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On The Welfare and Distributional Effects of Implementing an Expenditure Fiscal Rule

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ON THE WELFARE AND DISTRIBUTIONAL EFFECTS OF IMPLEMENTING AN EXPENDITURE FISCAL RULE†

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Abstract

The aim of this paper is to study the welfare and distributional effects of implementing an expenditure fiscal rule in a developing country. Particularly, I want to measure these effects in a transitional dynamics environment, triggered by a negative and transitory aggregate shock. To accomplish this objective, I use an environment of incomplete markets and heterogeneous agents, following closely the frameworks developed by Aiyagari (1994) and Huggett (1993) and the spirit of Aguirre (2015). I found that, on average, agents will prefer an expenditure fiscal rule framework rather than a discretionary counter-cyclical fiscal policy. In the benchmark case, the government uses transfers to increase the consumption of agents during the aggregate shock. However, in the long run the opposite effect brought by the interest rate mechanism is stronger. Nevertheless, this effect is not the same for all agents: richer agents benefit from the increase in the interest rate while poorer agents will lose. With the fiscal rule, transfers decrease during the aggregate shock before coming back at their original level. As the interest rate does not change, the only effect is the one brought by transfers. The total effect is then smaller than in the benchmark case.

Keywords: Heterogeneous Agents, Fiscal Policy, Fiscal Rules, Welfare, Wealth Distribution.

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1 Introduction

In the aftermath of the 2008-2009 international crisis, many governments experienced important fiscal deterioration as a result of stimulus packages implemented to face this critical economic downturn. Consequently, expenditure fiscal rules have had an increasing popularity, especially as a mechanism to correct unfavorable fiscal positions resulting from fiscal stimulus. According to IMF’s FAD\(^1\) Fiscal Rules database, 29 countries currently have an expenditure fiscal rule in place, of which 14 are advanced economies and 15 are emerging and developing economies. Additionally, more than one third of these rules were introduced since 2009. After the global crisis, 15 rules were implemented, of which 90% (13) were expenditure fiscal rules, and 70% of these new expenditure fiscal rules (9) were implemented in developing countries.

Some authors have studied the harmful compositional effects that fiscal rules have had on spending (Dahan and Strawczynski (2010), Cordes et al. (2015)). One of the most common side effect relates to the risk that policymakers cut high quality spending in order to achieve compliance of the rule. Examples of this high quality spending are transfers and investment in public goods. These are important spending lines, in developing countries, to shorten wealth inequality and improve social welfare. To the best of my knowledge, no studies have taken into account the link between expenditure fiscal rules and wealth distribution and welfare.

The goal of this paper is to study the welfare and distributional effects of implementing an expenditure fiscal rule in a developing country. Particularly, it measures these effects in a transitional dynamics environment, triggered by a negative and transitory aggregate shock\(^2\) and, to accomplish this objective, an environment of incomplete markets and heterogeneous agents is used. This framework allows to generate endogenous wealth distribution and welfare measures. It follows the classic literature of heterogeneous agents (Aiyagari, 1994; Huggett, 1993) and the spirit of Aguirre (2015)\(^3\), and its main contribution is shedding light on the welfare and distributional consequences of fiscal expenditure rules. Additionally, idiosyncratic unemployment shocks are modeled in a novel way.

This model consists in a general equilibrium model with four sectors: i) households, ii)...

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\(^1\)Fiscal Affairs Department.

\(^2\)Expenditure fiscal rules are most commonly implemented during bad economic times. Almost all countries in IMF’s data base adopted them in moments preceded by a negative change in output gap and were recently introduced in response to the financial crisis (Cordes et al., 2015).

\(^3\)Aguirre (2015) investigates the quantitative effects on welfare and wealth distribution of implementing a structural balance fiscal rule in the case of the Chilean economy.
government, iii) production and iv) external sector. Regarding households, I use standard CRRA utility function in order to keep tractability and assure concavity. The disposable income of agents is composed by wage net of taxes, returns on savings and the lump sum transfers of the government. The latter element of disposable income is the one that captures the direct effect of government fiscal framework on households. Government finances these lump sum transfers and the debt interest payment through a tax on wages. Concerning the external sector, this economy will be an international price taker, then domestic interest rate consists in the international interest rate plus a spread. This spread depends on the debt-to-GDP ratio, which reflects an international risk premium that the economy faces because of the probability for government to make default. Finally, I use a competitive environment for the productive sector with a Cobb-Douglas production function of a representative firm to obtain wages and capital hiring.

The quantitative exercise in this paper is carried out as follows. I model a benchmark case which simulates a representative developing country in which a discretionary fiscal policy has been carried out to face this deep economic downturn, before and after a deep recession (e.g. the international global crisis). Second, as a counter-factual scenario, I model the same economic downturn but in the context of a fiscal policy characterized by an expenditure fiscal rule. This expenditure rule consists in adjusting current spending in order to maintain the debt-to-GDP ratio constant. This fiscal rule is considered an expenditure fiscal rule with a debt objective, as public expenditure is the only budgetary aggregate that is adjusted to accomplish the rule. I chose this rule because the combination of expenditure and debt rule is commonly used in developing countries (Cordes et al., 2015). In the exercises that I propose in this paper, the transitional dynamics start in the same initial steady state, but reach a different one, depending on the fiscal policy framework.

I want to capture two important transmission mechanisms in the model: transfers and interest rate. Changes in public expenditure will affect the disposable income of agents via changes in transfers: an increase (decrease) in transfers will increase (decrease) the disposable income of agents. Non-constrained agents will smooth consumption and increase (decrease) savings, while constrained ones will increase (decrease) consumption. This is a direct effect or a first-round effect. As long as taxes remain constant and the economy suffers from a negative shock, an increase in transfers will increase public debt. Thus, as a second-round effect, there is an increase (decrease) in the interest rate via risk premium because of an increase in government’s debt. This increase (decrease) in interest rate has a twofold effect. On the one hand, it leads to an increase (decrease) in disposable income through changes
in savings’ returns; but, on the other hand, there is less (more) capital hiring. As a result of this latter effect, marginal productivity of labour as well as wages and disposable income decrease (increase).

These two mechanisms will have a final net effect on disposable income, which will have effects on welfare through changes in consumption. In order to quantify the welfare effect of the different fiscal policies, I use a measure of the consumption equivalent variation. I quantify the welfare effect of a given policy framework for an individual by asking: by how much the consumption has to change in all future periods and in the initial steady state, so that the expected utility equals the one after the transition, under a specific policy framework? In other words, by how much, in consumption terms, the agents benefit or lose from a specific fiscal policy framework in an economic downturn context? In order to study the distributional effects, I compute the Gini coefficient in the initial steady state, and I compare it with the Gini coefficients that result from the transition of each policy framework.

The model is calibrated to match macroeconomic features of the Costa Rican economy. First, it is a developing country that presents the behaviour that I want to capture in the benchmark case (counter-cyclical fiscal policy during international crisis). Second, it is a country in which government expenditure in transfers (mainly education and health) is very important. Third, it is about to introduce an expenditure fiscal rule.

The quantitative exercises show that, as a result of a deep economic downturn, there is a loss in welfare regardless of the fiscal policy framework. Nevertheless, losses are lower in a fiscal policy framework characterized by an expenditure fiscal rule. This is because the benefits from the counter-cyclical fiscal policy framework are overcome by the negative effects brought by the resulting higher debt-to-GDP ratio and interest rate. This increase in interest rate decreases capital hiring, output, wages and employment. In the case of the fiscal rule, despite the negative effects in consumption during the aggregate shock, the stability of the debt-to-GDP ratio and interest rate ensures convergence to the initial steady state and finally a recovery of the initial levels of capital hiring, output, wages and employment.

The average welfare of agents decreases by 2.65% in consumption-equivalent units in the case of the fiscal rule, and declines by 3.06% with the counter-cyclical fiscal policy. Additionally, poor unemployed agents lose more in consumption-equivalents units in the benchmark case (4.95%) than in the expenditure fiscal rule policy framework (3.5%). Nevertheless, rich employed agents gain 0.42% in consumption-equivalent units in the benchmark case, but in

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4 According to OECD (2016) the public spending in Costa Rica is 6.9% of GDP (well above OECD average), and health spending is 10% (also above OECD average).
a fiscal rule case they lose 0.7%. The effects on welfare are different across agents because of their level of assets. On the one hand, due to the magnitude of their savings, richer agents benefit from the income effect of the higher interest rate in the benchmark case. On the other hand, the wages and transfers are more important in the disposable income of poor agents and, in the long run, these components decrease because of higher debt and interest rate in the benchmark case.

