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Information Transmission in the Financial Advice: A Two Dimensional Approach

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Information Transmission in Financial Advice: A Two Dimensional Approach*

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1 Introduction

Advice is an opinion or recommendation offered as a guide to action. An advisor gives advice to a decision maker in order to guide his decision by providing information. Furthermore, advice is given in situations in which informational gaps exist between the agents in a market. In particular, agents in Financial markets have large information asymmetries (Brealey, Leland and Pyle, 1977), so advice is important in this market. Hung et al. (2008) show that 73 percent of all U.S. retailer investors consult financial advice before buying stocks and Chater et al (2010) shows that this number rises to 80 percent in the European Union. Nearly half of Canadian households report using financial advisors (The Investment Funds Institute of Canada 2012), and roughly 80% of the $876 billion in retail investment assets in Canada reside in advisor-directed accounts (Canadian Securities Administrators 2012).

Information asymmetries also make it difficult for the advisor to prove the truthfulness of his advice, so the decision maker takes advice only if there is trust between

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the pair. This may generate an incentive for the advisor to misreport his private information (Sobel, 1985). This incentive increases with the advisor bias regarding the decision-maker’s preferred action (Crawford and Sobel, 1982). The economic literature has focused on three different factors that generate this bias: commissions (Dugar and Nathan, 1995 and 1996, Womack, 1996, Stoughton et al, 2011), kickbacks (Stocken and Morgan, 2003), and career incentives (Lin and McNichols, 1998) that advisors receive in order to sell certain assets. These incentives to misreport have been recognised by the U.S. Securities and Exchange Commission (SEC), who have listed four areas of conflict of interest for the financial advisor: investment banking, brokerage commissions, analyst compensation and ownership interests in the company.

There is diverse evidence that such conflicts of interest exist and are important for the financial industry. First, the firms that provide financial services have placed a great emphasis on the construction of a “Great Wall” between the brokering and the other areas of the firm. Second, Christoffersen, Evans, and Musto (2013) shows that brokers are more likely to sell funds that earn them higher commissions and Michaely and Womack (1999) shows that brokerage analysts frequently recommend companies that their firm have recently taken public and those recommendations underperform the recommendations of unaffiliated brokers. Third, analysts and firms must abide by disclosure rules set by the New York Stock Exchange (NYSE) and the Financial Industry Regulatory Authority (FINRA). Fourth, in a recent victory of the State of New York versus Merril Lynch & Co., the latter agreed to pay $100 million and reform its stocks research process to settle allegations. (SEC, US Department of Treasury (2009)).

Despite the existence of bias and the reduced performance of managed portfolios, Foerster, Linmainmaa, Melzer and Previtero (2015) found evidence that financial advisors exert substantial influence over their clients’ asset allocation, which is proof of the existence of information transmission between advisor and investor. Michaely and Womack (1999) argue that the market does not recognize the full extent of the financial advisor bias, fostering the information transmission between advisor and investor.

In this work we study communication between investors and financial advisors; particularly, the conditions under which there is information transmission between the two and how a biased advisor behaves. We assume that advisors are anonymous in the sense that before interaction all the advisors are equal. This could be interpreted as the
relation between an investor and an advisor after the investor has selected an advisory firm but cannot distinguish between advisors. The novelty of our model is that the investor not only has uncertainty about the advisor bias (Benabou and Laroque, 1992, Morgan and Stocken, 2003), but also about his skill. This new uncertainty allows the existence of recommendations with different levels of confidence, which is what we see in Stock Recommendations (See Figure 1).

In our model there are two agents: a financial advisor (A) and an investor (R). There are two risky-assets and only one of them will be profitable. The investor must decide the investment in each asset, but before communication with the advisor she assigns to each asset the same probability to be profitable. The advisor receives a soft private signal about which will be the profitable asset, so he has better information about the profitable asset than the investor. Furthermore, the advisor has private information about the noise of his signal and is aware of his bias with respect to the investor’s preferences. The advisor and investor communicate through a costless message with two dimensions: which asset the advisor recommends and his confidence about that recommendation. The message is costless in the sense that for the advisor sending different messages does not have different costs. The investor sees the message and then chooses his investment. We solve this model for a single interaction.

Our model is an appropriate framework to represent the advisor-investor relation for four reasons. First, the advisor’s private information in financial markets is not verifiable by investors, because many of them do not have financial formation. Second, the advisor does not face a direct cost from recommending one thing or another - financial advice is cheap talk. Third, in financial markets there are advisors with different and difficult to identify skill levels. Fourth, the asset recommendations have two components that we use in the message of our model, specifically, the recommendations about an asset are of the form: strong buy, buy, hold, sell, strong sell. For example, Figure 1 represents 27 analyst recommendations over the Wal-Mart stock in July. Thirteen of them recommended strong buy, one recommended sell, twelve recommended hold, none recommended sell and one of them recommended strong sell.

Our model has four principal results. The first two results deal with the level of information transmission in equilibrium and the last two are about the behaviour of a biased advisor. First, when investor trust in the advisor is low enough, no information is transmitted through financial advice. Second, when the investor trust in the advisors
is not high enough, there is information transmission only if the unbiased advisors are sufficiently skilled. These two results show us that what’s important for information transmission in financial advice is not the number of unbiased advisors, but the amount of information they have. Third, if investor trust in the advisors is high enough, a deceitful advisor will always say that his confidence is high. Fourth, even in a single-period interaction, an advisor that wants to deceive the investor may send a message with low confidence in order to gain credibility.

Finally, in equilibrium, both messages provide different information about which type of advisor is giving the recommendation. In that sense, when the investor trust is high, the strong recommendations provide more information to the investor about the profitability of that asset than a weak recommendation, so the investor will invest more after receiving a strong recommendation. But when the investor trust is low, both messages provide the same information about the profitability of the asset, so the investment after receiving a strong recommendation is the same than after receiving a weak recommendation, despite that both message exist in equilibrium and provide different information to the investor about which type of advisor is sending the recommendation.
2 Related Literature

Our model is related to the cheap talk literature. In that sense, the closest work to ours is Benbaou and Laroque (1992), which also considers the incentives of analysts to misreport their information in a cheap talk model of financial advice. Our work differs from theirs in two respects. First, in our model the noise of the advisor signal is private information of the advisor whereas Benabou and Laroque (1992) treat the noise in the advisor signal as common knowledge. Second, in their model the advisor can only send two possible messages (which state will occur), but in our model the advisor can send four possible messages (which state will occur and his confidence about it).

Our paper is also related to Morgan and Stocken (2003). They analyse information transmission in the market for financial advice where the investor is uncertain about advisor bias. Our work differs from theirs in three respects. First, in their model the biased advisor wants to induce the investor to overvalue the profitability of all assets, while in our model the biased advisor wants to sell the unprofitable asset to the investor. Second, in their work, analyst skills are common knowledge. Third, they use messages where the advisor tells the investor which will be the future price of the asset, but without telling him his confidence about that.

Related works in the reputational cheap talk literature are Sobel (1985), who solves a model where the advisor receives a noiseless signal, and Morris (2001), who solves a model where the advisor always wants the decision maker to do the same action. Chakraborty and Harbaugh (2010) solves a multidimensional cheap talk model where the different dimensions of the message are about different actions that the decision maker must take. This is different than our model because our two dimensional approach of the message is about the same action, where one dimension is about the recommended action and the other dimension is about the confidence on that recommendation.

