Fiscal Policy, the Real Exchange Rate and the Current Account Under Rational Expectations: A Mundellian Framework

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1. Introduction

Recent international developments have focused attention once again on the relationship between fiscal policy, the real exchange rate (R), and the current account (CA). Early models like Mundell (1963) have no role for expectations and center on steady state effects of fiscal policy. Kouri (1976) includes dynamics and the government, but in working with a one sector model has by contraction a constant R. On the other hand, Dornbusch and Fischer (1980) study the interactions between CA and R under static expectations, and between CA and the nominal exchange rate (E) under perfect foresight, but do not have a government sector on their model. Finally, both Hodrick (1980) and Sachs and Wyplosz (1983) present a dynamic framework dealing with fiscal policy; the former uses adaptive expectations and does not study R, while the latter two authors assume perfect foresight and consider bond financed deficits.

This paper is set in a modified Mundell-Dornbusch-Fischer scenario where the home country is specialized in production and full employment exists. The inclusion of a fiscal sector which is allowed to run money financed deficits has implications for the adequate specification of goods demands and the current account. In addition, asset demands are made dependent on return and wealth, thereby stressing portfolio behavior rather than transactions type motives; this framework seems to fit better in a world of widely fluctuating exchange rates which reflect to a large extent portfolio reallocations by agents. An important feature of this framework is the existence of equilibrium in
goods and asset markets at every point in time, while the current account balances only in the long run. This illustrates the difference between temporary and steady state equilibrium.

The outline of the paper is as follows. Section 2 introduces the model and provides a discussion of its underlying assumptions. In section 3 a balanced budget expansion is considered under rational expectations and a condition is derived under which the model presents saddle path stability. The impact effect of the fiscal action, but not its long run result, is shown to depend crucially on asset substitutability. In section 4 money financed deficits are studied and this time both the impact effect and the long run outcome of the fiscal action are critically influenced by substitution between assets. Section 5 states the main conclusions of the paper.

2. The Model

We consider an economy specialized in the production of an exportable good whose price \( P \) adjusts instantaneously to clear the market, and is influenced by internal demand conditions. The nominal exchange rate is fully flexible and the country faces a price for the importable good in world markets, which together with purchasing power parity implies

\[
(1) \quad P_M = E \cdot P_M^* 
\]

where \( P_M \) = internal price of the importable, \( E \) = nominal exchange rate and \( P_M^* \) = world price of the importable.
Therefore, the real exchange rate is defined as

\[ R = E P_M^*/P \]

This is a typical Mundellian scenario in terms of the choice of goods; however, the model will present fully flexible prices and a constant, full employment level of output, departing in this way from Mundell.

We notice from the outset one limitation of this framework, regarding the fact that the real exchange rate is exactly the inverse of the terms of trade. In more general models, these two variables will not have such a fixed link. The only way of overcoming this problem is having a structure which at least includes three goods - importables, exportables and nontradeables - , an avenue which will not be pursued here. Another characteristic of this framework is that the terms of trade are endogenous, which can be justified for small economies selling differentiated products abroad.

Wealth will be held in our model in the form of domestic money and a foreign real bond which is a perpetuity giving one unit of the imported good per period. The return of this foreign bond in terms of the home good is the world interest rate plus the expected rate of nominal exchange rate depreciation. In this framework, even if domestic residents were allowed to hold foreign currency they would never do so because while having the same characteristics of the bond it gives a consistently lower return.

Consequently, we define real financial wealth as
(3) \( w = m + Rf/r^* \)

where \( w = \) real wealth measured in terms of the exportable, \( m = M/P, \)
\( f = \) number of units of the foreign asset owned by domestic residents and
\( r^* = \) international rate of interest (a real rate, assuming away interna-
tional inflation).

Asset demands depend on return and wealth in an analogous
fashion to that used in Sidrauski (1967) and Dornbusch (1975).

(4) \( m^d = h(r^* + (E/E)^e) \) \( w = \) \( m \quad h' < 0 \)

(5) \( (Rf/r^*)^d = k(r^* + (E/E)^e) \) \( w = Rf/r^* \quad k' > 0 \)

where the usual adding-up constraints apply \( (h+k=1; \ h' + k'= 0) \)

Since the return on the foreign bond is at the same time the
opportunity cost of holding money, the signs of \( h' \) and \( k' \) are then ex-
plained.

The government sector is introduced through its budget cons-
traint, which allows for money financed deficits. This type of fiscal
behavior has received support in the literature dealing with semi-
industrialized economies, particularly in Bruno (1979), McKinnon and

\[ g = t + M/P \quad \text{or} \]
\[ (6) \quad g = t + \theta m \]

where \( g = G/P, \ t = \) direct taxes measured in terms of the exportable and
\( \theta = \) rate of monetary expansion.

Since many models undertake exercises with exogenous shifts
in fiscal and monetary policy, it is important to realize the interdependence presented in equation (6); the government can choose independently only two of its three instruments (expenditure, taxes and the rate of monetary expansion). In particular, we assume that the government sets $t$ and $\theta$, letting $g$ adjust to satisfy the budget constraint$^1$.