The changes in the interest rate generate income and substitution effects that also affect the distribution of wealth. Regarding the income effect, the rise in the interest rate increases disposable income of agents, which allows them to consume and save more. Additionally, there is a substitution effect that increases the price of present consumption, and increases savings even more. The magnitude of these effects depends on the level of assets that agents hold. For the richer ones, these effects are larger.

As a result of these different effects across agents, the Gini coefficient worsens in the counter-cyclical fiscal policy case, but not in the case of an expenditure fiscal rule. Since debt-to-GDP ratio remains constant in this latter case, the income and substitution effects of interest rate are turned off. However, in the benchmark case, these effects favour rich agents. In terms of wealth distribution, the Gini coefficient in the initial steady state is 0.50, and the resulting Gini coefficients are 0.56 and 0.50 for the benchmark case and the expenditure fiscal rule case, respectively.

The main conclusion of the paper is that discretionary counter-cyclical fiscal policy generates, in average, greater welfare losses in the long run compared with an expenditure fiscal rule framework. Moreover, the discretionary counter-cyclical fiscal policy generates an income and substitution effect of the interest rate, which benefit the rich agents the most and generate more wealth inequality.

New studies shed some light on the welfare effects of fiscal rules in general (e.g. Garcia et al., 2011; Landon and C. Smith, 2015; Ojeda-Joya et al., 2016; Aguirre, 2015)\(^5\), but none of them study the specific case of expenditure fiscal rules. Additionally, solely Aguirre (2015) explores the distributional effects of a structural balance fiscal rule for the Chilean case. Thus, as aforementioned, the contribution of this paper is to study welfare and distrib-

\(^{5}\)Garcia et al. (2011) use a Dynamic Stochastic General Equilibrium (DSGE) model to measure welfare gains of implementing a structural surplus rule. It is a theoretical work and not a specific application to a country, and they do not study distributional effects. Landon and C. Smith (2015) use Monte Carlo techniques to examine the impact on welfare of five types of government expenditure rules, using a VAR approach for Canadian provinces. Ojeda-Joya et al. (2016) use a DSGE to study the welfare effects of commodity shocks under alternative fiscal policies. Aguirre (2015) uses a heterogeneous agents and incomplete markets environment to study the welfare effects of implementing a Structural Budgeted Balance Rule.
butional effects of expenditure fiscal rules. These have not been studied until now.

The rest of the paper is structured as follows: second section consists in a literature review. Third section offers a brief review on expenditure fiscal rules and the Costa Rican case. Fourth section develops the theoretical framework and fifth section the transmission mechanisms and the exercise proposed. Sixth section describes the calibration. Seventh section exposes the results. Finally, eighth section presents the conclusions.

2 Literature Review

Incomplete markets models with heterogeneous agents seem to be a natural environment to analyse fiscal policy effects. Since Aiyagari (1994) and Huggett (1993), this framework has been used to study the effects of fiscal policy because it allows to model Non-Ricardian households agents in a more flexible way than alternatives, like a Dynamic Stochastic General Equilibrium model with rule of thumb for Non-Ricardian agents. Additionally, these kind of models are able to generate a non-trivial distribution of wealth, which is a very important feature when analysing fiscal policy.

The majority of fiscal policy papers study the effects of taxes, but just a few of them analyse the expenditure side. Regarding expenditure and debt studies, Aiyagari, Marcet, et al. (2002) study a benevolent government who accumulates assets as a precaution against adverse shock on expenditure. Aiyagari and McGrattan (1998) use a heterogeneous agents and incomplete markets model to study why there is a bias of governments to accumulate debt and they point at the incentives to accumulate precautionary savings by households as a reason. Finally, Aguirre (2015) studies the welfare and distributional effects of a structural balance fiscal rule. He finds that low skilled and poor agents benefit the most from the implementation of this fiscal rule because of its counter-cyclicality nature of spending.

Regarding welfare and distributional effects of fiscal rules, there are several methodologies used by authors to measure the effects of the implementation of fiscal rules. Empirical methodologies as well as theoretical exercises have been implemented. Garcia et al. (2011) study the effects on welfare of a structural surplus rule in a DSGE environment. These authors found that under this kind of rule, credit constrained agents benefit in welfare terms. On the other hand, Landon and C. Smith (2015) use a Monte Carlo technique to examine the impact on welfare and government spending stabilization performance of five types of expenditure fiscal rules. They use data of the Canadian provinces, and found that an expenditure fiscal rule related to a balanced budget rule objective performs the best. Ojeda-
Joya et al. (2016) use a DSGE framework to study the welfare effects of commodity shocks under alternative fiscal rules in a small open commodity-rich economy. These alternative fiscal rules are captured in the degree of pro-cyclicality of fiscal policy. They encountered that non-Ricardian agents prefer counter-cyclical fiscal policy while Ricardian agents prefer a pro-cyclical one.

Finally, Aguirre (2015) uses an environment of heterogeneous agents and incomplete markets to study welfare and distributional effects of implementing a structural budget balance rule. The aim of the paper is to measure the welfare and distributional effects of implementing this kind of rule in a commodity-rich economy. In this model, the economy is subject to systematic aggregate shocks and productivity idiosyncratic shocks. The author found that, on average, agents are better in terms of consumption equivalents with the fiscal rule than without it.

In Aguirre (2015), the introduction of a fiscal rule generates a decrease in the effects of aggregate risk because of the insurance role of the rule. As long as there are fiscal policy increases transfers in bad times, there is a reduction in precautionary savings of the poor low-skilled agents, therefore increasing their consumption. This generates welfare benefits of the fiscal rule for poor agents. Nevertheless, this decrease in savings generates an increase in interest rate. As a result, there is less capital accumulation, and this generates a fall in output of 0.2% and a reduction of wages of 0.2%. The fall in wages decreases consumption of agents, but this fall is compensated by the increase of consumption generated by the less precautionary savings aforementioned. On the other hand, rich agents benefit from the increase in the interest rate, so they benefit from switching to a structural balance fiscal rule, but their benefits are lower than the ones of poor agents.

As Aguirre (2015), I analyse the effects that the interest rate and government transfers generate in each fiscal policy framework. Fiscal policy generates changes in interest rate through debt accumulation. The way in which the debt is accumulated depends on the public expenditure patterns, and the public expenditure pattern depends on the fiscal policy framework in place. I focus my attention on the way transfers and interest rate interact together in a discretionary counter-cyclical fiscal policy case and in a framework of fiscal policy characterized by an expenditure fiscal rule.

They define a parameter $\phi$, which value will determine the degree of pro-cyclicality. If $\phi = -1$ fiscal policy is counter-cyclical, if $\phi = 0$ it is neutral, if $\phi = -1$ it is procyclical and there is a historical scenario where $\phi = 0.81$ which reflects the Colombian case, country for which the model was calibrated.
3 A Brief Review on Expenditure Fiscal Rules

Fiscal rules are institutional mechanisms implemented by countries in the search of discipline and fiscal credibility (IMF, 2009). Specifically, they are numerical limits on budgetary aggregates which act as long-lasting constraints on fiscal policy actions (Schaechter et al., 2012). In 1990, only four countries had fiscal rules. As mentioned in the introduction, according to the IMF’s FAD Fiscal Rules data base, there are 89 IMF’s members using fiscal rules of any kind nowadays. Of the total, 29 countries use expenditure fiscal rules of which 14 are advanced economies, 14 emerging economies and one low-income country. More than one third (13) of these 29 rules were implemented after the global financial crisis of 2008 and 9 of them were implemented in developing and emerging economies.

This reflects the countries’ need to correct the fiscal imbalance resulting from fiscal stimulus, in order to face this critical economic downturn. Nowadays, expenditure fiscal rules are widely implemented in developing countries, mainly because: i) they are simple to operate and communicate, and ii) they are used to lock fiscal adjustment. The international 2008-2009 economic crisis left a legacy of fiscal imbalance that had to be corrected, and this is why these characteristics of expenditure rules are so appealing for these countries. Additionally, expenditure rules are commonly combined with a debt objective in developing countries (Cordes et al., 2015), as a way to ensure fiscal sustainability.

The use of expenditure fiscal rules in emerging countries has grown after the international crisis. Figure 1 shows the increase in the use of expenditure fiscal rules, which is remarkable in emerging economies. Because of this, it is significant to stress the importance of studying the effects of expenditure rules in an emerging and developing country context.