3 The Model

There are two agents: an investor (R) and a financial advisor (A). There are two risky assets: $y$ and $z$. One of them will pay $1 at the end of the period, while the other will pay nothing. Therefore there are two possible states of nature ($e$) defining which
Table 1: Risky Assets Payoffs

<table>
<thead>
<tr>
<th>Asset</th>
<th>State</th>
<th>( y )</th>
<th>( z )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( y )</td>
<td>1</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>( z )</td>
<td>0</td>
<td>1</td>
<td></td>
</tr>
</tbody>
</table>

of the risky asset is the profitable one, with some abuse of notation, we will call the state as the profitable risky asset of that state, that is, \( e \in \{ y, z \} \). Table 1 sums up the payoff of each asset in each state.

Both states are equally likely. Asset \( y \) and \( z \) cost \( \frac{1}{2} \). R has $1 to invest and she can buy any non negative amount of each asset. There is also a risk-free asset (\( f \)) that pays zero interest (keeping the money in the pocket) and the investor (R) can borrow at a constant interest rate \( 0 < k < \frac{1}{2} \). Therefore, R has to choose the investment amount in asset \( y \) (\( i^y \in [0, \infty) \)), in asset \( z \) (\( i^z \in [0, \infty) \)) and in the risk-free asset (\( i^f = 1 - i^y - i^z \)). We assume that if the investor is indifferent between investing in two portfolios, she chooses the portfolio with the bigger investment in the risk-free asset.\(^1\)

R’s utility, \( U_R(w_R) \), depends on her wealth (\( w_R(i^y, i^z) \)) at the end of the period, itself determined by the investment decision (\( i^y, i^z \)) and the state:

\[
w_R = \begin{cases} 
(1 - \frac{1}{2}i^y - \frac{1}{2}i^z) + i^y & \text{if } e = y \\
(1 - \frac{1}{2}i^y - \frac{1}{2}i^z) + i^z & \text{if } e = z 
\end{cases}
\]

\( U_R(w_R) \) is increasing in \( w_R \), strictly concave, twice differentiable and \( U_R'(0) = \infty \) and \( U_R'(\infty) = 0 \).

There are three possible types of financial advisor (A): Unbiased Unskilled (UU), Unbiased Skilled (US) and Biased Skilled (B). We will determine the advisor type by \( \tau \in \{UU,US,B\} \). In what comes next, we will refer to the Biased Skilled advisor simply as biased A. All agents behave strategically.

One characteristic that distinguishes between types of financial advisors is their

\(^1\)This assumption could be interpreted as that an investment in both assets has a very small probability to pay nothing, while the risk-free asset is secure. We use this assumption to simplify the investment decision of the investor, making her invest in only one of the assets at the same time and therefore simplify the explanation of the equilibrium.
skill level. Financial advisors may be skilled or unskilled. There are two types of
financial advisors that are skilled: unbiased skilled (US) and biased skilled (B). Each
of them receives a signal \( (s^{US}, s^{B} \in \{y, z\}) \) that contains perfect information about
the profitable asset. That is, \( \text{Prob}[s^{US} = y|e = y] = \text{Prob}[s^{US} = z|e = z] = \text{Prob}[s^{B} = y|e = y] = \text{Prob}[s^{B} = z|e = z] = 1 \). On the other hand, there is only one type
of financial advisor that is unskilled: the unbiased unskilled (UU) one. He receives a
noisy signal \( (s^{UU} \in \{y, z\}) \) about the profitable asset of that period, where \( \text{Prob}[s^{UU} = y|e = y] = \text{Prob}[s^{UU} = z|e = z] = p^{U} \) and \( \frac{1}{2} < p^{U} < 1 \), where \( (1 - p^{U}) \) is the noise of
the unskilled signal. \( s^{US}, s^{UU} \) and \( s^{B} \) are soft information (unverifiable by R).

This means that all types of financial advisors (F) have private information about
the profitable asset, but while the one that is unskilled (UU) has imperfect information,
the skilled ones (US,B) know exactly which state will occur. Financial advisors know
their own skill level, while the investor (R) does not. This information structure is
common knowledge.

Another characteristic that distinguishes between types of financial advisors is their
bias. A may be biased or unbiased. A knows his bias, but R does not. There are two
types of A that are unbiased: unbiased unskilled (UU) and unbiased skilled (US). Both
unbiased types have the same utility than the investor: \( U^{UU}(i^{y}, i^{z}) = U^{US}(i^{y}, i^{z}) = U^{R}(i^{y}, i^{z}) \). This represents financial advisors that want the investor (R) well-being.

There is one type of A that is biased: the biased skilled (B), with the u

The biased A is shown to be better off when he induces the investor to buy the
wrong asset, i.e., the asset that is unprofitable. Here we represent advisors that have
incentives to get rid of bad assets or to increase the demand for assets that their firm
has taken public and are not performing well.

The proportion of Unbiased Unskilled (UU) is \( \mu \), the proportion of Unbiased Skilled
(US) is \( \mu \) and the proportion of Biased Skilled (B) is \( 1 - 2\mu \). This is common knowledge
and is summed up in Table 2.

Let \( \mu \) represent investor trust. It reflects the confidence of the investor in the fact
that the firm’s financial advisors have incentives aligned with his own.

Before R makes an investment decision, A can send a costless message \( M \) to R. The
form of the message is \( M = (x, c) \), where \( x \in \{y, z\} \) represents the assets that A
advises the investor to buy and \( c \in \{\text{Low}, \text{High}\} \) represents the confidence about his
advice. The message does not directly affects any payoff. A message where \( c = \text{High} \)
Table 2: Proportion of advisor types.

<table>
<thead>
<tr>
<th></th>
<th>SKILLED</th>
<th>UNSKILLED</th>
</tr>
</thead>
<tbody>
<tr>
<td>UNBIASED</td>
<td>$\mu$</td>
<td>$\mu$</td>
</tr>
<tr>
<td>BIASED</td>
<td>$1 - 2\mu$</td>
<td>0</td>
</tr>
</tbody>
</table>

will be called a high confidence message and a message where $c = Low$ will be called a low confidence message.

The advice about the asset is truthful if the advisor recommends the signaled asset ($x = s^\tau$) and untruthful if he advises the not signaled one ($x \neq s^\tau$), with $\tau \in \{UU, US, B\}$. The same applies to his confidence report, both types of skilled A (US, B) report his confidence truthfully if they send a high confidence message ($c = High$) and the unskilled unbiased A type reports his confidence truthfully if he sends a low confidence message ($c = Low$).

### 3.1 Timing

The model could be summarized in the next timing:

0. Nature selects one type of A.

1. Nature chooses $e \in \{y, z\}$.

2. Nature sends the signal ($s^t \in \{y, z\}$) to the advisor.

3. A sends a message $M = (x, c)$ with $x \in \{y, z\}$ and $c \in \{Low, High\}$.

4. R receives $M = (x, c)$ and chooses $i^y, i^z$.

5. $e$ is revealed and the payoffs are received by R and A.
4 Information Transmission

The financial advisor (A) has to make two decisions: whether or not to advise the signaled asset and whether to do so with high or low confidence. We will focus on a symmetric equilibrium, in the sense that a strategy will not depend on whether the signaled asset was $y$ or $z$. For each type of advisor ($\tau$), we define:

- $q(\tau) \in [0,1]$, represents the probability that an advisor of type $\tau$ advises the signaled asset ($x = s^\tau$), independent of whether the signaled asset is $y$ or $z$.

- $r(\tau) \in [0,1]$, represents the probability that an advisor of type $\tau$ reports that his confidence is high, independent of whether the signaled asset is $y$ or $z$.

