in the presence of uncertain water inputs but future developments may enrich this work. First, we identify the need to improve in the production function. Water-crop relations are still under development to capture the carryover effects that have been presented. A more solid agronomic knowledge will allow for a more complete understanding of the intertemporal effects. A second way is the extension of the problem to consider other options of farmer's adaptations considering both permanent and annual crops. Also, replanting is not included so it may be aggregated to the present analysis. Finally, access to water markets could also be explored. The present research establishes a simple and basic theoretical basis for decision takers to deal with perennials in the presence of climate change.

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Chapter 2

Water storage decisions under water uncertainty: a dynamic economic approach

October 2023

Contents

Abstract

New and efficient water management strategies are imperative to develop in the context of climate change since it will impact the mean, variance and spatial distribution of the supply. Also, the demand and competition between the different users is increasing. This scenario challenges the current practices like reservoir operation. Hydraulics has developed sophisticated mechanisms to formulate more flexible and adaptive operational rules but human behavior is not taken into account. The present research addresses the question of whether dams are a useful adaptation strategy for coping with the challenges of climate change. The novelty of the proposal is the development of a robust structure for water storage analysis. We complement previous literature of rational expectations in a poorly developed market like water storage markets. Using a dynamic economic approach, we develop optimal storage rules that are based on expected future water resources instead on historical records. Prices may not be observable, so we do not consider them. Instead, we reinterpret the resulting equilibriums to capture the marginal value of water. With this strategy we can find the long-run equilibrium for the storage policy and the marginal value of water. Our results indicate that the effect of having a bounded storage like a dam, limits not only the physical capacity for water storage but also affects the moment when storage begins. So, even when the dam capacity may never be reached, ignoring it in the analysis will overestimate the marginal value of water and mislead the decisions of water use. We then analyzed climate change effects through a decrease in the mean values of the water inflows and an increase in its variance. Also, non-stationarity was addressed. For the first effect, we found that the role of the dam is relevant when less water is expected on average. The operation rule should be adapted to the new projections and storage will start with lower levels of water availability. If no adaptation is allowed, the dam will be maintained at its full capacity. In the second case, changes in the variance present no changes in the storage rule, so the role of the dam is not affected. Finally, non-stationarity shows different trajectories towards the long run equilibrium of the storage policy. Climate change will affect these trajectories by accelerating or decreasing its convergence to the long-run equilibrium. Further work may be developed to include the interaction of storage with other management practices such as water markets or groundwater use. Also, this framework can contribute to the determination of the optimal capacity of the reservoir and the cost-benefit analysis that takes place when a new infrastructure is projected.

1. Introduction

A vast amount of literature has been developed on the climate change effects on water resources. Changes in means and variances and its spatial distribution are expected in the future (IPCC, 2008). In parallel, economic development is increasing the competition between the different uses like human consumption, agriculture and energy generation (OECD, 2011). In this context of a changing climate and increasing resource pressure, it is imperative to develop efficient mechanisms that help secure the water availability and allow for its allocation and reallocation. Historically water management strategies have been based on past hydrological records but in this context, those records are not a good guide to future conditions and so the current practices may not be robust (Draper & Lund, 2004; IPCC, 2008).

In the past, farmers have developed different strategies to cope with water supply variability and one of the is building water storage infrastructure as dams or reservoirs (Kiparsky, Milman and Vicuña 2012; Hansen, Libecap and Lowe 2011). Their construction is a planned decision that can have a positive impact in the likelihood of a successful harvest during extreme events (Hansen et al., 2011). Historically dams are one of the first interventions when a basin is beginning its development with different objectives like supply stabilization for different water demands and flood control. Negative impacts may also occur when this kind of infrastructure is built. Their success depends not only on the proper design and operation but also on the previous analysis of the possibility of the basin closure (S. Vicuña, Alvarez, Melo, Dale, & Meza, 2014). This phenomenon occurs when the available water resources fall short of meeting all the demands due to basin overdevelopment. In the presence of extreme events, the agents' exposure may increase since their portfolio of adaptation options is reduced.

Most of reservoir operation is determined by a regulation based on historical hydrologic records (Georgakakos et al., 2012). Many authors have described that this fixed rule will not be enough to cope with the challenges that climate change poses (Draper & Lund, 2004; Georgakakos et al., 2012; Sebastian Vicuña, Dracup, Lund, Dale, & Maurer, 2010)

Furthermore, there are other challenges than the ones related to the climate and hydraulics since the physical impacts will influence the human behavior (Rehana and Mujumdar 2014; Olmstead, Fisher-Vanden and Rimsaite 2016; Olmstead 2010). So, human adaptation is also observable as a rational response from water users like farmers. To completely understand the impacts of climate change in water management as dam operations, these rational decisions and their dynamics must be considered.

The present analysis addresses the question of whether dams are a useful adaptation strategy for coping with the challenges of climate change. To do so we develop an economical conceptual framework that relies on the optimization of expected benefits in time instead of only historical physical records. This strategy allows us to characterize an optimal policy for water storage and the marginal value of water. We model a representative agent as a benefit maximizer that chooses the interannual allocation of water by managing a reservoir. His decisions constitute a dynamic stochastic process since there is uncertainty in the future availability of water and the action taken in one period determines the future options. We then include changes in the water inflows to analyze the effects of different climate change projections.

2. Literature review

This research considers water as a storable good similar to a commodity. So, rather than focusing on historical records to deduce a hydrological rule of operation, we consider the maximization of expected benefits as the key to have an optimal storage policy.

Modeling the storage of commodities has a long tradition in the finance and macroeconomics literature, even in conditions of uncertainty. Scheinkman and Schechtman (1983) developed a simple model of production and storage, simulating a market for agricultural products. They assumed rational expectations of the storage speculators and risk-neutral producers with the possibility of supply responses in a partial equilibrium context in both a finite and infinite horizon problem. Following the same approach, Deaton and Laroque (1992) prove the existence of a stationary rational expectation equilibrium when the demand for consumption is linear. Since the stocks are non-negative, the solution for the price behavior is non-linear. Cafiero et al. (2015) extended this result to situations where the production shocks may be unbounded. All previous studies treat uncertainty as a stationary stochastic shock that influences production. Also, these studies consider a fixed cost of storage or even a strictly convex cost function, but there is no inclusion of an upper bound for the amount stored.

Even though Williams and Wright (1991) specifically mention that the traditional storage model does not apply to water, in recent efforts, their approach has been used for determining reservoirs' maximum capacity (Brennan, 2008; Xie et al., 2017) and water reallocation impacts (Truong, 2012). Also, Xie and Zilberman (2017) developed a water storage model that directly extends the storage model in competitive markets. Working with an infinite horizon problem that maximizes the benefits of using water, the authors deal with the non-linearity in the marginal value for water using an indicative function. Similarly, Truong (2012) formulates a model that maximizes the total expected present water revenues for an infinite horizon and considers that the formulation satisfies the Bellman equation assuming that the value function is differentiable. On the other hand, Brennan (2008; 2010) applies the same Euler Equations developed for commodities to assess the economic significance of a missing market for water storage and analyzes its potential empirically. The most significant difference between water and other commodities previously analyzed, is that water has a minimum storage of zero and an upper bound. This difference translates into a new restriction to the traditional mode, implying that the value function will be non-linear and its differentiability will not be assured. Oglend and Kleppe (2017) extended the traditional storage model to consider this upper bound on commodities with a fixed maximum capacity, like water or oil. They study the effects of this limit on price and its volatility. Guerra et al. (2021) make some technical precisions on the model of Oglend and Kleppe (2017) to ensure its convergence and the existence of a solution.

We developed a theoretical framework for analyzing the optimal use of a dam when water inflows are uncertain. Our context considers a representative agent as a benefit maximizer of using and storing water subject to a maximum capacity of a reservoir. In order to capture the long-term equilibrium and also non-stationarity conditions, we consider both an infinite and finite time horizon. The first approach is based on the traditional storage model developed by Wiliams and Wright (1991), but we include an upper bound to consider the fixed capacity of the dam similar as Oglend and Kleppe (2017) and Guerra et al. (2021). With this strategy we can find an optimal policy for the reservoir operation. Also, we will characterize the marginal value of water[1]. Both results are functions that depend on the water availability. They constitute an equilibrium for stationary conditions and we will call them the base case. Afterwards, we analyze the

[1] Example: The value of water is related to the benefits on using for irrigation in an agricultural context.

climate change impacts through changes in mean and variances of the water inflows. We calculate new optimal paths for storage and its related marginal value function. Also, we test the effects of the inflow changes in an already fixed policy. At the end, we complement the analysis to include non-stationarity inflows with a finite horizon model. This second approach considers stochastics dynamic programming to resolve the same maximization problem.

The novelty of the proposal is the development of a robust structure for water storage analysis. Using a dynamic economic approach, we develop optimal storage rules that are based on expected future water resources instead on historical records. We complement the work done by Guerra et al. by applying the rational expectations approach in a poorly developed market like water storage markets. Prices may not be observable, so we do not consider them. Instead, we reinterpret the resulting equilibriums to capture the marginal value of water. We then show how this model serves to the question of analyzing a reservoir and its benefits in a changing context as climate change.

3. Water storage model

The problem we address first is the theoretical way to find the optimal policy of how much water to use and store in each period. We assume that the decisions are taken by a representative agent that acts as benefit maximizer who manages the reservoir designed for interannual stabilization of water supplies.