The production side of the model is highly simplified. With labor as the only productive factor, no population growth and fully flexible wages and prices, we have a given level of output of the exportable ($y$), the only good produced at home. At every moment in time our price flexibility assumptions assure equality between supply and demand in this market; the latter is defined as

$\text{(7) } y = c^x(y-t, R, m + Rf/r^s) + x(R) + g$

where $c^x(\ )$ = domestic consumption of the exportable and $x(\ )$ = exports.

As shown above, domestic demand for the exportable depends on financial wealth and disposable income, reflecting both life cycle considerations and some type of liquidity constraint, which is responsible for the absence of human wealth from equation (7)$^2$. This asymmetry between the influence of the two types of wealth -although perhaps

$^1$Government spending is endogenous whenever the rate of money expansion is positive because the authority can determine $\theta$, but not the value of deficit financing.

$^2$The formulation in (7) may be justified for a population composed of two types of agents: those who derive their income mainly from the payment to their labor services and those whose income is primarily determined by the return on their assets.
extreme-captures the fact that liquidity constraints tend to fall heavily on households with a higher proportion of human assets\(^3\). As for the other variables affecting demand, \( R \) being the relative price of the importable to the exportable has a separate influence on \( c^X \) and \( x \) through the substitution effect. It is explicit in (7) that government spending falls entirely on the domestic good.

We also notice that disposable income, although affecting private consumption, is an exogenous variable of the model. With a constant level of output, this argument implies that the government controls not only nominal, but also real taxes. This is a realistic assumption for economies in which taxes are mainly based on flows and therefore keep pace with inflation. However, if taxes on the value of stocks are an important source of government revenue, the tax system tends to be less indexed, because of lags in the adjustment of its base (i.e., the legal valuation of stocks). In our highly simplified scenario, it is sufficient to assume that taxes are a fraction of labor income (which is equal to the value of output) and that workers supply labor inelastically.

Finally, the current account in this model without investment will be equal to total savings, which includes both the private sector (\( S^P \)) and the government (\( S^G \)).

---

\(^3\)We must also acknowledge that the inclusion of human wealth as another variable influencing consumption makes the model intractable because the appropriate rate of discount involves one of our differential equations.
(8) \( CA = SP + G \)

from (6)

(9) \( G = t - g = -\delta m \)

Private savings need be consistent with the consumption specification and must not violate the identity

(10) \( y - t + RF = c^X(y-t, R, w) + c^M + SP \)

We notice that domestic consumption of the exportable is a function of \( R \), wealth and disposable labor income, but not of the flow of interest payments ( RF ). Therefore, if we further write down private savings as independent of RF (i.e. \( SP = S(y-t, w) \)) we will be implicitly assuming that all interest payments are used for consumption of the importable. This is exactly the assumption used by Dornbusch and Fischer when they specify \( S = S(w) \), but it does not seem very appealing for at least two reasons:

(i) Even if the model is ad-hoc, this feature introduces an asymmetry between \( c^X \) and \( c^M \) that can neither be justified on theoretical grounds, nor -as far as we know- on empirical grounds.

(ii) The fact that all interest income is spent regardless of what is happening to wealth seems to contrast with the life cycle aspects of the consumption function.

Therefore, we will assume that interest payments on the foreign asset are saved and so private savings take the form

(11) \( SP = S(y-t, w) + RF = S(y-t, m + RF/r^*) + RF \)

\( S_y > 0, S_w < 0 \)
Replacing (9) and (11) in (8)

(12) \[ CA = S(y-t, m + Rf/r^*) + Rf - \delta m \]

And noting that CA also equals the net accumulation of foreign assets

(13) \[ Ch = Rf/r^* \]

from (12) and (13)

(14) \[ f = r^*/R \left( S(y-t, m + Rf/r^*) + Rf - \delta m \right) \]

In what follows we study the effects of fiscal actions on the economy under rational expectations. As a matter of presentation we will first analyze steady state results and then dynamic adjustment; this strategy seems to facilitate the understanding of the transition period.

3. A balanced budget expansion under rational expectations.

3.1 Solution of the Model

In order to concentrate on the effects of a balanced budget fiscal expansion we study it starting from a situation in which there is no government deficit (i.e. \( \delta = 0 \)). It may also be noticed that if initially the fiscal sector finances part of its budget via money, an increase in taxes, \( dt \), will not lead to an equal increase in government spending because of the general equilibrium effects of higher direct taxes on wealth, which in turn affect tax revenues.

Rational expectations in a non-stochastic economy are equiva-
lent to perfect foresight. In our context this implies that the expected rate of nominal depreciation of the domestic currency equals the actual rate, or

(15) \( \frac{\dot{E}}{E} = \left( \frac{\dot{E}}{E} \right)^E \)

In steady state all real variables will be constant and so nominal variables will be growing at the same rate. In particular, the real money stock and the real exchange rate will be unchanging; thus

(16) \( \theta = \Pi = \frac{\dot{E}}{E} = 0 \)

where \( \Pi \) is the rate of change in the price of the exportable and the world price of the importable has been taken as given.

