VISCOUS DAMPER OPTIMIZATION IN MULTISTOREY BUILDING STRUCTURES

FELIPE SAITUA

Thesis submitted to the Office of Research and Graduate Studies in partial fulfillment of the requirements for the Degree of Master of Science in Engineering

Advisor:
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Santiago de Chile, June, 2017

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To my family
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RESUMEN

En esta investigación se estudia la distribución óptima en altura de disipadores viscosos instalados en edificios. La excitación sísmica es modelada como un proceso estocástico (ruído blanco filtrado) y las estadísticas de la respuesta estructural son obtenidas a través de un análisis en formulación espacio-estado. Para aplicaciones que involucran dispositivos no lineales se aplican técnicas de linealización estadística. Tres consideraciones de diseño práctico son abordadas: (i) cuantificación realista del costo de los disipadores, basada en la capacidad de fuerza del disipador en vez de en su coeficiente de amortiguamiento; (ii) análisis de distintas configuraciones de soporte de los disipadores que permiten conectarlos entre pisos no consecutivos de manera de maximizar la disipación de energía; y (iii) incorporación explícita en la optimización del costo de reforzamiento de columnas que podría ser requerido para soportar las fuerzas inducidas por los disipadores a través del sistema de soporte. Cinco funciones objetivo son definidas para tratar estas consideraciones, todas cuantificadas en términos del costo de los disipadores. El impacto de la linealización estadística cuando las funciones objetivo incluyen valores extremos es analizado. El problema de diseño óptimo considera la respuesta estructural como restricción, exigiendo que un nivel de reducción de respuesta objetivo sea obtenido a través de la acción de los disipadores. Una extensión hacia una formulación de diseño multi-objetivo también es discutida. Este marco teórico es ilustrado con el diseño de un sistema de disipación viscosa aplicado a un edificio chileno real de 26 pisos, considerando una excitación sísmica compatible con la amenaza sísmica regional. Los resultados demuestran que la optimización que explícitamente minimiza funciones de costo deviene beneficios económicos sustanciales, y que disipadores conectados entre pisos no consecutivos contribuyen considerablemente a la reducción de costos del sistema de disipación.

Palabras clave: optimización de dispositivos viscosos; disipación de energía; excitación estocástica; estadísticas de segundo orden; reforzamiento de columnas; pisos no consecutivos
ABSTRACT

This research discusses the optimal height-wise distribution of viscous damper in multistorey structures. Seismic excitation is modeled as a stochastic process (filtered white noise), and response statistics for linear structural systems are obtained through state-space analysis. For applications involving nonlinear dampers statistical linearization principles are employed. Emphasis is placed on three practical design issues: (i) realistic quantification of damper upfront cost, based on damper force capacity rather than merely utilizing the damping coefficient; (ii) investigation of different bracing configuration schemes that allow dampers to be connected between non-consecutive storeys in order to maximize energy dissipation; and (iii) explicit incorporation in the optimization of the cost of column strengthening that might be required to accommodate the damper forces exerted on them through the bracing system. Five objective functions are defined to address these points, all quantified in terms of damper cost. The impact of the statistical linearization when the objective function includes peak response quantities is discussed. The optimal design problem considers primarily the structural performance as a constraint, requiring that a specific level of vibration suppression be achieved through the damper implementation. An extension to a multi-objective design setting is also discussed. The proposed framework is illustrated with the design of a supplemental viscous damping system for an actual Chilean 26-storey building, considering an excitation that is compatible with the regional seismic hazard. Results demonstrate that optimizing for cost leads to substantial economic benefits and that dampers connected across multiple storeys contribute to considerable cost reduction.

Keywords: viscous damper optimization; energy dissipation; stochastic excitation; second-order statistics; column strengthening; non-consecutive storeys
1. INTRODUCTION

Disasters caused by recent earthquakes have caught the attention of researchers and have urged them to develop practical and efficient strategies to overcome the effects on the built environment of these catastrophic events. In this context, structural engineers have played a crucial role developing new seismic protection devices that can drastically reduce structural demand to allowable levels and thus provide comfort and safety to users. Active, semi-active and passive devices have been designed and extensively tested, and nowadays there is a solid agreement on their benefits. Passive devices have been especially highlighted by the scientific community and professional practitioners; since there is no need of an external source of power, these devices are exceptionally reliable and usually cost-effective. Among them, fluid viscous dampers are of particular relevance as the modelling process is comparatively simple and there is a robust background of military and structural applications (on new and existing structures) that validate their effectiveness (Symans et al., 2008). The Torre Mayor Building in Mexico, the Virginia Power North Ana Nuclear Station in the United States and the Deloitte Building in Chile are just a few of many examples in which these devices have been successfully implemented.

Figure 1-1 shows a schematic diagram of a viscous damper device. When the piston rod moves from left to right, silicon-like fluid flows from the right chamber to the left one through small orifices in the piston head. In this passing of fluid energy is dissipated by heat due to friction between the liquid and the piston head. This movement of liquid implies a viscous force (due to pressure differential across the piston head) proportional to piston velocity, and a restoring force due to liquid compressibility. The latter force induces additional stiffness to the system which is usually undesirable, so the presence of the accumulator is needed. Experimental testing (Constantinou & Symans, 1992) have shown no measurable added stiffness below a cut-off frequency of 4Hz, which is sufficiently high for most structural engineering applications. Moreover, the existence
of this cut-off frequency is beneficial because in this manner dampers can provide only additional damping to the fundamental mode and additional damping and stiffness to higher modes, which usually suppress the influence of the latter modes on the response of the structure (Constantinou & Symans, 1993).

![Fluid Viscous Damper Device](source: Constantinou & Symans (1993))

Figure 1-1: fluid viscous damper device (source: Constantinou & Symans (1993))

One aspect of viscous dampers that requires special attention is the height-wise distribution of the devices along the storeys of the building in which they are installed. Unlike other devices such as base isolators or frictional pendulum systems, these dampers can be installed in practically any location of the structure in which their ends are exposed to sufficient relative velocity to engage in energy dissipation. The most common way to connect dampers to the structure is to attach one end at a given floor level and the other end at the next (either upwards or downwards) floor level, but many other configuration schemes can be implemented. Therefore, the effectiveness of a viscous damper system depends drastically on the way they are installed in terms of location.
Figure 1-2 presents a brief example of the previous claim. The same 12-storey building model of fundamental period equal to 1 second is equipped with viscous dampers installed according to four different height-wise distributions: explicit optimization distribution (i.e. posing a formal optimization problem to minimize a specific objective function), random distribution (assign damper location and size randomly), uniform distribution (all dampers are equal) and stiffness-proportional distribution (the size of a damper in a given storey is proportional to the lateral stiffness of that storey). In all cases the proposed distribution indicates the damping coefficient of each viscous damper ($c_{di}$) and as a way to compare distributions the total damping coefficient $C_o$ ($C_o = \sum c_{di}$) has been set equal to the same value. If the aim of the designer is to reduce average interstorey drift response then results indicate that explicitly optimizing the height-wise distribution leads to an average interstorey drift $f_{e^*}^o$. When comparing this value to what is obtained following random, uniform and stiffness proportional distributions a 17, 14 and 13% improvement utilizing the explicit solution is found, respectively. This simple verification indicates that effectiveness is indeed susceptible to how the damping coefficient of each damper is calculated and that there are different levels of sophistication from which a designer can determine optimal design. Differences on effectiveness are even more relevant when nonlinear or more complex quantities are considered for cost measurement or constraint definition, which is the case for instance of viscous damper upfront cost (i.e. the direct cost of purchasing a damper from the manufacturer).
Four different distribution schemes are considered motivated by this realization. Various researchers have focused on developing effective criteria to determine the best way to allocate to different locations/storeys the total damping coefficient (i.e., total damper capacity) for supplemental viscous damping system. Their efforts can be categorized mainly in three groups of distribution schemes. The first one consists of distributions that assign total damping coefficient based on predefined and simplified guidelines. In this group, the uniform, stiffness proportional, storey shear proportional and story shear strain energy distributions can be found (Hwang, Lin, & Wu, 2013; Landi, Conti, & Diotallevi, 2015; Whittle, Williams, Karavasilis, & Blakeborough, 2012). From all these alternatives, this work will further discuss the uniform distribution, since it is probably the most popular approach in professional practice. In a supplemental damping system of $n_d$ dampers this distribution indicates that the design vector $c_d^*$ is simply:

![Figure 1-2: sensitivity of fluid viscous damper performance to height-wise distribution.](image)
that is, every damping coefficient $c_{di}^*$ has the same value, which is the $n_d$-th part of the total $C_o$ value needed to reach the target performance. Consequently, no iterations or steps are needed to obtain the design vector once the total $C_o$ is decided. The choice of $C_o$ can be seen as an optimization problem itself; it is the selection the minimum possible value of $C_o$ such that the structure behaves as desired. Due to the simplicity, this task can be achieved by simple iterations of $C_o$ until the structure performance reaches the target level. This is the simplest method available and usually leads to dampers with low individual forces, but since there is no optimization of the height-wise distribution this comes at expense, as will be clearly shown later, of a higher total damper cost.

In the second group of distribution schemes, algorithms sequentially allocate a fraction of a predefined total damping capacity at a location where a certain performance index is maximized. In this group, the Sequential Search Algorithm (SSA), the Simplified Sequential Search Algorithm and an Energy-Based Sequential Algorithm (J. L. Lin, Bui, & Tsai, 2014; López-García, 2001; Zhang & Soong, 1992) are the most acknowledged methods. In this work, the SSA is used as a comparison method as involves a reasonable equilibrium between computational efficiency and simplicity. This allocation procedure finds its underlying philosophy on the Steepest Descent Method and can be summarized as following:

a) Select a $C_o$ value (based on some other performance criterion) and a $n$ value to define the unit damping coefficient value $c_{di} = C_o / n$.

b) Calculate the second-order response statistics of the structure without dampers.

c) Find the location where the root mean square interstorey drift response is maximum.

d) Allocate a damper of damping coefficient $c_{di}$ at that location.
e) Calculate the response of the current structure (equipped with the new damper), according to the performance quantification defined.

f) Repeat from c) to e) until $C_0$ has been completely allocated.

Although straightforward, this method is still distant from practitioners. Depending of the performance quantification chosen, step e) can be cumbersome for most engineers (if it implies, for instance, nonlinear time history analysis or calculation of second-order statistics of the structural response). However, the mathematical formulation proposed in this work should provide a simple and direct method to calculate these responses, and therefore this method could arise as a possible alternative for supplemental damping design.

In the third group of distribution schemes, methods establish a formal optimization procedure based on specific performance objectives and constraints. Many authors have considered this approach exploring different formulations and solution alternatives: transfer function performance metrics (Takewaki, 1997), genetic algorithms (Singh & Moreschi, 2002), gradient-based algorithms (Lavan & Levy, 2006), random vibration theory (Gidaris & Taflanidis, 2015) and more. Once problem is formulated any appropriate numerical optimization algorithm can be used to solve it. In this study, an explicit, general-purpose algorithm is implemented to solve the associated optimization problem. The solver is called glcCluster (hereafter, explicit algorithm) and is a hybrid algorithm for constrained mixed-integer global optimization. It uses a combination of the DIRECT algorithm (Jones, Perttunen, & Stuckman, 1993) and a clustering algorithm (Holmström, Göran, & Edvall, 2010). The theory underling the algorithm is complex and it is not the purpose of this investigation to explain it, but it is indeed necessary to emphasize why the utilization of this tool is convenient for the objectives of this study.

Before justifying the election of the algorithm, it is important to describe in general terms the characteristics of the problem treated in this investigation (described in detail later on this thesis). The design optimization problem is non-convex, non-linear and large-
dimensional. The objectives functions considered for the analysis are mostly convex, which is beneficial in terms of solution algorithm procedure, but the utilization of the maximum function on the constraint creates a non-convex (and non-smooth) domain. Nonlinearities are present in both the constraint and objective function, as cost metrics and structural performance calculation are indeed nonlinear relations. The utilization of many possible damper locations among the height of the structure combined with the latter issues complicates even more the solution procedure, as dimensionality on the design vector can impact the efficiency of the solver. From all these considerations stem the complexity of the design problem that the explicit algorithm can straightforwardly manage due to the reasons described next.

First, this explicit algorithm is known as one of the most robust algorithms to solve global optimization problems. As this study is focused on high-rise buildings, the non-convex nature of the problem and the large number of storeys can create a dimensionality issue when the solver tries to reach an optimal solution. In these cases, it is common that manifold solutions reach near-optimal values so the solver needs to account for global minima appropriately. In this context, studies have compared the efficiency of many global optimizers and have concluded that the explicit algorithm is, in most of the tests, the best algorithm available (Rios & Sahinidis, 2013). Moreover, the environment in which it is implemented, the TOMLAB Optimization Software (Holmström et al., 2010) is a well-known toolbox which has been extensively used in engineering applications. In fact, the explicit algorithm has been used as a research tool in other viscous dampers studies (Gidaris, 2015; Taflanidis & Scruggs, 2010) and has shown good performance. Second, the TOMLAB toolbox is implemented in MATLAB (The Mathworks Inc., 2016), which is a robust, recognized computer software frequently used by practitioners. Since one of the aims of this investigation is to provide an optimization procedure that can be easily implemented in professional practice, this is a crucial advantage. Third, as a general-purpose optimization algorithm it can straightforwardly account for nonlinearities
in the objective function and constraints, which is important as performance metrics and cost functions are certainly nonlinear.

Regardless of the chosen optimization method, it is crucial to select an appropriate way to quantify the performance of the structure and of the supplemental damping system. The adopted approach defines the extent and the implementation of the optimization method. In this context, there are many alternatives to define performance, which differ either in the philosophy underlying the selection of the response quantity or in the method to calculate the response. For the former criterion, the designer can aim at a specific response reduction with respect to the structure without dampers (for instance, design for a 40% reduction of maximum interstorey drift) or to restrict an engineering demand parameter to a fixed allowable level (for instance, to constrain the interstorey drift to 2‰). For the latter criterion, performance can be computed as an average of time-history analysis results or can be given by comprehensive risk analysis, among other intermediate alternatives in terms of complexity. In this study, performance is quantified as a specific response reduction with respect to the structure without the supplemental damping system and the performance is obtained using Random Vibration Theory (RVT). The first choice provides a useful way to compare structural behavior independent of certain predefined threshold for structural demand parameters (which can vary from one context to another), and the second is based on a reasonable equilibrium between theoretical rigorousness and reasonably simple computational implementation.

The idea of applying RVT in the field of structural dynamics is to represent the earthquake excitation as a stochastic process, and as a result the structural response is a stochastic process as well, from which one can calculate statistical indexes (mainly means and variances) to quantify performance. A stochastic process is a set of random variables related to a similar phenomenon that are functions of the independent variable time. Consider a stochastic process \( x(t) \) with different realizations \( x_1(t), \ldots, x_n(t) \) as seen in Figure 1-3. The characterization of the process requires the multivariate probability density function \( p(x_{t1}, \ldots, x_{tm}) \), where \( x_{ti} \) is also a random variable of sample values in
each realization of $x(t)$ at the instant $t_i (x_{t_1}, x_{t_2}, ..., x_{t_n})$. However, the calculation of
the first two probability density functions is usually enough to describe the process
accurately (Clough & Penzien, 2003).