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7 Germany, Indonesia, Japan and the United States.
8 Australia, Denmark, Finland, France, Germany, Greece, Israel, Japan, Luxembourg, Netherlands, Singapore, Spain, Sweden and United States.
9 Botswana, Brasil, Bulgaria, Colombia, Croatia, Ecuador, Georgia, Lithuania, Mexico, Namibia, Peru, Poland, Rumania, Russia
10 Mongolia
11 Croatia, Ecuador, Georgia, Greece, Japan, Mexico, Mongolia, Namibia, Poland, Romania, Russia, Spain and United States
12 It is important to mention that, according to the FAD Fiscal Rule data base, Argentina, Iceland and Kosovo abandoned the expenditure fiscal rule after 2008.
3.1 Expenditure Fiscal Rules

According to Cordes et al. (2015), expenditure rules typically take the form of a cap on nominal or real spending growth. They are frequently used in combination with debt objectives in developing countries as a way to ensure fiscal sustainability (Cordes et al., 2015). Moreover, in most of the cases, the implementation of expenditure fiscal rules was preceded by a negative change in the output gap. This is because expenditure fiscal rules are generally used as an expenditure brake in bad economic times (Cordes et al., 2015). As commented earlier, more than one third of expenditure fiscal rules were adopted in response to the economic crisis.

The latter motivates the study of welfare and distributional effects of the implementation of this kind of rules. If the economy is going through a deep economic downturn, a cut in transfers could harm welfare in terms of consumption. But on the other hand, an expenditure fiscal rule could help in terms of macroeconomic stability (which is typically the reason to adopt it), through the taming of debt growth.

The evidence about effects of fiscal rules in spending composition shows that expenditure fiscal rules reduce the participation of transfers and investment in total spending. Dahan and Strawczynski (2010) found that the ratio of social transfers to government consumption declined more rapidly in countries with fiscal rules compared to countries without them.
Cordes et al. (2015) found a decrease in public investment in countries with expenditure fiscal rules. This effect is statistically significant only for emerging economies. This author points that transfers and investment in public goods are often the easiest to cut to accomplish rule’s objectives. But at the same time, he warns about how these expenditure items are expected to contribute to long-term growth. Additionally, it is worth mentioning that transfers and investment in public goods are important spending items of the governments to shorten wealth inequality and improve social welfare in developing countries.

On the other hand, Cordes et al. (2015) found that countries with expenditure rules have higher compliance rates compared to countries with other rules. The explanation is that the country complies more often if the target is directly under control of the government. About long-term sustainability, empirical evidence presented by Cordes et al. (2015) shows that countries with expenditure rules have on average higher primary balances and also lower primary spending. Therefore, the evidence shows that expenditure fiscal rules are relatively effective when compared to others’ rules. This contributes to the macroeconomic stability of the country, and could have positive effects on welfare because the economic cycle is less volatile.

### 3.2 Costa Rica

In this paper, I concentrate my attention on the Costa Rican case. During the international downturn of 2008-2009, Costa Rica’s government implemented counter-cyclical fiscal policy, which consisted in increasing the public spending via transfers. As a result, debt-to-GDP ratio rose uninterruptedly from 2008 to 2016. This increment in debt-to-GDP ratio led Costa Rica to lose the investment grade in its risk qualification. This unsustainable path of debt requires a fiscal consolidation to ensures fiscal sustainability.

Costa Rican government wants to implement the expenditure fiscal rule in order to lock in a fiscal adjustment. This Costa Rican expenditure fiscal rule has two important characteristics of emerging and developing economies: i) it defines caps on expenditure growth according to the GDP growth and the level of debt-to-GDP ratio, and ii) it is going to be implemented to achieve a fiscal consolidation after a counter-cyclical fiscal policy.

These characteristics of the Costa Rican case represent the typical emerging and developing economy problem after international crisis. That’s why I use this economy to calibrate important parameters of the model.

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13 Although, there could be a reverse causality problem in these estimations, because the presence of the rule could represent social preferences that are the true cause of outcomes.
4 The Model

This section describes the economic environment that characterizes this economy. The model is constructed in order to address the specific features of a developing economy discussed above.

4.1 Preferences

The economy is inhabited by a continuum of infinitely lived agents of measure 1. It is assumed that agents will have additive-separable preferences across time, with a subjective discount factor $\beta$ which is common across agents and constant over time. Agents determine their sequence of consumption $\{c_{t+j}\}_{j=0}^{\infty}$ according to discounted expected utility

$$E_t \left\{ \sum_{j=0}^{\infty} \beta^j u(c_{t+j}) \right\}$$

Where the function $u(\cdot)$ is defined by a CRRA\textsuperscript{14} utility function as the following

$$u(c) = \frac{c^{1-\sigma}}{1-\sigma}$$

Where $\sigma$ is the relative risk aversion coefficient. This functional form maintains simplicity and tractability of the model and its concavity is an important feature to ensure a fixed point in the recursive problem.

4.2 Household’s Problem

During each period, households derive utility solely from consumption of $c$ units of goods and accumulate assets that are subject to a borrowing constraint $a \geq 0$. Following literature about monetary and fiscal policy in an environment of heterogeneous agents, households face idiosyncratic employment shocks in this model.\textsuperscript{15} Unemployment shocks are represented by $\epsilon \in E = \{1, 0\}$ that follows a Markov process with transition probability $\pi(\epsilon'|\epsilon)$. If $\epsilon = 1$ households are employed, and unemployed otherwise. This parameter $\epsilon$ is random and statistically independent across consumers. It is assumed that $\epsilon$ satisfies the law of large numbers, then the total fraction of agents with $\epsilon = 1$ is known with certainty.

\textsuperscript{14}Constant Relative Risk Aversion

\textsuperscript{15}Imrohoroglu (1989), Krusell and A. Smith (1998), Krusell, Mukoyama, et al. (2009) and Aguirre (2015) are some examples of papers that use these kind of idiosyncratic shocks for policy analysis.
In the literature, it is usual that these unemployment shocks change with respect to the state of the economy. For example, İmrohoğlu (1989) proposed that unemployment shocks’ transition matrix changes with respect to the state of the economy. In his simple model, the author assumes two aggregate states and a first order transition matrix for the unemployment shocks in each of them. Since then, this strategy has been broadly used. Nevertheless, this approach is very inflexible since unemployment simply jumps from one state to another without intermediate points. Additionally, this approach was developed for models in which aggregate states of the economy stochastically change. But it is not useful with deterministic cases, like in this document.

In this paper, I propose that unemployment shocks depend on the output gap. For that purpose, I assume that there is a one-to-one mapping from unemployment to idiosyncratic transition probabilities. Hence, as long as unemployment moves, transitional probabilities will also move. First, I will suppose that unemployment depends on the output gap. Then, unemployment can be described in the following way

$$u_t = u_{ss} + \kappa (y_{ss} - y_{t-1})$$

Where $u_t$ is unemployment, $u_{ss}$ is the unemployment rate in steady state, $y_{ss}$ is the level of production in the steady state, $y_{t-1}$ is the level of production in the period $t-1$ and the parameter $\kappa$ is the response of unemployment to deviations of the product from the steady state.

Second, as long as there is a continuum of agents of mass one, we can say that the percentage of people that lose their job in a specific period is equal to the exit rate $x_t$, and the finding rate $f_t$ is the proportion of people that find a job. Therefore, total employment is the proportion of people that maintain their job $(1 - x_t)$ plus the proportion of people that find a job $f_t$. We can represent total employment as follows

$$N_t = 1 - x_t + f_t$$

Additionally, I assume that the exit rate remains constant overtime following the findings of Shimer (2012), so we can eliminate the subscript $t$ for the variable $x$. From equation (4) and knowing that $N_t = 1 - u_t$, we can find a expression for $f_t$ in function of $u_t$, as following

---

\(^{16}\)It is possible that unemployment jumps from very high values to very low values, and that could be infeasible in reality.

\(^{17}\)Shimer (2012) finds that from 1990 to 2010, the employment exit probability is acyclical. Meanwhile, employment finding probability accounts for substantial fluctuation.
\[ f_t = x - u_t \quad (5) \]

From equation (5) we can compute a finding rate every period \( t \). Finally, the first-order Markov transition function of unemployment shocks for every period \( t \) can be written as follows

\[
\pi = \begin{bmatrix} 1 - x & x \\ f_t & 1 - f_t \end{bmatrix} \quad (6)
\]

Where the first row contains the probabilities that employed agents stay in employment \((1 - x)\) and the probability they transit to unemployment \((x)\). The second row contains the probability that unemployed agents transit to employment \((f_t)\) and the probability they stay in unemployment \((1 - f_t)\). As for the transition, we can compute a sequence of transition matrices for a transition period.