We will represent long run equilibrium as a situation in which the level of foreign assets and the real exchange rate have stationary values; when this occurs all other real variables determined in the model are also in steady state.

Our initial concern will be to obtain equations of motion for both \( R \) and \( f \); following treatments like Sidrauski (1967) and Calvo and Rodriguez (1977) we use equations (4), (5) and (15) to express equilibrium in the asset markets as

(17) \( \frac{m}{(RF/r^*)} = L(r^* + \frac{\dot{E}}{E}) \quad L' < 0 \)

inverting (6')

(18) \( \frac{\dot{E}}{E} = J \left\{ \frac{m}{(RF/r^*)} \right\} - r^* \quad J' < 0 \)

We notice that \( J' \) is a parameter identifying asset substitutability. If \(-J'\) is high, a small variation in the ratio of assets held
by agents requires a big change in expected returns; thus, assets are poor substitutes. Conversely, if $-J'$ is low assets are highly substitutable.

Also

\[(19) \frac{\dot{R}}{R} = \frac{\dot{E}}{E} - \Pi \]

and since $\frac{\dot{m}}{m} = -\Pi$, we arrive to

\[(20) \frac{\dot{R}}{R} = J \left[ \frac{m}{(RF/r^*)} \right] + \frac{\dot{m}}{m} - r^* \]

Now, as the market for the exportable clears at every point in time, we can obtain a reduced form relationship between $R$, wealth and taxes which will be always fulfilled

\[(21) w = w(R, t), \text{ where } w_R, w_t < 0 \]

From (21) and the definition of wealth

\[(22) m = w(R, t) - RF/r^* \]

and differentiating (46) with respect to time ($T$)

\[\frac{1}{m} \frac{dm}{dT} = \frac{1}{m} \left( w_R \frac{dR}{dT} - \frac{R}{r^*} \frac{df}{dT} - \frac{f}{r^*} \frac{dR}{dT} \right) \]

\[\frac{\dot{m}}{m} = \frac{R}{w(R, t) - RF/r^*} \left( w_R \frac{\dot{R}}{R} - \frac{\dot{f}/r^*}{r^*} - (f/r^*) \frac{\dot{R}}{R} \right) \]

Substituting (22) and (23) into (20)

\[(24) R = \Phi \left[ J \left( \frac{w(R, t) - RF/r^*}{RF/r^*} \right) - \frac{RF}{r^*(w(R, t) - RF/r^*)} - r^* \right] \]

where $\Phi = \frac{R}{w(R, t) - Rw_R} > 0$
This is our first dynamic equation; the second one is obtained by substituting (21) into (14) and setting $\theta = 0$

\[ \dot{\theta} = \frac{r^*}{R} \left( S(w(R,t), y-t) + Rf \right) \]

Now, investigating the slope of the $\dot{\theta} = 0$ locus in $(R, \theta)$ space, around the steady state (denoted by a bar over the variable)

\[ \frac{\partial \dot{\theta}}{\partial R} (R, \theta) = r^* > 0 \]

\[ \frac{\partial \dot{\theta}}{\partial f} (R, \theta) = \frac{r^*}{R} \left( S_{\theta} w_R + f \right) > 0 \]

\[ \implies \frac{dR}{df} \bigg| \dot{\theta} = 0 = \frac{-R}{(S_{\theta} w_R + f)} < 0 \]

To investigate the $\dot{\theta} = 0$ locus we start by replacing (25) in (24) to obtain

\[ (24') \dot{R} = \phi \left( \frac{(w(R,t) - Rf/r^*)}{Rf/r^*} - \frac{S(w(R,t), y-t) + Rf}{w(R,t) - Rf/r^*} \right) - r^* \]

and so, always evaluating around the steady state

\[ \frac{\partial \dot{R}}{\partial R} (R, \theta) = \phi \left[ \frac{J'(Rw_R - w(R,t))r^*}{R^2 f} - \frac{(S_{\theta} w_R + f)}{w(R,t) - Rf/r^*} \right] \]

\[ \frac{\partial \dot{R}}{\partial f} (R, \theta) = \phi \left[ -J'r^* w(R,t) \frac{R}{Rf^2} - \frac{R}{w(R,t) - Rf/r^*} \right] \]

Since it is not possible to sign expressions (28) and (29) the $\dot{R} = 0$ locus remains ambiguously sloped. This indeterminacy is due
to the particular type of expectation formation hypothesis used. If instead we had worked with some ad-hoc assumption about expectations like \( R = \lambda (R - R) \), which has been widely used in the literature, life would be much easier, but the agents would not be taking advantage of the information provided by the structure of the economy.