![Stochastic Process](image)

**Figure 1-3:** stochastic process $x(t)$ (source: Clough & Penzien (2003))

In some cases, a stochastic process does not depend on the absolute time in which
statistics quantities are being measured but on the time interval between instants $t_i$ and $t_j$.
These processes are called stationary, which in simple words means that statistics are
independent of time shifts. An immediate consequence of this property is that the mean
$\mu_x(t)$ and the variance $\sigma^2_x(t)$ of the process are constant values:

\[
\mu_x(t) = \mathbb{E}[x(t)] = \mathbb{E}[x(t + \tau)] \quad \text{if } \tau = -t
\]

\[
\rightarrow \mu_x(t) = \mathbb{E}[x(t)] = \mathbb{E}[x(0)] = \mu_x \quad \text{(constant)} \quad \forall t \tag{2}
\]
Same reasoning applies for $\sigma_x^2 = \mathbb{E}[x(t)^2] - \mu_x^2$. Therefore, the assumption of stationarity allows to model earthquake excitation with constant mean and variance. Moreover, ground motion corresponds to a zero-mean Gaussian stochastic process, so $\mu_x = 0$ and $\sigma_x^2 = \mathbb{E}[x^2]$.

Once the earthquake excitation is defined as a zero-mean stationary Gaussian stochastic process, the objective is to statistically characterize the behavior of a structural system exposed to this ground motion. It can be demonstrated that if the input of a linear stable system is a zero-mean stationary Gaussian stochastic process, then its output is also zero-mean, stationary, and Gaussian (Clough & Penzien, 2003). As for the statistics of this output, let us consider the following linear system in space-state representation:

$$
\dot{x} = Ax + B f \\
y = Cx + D f
$$

(3)

Knowing that the input is zero mean ($\mathbb{E}[f] = 0$), by applying the $\mathbb{E} [\cdot]$ operator is simple to note that $\mu_x = \mu_y = 0$, i.e., the state vector and the output vector are also zero-mean processes. As for the second-order statistics, if $f = w$ is a zero-mean Gaussian white noise of spectral density matrix $S_w$, it can be shown that the covariance matrix $P_{xx}$ of the state vector $x$ is given by the solution of:

$$
P_{xx}A^T + AP_{xx} + 2\pi BS_wB^T = 0
$$

(4)

which is an algebraic Lyapunov equation (matrix equation). Moreover, if $D = 0$ (zero matrix) and since $y$ is a linear transformation of $x$, the output is also Gaussian with covariance matrix $P_{yy} = CP_{xx}C^T$ (Lutes & Sarkani, 2004). Following this procedure, the variances of the output of a structural system (for instance: interstorey drifts, floor accelerations, or damper forces) can be simply and elegantly calculated, and then used to state the optimization problem.
The simplicity of the formulation in (3) can be further exploited to create more realistic ground motions, applying filters to white noise excitation to generate ground accelerations with specific frequency content. To do so, a secondary space-state representation can be formulated:

\[
\begin{align*}
\dot{x}_f &= A_f x_f + B_f w \\
\mathbf{a}_g &= C_f x_f
\end{align*}
\]

in which space state matrices \(A_f, B_f\) and \(C_f\) account for the equations of motion of an specific filter (for instance: Kanai-Tajimi or Clough-Penzien filter) and \(\mathbf{a}_g\) is the resulting vector of earthquake ground accelerations. Then, both space-state representations can be combined into an augmented representation and the aforementioned procedure can be applied to obtain the statistical quantification of the structural performance.

One of the main limitations of this formulation is that the dynamical system must be linear. Therefore, to account for structural and supplemental nonlinearities an approximation has to be adopted. Usually, inclusion of any seismic protection device protects structural components from developing inelastic deformations, so the assumption of linear structural matrices is accurate. However, viscous dampers devices are usually nonlinear so their constitutive model must account for its nonlinear nature. To do so, one can apply the statistical linearization procedure. Consider a nonlinear single degree of freedom (SDOF) system with equation of motion:

\[
m\ddot{y} + c\dot{y} + k\dot{y} + f(y, \dot{y}) = p(t)
\]

where \(p(t)\) is an stochastic external force, \(f(y, \dot{y})\) is a nonlinear force and \(y\) is the displacement of the nonlinear system. The purpose of the statistical linearization is to replace (6) with an equivalent linear system of displacement \(s\) such that the nonlinear force is approximated by \(f(s, \dot{s}) = k_{eq}s + c_{eq}\dot{s}\) and then (Acciani, Di Modugno, Abrescia, & Marano, 2015):
\[ m\ddot{s} + c\dot{s} + ks + c_{eq}\dot{s} + k_{eq}s = p(t) \quad (7) \]

such that the error between solutions of (6) and (7) \( e_a \) is minimized in a mean-square sense. The \((c_{eq}, k_{eq})\) pair that minimizes \( e_a \) needs to satisfy:

\[
\frac{\partial \mathbb{E}[e^2_a]}{\partial c_{eq}} = 0 \quad \frac{\partial \mathbb{E}[e^2_a]}{\partial k_{eq}} = 0 \quad (8)
\]

which leads to:

\[
c_{eq} = \frac{\mathbb{E}[f(s, \dot{s})\dot{s}]\mathbb{E}[s^2] - \mathbb{E}[f(s, \dot{s})s]\mathbb{E}[s\dot{s}]}{\mathbb{E}[\dot{s}^2]\mathbb{E}[s^2] - (\mathbb{E}[s\dot{s}])^2}
\]

\[
k_{eq} = \frac{\mathbb{E}[f(s, \dot{s})s]\mathbb{E}[\dot{s}^2] - \mathbb{E}[f(s, \dot{s})\dot{s}]\mathbb{E}[s\dot{s}]}{\mathbb{E}[\dot{s}^2]\mathbb{E}[s^2] - (\mathbb{E}[s\dot{s}])^2}
\quad (9)
\]

In this study, \( s \equiv v \) is the damper deformation (along its axis), and the nonlinear force is given by:

\[
f(s, \dot{s}) \equiv f(v, \dot{v}) = f(\dot{v}) = c_d|\dot{v}|^\alpha \text{sgn}(\dot{v}) \quad (10)
\]

Besides, from the RVT it is known that the statistic \( \mathbb{E}[v\dot{v}] \) is:

\[
\mathbb{E}[v\dot{v}] = \frac{1}{2} \frac{d}{dt} \mathbb{E}[v^2] = 0 \rightarrow \mathbb{E}[v\dot{v}] = 0 \quad (11)
\]

since \( \mathbb{E}[v^2] = \mu^2 + \sigma_v^2 \) is a constant for second moment stationary processes. Therefore, noting that \( \text{sgn}(\dot{v})\dot{v} = |\dot{v}| \), the expression for \( c_{eq} \) simplifies to:

\[
c_{eq} = c_d \frac{\mathbb{E}[|\dot{v}|^\alpha \text{sgn}(\dot{v})\dot{v}]}{\mathbb{E}[\dot{v}^2]} = c_d \frac{\mathbb{E}[|\dot{v}|^{\alpha+1}]}{\mathbb{E}[\dot{v}^2]} \quad (12)
\]

Defining \( \mathbb{E}[\dot{v}^2] \equiv \sigma_{\dot{v}}^2 \) and computing \( \mathbb{E}[|\dot{v}|^{\alpha+1}] \) appropriately (Di Paola, La Mendola, & Navarra, 2007), the statistical linearization of the viscous damper coefficient is:
where $\Gamma(\cdot)$ is the Gamma function. This expression gives the linear coefficient $c_{eq}$ that can be implemented in the aforementioned formulation to calculate response statistics. Since viscous dampers are purely velocity-dependent devices it can be proven that $k_{eq}$ is zero.

Considering the aforementioned nature of damper height-wise distribution and the method to calculate the response, the purpose of this study is to develop an optimization framework based on an explicit optimization algorithm. The main issues of this research are: 1) it is focused on professional practice application, so the straightforward methodology to calculate response statistics based on space-state representation is utilized; 2) although globally applicable, the illustrative example shown in this investigation considers particular aspects of Chilean seismic and structural characteristics; and 3) with the intention to create a procedure that can have an impact on practitioners, real design considerations are directly incorporated into the optimization. This latter particularity consists of three key design issues that are further explained in the following paragraphs.

The first issue is to incorporate into the optimization problem an accurate representation of the viscous damper upfront cost. Either treated as objective function or budget constraint, damper cost is usually simplified to quantifications which do not directly relate to cost. Probably the most common alternative to do so is to measure the total damping coefficient of the system $C_o$ (Taflanidis & Scruggs, 2010) or the added damping ratio per storey (Marano, Trentadue, & Greco, 2007). Clearly this approach presents the advantage of simplicity, but damper cost is related to peak damper forces and not to damping coefficients. A further step in sophistication is to measure the total root mean square damper forces (Gidaris & Taflanidis, 2015) as the inclusion of the velocity
component induces a more realistic quantification of damper forces. However, when minimizing the root mean square values there is no consideration for peak forces, which is what actually determines cost. Moreover, is it known that cost is a nonlinear function of damper peak force (Gidaris & Taflanidis, 2015), so a further step in refinement is needed to correctly account for cost. It is important to mention that there are even more sophisticated approaches that incorporate, for instance, prototyping cost and cost due to the number of damper locations (Pollini, Lavan, & Amir, 2016), but with the intention to maintain a reasonable balance between accuracy and professional practicality these latter considerations are not addressed in this investigation.

The second issue is to account for damper configuration schemes in which dampers are attached at non-consecutive (i.e. non adjacent) floor levels. Usually, dampers are anchored at a given floor level and at an adjacent floor level, thus energy dissipation depends on the interstory velocity at that storey. However, since for harmonic input energy dissipation is proportional to the square of damper deformation, it seems reasonable to analyze the effect of connections between non-consecutive floor levels between which relative deformation is larger than between adjacent floor levels. For instance, dampers anchored at the base level and at the second floor level, at the first floor level and at the third floor level, and so on, i.e., dampers anchored two floor levels apart. Figure 1-4 shows a picture of the Plaza Talca Building in which dampers are connected in this fashion. In Chile, this type of configuration has been implemented successfully in this building and in many others such as, for instance, the Las Condes Capital Building in which viscous dampers are connected every three floor levels. Examples around the world are less common, but the Torre Mayor Building in Mexico is another case where dampers are installed in this manner. From a theoretical perspective, this arrangement has not been deeply analyzed yet by the scientific community. In one of the few researches that mention the issue it is emphasized that these configuration schemes are more efficient than consecutive configurations at no expense of larger forces (Silvestri & Trombetti, 2004),
although a theoretical quantification of the benefits with respect to the consecutive configuration scheme it not provided.

Figure 1-4: scheme in which dampers are connected every two storeys (source: rba-global.com)

The third issue is to include in the optimization a penalization due to the additional force demands imposed by supplemental viscous dampers on structural members, specially columns. This consideration has been widely discussed and analyzed (Karavasilis, 2016; Seo, Karavasilis, Ricles, & Sause, 2014; Wang, Lai, Schoettler, & Mahin, 2015; Whittle, Williams, & Blakeborough, 2012), although applications on heightwise distribution optimization are very limited. Lavan (2015) proposed a formulation in which allowable stresses of structural members are accounted for, but only limiting them to specific maximum allowable values and only in the supporting braces. The importance
of this effect can be appreciated in Figure 1-5. In an interior frame, the axial load demand on a column is relatively constant (at the gravity load level) but the bending moment varies considerably, Figure 1-5a. With the inclusion of a seismic protection device bending moments decrease (because drifts are reduced) but axial loads increase. If the tip of the new P-M demand is closer to the nominal capacity curve than the original curve, the inclusion of these devices might be unproductive and detrimental to structural performance of key structural members, particularly columns (Constantinou & Symans, 1993).

Figure 1-5: P-M diagram without (a) and with (b, c and d) seismic protection devices (source: Constantinou & Symans (1993))
Designers usually assume that this risk is suppressed when linear viscous dampers are used since peak elastic forces are supposedly completely out-of-phase with peak viscous damper forces. However, this assumption might not be entirely accurate. Consider the chevron bracing scheme shown in Figure 1-6. The axial load imposed on the braces due to a linear damper force is $|N_{f1}| = |N_{f2}| = f_d / \cos \theta$, and produces an additional axial load demand on the adjacent column $N_f(\dot{v}) = 0.5f_d \tan \theta = 0.5c_d \tan \theta \dot{v}$. According to Goel (2002) the total axial load demand on the adjacent right column is $N(v, \dot{v}) = N_e(v) + N_f(\dot{v})$, where $N_e$ is an elastic component and $N_f$ is the viscous component due to the damper force acting through the bracing system. In his findings, Goel (2012) showed that the peak value of $N(v, \dot{v})$ occurs in the interim phase between maximum displacement and maximum velocity, so the out-of-phase idealization usually assumed for viscous dampers is not valid when inclination exists, which is usually the case (either in the bracing system or the damper itself). The specific values that $N_e$ and $N_v$ can take depends on the structural system characteristics and on the supplemental damping system configuration. Moreover, nonlinear dampers are commonly preferred in professional practice rather than linear ones, so the out-of-phase idealization is even more inaccurate in those cases.

Figure 1-6: additional axial load demand due to viscous damper force
1.1. **Hypothesis**

Inclusion of realistic damper cost and additional axial load demand on columns in the optimization has an impact on the optimal height-wise distribution, and configuration schemes in which dampers are connected to non-consecutive storeys improve the efficiency of viscous damper.

1.2. **Objectives**

1.2.1. **General objective**

To develop an explicit optimization method for height-wise distribution of viscous dampers that can account for real design considerations and can be utilized in professional practice.

1.2.2. **Specific objectives**

i) To implement and computationally validate the space-state representation of a dynamic system and the calculation of second-order response statistics based on RVT.

ii) To state a formal optimization problem with different alternatives of objective functions to compare the relative effect of real design considerations.

iii) To implement an explicit optimization method that can straightforwardly manage and solve the optimization problem stated.

iv) To apply the explicit optimization method to compare total cost of height-wise distributions in which: a) cost function is realistic as opposed to simplified; b) column strengthening cost is considered as opposed to not considered; and c) dampers are connected consecutively as opposed to non-consecutively.

v) To validate explicit optimization convenience by comparing results with those given by simplified height-wise distribution.

vi) To validate RVT optimization results through linear time-history analysis.
1.3. **Methodology**

The space-state representation of the dynamic system is computationally implemented in MATLAB software, and the explicit optimization method is included in the formulation through the TOMLAB toolbox. Mass and stiffness matrices are obtained from a real 3D finite element model of a Chilean high-rise reinforced concrete building, and a five percent damping ratio is assumed in all modes to obtain the inherent damping matrix. Later, parameters of a Clough-Penzien Power Spectral Density Function (PSDF) are fitted through nonlinear least squares fit from empirical PSDFs of eighteen ground motions recorded during the $M_{w}$ 8.8 2010 Maule (Chile) Earthquake. Then, optimization results for linear viscous dampers are analyzed in terms of realistic damper cost, column strengthening cost and non-consecutive damper configuration schemes, and convenience of explicitly stating the optimization problem is verified contrasting performances with those of the Uniform and SSA height-wise distributions. A multi-objective formulation approach, in which cost and performance are set as competing objectives, is discussed and analyzed. Then, a comparison of height-wise distribution optimization using linear and nonlinear viscous dampers is performed in terms of structural behavior and cost. Finally, to validate the accuracy of the proposed method, stochastic representation of structural performance obtained from the optimization is compared to deterministic representation obtained through linear time-history analysis using Simulink.