This representative actor maximizes the sum of benefits of using the water over time for an infinite horizon. At the beginning of every period t, the total water availability is z_t that is either consumed (c_t) - generating a benefit of $\Pi\ (c_t)$- or stored (x_t). The reservoir has maximum capacity given and fixed of $\bar{x}$, thus x_t has both a lower and upper bound $(0 \leq x_t \leq \bar{x})$. Each period, the decision of how much water to storage is reversible, so x_t is the stock of water. The next period starts and a new realization of the stochastic inflow (w_t) is realized. So, z_{t+1} includes x_t multiplied by a loss of *d* (evaporation, leaks) and the new stochastic inflow (w_t). Given β as the intertemporal discount factor, the problem of the benefit maximizer agent can be stated as:

$$\underset{\{x_t\}}{Max}\ E_0 \left[\sum_{t=0}^{T} \beta^t\ \Pi\ (c_t)\right] \qquad \text{(Eq. 1)}$$

subject to $\forall t \geq 0$:

- Water balance : $z_{t+1} \equiv c_{t+1} + x_{t+1} \equiv (1-d)x_t + w_{t+1}$
- Reservoir restriction: $0 \leq x_t \leq \bar{x}$
- Inflows' distribution: $w_t \sim f_w(\mu_t, \sigma_t)$
- Border conditions : z_0 given.

In the next sections we developed the previous problem considering two decision horizons T: infinite with $T = \infty$ and finite where $T < \infty$.

3.1 Infinite horizon

Following previous literature for a problem with infinite horizon, the maximization problem can be rewritten using a rational expectation context (Brennan, 2008, 2010; Deaton & Laroque, 1992; Guerra Vallejos et al., 2021; Oglend & Kleppe, 2017) . Since we are analyzing water resources, the market may not be well developed or competitive, so prices may not be observable. Instead, we use the marginal value

$MB_t \equiv \Pi'(c_t)$ which is related to the benefit from using water or storing it. The decision on the amount of water to store (x_t) is given by the relation between the marginal benefit that is obtained in the present time and the expected marginal benefit of the next period:

$$MB_t \lesseqgtr \beta(1-d)\, E_t[MB_{t+1}] \qquad \text{(Eq. 2)}$$

There are three possible cases:

a) If $MB_t - \beta(1-d)\, E_t[MB_{t+1}] > 0$, i.e., the expected benefits of the next period are less than the benefits of using the water in the present, then the storage will be zero ($x_t = 0$).
b) If $MB_t - \beta(1-d)\, E_t[MB_{t+1}] = 0$, i.e., the expected benefits of the next period are equal to the benefits of using the water in the present, then the storage will be positive ($0 < x_t < \bar{x}$).
c) If $MB_t - \beta(1-d)\, E_t[MB_{t+1}] < 0$, i.e., the expected benefits of the next period are greater than the benefits of using the water in the present, then the storage will be the maximum possible ($x_t = \bar{x}$). This situation is only attainable if the amount of water available is higher than the maximum capacity of the reservoir ($z_t > \bar{x}$). Therefore, in this case $x_t = \min(\bar{x}, z_t)$.

Given $F(c_t)$ as the inverse demand for water consumption and denoting $MB_t \equiv MB(z_t)$ we can resume the three previous situations as a function with three intervals:

$$MB(z_t) = \begin{cases} \mathrm{F}(z_t) & \text{if } x_t = 0 \\ \beta(1-d)\, E_t[MB(z_{t+1})] & \text{if } 0 < x_t < \bar{x} \\ \mathrm{F}(z_t - \bar{x}) & \text{if } x_t = \bar{x} \end{cases} \qquad \text{(Eq. 3)}$$

So, $MB(z_t)$ will have two kinks, MB^* when $z_t = z^*$ and MB^{**} if $z_t = z^{**}$, with $0 \le z^* \le z^{**}$.

In the first interval, when $0 \le z_t < z^*$, all water available will be consumed ($c_t = z_t$), so MB will equal the inverse demand $\mathrm{F}(c_t) = \mathrm{F}(z_t)$. At the opposite side, when $z_t > z^{**}$, storage reaches its maximum, so MB will equal the inverse demand displaced by $\bar{x}$. In the middle, storage is positive.

Another way of looking at the marginal benefit is through the following expression:

$$MB(z_t) = \min\ \{\ \mathrm{F}(z_t - \bar{x}),\ \max\{\ \mathrm{F}(z_t),\ \beta(1-d)\, E_t[MB\ (z_{t+1})]\ \}\ \} \qquad \text{(Eq. 4)}$$

The previous equation is an extension of the storage model with no maximum capacity described in Williams and Wright (2005) [2], similar as the proposals of Oglend and Kleppe (2017) and Guerra et al. (2021)[3].

Inserting $z_{t+1} = w_t + (1-d)x_t$ and $x_t = z_t - \mathrm{F}^{-1}(MB(z_t))$:

$$MB(z_t) = \min\left\{\mathrm{F}(z_t - \bar{x}), \max\left\{\mathrm{F}(z_t), \beta(1-d)E_t MB\left(w_t + (1-d)\left(z_t - \mathrm{F}^{-1}(MB(z_t))\right)\right)\right\}\right\} \qquad \text{(Eq. 5)}$$

If we assume that the stochastic inflows w_t are i.i.d. then the marginal benefit function becomes:

[2] In their work, the marginal benefit of a commodity is equivalent to its price and the function that describes it is given by: $p_t = \max\ \{\ \mathrm{F}(z_t),\ \beta(1-d)\, E_t(p_{t+1})\ \}$.

[3] Authors assume a competitive market for commodities so instead of marginal benefit, they use prices.

$$MB(z) = \min\left\{F(z-\bar{x}), \max\left\{F(z), \beta(1-d)\, Ef\left(w+(1-d)\left(z-F^{-1}\big(MB(z)\big)\right)\right)\right\}\right\} \quad \text{(Eq. 6)}$$

Oglend and Kleppe (2017) define a similar function, with a difference in the last term. This implies a very different convergence in the solution which is discussed by Guerra et al. (2021)[4]. These authors propose a set of conditions in order to have a robust solution for the previous problem. In their work they also developed a bound model that incorporates free disposal. This assumption is not considered in our analysis. Hence, we prove the existence of a unique solution for Eq 6. following a similar logic as Deaton and Laroque (1992) shows for the traditional storage model and Guerra et al. (2021) uses in their bounded model. The details are in the Appendix. The solution is a numerical function $MB(z_t)$ that characterizes the long-term equilibrium with the storage policy $x(z_t)$. Since neither of them is an analytical function, we simulate them to analyze its properties. The existence of a solution depends on the stationarity of the inflows, so to complement the present analysis, a finite horizon model is described next that can capture non-stationarity situations.

3.2 Finite horizon

Looking at the case of Eq.1 with a finite horizon T we will be able to observe the trajectories towards the long-term equilibrium solution presented in the previous section and include a non-stationarity analysis.

Following optimal control terminology, c_t and x_t are the action or decision variables and z_t is the state variable. Given the relation between c_t and x, once one of them is specified, the other is determined. Eq.1 can be reformulated as one with only one decision variable where the objective is to find the optimal value function $V(z_0)$:

$$V(z_0) = \max_{\{x_t\}} E_0 \left[\textstyle\sum_{t=0}^{T} \beta^t\, \Pi\,(z_t - x_t)\right] \qquad \text{(Eq.7)}$$

subject to the same conditions as Eq.1. z_0 and T are given and known.

The $MB(z_t)$ and the storage policy $x(z_t)$ are resolved for every period using a backward strategy. The Kuhn-Tucker conditions of Eq.1 are resumed in the following inequalities:

$$\begin{array}{lcll} MB(z_t - x_t) & \geq & (1-d)\beta\, E_t\, [\; V_t'\,(\bar{x},\, (1-d)x_t + w_{t+1})\;] & \text{when } x_t = 0 \\ & = & & \text{when } 0 < x_t < \bar{x} \\ & \leq & & \text{when } x_t = \bar{x} \end{array}$$

At the left side we have the MB of consuming water and at the right is the MB of storing water. The three intervals are consistent with the development shown in the infinite horizon analysis.

We will model different periods and compare the results of this trajectories with the equilibrium given by the infinite approach. Also, the variation of the inflow distribution will be addressed to characterize the impacts of climate change in both MB of water and the storage policy $x(z_t)$. The Appendix shows an example for three periods.

[4] Authors assume a competitive market for commodities so instead of marginal benefit, they use prices.

4. Results

4.1 Base case simulations

Assuming a linear inverse demand for water consumption, we simulate the function $MB(z_t)$ and the storage policy $x(z_t)$ given a water inflow (shocks) distribution. We simulate both infinite and finite horizons.