Nevertheless, we will proceed to analyze the local stability of the system around steady state, to see if it makes sense to study the effects of shocks to the economy using this model. The linearized system in matrix form looks like,

\[
\begin{bmatrix}
\dot{R} \\
\dot{\epsilon}
\end{bmatrix} = 
\begin{bmatrix}
R - R \\
\epsilon - \epsilon
\end{bmatrix} \phi \begin{bmatrix}
\frac{J^* r^*}{R f} (R w_R - w(R,t)) - \frac{(S w_R + f)}{w(R,t) - R f / r^*} \\
- \frac{J^* r^* w(R,t) - R}{R f^2} - \frac{R}{w(R,t) - R f / r^*}
\end{bmatrix}
\]

where

\[
Z = \begin{bmatrix}
r^* (S w_R + f) \\
r^*
\end{bmatrix}
\]

Recalling that \(|Z| = z_1 z_2\), where \(z_1\) and \(z_2\) are the two roots of the system, we need that \(|Z| < 0\) for saddle path stability. After performing the necessary computations we arrive to
(31) \[ |Z| = \frac{\phi J' r^* w(R,t)}{R^2 \left( R^* S_w w(R,t) + r^* Rf \right)} \]

Thus, stability requires that \( r^* S_w w(R,t) + r^* Rf < 0 \).

Rearranging terms this condition may be expressed as

(32) \[ r^* \frac{(Rf/r^*)}{w(R,t)} \leq -S_w \]

Recognizing that \( S_w = -c_w \), we can rewrite (32) as

(32') \[ r^* \frac{(Rf/r^*)}{w(R,t)} < c_w \]

The interpretation of (32') follows. If wealth increases marginally, \( (Rf/r^*)/w(R,t) \) is the proportion allocated to the foreign asset; this will produce an incremental flow of interest payments \( r^* (Rf/r^*)/w(R,t) \) which go into savings. On the other hand, \( c_w \) of the wealth increase is devoted to consumption. The first effect tends to improve the CA while the second one tends to worsen it. On the whole we require the wealth effect on spending to be greater than the interest payments effect so that the CA deteriorates following the initial wealth increase. Otherwise wealth would grow without bound.

We notice that life cycle models where agents have finite lives tend to result in the interest rate being smaller than the marginal propensity to consume out of wealth. This is more than we need for stability.
3.2 Steady state effects.

In steady state we have a system of three equations (goods market equilibrium, asset markets equilibrium and CA=0) in three unknowns \((w, R \text{ and } f)\). After substituting the asset market equilibrium condition in the CA=0 equation it gets reduced to a two-equation system in \(R, w\) and exogenous variables

\begin{align*}
(33) \quad y &= cX(y-t, R, w) + x(R) + t \\
(34) \quad 0 &= S(y-t, w) + r^*(1-h)w 
\end{align*}

Equilibrium is presented in \((R, w)\) space as the intersection of the locus \(yy\) (goods market clearing condition) and \(CA=0\) in figure 1. The schedule \(yy\) is downward sloping because a decrease in \(R\), starting from steady state, will leave an excess supply of goods thereby requiring an increase in wealth to clear the market. On the other hand, CA balance depends only on wealth as evident from equation \((34)\).

The balanced budget expansion provokes a decline in disposable income, depressing private savings and consumption. Starting from steady state at \(E\) in figure 2, if the stability condition is met, wealth should decrease to restore CA equilibrium. In addition, the \(yy\) schedule will shift downwards to avoid an excess demand of goods after the increase in government spending.

At first sight the only unambiguous long run effect seems to be a decrease in wealth and thus, since asset returns are unchanged, a drop in the value of both assets in the same proportion. However, it can be shown that \(R\) will be higher and \(f\) lower in the new steady state
if the following condition is met

\[(35) \ S_Y > S_w w_t \] where

\(S_Y\) = marginal propensity to save out of disposable income

\(S_w\) = marginal propensity to save out of wealth

\(w_t = d w/d t < 0\) from the goods market equilibrium condition.

Some intuition for (35) follows. The tax increase lowers disposable income and savings and thus tends to produce a CA deficit at the same time as an excess demand for exportables. To restore balance in the goods market a combination of \(w\) and \(R\) decline is required; since a decrease in \(w\) is also needed to clear the CA, \(w\) will unambiguously go down. Now, if \(S_Y\) is greater than \(S_w w_t\) the necessary drop in \(w\) to clear the CA (EC' in figure 3) will be bigger than the one needed to equilibrate the market for the exportable (EG) at the initial \(R\). Therefore, to avoid an excess supply of the domestic good, \(R\) will have to be higher in the new steady state. But if at \(E'\) \(R\) is higher, \(w\) is lower and the opportunity cost of holding money is unchanged, equilibrium in the foreign assets market is only possible with a lower level of \(f\), thereby requiring a CA deficit during transition.
3.3. **Dynamics**

Bearing in mind that the $R=0$ locus has an indeterminate slope, we have two possible configurations for the phase diagram in $(R,f)$ space, as shown in figure 3. In both cases the saddle path implies the coincidence of CA surplus with $R$ appreciation and CA deficit with $R$ depreciation. It will be assumed that the economy lies always in some point of the saddle path; this characteristic is an explicit outcome of optimizing-agents models.