1.4. **Results and conclusions**

Based on results obtained from a case study analyzed in next section, the following conclusions can be drawn:

i) An optimization framework in which objective functions are based of cost metrics are substantially more beneficial in terms of cost reduction than simplified design alternatives. The inclusion of a nonlinear relation between cost and damper capacity, and of the column strengthening cost lead to a design that is more realistic
and accurate. Inclusion of maximum feasible damper force capacity is also a cause of differences between design vectors when optimizing height-wise distributions.

ii) For the same level of energy dissipation, dampers connected every $n_c$ storeys develop forces that are $n_c$ times lower than forces developed in dampers connected consecutively. As cost metrics are force-based functions, this conclusion implies that cost is reduced by approximately the same factor $n_c$ with respect to the cost of a supplemental system in which dampers are connected consecutively.

iii) The explicit optimization algorithm used in this research allow the supplemental viscous damping system to achieve the target reduction at the expense of a lower cost than those of the Uniform and SSA distributions. Differences are more relevant when nonlinearities in cost objective functions are more significant.

iv) Optimization in which nonlinear dampers are considered are economically beneficial and lead to peak forces that are usually lower than those in linear dampers. The latter finding is important not only due to economic issues but also as a feasibility requirement; optimal design of linear dampers usually leads to extremely large dampers that might exceed the commercially available maximum damper force capacity.

v) Peak responses approximated by stochastic stationary analysis (obtained in the optimization procedure) correlate well with non-stationary time-history analysis peak responses. Therefore, RVT analysis combined with the explicit optimization is a valid, accurate and fast tool for design of supplemental viscous damping systems.

1.5. Future research

i) Consecutive and non-consecutive configuration schemes: in this investigation, non-consecutive distribution schemes allow dampers to connect non-consecutively only in a fixed number of floor levels (i.e. only every two storeys or only every three storeys). However, on the same design dampers can be allocated non-consecutively and also consecutively, so an extension of possible damper
locations can be implemented. This modification implies a considerably higher computational effort, as design vector in the optimization increases nearly by a factor of the maximum number of storeys apart that dampers can possibly be connected to in the design (i.e., if dampers can be connected at most every two storeys then this modification implies that design vector should be nearly doubled to account for all possible locations).

ii) **Counteraction effect of additional axial load demand due to damper forces:** as analyzed in other studies (Whittle, Williams, & Blakeborough, 2012), the additional axial load imposed by dampers can be reduced if an appropriate distribution of dampers among different bays of the same storey is considered. A suppression of additional axial load demand can occur if the tensile load in one column counteracts with the compression load generated at the same column in the adjacent bay of a different storey. Therefore, the optimization procedure could include this concept and perform a “width-wise” optimization across the bays of a given storey.

iii) **Different bracing configurations:** in this investigation, connectivity matrix and assumption made for column strengthening cost are based on a chevron bracing. However, other bracing configuration schemes can be analyzed to further validate conclusions reached and to estimate the effect (if any) of these changes on the optimization. Moreover, bracing flexibility can also be accounted for in the optimization problem as has been done in recent studies (Pollini, Lavan, & Amir, 2017).

iv) **Limitation to maximum force per storey:** the maximum force demand that a single storey can withstand \( (F_o) \) is limited. This limitation derives directly from structural and architectural constraints: the number of bays in which dampers can be installed, the space available on each bay to install a damper, force concentration on key structural members and so on. Hence, the cost and performance that an optimization design can reach is a function of \( F_o \). To study this effect, a triple-objective (cost, performance and \( F_o \)) optimization problem can be stated. As a
result, the Pareto surface of these objective can illustrate the trade-off relations between them and therefore provide a useful decision tool to the designer.

v) **Further sophistication to damper cost function:** although this study presents an accurate representation of damper cost, other elements can be considered when estimating cost. For instance, the additional cost of manufacturing a large number of different damper sizes, the cost of prototyping each type of damper (Pollini et al., 2016) and the additional cost of the bracing system when dampers are connected to non-consecutive storeys (for instance, due to slab perforation when the bracing system passes through a floor level).

vi) **Effect of natural period of structure respect to ground motion frequency content:** instead of analyzing real structures, a parametric analysis could be done considering theoretical structures with a specific natural period and study their performance on the optimization respect to the earthquake acceleration modeled. The structure with a natural period closer to the predominant frequency range of the excitation is supposedly more affected than the rest, and therefore solutions obtained on the optimal design vector can vary. This is especially relevant when nonlinear performance and cost measures are considered.

vii) **Inclusion of first passage theory to estimate performance:** the RVT approach used in this investigation is based on the second-order response statistics to quantify performance, which is simple and convenient. However, more comprehensive and reliable tools in this field can be adopted. Perhaps the more appropriate one for design purposes is the first passage theory. Based on this concept one can estimate the rate at which a stochastic process exceeds a specific threshold, and therefore calculate the probability of this stochastic process not to surpass an acceptable performance level (for instance, the probability of not exceeding a 0.45g floor acceleration). This modification implies a better representation of the performance constraints and possibly an extension of the cost function by including the repair cost (measured through the probability of failure of the system).
2. VISCOS DAMPER OPTIMIZATION IN MULTISTOREY BUILDING STRUCTURES

2.1. Introduction

An increasingly popular approach to attenuate the effects of large earthquakes on the built environment consists of equipping structures with passive energy dissipation devices. Among them, fluid viscous dampers are of special relevance; their proven efficacy and modeling simplicity make them an attractive seismic protection device for new and existing buildings (Constantinou & Symans, 1993; Mahin, Lai, Schoettler, & Wang, 2015; Symans et al., 2008). The effectiveness of such dampers in reducing the seismic response of multistorey buildings is sensitive to their height-wise distribution (Singh & Moreschi, 2002; Whittle, Williams, Karavasilis, et al., 2012), and a variety of relevant optimization criteria have been proposed in the literature. A first group of distribution schemes, such as the uniform, the stiffness proportional, or the storey shear proportional (Landi et al., 2015; Whittle, Williams, Karavasilis, et al., 2012), distribute the total damping coefficient (i.e., the sum of the damping coefficients of all dampers) according to pre-selected simplified criteria, with the total damping coefficient chosen so that a specific performance is achieved, for example a specific increase of the damping ratio in some chosen mode (typically the fundamental mode). Somewhat more sophisticated alternatives, still belonging to this group, are the storey shear strain energy distribution schemes (Hwang et al., 2013), which have been shown to provide a good compromise between efficiency of damper application and implementation simplicity (Landi et al., 2015). In a second group of distribution schemes dampers (representing a portion of the total damping coefficient) are sequentially placed at the location (i.e., storey) where the value of a specific performance index reaches a maximum. Examples are the Sequential Search Algorithm (SSA) (Zhang & Soong, 1992), the Simplified Sequential Search Algorithm (López-García, 2001) and an Energy-Based Sequential Algorithm (J. L. Lin et al., 2014), with performance indexes given by interstorey drift, interstorey velocity, and dissipation rate of elastic strain energy, respectively. Finally, a
third group of distribution schemes establish a formal optimization procedure based on some chosen performance objectives, incorporated in the optimal design through proper selection of the objective and constraint functions. For instance, Takewaki (1997) derived optimality criteria and proposed an optimization scheme that minimizes the sum of the amplitudes of transfer functions at the undamped fundamental natural frequency. Singh and Moreschi (2002) used genetic algorithms to reduce the root mean square response of base shear and floor accelerations. (Lavan and Levy (2006) proposed an equivalent problem approach to minimize the total damping coefficient needed to keep interstorey drifts within allowable levels, and presented a gradient-based solution procedure for an ensemble of ground motions. Gidaris and Taflanidis (2015) considered the minimization of the sum of root mean square responses of damper forces constraining interstorey drifts and floor accelerations to target performance levels.

A variety of different performance quantifications were adopted in the aforementioned studies. Performance is frequently described with respect to that of the uncontrolled structure (Lavan, 2015; Singh & Moreschi, 2001), i.e., targeting a specific improvement, though frameworks that evaluate globally the favorability of the damper implementation also exist (Gidaris & Taflanidis, 2013; Park, Koh, & Hahm, 2004; Shin & Singh, 2014). Simplified approaches adopt a modal analysis philosophy, emphasizing the damping ratio (or transfer function) at the fundamental mode. Other methodologies use time-history analysis and peak response quantities (interstorey drifts and absolute floor accelerations) to evaluate structural performance, the quantification of which ranges from simple aggregation over an ensemble of ground motions representing future excitations (T. K. Lin, Hwang, & Chen, 2016) to comprehensive risk analysis through probabilistic frameworks that might even include life-cycle cost considerations (Gidaris & Taflanidis, 2015). Between these two extremes in terms of complexity, i.e. simplified modal analysis and comprehensive time-history analysis, another wide range of approaches evaluate performance using random vibration theory, modeling the seismic excitation as a stationary stochastic process. In this case the response is typically
quantified in terms of variance or root mean square (RMS) values, though more advanced quantifications such as first-passage probability have also been suggested (Marano et al., 2007; Taflanidis & Scruggs, 2010). In most of these investigations emphasis was placed on linear dampers, though studies that discuss nonlinear damper implementations also exist. When the latter are combined with a stochastic representation of the excitation, typically statistical linearization techniques are adopted (Di Paola et al., 2007; Enrico Tubaldi & Kougioumtzoglou, 2015) to simplify the evaluation of the response of interest.

This study investigates the optimal height-wise distributions of viscous dampers in multistorey structures emphasizing design considerations that are relevant in practical applications but have not been yet fully explored in past studies. While many of these considerations are of general relevance, focus is placed here on Chilean reinforced concrete multistorey buildings. Three main topics are addressed: the cost of the supplemental dampers, the bracing characteristics of the damping system, and the forces on structural members due to the supplemental damping system.

Regarding the first aforementioned topic, though the importance of explicitly incorporating the upfront damper cost has been demonstrated (Gidaris & Taflanidis, 2015; Pollini et al., 2016, 2017), this cost is commonly ignored or is only approximately addressed in simplified design frameworks. In these latter cases usually the total damping coefficient is considered (Singh & Moreschi, 2002), but this quantity does not explicitly indicate the damper cost, which is actually more related to the force capacity rather than to the damping coefficient.

In terms of bracing schemes, emphasis has been placed solely on configurations where bracing terminals are anchored at consecutive (i.e., adjacent) floor levels. Such approach might not be suitable, though, for stiff buildings where interstorey velocities might not be large enough for efficient energy dissipation through supplemental viscous dampers (Baquero Mosquera, Almazán, & Tapia, 2016). This is typically the case of Chilean residential buildings where the lateral force resisting system is made up of stiff
reinforced concrete shear walls. While many studies have shown the efficacy of bracing schemes that amplify the interstorey displacement between adjacent floor levels (Baquero Mosquera et al., 2016), very few researches have considered supplemental dampers attached to braces connecting non-consecutive (i.e., non-adjacent) floor levels. Among researchers that mention this issue, Silvestri and Trombetti (2004) remark that the performance of dampers connecting non-consecutive floor levels is more efficient, but they do not provide a comprehensive theoretical framework for analysis/design.

Finally, the additional force demands imposed by supplemental devices on structural members have been widely discussed in the literature but are consistently ignored in height-wise damper optimization. Symans and Constantinou (1998) showed that, despite the reduction of interstorey drifts and inelastic deformations, supplemental devices might induce significant axial forces in columns and introduce local failures (for example due to buckling). This issue is usually ignored in design when the supplemental dampers are viscous devices because their peak forces are supposedly out of phase with the peak forces imposed by the seismic excitation (i.e., at the peak displacement response). However, such assumption might not be entirely true because of damper nonlinearity, bracing flexibility, structure nonlinearity (Wang et al., 2015) and damper and/or bracing inclination (Goel, 2002), among other reasons. Moreover, even assuming that the out-of-phase idealization is true, force demands on structural members during the interim phase between force and displacement peaks might still be significant (Whittle, Williams, & Blakeborough, 2012). This issue is especially critical in high-rise buildings (Wang et al., 2015) where forces on the columns at the bottom of the building are large. Recent experimental studies (Karavasilis, 2016; Seo et al., 2014) have validated some of these concerns, showing that steel moment resisting frames equipped with viscous dampers have a unique failure mode (different from the one of the bare frame) characterized by a soft storey mechanism where plastic hinges develop at the ends of columns due to the high axial force demands imposed by the supplemental dampers. All these remarks indicate that the actual cost of a supplemental damping system should also include possible
strengthening of structural members, especially columns. This issue was only partially addressed by (Lavan, 2015), who proposed a formulation in which allowable stresses of structural members can be accounted for as an optimization constraint.

This investigation establishes a versatile, yet simple (and therefore appropriate for practical design applications) optimization framework for the height-wise distribution of viscous dampers that addresses all three aforementioned issues. Stochastic response under stationary conditions is assumed to obtain a faithful description of the structural performance. A state space formulation is adopted to obtain response statistics, and statistical linearization is used when nonlinear dampers are considered. Alternatives to calculate peak damper forces when such linearization is employed are discussed. Different objective functions are formulated to address the issues discussed above within the optimization approach, and comparisons between the corresponding results indicate the relative importance of such issues in practical design applications. In the next section the problem formulation is presented giving special attention to calculation of second order response statistics and to a formulation that can accommodate non-consecutive bracing schemes. In Section 2.3 the cost functions and performance constraints are described in detail, and in Section 2.4 the optimization problem and its solution are discussed. In Section 2.5 a comprehensive case study of a real Chilean high-rise building is presented and evaluated.

2.2. Problem formulation and calculation of response

2.2.1. Equations of motion and space state representation

Consider a $n_s$-storey building structure equipped with $n_d$ viscous dampers (linear or nonlinear) connecting different floor levels as seen in Figure 2-1. The distinction between the different bracing schemes will be examined later in this paper. Though the design framework discussed here can be extended to three-dimensional building models, focus will be placed herein on planar structures.
The equation of motion of the structure is given by:

\[ M_s \ddot{r} + C_s \dot{r} + K_s r + T_s^T f_d = -M_s R_s a_g \]  

(14)

where \( M_s, C_s \) and \( K_s \) are the mass, damping and stiffness matrices of the structure, respectively, \( r \) is the vector of floor displacements relative to the base of the structure, \( R_s \) is the influence vector for earthquake ground acceleration \( a_g \), \( f_d \) is the vector of damper forces and \( T_s \) is the connectivity matrix that relates the velocities of the global degrees of freedom \( \dot{r} \) to the vector of relative velocities between the ends of each damper \( \dot{v} = T_s \dot{r} \).