Regarding other elements of the households’ problem, the disposable income of agents is composed of three sources: i) the wage net of taxes \((1 - \tau)\omega\), ii) the returns on savings \((1 + r)a\) and iii) the lump sum transfers \( T \) from the government. With this disposable income, the individuals decide which will be their levels of consumption \( c \) for this period and savings \( a' \) for the next period (control variables), given the stock of savings \( a \) and the idiosyncratic shock received in the present period (individual state variables), where \((a, \epsilon) \in Q = A \times E = [0, \infty) \times \{0, 1\}\). The aggregate state variable will be different weather we are studying the steady state or the transition. Regarding the steady state, the aggregate state variable is the cross-section distribution over individual state variables \( \Phi (a, \epsilon) \in \mathcal{M} \).\(^{18}\)

Regarding the transition, the aggregate state variable is time \( t \). From now on, the notation will correspond to the transition case for exposition purposes. The household’s problem is as follows\(^{19}\)

\[
V_t (a, \epsilon) = \max_{c \geq 0, a' \geq 0} \left\{ u(c) + \beta \sum_{\epsilon' \in E} \pi (\epsilon' | \epsilon) V_{t+1} (a', \epsilon') \right\} \quad (7)
\]

\(^{18}\)\(\mathcal{M} \) is the set of all probability measures on the measurable space \( M = (Q, B(Q)) \) with \( B(Q) = B(A) \times P(E) \), where \( B(A) \) is the Borel \( \sigma \)-algebra of \( A \) and \( P(E) \) is the power set of \( E \).

\(^{19}\)The recursive formulation of the household’s problem and definition of stationary competitive equilibria for the steady state case is presented in the Annex 2. These definitions are important for the computation of the transition, but it is considered that the recursive definition of the transition is sufficient for the analysis in this section.
\[ c + a' = \epsilon (1 - \tau) \omega_t + (1 + r_t) a + T_t \]  

(8)

Solving the first order conditions implied by recursive problem\(^{20}\) we can obtain the following Euler’s equations

\[
\begin{align*}
    u_c(c) & \geq \beta \left\{ 1 + r_t \right\} \sum_{\epsilon \in E} \pi_\epsilon(\epsilon'|\epsilon) u_c(c') \\
    & = \text{if } a' > 0
\end{align*}
\]  

(9)

Equation (9) tells us that some agents will smooth consumption across time, equating the marginal utility of consumption and the discounted expected value of the marginal utility of consumption in the next period. Also, equation (9) is fulfilled with inequality for agents which budget constraint is active, in other words: restricted agents cannot smooth consumption across time.

### 4.3 Production

The aggregate production function is defined as a standard Cobb-Douglas function as follows

\[ Y = z K_t^\alpha N_t^{1-\alpha} \]  

(10)

Where \( K \) is aggregate capital, \( N \) is aggregate labour supply, \( \alpha \) is the elasticity of capital and \( z \) is an aggregate shock. With this functional form we can assure decreasing returns to each factor and constant returns to scale. I assume a perfect competition environment, then a representative firm can be used and there are no profits. With this production function, the representative firm will hire capital and labour according to the following optimality conditions

\[
\begin{align*}
    MP_K &= z\alpha \left( \frac{N_t}{K_t} \right)^{1-\alpha} - \delta = r_t \\
    MP_L &= z(1-\alpha) \left( \frac{K_t}{N_t} \right)^\alpha = \omega_t
\end{align*}
\]  

(11)\( ^{20}\)

From equation (11), firms will hire capital until marginal productivity equates the do-

\(^{20}\) Annex 3 contains the resolution of the recursive problem specified in equations (22) and (8).
mestic interest rate. Since this is a small open economy, the domestic interest rate depends on factors different than firms’ decisions, as described in the external sector subsection. On the other hand, wage is determined by firms’ decisions, specifically, it is equal to the marginal productivity of labor (equation (12)).

4.4 Fiscal Policy

The government follows a budget balance described by the following equation

\[ B_t' + \tau \omega_t N_t = (1 + r_t) B_t + T_t \]

(13)

Where \( B_t \) is the stock of public debt, \( \tau \) is the tax rate and \( T_t \) are lump sum public transfers. I assume that the government’s revenues come solely from labour income tax.\(^{21}\) Taxes will remain constant over time and fiscal policy will only be carried out through expenditure. Additionally, it is assumed that the public spending in transfers is exogenously determined by a determined fiscal policy framework (described in section 5.1).

Rationalizing all the adjustments via transfers reflects the typical behaviour of emerging and developing countries, because, as pointed out in section three, this type of economies adjust investment and transfer spending items to meet the rule’s objectives, meanwhile advanced economies do not. Then this is a specific feature that accounts for developing economies.

4.5 External Sector

Since I want to capture the features of an emerging and developing country, it is necessary to assume that it is a small open economy. Then, this economy will be international price taker, particularly it will be taker of the international interest rate. The domestic interest rate of this economy is the international interest rate plus a spread.

\[ r_t = r^* + s_t \]

(14)

International interest rate \( (r^*) \) will be given and the interest rate spread accounts for a specific country risk premium. I will assume that interest rate spread depends solely on the government’s debt-to-GDP ratio \( b_t \), as follows

\(^{21}\)This assumption is made for computational purposes.
\[ s_t = \rho b_t \]  \hspace{1cm} (15)

Where \( b_t \) is the debt-to-GDP ratio, \( \rho \) is the corresponding coefficient. Additionally, it is important to mention that all agents in the economy will face the same domestic interest rate. First, firms will face the same risk as the government because of two reasons: the same macroeconomic environment and transfer risk (Durbin and Ng, 2005). With respect to the same macroeconomic environment, there are exogenous shocks that affect both the government and the firms in the same proportions. For example: a recession could worsen firms and government’s repayment capacity. With respect to the transfer risk, governments have the capacity to transfer its repayment problems to the firms via an increase in taxes, imposing price controls or confiscating firm’s assets (expropriation). If the repayment capacity of the government falls, then it is prone to use some of these mechanisms. Then, I assume that if government makes default, then firms also do it. Second, I assume that agents in this economy will always prefer the domestic interest rate as long as it is greater than the international rate.\(^{22}\)

### 4.6 Welfare Analysis

I want to compute the welfare effects during the transition that is proposed. For welfare measure, this paper will follow closely Conesa and Krueger (1999). Given the form of the utility function, welfare measure for an individual of type \((a, \epsilon)\) could be computed in the following form

\[
g(a, \epsilon) = \left[ \frac{V_1(a, \epsilon)}{V_0(a, \epsilon)} \right]^\frac{1}{1-\sigma} - 1 \hspace{1cm} (16)
\]

The variable \( g(a, \epsilon) \) quantifies by how much the consumption has to be changed in all future periods and in the initial steady state, so that the expected utility equals the one after the transition under a specific policy framework. In other words, by how much, in consumption terms, the agents benefit or lose from a specific fiscal policy framework. For example, \( g(a, \epsilon) = 0.1 \) implies that the initial steady state has to be increased in 10% in order for the expected utility to equal the one after transition.

\(^{22}\)This assumption is necessary to avoid modelling default and its implications in the saving decision of the agents. Also, there are other papers which have similar economic environment and implicitly have the same assumption (i.e. Aguirre (2015))
Specifically, \( g(a, \epsilon) \) compares the value function \( V_1 \) in the period 1 of the transition \textit{vis-à-vis} value function in the steady state \( V_0 \). The value function in period 1 is the one that takes into account all the information in all the periods of the transition. Additionally, the aggregate measure that I use is the following

\[
g^* = \int \left[ \frac{V_1(a, \epsilon)}{V_0(a, \epsilon)} \right]^{1 - \sigma} d\Phi_0 - 1 \tag{17}
\]

The variable \( g^* \) will be the weighted average of welfare that results from a specific fiscal policy framework. The variable \( g^* \) will compute the average of all individuals of type \((a, \epsilon)\) weighted by the proportion of population with the amount of assets \(a\).

5 Transmission Mechanisms and Exercise

In this section I analyse the transmission mechanisms that operate in the model. For that purpose, I presume there is a counter-cyclical fiscal policy of increasing government transfers. First of all, from equation (8) we know that an increase in public spending will rise the agent’s disposable income. As a result of this rise in disposable income, we know from equation (9) that non-liquidity constrained agents will increase their consumption and also their savings in order to smooth consumption across time. On the other hand, liquidity constrained agents will increase only their consumption.

