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# Massive 3d gravity and Bigravity

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**Abstract.** We review some results concerning bigravity and multigravity. Bigravity was considered for the first time by Isham, Salam and Strathdee almost 40 years ago. We update some results and in particular consider the three-dimensional version which has some interesting features.

## 1. Introduction

In this contribution we shall discuss several properties of multigravity. The first example of a multigravity theory was considered almost 40 years ago by Isham-Salam and Strathdee [1]. Their action has two metrics  $g_{(1)}$  and  $g_{(2)}$  and has the form

$$I[g_1, g_2] = \int [\sqrt{g_1}R_1 + \sqrt{g_2}R_2 - V(g_1, g_2) + L(g_1, g_2, \text{matter})] \quad (1)$$

Let us discuss some properties and motivations to consider this action and its generalizations with  $N$  metrics.

- The multigravity version,  $g_1, g_2, g_3, \dots, g_N$ , is the natural (but incomplete) “spin 2 Yang-Mills” theory.
- For the particular action (1), the 4d linear spectrum contains a massive and a massless gravitons. Thus, this action provides a covariant action for a massive graviton.
- The action (1) can be written in any spacetime dimension  $d$ . At  $d = 3$ , the spectrum contains a single massive graviton (the massless particle is trivial). This provides more challenges for AdS<sub>3</sub>/CFT<sub>2</sub>. At the same time provides yet another action for a massive graviton in three dimensions besides TMG [2] and NMG [3, 4] (see also [5]).
- As other alternative theories, bigravity has interesting cosmological applications in cosmology. The second metric can be seen as a ‘matter field’ providing an alternative to dark matter [6, 7, 8].

Bigravity has received intermittent but consistent attention since it was first presented. For some recent work see [9, 10, 11, 12, 13] (an incomplete list!).

The original work presented here was developed in several collaborations, in particular, [14, 8, 7, 15].

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An important property of multigravity is the fact that an interaction potential preserving the full  $diff_d^N$  invariance does not exist [16]. Naively, one could motivate multigravity as an analogy with Yang-Mills theory extending the action of one Maxwell field to  $N$  fields,

$$\int F_{\mu\nu} F^{\mu\nu} \rightarrow \int F_{1\mu\nu} F_1^{\mu\nu} + F_{2\mu\nu} F_2^{\mu\nu} + \dots + \mathbf{interactions}$$

It is extremely tempting to do the same with the spin two field:

$$\int \sqrt{g}(R - 2\Lambda) \rightarrow \int \sqrt{g_1}(R_1 - 2\Lambda_1) + \sqrt{g_2}(R_2 - 2\Lambda_2) + \dots + \mathbf{interactions}$$

However, for spin 2, the interactions do not have the same nice structure. The  $N$ -Gravity action functional

$$I[g_{\mu\nu}^{(a)}] = \sum_{a=1}^N \int \sqrt{g^{(a)}} R(g^{(a)}) \quad (2)$$

is invariant under independent transformations on each metric

$$\delta g_{\mu\nu}^{(a)} = \partial_\mu \xi_{(a)}^\alpha g_{\alpha\nu}^{(a)} + \partial_\nu \xi_{(a)}^\alpha g_{\mu\alpha}^{(a)} + \xi_{(a)}^\alpha \partial_\alpha g_{\mu\nu}^{(a)}, \quad a = 1 \dots N. \quad (3)$$

However, no interaction between the different metrics exists that preserves this full diff invariance. Only the diagonal subgroup can be preserved. This is a perturbative theorem. For weak fields,

$$g_{\mu\nu}^{(a)} = \eta_{\mu\nu} + h_{\mu\nu}^{(a)} \quad (4)$$

no cross potential  $V(h_{\mu\nu}^{(a)})$  preserving the full  $N$  dimensional symmetry exists.