Figure 2-1: \( n_s \)-storey structure with consecutive \( (n_c = 1) \) and non-consecutive \( (n_c > 1) \) bracing schemes

As discussed in the introduction \( a_g \) will be modeled as a stationary excitation in the design framework and a state space formulation will be adopted to obtain the response
statistics. To accommodate this, let \( x_{st} = (r \quad \dot{r})^T \) denote the state vector of the structure, then (14) can be expressed as:

\[
\dot{x}_{st} = A_{st}x_{st} + B_{st}f_d + E_{st}a_g
\]

(15)

where \( z \) is the vector of outputs for the design problem and \( A_{st}, B_{st}, C_{st}, D_{st} \) and \( E_{st} \) are space state matrices defined in the Appendix. The zero-mean stochastic stationary ground acceleration \( a_g \) is modeled as a filtered white noise, and is given by:

\[
\dot{x}_f = A_f x_f + E_f W(t)
\]

\[ a_g = C_f x_f \]

(16)

where \( W(t) \) denotes a white noise sequence with spectral intensity \( S_w \), \( x_f \) is the state vector of the filter and \( A_f, C_f \) and \( E_f \) are the associated space-state matrices defined in the Appendix. Combining (15) and (16) the space state equations that govern the augmented system (excitation and structural model) are:

\[
\dot{x}_{st} = A_{st}x_{st} + B_{st}f_d + E_{st}C_f x_f
\]

\[
\dot{x}_f = A_f x_f + E_f W(t)
\]

(17)

The force demand on the \( i \)-th viscous damper of the system is given by:

\[
f_{di} = c_{di} |\dot{v}_i|^{\alpha_i} \text{sgn}(\dot{v}_i)
\]

(18)

where \( \alpha_i, c_{di} \) and \( \dot{v}_i \) are the viscous exponent, damping coefficient and relative velocity of the \( i \)-th viscous damper, and \( \text{sgn}(\cdot) \) is the signum function. For linear dampers \( \alpha_i = 1 \) whereas for nonlinear dampers statistical linearization will be employed to replace the nonlinear force with an equivalent linear one. In this case, the equivalent damping coefficient of the \( i \)-th viscous damper \( c_{eqi} \) is given by (Di Paola et al., 2007):
where $\Gamma(\cdot)$ is the Gamma function and $\sigma_{\dot{v}_l}$ is the standard deviation of $\dot{v}_l$. The linearized damper force is then $f_{dl} = c_{eqi} \dot{v}_l$. The damper force vector $f_d$, with elements $f_{di}$, can be expressed as:

$$f_d = K(c_{eq}) \dot{v}$$

(20)

where $K(c_{eq})$ is a diagonal matrix with the equivalent damping coefficient $c_{eqi}$ of each damper. Relative velocities are given by $\dot{v} = T_s \dot{r} = L_{st} \dot{x}_{st}$, where $L_{st}$ is the connectivity matrix that relates the state vector $x_{st}$ to the relative velocities $\dot{v}$. Note that the dependence of $K$ on $c_{eq}$ is explicitly emphasized.

We can finally formulate the augmented state-space system representation. Let $x = (x_{st} \quad x_f)^T$ be the augmented state vector of the system. Combination of (17) and (20) gives:

$$\dot{x} = A(c_{eq})x + E_w W(t)$$

$$z = C(c_{eq})x$$

(21)

where $A(c_{eq}), C(c_{eq})$ and $E_w$ are space-state matrices of the augmented representation and

$$A(c_{eq}) = A_a + B_a K(c_{eq}) L_a \quad C(c_{eq}) = C_a + D_a K(c_{eq}) L_a \quad L_a = (L_{st} \quad O_{n_d x n_f})$$

(22)

where $n_f$ is the dimension of $x_f$ and $O_{n_d x n_f}$ is a $n_d$ by $n_f$ matrix of zeros. Expressions for $A_a, B_a, \ C_a, D_a, E_w$ and $L_{st}$ can be found in the Appendix. The dependence of the various matrices on $c_{eq}$ is explicitly noted.
2.2.2. Second-order statistics or the response

Under the assumptions stated in the previous section, output \( z \) is a zero-mean, Gaussian (for linearized system) stochastic sequence with covariance matrix given by:

\[
R_{zz} = \mathbb{E}[zz^T] = C(c_{eq})P(c_{eq})C(c_{eq})^T
\]

(23)

where the covariance matrix of the state vector \( P(c_{eq}) \) is determined through the solution of the algebraic Lyapunov equation (Lutes & Sarkani, 1997):

\[
A(c_{eq})P(c_{eq}) + P(c_{eq})A(c_{eq})^T + 2\pi S_w E_w E_w^T = O
\]

(24)

Matrix \( R_{zz} \) provides the information on the second order statistics of the output \( z \). The \( k \)-th element of its diagonal is the variance \( \sigma_{z_k}^2 \) of the \( k \)-th output, i.e.:

\[
\sigma_{z}^2 = \text{diag}(R_{zz})
\]

(25)

In this study, interstorey drifts \( \delta \), absolute floor accelerations \( a_t \) and the summation of all damper forces above a certain storey level \( F_D \) are defined as outputs of \( z \). For the sake of brevity, quantity \( F_D \) will be subsequently referred to as “cumulative damper force”. All these vectors are defined separately at each storey. The axial loads on columns \( N_f \) due to dampers forces can be related to vector \( F_D \), a relationship that will be further discussed in Section 2.3. Vector \( \sigma_{z}^2 \) can be decomposed into the vectors of variances for each output as:

\[
\sigma_{z}^2 = \begin{pmatrix}
\sigma_{\delta}^2 \\
\sigma_{a_t}^2 \\
\sigma_{F_D}^2
\end{pmatrix}
\]

(26)

The variances of the damper relative velocities are also needed in the problem formulation and are related to the state covariance matrix \( P(c_{eq}) \) by:
\[ \sigma_v^2 = L_a^T P(c_{eq}) L_a \]  

(27)

Using these variances, the RMS damper forces can be obtained as:

\[ \sigma_{f_d} = K(c_{eq}) \sigma_v \]  

(28)

When statistical linearization is employed, variances \( \sigma_v^2 \) are also used to calculate the equivalent damping coefficient given by (19). In this case a cyclic dependence exists: \( c_{eq} \) depends on \( \sigma_v \) based on (19) but based on (27) and the dependence on \( P(c_{eq}) \) the latter also depends (implicitly) on \( c_{eq} \). An iterative process is therefore needed to solve for the equivalent damper coefficients and the state covariance matrix. In the context of the optimization established later in this manuscript this is avoided by adopting the equivalent damping coefficients as the primary design variables, and then back-deriving the actual damping coefficients through (19). In this case there is no iterative process required. Once \( c_{eq} \) is known, \( \sigma_v \) can be solved for by calculating the second order statistics, and finally \( c_{dl} \) can be computed based on (19) as:

\[ c_{dl} = \frac{c_{eq} \sqrt{2\pi} \sigma_v^{1-\alpha}}{2^{1+\frac{\alpha}{2}} \Gamma \left( 1 + \frac{\alpha}{2} \right)} \]  

(29)

### 2.2.3. Non-consecutive damper bracing scheme

One of the objectives of this work is to analyze the response considering bracing schemes where brace terminals are anchored not only at consecutive floor levels but also at non-consecutive floors as well, i.e., braces where one terminal is anchored at floor level \( i \) and the other terminal is anchored at floor level \( i + n_c \), with \( n_c > 1 \). For instance, if \( n_c = 2 \), then braces are such that terminals will be located at the base level and at the second floor level, at the first floor level and at the third floor level, and so on. Figure 2-1 shows a schematic representation of \( n_c = 1 \) and \( n_c > 1 \). While some implementations of
such bracing schemes do exist, especially in Chile, their potential benefits have not been fully examined.

To theoretically demonstrate the advantages of the $n_c > 1$ approach, let's examine a simplified analytical comparison. We will consider here the case of a single damper, although the philosophy underlying this reasoning can extend directly to systems equipped with multiple dampers. We will constrain the discussion to harmonic excitation with frequency $\omega$. Denoting the relative displacement between the ends of the damper as $v_o$, the energy dissipated $E_d$ by a single damper is given by (Symans & Constantinou, 1998):

$$E_d = c_d \omega^\alpha v_o^{1+\alpha} \frac{2^{\alpha+2} \Gamma^2 \left(1 + \frac{\alpha}{2}\right)}{\Gamma(2 + \alpha)} = c_d \omega^\alpha v_o^{1+\alpha} F(\alpha)$$  \hspace{1cm} (30)

where $\alpha$ denotes the viscous exponent and $c_d$ the damping coefficient. Assuming that lateral displacements varies linearly with height and that axial deformations of the dampers are linearly related to interstorey deformations, the peak deformation of the damper $v_{on_c}$ when the bracing scheme is such that $n_c > 1$ is given by:

$$v_{on_c} = n_c v_{o1}$$  \hspace{1cm} (31)

where $v_{o1}$ is the peak damper deformation when $n_c = 1$.

Same energy dissipation capability is needed to achieve the same level of vibration suppression independent of the value of $n_c$ (this can be viewed as equivalent to establishing the same improvement in damping ratio). Therefore, for a given performance level, when $n_c > 1$ the viscous damper coefficient $c_{dn_c}$ must satisfy:

$$E_{d1} = E_{dn_c} \rightarrow c_{d1} \omega^\alpha v_{o1}^{1+\alpha} F(\alpha) = c_{dn_c} \omega^\alpha (n_c v_{o1})^{1+\alpha} F(\alpha)$$  \hspace{1cm} (32)

which leads to:
\[ c_{dn_c} = \frac{c_{d1}}{n_c^{1+\alpha}} \]  \hspace{1cm} (33)

This result shows that, if the energy dissipation (and therefore performance) is fixed, a \( n_c > 1 \) bracing scheme leads to a total damping coefficient \( C_{on_c} = \sum c_{dn_c} \) that is \( n_c^{1+\alpha} \) times smaller than the total damping coefficient \( C_{o1} = \sum c_{d1} \) of the \( n_c = 1 \) bracing scheme. Alternatively, (33) indicates that for a given a total damping coefficient \( C_o \) the energy dissipated by a \( n_c > 1 \) bracing scheme is \( n_c^{1+\alpha} \) times higher than the energy dissipated by the \( n_c = 1 \) bracing scheme.

By itself, the value of \( C_o \) is not of significant practical interest because, as mentioned earlier, it has no direct relation with the damper cost. In fact, the relevant quantity directly related to cost estimation is the force capacity of the damper, which obviously must be greater than the force \textit{demand} on the damper. Based on the assumption that lateral displacements vary linearly with height, so do velocities under harmonic excitation of the same frequency. Therefore, when \( n_c > 1 \) the relative velocity between damper terminals is approximately \( n_c \) times greater than the relative velocity when \( n_c = 1 \), that is:

\[ \dot{v}_{n_c} = n_c \dot{v}_1 \]  \hspace{1cm} (34)

Using (18), (33) and (34) damper forces when \( n_c > 1 \) are given by:

\[ f_{dn_c} = c_{dn_c} |\dot{v}_{n_c}|^\alpha = \frac{c_{d1}}{n_c^{1+\alpha}} |n_c \dot{v}_1|^\alpha = \frac{1}{n_c^{1+\alpha-\alpha}} c_{d1} |\dot{v}_1|^\alpha = \frac{f_{d1}}{n_c} \]  \hspace{1cm} (35)

which means that, with respect to the case where \( n_c = 1 \), damper forces are reduced by a factor of \( n_c \) regardless of the value of \( \alpha \). Since damper forces are directly related to damper cost, this result shows that \( n_c > 1 \) bracing schemes might induce a significant reduction of damper cost.
2.3. Cost function and performance functions

The design objective is to identify the damping coefficient vector $c_d$ (made up of the damper coefficients of all supplemental dampers), that explicitly establishes optimality with respect to some chosen objectives and constraints. To simplify the optimization, the equivalent damping coefficient $c_{eq} = \{c_{eqi}, i = 1, \ldots, n_d\}$ is posed as the design optimization vector as discussed earlier, and then (29) is applied to determine $c_d$ (note that if dampers are linear, $c_d = c_{eq}$). To describe the objectives and constraints that define the problem, preference will be given to simple measures (expressed through second order statistics that are easy to calculate as detailed in Section 2.2) that can be related to the cost of implementation of the supplemental damping system. Consideration about the established vibration suppression level will be incorporated through performance functions related to the second order statistics of the response.

2.3.1. Cost functions

Three different cost functions are considered. The first and simplest one is the sum of RMS damper forces, i.e.:

$$J_1(c_{eq}) = \sum_{i=1}^{n_d} \sigma_{f_{di}}$$

(36)

where $\sigma_{f_{di}}$ is the RMS force demand on the $i$-th damper and $J_1$ has units of force. This measure facilitates a closer connection to the damper cost (Gidaris & Taflanidis, 2015) than the commonly adopted total damping coefficient (sum of each damping coefficient $c_{di}$) (Singh & Moreschi, 2002). However, damper cost is actually related to damper force capacity, which is a function of the peak damper force rather than of the RMS response. Moreover, this cost is not linearly related to damper force capacity (Gidaris & Taflanidis, 2015). The second cost function incorporates these considerations, and is given by:
\[ J_2(c_{eq}) = 96.88 \sum_{i=1}^{n_d} f_{doi}^{0.607} = C_{up}(c_{eq}) \]  

(37)

where \( f_{doi} \) is the peak force demand on the \( i \)-th damper (in units of kN) and \( J_2 \) is expressed in United States Dollars (USD). For linear dampers peak force quantities can be approximately related to RMS values as \( f_{doi} = p_f \sigma_{f_{dpi}} \), where \( p_f \) is the peak factor, relating the mean of the peak damper force (or peak of any stochastic variable) to its standard deviation. In this study this factor is taken as 2, corresponding to a single degree of freedom oscillator with 5% damping vibrating for 20 cycles based on up-crossing rate (Der Kiureghian, 1980) (representing period of vibration for stochastic variable). The 20 cycles were chosen to accommodate example considered later, based on strong ground motion duration characteristics for the region and the fundamental period of the building structure. For nonlinear dampers a modification is warranted since accuracy of statistical linearization typically reduces for peak response quantities, compared to the accuracy of average/RMS response quantities (Roberts & Spanos, 2003). Since intent of developed framework is to have practical utility, a simplified approximation is adopted to estimate peak damper forces. These are assumed equal to the forces developed for a specific peak velocity, and therefore are given by:

\[ f_{doi} = c_{di} (p_f \sigma_{vi})^\alpha \]  

(38)

The relationship in (37) between the damper force capacity and the damper cost was derived from the analysis of available commercial information (Gidaris & Taflanidis, 2015) and indicates that the cost of a marginal increment in force capacity is smaller in dampers having a large force capacity than in dampers having a small force capacity. Similar nonlinear functions have been considered in other studies (Pollini et al., 2017).