On the other hand, the investor (R) must choose her investment in each asset ($i^y, i^z, 1 - i^y - i^z$). R receives the message before she makes an investment decision, so after the message and before the investment, one asset is recommended and one is not recommended. We will refer to the assets using this distinction that the message generates. Therefore, the investment decision is determined by investment in the advised asset ($i^a$) and the investment in the non-advised one ($i^n$):

- $i^a$ is the investment in the recommended asset, independently if $y$ or $z$ are recommended.

- $i^n$ is the investment in the non-recommended asset, independently if $y$ or $z$ are recommended.

We will focus on an honest strategy in the sense used by Sobel (1985) and Benabou & Laroque (1992), where the unbiased types of A (UU,US) recommend the asset and report their confidence truthfully. That is, both unbiased types advise the signaled asset ($x = s^\tau$, $\tau \in \{UU,US\}$), the unbiased skilled type (US) reports that his confidence is high ($r(US) = 1$) and the unbiased unskilled type (UU) reports that his confidence is low ($r(UU) = 0$), for any strategy of the investor (R) and the biased A. Starting from this honest strategy of the unbiased types, we will see the optimal responses of R and the biased A and finally, given those optimal responses, we will test to see if the

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2The reason for the focus on this type of equilibrium is that in our model the assets are anonymous, in other words, the name of the asset does not provide any information about their future payments.
honest strategy is optimal for both unbiased types.

**DEFINITION 1**: An honest strategy is a strategy in which both unbiased types advise the signalled asset and report their confidence truthfully. The honest strategy of the unbiased skilled A is \((q(US) = 1, r(US) = 1)\) and the honest strategy of the unbiased unskilled A is \((q(UU) = 1, r(UU) = 0)\). Therefore, the message that each of them send is:

\[
M = (s^{UU}, Low) \quad \text{for the unbiased unskilled type}
\]

\[
M = (s^{US}, High) \quad \text{for the unbiased skilled type}
\]

An honest equilibrium is one in which unbiased types play honest strategies.

A high confidence message will be represented by \(M^h\) and a low confidence message will be represented by \(M^l\).

Given the honest strategy, only two messages can come from unbiased advisors: \(M^h\) (a high confidence message) and \(M^l\) (a low confidence message). Any message that differs from these two will be interpreted by R as coming from a biased A, and will not be believed. Because the advisor has two possible skill levels, a message with two possible levels of confidence is rich enough to separate the two types of unbiased advisors in an honest strategy.

The existence of two possible levels of confidence in the advise is useful for information transmission between A and R, because R is not sure of the advisor skill-level. Confidence is the only component in the message that provides information to R about which type of advisor is sending the message. The advised asset could be \(y\) or \(z\), but does not provide any information to R about which type of advisor is sending the message, because both unbiased types recommend \(y\) and \(z\) only when signaled, and they do so with the same likelihood. On the other hand, the confidence with which the message is reported provides information about the advisor’s type because a high (low) confidence message cannot come from an unbiased unskilled (skilled) advisor. Again, \(r(B)\) represents the probability that the biased A sends a high confidence message and \(1 - r(B)\) represents the probability that the biased A sends a low confidence message. Because \(r(UU)\) and \(r(US)\) are fixed, hereafter, we refer to \(r(B)\) just as \(r\).

Once the biased A has chosen the type of message, he will optimally choose a strategy to advise the signaled asset. This strategy depend on whether he is sending a
low or high confidence message. With some abuse of notation, we will define \( q^h(B) \) as the probability that the biased A reports the signaled asset when he is sending a high confidence message and \( q^l(B) \) as the probability that the biased A reports the signaled asset when he is sending a low confidence message. Because \( q(UU) \) and \( q(US) \) are fixed, hereafter, we refer to \( q^h(B) \) and \( q^l(B) \) simply as \( q^h \) and \( q^l \).

After the investor (R) receives \( M \in \{M^l, M^h\} \), she updates (using Bayesian updating) the probability that the advised asset is indeed the profitable one. That is, \( \text{Prob}[x = y | e = y, M^h, q^h, r, \mu] = \text{Prob}[x = z | e = z, M^h, q^h, r, \mu] = \text{Prob}[x \text{ is profitable} | M^h, q^h, r, \mu] \) is the updated probability that the advised asset will be profitable when R receives a high confidence message and \( \text{Prob}[x = y | e = y, M^l, q^l, r, \mu] = \text{Prob}[x = z | e = z, M^l, q^l, r, \mu] = \text{Prob}[x \text{ is profitable} | M^l, q^l, r, \mu] \) is the updated probability that the advised asset will be profitable when R receives a low confidence message\(^3\). These are:

\[
\begin{align*}
\text{Prob}[x \text{ is profitable} | M^l] &= \frac{\mu p^U}{(1-2\mu)(1-r)} + \frac{(1-2\mu)(1-r)q^l}{(1-2\mu)(1-r) + \mu} \\
\text{Prob}[x \text{ is profitable} | M^h] &= \frac{\mu}{(1-2\mu) r + \mu} + \frac{(1-2\mu) r q^h}{(1-2\mu) r + \mu}
\end{align*}
\]

\(^3\)The equality in these probabilities comes from the assumption that A is playing a symmetric strategy.

Given \( \text{Prob}[x \text{ is profitable} | M] \), with \( M \in \{m^S, m^U\} \), R chooses investment amounts in the recommended \((i^a)\) and the non recommended \((i^n)\) assets:

\[
\max_{i^a, i^n \in [0, \infty)} E[U^R(w^R(i^a, i^n))]
\]

The optimal investment amount in each asset \((i^a, i^n)\) satisfies:

\[
\frac{U^R(i^n)}{U^R(i^a)} = \frac{\text{Prob}[\cdot | M]}{1 - \text{Prob}[\cdot | M]}
\]

with \( M \in \{M^l, M^h\} \).

Where \( \text{Prob}[\cdot | M] \) is the probability that the recommendation is correct (i.e., that the advised asset is the profitable one). An equilibrium has information transmission if and only if \( \text{Prob}[\cdot | M] \neq \frac{1}{2} \). A message is believed to be true if \( \text{Prob}[\cdot | M] > \frac{1}{2} \), is believed to
be false if $\text{Prob}[\cdot | M] < \frac{1}{2}$ and is believed to not convey any information if $\text{Prob}[\cdot | M] = \frac{1}{2}$. Finally, the level of information transmission is given by $|\text{Prob}[\cdot | M] - \frac{1}{2}|$.