As long as the government increases spending and taxes remain constant, it is expected for the debt-to-GDP ratio to increase. As a result, the interest rate spread rises according to equation (15) and therefore the domestic interest rate also increases. This rise in the interest rate will have a second round effect on the economy. This can be divided into two types: i) effects on consumer decision and ii) effects in production.

\textbf{Effects on consumer decisions:} the effects on the consumer decisions can be analysed from equation (9). On the one side, there is an income effect for non-liquidity constrained agents. As a response of a rise in interest rate, the return on savings increases and therefore the disposable income of the consumers also increases. This increments the consumption and savings of non-constrained agents. On the other hand, there is a substitution effect because an increase in the interest rate increases the price of present consumption which leads the agents to decide to save even more. The net effect will unambiguously be a rise in the savings of non-liquidity constrained agents.

\textbf{Effects on production:} from equation (11) we can see that a rise in the domestic
interest rate will increase the optimal marginal productivity of capital and consequently there will be less hiring of capital. This lower hiring of capital impacts negatively the marginal productivity of labour as shown in equation (12), and therefore the level of wages. This lower level of wages decreases the disposable income of agents and therefore its consumption and savings.

Therefore, we can identify a trade off brought by fiscal policy: on the one hand increasing transfers has a direct positive effect on disposable income and an indirect effect on interest rate. But on the other hand increasing spending has a crowding out effect, and firms will hire less capital, impacting negatively the disposable income of agents. The figure 2 summarizes the transmission mechanisms.

Figure 2: Transmission Mechanisms

5.1 The Exercise

The exercise proposed in this paper consists in the computation of a transition path triggered by a negative aggregate shock. This negative shock is transitory, but depending on the fiscal policy framework the economy reaches a different steady state. The idea is to compute two different fiscal policy frameworks: i) a benchmark case characterized by a discretionary counter-cyclical fiscal policy; and ii) an expenditure fiscal rule associated with a debt objective in which government adjusts expenditure in order to maintain the debt-to-GDP ratio constant.

Regarding the benchmark case, it tries to capture the response showed by governments of developing countries to the 2008-2009 international economic crisis. As a response to the
negative shock in the benchmark case, the government will increase transfers and, therefore, will accumulate debt. After the aggregate shock, the government will adjust spending in order to maintain the resulting debt-to-GDP ratio constant. This represents the widespread behaviour of governments during and after the international economic crisis. As long as the debt-to-GDP ratio reaches a new level, the interest rate will change, and then the steady state will by a new one.

As a counter-factual case, I simulate the same aggregate shock, but changing the fiscal policy framework to one characterized by a fiscal policy rule. With this fiscal rule, I want to capture the widespread practice of combining expenditure fiscal rules with a debt objective. As aforementioned, emerging and developing economies usually combine expenditure rules with a debt target. I set the debt objective to maintain the debt-to-GDP ratio constant to the initial steady state level. As a result, the government has to adjust transfers in order to meet this objective. The final steady state is the same, as long as the debt-to-GDP ratio and the interest rate remain constant.

Considering that the debt-to-GDP ratio has to be constant in the fiscal rule case, then we can know the transfers-to-GDP ratio required to meet this objective. We can start by defining the budget balance of the government (equation 13) as percentage of GDP as following

\[ b_{t+1}(1 + \gamma_t) + p_t = (1 + r_t)b_t + t_t \]

(18)

Where \( p_t \) stands for \( \tau \omega N \) as percentage of GDP, \( b_t \) stands for debt-to-GDP ratio, \( \gamma_t \) is the GDP growth and \( t_t \) stands for transfers as percentage of GDP. Isolating the variable \( t_t \) we get the following expression

\[ t_t = (b_{t+1} - b_t)(1 + \gamma_t) + p_t - (r_t - \gamma_t)b_t \]

(19)

As long as the debt-to-GDP remains constant, then we can express the expenditure path as a percentage of GDP, as follows

\[ t_t = p_t - (r_t - \gamma_t)b_t \]

(20)

5.2 Transition Path

The exercise will consist in comparing the welfare gains and distributional effects of implementing a discretionary counter-cyclical fiscal policy \( (R_1) \) with respect to an expenditure
fiscal rule policy framework \( (R_2) \). The transition path is defined as follows.\(^{23}\)

**Definition 1.** Given the initial distribution \( \Phi_0 \) and a fiscal policy framework \( R_j \), a competitive equilibrium is defined by a sequence of individual functions for the households \( \{ V_t, c_t, a_{t+1} : Q \times M \rightarrow R \}_{t=0}^{\infty} \), a sequence of production plans for the firm \( \{ N_t, K_t \}_{t=0}^{\infty} \), factor prices \( \{ \omega_t, r_t \}_{t=0}^{\infty} \), government transfers \( \{ T_t \}_{t=0}^{\infty} \), government debt \( \{ B_t \}_{t=0}^{\infty} \) and a sequence of measures \( \{ \Phi_t \}_{t=1}^{\infty} \), such that, for all \( t \),

1. Given \( \{ \omega_t, r_t \} \) and \( \{ T_t, B_t \} \) the functions \( \{ V \} \) solve Bellman’s equation for period \( t \) and \( \{ a_{t+1}, c_t \} \) are associated policy functions.

2. The prices \( \omega_t \) and \( r_t \) satisfy

\[
 r_t = F_K(K_t, N_t) - \delta \\
 \omega_t = F_L(K_t, N_t)
\]

3. Government Budget Constraint,

\[
 B'_t + \tau \omega_t N_t = (1 + r_t) B_t + T_t 
\]

4. Market clearing,

\[
 N_t = \int \epsilon_t d\Phi_t \\
 \int c_t(a_t, \epsilon_t) d\Phi_t + \int a_{t+1}(a_t, \epsilon_t) d\Phi_t = F(K_t, N_t) + (1 - \delta) K_t
\]

5. Aggregate law of motion

\[
 \Phi_{t+1} = \Gamma_t(\Phi_t)
\]

\(^{23}\)Annex 1 contains the demonstration that recursive problem defined in equation (22) has a fixed point, therefore we can assure that equilibrium exists and is unique.
Definition 2. A stationary equilibrium is one such that all elements of the equilibrium that are indexed by $t$ are constant over time.

The functions $\Gamma_t$ can be defined in the following way. Define Markov transition functions $Z : Q \times B(Q) \to [0, 1]$, induced by the transition probabilities $\pi$ and the optimal policy $a_{t+1}(a, \epsilon)$ as

$$Z((a, \epsilon), (A, E)) = \sum_{\epsilon' \in E} \begin{cases} 
\pi(\epsilon' | \epsilon) & \text{if } a_{t+1}(a, \epsilon) \in A \\
0 & \text{Otherwise}
\end{cases}$$

For all $(a, \epsilon) \in Q$ and all $(A, E) \in B(Q)$. Then

$$\Phi_{t+1}(A, E) = (\Gamma_t(\Phi_t))(A, E) = \int Z((a, \epsilon), (A, E)) \Phi(da \times d\epsilon)$$

For all $(A, E) \in B(Q)$. These transition functions tell us what are the savings of the agents each period given the probability of receiving an idiosyncratic shock. Then the distribution of assets could be computed for each period.

6 Calibration

For the reasons described in section 3.2, we calibrate the model for the Costa Rican economy in annual basis. The idea is to calibrate the steady state with 2008 data, which is the year that corresponds to two important events: i) it is the year right before the international financial crisis hit the Costa Rican economy and ii) it is the year when the Costa Rican government achieved its minimal debt-to-GDP ratio (24%) before it started to rise uninterruptedly. The data I use correspond to Central Government ones.