A final, new cost function \( J_3 \) is introduced in this study to account for the cost of column strengthening due to the additional forces imposed by the damping system, \( C_{cs}(c_{eq}) \). Based on discussion in Section 2.1, this cost is assessed as a function of the ratio
of the additional axial load due to damper forces to the initial axial load capacity (i.e. the axial load capacity of the structure without dampers). This ratio indicates the relative degree of strengthening required. In turn, the additional axial load on columns due to damper forces is a function of the cumulative damper force (i.e., the sum of forces in dampers located above a given storey). This cost function is then given by:

$$J_3(e_{eq}) = 96.88 \sum_{i=1}^{n_d} f_{doi}^{0.607} + \omega_N \sum_{j=1}^{n_s} \frac{\gamma F_{Doj}}{N_{oj}} = C_{up}(e_{eq}) + C_{cs}(e_{eq})$$  \hspace{1cm} (39)$$

where $F_{Doj}$ is the peak cumulative damper force at the $j$-th storey, $N_{oj}$ is the initial (i.e. of the structure without dampers) axial load capacity of the columns at the $j$-th storey and $J_3$ is expressed in USD. The parameter $\omega_N$ (also expressed in USD) is a constant introduced to: (i) convert damper forces to axial forces at each storey (i.e., it considers the geometric variability of different damper bracing alternatives as discussed next); and (ii) quantify the cost of the strengthening required relative to the initial axial load capacity. Weight $\gamma$ is a constant modification factor introduced for nonlinear dampers to approximate peak response statistics. As mentioned in Section 2.2, the axial force on the columns at a given storey, $N_{fj}$, is proportional to the cumulative damper force at the same storey, $F_{Dj}$, with the scaling factor related to the specific characteristics of the bracing scheme. For instance, for the chevron bracing scheme shown in Figure 2-1, $N_{fj}$ and $F_{Dj}$ are related to each other by the bracing inclination angle $\theta$ (assumed equal at all stories) so that $N_{fj} = 0.5F_{Dj}\tan \theta$. Hence, for the sake of conceptual clarity, from now on the quantity $F_{D}$ will be referred to as “axial load”, and $F_{Dj}$ represents the axial load of the columns on the $(j - 1)$-th storey (for instance, $F_{D1}$ is the axial load over the foundation and $F_{D2}$ the axial load over the first storey columns). Peak axial forces are calculated through use of the peak factor as $F_{Doj} = p_f F_{Dj}$, whereas for nonlinear dampers a modification is established through $\gamma$ to consider that statistics for $F_{Doj}$ are based on linearization for the damper forces (peak forces approximated $f_{doi} = p_f \sigma_{f_{loi}}$) whereas, as explained above, peak
damper forces are better approximated by (38). Modification factor for the axial load at each storey $\gamma$ is obtained by calculating the ratio of peak force under these two assumptions:

$$\gamma = \frac{c_d (p_f \sigma_y)^\alpha}{p_f \sigma_{f_d}} = \frac{p_f^{g-1} \sqrt{2\pi}}{2^{1+\frac{\alpha_i}{2}} \Gamma \left(1 + \frac{\alpha_i}{2}\right)} = \gamma(\alpha)$$

which is a function only of the viscous exponent $\alpha$. Since this weight is same for all dampers at all stories, this modification accurately adjusts for the cumulative effect of the peak damper force at each storey. For linear dampers $\gamma = 1$ whereas for nonlinear dampers $\gamma < 1$ (and therefore axial load is reduced).

To evaluate $N_{oj}$ it is assumed that column cross-sections change every $n_r$ stories, hence so does the initial axial load capacity. The initial axial load capacity at the foundation level is assumed equal to a typical axial load on a column due to gravity loads on a single storey, denoted $\bar{N}$, times the number of stories $n_s$. Cost $C_{cs}(c_{eq})$ is then ultimately proportional to the $\omega_N/N$ ratio. Since, to the best of the authors’ knowledge, no reliable information on the value of factor $\omega_N$ is available, a parametric analysis will be performed.

### 2.3.2. Maximum feasible damper force capacity

The cost function $C_{up}(c_{eq})$ discussed in the previous section does not incorporate considerations about the maximum feasible damper force capacity $f_{max}$ of commercially available dampers. Of course larger values of $f_{dol}$ can be accommodated by using multiple dampers per storey, with the capacities of each damper limited to $f_{max}$. In this case, though, the cost per damper is related to each individual force capacity and not the total capacity per storey. To address this issue, the following modification for $C_{up}(c_{eq})$, needed in (37) and (39), is proposed, assuming that if at some storey $f_{dol}$ exceeds $f_{max}$ then such demand will be accommodated using multiple, equal capacity dampers, the number of
which is the smallest number needed so that the force capacity of each damper is smaller than \( f_{max} \):

\[
C_{up}^m(c_{eq}) = 96.88 \sum_{i=1}^{n_d} \left[ \frac{\tilde{f}_{doi}}{f_{max}} \right] \left( \frac{\tilde{f}_{doi}}{f_{max}} \right)^{0.607}
\]

(41)

where \( \lceil \cdot \rceil \) is the ceiling function. The modified cost functions when this updated relationship is used for the damper cost will be denoted \( J_{2m}(c_{eq}) \) and \( J_{3m}(c_{eq}) \), respectively.

2.3.3. Performance function

The performance function can be described based on second order statistics of the output, \( h(\sigma_z) \), while different approaches can be taken for the exact functional form of this function. The simplest approach is to define performance as an average measure of the output vector \( z \) (i.e. sum of variances). However, since seismic performance is typically expressed in terms of occurrence of different failure modes, quantified by engineering demand parameters exceeding certain thresholds, it is more reasonable to define performance in terms of the maximum normalized variance. The latter is directly related to the probability of occurrence of any failure mode in the structure (Taflanidis & Scruggs, 2010). This approach leads to the following definition for the performance function:

\[
h(\sigma_z) = \max_k \left( \lambda_k \frac{\sigma_{zk}}{\beta_k} \right)
\]

(42)

where \( \lambda_k \) and \( \beta_k \) are the relative importance and normalization constant of the \( k \)-th output of \( z \), respectively.
2.4. Design optimization

The design optimization formulation requires adoption of appropriate objective and constraint functions. Among the different functions discussed in the previous section, preference in this work is given to the cost functions as the objective function, and to the performance function as the constraint function. This ultimately leads to a design problem searching for the minimum cost that can guarantee a specific performance level. A multi-objective formulation that incorporates both these considerations as explicit objectives will also be examined.

2.4.1. Single-objective design

The optimization problem is initially formulated as a single-objective, constrained optimization approach, in which cost metric \( f_l \) [where \( l \) refers to different alternatives according to (36), (37) and (39), perhaps with the modification described in (41)] is the objective function to be minimized and the performance function in (42) is the constraint. At the design stage focus is placed on the interstorey drift performance, leading to \( \lambda_k = 0 \) for performance outputs associated with accelerations and axial loads (i.e. related to vectors \( \sigma_a \) and \( \sigma_F \)) and \( \lambda_k = 1 \) for the outputs associated with the interstorey drifts (i.e. related to \( \sigma_\delta \)). The threshold vector \( \beta \) is chosen as the maximum RMS interstorey drift response of the structure without dampers, and from that vector the maximum value \( \bar{\sigma}_{\delta_0} \) is selected to formulate the performance function. This constraint ultimately facilitates the identification of the damper distribution providing a target reduction of the interstorey drift response with respect to that of the uncontrolled structure. The optimization problem is expressed as:

\[
\begin{align*}
\text{min} \quad & f_l(c_{eq}) \\
\text{subject to} \quad & \max_j \left[ \frac{\sigma_{\delta_j}(c_{eq})}{\bar{\sigma}_{\delta_0}} \right] \leq c_{tg} 
\end{align*}
\]
where \( \sigma_{\delta j} \) is the RMS interstorey drift response at the \( j \)-th storey of the structure equipped with dampers and \( c_{tg} \) is the target performance (vibration suppression ratio with respect to the uncontrolled structure).

The optimization problem posed in (43) is a nonlinear constrained optimization problem with potentially multiple local minima (non-convex characteristics). The challenges in this optimization problem stem from multiple sources: nonlinearities in the objective functions and constraints (use of max function for the latter creates a non-smooth problem), and trade-off in the performance between the capacity of dampers at different floors. To address these challenges a global optimization algorithm is adopted and implemented through the TOMLAB optimization environment (Holmström et al., 2010), a powerful optimization toolbox that has been extensively used in engineering applications. The chosen solver seeks the optimum solution initially through a derivative-free, direct search phase (to avoid problems with multiple local minima), and then refines the solution through a gradient-based phase (to converge to feasible solutions in the boundary of the constraints). This solver can efficiently account for nonlinearities and existence of multiple local minima even for large dimensional design vectors, which is crucial for application to high-rise buildings.

In the case of nonlinear dampers, cost function \( J_1 \) does not change in the sense that linear and nonlinear damper distributions having the same values of \( c_{eqt} \) (equal to \( c_{dl} \) for linear dampers) lead to the same value of \( J_1 \). The \( J_2 \) and \( J_3 \) objective functions [and their modifications according to (41)], though, do take different values depending on whether dampers are a linear or nonlinear. The difference stems from the modification established to calculate the peak damper force given by (38), and from the modification factor used to calculate axial loads given by (40). If these modifications were not established, and peak forces were calculated using direct statistical linearization, then, again, no differences would exist.
2.4.2. Extension to multi-objective optimization

The design problem is extended now to a multi-objective problem, explicitly incorporating \( h(\sigma_\delta) \) as objective function, rather than merely as a constraint. This can be equivalently considered as examining the single-objective design problem discussed in the previous section with different targets for the response reduction. This then leads to the following dual-objective optimization:

\[
c_{eq}^* = \arg\min_{c_{eq} \in \mathbb{R}^{n_d}} \left( J_l(c_{eq}), h(\sigma_\delta) \right)
\]  

(44)

Since the objectives considered “compete” against each other there is no solution that simultaneously minimizes both of them. Instead, solution of this bi-objective optimization problem leads to a set of dominant designs. A design is termed dominant, and belongs in the Pareto set of optimal solutions, if there is no other design that can simultaneously improve both performance objectives. The representation of the Pareto set in the objective space corresponds to the Pareto front. This multi-objective problem (44) is solved here through the epsilon-constrain approach (Mavrotas, 2009), a popular algorithm for finding solutions of bi-objective optimization problems due to its ability to identify non-convex parts of the Pareto front. Ultimately this leads to a formulation identical to (43), as a single-objective, constrained optimization problem with different thresholds \( c_{tg} \) associated with the constraint. Systematic variation of \( c_{tg} \) leads to identification of the entire Pareto front.

2.5. Case study

The design framework is illustrated next for the retrofit of an existing 26-storey Chilean building with viscous dampers (Ugalde & López-García, 2017). The chosen building, having base dimensions of 13 by 49 meters, total seismic weight of 1169 tonf and typical storey-height of 2.52 meters, is a typical example of Chilean high-rise building (in terms of structural properties and modal characteristics).
The excitation and structural model characteristics are discussed, and then the various design implementations are examined in detail. With respect to the latter, initially focus is placed on impact of the objective function selection, looking at the $n_c = 1$ case. Five different design cases are examined with respect to objective function definition, these correspond to optimization for objective functions $J_1, J_2, J_{2m}, J_3$ and $J_{3m}$, and will be referenced, respectively, as $D_1, D_2, D_{2m}, D_3$ and $D_{3m}$. Then the impact of bracing schemes in which dampers are non-consecutively attached ($n_c > 1$) is examined, followed by a comparison between the proposed optimization scheme to simplified damper distribution approaches. Results for the multi-objective design formulation are then presented and the nonlinear damper optimization is assessed, focusing specifically on comparison to linear optimization performance and the benefits of explicitly considering nonlinearities when designing a supplemental damping system. Finally, time-history analysis is performed to validate trends obtained from the stationary analysis against patterns observed under real (recorded) ground motions. Unless otherwise indicated, the target reduction of interstorey drift response is set equal to 40% i.e., the quantity $c_{tg}$ in (43) is set chosen to be 0.60. Based on plan dimensions for the considered structure, design loads and number of storeys, axial load $\bar{N}$ is set equal to 35 tonf and quantity $n_r$ is set equal to 5. Parameter $\omega_N$ needed for column strengthening cost is set equal to 1000 USD following a parametric analysis discussion detailed later. Based on commercially available data maximum force capacity $f_{max}$ of a single damper is set equal to 815 tonf (Taylor Devices Inc., 2015). Note that tonf is used herein to describe forces, as is the standard in the Chilean region. Conversion to kN, if desired, can be established by multiplying current force values by $g$ constant.

2.5.1. Excitation model

The Clough-Penzien filter, corresponding to a modification of the traditional Kanai-Tajimi filter, is adopted to represent seismic excitations within the proposed
framework. The Power Spectral Density Function (PSDF) of the filter is given by (E. Tubaldi, Barbato, & Dall’Asta, 2014):

\[
S_{CP}(\omega) = S_w \frac{\omega_g + 4\xi_g^2\omega^2\omega_g^2}{(\omega_g^2 - \omega^2)^2 + 4\xi_g^2\omega^2\omega_g^2} \frac{\omega^4}{(\omega_p^2 - \omega^2)^2 + 4\xi_p^2\omega^2\omega_p^2}
\]  

(45)

where \(\omega_g\) and \(\xi_g\) are the frequency and damping ratio of the main filter (this represents the Kanai-Tajimi filter), \(\omega_f\) and \(\xi_f\) are the same parameters of the second filter (used to attenuate low frequency content) and \(S_w\) is the spectral intensity scaling. The state-space representation of this PSDF is presented in Appendix A. The parameters of the filter are chosen through calibration to a suite of ground motions that are representative of the Chilean seismic hazard. Ground motions recorded during the \(M_w 8.8\) 2010 Maule (Chile) earthquake are chosen for this purpose. From the available ground motions, only the ones recorded on what is defined as Soil Type II (soft rock or stiff soil) in the Chilean seismic design code NCh433 (INN, 2013) are considered. Soil Type II is the soil type at the location of the considered building. The 18 selected ground motions are summarized in Table 2-1. The maximum displacement, pseudo-velocity and pseudo-acceleration spectra of the selected records can be found on Appendix B.
Table 2-1: List of selected ground motions

<table>
<thead>
<tr>
<th>Station</th>
<th>PGA [g]</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>N-S component</td>
<td>E-W component</td>
</tr>
<tr>
<td>Constitucion</td>
<td>0.35</td>
<td>0.54</td>
<td></td>
</tr>
<tr>
<td>Hualane</td>
<td>0.38</td>
<td>0.38</td>
<td></td>
</tr>
<tr>
<td>Llolleo</td>
<td>0.67</td>
<td>0.33</td>
<td></td>
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<tr>
<td>Matanzas</td>
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<td>0.34</td>
<td></td>
</tr>
<tr>
<td>Papudo</td>
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<td>0.30</td>
<td></td>
</tr>
<tr>
<td>Maipu</td>
<td>0.56</td>
<td>0.49</td>
<td></td>
</tr>
<tr>
<td>Penalolen</td>
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<td>0.29</td>
<td></td>
</tr>
<tr>
<td>Talca</td>
<td>0.22</td>
<td>0.47</td>
<td></td>
</tr>
<tr>
<td>Vina Centro</td>
<td>0.22</td>
<td>0.33</td>
<td></td>
</tr>
</tbody>
</table>

The proposed calibration has two steps: i) selection of the main filter parameters so that the normalized PSDF of the Clough-Penzien filter matches the average normalized PSDF of the ground motions in Table 2-1 and ii) selection of intensity parameter $S_w$ so that the ground acceleration has a specific standard deviation $\sigma_g$. Normalization is established so that the area under the PSDF is unity, in other words the variance of the normalized stochastic excitation is unity. The one-sided PSDF of each accelerogram is first calculated numerically, it is then normalized, and finally all normalized PSDFs are averaged following procedure described in (Mohraz & Sadek, 2001) to obtain the normalized hazard PSDF $S^n(\omega)$. The parameters of the Clough-Penzien filter are then fitted through nonlinear least squares optimization to match $S^n(\omega)$. Resulting values are $\omega_g = 16.46$ rad/s, $\xi_g = 0.5969$, $\omega_p = 6.845$ rad/s and $\xi_p = 0.4753$. The averaged PSDF $S^n(\omega)$ and the fitted PSDF $S_{CP}(\omega)$ are plotted in Figure 2-2.
The value of parameter $S_w$ is finally defined in such a way that the (assumed stationary) ground acceleration random process has a specific standard deviation $\sigma_g$ (Acciani et al., 2015):

$$S_w = \frac{2\sigma_g^2 \xi_g \xi_p (w_g^4 + 4\xi_g \xi_p \omega_p^3 \omega_p + 2(2\xi_g^2 + 2\xi_p^2 - 1)\omega_p^2 \omega_p^2 + 4\xi_g \xi_p \omega_g \omega_p^3 + \omega_p^4)}{\pi \omega_g^2 \left((1 + 4\xi_g^2)\xi_p \omega_p^2 + \xi_g (1 + 16\xi_g^2 \xi_p^2)\omega_g^2 \omega_p + 16\xi_g^4 \xi_p \omega_g \omega_p^3 + 4\xi_p^3 \omega_p^3\right)}$$ (46)

The target $\sigma_g$ is calculated based on a target Peak Ground Acceleration (PGA), with the latter defined based on NCh433 provisions for Seismic Zone 2 (where Santiago City is located) as 0.30g. Assuming a peak factor $p_f = PGA/\sigma_g = 2$ (same value adopted earlier), the target $\sigma_g$ is 0.15g and then (46) gives $S_w = 0.0200$ m$^2$/s$^3$.