4.1.1 Infinite horizon

We present the results using the infinite approach. The next figure presents the long run equilibrium for both $MB(z_t)$ and $x(z_t)$[5]:

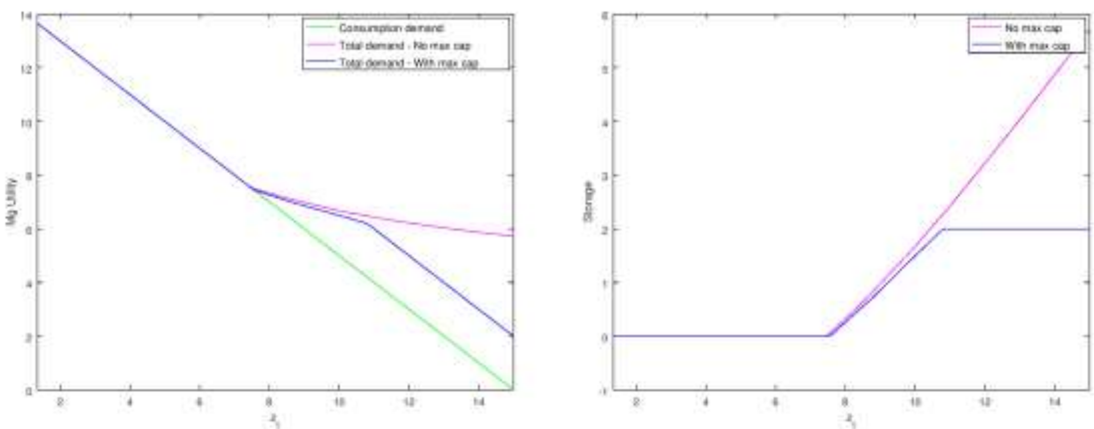


Figure 1: Marginal benefit of water $MB_t(z_t)$ (left) and storage policy $x_t(z_t)$ (right).

In the left graph, the $MB(z_t)$ when storage is upper bounded (Oglend, Guerra) is shown in blue. The three intervals explained in Eq.3 can be clearly identified. The first interval is when all water is consumed. In that section, $MB(z_t)$ equals the inverse demand for water consumption $F(z_t)$, the green line. This lasts up to the first kink ($MB^* = 7.4, z^* = 7.55$) that marks the start of storage. In this second interval, part of the water is consumed and part is stored. The third interval starts in the second kink ($MB^{**} = 6.20, z^{**} = 10.79$). When the water available is higher than this point, the maximum capacity of the reservoir has been reached. So, even if storing may bring more benefits, it is physically restricted. From that point, the amount of water stored is kept fixed at $\overline{x}$ and the rest of water is consumed. Hence, the $MB(z_t)$ is parallel to $F(z_t)$.

The same results are observed when the storage policy $x(z_t)$ is analyzed (right graph). In blue we see the intervals and the same kinks that are related with the $MB(z_t)$: no storage, positive storage and the maximum capacity. The differences between the traditional storage model and the one with an upper limit can be clearly identified in the first kink. The amount of water stored is higher for the first case, so using this policy for a bounded situation like a dam will not be optimal.

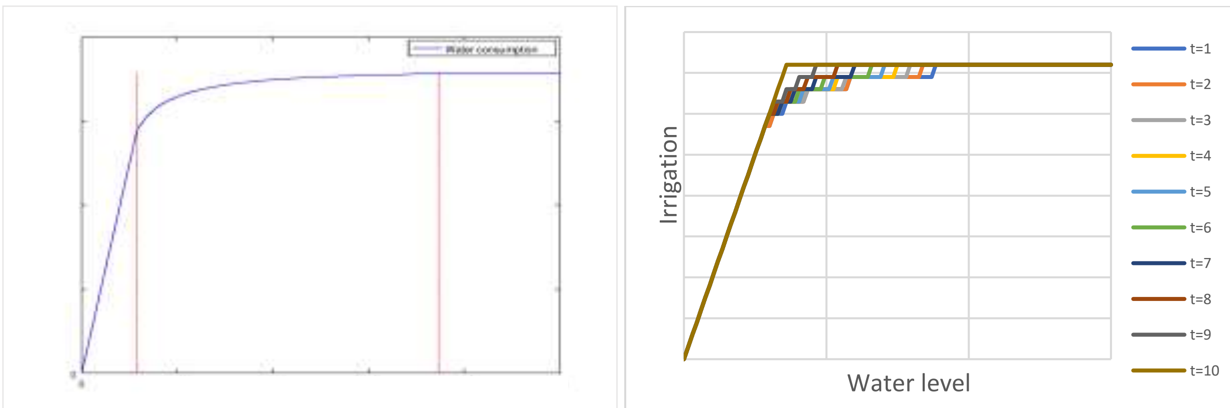
Just for comparison we include in magenta the case of the classical storage model developed by Williams. Two relevant differences between this traditional model and ours are worth commenting on. The first one refers to the first kink. The values in the classical model are $MB^o = 7.53$ and $z^o = 7.46$. Since $z^o < z^*$, storage starts when less water is available. Therefore $MB(z_t)$ is greater for the unbounded case. So, the

[5] Simulation parameters: $F(z) = 15 - z, \overline{x} = 2, \beta = 0.95,\ d = 0$ and $w_t \sim LogNormal(\mu = 2,\ \sigma^2 = 0.05)$.

$MB(z_t)$ could be overestimated when the capacity is fixed but is not considered in the analysis. Even if the maximum capacity is large enough and may never be reached, it affects the start of storage so it should never be ignored. The second difference is the existence of a second kink. This point is related to the maximum capacity $\overline{x}$, so it doesn't appear in the traditional model.

4.1.2 Finite horizon

We consider $T = 10$ to observe the trajectories of the decision variables. The results for the irrigation policy $c(z_t)$ are presented in the next graphs:



(a) (b)

Figure 2: Irrigation policy for: (a) infinite horizon, (b) 10-period analysis without trends in the inflow.

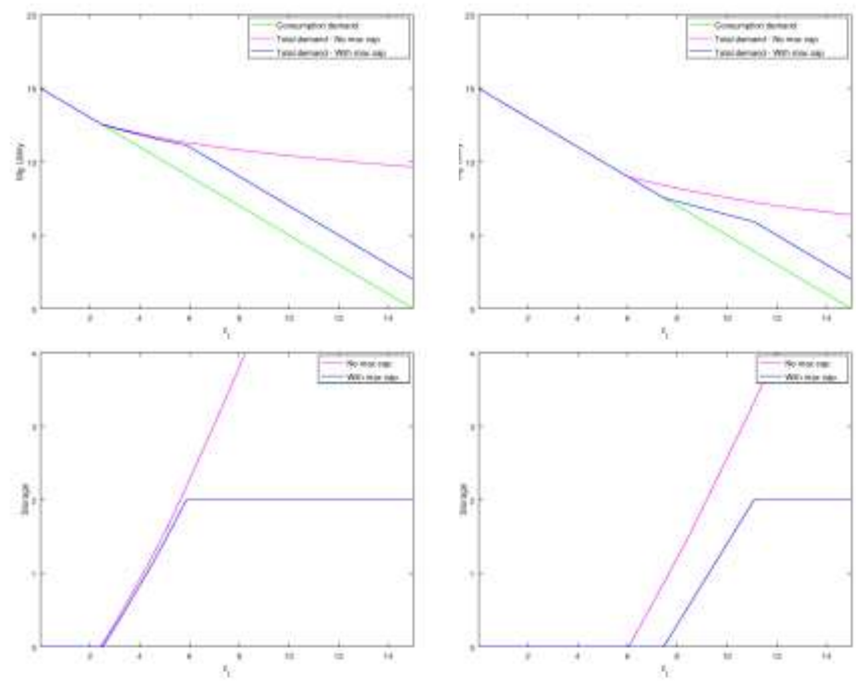
The long-term policy will be the equilibrium policy that is found with the infinite horizon (Figure 2a). In the finite solution, the policy presents multiple kinks that depend on how many periods are analyzed (Figure 2b). In the final period, only one kink appears since no water will be stored. But, when more periods are included, the kinks are multiplied and storage appears. These results imply that the decision of how much water to use or to store is not the same in all periods and follows a trajectory. More details of the kinks' origins are presented in the Appendix where the model using a three-period example is developed.

4.2 Climate change effects

4.2.1 Changes in the long run equilibrium

As we pointed out previously, water resources are expected to vary with climate change. For the purpose of this analysis, we focus on precipitations since it's highly correlated with dam inflows. Future projections consider changes in annual averages and in the occurrence of extreme events. So, we change the mean and variance of the water inflow distribution. To better understand the effect of each change we run two scenarios: one where only the expected value of the inflow (shock) was decreased and a second where only the variance of the shock was increased. In both scenarios, the demand, the maximum capacity and the other parameters are kept the same.