Given that the cost of each asset is $\frac{1}{2}$, that the assets payoffs are correlated and the assumption that if the investor is indifferent between two portfolios she chooses the one with the bigger investment in the risk-free asset, we can characterize her investment decision in the following way:

**LEMMA 1:**

1. *When $\text{Prob}[\cdot | M] > \frac{1}{2}$, $i^a(\text{Prob}[\cdot | M])$ is continuous, differentiable and increasing in $\text{Prob}[\cdot | M]$.*

   *When $\text{Prob}[\cdot | M] < \frac{1}{2}$, $i^a(\text{Prob}[\cdot | M])$ is continuous, differentiable and decreasing in $\text{Prob}[\cdot | M]$.*

2. *The investment in each asset is:*

   $$
   \begin{cases}
   i^a = \infty, i^n = 0 & \text{if } \text{Prob}[\cdot | M] = 1 \\
   i^a > 0, i^n = 0 & \text{if } \frac{1}{2} < \text{Prob}[\cdot | M] < 1 \\
   i^a = 0, i^n = 0 & \text{if } \text{Prob}[\cdot | M] = \frac{1}{2} \\
   i^a = 0, i^n > 0 & \text{if } 0 < \text{Prob}[\cdot | M] < \frac{1}{2} \\
   i^a = 0, i^n = \infty & \text{if } \text{Prob}[\cdot | M] = 0
   \end{cases}
   $$

3. *$\text{Prob}[\cdot | M]$ depends on 2 parameters ($\mu$ and $p^U$) and on the strategy of the biased $A(q^h, q^l, r)$.***

   *Proof: See Appendix.*

This lemma states three things. First, if the investor (R) believes that the message is true ($\text{Prob}[\cdot | M] > \frac{1}{2}$), she invests a positive amount in the recommended asset and any amount in the non-recommended one. If R does not believes the message ($\text{Prob}[\cdot | M] < \frac{1}{2}$), she invests a positive amount in the non-recommended asset and any amount in the advised one. Finally, if R believes that the message does not convey any information ($\text{Prob}[\cdot | M] = \frac{1}{2}$), she does not invest anything in the risky-assets and keeps all the money in her pocket (risk-free asset). Second, when R is investing a positive amount in a risky asset, the invested amount in that asset is increasing in the
probability that the asset is profitable. Third, given the parameters of the model and the honest strategy of the unbiased types, the probability that a message is correct is given by the strategy of the biased A \((r, q^h, q^l)\).

We now study the optimal strategy for the biased A, given the honest strategy of the unbiased type of advisors and investment decision of R. Because the unbiased types of advisors are playing an honest strategy, the biased A knows exactly which \(Prob[\cdot | M^h, \mu, q^h, r]\) and \(Prob[\cdot | M^l, \mu, q^l, r]\) will induce with the strategy \((q^h, q^l, r)\). The biased A wants to induce R to invest the largest possible amount in the unprofitable asset and to do so he wants to make the information transmission of the message as high as possible.

The biased A faces a trade-off between the probability of advising the signaled asset and the probability of each message being correct. When the investor believes that the high (low) confidence message is true for a given value of \(q^h (q^l)\), the biased A only gets a positive payoff when he advises the non-signaled asset. In this case, a higher \(q^h (q^l)\) makes the message more credible, so when the biased advisor advises the non-signalled asset he gets a higher payoff, but this occurs with a lower probability. The biased A faces the same trade-off when the investor believes that the high (low) confidence message is false for a given value of \(q^h (q^l)\), a lower \(q^h (q^l)\) makes the message less credible, so when the biased advisor advises the signaled asset he gets a higher payoff, but this occurs with a lower probability. Lemma 2 characterizes this trade-off in equilibrium.

**LEMMA 2:** If there is an equilibrium in which the biased A reports the signaled state with a positive probability (i.e., \(q^h > 0\) or \(q^l > 0\)), then that equilibrium does not have information transmission. In equilibrium, \(Prob[\cdot | M] \geq \frac{1}{2}\).

Proof: See Appendix.

If for a given \(q^h (q^l)\) the investor believes that the message is false, the biased advisor obtains a positive payoff when he advises the signaled asset and a zero payoff when he advises the non-signaled one. In this case, the negative effect of an increase of \(q^h (q^l)\) in the amount of the positive payoff is smaller than the positive effect on the probability of getting the positive payoff. This is true for any \(q^h (q^l)\) such that \(Prob[\cdot | M^h, q^h] < \frac{1}{2}\) (\(Prob[\cdot | M^l, q^l] < \frac{1}{2}\)). Therefore, it is never optimal for biased advisor to play a \(q^h < 1\)
($q^l < 1$) such that the investor believes that the message is false. On the other hand, if for a given $q^h$ ($q^l$) the investor believes that the message is true, the biased advisor obtains a positive payoff when he advises the non-signaled asset and a payoff of zero when he advises the signaled one. The negative effect of a decrease of $q^h$ ($q^l$) in the amount of the positive payoff is smaller than the positive effect on the probability of getting the positive payoff. This is always true when the investor believes that the message is true. Therefore, a $q^h > 0$ ($q^l > 0$) is never optimal for the biased A if the investor believes that the message is true. The difference between the two cases is that the biased advisor can always make the investor believe that the message is true if he advises the signaled asset with a high enough probability (high $q^h, q^l$), but he can not always convince the investor that the message is false. When investor trust is high enough, even when $q^h = 0$ ($q^l = 0$), the investor believes that the message is true. So, $q^h = 1$ ($q^l = 1$) cannot be part of an equilibrium because the investor will believe the message is true and the biased A therefore has an incentive to reduce $q^h$ ($q^l$). Still, $q^h = 0$ ($q^l = 0$) can be part of an equilibrium, because if investor trust is high enough, he still believes that the message is true and the biased F does not have incentive to increase $q^h$ ($q^l$). Therefore, $0 < q^h < 1$ ($0 < q^l < 1$) is an optimal strategy for the biased A only when the investor believes that the message does not convey any information, because only in that case does the biased A not have an incentive to increase (when the message is believed to be false) or decrease (when the message is believed to be true) it.

Lemma 2 states two things. First, that when information transmission exists, the biased A always advises the non-signaled asset ($q^l = 0, q^h = 0$). Second, in equilibrium, $\text{Prob}[\cdot | M] \geq \frac{1}{2}$, that is, messages are either believed (to be true) or to not convey information, but never are believed to be false.

The first statement of Lemma 2 implies that when information transmission takes place, the respective probabilities that the messages are correct are:

$$\text{Prob}[x \text{ is profitable} | M^l] = \frac{\mu p^U}{(1 - 2\mu)(1 - r) + \mu}$$

$$\text{Prob}[x \text{ is profitable} | M^h] = \frac{\mu}{(1 - 2\mu) r + \mu}$$

(4')
The second statement of Lemma 1 allows us to label those probabilities as “credibility”, because for the biased A the credibility of each type of message is all that matters. The credibility has two components:

1. The probability that the message was sent by an unbiased advisor. We call this the trust component. In the low confidence message, this component is \( \frac{\mu}{(1-2\mu)(1-r)+\mu} \) and in the high confidence message is \( \frac{\mu}{(1-2\mu) r+\mu} \).

2. The probability that a message sent by an unbiased advisor is correct. We call this the skill component. In the low confidence message this is \( pU \) and in the high confidence message it is 1.

The biased A cannot affect the skill component, but he can affect the trust component. The higher the probability that a biased A sends a type of message, the lower the likelihood that the corresponding type of message is coming from an unbiased advisor. Therefore, a larger \( r \) makes the trust component of the high confidence message smaller and the trust component of the low confidence message larger.

**Lemma 3:** The biased A will always send a high confidence message with a probability higher than \( \frac{1}{2} \) (In equilibrium, \( r > \frac{1}{2} \)).

Proof: See Appendix

This Lemma states that a larger proportion of biased advice will be conveyed using high confidence messages. The reason is that the high confidence message has a bigger skill component than the low confidence message \( (pU < 1) \) and when \( r = \frac{1}{2} \) the trust component of both messages is the same \( (2\mu) \). In this way the high confidence message is more credible and the advisor obtains a larger payoff by sending the high confidence message than the low confidence one. Therefore, the biased A is better-off displaying high confidence with a probability higher than \( \frac{1}{2} \).

Proposition 1 characterizes the honest equilibrium and the information transmission in it for the possible combinations of the model parameter values \( (\mu, pU) \).