For the risk aversion coefficient, capital elasticity and depreciation rate, I pick $\sigma = 2$, $\alpha = 0.33$ and $\delta = 10\%$. Regarding the tax rate, I chose $\tau = 15\%$ which represents the total revenue of the Central Government as a percentage of GDP in 2008. For the parameter of equation (15) I chose $\rho = 0.11$, which is the value calculated by Kumar and Baldacci (2010). The debt-to-GDP ratio is set in 24%. These values and equation (15) imply an interest rate spread of 2.04%. The international real interest rate is set in 3%, which is the real interest rate for the United States in 2008 according to IMF data. Using equation (14), this international real interest rate and the spread imply a domestic real interest rate of 5.64%. Unemployment rate is set in $u_{ss} = 5\%$, which is the rate of open unemployment
calculated with the ENAHO\textsuperscript{24} survey in Costa Rica. This unemployment rate implies a level of employment of $N = 0.95$. On the other hand, in order to calibrate the finding rate it is necessary to use the unemployment duration. Following İmrohorogIu (1989), duration and finding probability have the following relationship

$$ D = \frac{1}{f} \quad (21) $$

The unemployment duration for Costa Rica in 2008 is 4 periods in an annual basis\textsuperscript{25}. Using equation (21) and the value of duration, the finding probability is $f = 0.25$. Using $f$, unemployment rate $u_{ss}$ and equation (5) we can obtain the exit rate $x = 0.30$. Table 1 contains the summary of the calibrated parameters for the initial steady state.

Table 1: Calibration

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Source</th>
</tr>
</thead>
<tbody>
<tr>
<td>Risk aversion</td>
<td>$\sigma$</td>
<td>2</td>
</tr>
<tr>
<td>Capital elasticity</td>
<td>$\alpha$</td>
<td>0.33</td>
</tr>
<tr>
<td>Depreciation</td>
<td>$\delta$</td>
<td>10%</td>
</tr>
<tr>
<td>Tax rate</td>
<td>$\tau$</td>
<td>15%</td>
</tr>
<tr>
<td>Elasticity of $s$</td>
<td>$\rho$</td>
<td>0.11</td>
</tr>
<tr>
<td>Debt-to-GDP</td>
<td>$B/Y$</td>
<td>24%</td>
</tr>
<tr>
<td>Spread</td>
<td>$s$</td>
<td>2.64%</td>
</tr>
<tr>
<td>Intern. int. rate</td>
<td>$r^*$</td>
<td>3%</td>
</tr>
<tr>
<td>Unemployment rate</td>
<td>$u_{ss}$</td>
<td>5%</td>
</tr>
<tr>
<td>Duration</td>
<td>$1/f$</td>
<td>4 periods</td>
</tr>
</tbody>
</table>

Table 1 contains the summary of the calibrated parameters for the initial steady state. The steady state was calculated with these parameters\textsuperscript{26}, from which I started the transition exercises. This steady state was found iterating over $\beta$. The steady state discount factor obtained was $\beta = 0.9149$. Table 2 contains the results for the steady state and the comparison with actual data. The model is considered good, because most of the predicted values of variables for which the model was not calibrated are close to actual data.

\textsuperscript{24}Encuesta Nacional de Hogares (National Household’s Survey).
\textsuperscript{25} Calculated by SEDLAC (CEDLAS (Universidad Nacional de la Plata) and The World Bank). http://sedlac.econo.unlp.edu.ar/eng/
\textsuperscript{26}The computational algorithm for the steady state is in Annex 3.
Table 2: Steady state

<table>
<thead>
<tr>
<th>Variable</th>
<th>Model</th>
<th>Data</th>
</tr>
</thead>
<tbody>
<tr>
<td>Transfers as % of GDP</td>
<td>$T/Y$</td>
<td>8.7%</td>
</tr>
<tr>
<td>Private Consumption % of GDP</td>
<td>$C_p/Y$</td>
<td>68.6%</td>
</tr>
<tr>
<td>Total Consumption % of GDP</td>
<td>$C/Y$</td>
<td>77.3%</td>
</tr>
<tr>
<td>Investment % of GDP</td>
<td>$I/Y$</td>
<td>29%</td>
</tr>
<tr>
<td>Net exports % of GDP</td>
<td>$NX/Y$</td>
<td>-6.3%</td>
</tr>
<tr>
<td>Gini coefficient</td>
<td>-</td>
<td>0.49</td>
</tr>
</tbody>
</table>

Own elaboration.*Current Expenditure.

For the transition exercises, I use four periods of aggregate negative shock. In the first period, there is a deep negative shock of 10% which represents the international economic crisis (Blanchard et al., 2010). After this period, I set three periods of slow growth (2% every period). I have calibrated the aggregate shocks $z$ in order for aggregate GDP to match the behaviour described. The counter-cyclical fiscal policy for these four periods was calibrated for the Costa Rican case. Thus, transfers increase by 1 percentage point of GDP in the first period, 2 percentage points of the GDP in the following two periods, and finally come back to its initial steady state level in the fourth period. The parameter $\kappa$ is set in 0.5 in order to match an unemployment rate of 8% in the last period of the aggregate shock, which is the 2012 value in Costa Rica (unemployment after international crisis). The results of the calibration in the negative aggregate shock period are presented in table 3.

Table 3: Calibration of the parameters of the transition

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Source</th>
</tr>
</thead>
<tbody>
<tr>
<td>Aggregate shock</td>
<td>$z$ 0.9879, 0.9959, 1.0041, 1.0082</td>
<td>Calibrated</td>
</tr>
<tr>
<td>Sens. of unemp. to output gap</td>
<td>$\kappa$ 0.5</td>
<td>Calibrated</td>
</tr>
<tr>
<td>Counter-cyclical fiscal pol.</td>
<td>$T/y$ 9.7%, 10.7%, 10.7%, 8.7%</td>
<td>Calibrated</td>
</tr>
</tbody>
</table>

Own elaboration.

7 Results

Regarding the benchmark case, in the first four periods of the transition, the debt-to-GDP ratio reaches 31.7% because of the counter-cyclical fiscal policy. This level of debt implies an interest rate of 6.5%. The government then adjusts the transfers in order to maintain this
new debt-to-GDP ratio (resulting from the counter-cyclical policy) constant, until reaching the final steady state. This has been the behaviour of the Costa Rican government in the last few years. The country is adjusting expenditure in order to prevent the debt-to-GDP ratio to enter in an unsustainable path. With the discretionary counter-cyclical fiscal policy, there is a new final steady state implied by this new level of debt. Table 4 shows the macroeconomic results of this final steady state.

Table 4: Final steady state in benchmark case

<table>
<thead>
<tr>
<th>Variable</th>
<th>Model</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Transfers as % of GDP</td>
<td>(T/Y)</td>
<td>8%</td>
</tr>
<tr>
<td>Private Consumption % of GDP</td>
<td>(C_p/Y)</td>
<td>70.8%</td>
</tr>
<tr>
<td>Total Consumption % of GDP</td>
<td>(C/Y)</td>
<td>78.8%</td>
</tr>
<tr>
<td>Investment % of GDP</td>
<td>(I/Y)</td>
<td>19.6%</td>
</tr>
<tr>
<td>Net exports % of GDP</td>
<td>(NX/Y)</td>
<td>1.6%</td>
</tr>
</tbody>
</table>

Own elaboration.*Current Expenditure.

On the other hand, in the expenditure fiscal rule case, since the debt-to-GDP ratio remains constant, the interest rate also remains constant. Considering that the interest rate does not change, the steady state stays the same before and after the transition.

### 7.1 Transition

Figure 3 shows the principal macroeconomic outcomes of the transition. In the benchmark case the transition lasts 29 periods; meanwhile, in the case of the expenditure fiscal rule the transition lasts 14 periods.\(^{27}\) The aggregate consumption decreases during the negative shock period, and decreases even more in the expenditure fiscal rule case due to lower transfers. As long as the government has to comply with the rule, then transfers have to decrease.

During the economic downturn, aggregate capital decreases due to aggregate shock. Nevertheless, the interest rate has a different behaviour depending on the fiscal policy framework in place, thus there are two different effects on capital. On the one hand, with the counter-cyclical fiscal policy, interest rate increases because of the increment in debt-to-GDP ratio. On the other hand, with the fiscal rule, the debt-to-GDP ratio has to remain constant, thus interest rate remains constant as well. As a result, capital decreases more in the benchmark case. As wages are a function of capital, they also decrease more in the benchmark case.

\(^{27}\)In order to clarify how I calculate the number of periods, is useful to remember the computational algorithm. See Annex 4.
In the benchmark case, there are two mechanisms working in an opposite direction as discussed in section 5. Transfers have a positive effect on disposable income meanwhile interest rate has a negative effect on it through lower wages. On the other hand, in the fiscal rule case wages also decrease, but the effect is lower compared to the benchmark case due to the stability of the interest rate. Additionally, in the fiscal rule case, transfers are reduced during the negative aggregate shock in order to comply with the objective. During the aggregate shock period, the net effect on disposable income is less harmful in the benchmark case than with the fiscal rule.