We start the analysis with the infinite approach. In the next figure the results for the new $x(z_{\mathrm{t}})$ and $MB(z_{\mathrm{t}})$ are presented:



(a) (b)

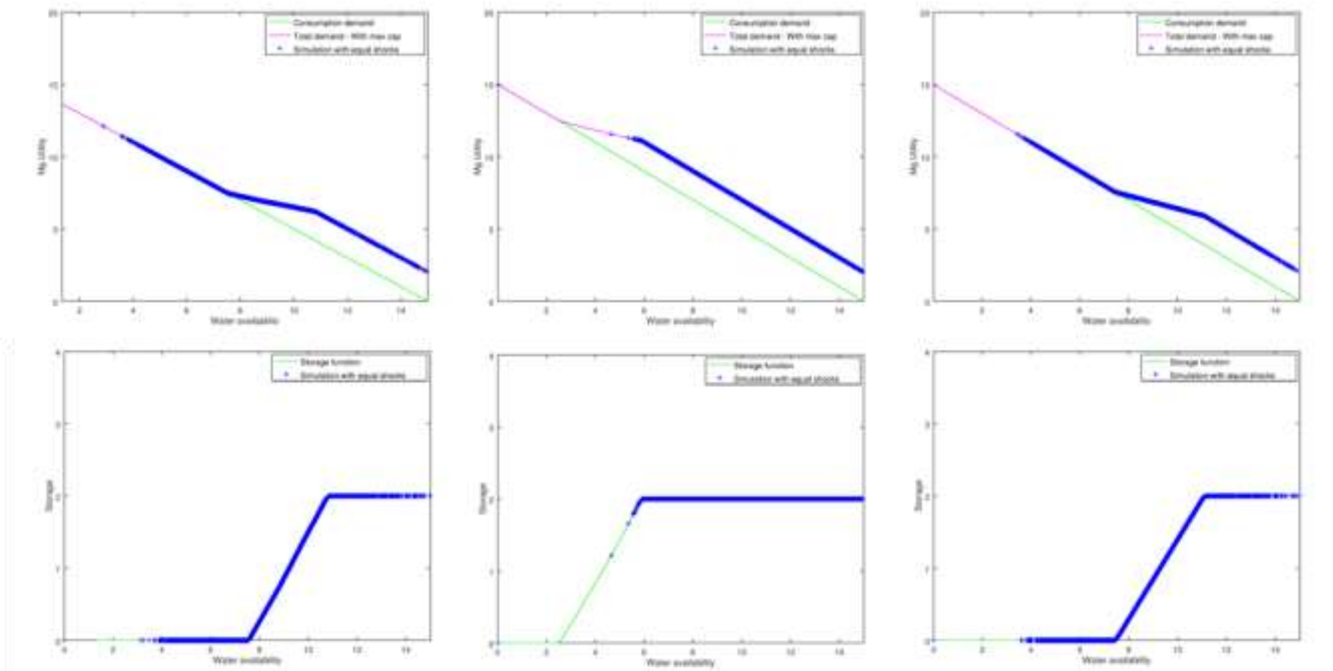
Figure 3: Simulation results for two scenarios of climate change: mean decrease of the inflow (a) and variance increase of the inflows (b). The functions are the $MB_{\mathrm{t}}(z_{\mathrm{t}})$ water (up) and storage policy $x_{\mathrm{t}}(z_{\mathrm{t}})$ (down).

Figure 3a shows that a decrease in the mean inflow will shift the first kink to the left ($z^{*} = 2.5$). This effect causes an up-shift of $MB(z_{\mathrm{t}})$. It is an intuitive result since less water is expected, so the *MB* water is higher. On the other hand, when the variance of the inflow is increased (Figure 3b), we do not observe any relevant variations from the base scenario. The first kink is very close, $z^{*} = 7.5$ vs. $z^{*} = 7.55$. But when the traditional model (in magenta) is compared with the bounded model, there are important differences. The gap that was discussed in the previous chapter becomes higher. Storage will start with significantly less water available, which will result in the loss of present benefits. So, using the traditional model for modeling a water dam will deliver misleading results.

When both scenarios of changes are compared, the results are very similar to the previous analysis since the modification of the variance does not differ from the base case equilibrium.

4.2.2 Inflow shocks

Previous results have established the optimal policies when different inflows are expected. Next we will show the effects of a change in the inflows once the optimal storage policy is defined. We consider separately an increase in the mean and the decreased in the variance of the inflows. The following figure shows the realization of the different inflows and the consequent *MB* and storage decision:



(a) (b) (c)

Figure 4: Impact of inflow realizations in the *MB* of water and storage policy: (a) base scenario, (b) decrease inflow's mean and (c) increase inflow's variance.

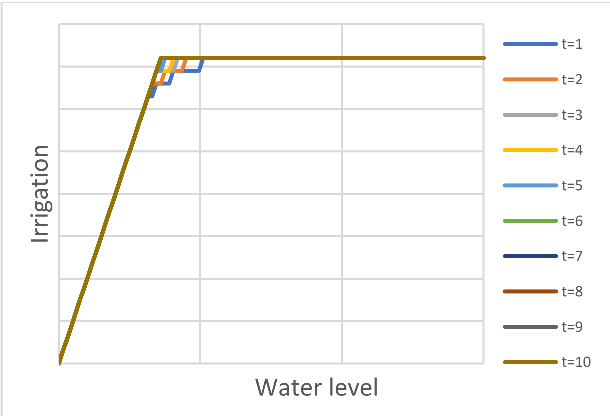
The column on the left shows the base case since the inflow realizations have the same distribution as the one used to compute the base case of $MB(z_t)$ and $x(z_t)$. As expected, we observe the correlation of the inflows' realizations and the functions, in the sense that the realizations are well distributed in the three intervals.

When the inflows mean decreases, the inflow realizations are concentrated in the third interval (Figure 4b). It is clearly shown that in most cases the decision will be to store water and maintain the dam at its full capacity. The effects include a higher water availability and the consequent low MB of water. This is a counterintuitive effect. Since the mean decreases, less water resources will be expected in the future. This scarcity is usually related to high marginal values, but when storage is present, the effect is the opposite as shown in the example.

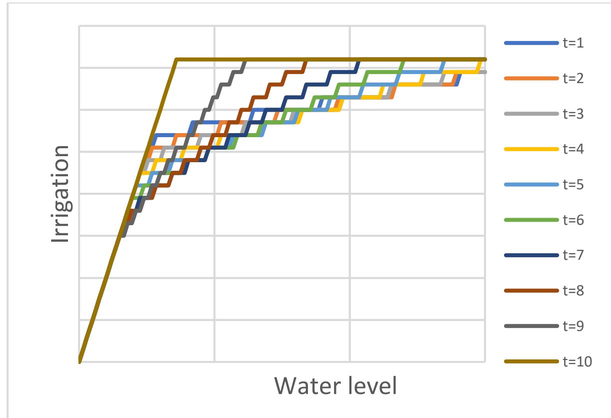
When the change occurs in the variance the inflows are very similarly distributed as in the base case. So, no impacts are identified.

4.2.3 Non stationarity

As pointed out in the previous chapter, using an infinite horizon approach limits the analysis to stationary inflows. So, we complement the preceding results using the finite horizon model. We consider $T = 10$ to observe the trajectories of the decision variables. The results for the irrigation policy $c(z_t)$ are presented in the next graphs:



(a)



(b)

Figure 5: Irrigation policy for 10-period analysis considering non-stationarity in the inflows: (a) positive trend in the mean and (b) negative trend in the mean.

Recalling the irrigation policy presented in the previous section (see Figure 2), we found that when the inflow has a positive trend, the trajectories of all periods converge (Figure 5a). The difference between them is reduced and in all periods the irrigation policy is the higher achievable. This is an expected result since the water availability is expected to increase in the future, so more water will be used in all periods. On the opposite side, when a negative trend is incorporated, the irrigation is different in each period (Figure 5b). Hence, the consequent water storage and its levels also differ between periods.

Given previous results we show that there exist different irrigation levels in each period, so there are different trajectories towards the long run equilibrium. Hence, the dam will be used accordingly to the use of water in each period.

We repeat the previous analysis but instead of varying the trend of the mean, we vary the trend in the variance. This change presents no difference between the base case. It is consistent with the previous analysis with infinite horizon where a change in the variance does not affect the base scenario.

5. Conclusions

The present research analyzes the use of a dam to study whether this infrastructure is useful to mitigate climate change effects. Extending the dynamic economic theory of commodities, we developed a theoretical framework that is based on the marginal value of water when storage is available. The water availability depends on uncertain inflows that are affected are affected by climate change.

To do so, we develop a model based on traditional storage model of commodities (Williams & Wright, 1991), but including an upper bound for storage (Guerra Vallejos et al., 2021; Oglend & Kleppe, 2017). In our case, prices may not be observed since water markets may not be developed for storage. But the amounts of water used and stored are known, so our results still apply in the analysis of the marginal value of water and the storage policy. Our results indicate that the effect of having a bounded storage limit not only the physical capacity for water storage but also affects the moment when storage begins. So, even when the dam capacity may never be reached, ignoring it in the analysis will overestimate the marginal value of water and mislead the decisions of water use.

Our model establishes a link between the marginal value of water and the inflow shocks, so climate change will have an effect on the marginal value of water. We analyze the impact of climate change on both functions through a variety of strategies. These include changes in the stationary distribution of the inflows and also trends in the expected means to simulate non-stationarity situations. We separate them to identify each effect more precisely.

We found that the role of the dam is relevant when less water is expected to be available in the future. When the expectancy of the inflows decreases, the operation rule may adapt and storage will begin with lower levels of water availability. If no adaptation is allowed, the dam will be maintained at its full capacity. Changes in the variance present no changes in the storage rule, so the role of the dam is not affected.

Finally, we find different trajectories towards the long run equilibrium of the water use policy. So, the decision of how much water to use or to store is not the same in all periods. These trajectories vary when non-stationary inflows are included. It converge is accelerated (decelerated) when a positive (negative) trend is incorporated.