Now, the no-go theorem says nothing about interactions preserving the diagonal subgroup. We shall consider the Isham-Salam-Strathdee family ( $g_1 = g, g_2 = f$ ),

$$\begin{aligned} I[g_{\mu\nu}, f_{\mu\nu}] = & \frac{1}{16\pi G} \int \left[ \sqrt{g}(R_g - 2\Lambda) + \sigma \sqrt{f}(R_f - 2\Lambda) \right. \\ & + \frac{\nu}{4} \sqrt{f}(g_{\mu\nu} - f_{\mu\nu})(g_{\alpha\beta} - f_{\alpha\beta})(f^{\mu\alpha} f^{\nu\beta} - f^{\mu\nu} f^{\alpha\beta}) \\ & \left. + \mathcal{L}_m(g_{\mu\nu} + \sigma f_{\mu\nu}, \Phi) \right] \quad (5) \end{aligned}$$

Observe the particular coupling to the matter Lagrangian. We shall not consider a matter lagrangian for each metric, as often done in the literature. We shall argue below that the coupling of normal matter to  $g_{\mu\nu} + \sigma f_{\mu\nu}$  is the most natural choice. The point is that  $g_{\mu\nu} + \sigma f_{\mu\nu}$  describes a massless graviton (see below) and therefore masses are still attracted by the action of spin 2 massless particle in this theory, just as in ordinary gravity.

Any modification of general relativity must have a regime where it reproduces Einstein gravity. The theory described by the action (5) has a phase in which is identical to GR. The equations of motion that extremize (5) ( $8\pi G = 1$ ) are,

$$\begin{aligned} G_{\mu\nu}(g) + \Lambda g_{\mu\nu} &= \nu(g - f)_{\mu\nu} + T_{\mu\nu} \\ \sigma [G_{\mu\nu}(f) + \Lambda f_{\mu\nu}] &= -\nu(g - f)_{\mu\nu} + \sigma T_{\mu\nu} \end{aligned}$$

where  $(g - f)_{\mu\nu}$  is a complicated combination of  $g_{\mu\nu}, f_{\mu\nu}$ . The important point here is that this term vanishes at  $f_{\mu\nu} = g_{\mu\nu}$ .

Thanks to the particular matter coupling to  $g_{\mu\nu} + \sigma f_{\mu\nu}$  (massless mode), these equations admit a class of solutions with

$$f_{\mu\nu} = g_{\mu\nu} \quad (6)$$

where  $g_{\mu\nu}$  is any solution to Einstein equations ( $8\pi G = 1$ ),

$$G_{\mu\nu} = T_{\mu\nu}. \quad (7)$$

We conclude that any solution  $g_{\mu\nu}$  to Einstein equations can be uplifted to be a solution of bigravity by choosing  $f_{\mu\nu} = g_{\mu\nu}$ . This is perhaps not too relevant. What is more important is the converse statement. All solutions to bigravity are of the form  $f_{\mu\nu} = g_{\mu\nu}$  plus a short range massive fluctuation.

This can be seen as follows. Consider the linear fluctuations around  $f_{\mu\nu} = g_{\mu\nu} = \bar{g}_{\mu\nu}$

$$g_{\mu\nu} = \bar{g}_{\mu\nu} + h'_{\mu\nu}, \quad f_{\mu\nu} = \bar{g}_{\mu\nu} + k'_{\mu\nu}. \quad (8)$$

The Isham-Salam-Strathdee action (5) becomes

$$I_{\text{linear}} \simeq \int [h^{\mu\nu} \mathcal{G}(h)_{\mu\nu} + h^{\mu\nu} T_{\mu\nu}] + \left[ k^{\mu\nu} \mathcal{G}(k)_{\mu\nu} - \frac{\nu}{4\sigma} (k^{\mu\nu} k_{\mu\nu} - k^2) \right] \quad (9)$$

where  $h_{\mu\nu}$  and  $k_{\mu\nu}$  are linear combinations of the fluctuations in (8)

$$h_{\mu\nu} = \frac{1}{1+\sigma} (h'_{\mu\nu} + \sigma k'_{\mu\nu}), \quad k_{\mu\nu} = \frac{1}{1+\sigma} (h'_{\mu\nu} - k'_{\mu\nu}). \quad (10)$$

and  $\mathcal{G}$  is the Pauli-Fierz operator.  $h_{\mu\nu}$  describes a massless graviton and is coupled to matter. On the other hand  $k_{\mu\nu}$  is the massive mode. Both are decoupled. Both actions are unitary and stable. Summarizing:

- $f_{\mu\nu} - g_{\mu\nu} \simeq k_{\mu\nu}$ =massive mode, short range. Thus,  $f_{\mu\nu} = g_{\mu\nu}$  to any desired scale, and we recover Einstein gravity coupled to matter.
- $g_{\mu\nu} + \sigma f_{\mu\nu} = \bar{g}_{\mu\nu} + h_{\mu\nu}$ =massless mode coupled to matter.