**Proposition 1:** If \( \mu \leq \frac{1}{4} \) the honest equilibrium does not include information transmission. If \( \frac{1}{4} < \mu \leq \frac{1}{3} \) the honest equilibrium does not have information transmission if \( pU < \frac{1-2\mu}{2\mu} \), but if \( pU > \frac{1-2\mu}{2\mu} \) it has information transmission, where the
biased $A$ sends high confidence messages with probability $r^*$. If $\frac{1}{3} < \mu < \frac{1}{2}$ the honest equilibrium always has information transmission and in such an equilibrium, $r < 1$ if $p^U \geq \frac{\mu}{(1-\mu)}$ and $r = 1$ if $p^U < \frac{\mu}{(1-\mu)}$.

Proof: See Appendix.

This equilibrium is represented graphically in Figure 2.

Figure 2: Proposition 1 (the x-axis represents $\mu$ and the y-axis represents $p^U$)

The combination of the possible $\mu, p^U$ generates three areas where the honest equilibrium has different characteristics: 1) An area where the honest equilibrium does not include information transmission, 2) An area where the honest equilibrium includes information transmission and the biased $A$ sends a low confidence message with a positive probability and 3) An area where the honest equilibrium has information transmission and the biased $A$ always sends a high confidence message.

The information transmission in the message, presented in Proposition 1, can be analyzed across different values of $\mu$.

When $\mu$ is low ($\mu \leq \frac{1}{4}$), the trust component of both messages is small enough in order to induce $R$ to not believe any message, making information transmission impossible.

When $\mu$ is at an intermediate level ($\frac{1}{4} < \mu \leq \frac{1}{3}$), the trust component is not enough to make the messages credible. $R$ will believe the messages only if the trust
component of the low confidence message is big enough to compensate for the small trust component, that is, when the noise of the unskilled signal is low enough (and $p^U$ is high).

When $\mu$ is high ($\mu > \frac{1}{3}$), the trust component of the messages is large for both messages, so without depending on the skill components, the messages will be credible. Therefore, there will be information transmission for every level of $p^U$.

Now, we analyze the equilibrium strategy of the biased A, who chooses three things: $q^l, q^h, r$. In the zone of information transmission, the biased A always advises the non-signaled asset, so $q^l, q^h = 0$.

4On the other hand, from Lemma 3, we only know that $r$ is between $\frac{1}{2}$ and 1, but it’s equilibrium value is given by $\mu$ and $p^U$, because these parameters affect the skill and the trust components of the messages.

The skill component of the high confidence message is higher than the low confidence message ($p^U < 1$) and this gap gets bigger as $p^U$ decreases. On the other hand, we know that in equilibrium $r > \frac{1}{2}$, so the trust component of the low confidence message is higher than the trust component of the high confidence message and this gap is increasing in $r$. This effect is greater as $\mu$ increases, because when there are only a few biased A, the proportion of biased A that send each type message makes only a slight difference in the probability that each type of message is coming from an unbiased advisor (trust component), but when the proportion of biased A is large, the proportion of them that send each type of message has a big impact on the trust component of each type of message.

When $\mu$ is high ($\frac{p^U}{1+p^U} < \mu < \frac{1}{2}$), even with $r = 1$ (every biased A sends high confidence messages), the gap in the trust component (between low and high confidence messages) is not enough to compensate the gap in the skill component, so the high confidence message is still more credible and $r = 1$ is an optimal strategy for the biased A. This occurs when skill component of the low confidence message is small ($p^U$ is small) and/or when there only a few biased advisors ($\mu$ is high).

When $\mu$ is not high ($0 < \mu \leq \frac{p^U}{1+p^U}$), $r = 1$ is not optimal for the biased A, because there are many biased A and/or the gap in the skill component between messages is small ($1 - p^U$ is small). Therefore, the gap in the trust component that $r = 1$ generates is larger than the gap in the skill component, a low confidence message is more credible and the biased A is better-off sending the high confidence message with a

4Lemma 2.
lower probability than 1. In equilibrium \( r \) is such that the gap in the trust component exactly compensates the gap in the skill component of the messages and both messages are equally credible, because the better information of the unbiased skilled advisor is exactly compensated by the lower probability of receiving advice from an unbiased advisor in the high confidence message. Therefore, when \( r < 1 \), an equilibrium with information transmission will be defined by an \( r^* \) such that \( \text{Prob}[\cdot|M^h, r = r^*, q^h = 0] = \text{Prob}[\cdot|M^l, r = r^*, q^l = 0] \):

\[
r^*(p^U, \mu, q^l = 0, q^h = 0) = \frac{1 - 2\mu + \mu(1 - p^U)}{(1 - 2\mu)(1 + p^U)}
\]

(5)

Finally, we need to check that this is an equilibrium, that is, that the unbiased types do not have incentives to deviate from their honest strategy.

We start with the area where the honest equilibria does not have information transmission. In that area, both messages \( (M^l, M^h) \) induce \( \text{Prob}[\cdot|M^l] = \text{Prob}[\cdot|M^h] = \frac{1}{2} \), so both unbiased types (UU,US) will not have any incentive to deviate in the confidence report of their honest strategy. On the other hand, the investor (R) believes that the message does not convey information and she will not invest in any of the asset. Therefore, both unbiased types will have no incentives to recommend the signaled asset and deviate from their honest strategy, because reporting the signaled asset has no effect on the payoff.

In the area where the honest equilibria includes information transmission and the biased A sends a low confidence message with a positive probability, both messages have the same probability to be correct, so both unbiased types do not have an incentive to deviate in the confidence report of their honest strategy. On the other hand, their message is believed by R and she invests a positive amount in the advised asset. Therefore, when both unbiased types advise the signaled asset, the investor buys the asset that they think is more likely to be profitable \(^5\). Then, both unbiased types do not have an incentive to deviate from reporting the signaled asset.

We now check the area where the honest equilibria has information transmission and the biased A always send high confidence messages. When R sees \( M^l \), she believes

\(^5\)Remember, the unbiased skilled advisor is sure about which asset is profitable, while the unbiased unskilled is not.
that \( \text{Prob}[\cdot|M^l] = p^U \), which is the optimal probability that the unbiased unskilled A would choose to induce, so he does not have any incentive to deviate from the low confidence report of his honest strategy. In this area, the high confidence message \((M^h)\) induces a higher \( \text{Prob}[\cdot|M] \) than the low confidence message \((M^l)\) and the unbiased skilled type wants to induce a high \( \text{Prob}[\cdot|M] \), so he will not deviate from reporting high confidence. On the other hand, the non-deviation of the unbiased types from advising the signalled asset follows the same logic. If information transmission exist, both unbiased types do not have an incentive to deviate from reporting the signalled asset.

5 Comparative Statics

5.1 Information Transmission

We analyze the effect of changes in \( \mu \) and \( p^U \) on the information transmission.

The impact of a change in investor trust on the level of information transmission is not monotonous. Figure 3 shows the level of information transmission \(|\text{Prob}[\cdot|m] - \frac{1}{2}|\) for corresponding levels of investor trust.

When \( \mu \) is high \((\frac{p^U}{1+p^U} < \mu < \frac{1}{2})\), all the biased types send high confidence messages, so a decrease in investor trust only decreases the trust component of the high confidence messages, but does not affect the trust component of low confidence messages. Therefore, a decrease in investor trust decreases the information transmission of both messages. In this range of \( \mu \), both types of messages have the same level of information transmission.

When \( \mu \) is intermediate \((\frac{1}{2(1+p^U)} < \mu \leq \frac{p^U}{1+p^U})\), the biased type sends both types of messages with a positive probability, so a decrease in the investor trust decreases the trust component of both type of messages. Therefore, a decrease in investor trust decreases the information transmission of both messages. In this range of \( \mu \), both types of messages have the same level of information transmission.