The present work could be extended to address the determination of the optimal capacity of the reservoir. It is realistic to expect a fixed maximum capacity in the short term, but in the long term, it may change. The capacity must be treated as a discrete decision variable, introducing a new non-linearity in the model. Also, not only the capacity has to be optimized but also the time this decision is made. Also, the model could include other strategies that are usually in place when storage decisions are taken. Groundwater management is a complementary alternative for water users when dealing with uncertainty. Aquifers may be considered as natural reservoirs. A generalization of the model developed could be used to model a basin including both superficial and underground resources. Pumping costs may be considered so the maximum capacity could be modeled as an increasing cost function instead of a fixed restriction.

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Appendix

I. Demonstration of the long run equilibrium existence

The demonstration is based on Deaton and Laroque (1992) Theorem 1 and is structured in 3 lemmas that will be used for the demonstration of the Theorem itself.

For simplicity we will name $MB(z) \equiv f(z)$. Suppose that the equilibrium of Eq.6 is given by a function $g(z)$, i. e.:

$$f(z) = \min\left\{\mathrm{F}(z-\bar{x}), \max\left\{\mathrm{F}(z), \beta(1-d)Eg\left(w+(1-d)\left(z-\mathrm{F}^{-1}\big(f(z)\big)\right)\right)\right\}\right\} \quad \text{(Eq. A1)}$$

Then, the SREE is a function f such that $f = g$.

The water availability z is defined in the set $X = \{z \in \mathbb{R}, z \geq \underline{w}\}$, so both f and g are defined in X.

We define the set $Y = \{(p,z) \,/\, z \in X,\ F(z) \leq p < p_1\}$ and let $G: Y \to \mathbb{R}$ such that:

$$G(q,z) = \beta(1-d)\, E\, g\left(w + (1-d)\big(z - \mathrm{F}^{-1}(q)\big)\right) \qquad \text{(Eq. A2)}$$

Given (q,z) as the marginal benefit and water availability today, G is the expected discounted marginal benefit for tomorrow.

Eq. A.2 can be rewritten as:

$$f(z) = \min\{\mathrm{F}(z-\bar{x}), \max\{\mathrm{F}(z), G(f(z),z)\}\} \qquad \text{(Eq. A3)}$$

Take T as the operator that assigns to a function g the function f that is the solution to the previous equation ($Tg = f$).

<u>Lemma 1.</u>

Suppose $g: X \to (p_0, p_1)$ is continuous and non-increasing. Then $G(q,z)$ is also continuous and non-increasing in both its arguments q and z.

<u>Proof Lemma1</u>.

Given the definition of $G(q,z)$ and the fact that the expectation of a continuous function is also continuous, $G(q,z)$ is continuous. A similar argument can be established for $G(q,z)$ to be non-increasing in both q and z.

▪

Two relevant facts are important to notice:

i) $G(F(z),z) = \beta(1-d)\, E\, g\left(w+(1-d)\big(z-\mathrm{F}^{-1}(F(z))\big)\right) = \beta(1-d)\, E\, g(w) = MB^*$

This is the maximum marginal benefit that is expected for the next period, since there is no storage and $G$is non-increasing in z. Also, MB^* does not depend on z.

ii) $G(p,z) \leq \beta(1-d)\, p_1 < p_1$

$G(p,z)$ is bounded.

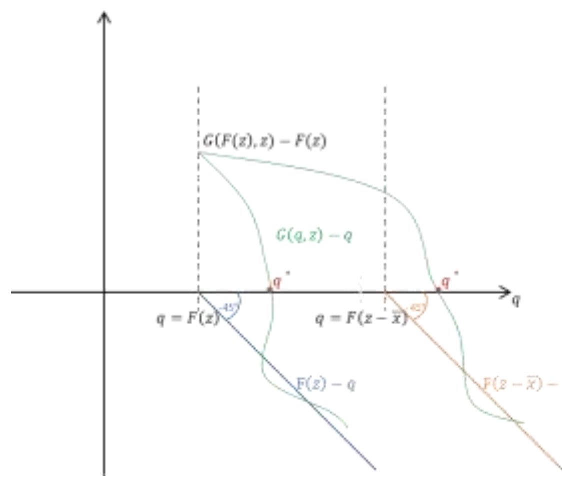
Returning to Eq. A3, given z we look for $f(z) \equiv q$ that is the root of:

$$\min\{\mathrm{F}(z-\bar{x})-q, \max\{\mathrm{F}(z)-q, G(q,z)-q\}\} = 0 \qquad \text{(Eq. A4)}$$

Given the domain of G, $q \geq F(z)$ and so $G(F(z),z)$ is the maximum marginal benefit expected. By Lemma 1, $G(q,z)$ is non-increasing, so $G(q,z)-q$ will be strictly decreasing. Both $\mathrm{F}(z)-q$ and $\mathrm{F}(z-\bar{x})-q$ are also strictly decreasing.

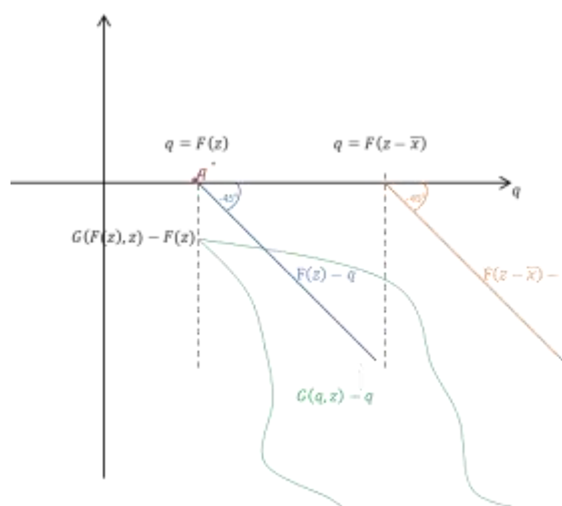
We will analyze graphically the 3 possible cases:

A. $G(F(z),z)-F(z) > 0$.



The function $G(q,z)-q$ starts from a positive value and varies up to a negative value with the increment of q up to p_1. It has therefore a unique zero (q^*) given by $G(q^*,z)-q^*$ or $F(z-\overline{x})$ depending on the relation between these two functions as the two green lines show.

B. $G(F(z),z)-F(z) < 0$



As opposed in Case A, the function $G(q,z) - q$ starts from a negative value so the root is equal to $F(z)$ and is unique.

C. $G(F(z), z) - F(z) = 0$

Following a similar argument than in the Case B, the root of Eq. A4 is also unique.

Resuming, for a given z, we have found that there exists a unique q that is the root of Eq. A4.

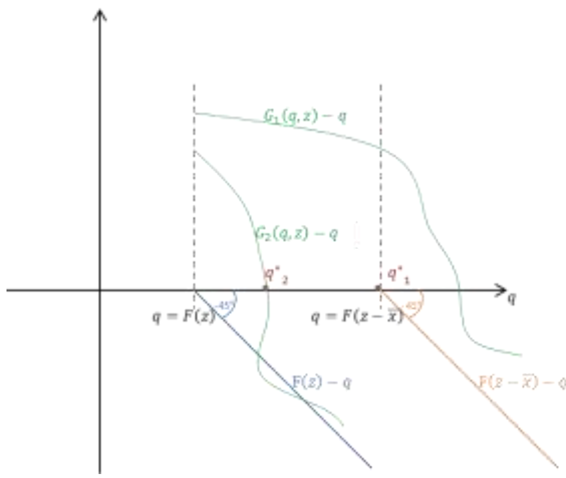
Lemma 2.

Given two functions g_1 and g_2, with $g_1 \geq g_2$. Then, $G_1 \geq G_2$ and also $TG_1(q,z) = f_1 \geq f_2 = TG_2(q,z)$, $\forall z \in X$.

Proof Lemma 2.

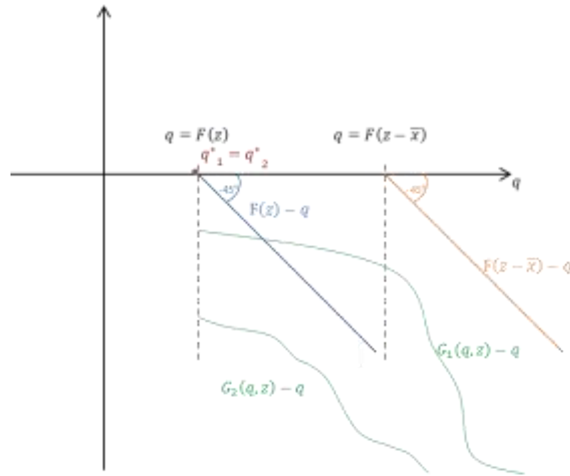
Given z, we can observe two scenarios:

i. $G_1(F(z), z) - F(z) \geq G_2(F(z), z) - F(z) > 0$:



Clearly $q_1^* \geq q_2^*$. They can take the same value if $G_1(q,z)$ and $G_2(q,z)$ are equal in that section of q.

ii. $0 \geq G_1(F(z), z) - F(z) \geq G_2(F(z), z) - F(z)$:



In this case, $q_1^* = q_2^* = F(z)$.

So, for any given z, we have demonstrate Lemma 2, which includes that T is an operator satisfying monotonicity.