The direction of movement of the two schedules after the fiscal expansion may be determined from

$$
\frac{\partial f}{\partial t} = \frac{r^*}{R} (S_w w_t - S_y) < 0
$$

(36)

$$
\frac{\partial R}{\partial t} (R, f) = \frac{J' w_t}{R E / r^*} - \frac{(S_w w_t - S_y)}{w(R, t) - RF / r^*} > 0
$$

(37)

The appropriate signs of (36) and (37) have been determined under condition (35), implying that $f=0$ shifts to the right and $R=0$ to the left, as shown in figure 4. Therefore, equation (35) is sufficient both to unambiguously determine the long run effects of the fiscal expansion and to identify the direction of movement of the two loci.  

In figure 4 we see that the impact effect on $R$ is ambiguous; in what follows intuition is provided for this result. From (36) and

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4An opposite assumption about the sign of equation (35) makes $f = 0$ to shift down but the $R = 0$ movement becomes ambiguous.
(37) it is apparent that asset substitutability -as captured by the parameter \( J' \)- influences the shift in the \( R=0 \) locus and does not affect the movement of the \( f=0 \) schedule. In particular, the poorer substitutes are money and foreign bonds (high \( -J' \)) the stronger the leftward shift in \( R=0 \) and the greater the chances of an \( R \) appreciation. Conversely, if \( -J' \) is low the possibilities of an \( R \) depreciation are enhanced.

What are the economics of this result? The initial excess demand of goods provoked by the expansionary fiscal policy requires an adjustment in \( R \) and \( w \) to clear the market. If \( R \) increases, \( w \) will have to decline more than proportionately and thus real money balances will have to decrease even more. But this is only possible if relative asset demands are responsive to a change in returns, and it requires \( (E/E) \) to increase. If, on the contrary, assets are very poor substitutes, the necessary rise in \( (E/E) \) to decrease money balances -and hence \( w \)- will exceed the possible range of outcomes given by the parameters of the model and the saddle path adjustment of the economy.

During transition the movement of the economy involves an increasing \( R \), a CA deficit and a falling wealth along the new saddle path which goes through point \( E' \). In the long run \( R \) is higher and \( f \) is lower.
4. A deficit expansion under rational expectations.

4.1 Solution of the model

When considering deficit spending under rational expectations
the rate of money growth turns positive and the equilibrium condition in
the goods market becomes

\[ y = c^x(y-t, R, w) + x(R) + t + \theta h(r^* + E/E)w \]

From above the reduced form equation for wealth is

\[ w = w(R, t, \theta, E/E) \]

where \( w_R, w_t, w_\theta < 0; w_E/E > 0 \)

and so

\[ m = w(R, t, \theta, E/E) - Rf/r^* \]

Now, if we attempt to solve the model using the same ap-
proach as in section 3.1, differentiation of (22') with respect to time
is needed; but this will leave us with a second order differential equa-
tion in the nominal exchange rate which enormously complicates the
analytical solution of the model.

Therefore, a different strategy will be pursued. We start by
differentiating the goods market clearing equation with respect to time,
to obtain

\[ 0 = BR + (X_{c_w} + \theta)m + (X_{c_w} R/r^*)f \]

where \( B = (X_{c_R} + X_{c_w} + c_w f/r^*) > 0 \)

Solving now for \( m \)
(39) \[ m = \left\{ -BR - (c_{w}^{X}/r^{*})f \right\}/(c_{w}^{X} + \theta) \]

using (39), (20) and the CA equation

\[
(40) \quad R = \phi^{*}\left\{ J\left(m/r^{*}\right) - (\theta + r^{*}) \frac{c_{w}^{X}}{m(\theta + c_{w}^{X})} \left\{ S\left(m + \frac{Rf}{r^{*}}\right)\frac{y-t}{y-t} + Rf - \phi m \right\} \right\}
\]

where \( \phi^{*} = \left\{ \frac{Rm(c_{w}^{X} + \theta)}{m(c_{w}^{X} + \theta) + BR} \right\} > 0 \)

To express our system of differential equations in terms of \( R, f \) and exogenous terms we linearize the goods market equilibrium condition and solve for \( m \) to get

\[
(41) \quad m = \left\{ (y-t)(1-c_{y}^{X})/c_{w}^{X}/(c_{w}^{X} + \theta) \right\}
\]

Substituting (41) in (40) and the CA equation we finally arrive to

\[
(40') \quad R = \phi^{*}\left\{ J \left\{ \frac{(y-t)(1-c_{y}^{X}) - BR}{(Rf/r^{*})(c_{w}^{X} + \theta)} \right\} - (\theta + r^{*}) \right\}
\]

\[
- \frac{c_{w}^{X}}{m(c_{w}^{X} + \theta)} \left\{ S\left(y-t, \frac{(y-t)(1-c_{y}^{X}) - BR}{c_{w}^{X} + \theta} + Rf/r^{*} \right) \right\}
\]

\[
+ Rf - \theta \left\{ \frac{(y-t)(1-c_{y}^{X}) - BR}{(c_{w}^{X} + \theta)} \right\}
\]
\[
(42) \quad \dot{f} = \frac{r^*}{R} \left[ S(y-t, \frac{(y-t)(1-c_Y^X)BR}{(c_W^X + \theta)} + RF/r^* \right] \\
+ RF - \Theta \left[ \frac{(y-t)(1-c_Y^X)BR}{(c_W^X + \theta)} \right]
\]