When \( \mu \) is low \((0 \leq \mu \leq \frac{1}{2(1+p^U)})\), there is no information transmission and a decrease in the investor trust does not affect the information transmission of any message. The intuition behind this is direct. Without loss of generality, we analyze the
Figure 3: Probability that the advised asset is profitable for each type of message (Thick blue for a low conviction message and thin red for high conviction) for different values of $\mu$, given $\frac{1}{2} < p^U < 1$:

![Figure 3: Probability that the advised asset is profitable for each type of message](image)

information transmission of a high confidence message. If before the change in the proportion of biased A there was no information transmission (i.e., $Prob[\cdot | M^h, q^{h*}] = \frac{1}{2}$), a decrease of $\mu$ decreases the trust component of the high confidence message, so $Prob[\cdot | M^h, q^{h*}] < \frac{1}{2}$. $q^{h*}$ is no longer part of an equilibrium, because the biased A has an incentive to advise the signaled asset with a higher probability. Therefore, the new equilibrium will be when $Prob[\cdot | m^S, q^{h'}] = \frac{1}{2}$, with $q^{h'} > q^{h*}$. In this case, the reduction of the trust component is compensated by an increase in the probability that the biased A advises the signaled asset, making the information transmission of the message constant with regard to marginal changes in $\mu$.

Information transmission is decreasing in the noise of the unskilled signal, because more noise (less $p^U$) decreases the skill component of the low confidence message. Therefore, the critical level of investor trust which encourages the existence of information transmission is decreasing in $p^U$. The information transmitted by the low confidence message is increasing in $p^U$. The information transmitted by the high confi-
dence message is not affected by a change in $p^U$ in the area where the biased F always sends high confidence messages, but is increasing in the zone in which the biased F sends a low confidence message with a positive probability.

5.2 Biased Advisor Strategy

We know that $q^l = 0$ and $q^h = 0$ in the zone of information transmission. Alternatively, $r$ depends of $\mu$ and $p^U$ in equilibrium. Figure 4 represents graphically the equilibrium probability that a biased A sends a high confidence message ($r$) for different values of $\mu$ and $p^U$.

Figure 4: $r^*(\mu, p^U)$ in a scale of greys between Black ($r^* = \frac{1}{2}$) and white ($r^* = 1$), for different possible values of $\mu$ and $p^U$:

As the noise of the unbiased unskilled signal decreases (larger $p^U$), the gap in the skill component gets smaller. To compensate, a smaller gap in the trust component between the messages is required. Therefore, in equilibrium, $r$ is decreasing in $p^U$.

**Corollary 1:** The effect of a change in the noise of the unskilled signal $(1 - p^U)$ in the proportion of low confidence messages is monotonous. The proportion of low confidence messages is decreasing in the noise of the unskilled signal $(1 - p^U)$.
The effect of a change in investor trust ($\mu$) on $r$ depends on whether or not investor trust is high:

When $\mu$ is high ($\frac{\mu'}{1+p'} < \mu < \frac{1}{2}$), a marginal change in $\mu$ does not affect $r$, because in this range of $\mu$ a high confidence message is more credible than a low confidence message. Therefore, a marginal change in $\mu$ does not change this condition and $r = 1$ remains optimal. In equilibrium, when $\mu$ is high, a change in $\mu$ does not affect $r$.

When $\mu$ is not high ($0 < \mu \leq \frac{\mu'}{1+p'}$), both message are equally credible and a positive marginal change in $\mu$ increases the trust gap. This breaks the equality, so a smaller $r$ is needed in order to recover. When $\mu$ is not high, $r$ is decreasing in $\mu$.

**Corollary 2:** The effect of a change in investor trust on the proportion of low confidence messages is not monotonous. In the region in which information transmission exists and the biased A sends a low confidence message with a positive probability, a decrease in investor trust ($\mu$) increases the proportion of low confidence messages. In the region in which information transmission exists and the biased A always sends a high confidence message, a decrease in the investor trust ($\mu$) decreases the proportion of low confidence messages.

A database of weekly stocks recommendations from more than 30 financial advisors is available over the last 10 years. This database could be used to construct a variable measuring the proportion of low confidence messages, and later used to empirically test both corollaries.

### 6 Conclusions

Our work investigates information transmission in financial advice. With this kind of advice, the private information of the advisor is soft, so that trust (in the advisor) is essential for information transmission. In accordance with this, our results show that information will be transmitted only if the trust in the advisor is sufficiently high. This trust is in two dimensions - trust regarding the advisor’s incentives, and also regarding his skills. In order for information transmission to exist, unbiased advisors must have enough information in order to compensate for the proportion of biased advisors. This result differs from Sobel (1985), Benabou and Laroque (1992), Morris (2001), Morgan and Stocken (2003) and Chakraborty and Harbaugh (2010), where the unique condition
for information transmission is the trust that the advisor has correct incentives. In
the same way, a strong recommendation induces a higher investment than a weak
recommendation only when the investor trust is high. However, if investor trust is low,
the investment decision does not depend on the strength of the recommendation.

Our model also provides insight about the behavior of a biased financial advisor.
When investor trust in the advisor is high enough, a biased advisor sends advice with
high confidence in order to induce the investor to invest more. On the other hand,
when the trust in the advisors is not sufficiently high, the advisor could send a low-
confidence message in order to gain credibility, even when the interaction is one-time
occurrence and reputation is not a concern.
7 Bibliography


http://www.sec.gov/investor/pubs/analysts.htm

Appendix

Proof Lemma 1:

\[
\max_{i^y, i^z \in [0, \infty)} \ Prob[e = y|M] U^R \left( 1 + \left( 1 - \frac{1}{2} \right) i^y - \frac{1}{2} i^z \right) \\
+ (1 - Prob[e = y|M]) U^R \left( 1 + \left( 1 - \frac{1}{2} \right) i^y - \frac{1}{2} i^z \right)
\]

\( (6) \)

\[
FOC(i^y) : \frac{Prob[e = y|M]}{(1 - Prob[e = y|M])} = \frac{U^R (1 + (1 - \frac{1}{2}) i^z - \frac{1}{2} i^y)}{U^R (1 + (1 - \frac{1}{2}) i^y - \frac{1}{2} i^z)}
\]

\( (7) \)

\[
FOC(i^z) : \frac{(1 - Prob[e = y|M])}{Prob[e = y|M]} = \frac{U^R (1 + (1 - \frac{1}{2}) i^y - \frac{1}{2} i^z)}{U^R (1 + (1 - \frac{1}{2}) i^z - \frac{1}{2} i^y)}
\]

\( (8) \)

Without loss of generality, let’s analyze the case where \( \frac{1}{2} \leq Prob[e = y|M] \leq 1 \). Because we assume symmetric strategies, \( 0 \leq Prob[e = y|M] \leq \frac{1}{2} \) is analogous for \( i^z \).

We are going to analyze the optimal portfolio for different values of \( Prob[e = y|M] \) between \( \frac{1}{2} \) and 1.

1. When \( Prob(E = y|M) = \frac{1}{2} \)

After R sees the message, she thinks that both risky-asset have the same probability to be profitable.

Equations (8) and (9) imply an optimal portfolio where the investment in each asset is the same: \( i^y = i^z \)

This optimal portfolio \( (i^y = i^z) \) payoffs could be replicate by one defined by \( (i^{y*}, i^{z*}) \), where \( i^{y*} = i^{z*} = 0 \). We assume that if the investor (R) is indifferent between two portfolios, she chooses the one with the bigger investment in the risk-free asset. So, when \( Prob[e = y|M] = \frac{1}{2} \), R will choose a portfolio given by \( (i^y = 0, i^z = 0) \), that is, she will not invest in any of the risky asset.