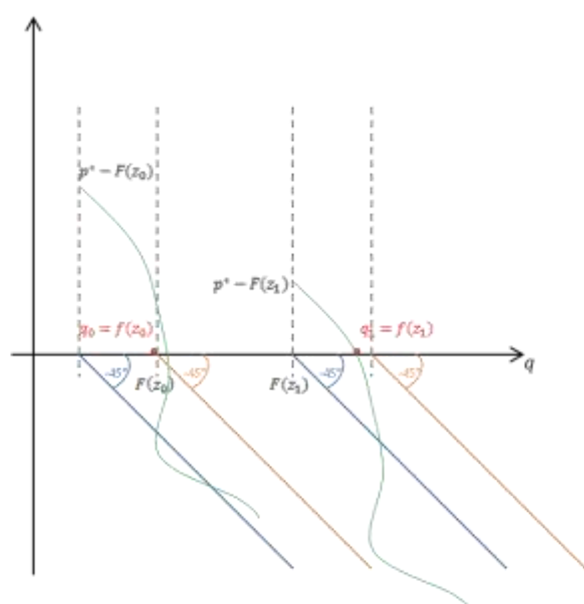
■

<u>Lemma 3.</u>

Suppose $z_0 > z_1$, then $f(z_0)$ <$f(z_1)$ and $f(z)$ is strictly decreasing.

<u>Proof Lemma 3.</u>

Up to now, z has been fixed but the same logic followed before can be used for this demonstration. Graphically we have:



p^* is constant and does not depend on z and $G(q, z_0) - q$ is the same function than $G(q, z_1) - q$, but shifted. So, if $z_0 > z_1$ then $q_0 = f(z_0) < f(z_1) = q_1$ and the function $f(z)$ is strictly decreasing. Also, there will be a unique z^* such that $p^* - F(z^*) = 0$.

In the illustration we have considered that $G(F(z_0), z_0) - F(z_0) > 0$ and $G(F(z_1), z_1) - F(z_1) > 0$, but the argument is similar when that condition is not attained.

▪

Theorem 1.

Under the following assumptions A1-A5, there exists a unique SREE $f(z)$ that is continuous and non-increasing.

- A1. Inflows are i.i.d. a support given by $[\underline{w}, \overline{w}]$.
- A2. The demand for water consumption $D(p)$: $(p_0, p_1) \rightarrow \mathbb{R}$ is continuous and strictly decreasing.
- A3. $F \equiv D^{-1}(p)$.
- A4. $F(\underline{w}) \in (0, +\infty)$ is the maximum marginal benefit.
- A5. $0 < \beta(1-d) < 1 \Rightarrow \beta < \frac{1}{(1-d)}$.

Proof Theorem 1:

a) Let $\mathcal{G} \equiv \{g: X \rightarrow (p_0, p_1), g\ continuous, g(\underline{w}) = F(\underline{w})\}$. Then, the operator T maps $\mathcal{G}$ into itself[6].

b) Define the metric $d(g_1, g_2) = \| g_1 - g_2 \| \equiv sup_{z \in X} |g_1(z) - g_2(z)|$ and suppose $a \geq 0$, scalar. If $g_1(z) \leq g_3(z) = g_2(z) + a, \forall z \in X$ then $(Tg_1)(z) \leq (Tg_3)(z) = T(g_2(z) + a), \forall z \in X$, since T is a monotonic operator.

Define $\gamma = \beta(1-d)$, then:

$$T(g_2(z) + a) \equiv \min\left\{\mathrm{F}(z - \bar{x}), \max\{\mathrm{F}(z), \gamma E g_2\big(w + (1-d)\big(z - \mathrm{F}^{-1}(Tg_2(z) + a)\big) + a\big)\}\right\}$$

$$\leq \min\left\{\mathrm{F}(z - \bar{x}), \max\left\{\mathrm{F}(z), \gamma E g_2\left(w + (1-d)\left(z - \mathrm{F}^{-1}\big(Tg_2(z)\big)\right) + a\right)\right\}\right\}$$

$$= \min\left\{\mathrm{F}(z - \bar{x}), \max\left\{\mathrm{F}(z), \gamma E g_2\left(w + (1-d)\left(z - \mathrm{F}^{-1}\big(Tg_2(z)\big)\right)\right) + \gamma a\right\}\right\}$$

$$\leq \min\left\{\mathrm{F}(z - \bar{x}) + \gamma a, \max\left\{\mathrm{F}(z) + \gamma a, \gamma E g_2\left(w + (1-d)\left(z - \mathrm{F}^{-1}\big(Tg_2(z)\big)\right)\right) + \gamma a\right\}\right\}$$

$$= \min\left\{\mathrm{F}(z - \bar{x}), \max\left\{\mathrm{F}(z), \gamma E g_2\left(w + (1-d)\left(z - \mathrm{F}^{-1}\big(Tg_2(z)\big)\right)\right)\right\}\right\} + \gamma a$$

$$= (Tg_2(z)) + \gamma a.$$

[6] *F(z)*, *F(z-x)* and *G(q,z)* are continuous, so *TG(z)* is also continuous.

Taking a) and b) and by Blackwell's contraction Theorem, T is a contraction mapping with modulus $\gamma = \beta(1-d)$. So, there is a unique fixed point f such that $d(f, Tf) = 0$ and it can be found by iteration starting from any function $g \in \mathcal{G}$.

⊗.

II. Finite horizon 3-period deterministic problem

We consider a quadratic benefit function of water consumption, implicating a linear demand $F(c_t) = a - bc_t,\ a, b > 0$. There is a constant inflow k and no evaporation ($d = 0$). The problem can be stated as:

$$V_t(z_t) = \max_{\{x_t\}} E_t\left[\sum_t^3 \beta^t \int_0^{z_t - x_t} \mathrm{F(q)dq}\right]$$

subject to

$$z_{t+1} = x_t + k \quad t = 1,2,3$$

$$0 \leq x_t \leq \bar{x} \qquad t = 1,2,3$$

Applying dynamic programming, the solution is found using a backward process. We start in $t = 3$, and assume that the optimal storage rule is zero, i.e., $x_3^* = 0$. Replacing in the value function, we obtain $V_3{}^*(z_3) = az_3 - \frac{b}{2}z_3{}^2$.

With this information we formulate the problem for $t = 2$:

$$V_2(z_2) = \max_{\{x_2\}} \int_0^{z_2 - x_2} \mathrm{F(q)dq} + \beta E_t[V_3{}^*(z_3)]$$

subject to

$$z_3 = x_2 + k$$

$$0 \leq x_2 \leq \bar{x}$$

Using the Kuhn-Tucker first order conditions we find an optimal storage function in intervals with two kinks (z^* and z^{**}):

$$x_2^*(z_2) = \begin{cases} 0 & z_2 \leq z^* = -A(1+\beta) \\ A^{7} + \dfrac{z_2}{1+\beta} & z^* < z_2 < z^{**} \\ \bar{x} & z_2 \geq z^{**} = (\bar{x} - A)(1+\beta) \end{cases}$$

Graphically:

[7] A is a constant depending on the parameters: $A = \frac{a(\beta-1)}{b(1+\beta)} - \frac{\beta k}{(1+\beta)}$.

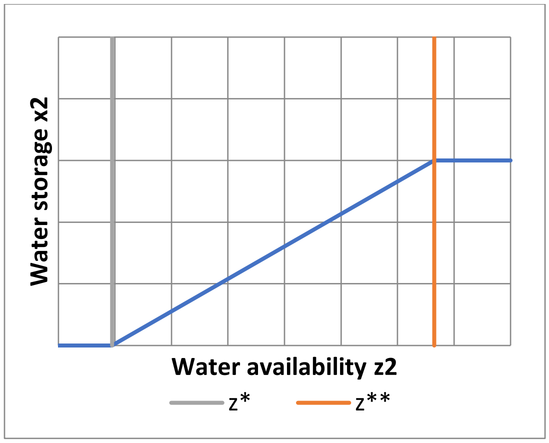


Figure 6: Optimal storage function for the second period.

This implies an optimal value function that is also defined in three intervals with the same kinks z^{*} and z^{**} as shown in the next figure:

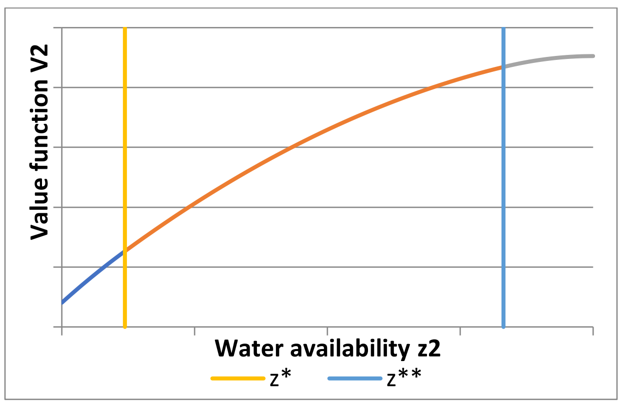


Figure 7: Optimal value function for the second period.

The solution is strictly concave in each of its sections and also continuous.

Analogously, we formulate and solve the problem for the first period $t = 1$:

$$V_1(z_1) = \max_{\{x_1\}} \int_{0}^{z_1 - x_1} \mathrm{F(q)dq} + \beta E_t[V_2{}^{*}(z_2)]$$

subject to

$$z_2 = x_1 + k$$

$$0 \leq x_1 \leq \bar{x}$$

In this case we have to take into account the three intervals of $V_2{}^{*}(z_2)$.

