A NEW TDR-WAVEFORM APPROACH
CAPABLE TO ESTIMATE SOIL MOISTURE
CONTENTS AT LARGE ELECTRICAL
CONDUCTIVITY RANGES

FELIPE ANDRÉS CRISTI MATTÉ

Thesis submitted to the Office of Research and Graduate Studies in partial fulfillment of the requirements for the Degree of Master of Science in Engineering

Advisor:
JOSÉ F. MUÑOZ PARDO

Santiago de Chile, September 2014
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FELIPE ANDRÉS CRISTI MATTÉ

Members of the Committee:

JOSÉ F. MUÑOZ PARDO
FRANCISCO SUÁREZ POCH
CRISTIÁN ORTIZ ASTETE
HÉCTOR JORQUERA GONZÁLEZ

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Dedicated to my family, for their support and love from the distant Patagonia.
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# TABLE OF CONTENTS

ACKNOWLEDGMENTS ........................................................................................................ iii

TABLE OF CONTENTS .......................................................................................................... iv

LIST OF TABLES ...................................................................................................................... vi

LIST OF FIGURES .................................................................................................................... vii

ABSTRACT ............................................................................................................................... ix

RESUMEN ............................................................................................................................... x

1 ARTICLE BACKGROUND ................................................................................................. 1
   1.1 Introduction .................................................................................................................... 1
   1.2 Objectives ...................................................................................................................... 2
   1.3 Literature review .......................................................................................................... 3
       1.3.1 Dielectric constant principles ............................................................................. 3
       1.3.2 TDR measurements in saline soils ..................................................................... 6
   1.4 Future Research ............................................................................................................ 8

2 A NEW TDR-WAVEFORM APPROACH CAPABLE TO ESTIMATE SOIL
   MOISTURE CONTENTS AT LARGE ELECTRICAL CONDUCTIVITY RANGE .......... 8
   2.1 Introduction .................................................................................................................. 8
   2.2 Materials and Methods ............................................................................................... 11
       2.2.1 Traditional analysis of TDR waveform ............................................................... 11
       2.2.2 New TDR-waveform approach ......................................................................... 14
       2.2.3 Experimental setup and drainage experiments ................................................. 15

iv
2.3 Results ................................................................................................................................. 18
  2.3.1 Experimental setup validation ................................................................. 18
  2.3.2 Moisture contents estimations ................................................................. 19
  2.3.3 Model Validation ......................................................................................... 25
  2.4 Discussion and conclusions ........................................................................ 26

REFERENCES ......................................................................................................................... 29
LIST OF TABLES

Table 2-1. Experimental conditions utilized in the experiments ........................................17

Table 2-2. Goodness-of-fit indicators of the three θ estimations for each experiment....21

Table 2-3. Sensitivity analysis of the coefficients of equation [12] obtained for the 0.4-M experiment, showing goodness-of-fit indicators and 95% confidence intervals. Values within parentheses represent the variation of the goodness-of-fit indicators from the original 0.4-M experiment. ........................................21
LIST OF FIGURES

Figure 1-1. Real and imaginary part of TDR measured apparent permittivity as function of water content, and Topp et al., (1980) adjustments (Bitelli et al., 2008)....5

Figure 1-2. TDR waveforms taken in solutions with different BEC (Jones et al., 2002)..7

Figure 2-1. TDR waveforms, obtained at a fixed moisture content ($\theta_{\text{exp}} = 0.037 \text{ m}^3 \text{ m}^{-3}$) but at different bulk electrical conductivities (BEC), reveal that the end of the sensors rods ($x_2$) becomes undetectable above 5 dS m$^{-1}$. This is the reason why traditional TDR analysis fails in highly conductive soils, resulting in inaccurate moisture content estimations. .................................................................13

Figure 2-2. Graphical representation of the variables proposed to determine $\theta$ based on the new methodology. ...........................................................................................................15

Figure 2-3. Experimental setup utilized to assess the new TDR-waveform approach. ...17

Figure 2-4. Volumetric moisture content determined by the gravimetric ($\theta_{\text{exp}}$) and by the Topp et al. (1980) ($\theta_{\text{Topp}}$) methodologies for a sand sample saturated with distilled water. .............................................................19