## 2. Bigravity and Cosmology

Linear bigravity is equivalent to Einstein gravity, plus a massive short-range decoupled graviton. Let us explore now other non-linear aspects of this theory. We shall see that an alternative to dark matter emerges.

In cosmology, the Universe goes through several phases: Radiation, matter, acceleration. In a FRW model,

$$ds^2 = -dt^2 + a(t)^2 d\vec{x} \cdot d\vec{x} \quad (11)$$

the dynamics of  $a(t)$  is given by the Friedmann equation,

$$\frac{\dot{a}^2}{a^2} = \frac{\Omega_r}{a^4} + \frac{\Omega_m}{a^3} + \Omega_\Lambda \quad (12)$$

The dark matter mystery is that fact that the observed baryonic  $\Omega_m$  is smaller than expected from Universe dynamics and CMB observations.

Let us show that bigravity contributes exactly at order  $\frac{1}{a^3}$  to the Friedman equation. Consider the homogeneous, isotropic ansatz in the bigravity theory,

$$\begin{aligned} ds^2 &= -dt^2 + a(t)^2 d\vec{x}^2, \\ df^2 &= -X(t)^2 dt^2 + Y(t)^2 d\vec{x}^2. \end{aligned}$$

Note that while  $g_{tt} = -1$   $X(t)$  must be left arbitrary in order to have a solution

The equation for  $a(t)$  (Friedmann equation) is

$$\frac{\dot{a}^2}{a^2} = \frac{\nu Y^3/X}{a^3} + \frac{\Omega_{baryons}}{a^3} + \frac{\Omega_r}{a^4} + \Omega_\Lambda \tag{13}$$

plus two equations for  $X(t)$  and  $Y(t)$  that we omit. See [6, 7, 8] for details. A remarkable property of bigravity is that near  $a = 0$  the combination  $\nu Y^3/X$  is constant. Thus we can set

$$\nu Y^3/X = \Omega_{dark\ matter}$$

and the contribution from  $f_{\mu\nu}$  to the Friedmann equation is just like pressureless matter; dark matter!

The dark matter interpretation can be taken much further. Fluctuations around Friedmann Universe can be introduced as

$$\begin{aligned} ds^2 &= -(1 + 2\Psi)dt^2 + a(t)^2(1 - 2\Phi)\delta_{ij}dx^i dx^j \\ df^2 &= -X(t)^2(1 + 2\Xi)dt^2 - Y^2\partial_i\beta dt dx^i \\ &\quad + Y(t)^2[(1 - 2\chi)\delta_{ij} + \partial_i\partial_j\mu]dx^i dx^j \end{aligned}$$

and have the expected properties for dark matter. See [8] for details (the action considered in this reference is slightly different from (5)).

### 3. Bigravity and black holes

The Schwarzschild-dS metric is a solution to the full set of equations

$$ds^2 = -\left(1 - \frac{2M}{r} - \Lambda r^2\right) dt^2 + \frac{dr^2}{1 - \frac{2M}{r} - \Lambda r^2} + r^2 d\Omega^2$$

provided we choose the second metric to be equal to  $g_{\mu\nu}$ . Thus, the action (5) does have black holes. A natural question is whether these are the most general set of black holes. The answer, first discussed in [17], is negative and an interesting family of other black holes does exist. The following metric is an exact solution to the action (5) [17]

$$\begin{aligned} ds^2 &= -\left(1 - \frac{2M}{r} + \Lambda_1 r^2\right) dt^2 + \frac{dr^2}{1 - \frac{2M}{r} + \Lambda_1 r^2} + r^2 d\Omega^2 \\ df^2 &= -\left(1 - \frac{2\mu}{r} + \Lambda_2 r^2\right) dt'^2 + \frac{dr^2}{1 - \frac{2\mu}{r} + \Lambda_2 r^2} + r^2 d\Omega^2 \end{aligned}$$

with  $t' = t + \xi(r)$  where  $\xi(r)$  is a function of  $r$  only (see [17] for the full metric).

- $\Lambda_1$  and  $\Lambda_2$  are integration constants. Only one combination of them is related to action couplings.
- The mass parameters  $M$  and  $\mu$  are independent integration constants.

The thermodynamical properties, the Kerr and Kerr-Newman extensions of these black holes are not known.