The optimal storage rule is also a function by intervals but in this case more kinks appear (z^{*}, z^{**} and z^{***}):

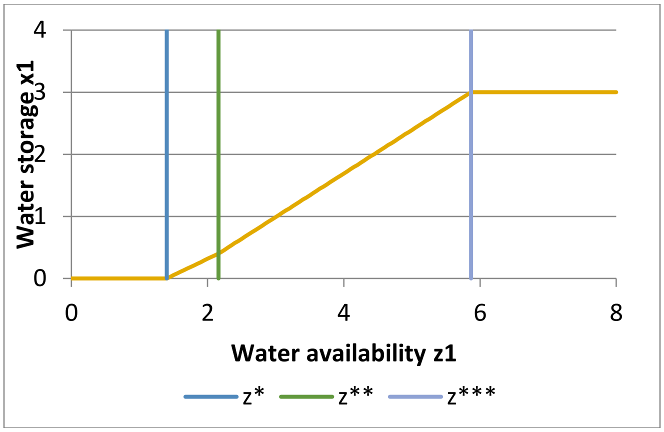


Figure 8: Optimal storage function for the first period.

The Value function will also be in intervals with three kinks, but continuous and strictly concave in each of them:

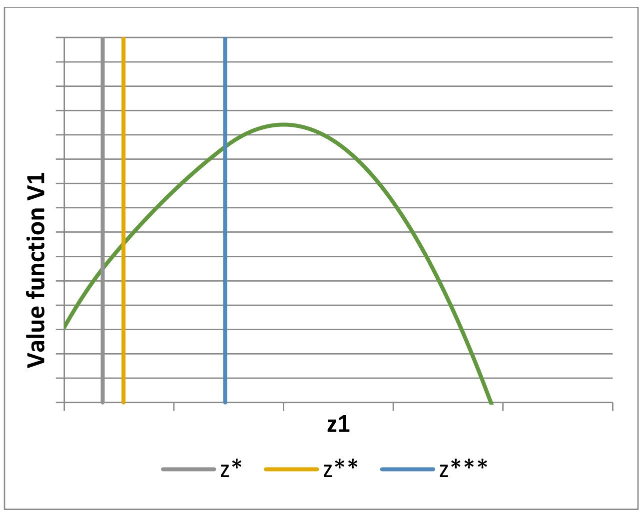


Figure 9: Optimal value function for the first period.

In an economic analysis like this is important to analyze the behavior of the marginal value of water. Due to the concavity of the value function the marginal value will be linear and strictly decreasing:

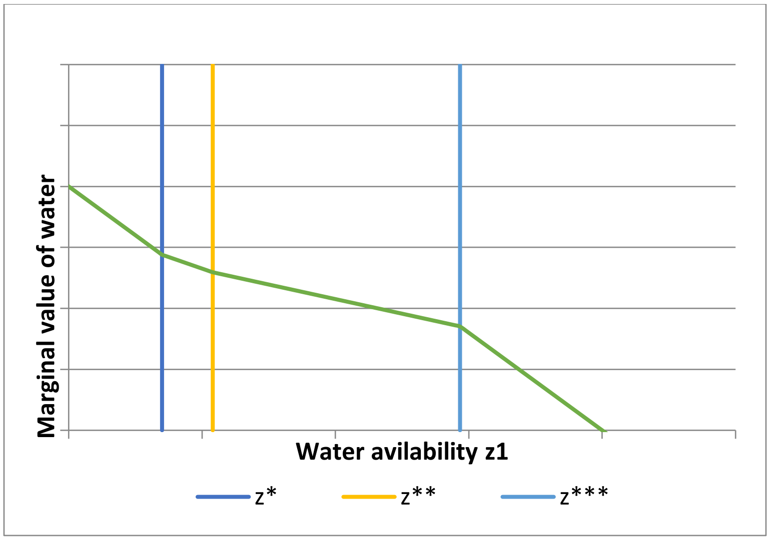


Figure 10: Marginal benefit function for the first period.

The previous results are as expected. First of all, the concavity of the value function is demonstrated. Secondly, the kinks are expanded when more periods are considered; even in this brief 3-period analysis is reflected. This implies a mathematical difficulty to expand the analytical solutions to a more realistic context with longer horizons.

Chapter 3
Economic modelling of crop choice under water scarcity
December 2023

Contents

Abstract
This article addresses the decisions of crop choice under water scarcity, focusing on low- to medium-income nations. The traditional approach of treating land as a fixed input falls short when considering water scarcity and institutional water allocation. The research develops a novel crop choice economic model that considers water, not land, as the primary decision variable for farmers. The literature review explores various econometric models and their applications in understanding agricultural supply responses and climate change impacts. The methodology employs a multinomial logistic structure to analyze water allocations for crop choices, utilizing data from the National Agricultural Census of Peru. Results from tobit, panel tobit and seemingly unrelated regression models are compared between water and land allocations for specific crops. Despite partial identification of profit influences, few statistically significant models are identified, revealing challenges in attributing effects to chosen variables. The conclusion emphasizes the model's contribution to understanding water allocation decision-making under data constraints, suggesting future research explore medium and long-term effects with improved data quality for a more comprehensive understanding of crop-mix allocation complexities under water scarcity.

1. Introduction

Crop choice and land use models have been widely used to understand the effects of a change policies, economics or even climate. Crop-mix models are frequently at farmers scale while land use models are national or sub-national. In both cases, econometric approaches are developed with requires a large and rich amount of data that matches allocations with weather and economic variables (Arora, Feng, Anderson, & Hennessy, 2020; Michael R Moore & Negri, 1992; Oczkowski & Bandara, 2013; Olen, Wu, & Langpap, 2016; Speir, Mamula, & Ladd, 2015). These requirements create a restriction for undeveloped countries, since datasets often lack completeness. And is in these low- or medium-income nations where agricultural patterns have a very important effect since their economies are highly depend on this activity (Melo & Foster, 2021). Understanding the decision in crop-mix allocations is fundamental to policy formulation.

A long tradition of land use models is based on the dual multioutput technology where jointness is incorporated by a fixed input in a profit maximization function (Chambers & Just, 1989). Fixed inputs are defined when its farm level quantity is the variable that determines the producer decisions. In the case of variable inputs, price replaces quantity in the producer's decisions. Land is usually considered as the fixed input and all other like water, fertilizer or labor are variable. The assumption behind is that there exists a market for all variables inputs and the producer can adjust the use of them in the short term. However, in at least two situations this doesn't apply for water. First, when water is allocated through institutional settings. Is this case, there is no market for water so it's not available to be bought or sold to adjust crops allocations. And secondly, in the presence of water scarcity. In times of droughts or in arid places, buying water is not an option. Even using underwater may not be available since it requires a prior investment to be used in times of irrigation season.

Crop choices will depend on both variable and fixed inputs and its prices (Boggess, Lacewell, & Zilberman, 1993). Optimal input allocation, including water, will depend on input and outprices and also yields and irrigation technology. These information is usually available in all countries.

So, understanding crop-mix allocation in the context of water scarcity is still missing for low to medium income nations. The objective of this research is to develop a simple crop choice model that captures the economic rationality of the decision makers based on water allocations. We propose a comparison on water and land allocations for completeness.

2. Literature review

Chambers and Just (1989) applied a dual approach to the decisions on the allocations of fixed inputs. They develop a multioutput profit maximization function that characterizes land as the fixed input that provides a source of jointness on the input decisions. Using this approach, Moore and Negri (1992) extended the question of whether land or water are fixed inputs. Analyzing producer data at national level on land and water allocations, the authors found that water should be treated as a fixed-input instead of a variable - one since due to the water entitlement context. Similarly, Moore and Dinar (1995) studied long-run farm behavior evaluating competing models of input use for surface water and land. Using the multiproduct firm theory, ten counties in California were studied using 1988 production info resulting that both water and land were quantity fixed so they should be treated as fixed inputs.

Boggess et al. (1993) develop a simple model to explain irrigation decisions and technology adoption. The authors maximize profits in order to choose which technology will be applied. Newer technologies may be more efficient and achieve higher yields but have higher fixed costs compared to traditional irrigation practices. Which technology will be optimal is determined by the marginal product of water and the price of water. The same model may be used to characterize crop instead of technologies choices.

Oczkowski and Bandara (2013) run a comprehensive systematic econometric study of agricultural supply response at the regional level in Australia. Applying the multioutput profit maximization function proposed by Chambers and Just (1989) on 50.000 land allocation data points of 64 commodities and 24 years they found that land allocations partially adjust to changes on prices, rainfall and other climate variables. Given the Australian institutional settings, the economic drives have small short-run effects on land-use allocations. Speir et. al (2015) follows the model developed by Moore and Negri (1992) considering water as a quantity fixed input. They estimate the impacts of water changes on crop and labor production at county scale in San Joaquin Valley (California). For their analysis they employed 22 years of a complete data set per county that included input and output prices for seven crop groups, total water availability and other external factors for six counties. They run regressions not only on the profit function but also to calculate labor, groundwater demands and crop supply. Among the results, they found that cotton was the most sensitive crop to water availability while permanent crops remained stable. Also, a structural change was identified in the 22 years related to annual crops and water supply.