2. When \( \frac{1}{2} < Prob[e = y|M] < 1 \):

After R sees the message, she thinks that the asset \( y \) is more likely to be profitable, but is not sure that will be profitable.

Equations (8) and (9) implies that in the optimal portfolio, the relation between the investment in each type of risky asset satisfies the following condition: \( i^y > i^z \)
Also, because we assume that $U R(0) = \infty$, another condition of the optimal portfolio is $i^y < 1$.

Any portfolio that satisfies the optimal condition $(0 \geq i^y < i^z < 1)$ could be replicated by a portfolio defined by $(i^{y*}, i^{z*})$, where $i^{y*} = i^y - i^z > 0$ and $i^{z*} = 0$. We assume that if $R$ is indifferent between two portfolios, she chooses the one with the greatest investment in the risk-free asset, so when $\frac{1}{2} < \text{Prob}[e = y|M] < 1$, the investment decision of $R$ is $(0 < i^y < 1, i^z = 0)$, that is, she invest only in the asset that is more likely to be profitable.

3. When $\text{Prob}[e = y|M] = 1$:

This means that $R$ is sure that $y$ will be the profitable asset, so will only invest in $y$.

The investor ($R$) has only $1$ to invest and she has to borrow (at rate $k$) in order to invest more than that. Therefore, there is a discontinuity on her utility function at $i^y = 2$. $R$ solves:

$$
\begin{cases}
\text{MAX } U^R(1 + (1 - \frac{1}{2})i^y) & \text{if } i^y \leq 2 \\
\text{MAX } U^R(2 + (1 - \frac{1}{2} - k)(i^y - 2)) & \text{if } i^y > 2 
\end{cases}
$$

But we assume that $0 < k < \frac{1}{2}$, so $(1 - \frac{1}{2} - k)$ is positive and the utility is strictly increasing in $i^y$. Therefore, if $R$ is sure about the state, she will invest $\infty$ in asset $y$ and nothing in asset $z$.

The three previous points characterize $R$ investment decision $(i^y, y^z)$ for different values of $\text{Prob}[e = y|M]$, which are the point 2 of Lemma 1.

Now we have to prove the points 1. and 3.

Again, without loss of generality, let’s assume that $\frac{1}{2} \leq \text{Prob}[e = y|M] \leq 1$.

Let’s now prove that $i^{y*}(\text{Prob}[e = y|M])$ is continuous in $\text{Prob}[e = y|M]$.

Let’s define:

$$
U^{R*}(\text{Prob}[e = y|M]) \equiv \max_{i^y \in [0, \infty)} U^R(i^y, \text{Prob}[e = y|M])
$$

and

27
\[
i^y(\text{Prob}[e = y|M]) \equiv \{i^y \in [0, \infty) \mid U^R(i^y, \text{Prob}[e = y|M]) = U^{Rs}(\text{Prob}[e = y|M])\}
\]

\[
U^R(i^y, \text{Prob}[e = y|M]) \text{ is strictly concave in } i^y, \text{ so } i^y(\text{Prob}[e = y|M]) \text{ is a function.}
\]

Because \(\text{Prob}[e = y|M] \subseteq \mathbb{R}, \ i^y \subseteq \mathbb{R}\) and \(U^R(i^y, \text{Prob}[e = y|M])\) is continuos, we know from Berge Maximum Theorem that \(i^y(\text{Prob}[e = y|M])\) is continuos (uhc implies continuity in a function). This proves part of the point 1 of Lemma 1.

We now prove that \(i^y(\text{Prob}[e = y|M])\) is increasing in \(\text{Prob}[e = y|M]\) when \(\frac{1}{2} \leq \text{Prob}[e = y|M] \leq 1\).

We know from point 2. of Lemma 1 that in this range of \(\text{Prob}[e = y|M]\), \(i^y \geq 0\) and \(i^z = 0\). Therefore,

\[
\frac{\partial^2 U(i^y, \text{Prob}[e = y|M])}{\partial i^y \partial \text{Prob}[e = y|M]} = U'(i^y/e = y) + \frac{1}{2} U'(i^y/E = z) > 0 \quad (10)
\]

(11) implies that \(U(i^y, \text{Prob}[e = y|M])\) is supermodular in \((i^y, \text{Prob}[e = y|M])\). By Topkins Theorem, \(i^y(\text{Prob}[e = y|M])\) is increasing in \(\text{Prob}[e = y|M]\), which proves the antoher part of point 1 in Lemma 1.

The proof of point 3 comes direct from equation (4).

**Proof Lemma 2:**

Let’s define:

\(V^h_s\): The payoff for the biased A when he is seding a high confidence message and advises the signaled asset.

\(V^h_n\): The payoff for the biased A when he is sending a high confidence message and advises the not signaled asset.

\(V^l_s\): The payoff for the biased A when he is sending a low confidence message and advises the signaled asset.

\(V^l_n\): The payoff for the biased A when he is sending a low confidence message and advises the not signaled asset.

Without loss of generality, let’s assume that the biased A is sending a high confidence message. The relevant decision then is \(q^S\).
Let’s assume that $q^{S*} > 0$ is an equilibrium strategy with information transmission. The information transmission implies one of two things: that the message is believed to be true ($\text{Prob}[x \text{ is profitable}|M^h, q^{h*} > 0] > \frac{1}{2}$) or that the message is believed to be false ($\text{Prob}[x \text{ is profitable}|M^h, q^{h*} > 0] < \frac{1}{2}$).

1. If the message is believed to be true ($\text{Prob}[x \text{ is profitable}|M^h, q^{h*} > 0] > \frac{1}{2}$):
   
   We know from Lemma 1 that $i^a > 0$ and $i^n = 0$.

   Therefore, $V^h_n > 0$ and $V^h_a = 0$.

   $q^{S*}$ is not an equilibrium.

2. If the message is believed to be false $\text{Prob}[x \text{ is profitable}|M^h, q^{h*} > 0] < \frac{1}{2}$.

   We know that $q^{h*} < 1$, because if $q^{h*} = 1$, $\text{Prob}[x \text{ is profitable}|M^h, q^{h*} = 1] > \frac{1}{2}$.

   When $\text{Prob}[x \text{ is profitable}|M^h, q^{h*} > 0] < \frac{1}{2}$, we know from Lemma 1 that $i^a = 0$ and $i^n > 0$.

   This implies that $V^h_n = 0$ and $V^h_a > 0$.

   Because $q^{S*} < 1$, $q^{S*}$ is not an equilibrium.

1. and 2. implies that cannot exist an equilibrium with information transmission where the biased A advises the signaled asset with a positive probability.

Therefore, if there is an equilibrium where the Biased A imitates the unbiased skilled type with a positive probability and advises the signaled state with a positive probability, that equilibrium does not have information transmission.

The proof when the biased A is imitating the unbiased unskilled type is analogous.