Some (modest) progress can be achieved in three dimensions. The full rotating metric analogous to the Isham-Storey black hole is given by [15]

$$ds^2 = -(-\Lambda r^2 - M)dt^2 + \frac{dr^2}{-\Lambda r^2 - M + \frac{J^2}{4r^2}} + Jdt d\phi + r^2 d\phi^2 \quad (14)$$

and  $df^2 = f_{\mu\nu} dx^\mu dx^\nu$  is given by:

$$df^2 = -\left(\frac{2r^2}{\lambda^2 \ell^2} - M_f\right) dt^2 + \frac{8r^2 \ell^2 (2r^4 \lambda^2 + 2r^2 (M_f \lambda^2 \ell^2 - \ell^2 M_g \lambda^2 - 2M_g \ell^2) + \ell^2 J_g^2) dr^2}{\lambda^2 (4r^4 - 4r^2 M_g \ell^2 + \ell^2 J_g^2)^2} + r^2 d\varphi^2 \\ + J_g dt d\varphi - \frac{4\ell r \sqrt{(r^2(2-\lambda^2) + \lambda^2 \ell^2 (M_g - M_f))(4r^2(2M_g - M_f \lambda^2) + J_g^2(\lambda^2 - 2))} dt dr}{\lambda^2 (4r^4 - 4r^2 M_g \ell^2 + J_g^2 \ell^2)}$$

where

$$2\nu\lambda^2 + 4\sigma\lambda\nu + \nu + \sigma = 0 \quad (15)$$

Both  $g_{\mu\nu}$  and  $f_{\mu\nu}$  have constant curvature (just like in NMG),

$$R(g)^{\mu\nu}{}_{\alpha\beta} = \Lambda_g \delta_{[\alpha\beta]}^{[\mu\nu]} \\ R(f)^{\mu\nu}{}_{\alpha\beta} = \Lambda_f \delta_{[\alpha\beta]}^{[\mu\nu]}$$

Both black holes differ from AdS only in their global properties. Again, the thermodynamical properties of these black holes are not known. Since the curvature of black holes is constant, this seems to indicate that the full solution can be build as identifications along some Killing vector. Isolating the particular vector is an interesting problem. One may also wonder whether the Cardy formula works in this context [18].

The asymptotic structure of the bigravity black holes present some interesting features. The  $r \rightarrow \infty$  leading terms in both metrics are

$$ds^2 \simeq -\frac{r^2}{\ell^2} dt^2 + \frac{\ell^2 dr^2}{r^2} + r^2 d\varphi^2 \\ df^2 \simeq \frac{1}{2} \left( -\frac{2}{\lambda^2} \frac{r^2}{\ell^2} dt^2 + \frac{\ell^2 dr^2}{r^2} + r^2 d\varphi^2 \right).$$

Thus, both metrics are asymptotically AdS, but with different “speeds of light”. Each metric has 6 Killing vectors but 4 are common to both: time translations  $t \rightarrow t + \epsilon$ , and  $SL(2, \mathfrak{R})$  isometries of Euclidean 2-dimensional  $AdS_2$ :  $SL(2, \mathfrak{R}) \times \mathfrak{R}$ . This is quite similar to the solutions discussed in [19]. A critical case occurs for

$$\lambda^2 = 2$$

where the full asymptotic  $SO(2, 2)$  symmetry is restored. A natural question in the context of 3d gravity is whether this group can be extended to the conformal group.

#### 4. $\lambda^2 = 2$ and Brown-Henneaux symmetry in 3d bigravity

Brown and Henneaux [20] discovered that the phase space of 3d AdS gravity is classified by an infinite number of charges  $L_n, \bar{L}_n$  satisfying the Virasoro algebra with central charge

$$c = \frac{3\ell}{2G}. \quad (16)$$

The goal of this section is to extend the results of Brown-Henneaux to bigravity. Let us first briefly review the results of [20].

All solutions to 3d gravity are diffeomorphic to AdS space

$$ds^2 = - \left(1 + r^2\right) dt^2 + \frac{dr^2}{1 + r^2} + r^2 d\varphi^2. \quad (17)$$

This statement seems to contradict the existence of an infinite number of charges classifying solutions. What happens is that, on open spaces, not all diffeomorphisms are un-physical. The Brown-Henneaux charges are asymptotic charges associated to non-trivial coordinate changes.