![Figure 2-2: Clough-Penzien PSDF fitted to empirical PSDF](image)

**2.5.2. Structural models**

Two 2D models are developed for the structure (one along each of the two main orthogonal axes $x$ and $y$), obtained by static condensation of the initial 3D structural
model. The degrees of freedom of the resulting 2D models are the lateral floor displacements of each of the 26 storeys. The 2D models are denoted 26X and 26Y, respectively, and their first and second vibration periods are $T_{1x} = 1.29$ sec., $T_{2x} = 0.30$ sec., and $T_{1y} = 1.51$ sec., $T_{2y} = 0.35$ sec., respectively. The modal participating mass ratios of the first and second mode are 0.69 and 0.18 for the 26X model, and 0.66 and 0.18 for the 26Y model. The inherent damping is assumed equal to 5% for all modes.

2.5.3. Optimization comparison for different cost functions for linear dampers connecting consecutive floors

Discussion on results starts by examining the impact of the selection of the cost function, looking initially at linear dampers connecting consecutive floors (i.e., $n_c = 1$). In this subsection all five different design cases are examined, $D_1, D_2, D_{2m}, D_3$ and $D_{3m}$, corresponding, recall, to objective functions $J_1, J_2, J_{2m}, J_3$ and $J_{3m}$, respectively. These correspond to the three different cost functions discussed in Section 2.3.1 ($J_1, J_2$ and $J_3$) and the modifications established for the latter two ($J_{2m}$ and $J_{3m}$) when considering the maximum force capacity of the dampers as detailed in Section 2.3.2. Results are summarized in Table 2-2, which for each design case presents: all cost functions $J_l$ evaluated at optimal design vector for each design case; column strengthening cost $C_{cs}$; maximum peak force over all dampers $\max(f_{do})$; damping ratio of the retrofitted structure at the fundamental mode $\xi$; mean RMS interstorey drift $\overline{\sigma_\delta}$ and floor acceleration $\overline{\sigma_{a_t}}$; maximum RMS drift $\max(\sigma_\delta)$ and floor acceleration $\max(\sigma_{a_t})$; and mean ratio per storey of RMS interstorey drifts $\sigma_\delta/\sigma_\delta^o$ and floor accelerations $\sigma_{a_t}/\sigma_{a_t}^o$ respect to the RMS values of the structure without dampers, which is a representation of the average response reduction per storey.

The normalized interstorey drift and floor acceleration RMS response values of model 26X and 26Y are shown in Figure 2-3 to Figure 2-6. The normalization is performed with respect to the response of the structure without any dampers, which corresponds to 1.24‰ for 26X and 1.30‰ for 26Y for interstorey drifts, and 0.36g for
both models for floor accelerations. Consistent with the performance objectives set in the design the maximum interstorey drift response of the retrofitted structures is always the same (and corresponding to the targeted reduction of 40% compared to the structure without the dampers), but not always at the same storey. The damper distribution has, therefore, an effect on the displacement patterns along the height of the structure. The acceleration distribution (considerable acceleration values at lower stories) also demonstrates the relative important contribution from the second mode for this structure. Note that for a peak factor $p_f$ equal to 2 the peak interstorey drift response of the model with no supplemental dampers is roughly equal to 2.5‰, lower than the threshold commonly assumed for damage to concrete structures (5‰). This is consistent with the observed behavior of the building during the 2010 Chile earthquake (no reported structural damages). However, the building (and many other similar structures) did suffer significant nonstructural damages for both drift and acceleration sensitive components, which is the reason why the reduction of the seismic response is nevertheless a relevant issue, even for the hazard level represented by the 2010 quake. Of course, larger seismic events can always arise and for them vibration suppression in terms of interstorey drift might also be a relevant for damages to structural components.

Looking at the results, we should point out first that, as expected, each design case minimizes the corresponding cost function, indicating that the challenging non-convex optimization is efficiently solved with the adopted techniques. Of course the important discussion is the relative comparison between the design cases across all performance objectives. Design case $D_1$ minimizes force demands on dampers (both $J_1$ and $\max(f_{do})$), but at the expense of relatively large values of cost-related quantities $J_2$ and $J_3$. When considering $D_2$, design leads to reductions in $J_2$ (with respect to that for $D_1$) equal to 44% in model 26X and 18% in model 26Y, although at the expense of large values of maximum peak damper force $\max(f_{do})$ that might not be feasible. Optimal design $D_3$ leads to higher values of upfront cost $J_2$ than $D_2$ but smaller values of total cost (i.e., including column strengthening) $J_3$. Considering both building models, $J_3$ is reduced on average by 24%.
with respect to $D_1$ and by 18% with respect to $D_2$, and column strengthening cost $C_{cs}$ is reduced by 36% and 53%, respectively. In all instances the maximum damper force $\max(f_{do})$ exceeds the feasible limit of 815 tonf for a single damper. This means that design cases $D_{2m}$ and $D_{3m}$ will lead to differences corresponding to their counterparts that do not explicitly incorporate maximum force capacity considerations. Results in Table 2-2 indicate that consideration of $f_{max}$ leads to designs $D_{2m}$ and $D_{3m}$ having not only smaller values of $J_{2m}$ and $J_{3m}$ (as expected) but also smaller values of $\max(f_{do})$ than those given by $D_2$ and $D_3$, respectively. This shows that consideration of the maximum force in the damper cost leads to a distribution that avoids excessively large dampers placed at a single storey. This trend is further discussed later, when looking explicitly at the distribution of the damper forces. Moreover, in $D_{2m}$ and $D_{3m}$ designs mean floor acceleration reduction per storey $\frac{\sigma_{a_t}}{\sigma_{a_o}}$ seems to be reduced up to 6% more respect to their corresponding counterparts, which is another advantage of the more uniform distribution established through considering the maximum force capacity. Overall, this discussion indicates that consideration of $f_{max}$ is indeed relevant in the formulation of the optimization problem.
Table 2-2: Responses for models 26X and 26Y for different design cases ($n_c = 1$).

<table>
<thead>
<tr>
<th></th>
<th>26X</th>
<th></th>
<th>26Y</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$D_1$</td>
<td>$D_2$</td>
<td>$D_2m$</td>
</tr>
<tr>
<td>$J_1$ [tonf]</td>
<td>2553</td>
<td>2829</td>
<td>2601</td>
</tr>
<tr>
<td>$J_2$ [$10^3$ USD]</td>
<td>161</td>
<td>90</td>
<td>117</td>
</tr>
<tr>
<td>$J_{2m}$ [$10^3$ USD]</td>
<td>187</td>
<td>165</td>
<td>150</td>
</tr>
<tr>
<td>$J_3$ [$10^3$ USD]</td>
<td>229</td>
<td>204</td>
<td>203</td>
</tr>
<tr>
<td>$J_{3m}$ [$10^3$ USD]</td>
<td>255</td>
<td>279</td>
<td>237</td>
</tr>
<tr>
<td>$C_{cs}$ [$10^3$ USD]</td>
<td>69</td>
<td>114</td>
<td>86</td>
</tr>
<tr>
<td>max($f_{do}$) [tonf]</td>
<td>1382</td>
<td>4775</td>
<td>2256</td>
</tr>
<tr>
<td>$\xi$ [%]</td>
<td>14.3</td>
<td>15.9</td>
<td>14.4</td>
</tr>
<tr>
<td>$\sigma_\delta$ [%]</td>
<td>0.66</td>
<td>0.67</td>
<td>0.67</td>
</tr>
<tr>
<td>max($\sigma_\delta$) [%]</td>
<td>0.74</td>
<td>0.74</td>
<td>0.74</td>
</tr>
<tr>
<td>$\sigma_{\alpha_t}$ [g]</td>
<td>0.13</td>
<td>0.14</td>
<td>0.13</td>
</tr>
<tr>
<td>max($\sigma_{\alpha_t}$) [g]</td>
<td>0.19</td>
<td>0.23</td>
<td>0.20</td>
</tr>
<tr>
<td>$\sigma_\delta/\sigma_{\delta o}$ [%]</td>
<td>59</td>
<td>60</td>
<td>60</td>
</tr>
<tr>
<td>$\sigma_{\alpha_t}/\sigma_{\alpha_{t o}}$ [%]</td>
<td>55</td>
<td>62</td>
<td>56</td>
</tr>
</tbody>
</table>

With respect to the overall damper efficiency the results in last seven rows of Table 2-2 show that similar vibration suppression is established through all five examined design cases, with the targeted reduction of 40% for maximum interstorey drift established with approximately 10% of supplemental damping ratio (as intrinsic damping ratio is 5%). Such a supplemental damping threshold has been set as a reasonable objective in many viscous damper applications (Hwang et al., 2013; Landi et al., 2015). With respect to the drift and acceleration reduction, the mean drift reduction per storey $\sigma_\delta/\sigma_{\delta o}$ and mean acceleration reduction per storey $\sigma_{\alpha_t}/\sigma_{\alpha_{t o}}$ take similar values as the target of 40%.
reduction of the maximum drift. This shows that the proposed design scheme establishes a similar vibration suppression across all storeys, not only at the storeys where interstorey drifts are maximized.

Figure 2-3: Normalized RMS interstorey drifts for model 26X
Figure 2-4: Normalized RMS floor accelerations for model 26X

Figure 2-5: Normalized RMS interstorey drifts for model 26Y
Discussion now moves to the distribution of damper across the height, with results shown in Figure 2-7 to Figure 2-9, all of them illustrating the strong influence of the target cost function on the optimized height-wise damper distribution. Figure 2-7 shows the optimal damping coefficient, Figure 2-8 the associated peak damper forces and Figure 2-9 the corresponding peak column axial loads. Results only for model 26X are reported due to space limitations (results for 26Y follow similar trends). $D_1$ leads to distribution where small dampers (quantified in terms of force capacity) are located at several stories. Minimization of upfront damper cost ($D_2$ design case) leads to a distribution where a single larger damper is located at a mid-height, but such distribution leads to larger axial loads on columns. Minimization of total upfront cost (damper cost plus cost of column strengthening, $D_3$ design case) leads to a distribution where again a larger damper is located at a mid-height storey but at a lower location (and with a slightly reduced capacity), resulting in smaller axial loads on columns. In both latter cases consideration of maximum damper force capacity (i.e., $D_{2m}$ and $D_{3m}$) leads to significantly different

Figure 2-6: Normalized RMS floor accelerations for model 26Y
damper distribution and axial loads on columns. For instance, the large damper at the 16th storey in $D_2$ is replaced by two smaller dampers at the 15th and 16th storeys in $D_{2m}$. In passing, it is noted that as shown in Figure 2-4 and Figure 2-6 floor accelerations tend to be larger when the optimized damper distribution consists of a few large dampers than when it consists of several smaller dampers, another possibly relevant consideration (not further explored in this investigation) for optimization of height-wise damper distribution.

![Figure 2-7: Optimized height-wise distributions for model 26X for different design cases](image)

Figure 2-7: Optimized height-wise distributions for model 26X for different design cases

Figure 2-10 illustrates the influence of the value of $\omega_N$ in (39) on $J_3$ for $D_3$. As expected, if the cost of column strengthening $C_{cs}$ is relatively low then total cost $J_3$ is essentially equal to the damper cost $C_{up}$. If the cost of column strengthening is higher, on the other hand, then total upfront cost is essentially equal to the cost of column strengthening. In all the examples shown in this paper the value of $\omega_N$ is set equal to 1000 USD, a reasonable choice within the range of values where both the damper upfront cost and the cost of column strengthening significantly contribute to the total cost $J_3$. 
Figure 2-8: Peak damper forces for model 26X for different design cases

Figure 2-9: Peak axial load demand on columns under optimal damper distribution for model 26X for the different design cases
Assessment moves next to examining dampers connected to non-consecutive floor levels (i.e., \( n_c > 1 \)), with results summarized in Table 2-3, focusing in this case on the performance improvement between \( n_c > 1 \) and \( n_c = 1 \) cases. To facilitate an easier presentation herein sub-index “1” refers to the case where \( n_c = 1 \) and sub-index "\( n_c \)" indicates the case where \( n_c > 1 \). For each of the design cases performance is reported only with respect to the corresponding cost function used in the optimization, for example when design case \( D_1 \) is examined the ratio of objective functions \( J_{l1}/J_{ln_c} \) corresponds to cost function \( J_1 \) (\( l = 1 \)). Comparison of performance across the different objectives and design cases has been already discussed in detail in the previous section. Other results reported in Table 2-3 include further comparison between \( n_c = 1 \) and \( n_c > 1 \) cases: fundamental mode damping ratio \( \xi_1/\xi_{nc} \); total damping coefficient ratio \( C_{o1}/C_{onc} \); and RMS damper velocities ratio \( \sigma_{vnc}/\sigma_{v1} \). Average results over the 5 design cases are also
considered for an easier comparison, and are reported as the quantity $\bar{D}_l$ in Table 2-3 (column with bold letters in that Table). Results clearly indicate that, for the same level of energy dissipation, when $n_c > 1$ the value of all cost functions is reduced by a factor approximately equal to $n_c$ with respect to that when $n_c = 1$. The fundamental mode damping ratio (representative of energy dissipation) remains essentially constant (i.e., $\xi_1/s_{n_c} \approx 1$), but the sum of damper coefficients $C_o$ is reduced by a factor roughly equal to $n_c^{1+\alpha} = n_c^2$ and damper velocities $\sigma_v$ increase by a factor roughly equal to $n_c$, which leads to an average decrease of damper forces (and consequently of costs) by a factor approximately equal to $n_c$. These results are consistent with the theoretical discussion in Section 2.2.3. Axial loads on columns are examined in Figure 2-11 focusing, again, on model 26X. Results show that axial load demands when $n_c > 1$ decrease by a factor approximately equal to $n_c$ with respect to those when $n_c = 1$. This observation can be more clearly appreciated at the foundation level. For instance, for $D_2$ the axial demand is roughly equal to 5100 tonf when $n_c = 1$, whereas when $n_c = 3$ the axial load demand is roughly equal to 1700 tonf. Overall, these results clearly highlight the benefits of bracing schemes where supplemental dampers are anchored at non-consecutive floor levels. Should point out, though, that the angle of the bracing scheme has not been explicitly considered in the formulation; if $n_c > 1$ is facilitated with steeper angles (closer to vertical) then the relative contribution of the damper forces to axial loads will increase.
Table 2-3: Effect of $n_c$ on different response quantities for different design cases