Figure 2-5. Comparison of moisture contents obtained using the traditional TDR interpretation ($\theta_{\text{Topp}}$), the calibrated estimation ($\theta_{\text{Topp, cal}}$) and the methodology proposed in this work ($\theta_{\text{model}}$), with the experimental moisture contents ($\theta_{\text{exp}}$). The bulk electrical conductivity (BEC) associated to the measurements is shown in the right y-axis. Each subfigure presents the results for soils with different electrical conductivities. a) Soil saturated with distilled water and b) 0.1 M NaCl............................................................22
Figure 2-6. Comparison of moisture contents obtained using the traditional TDR interpretation ($\theta_{\text{Topp}}$), the calibrated estimation ($\theta_{\text{Topp, cal}}$) and the methodology proposed in this work ($\theta_{\text{model}}$), with the experimental moisture contents ($\theta_{\text{exp}}$). The bulk electrical conductivity (BEC) associated to the measurements is shown in the right y-axis. Each subfigure presents the results for soils with different electrical conductivities. a) Soil saturated with 0.2 M NaCl solution and b) 0.3 M NaCl.

Figure 2-7. Comparison of moisture contents obtained using the traditional TDR interpretation ($\theta_{\text{Topp}}$), the calibrated estimation ($\theta_{\text{Topp, cal}}$) and the methodology proposed in this work ($\theta_{\text{model}}$), with the experimental moisture contents ($\theta_{\text{exp}}$). The bulk electrical conductivity (BEC) associated to the measurements is shown in the right y-axis. Each subfigure presents the results for soils with different electrical conductivities. a) Soil saturated with 0.4 M NaCl Solution and b) 0.6 M NaCl.

Figure 2-8. Comparison of the estimated moisture content using the traditional methodology ($\theta_{\text{Topp}}$) and the proposed model ($\theta_{\text{predicted}}$) previously calibrated on a previous 0.4-M experiment, with the experimental moisture content ($\theta_{\text{exp}}$, x-axis). The bulk electrical conductivity (BEC) associated to the measurements is also shown (right y-axis).
ABSTRACT

Time domain reflectometry (TDR) has been widely used by the scientific community as a reliable method to indirectly measure the moisture content ($\theta$) of soils, and in most soils TDR can provide observations of $\theta$ at high temporal resolution with acceptable accuracy. This technique induces an electrical wave in waveguides inserted into the soil, estimates the soil bulk dielectric constant ($\varepsilon$) based on an interpretation of the reflected electromagnetic signal, and then relates $\varepsilon$ with $\theta$. In electrically conductive soils, the reflected signal can be highly attenuated by the effect of the soil’s bulk electrical conductivity, resulting in very large errors in the estimation of $\theta$; the traditional TDR methodology is thus subject to large errors and uncertainties. This thesis presents a new waveform interpretation methodology based on different variables than those used in the traditional TDR methodology. This novel approach extends the applicability of TDR sensors, doubling the actual electrical conductivity range with reliable and accurate measures of $\theta$. The new approach makes it possible to more accurately measure soil moisture contents in settings that have traditionally been difficult to observe.

Keywords: Moisture content, TDR, electrical conductivity, waveform, bulk dielectric constant.
La reflectometría de dominio temporal (Time Domain Reflectometry, TDR) ha sido ampliamente utilizada por la comunidad científica como un método indirecto de medición del contenido de humedad (θ) de los suelos. Esta metodología es capaz de proporcionar observaciones de θ con una alta resolución temporal y con una precisión aceptable. El funcionamiento de esta metodología consiste en la estimación de la constante dieléctrica aparente del suelo (ε), basada en una interpretación de la reflexión de la señal electromagnética (comúnmente denominada “waveform”) emitida por el sensor, para luego relacionar ε con el contenido de humedad del suelo. En suelos eléctricamente conductivos, la señal electromagnética puede verse fuertemente atenuada debido al efecto de la conductividad eléctrica aparente (bulk electrical conductivity, BEC), lo que impide la detección del final de las varillas del sensor, y por lo tanto la estimación de ε, resultando en errores significativos en las mediciones del contenido de humedad.

Esta tesis presenta una nueva metodología de interpretación de la onda reflejada del sensor basada en la detección distintas variables que las utilizadas por la metodología tradicional de TDR. Este novedoso enfoque metodología puede extender la aplicabilidad de sensores TDR, duplicando el rango de conductividad eléctrica actual con mediciones precisas y confiables. El nuevo enfoque extiende la aplicabilidad del sensor TDR, doblando el rango actual de conductividad eléctrica con mediciones confiables y precisas de θ. La nueva metodología presentada en esta tesis abre una nueva ventana investigación de las mediciones del contenido

**Palabras clave:** contenido de humedad, TDR, conductividad eléctrica, forma de la onda, constante dieléctrica aparente.
1 ARTICLE BACKGROUND

1.1 Introduction

Topp et al. (1980) introduced a simple relationship for determining the volumetric moisture content (θ) from the estimation of the dielectric constant of the soil based on the analysis of an electromagnetic pulse transmitted by a coaxial cable ending in multiple metallic rods. The introduction of this methodology has since been widely used in diverse science disciplines, such as agronomy, hydrology, mining and environment engineering, providing accurate and reliable measurements of θ in a noninvasive, fast and low-cost technology, with typical errors smaller than 0.013 m$^3$m$^{-3}$.