In recent econometric developments, Arora et al. (2020) study the effects of climate change on land allocations by including an explicit crop-specific yield-weather function in the profit function. Water is treated as a variable input and has impact on profits through the level of production. Melo and Foster (2021) proposed a similar approach but adapted to medium and low-income countries, where land allocations matched to weather specific data is not available. Profits were constructed using average regional data for ten fruits, vines, field crops, forestry, pasture and fallow land. Their results indicate a relative low impact of climate change on Chilean agriculture, that may be explained by the possibility of changing the crop-mix by the farmers, at least at the long-run.

In this research farmer´s decisions are modeled, assuming they depend on the relative profitability of each activity (Arora et al., 2020; Melo and Foster, 2021). Our analysis is based on Peru that accounts for the same data problems as Chile so different strategies were taken to form a dataset that matches crop allocations and profits. A novelty of our proposal is the consideration of water instead of land allocations as the decision variable of the farmers, following previous findings where water is a fixed input (M. R. Moore & Dinar, 1995; Michael R Moore & Negri, 1992; Speir et al., 2015). We also construct the model for land allocations to compare the results.

3. Methodology

3.1. Model description

The following econometric model analyzes the water allocations that determine crop-choices of a group of farmers. We call this group a conglomerate. Assuming individual decisions are driven by the maximization of the expected benefits per hectare, the decision of how much water to use for each crop is determined by the relative net incomes of each crop.

The water used to irrigate a crop $\boldsymbol{i}$ in a conglomerate $\boldsymbol{c}, \boldsymbol{W_{ic}}$, depends on the net benefits of all crops ($\boldsymbol{i}$ + $\boldsymbol{j}$) and other variables, $\boldsymbol{x_c}$, like location, source of water, irrigation technology, among others. We model $\boldsymbol{W_{ic}}$ as the water share and asume that it has a multinomial logistic structure (Arora et al., 2020; Melo and Foster, 2021; Oczkowski and Bandara, 2013):

$$W_{ic} = \frac{exp[f(x_c/\beta_i)]}{\sum_{j\in J} exp\left[f\left(x_c/\beta_j\right)\right]}$$

Taking one crop as a reference crop, the relationship between the shares becomes linear and the parameters β_i can be estimated:

$$\ln\left(\frac{W_{ic}}{W_{1c}}\right) = f(x_c/\beta_i) - f(x_c/\beta_1) = \beta_i{}'x_c - \beta_1{}'x_c = \alpha_i{}'x_c$$

Previous theory is applied to water shares as follows. Water used to irrigate a crop i in conglomerate c on year t is estimated as the area multiplied by the irrigation rate:

$$w_{ict} = \sum_{s=1}^{2} \sum_{x=1}^{2} ha_{icxts} * rate_i$$

The subindexes *x* corresponds to the irrigation technology, superficial or drip, and *s* to the water source, superficial or underground. Using crop 1 as the reference crop, the water share W_{ict} is estimated as:

$$W_{ict} = w_{ict}/w_{1ct}$$

Turning on the independent variables, water share depends on the net benefits π_{ict}. We construct the net benefits using crop prices, yields and costs. A second explanatory variable is the total water available on the conglomerate. We also include dichotomic variables as irrigation technology, the source of water and the district where the conglomerate is located.

3.2. Data description

We applied the previous model using the National Agricultural Census of Peru. This census collects crop data from producers. Each producer represents an agroeconomic unit (UA). In a UA, more than crop may exist, also irrigation technologies may differ between them. The source of water, superficial or underground, may vary also in a UA. Given the quality of the data, UA data was aggregated into conglomerates. We consider years 2016, 2017 y 2019 and avoid using 2018 since the variables collected that year didn't correspond to the series of the other years.

Data recollected on prices, yield and costs for each crop was very irregular so technical sheets were constructed for each crop grown in a tract for each year. We use these estimations to reconstruct the net benefits per crop.

4. Results

Water shares models were estimated using tobit, panel tobit and SUR regressions. Similar models were run but estimating land shares for comparison. Table 1 shows the results of water allocations *Wi*, and land allocations *Li* for each crop *i*: asparagus (1), avocados(2), onion (6) and pomegranade (7). The indpendet variables are: *Pi:* indicates the relative profit of each crop *i* to grapes; Lnw: is the natural logarithm of total water availability; *lIrrigation* and *lsource:* are the dummies variables referred to superficial and drop irrigation and superficial and underground water source; *Ldis:* refres to different districts to explore if localization effects where present. Grapes was elected as the reference crop because it is the most frequent crop between tracts and years an also the area dedicated to it is the highest. Different lags for water availability were explored with similar results.

The average and sd of estimates α_i are presented in the first and second row of each explanatory variable. Only tobit regression results are displayed since no differences were found using the other formulations.

We found very few statistically significant models. No differences were shown between water and land allocations. Own profits are expected to have a positive relation, this was found only in the case of onions. Cross profits are expected to have a negative effect which is found on asparagus and avocados. Water availability shows a negative impact which is contrary to expectation since more water should imply higher allocations. The effects of irrigation technology and the source of water were not important given previous results. The same applies for districts.

Table 1: Water and land shares allocation among crops.

Variable	W1	W2	W6	W7	L1	L2	L6	L7
P1	-0.2	**-0.14**	**-0.16**	**-0.14**	-0.21	**-0.15**	**-0.15**	**-0.12**
	0.11	0.09	0.1	0.09	0.11	0.09	0.1	0.1
P2	**-0.74**	-0.49	**-0.91**	**-0.64**	**-0.78**	-0.52	**-0.9**	**-0.49**
	0.57	0.49	0.51	0.5	0.57	0.5	0.53	0.54
P6	0.03	0.02	**0.02**	0.01	0.03	0.02	**0.02**	0.01
	0.02	0.02	0.02	0.02	0.02	0.02	0.02	0.02
P7	0.63	0.2	0.4	**0.29**	0.66	0.22	0.38	**0.01**
	0.49	0.42	0.44	0.43	0.48	0.43	0.45	0.46
lnw	**-0.03**	**-0.07**	**-0.09**	**-0.08**	**-0.03**	**-0.07**	**-0.09**	**-0.08**
	0.01	**0.01**	**0.01**	**0.01**	**0.01**	**0.01**	**0.01**	**0.01**
_Iirrigati~2	-0.15	**0.42**	**0.81**	**0.61**	-0.09	**0.48**	**0.84**	**0.78**
	0.2	**0.17**	**0.18**	**0.17**	0.2	**0.17**	**0.18**	**0.19**
_Isource_2	0.15	-0.01	-0.26	-0.1	0.11	-0.05	-0.28	-0.05
	0.2	0.17	0.18	0.18	0.2	0.17	0.18	0.19
_Idis~110102	**-1.13**	**-0.96**	-0.69	**-0.88**	**-1.09**	**-0.92**	-0.67	**-0.85**
	0.45	**0.39**	0.4	**0.39**	**0.44**	**0.39**	0.41	**0.42**
_Idis~110104	0.01	-0.04	0.14	0	0.02	-0.02	0.15	0
	0.39	0.33	0.35	0.34	0.38	0.34	0.36	0.36
_Idis~110105	-0.5	-0.16	-0.77	**-0.97**	-0.54	-0.23	-0.81	**-0.99**
	0.5	0.43	0.45	**0.44**	0.5	0.43	0.46	**0.47**
_Idis~110107	-0.45	-0.45	-0.26	-0.4	-0.43	-0.44	-0.25	-0.21
	0.44	0.39	0.4	0.39	0.44	0.39	0.41	0.42
_Idis~110108	-0.13	-0.39	0.05	0.05	-0.16	-0.37	0.12	0.12
	0.39	0.34	0.35	0.34	0.38	0.34	0.36	0.37
_Idis~110110	-0.86	0.3	-0.47	-0.65	-0.85	0.27	-0.47	-0.65
	0.5	0.43	0.45	0.44	0.49	0.43	0.46	0.47
_Idis~110111	0.34	-0.33	-0.28	-0.33	0.31	-0.35	-0.3	-0.28
	0.34	0.3	0.31	0.3	0.34	0.3	0.31	0.32
_Idis~110112	**1.24**	0.62	0.39	0.23	**1.26**	0.65	0.4	0.24
	0.63	0.54	0.56	0.55	**0.62**	0.55	0.58	0.59
	0.97	0.23	0.4	0.24	1	0.24	0.36	-0.15
_cons	0.83	0.72	0.75	0.73	0.83	0.73	0.77	0.79
Statistics								
N	516	516	516	516	516	516	516	516
r2_p	0.02	0.05	0.08	0.07	0.02	0.06	0.08	0.06

*Crops: 1: asparagus, 2: avocados, 6: onion and 7: pomegranade.

5. Conclusions

We have developed a simple crop-mix model that allocates water when data is not fully available. Profits influence could be partially identified, both own and cross effects, but the models lack significance. Also, no differences where identified between water or land shares models. Even though our results are quite similar to the effects shown by Oczkowski and Bandara (2013) who found that land allocations didn't adjust completely to economic inputs. Given institutional settings may explain the lack of flexibility in the input allocations.

So, the effects we wanted to understand were not identified. One reason may be the horizon of the model we run. We consider short-term effects when the adaptation of farmers to change the crop pattern may refer to a long-run response. Also, problems with the data used may also apply. Since we construct technical sheets for crop yields and crop costs, we had undermined the richness of data variability. Without this variability, no effect may arise in the regressions. Further research may include a better dataset with longer periods to explore both a medium and long-run effects and also the variance among the independent variables.

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