Tedious computations—and a bit of patience—allow us to obtain the elements of the transition matrix of this system, evaluated as usual around steady state.

\[
(43) \quad \dot{\Theta} = \dot{R} = \frac{\Theta}{R} \left[ 1 - S(y-t, \frac{r^*J'(y-t)(1-c_Y^X)}{RF(c_W^X + \theta)} \right] \\
- \frac{c_W^X}{m(c_W^X + \theta)} \left[ \frac{(\Theta - S_W^X)B}{(c_W^X + \theta)} + f(S_W^X/R^* + 1) \right]
\]

\[
(44) \quad \dot{\Theta} = \dot{R} = \frac{\Theta}{R} \left[ 1 - S(y-t, \frac{r^*J'[R(c_W^X f/r^*) - (y-t)(1-c_Y^X)}{RF(c_W^X + \theta)} \right] \\
- \frac{c_W^X}{m(c_W^X + \theta)} \left[ \frac{(\Theta - S_W^X)RC_W^X/r^*}{(c_W^X + \theta)} + R(S_W^X/R^* + 1) \right]
\]
\[ \frac{\partial f}{\partial R} (R, f) = \frac{r^*}{R} \left\{ \frac{(\theta - S_w)B}{(c_w^x + \theta)} + f\left(S_w/r^* + 1\right) \right\} \]

\[ \frac{\partial f}{\partial f} (R, f) = \frac{r^*}{R} \left\{ \frac{(\theta - S_w)Rc_w^x/r^*}{(c_w^x + \theta)} + R\left(S_w/r^* + 1\right) \right\} \]

Equations (43)-(46) define our transition matrix which we now call $Z'$. After some algebra it is possible to show that

\[ |Z'| = \frac{r^*J'(c_w^x + x_R)}{(c_w^x + \theta)Rf} \left[ m(\theta - S_w) - (Rf/r^*)S_w + r^* \right] \]

Saddle-path stability requires $|Z'|$ to be negative, which will occur if

\[ r^*(Rf/r^*)/w < \theta m/w - S_w \]

The above equation is similar to condition (32) except for the appearance of $\theta m/w$ on the right hand side. The interpretation is analogous as before. Out of a one-dollar increase in wealth $(Rf/r^*)/w$ goes to the external asset provoking and incremental flow of interest payments $r^*(Rf/r^*)/w$, which goes into savings. On the other hand, the wealth increase rises private consumption by $c_w$, and government spending by $\theta m/w$ (since $m/w$ of the dollar goes into money). Once again, for stability we require the total spending effect to be higher than the sa-
vings effect. Thus, the presence of deficit financing by the government further enhances the prospects of stability by increasing the spending effect.

4.2 Steady state effects

To analyze steady state results we work once more in \((R, w)\) space. The system of equations describing equilibrium is analogous in every respect to (33) and (34) except that now we start from a positive rate of money expansion. Therefore goods market equilibrium and CA balance (where appropriate substitution has been made of the asset markets equilibrium condition) look like (49) and (50) respectively

\[(49) \ y = c^x (y-t, R, w) + x(R) + t + \theta h(r^*+\theta)w \]

\[(50) \ 0 = S(y-t, w) + r^* \left[1 - h (r^*+\theta)\right] - \theta h(r^*+\theta)w \]

We will assume that the government always operates in the range of inflation taxes smaller than the revenue maximizing one. That is, an increase in the rate of money growth will lead to a rise in fiscal revenue; otherwise the government could increase its income by simply printing less money each period. This assumption will be referred to as the "seigniorage" condition, which requires that the elasticity of money demand with respect to the rate of money growth be less than one in absolute value; thus

\[(51) \ -\eta_{m\theta} = -\theta h'(r^* + \theta)/h (r^* + \theta) < 1 \]

To figure out how the CA=0 locus shifts when \(\theta\) goes up it is necessary to determine both the effect of the increase in \(\theta\) on the CA
and the adequate wealth response to maintain CA equilibrium.

We start by noticing that

(52) \( \partial f / \partial w = S_w - \theta h(r^* + \theta) + \left[ 1 - h(r^* + \theta) \right] r^* \)

which is negative under the stability condition.

With respect to the effect of \( \theta \) on the CA we can determine that

(53) \( \partial f / \partial \theta = -r^* h'(r^* + \theta) w - h(r^* + \theta) w - \theta h'(r^* + \theta) w \)

which is an ambiguous expression opening up two possible cases:

(A) The increase in \( \theta \) may provoke a CA surplus if the portfolio effect away from money in the private sector (which also adversely affects government spending by decreasing the inflation-tax base) is more powerful than the direct effect of the \( \theta \) increase in government dis-savings. This amounts to

(54) \( \partial f / \partial \theta > 0 \implies -(r^* + \theta) h'(r^* + \theta) / h(r^* + \theta) > 1 \)

Combining (54) with the seigniorage equation (51) we arrive to the following condition for the elasticity of money demand with respect to \( \theta \)

(55) \( 1 + r^* h'(r^* + \theta) / h(r^* + \theta) < | m_{\theta} | < 1 \)