Figure 3: Macroeconomic outcomes in transition

Nevertheless, we have to study the total effect until the economy reaches the new steady state. As a result of the counter-cyclical fiscal policy, the debt-to-GDP ratio rises, and the government adjusts the transfers downwards in order to avoid the debt-to-GDP ratio to enter in a unsustainable path. Therefore, after the counter-cyclical fiscal policy, interest rate stays in 6,5%. This higher level of interest rate brings down the aggregate capital to
a lower steady state level. Which implies lower wages and lower production in the steady state. Finally, lower production increases unemployment (following equation (3)). Thus, after the counter-cyclical fiscal policy, the disposable income of agents in the benchmark case is affected negatively by lower wages, lower transfers and more unemployment; but is affected positively by an increase in returns on savings.

The aggregate results of the entire transition are reflected by aggregate consumption and aggregate savings. Specifically for the benchmark case, aggregate consumption decreases, which is consistent with the lower wages, lower transfers and higher unemployment. But, additionally, the substitution effect brought by the higher interest rate increases the price of present consumption, and reduces it even more. The higher interest rate also implies a positive income effect which increases the disposable income, but it seems to be small in aggregate terms compared to other negative effects in disposable income. On the other hand, aggregate savings increase during the transition in the benchmark case because of this substitution effect brought by the interest rate. Nevertheless, savings have a hump shape during the transition. This is probably because of the negative effects brought by lower wages, transfers and higher unemployment, which leads some agents to disaccumulate savings in the final periods of the transition. As a final result of the transition, aggregate savings increase by 4.9% in the final steady state, whereas aggregate consumption decreases by 3.8%.

In the fiscal rule case, debt-to-GDP remains constant throughout the whole transition due to the fiscal rule. This means that the interest rate remains constant, and the aggregate capital progressively converges back to its initial steady state, and consequently the output also converges to the initial steady state. Regarding the fiscal policy, as long as the GDP grows, transfers also grow, consistently with the debt-to-GDP ratio objective. Additionally, due to the GDP convergence to the initial steady state, unemployment also comes back to its initial steady state level. The recovery of transfers, wages and unemployment mitigates the negative effects of the negative aggregate shock on the agents’ disposable income. Moreover, the stability in the interest rate generated by the expenditure fiscal rule avoids additional negative effects on aggregate capital and turns off substitution and income effects caused by interest rate fluctuations.

7.2 Welfare and Distributional Effects

Regarding distributional effects, the benchmark case results in a more unequal distribution of wealth with respect to the distribution in the steady state. This can be seen in figure
4 because of the downward movement of the Lorenz curve. In contrast, the distribution of wealth remains almost constant in the expenditure fiscal rule case. Specifically, the Gini coefficient in the initial steady state is 0.5. This coefficient moves to 0.55 in the benchmark case, and stays in 0.5 in the case of the fiscal rule.

This result suggests that the increase in aggregate savings observed in the benchmark case is driven mainly by the very rich agents. As discussed in section 5, the increment in the interest rate implies substitution and income effects that lead agents to save more. Nevertheless, the magnitude of these effects depends on the level of assets owned by the agents.

Figure 4: Lorenz curve.

![Lorenz curve diagram](image)

In the benchmark case, the first four quintiles accumulate 47.6% of wealth after the transition, whereas in the initial steady state they accumulate 58.8%. This change in the cumulative share of wealth is due to the decrease in the disposable income of agents in these quintiles, which leads agents to disaccumulate savings. Therefore, the negative effects (lower wages, lower transfers and higher unemployment) are more important than the substitution and income effects of interest rate in these quintiles. On the other hand, the fifth quintil
accumulates 52.4\% of wealth after the transition in the benchmark case, compared to more than 41.2\% in the initial steady state. This result suggests that, for richer agents, the magnitude of the income and substitution effect is very relevant, since they accumulate 11.2 additional percentage points of total wealth after the transition.

The distributional effects of the expenditure fiscal rule are nearly irrelevant. After the transition, the share of wealth of the fifth quintile is 41.5\%, barely different from the initial steady state (41.2\%). This is mainly because the interest rate remains constant, then the substitution and income effects are turned off and the level of wages is not negatively affected by downward changes in capital hiring. Additionally, the level of transfers comes back to the initial steady state level due to the recovery of the economy. Thus, disposable income barely changes at the end of the transition in this case.

Regarding welfare effects, as mentioned in section 4, they are measured by the equivalent consumption variation described in equation (17). The weighted average of consumption equivalent variation measure \( g \) is 3.06\% and -2.65\% in the benchmark and the fiscal rule case, respectively. Hence, in the expenditure fiscal rule case the losses are lower than in the benchmark case. According to the results presented above, this loss of welfare is due to the lower level of aggregate consumption. Nevertheless, as in the wealth distribution case, it is necessary to study welfare effects depending on the level of assets owned by agents.

Figure 5 shows the equivalent consumption variation measure \( g \) for every level of assets. In the benchmark case, rich agents have a higher benefit than in the expenditure fiscal rule case; actually, some of them have a positive \( g \). As mentioned before, richer agents benefit from the income effect of the higher interest rate, because as long as interest rate increases, their disposable income increases. In this case the magnitude of their assets allows them to increase their disposable income significantly, because the income effect is stronger than the negative effects in wages and transfers.
On the other hand, poor agents are in a worse situation in the benchmark case. As aforementioned, the negative effect on wages, transfers and unemployment is larger than the benefits brought by the higher interest rate. The result is that these agents experience a decrease in their disposable income and savings, and therefore in their consumption. Finally, it is important to mention that unemployed constrained agents lose less welfare in the benchmark case. Their loss in the benchmark case is 6.7%, and 8.04% in the expenditure fiscal rule case. This is probably because of the positive effects of the counter-cyclical fiscal policy, which increment the welfare of the very poor agents. Nevertheless, constrained agents represent a little percentage of population (0.2%), so we can say that positive effects of the counter-cyclical fiscal policy are not significant.

We can measure the consumption equivalent variation by quintile of wealth. Table 5 shows measure $g$ by quintil and unemployment status. As we can note, the first three quintiles will prefer the expenditure fiscal policy rather than the benchmark case, no matter if the agents are employed or unemployed. On the other hand, both employed and unemployed agents from the two richer quintiles prefer the counter-cyclical fiscal policy.

On the one hand, in the benchmark case it is expected for poor agents to be in a worse
situation because of their resulting lower disposable income in the transition. The rich agents mitigate the harmful effects of lower wages and transfers with the greater returns in assets. On the other hand, in the expenditure fiscal rule case, negative effects in welfare of the transition are lower because of the interest rate’s stability. The implementation of this fiscal policy framework mitigates the fall in aggregate capital and therefore in wages. The fiscal rule leads the output back to the initial steady state level. As long as there is a recovery in economic activity, transfers start to converge back to their initial steady state. Therefore, stability generated by the expenditure fiscal rule benefits the poorer agents due to the recovery of their disposable income after the negative aggregate shock.

Finally, although the counter-cyclical fiscal policy contributes more to alleviate the deep economic downturn, in the long run the expenditure fiscal rule has better outcomes in terms of welfare and distribution. As a general conclusion, if the decision of imposing a fiscal rule has to be put under voting, probably the majority will vote yes and will prefer the expenditure fiscal rule.

### 8 Conclusions

In this paper, I assess quantitatively the welfare and distributional effects of implementing an expenditure fiscal rule compared to a discretionary counter-cyclical fiscal policy (benchmark case). The benchmark case was built to mimic the reactive fiscal policy of emerging and developing countries to face the international crisis of 2008-2009. The idea was to evaluate, in terms of welfare and wealth distribution, the expenditure fiscal rule as a counter-factual fiscal policy framework.

The exercise consists in a transition triggered by a deep negative aggregate shock. I
build a heterogeneous agents and incomplete markets model to account for the welfare and
distributional effects of the two fiscal policy frameworks. This model relies on two transition mechanisms: Transfers and interest rate. On the one hand, there is a positive effect of increasing transfers on the disposable income, meanwhile the interest rate effect is twofold: it increases disposable income because of the returns on savings but decreases disposable income because of a negative effect on wages.

The results show that the first three quintiles of the wealth distribution prefer the expenditure fiscal rule than the counter-cyclical fiscal policy. This means that, despite the alleviating effects of the discretionary fiscal policy during the negative aggregate shock, the long run effects worsen the welfare of these agents. In the long run, the higher debt and higher interest rate generate harmful effects on the disposable income of the agents of these quintiles, due to less capital hiring, less production and therefore more unemployment. Additionally, these agents own low level of assets, therefore the benefits on their disposable income that come from the higher interest rate are not significant. The net effect is that in the long run they lose welfare, as long as they lose disposable income and disaccumulate savings.