<table>
<thead>
<tr>
<th></th>
<th>26X</th>
<th></th>
<th></th>
<th></th>
<th></th>
<th>26Y</th>
<th></th>
<th></th>
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<tbody>
<tr>
<td></td>
<td>$D_1$</td>
<td>$D_2$</td>
<td>$D_{2m}$</td>
<td>$D_3$</td>
<td>$D_{3m}$</td>
<td>$\overline{D_l}$</td>
<td>$D_1$</td>
<td>$D_2$</td>
<td>$D_{2m}$</td>
<td>$D_3$</td>
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<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\xi_1/\xi_{n_c}$</td>
<td>1.00</td>
<td>0.96</td>
<td>0.85</td>
<td>0.95</td>
<td>1.07</td>
<td><strong>0.96</strong></td>
<td>0.98</td>
<td>1.06</td>
<td>1.04</td>
<td>0.82</td>
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<tr>
<td>$C_{o1}/C_{on_c}$</td>
<td>3.95</td>
<td>4.16</td>
<td>2.78</td>
<td>3.41</td>
<td>4.54</td>
<td><strong>3.77</strong></td>
<td>4.41</td>
<td>4.68</td>
<td>4.33</td>
<td>2.36</td>
</tr>
<tr>
<td>$\bar{\sigma}<em>{vn_c}/\bar{\sigma}</em>{v1}$</td>
<td>2.01</td>
<td>2.19</td>
<td>1.83</td>
<td>2.20</td>
<td>1.97</td>
<td><strong>2.04</strong></td>
<td>2.22</td>
<td>2.09</td>
<td>1.99</td>
<td>2.03</td>
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<tr>
<td>$J_{i1}/J_{in_c}$</td>
<td>1.98</td>
<td>1.89</td>
<td>1.64</td>
<td>1.71</td>
<td>2.09</td>
<td><strong>1.86</strong></td>
<td>1.95</td>
<td>2.09</td>
<td>2.15</td>
<td>1.52</td>
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<td>$n_c = 3$</td>
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<tr>
<td>$\xi_1/\xi_{n_c}$</td>
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<td>1.02</td>
<td>0.95</td>
<td>1.03</td>
<td>0.98</td>
<td><strong>0.99</strong></td>
<td>0.97</td>
<td>1.12</td>
<td>1.00</td>
<td>0.89</td>
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<tr>
<td>$C_{o1}/C_{on_c}$</td>
<td>8.68</td>
<td>7.51</td>
<td>9.00</td>
<td>8.30</td>
<td>7.93</td>
<td><strong>8.28</strong></td>
<td>9.71</td>
<td>10.71</td>
<td>8.39</td>
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<tr>
<td>$\bar{\sigma}<em>{vn_c}/\bar{\sigma}</em>{v1}$</td>
<td>3.03</td>
<td>2.70</td>
<td>3.34</td>
<td>2.87</td>
<td>2.96</td>
<td><strong>2.98</strong></td>
<td>3.31</td>
<td>2.84</td>
<td>2.76</td>
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<tr>
<td>$J_{i1}/J_{in_c}$</td>
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<td>2.93</td>
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<td>2.89</td>
<td>3.14</td>
<td>2.94</td>
<td>2.34</td>
</tr>
</tbody>
</table>

Figure 2-11: Effect of $n_c$ on peak axial loads on columns for model 26X for three different design cases
2.5.5. Comparison to Uniform and SSA height-wise distributions

To investigate the benefits of height-wise damper distributions given by the proposed explicit optimization approach, two other well-known height-wise distribution schemes are considered here. The first approach is the Uniform distribution, i.e., same damper capacity in all storeys. The second approach is the SSA, where dampers are placed sequentially at the storey where the value of a performance index (interstorey drift in this study) reaches a maximum. Implicitly, the algorithm assumes that the gradient of the objective function takes its maximum value at the storey having the largest value of the performance index (Aguirre, 2015). The computation of the performance index is based on the second order statistics of the response. In both schemes the same target performance is adopted, i.e. reduction of drift corresponding to $c_{tg}$.

The comparison is based on linear dampers. Results are summarized in Table 2-4, which shows the ratio of the value of the cost function when the damper distribution is formally optimized to the value of the same cost function when the damper distribution is either the Uniform or that given by the SSA. The superscripts $U$ and $SSA$ are used, respectively, to describe these two simplified distributions. Similar to Table 2-3, focus is only on the corresponding cost function for each design approach. Results clearly indicate that the proposed explicit design approach leads to noticeably smaller values of cost functions in nearly every case, regardless of the value of $n_c$. On average, formal optimization reduces cost by 48% with respect to the Uniform distribution and by 21% with respect to the SSA distribution. Differences are more noticeable when nonlinearities in the target cost function are more relevant and no constraints exist for maximum damper forces (so scheme is allowed to benefit more from an imbalanced distribution), i.e. bigger differences in the $D_2$ design case rather than $D_{zm}$. While cost is not explicitly considered in its formulation, the SSA is generally more cost-effective than the Uniform distribution, most likely because, albeit simplified and sequential in nature (rather than targeting total
still incorporates an explicit damper distribution based on optimality criteria and structural performance, whereas the Uniform distribution does not.

Table 2-4: Performance ratios $J_i/J_i^U$ and $J_i/J_i^{SSA}$ of optimal to simplified damper distribution schemes for different design cases and $n_c$ values

<table>
<thead>
<tr>
<th>$n_c$</th>
<th>26X</th>
<th>26Y</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$D_1$</td>
<td>$D_2$</td>
</tr>
<tr>
<td>UNIFORM ($J_i/J_i^U$)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>0.90</td>
<td>0.34</td>
</tr>
<tr>
<td>2</td>
<td>0.90</td>
<td>0.34</td>
</tr>
<tr>
<td>3</td>
<td>0.91</td>
<td>0.37</td>
</tr>
<tr>
<td>SSA ($J_i/J_i^{SSA}$)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>0.93</td>
<td>0.69</td>
</tr>
<tr>
<td>2</td>
<td>0.93</td>
<td>0.70</td>
</tr>
<tr>
<td>3</td>
<td>0.96</td>
<td>0.93</td>
</tr>
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</table>
Figure 2-12: Comparison between proposed (design case $D_{3m}$ depicted) and simplified distributions for normalized RMS interstorey drifts for model 26X

Figure 2-13: Comparison between proposed (design case $D_{3m}$ depicted) and simplified distributions for normalized RMS floor accelerations for model 26X
Different response quantities for model 26X with \( n_c = 1 \) are shown in Figure 2-12 to Figure 2-15, focusing on \( D_{3m} \) design case (same patterns hold for the other ones). Figure 2-12 illustrates the normalized interstorey drifts and Figure 2-13 the normalized floor acceleration for the proposed optimization distribution and simplified distributions following similar normalization format as the one discussed in Section 2.5.3. Figure 2-14 shows same comparison approach for peak damper forces per storey and Figure 2-15 for peak axial loads on columns. Figure 2-12 indicates that, given a target response reduction (40%), the interstorey drift response is essentially the same in most storeys regardless of the damper distribution, but Figure 2-13 shows some important differences in the floor acceleration response. As observed before, floor accelerations tend to be larger when the damper distribution is such that supplemental damping is concentrated at relatively few stories. This is the case of both the SSA distribution and the distribution given by explicit optimization. However, since the latter aims at minimizing the total cost \( J_{3m} \), most of the “large” dampers are placed at stories below those where the “large” dampers of SSA distribution are located (Figure 2-14), resulting in much smaller axial loads on columns (Figure 2-15), even smaller than when the damper distribution is simply the Uniform.
Figure 2-14: Comparison between proposed (design case $D_{3m}$ depicted) and simplified distributions for peak damper forces for model 26X

Figure 2-15: Comparison between proposed (design case $D_{3m}$ depicted) and simplified distributions for peak axial loads on columns for model 26X
2.5.6. Multi-objective optimization

In all applications examined so far the target performance is a 40% reduction (i.e., $c_{tg} = 0.60$) of the interstorey drift response with respect to the response with no supplemental dampers. In this subsection results obtained considering other levels of target performance (i.e., other values of factor $c_{tg}$) are examined, and possible relevance to optimization of height-wise damper distributions is investigated. This is established in the context of the multi-objective scheme discussed in Section 2.4.2. The Pareto front for target performance ratio $c_{tg}$ and total cost $J_3$ for $D_3$ design case for linear dampers is shown in Figure 2-16, in this instance for model 26Y. Results demonstrate the competing nature of the two objectives in the sense that increasingly smaller values of $c_{tg}$ are achieved at the expense of increasingly larger values of $J_3$, and vice versa. As common with many multi-objective problems, close to the minimums of each of the objectives (i.e. towards the ends of the front) further improvement of the corresponding objective (i.e. the one that is close to its minimum) come at a significant deterioration of the other objective. This observation indicates that a “balanced” design criterion, i.e., a design where neither competing objective reaches its minimum value (graphically, a mid-point of the Pareto front) might offer a more practical solution.
Figure 2-16: Pareto front of $J_3$ and $c_{tg}$ for model 26Y and design case $D_3$

Figure 2-17: Total damping ratio $\xi$ as function of $c_{tg}$ for model 26Y and design case $D_3$
Figure 2-16 also confirms that the previously described cost reductions achieved with bracing schemes anchored at non-consecutive floor levels (i.e., \( n_c > 1 \)) can also be achieved for other target performance ratios. Further insight is provided by Table 2-5, which shows the ratio of the value of total cost \( J_3 \) when \( n_c = 1 \) (\( J_{31} \)) to the value of total cost when \( n_c > 1 \) (\( J_{3n_c} \)). The design case is again \( D_3 \). The average value of \( J_{31}/J_{3n_c} \) over the whole range of target performance ratios is equal to 1.8 for \( n_c = 2 \) and 2.7 for \( n_c = 3 \), which is consistent with the analysis made in Section 2.2.3 in the sense that when \( n_c > 1 \) the cost is reduced by a factor equal to \( n_c \) with respect to that when \( n_c = 1 \), independent of the level of target vibration suppression. Noticeably, if only a range of intermediate values of \( c_{tg} \) is considered (say, 0.30–0.60) the average values of the \( J_{31}/J_{3n_c} \) ratio are equal to 1.9 and 2.9 for \( n_c = 2 \) and 3, respectively (i.e., they are even closer to \( n_c \)).

Figure 2-17 depicts the total damping ratio at the fundamental mode of the structural system when aiming at different levels of target reductions \( c_{tg} \), for different values of \( n_c \). As expected, damping ratio increases with the level of suppression required, and the lower the value of \( c_{tg} \) the higher the marginal increment required in total damping ratio to achieve it. The reported variation may serve also as a useful guide when needing to decide on target damping ratio; this selection can be made based on the desired level or vibration suppression. As for the variation with respect to \( n_c \), it can be seen that regardless of the value of \( c_{tg} \) the three curves are practically equal to each other, which further confirms that allowing dampers to connect non-consecutively reduces cost (check Figure 2-16) but produces the same level of energy dissipation, irrespective of the target performance level.
Table 2-5: Values of the $J_{31}/J_{3n_c}$ ratio for different values of $c_{tg}$ for model 26Y

<table>
<thead>
<tr>
<th>$n_c$</th>
<th>$c_{tg}$</th>
<th>0.10</th>
<th>0.20</th>
<th>0.30</th>
<th>0.40</th>
<th>0.50</th>
<th>0.60</th>
<th>0.70</th>
<th>0.80</th>
<th>0.90</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>1.85</td>
<td>1.40</td>
<td>1.66</td>
<td>1.91</td>
<td>1.78</td>
<td>1.99</td>
<td>2.04</td>
<td>2.07</td>
<td>1.46</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>2.77</td>
<td>2.07</td>
<td>2.87</td>
<td>2.80</td>
<td>2.77</td>
<td>2.83</td>
<td>3.16</td>
<td>2.65</td>
<td>2.09</td>
<td></td>
</tr>
</tbody>
</table>

Figure 2-18: $J_3$ and decomposition to its components v/s $c_{tg}$ for $n_c = 1$ and model 26Y
Figure 2-19: Ratio of $J_{3m}$ to $J_3$ for $D_3$ design case v/s $c_{tg}$ for $n_c = 1$ and model 26Y

Figure 2-18 and Figure 2-19 finally report different cost-related metrics as a function of $c_{tg}$ for $n_c = 1$. Figure 2-18 shows the variation of $C_{up}$ and $C_{cs}$ in design case $D_3$, from which it can be seen that the difference between both cost functions increases as the level of targeted vibration suppression increases. This trend shows that even when $J_3$ penalizes the inclusion of dampers that increment axial load substantially, at a high level of target reduction the inclusion of bigger dampers is compulsory to achieve the energy dissipation required, and therefore $C_{up}$ has a more relevant contribution to $J_3$ at low values of $c_{tg}$. Figure 2-19 shows the ratio of cost functions $J_{3m}(D_3)$ to $J_3(D_3)$ that corresponds to $D_3$ optimal design for different values of $c_{tg}$. This case represents the inaccuracy (in terms of cost) when purchasing commercially available dampers (i.e., being forced to adhere to a maximum force capacity limit) based on optimizations that did not explicitly consider the limitation of $f_{max}$ (i.e., design case $D_3$). It can be noted that for a low level of vibration suppression (below 10%) both costs are actually the same, as peak damper forces are not exceeding $f_{max}$. However, even for modest levels of $c_{tg}$ (20% reduction...
and higher) cost increases up to 30%, which further emphasizes the importance of an accurate description of damper upfront cost and feasible damper capacity.