If this is the case the CA=0 locus will shift to the right; since the yy schedule unambiguously moves downwards the new equilibrium will involve a higher level of wealth and a lower R, as shown in figure 5A. Thus, portfolio composition and level effects operate in the same direction: towards a higher level of the foreign asset.
(B) The second possibility is that the higher $\theta$ produces a deficit in the CA, which arises if the direct effect of higher government dissavings dominates; that is

$$\frac{\partial f}{\partial \theta} < 0 \implies -(r^* + \theta)h'(r^* + \theta)/h(r^* + \theta) < 1$$

With equation (56) the seigniorage condition (51) -which also has to be met- becomes nonbinding and we can rewrite (56) as

$$|\eta_{m\theta}| < 1 + r^*h'(r^* + \theta)/h(r^* + \theta)$$

Under (57) the CA=0 locus will shift to the left since a lower level of wealth is required to increase savings and thus to equilibrate the CA. In the new steady state wealth will unambiguously be lower; with respect to $R$, it will be higher if at the initial $R$ the wealth decline necessary to balance the CA is higher than the one needed to clear the goods market, and vice versa. In figure 5B we show the case in which $R$ goes down. We notice that now portfolio composition and level effects operate in opposite ways, rendering ambiguous the effect on the CA.

In comparing equations (55) and (57) it is apparent that in case (A) money demand is more inelastic than in (B). Or, alternatively, case (A) corresponds to higher asset substitutability than (B).

4.3. Dynamics

To study dynamics in this framework we initially concentrate on the $f=0$ schedule. It is possible to show that both (45) and (46) are positive, and so that the $f=0$ locus is downward sloping, if the rate of money growth is not too high. This assumption receives support from our
seigniorage condition because too high a $\theta$ will make the economy cross the revenue maximizing point of the inflation tax\(^5\).

With respect to the $R=0$ locus any slope seems plausible, giving rise to the two possible configurations shown in figure 4. Therefore, the dynamics under deficit financing are analogous as those obtained for direct tax financing.

A deficit expansion shifts the $f=0$ locus to the left. The movement in the $R=0$ schedule is this time ambiguous, opening up two possible cases:

(i) If asset substitutability is high the $R=0$ locus will shift to the right, as presented in figure 6. In both cases (6A and 6B) the final level of $R$ is clearly lower; with respect to the CA, even though a surplus is the likely outcome, a deficit during transition can not be ruled out.

(ii) If asset substitutability is low, the $R=0$ schedule will shift to the left, as in figure 7, opening up the possibility of a higher $R$ in the new steady state. This time the CA is likely to present a deficit.

More interesting is, perhaps, the instantaneous effect of the deficit on $R$. If asset substitutability is low, an impact $R$ appreciation is the only possible outcome, fully agreeing with the result obtained for a balanced budget expansion. To explain this result we will

\(^5\)Nevertheless, it is not possible to show that the seigniorage condition unambiguously implies a positive sign for (45) and (46).
take a slightly different perspective than in section 3.3, centering now on asset markets; since the temporary equilibrium involves both goods and assets market clearing at every point in time, the perspectives are not only compatible but also complementary. Let us assume that \( R \) depreciates after the fiscal expansion; to equilibrate the goods market wealth necessarily has to fall on impact. But higher \( R \) and lower \( W \) requires an increase in the return of the foreign asset to clear this market, since \( f \) and \( r^* \) are given. Now, if assets are weakly responsive to the rate of return, the required upward jump in \((E/E)\) to clear the foreign bond market will exceed the possible range of outcomes given by the parameters of the model and the saddle path adjustment of the economy. Therefore, under low asset substitutability \( R \) will have to appreciate on impact.

Through time rationality assures that no more jumps will occur and the perfect foresight path indicates once more an inverse movement in \( CA \) and \( R \).

The ambiguities present in our results are precisely the outcome of having interdependent fiscal and monetary policies under perfect foresight. Indeed, if the two policies are independent and the fiscal budget is totally financed with taxes on labor income (i.e. no deficits), the modified dynamic system is now described by

\[
\dot{f} = \frac{r^*}{R} \left\{ S \{ w(R, t), y-t \} + Rf \right\}
\]
\[ R = \phi \left[ J \left( \frac{w(R,t)-Rf/r^*}{Rf/r^*} \right) - S \left( \frac{w(R,t),y-t+Rf}{w(R,t)-Rf/r^*} \right) -(\theta + r^*) \right] \]

The Jacobian of (14') and (24") is a slightly modified version of matrix \( Z \) and the new system is saddle-path stable provided condition (48) is met.

If the authority announces a higher rate of money growth maintaining a passive fiscal policy, the \( R=0 \) locus will unambiguously shift to the right under the two possible configurations of the phase diagram and the \( \dot{f}=0 \) schedule will not be affected, as shown in figure 8.

In both cases the impact effect is an instantaneous depreciation of \( R \), which necessarily requires a fall in wealth to maintain equilibrium in the goods market; for given \( f \) and \( r^* \), real money balances will have to fall and \( (\dot{E}/E) \) will go up to clear asset markets. During transition the CA is in surplus, wealth rises and \( R \) falls. The long run equilibrium involves a lower \( R \) and a higher \( f \).