On the other hand, richer agents prefer the discretionary counter-cyclical fiscal policy framework because of the benefits from the resulting higher interest rate. The magnitude of the savings of these agents is so relevant that the increase in the disposable income due to the return on their savings overcomes the negative effects of lower wages and lower transfers. In the benchmark case, these agents could increase their savings, generating a deterioration in the wealth distribution.

The results show that the Gini coefficient goes from 0.5 in the initial steady state to 0.55 after the transition in the benchmark case. On the other hand, in the expenditure fiscal rule framework, the Gini coefficient does not change. In welfare terms, the weighted average of the consumption equivalent variation is -3.06\% and -2.65\% in the benchmark case and in the expenditure fiscal rule case, respectively. The welfare analysis by quintiles show that the three first quintiles are worse in the benchmark case, and prefer the stability brought by the expenditure fiscal rule. We can conclude that expenditure fiscal rule performs better in terms of welfare and wealth distribution compared to a discretionary counter-cyclical fiscal policy.

Finally, it is important to mention these results could be improved by adding systematic aggregate shocks in the fashion of Krusell and A. Smith (1998), instead of an exogenous aggregate shock. Additionally, more types of expenditure rules can be added to the analysis.
order to compare performance. Finally, the way transfers are distributed to the population can also be changed, in order to capture the social policy programs that focuses transfers to the first two quintiles rather than in a lump sum way.
References


Kumar, M.S. and E. Baldacci (2010). *Fiscal deficits, public debt, and sovereign bond yields*. International Monetary Fund.


Annex

Annex 1: Demonstration of a fixed point

We want to prove that the problem has an unique fixed point. Let us consider the following Bellman equation:

\[
V_t(a, \epsilon) = \max_{c \geq 0, a' \geq 0} \left\{ u(c) + \beta \sum_{e' \in E} \pi(e'|\epsilon) V_{t+1}(a', e') \right\}
\]  

(22)

Let us define \( q = (a, \epsilon) \). We have,

\[
c = [\epsilon(1 - \tau)\omega + (1 + r)a + T - a']
\]

(23)

Where I define

\[
y(q) = \epsilon(1 - \tau)\omega + (1 + r)a + T
\]

So we can rewrite the Bellman equation as follows:

\[
V(q) = \max_{c \geq 0, a' \geq 0} \left\{ u(y(q) - a') + \beta \sum_{e' \in E} \pi(e'|\epsilon) V_{t+1}(q') \right\}
\]  

(24)

Let us now define the operator \( T \) as

\[
(TV)(q) = \max_{c \geq 0, a' \geq 0} \left\{ u(y(q) - a') + \beta \sum_{e' \in E} \pi(e'|\epsilon) V_{t+1}(q') \right\}
\]  

(25)

If \( T \) is a contraction mapping, then there will exist a unique function \( V \) such that,

\[
V(q) = (TV)(q)
\]  

(26)

In order to demonstrate that this is true, we need to prove that \( T \) is monotonic and satisfies the discounting property (Blackell Theorem).

1. Monotonicity: Let us considerate two value functions, \( V \) and \( W \), such that \( V(q) \geq W(q) \) for all \( q \in Q = A \times E = [0, \infty) \times \{0, 1\} \). We want to show that \( TV(q) \geq TW(q) \).

\[
(TV)(q) = \max_{c \geq 0, a' \geq 0} \left\{ u(y(q) - a') + \beta \sum_{e' \in E} \pi(e'|\epsilon) V_{t+1}(q') \right\}
\]
\[ \geq \max_{c \geq 0, a' \geq 0} \left\{ u(y(q) - a') + \beta \sum_{\epsilon' \in E} \pi(\epsilon'|\epsilon) W_{+1}(q') \right\} = TW(q) \]

Therefore we shown monotonicity.

2. Discounting: Let us consider a candidate value function, \( V \) and a positive constant \( d \).

\[
T(V + d) = \max_{c \geq 0, a' \geq 0} \left\{ u(y(q) - a') + \beta \left( \sum_{\epsilon' \in E} \pi(\epsilon'|\epsilon) V_{+1}(q') + d \right) \right\}
\]

\[
= \max_{c \geq 0, a' \geq 0} \left\{ u(y(q) - a') + \beta \sum_{\epsilon' \in E} \pi(\epsilon'|\epsilon) V_{+1}(q') + \beta d \right\} = TV + \beta d
\]

Therefore, the Belmann equation satisfies discounting. The operator \( T \) is a contraction mapping. The space \( Q \) is a complete metric space because it is the Cartesian Product of two complete metric spaces. As the utility function is a real value function, continuous, concave and bounded and that the space \( Q \) associated with a metric is a complete metric space, we can then conclude that the value function exists and is unique: there is an unique fixed point, therefore the solution to the problem exists and is unique.

Annex 2: Recursive formulation and stationary competitive equilibria for steady state

Recursive formulation of household’s problem

\[
V(a, \epsilon; \Phi) = \max_{c \geq 0, a' \geq 0} \left\{ u(c) + \beta \sum_{\epsilon' \in E} \pi(\epsilon'|\epsilon) V(a', \epsilon'; \Phi') \right\} \quad (27)
\]

s.t.

\[
c + a' = \epsilon(1 - \tau) \omega + (1 + (1 - \tau) r) a + T \quad (28)
\]

Equilibrium Definition

Definition 1. For a given fiscal rule \( R_j \) with \( j \in \{1, 2\} \), a stationary competitive equilibrium is a value function \( V : Q \times M \rightarrow R \), policy functions \( a' : Q \times M \rightarrow R \) and \( c : Q \times M \rightarrow R \), pricing functions \( r^* : M \rightarrow R \), \( \omega^* : M \rightarrow R \) and a probability measure \( \Phi^* \) such that
1. \( V \) satisfies the household’s Bellman equation and \( a', c \) are associated policy functions, given \( r^* \) and \( \omega^* \).

2. Given \( r^* \) and \( \omega^* \), \( K \) and \( N \) satisfy

\[
\begin{align*}
  r &= F_K(K, N) - \delta \\
  \omega &= F_N(K, N)
\end{align*}
\]

3. Market clearing,

\[
N = \int \epsilon d\Phi^*
\]

\[
\int c(a, \epsilon) d\Phi^* + \int a'(a, \epsilon) d\Phi^* = F(K, N) + (1 - \delta)K
\]

4. For all \((A, E) \in B(Q)\),

\[
\Phi^*(A, E) = \int Z((a, \epsilon), (A, E)) d\Phi^*
\]

**Annex 3: Computational algorithm for the steady state**

1. Fix \( \beta \in (0, \frac{1}{1 + r}) \) and solve the household recursive problem. Obtain \( v_{\beta}, a'_{\beta}, c_{\beta} \).

2. The policy function \( a'_{\beta} \) and \( \pi \) a Markov transition function \( Q_{\beta} \). Compute the unique stationary measure \( \Phi_{\beta} \) associated with this transition function.

3. Compute excess demand for capital

\[
d(\beta) = K_{ss} - Ea_{\beta}
\]

Where

\[
K_{ss} = \left( \frac{\alpha z}{r + \delta} \right)^{1 - \alpha} N
\]

If \( d(\beta) \approx 0 \), we have a SRCE. If not, update \( \beta \) from 1 anew. Since \( E_{a, \beta} \) is increasing in \( \beta \), then \( d(\beta) \) is strictly decreasing. So, if \( d(\beta) > 0 \ (< 0) \), increase (decrease) \( \beta \).
Annex 4: Computational algorithm for the transition

1. Fix the number of periods of transition $T$.

2. Compute the initial and final steady state.

3. Make a guess of the sequence of stock of debt $B_t^g$. This sequence implies a sequence of spread $s_t$ for each $t$, with which I can compute a sequence of stock of capital $K_g$.

4. With a sequence of capital I can compute a sequence of wages $\omega_g$, and with a sequence of employment $N_g$ I can compute a sequence of government income.

5. With a sequence of government income and a sequence of interest rate, I can compute a sequence of stock of debt $B_t^{\text{new}}$.

6. Check if $|B_t^{\text{new}} - B_t^g| < \epsilon$. If not, adjust the level of debt $B_t^g = B_t^{\text{new}}$ in 3. If yes, go to 7.

7. Check if $|B_T^{\text{new}} - B_T^g| < \epsilon$. If yes, we are ready. If not, go to one and increase $T$. 