2.5.7. Nonlinear dampers

The final implementation variation examined is nonlinear dampers, considering a value of the viscous exponent \( \alpha \) equal to 0.35. Focus is on comparison between linear and nonlinear implementations and discussion pertains initially to dampers connected to consecutive floor levels (i.e., \( n_c = 1 \)). The superscript \( nl \) is used to describe the nonlinear damper implementation cases. Summary results are shown in Table 2-6 and in Figure 2-20 and Figure 2-21, in all instances only for model 26X. Table 2-6 reports the ratio of the value of cost when dampers are nonlinear \( J_1^{nl} \) to the value of cost when dampers are linear \( J_1 \). The parameters reported on the rest of the table are the same as the first seven rows of Table 2-2 (simply in this instance for nonlinear damper implementation), expressing different force-based cost metrics. First, the values obtained for the \( J_1^{nl} / J_1 \) ratio indicate that, for a given level of structural performance, nonlinear dampers are more cost effective. This observation is consistent with what was reported in previous studies on nonlinear viscous dampers (W. H. Lin & Chopra, 2002). As mentioned in Section 2.4.1 the value of the \( J_1 \) metric is independent of whether the dampers are linear or nonlinear (optimal distribution with respect to equivalent viscous damping is the same), which is why the \( J_1^{nl} / J_1 \) ratio is equal to unity. This does not mean of course that other metrics and quantities (reported in this Table or in the follow up figures) are the same between linear and nonlinear damper implementations. For the other cost metrics the average cost reduction is 15%, reaching a maximum of 24% for the \( J_{3m} \) cost function (where nonlinearity is extremely relevant). Figure 2-20 includes linear and nonlinear peak forces for \( D_1, D_2 \) and \( D_3 \) design cases and illustrates that the differences between linear and nonlinear damper implementations can be indeed very relevant. Once again, for each design case the proposed optimization algorithm minimizes, as desired, the corresponding cost function, even when additional complexities created by the damper nonlinearities
exist. As for peak damper forces, it can be noted from that for every design case except $D_{2m}$ the force demand is reduced considerably, which is important in terms of feasibility when only dampers with reduced force capacity are available, and the bays that dampers can be incorporated (to facilitate the desired force demand) are limited. This can be further verified on Figure 2-20. More importantly, for the more realistic (in terms of what has been defined in this paper) design case, $D_{3m}$, this value is reduced by 41%. Figure 2-21 includes peak axial loads for the linear and nonlinear damper implementations and further illustrates the benefits of nonlinear dampers, as axial load demand is considerably reduced in most design cases. This shows that beyond the maximum peak forces (reported in Table 2-6) the average peak damper forces across the height of the structure (impacting the axial loads in Figure 2-21) also reduce. For the more realistic design case $D_{3m}$, the column strengthening cost is reduced by a factor of 54%.

Table 2-6: Values of the $J_1^{nl}/J_1$ ratio for different optimized distributions for model 26X

<table>
<thead>
<tr>
<th>$J_1^{nl}/J_1$</th>
<th>$D_1$</th>
<th>$D_2$</th>
<th>$D_{2m}$</th>
<th>$D_3$</th>
<th>$D_{3m}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$J_1$ [tonf]</td>
<td>1.00</td>
<td>0.84</td>
<td>0.84</td>
<td>0.80</td>
<td>0.76</td>
</tr>
<tr>
<td>$J_2$ [$10^3$ USD]</td>
<td>137</td>
<td>76</td>
<td>77</td>
<td>102</td>
<td>110</td>
</tr>
<tr>
<td>$J_{2m}$ [$10^3$ USD]</td>
<td>152</td>
<td>128</td>
<td>126</td>
<td>153</td>
<td>137</td>
</tr>
<tr>
<td>$J_3$ [$10^3$ USD]</td>
<td>189</td>
<td>168</td>
<td>163</td>
<td>128</td>
<td>134</td>
</tr>
<tr>
<td>$J_{3m}$ [$10^3$ USD]</td>
<td>205</td>
<td>220</td>
<td>213</td>
<td>180</td>
<td>161</td>
</tr>
<tr>
<td>$C_{cs}$ [$10^3$ USD]</td>
<td>53</td>
<td>92</td>
<td>87</td>
<td>27</td>
<td>24</td>
</tr>
<tr>
<td>max($f_{do}$) [tonf]</td>
<td>1057</td>
<td>3871</td>
<td>3628</td>
<td>2873</td>
<td>1436</td>
</tr>
</tbody>
</table>
Figure 2-20: Comparison for linear and nonlinear damper implementations of peak damper forces for model 26X

Figure 2-21: Comparison for linear and nonlinear damper implementations of peak axial loads for model 26X
Table 2-7: Effect of $n_c$ on the $J_{3m}$ cost function when dampers are nonlinear ($\alpha = 0.35$)

<table>
<thead>
<tr>
<th></th>
<th>$n_c$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>2</td>
</tr>
<tr>
<td>$\xi_1/\xi_{n_c}$</td>
<td>1.06</td>
</tr>
<tr>
<td>$C_{o1}/C_{on_c}$</td>
<td>2.83</td>
</tr>
<tr>
<td>$\bar{\sigma}<em>{o</em>{n_c}}/\bar{\sigma}_{o_1}$</td>
<td>1.98</td>
</tr>
<tr>
<td>$J_{11}/J_{1n_c}$</td>
<td>2.02</td>
</tr>
</tbody>
</table>

Results for connection to non-consecutive floor levels (i.e., $n_c > 1$) are reported in Table 2-7, in format similar to Table 3, considering in this instance only the nonlinear damper distribution that minimizes function $J_{3m}$ (i.e., $D_{3m}^{nl}$). Results are again consistent with the theoretical analysis of Section 2.2.3. For a given level of energy dissipation (i.e., $\xi_1/\xi_{n_c} \approx 1$) supplemental damping $C_o$ is reduced by a factor equal to 2.83 for $n_c = 2$ and equal to 4.21 for $n_c = 3$, values that are very similar to the anticipated values $n_c^{1+\alpha}$, i.e., $2^{1.35} = 2.55$ when $n_c = 2$ and $3^{1.35} = 4.41$ when $n_c = 3$. Damper RMS velocities $\sigma_{\psi}$ increase by a factor roughly equal to $n_c$, and cost $J_{3m}$ decrease by a factor also roughly equal to $n_c$. Thus, as predicted, the benefits of bracing schemes where $n_c > 1$ are essentially the same regardless of whether the dampers are linear or nonlinear.

### 2.5.8. Time history analysis validation

So far the performance comparisons were established considering the RMS response under stationary excitation, with peak quantities approximated through the peak factor. Here the response of the structure considering time-history analysis for a set of recorded ground motions is examined for validation purposes. This set is the one utilized in Section 2.5.1 for establishing the stochastic excitation model. Each record is scaled so that the 5%-damped pseudo-acceleration spectral ordinate at the fundamental period of the structural model is equal to that indicated by the Chilean seismic design code NCh433.
The average PGA value of the scaled records is 0.26g, which is somewhat less than the 0.30g value indicated by NCh433 (recall that 0.30g value was considered when defining factor $S_w$ for the stochastic excitation). Overall this setting facilitates consistency in the modeling between the time-history and stochastic analyses.

The response of the 26X model equipped with the optimal damper height-wise distribution according to $D_1$, $D_2$ and $D_3$ designs for linear dampers with $n_c = 1$ is obtained by linear time history analysis and compared to the response obtained through the stochastic analysis in Figure 2-22 for normalized peak interstorey drifts and in Figure 2-23 for normalized peak floor accelerations. Results corresponding to stochastic stationary analysis are denoted by SS whereas those given by time history analysis are denoted by TH. Normalization is always performed with respect to the response of the uncontrolled structure. For the TH case the normalization thresholds are 1.897‰ for interstorey drifts and 0.695g for floor accelerations. The results in Figure 2-22 show that performance in both instances (TH or SS) is similar with an interstorey drift response reduction of roughly 40%. This observation is consistent with the discussions reported in other studies (Lavan, 2015). With respect to normalized floor acceleration response (Figure 2-23) TH and SS results are quantitatively similar to each other (essentially identical in some instances cases), although qualitatively less than the interstorey drift response (specially for $D_3$ design). Setting apart the response at a few mid-height floor levels, overall the performance predicted by SS modeling is similar to the one obtained by TH analysis.
Figure 2-22: Comparison between TH and SS for normalized peak interstorey drifts for model 26X

Figure 2-23: Comparison between TH and SS for normalized peak floor accelerations for model 26X
Figure 2-24: comparison between peak damper forces for model 26X

Figure 2-25: comparison between peak axial loads for model 26X
A comparison in terms of damper forces is shown in Figure 2-24. Results show significant quantitative similarities. This verification is crucial, since quantification of peak damper forces is the input for most objective functions described earlier. Axial loads on columns are shown in Figure 2-25. While quantitative differences are now more significant, results have still strong qualitative similarities. An important observation, stemming ultimately from the qualitative similarity, is that the damper distribution that, based on stochastic analysis, minimizes axial loads on columns ($D_3$ design) is also the one that minimizes such quantity when the response is assessed by time history analysis. The difference might be simply attributed to the fact that a different (slightly) lower value of the peak factor should have been used. This depicts the importance of carefully selecting the peak factor for calculating peak forces in stationary analysis starting from RMS responses.

2.6. Conclusions

In this research a novel optimization framework for the height-wise distribution of supplemental viscous dampers in multistorey buildings that accounts for practical design considerations was presented. The dynamics of a linear $n_s$-storey building equipped with $n_d$ viscous dampers were described by a space state representation with which it is possible to straightforwardly: (i) calculate second order response statistics; (ii) approximate nonlinear damper forces through statistical linearization; (iii) model ground motion as a filtered Gaussian stationary white noise; and (iv) account for bracing schemes anchored at non-consecutive floor levels. Five cost functions that account (with different sophistication levels) for different relationships between cost and damper force capacity, strengthening of columns, and maximum feasible damper force capacity were considered. The performance function was defined in terms of the interstorey drift response, and was used as a constraint function by imposing a target reduction of the response in comparison with the uncontrolled structure. A single-objective, cost-minimization optimization problem was first analyzed, and then a multi-objective optimization approach in which
cost and performance were posed as two desired but competing objectives was investigated.

A 26-storey actual Chilean high-rise reinforced concrete building was studied as an illustrative example with earthquake hazard modeled according to the 2010 Maule earthquake (in terms of frequency content) and Chilean seismic code provisions (in terms of intensity). Results show the following trends:

- Explicit optimization of the damper distribution considering realistic cost metrics leads to significant savings with respect to damper distributions optimized for simplified cost metrics.
- Consideration of the cost of column strengthening has a significant impact on damper distributions, when compared to approaches that minimize only the cost of the dampers.
- Similarly, consideration of feasible damper maximum force capacity also has a significant influence on the optimal damper distribution.
- Multi-objective design setting, considering both cost and performance improvement as competing objectives, can provide valuable insight into the advantages of retrofitting schemes targeting different levels of vibration suppression.
- For a given level of energy dissipation, supplemental damping systems in which dampers are connected every $n_c$ floor levels ($n_c > 1$) reduce cost by a factor approximately equal to $n_c$ with respect to damping systems in which dampers are connected at consecutive floor levels ($n_c = 1$). This finding is independent of the level of target response reduction $c_{tg}$ (as seen in the multi-objective analysis) and of the viscous exponent $\alpha$ (as seen in the nonlinear damper analysis).
- The proposed optimization framework leads to significant savings with respect to simplified damper distribution schemes, particularly when the cost is a nonlinear function of response quantities.
Validation against real (recorded) time histories shows that performance (vibration suppression, peak damper forces) has similar characteristics to the predictions obtained through approximation by stationary stochastic dynamics.

For nonlinear damper applications, reliance on the linearized statistics for calculating peak response quantities leads to identical results (in terms of structural performance and equivalent linear characteristics) as the linear damper case. Explicit consideration of these nonlinearities for peak response calculation, though, produces results that can be more cost-effective, particularly when the optimal damper distribution consists of a few dampers, experiencing ultimately large force demands.

In general, this research shows the advantages of damper distributions optimized considering, in an explicit manner, practical (and relevant) design issues. In particular, cost functions defined in terms of damper force capacity and bracing schemes anchored at non-consecutive floor levels are of special relevance. Proper consideration of damper nonlinearities can also be of importance.
REFERENCES


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Seismic Design Handbook (pp. 47–124).


APPENDICES
APPENDIX A: DETAILS OF SPACE-STATE REPRESENTATION

The state space matrices in equation (15) are given by:

\[
A_{st} = \begin{pmatrix}
O_{n_s \times n_s} & I_{n_s \times n_s} \\
-M_s^{-1} K_s & -M_s^{-1} C_s
\end{pmatrix},
B_{st} = \begin{pmatrix}
O_{n_s \times n_d} \\
-M_s^{-1} T_s
\end{pmatrix},
E_{st} = \begin{pmatrix}
O_{n_s \times 1} \\
-R_s
\end{pmatrix}
\]

(47)

whereas matrices \(C_{st}\) and \(D_{st}\) are given by:

\[
C_{st} = \begin{pmatrix}
H_f T_r & O_{n_s \times n_s} \\
-M_s^{-1} K_s & -M_s^{-1} C_s \\
O_{n_d \times n_s} & O_{n_d \times n_s}
\end{pmatrix},
D_{st} = \begin{pmatrix}
B_{st} \\
T_f
\end{pmatrix}
\]

(48)

Matrix \(H_f\) normalizes interstorey displacements by storey height (diagonal matrix with the inverse of the corresponding storey heights) and \(T_f\) is an upper triangular matrix with unity values such that at each storey \(F_D = T_f f_d\). For the Clough-Penzien filter used in Section 2.5 the corresponding matrices of the formulation given by (16) are:

\[
A_f = \begin{pmatrix}
0 & 1 & 0 & 0 \\
-\omega_g^2 & -2\xi_g \omega_g & 0 & 0 \\
0 & 0 & 0 & 1 \\
\omega_g^2 & 2\xi_g \omega_g & -\omega_p^2 & -2\xi_p \omega_p
\end{pmatrix},
E_f = \begin{pmatrix}
0 \\
-1 \\
0 \\
0
\end{pmatrix}
\]

(49)

\[
C_f = \begin{pmatrix}
\omega_g^2 & 2\xi_g \omega_g & -\omega_p^2 & -2\xi_p \omega_p
\end{pmatrix}
\]
The matrix relating damper relative velocities to the state vector $\mathbf{x}$ is:

$$
L_{st} = (O_{n_d \times n_s} \quad T_s)
$$

whereas the matrices described in (22) needed for the augmented representation in (21) are:

$$
A_a = 
\begin{pmatrix}
A_{st} & E_{st}C_f \\
O_{n_f \times 2n_s} & A_f
\end{pmatrix}

B_a = 
\begin{pmatrix}
B_{st} \\
O_{n_f \times n_d}
\end{pmatrix}

C_a = 
\begin{pmatrix}
C_{st} & O_{2n_x \times n_f}
\end{pmatrix}

D_a = D_{st}

E_w = 
\begin{pmatrix}
O_{2n_x \times 1} \\
E_f
\end{pmatrix}
$$
APPENDIX B: MAXIMUM DISPLACEMENT, PSEUDO-VELOCITY AND PSEUDO-ACCELERATION SPECTRA FOR SELECTED RECORD

Figure B-1: Maximum displacement spectrum of selected records
Figure B-2: Pseudo-velocity spectrum of selected records

Figure B-3: Pseudo-acceleration spectrum of selected records