The distinction between trivial “un-physical” and non-trivial “physical” changes of coordinates was systematized by Regge and Teitelboim [21]. The transformations  $x'^{\mu} = x^{\mu} + \epsilon^{\mu}(x)$  are generated by the Hamiltonian constraint plus a boundary term

$$H[\epsilon] = \int_{\Sigma} \epsilon^{\mu} \mathcal{H}_{\mu} + \int_{\partial\Sigma} \epsilon^{\mu} q_{\mu}; \quad \mathcal{H}_{\mu} = 0, \quad q_{\mu} \neq 0 \quad (18)$$

- (i) If the boundary term vanishes,  $\int_{\partial\Sigma} \epsilon^{\mu} q_{\mu} = 0$ , then one has a true “gauge transformation” generated by a constraint.
- (ii) Conversely, if  $\int_{\partial\Sigma} \epsilon^{\mu} q_{\mu} \neq 0$  one has a fake “gauge transformation” not generated by a constraint. It does change the physical state.  $Q \equiv \int_{\partial\Sigma} \epsilon^{\mu} q_{\mu}$  is the Noether charge associated to the non-trivial transformation.

The set of “physical” coordinate changes depends on boundary conditions and is in general a non-trivial task to isolate them. We shall consider here a simple example that exhibits the richness of the Brown-Henneaux calculation.

Consider the following three-dimensional metric

$$ds^2 = r^2 dz d\bar{z} + \frac{dr^2}{r^2} + \underbrace{T(z) dz^2 + \bar{T}(\bar{z}) d\bar{z}^2}_{\dots},$$

$T$  and  $\bar{T}$  are ‘fluctuations’ on the background given by the first two terms. The background is just AdS<sub>3</sub> space (17) in Poincare (light-like) coordinates. The asymptotic Einstein equations impose on the fluctuations the conditions

$$\bar{\partial}T = 0, \quad \partial\bar{T} = 0.$$

Consider the set of coordinate changes that preserve that background (but allowing the fluctuations  $T$  and  $\bar{T}$  to transform). The non-trivial, unphysical, coordinate changes are conformal transformations

$$z \rightarrow f(z), \quad \bar{z} \rightarrow g(\bar{z}).$$

As a concrete example, consider the chiral case ( $\bar{T} = 0$ ) and act with the coordinate change  $z = f(z')$

$$\begin{aligned} ds^2 &= r^2 dz d\bar{z} + \frac{dr^2}{r^2} + T(z) dz^2 \\ &= r^2 \partial' f dz' d\bar{z} + \frac{dr^2}{r^2} + T(z) (\partial' f)^2 dz'^2 \end{aligned}$$

We can eliminate the Jacobian  $\partial' f$  by doing

$$r^2 \partial' f = r'^2, \quad \bar{z} = \bar{z}' - \frac{1}{2r'^2} \frac{\partial'^2 f}{\partial' f}.$$

and go back to exactly the metric we started from

$$ds^2 = r'^2 dz' d\bar{z}' + \frac{dr'^2}{r'^2} + T'(z') dz'^2,$$

where –note the Schwarz derivative–

$$T'(z') = T(z) (\partial' f)^2 - \frac{1}{2} \{f, z'\}.$$

This would all be meaningless –just a coordinate redefinition– if it wasn't for the fact that  $z = f(z')$  is not trivial. The actual value of the central charge needs a bit more work. The formula  $\delta T = [T, T]$ , also proved by Brown-Henneaux [22], normalizes  $T$  and one finds,

$$c = \frac{3\ell}{2G}. \quad (19)$$

This result became strongly entangled with black hole thermodynamics after the important papers [18, 23]. The idea is as follows. The black hole is asymptotically anti-de Sitter with  $\ell MG = L_0 + \bar{L}_0$ ,  $JG = L_0 - \bar{L}_0$ . Due to the AdS/CFT correspondence one expects a CFT<sub>2</sub> with a central charge equal to (19) (up to sub-leading quantum corrections). Next, for **any** unitary modular invariant 2d CFT, the number of states consistent with given (large)  $L_0, \bar{L}_0$  is (Cardy formula)

$$\rho(T, \bar{T}) = e^{2\pi\sqrt{\frac{c}{6}L_0 + 2\pi\sqrt{\frac{c}{6}\bar{L}_0}}.$$