**Proof Lemma 3:**

When there is information transmission, form lemma 2 we know that $q^l = 0$ and $q^h = 0$. If the unbiased A sends a low confidence message with the same probability than a high confidence message ($r = \frac{1}{2}$) and there is information transmission:

$$\text{Prob}[x \text{ is profitable}|M^l] = 2\mu p^U > \frac{1}{2}$$

$$\text{Prob}[x \text{ is profitable}|M^h] = 2\mu > \frac{1}{2}$$
So the payoffs of the biased A are:

\[ V_n^l = i^n(2\mu p^U) > 0 \]
\[ V_n^h = i^n(2\mu) > 0 \]

Because \( p^U < 1 \), \( V_n^h > V_n^l \). Therefore, \( r = \frac{1}{2} \) is not part of an equilibrium. We also know, that \( \text{Prob}[x \text{ is profitable}|M^l] \) is increasing in \( r \) and \( \text{Prob}[x \text{ is profitable}|M^h] \) is decreasing in \( r \), so the equilibrium \( r \) is greater than \( \frac{1}{2} \).

**Proof Proposition 1:**

We are going to analyze the existence and the form of an honest equilibrium with information transmission. We know from Lemma 3 that \( \frac{1}{2} < r \leq 1 \) and from Lemma 2 that an equilibrium has information transmission only if \( q^l = q^h = 0 \). Therefore, we have two possible equilibria with information transmission:

(a) \( (q^l = 0, q^h = 0, r = 1) \)
(b) \( (q^l = 0, q^h = 0, \frac{1}{2} < r < 1) \)

We will check the existence of (a) and (b) for different values of \( \mu \) and \( p^U \).

1. \( \frac{1}{3} < \mu < \frac{1}{2} \):

(a) \( (q^l = 0, q^h = 0, r = 1) \):

This strategy of the biased A implies that \( \text{Prob}[x \text{ is profitable}|M^h, q^h = 0, r = 1] = \frac{\mu}{1-\mu} > \frac{1}{2} \quad (\forall \mu > \frac{1}{3}) \).

\( V_n^h(q^h = 0) = i^n(\frac{\mu}{1-\mu}) > 0 \) and \( V_a^h(q^h = 0) = 0 \). So, \( q^S = 0 \) is part of an equilibrium.

Also, \( \text{Prob}[x \text{ is profitable}|M^l, q^l = 0, r = 1] = p^U \).

\( V_n^l = i^n(p^U) \) and \( V_a^l = 0 \). So, \( r = 1 \) is part of an equilibrium if and only if:

\[ \frac{\mu}{1-\mu} > p^U \]

Therefore, if \( \mu > \frac{1}{3} \) and \( \frac{\mu}{1-\mu} > p^U \), then \( (q^S = 0, r = 1) \) is an optimal strategy for the biased A.

(b) \( (q^l = 0, q^h = 0, \frac{1}{2} < r < 1) \):

Because the biased A is randomizing between each type of message, the payoff of each message must be the same.
The $r^*$ that makes $V^h_n(r) = V^l_n(r)$ is:

$$r(p^U, \mu) = \frac{1 - 2\mu + \mu(1 - p^U)}{(1 - 2\mu)(1 + p^U)}$$

This implies that $\text{Prob}[x \text{ is profitable} | M^h, q^h = 0, r = r^*] = \text{Prob}[x \text{ is profitable} | M^l, q^l = 0, r = r^*] = \mu(1 + p^U) > \frac{1}{2}$ ($\forall \mu > \frac{1}{3}, p^U > \frac{1}{3}$).

This means that when $r < 1$, there is also information transmission.

The deviation is to always send the high confidence message ($r = 1$), so $r^* < 1$ is equilibrium if and only if: $p^U > \frac{\mu}{1 - \mu}$.

Therefore, when $\frac{1}{3} < \mu < \frac{1}{2}$, there always exist information transmission in the honest equilibrium. If $p^U < \frac{\mu}{1 - \mu}$ the biased A always send high confidence messages and if $p^U > \frac{\mu}{1 - \mu}$ the biased A sends low confidence messages with a positive probability.

2. $\mu = \frac{1}{3}$:

(a) ($q^l = 0, q^h = 0, r = 1$):

This strategy of the biased A implies that $\text{Prob}[x \text{ is profitable} | M^h, q^h = 0, r = 1] = \frac{\mu}{1 - \mu} = \frac{1}{2}$

So, from Lemma 1, $V^h_n(q^h = 0) = 0$ and $V^l_n(q^h = 0) = 0$. So, $q^h = 0$ is part of an equilibrium.

Also, $\text{Prob}[x \text{ is profitable} | M^l, q^l = 0, r = 1] = p^U$ and $p^U > \frac{1}{2}$. So, $V^l_n = r^a(p^U) > 0$. Therefore, $r = 1$ is not part of an equilibrium, because it is better for the biased A to send a low confidence message with a higher probability.

(b) ($q^l = 0, q^h = 0, \frac{1}{2} < r < 1$):

This strategy implies $\Rightarrow \text{Prob}[x \text{ is profitable} | M^h, q^h = 0, r = r^*] = \text{Prob}[x \text{ is profitable} | M^l, q^l = 0, r = r^*] = \mu(1 + p^U) > \frac{1}{2}$ ($\forall p^U \in (\frac{1}{2}, 1)$).

From Lemma 1, $V^h_n(q^h = 0, r < 1) = V^l_n(q^l = 0, r < 1) > 0$ and $V^h_n(q^h = 0, r < 1) = V^l_n(q^l = 0, r < 1) = 0$. So $q^h = q^l = 0$ are part of an equilibrium.

So, $r^* < 1$ is part of an honest equilibrium with information transmission. So if $\mu = \frac{1}{3}$, the honest equilibrium has information transmission and the biased A sends low confidence messages with a positive probability.
3. \( \frac{1}{4} < \mu < \frac{1}{3} \):

(a) \((q^l = 0, q^h = 0, r = 1)\):

The biased A strategy implies that \( \text{Prob} \{ x \text{ is profitable} | M^h, q^h = 0, r = 1 \} = \frac{\mu}{1-\mu} < \frac{1}{2} \)

Form Lemma 1, \( V^{h}\_n(q^h = 0) = 0 \) and \( V^{h}\_a(q^h = 0) > 0 \)

So, \( q^h = 0 \) is not part of an equilibrium.

(b) \((q^l = 0, q^h = 0, \frac{1}{2} < r < 1)\):

The biased A strategy implies that \( \text{Prob} \{ x \text{ is profitable} | M^h, q^h = 0, r = r^* \} = \text{Prob} \{ x \text{ is profitable} | M^l, q^l = 0, r = r^* \} = \mu(1 + p^U) \)

\( \mu(1 + p^U) > \frac{1}{2} \) if and only if \( p^U > \frac{1-2\mu}{2\mu} \).

So, \( V^n_l = V^n_h > 0 \) if and only if \( p^U > \frac{1-2\mu}{2\mu} \).

If \( \frac{1}{4} < \mu < \frac{1}{3} \) and \( p^U > \frac{1-2\mu}{2\mu} \) the honest equilibrium has information transmission and the biased A sends low confidence messages with a positive probability.

If \( \frac{1}{4} < \mu < \frac{1}{3} \) and \( p^U \leq \frac{1-2\mu}{2\mu} \) the honest equilibrium does not have information transmission.

4. \( 0 < \mu \leq \frac{1}{4} \):

(a) \((q^l = 0, q^h = 0, r = 1)\) is never part of an equilibrium, for the same reason than in case 3. (a).

(b) \((q^l = 0, q^h = 0, \frac{1}{2} < r < 1)\):

Following 3. (b), this equilibrium has information transmission if and only if: \( p^U > \frac{1-2\mu}{2\mu} \). This is impossible, because \( \frac{1}{2} < p^U < 1 \) and \( 0 < \mu \leq \frac{1}{4} \).

Therefore, if \( 0 < \mu \leq \frac{1}{4} \), the honest equilibrium does not have information transmission.