Intuition for the ambiguous CA movement in the inter-dependent-policies scenario may be enlightened by the above exercise. A higher rate of money growth had contrasting effects on the CA in that framework; on the one hand, government dissaving goes up (allowing for the seigniorage condition) but on the other hand there are changes in the level and composition of wealth of private agents. The first effect tends to produce a CA deficit while the latter one pushes the economy in the direction of a CA surplus. The final outcome will obviously rely on
the relative strength of these two forces, which in turn depends on the substitutability between assets. If money demand has a low response to changes on its opportunity cost, for a given increase in $\theta$, government dissaving will go up by more and the portfolio effect on agents will be weaker in comparison to the case of a high responsiveness of real balances. In this situation a CA deficit will arise, and vice versa when asset substitutability is high.

But when fiscal and monetary policy are independent, an increase in $\theta$ necessarily produces a CA surplus because now government dissavings are nonexistent and the only remaining effect on the CA is the portfolio shift. This result of an unambiguous CA surplus following the increase in the rate of money growth is rather unusual in the literature, but agrees with Calvo and Rodriguez's (1977) model in which tradeable and nontradeable goods are both produced and consumed at home and the asset market configuration is similar, but not the same, as ours.

5. Conclusion

This model is set in a modified Mundell-Dornbusch-Fischer scenario where the government has been explicitly modelled. The inclusion of a fiscal sector which finances its expenses by direct taxes and money creation has important implications for the specification of goods demands and the CA. This interdependence between fiscal and monetary policy has received support in the literature dealing with semi-industrialized countries. Portfolio aspects - a crucial feature of the
model - are introduced through asset demands dependent on return and wealth as in Sidrauski and Dornbusch (1975).

The model is shown to present saddle path stability if a condition consistent with some previous literature (i.e. Sachs and Wyplosz (1983), Blanchard (1983)) is met. It is also shown that deficit financing enhances the perspective for stability.

A balanced budget expansion causes an ambiguous impact effect on R which crucially depends on asset substitutability. In particular, if money and the foreign bond are poor substitutes R necessarily appreciates. Through time the economy is likely to face a CA deficit coupled with R depreciation. In the long run wealth is lower and conditions are discussed under which the level of R is higher and the holdings of the foreign asset are lower.

Therefore, and in contrast to Kouri (1976), the impact effect of the fiscal action under rational expectations does not necessarily go in the same direction as the long run effect. With low asset substitutability the economy will experience in this scenario a short run R appreciation coupled with a long run depreciation.

If deficit financing is considered, not only the impact effect but also steady state results remain crucially dependent on asset substitutability. Indeed, when money holdings are responsive to changes on their opportunity cost, the strength of portfolio composition effects will drive the CA to a surplus and R will appreciate in the long run. Regarding the impact effect, if assets are poor substitutes R may per-
fectly depreciate, as was the case under a balanced budget expansion. Once again, short run results not necessarily predict well steady state effects.

However, if fiscal and monetary policies are treated as independent all the ambiguities go away. An announced increase in the rate of money expansion causes a short run R depreciation matched by long run appreciation and CA surplus during transition. The new steady state involves a lower R and a higher level of the foreign asset.

This unambiguous result is explained by recalling that a higher rate of money growth does not feed into government dissaving in this context and thus it affects the CA only through portfolio changes. Since in the long run wealth is necessarily higher, substitution and wealth effects go in the same direction: towards higher holdings of foreign assets. In contrast, an increase in the rate of money growth in the interdependent-policies framework has contradictory effects on the CA. On the one hand it increases fiscal dissaving tending to worsen the CA; on the other hand it produces a portfolio shift, leading the CA to a surplus. Which effect dominates will depend on the magnitude of asset substitutability and thus the long run outcome on wealth and R rests ambiguous.

Finally, and perhaps needless to say at this point, Mundell's clear-cut result of R appreciation and CA deficit following a fiscal expansion no longer holds in a portfolio model where rational expectations and money financed fiscal deficits are key features.
Figure 5

A

\[ R \]

\[ \text{CA} = 0 \quad \text{CA}' = 0 \]

\[ \text{E} \]

\[ \text{E}' \]

\[ Y \]

\[ W \]

B

\[ R \]

\[ \text{CA} = 0 \quad \text{CA}' = 0 \]

\[ \text{E} \]

\[ \text{E}' \]

\[ Y \]

\[ W \]

Figure 6

A

\[ R \]

\[ \dot{R} = 0 \quad \dot{R}' = 0 \]

\[ \text{E} \]

\[ \text{E}' \]

\[ \dot{f} = 0 \quad \dot{f}' = 0 \]

\[ f \]

B

\[ R \]

\[ \dot{R} = 0 \quad \dot{R}' = 0 \]

\[ \text{E} \]

\[ \text{E}' \]

\[ \dot{f} = 0 \quad \dot{f}' = 0 \]

\[ f \]
REFERENCES