Plugging the black hole values for  $L_0, \bar{L}_0$  and obtain exactly Bekenstein-Hawking entropy [18, 23].

$$\rho(M, J) = e^{\frac{2\pi r_+}{4G}}.$$

We would like now to extend the Brown-Henneaux analysis to bigravity. The details of this problem can be found in [15]. The bigravity Brown-Henneaux asymptotic solution is,

$$\begin{aligned} ds^2 &\sim \frac{dr^2}{r^2} + r^2 dz d\bar{z} + T(z) dz^2 + \bar{T}(\bar{z}) d\bar{z}^2 \\ df^2 &\sim \frac{1}{2} \left( \frac{dr^2}{r^2} + r^2 dz d\bar{z} + Q(z) dz^2 + \bar{Q}(\bar{z}) d\bar{z}^2 + \frac{dr}{r} (P(z) dz + \bar{P}(\bar{z}) d\bar{z}) \right) \end{aligned}$$

where the Einstein  $\times$  Einstein equations imply

$$\bar{\partial}T = \bar{\partial}Q = \bar{\partial}P = 0, \quad \partial\bar{T} = \partial\bar{Q} = \partial\bar{P} = 0,$$

Note that  $P(z)$  and  $\bar{P}(\bar{z})$  can be set to zero by a *trivial* coordinate redefinition, but not simultaneously in both metrics. In this sense  $P, \bar{P}$  are physical. Furthermore, due to the  $1/r$  slow fall-off they appear in the charges.

The set of non-trivial coordinate transformations that preserve the asymptotic conditions and have non-zero charges are conformal maps  $z \rightarrow f(z) \simeq z + \epsilon(z)$ . The metric fluctuations transform as

$$\begin{aligned} \delta Q &= -\epsilon \partial Q - 2\partial\epsilon Q + \frac{1}{2} \partial^3 \epsilon + P \partial^2 \epsilon \\ \delta P &= -\epsilon \partial P - \partial\epsilon P \end{aligned}$$

and the same for the  $\bar{z}$ - sector.



Note that  $P$  is (1,0) conformal field and  $\partial P$  is (2,0), as expected. On the other hand  $Q$  is almost a (2, 0) field. Interestingly the combination  $Q + \partial P$  is a (2, 0) field.

We can now compute the charges associated to these transformations and confirm that they are finite. Since the potential in the action does not have derivatives, the charges are the sum of two ADM functionals, one from each metric. The “holomorphic” charge is

$$\begin{aligned} Q &= Q_g + \sigma Q_f \\ &= \overbrace{T(z)} + \overbrace{Q(z) + \partial P(z)}, \end{aligned}$$

The total central charge is (computed by usual methods)

$$c = \frac{3\ell}{2G} \frac{1}{\sqrt{1 + 2\sqrt{2}\nu}} \left(1 + \frac{\sigma}{\sqrt{2}}\right) \quad (20)$$

Now, recall that the action (5) had another AdS background with  $g_{\mu\nu} = f_{\mu\nu}$ . (Observe the factor  $\frac{1}{2}$  in  $f_{\mu\nu}$  above: the black holes displayed here are not fluctuations on the background with  $f_{\mu\nu} = g_{\mu\nu}$ .) The central charge on the “strongly coupled” background with  $f_{\mu\nu} = g_{\mu\nu}$  is

$$c_0 = \frac{3\ell}{2G}(1 + \sigma). \quad (21)$$

In the whole range of (allowed) parameters it is observed that

$$c_0 > c. \quad (22)$$

Thus, there might be a flow interpolating these two CFT’s.

## 5. Conclusions and open problems

Multi-gravity is an interesting theory highly unexplored and full of open problems. I hope I have succeed in this Contribution to motivate this theory and explain its main features. I would like to end by mentioning some open problems.

- (i) Does the Cardy formula apply in three-dimensional bigravity?
- (ii) What is the entropy and temperature for bigravity black holes?
- (iii) Can one classify solutions with spherical symmetry? Birkhoff theorem?
- (iv) How different is New Massive gravity from bigravity ( $d = 3$ )
- (v) “Chiral” gravity. At  $d = 3$  the choice

$$\int \sqrt{g}R(g) - \sqrt{f}R(f)$$

gives a massive vector. The central charges are zero, and the dual theory should be a log CFT. Note that an action of this type has recently been considered in [24].

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