However, subsequent studies have shown that the measurement of ε is affected by other factors such as salinity (Nadler et al., 1999; Wyseure et al., 1999; Friedman, 2005), reporting an overestimation of θ as the electrical conductivity increases, and also limiting the use of this sensor only for soil bulk electrical conductivities lower than 5 dSm$^{-1}$. Furthermore, the soil temperature can also affect the measurements of ε, as reported by Or and Wraith, (1999), and finally, the mineralogy of the soil may have influence on ε depending on its porosity (Regalado et al., 2003). These studies highlight the need for soil-specific calibrations of the ε–θ relationship in order to avoid these limitations.

Currently, one of the most important limitation when using TDR methods is the difficulty to obtain reliable and accurate estimates of θ in highly conductive soils (Lekshmi et al., 2014), at present limited at 5 dSm$^{-1}$ for the best case of short and high frequency sensors (Benor et al., 2013).

In this thesis, a new method of interpretation of the TDR’s reflected waveform is developed, based on new variables that have not been used on any of the traditional methodologies. Also, these new variables incorporate the effect of the soil bulk electrical conductivity, thus creating a robust model capable measure θ in soils with a saturated bulk electrical conductivity (BEC) up to 11 dSm$^{-1}$. 
This document is structured as follows: section 1.2 states the main objectives of this investigation. Section 1.3 presents a brief review of the basic principles of the measurement of dielectric constant and the main advances related with the improvement of $\theta$ measurements by TDR sensors. Then, section 1.4 outlines the suggested future work of this research.

Subsequently, chapter 2 contains the main article of this thesis. Section 2.1 introduces the current context of moisture content measurement and its importance in geophysical processes. Section 2.2, describes the experimental setup and the waveform interpretation methodology used in the experiments. In section 2.3 the main results are presented for each one of the experiments, and also a detailed analysis of the applicable electrical conductivity range of the traditional methodology. Finally, section 2.4 discuss the major conclusions of this thesis, and their implications to measurements.

1.2 Objectives

The main objective of this thesis was to develop a new methodology to use a TDR sensor widely used by the scientific community that could be able to estimate moisture content in highly conductive saline soils and thus extend the electrical conductivity range, currently limited at 5 dSm$^{-1}$ for the best case of high frequency TDR sensor available in the market.

The specific objectives of this work were: 1) to evaluate the performance of common TDR sensor in highly conductive sandy soils, and verify the electrical conductivity range where it can measure moisture content accurately. 2) to determine new variables from the reflected TDR waveform with no direct relationship with dielectric properties of the soil, which is calculated based on the endpoint of the rods and thus it is greatly affected by high attenuation levels of the electromagnetic signal, and also variables well correlated with moisture content in order to developed a new relationship between them and soil moisture content.
1.3 Literature review

1.3.1 Dielectric constant principles

Dielectric constant (or dielectric permittivity) of a media can be expressed as a complex variable composed by a real part ($\varepsilon'$), referred to the stored energy in the medium, and an imaginary part ($\varepsilon''$) related to the dielectric losses or energy dissipation. This variable can be expressed as (Von Hippel, 1953).

$$\varepsilon = \varepsilon' - i\varepsilon''$$  

[1]

Where $i = \sqrt{-1}$. Furthermore, $\varepsilon''$ can be decomposed in two main components (Robinson et al., 2003):

$$\varepsilon = \varepsilon' - i \left( \varepsilon''_{\text{rel}} + \frac{\sigma_{dc}}{\varepsilon_0 \omega} \right)$$  

[2]

Where $\varepsilon''_{\text{rel}}$ denotes the losses produced by a dipole moment relaxation, and $\frac{\sigma_{dc}}{\varepsilon_0 \omega}$ represent the conductive losses given by the conductivity on the liquid phase. $\sigma_{dc}$ is the electrical conductivity at zero frequency (Sm$^{-1}$), $\varepsilon_0$ is the permittivity in vacuum ($\varepsilon_0 = 8.85 \times 10^{-12}$ Fm$^{-1}$), and $\omega$ is the operating frequency of the sensor.

The main assumption of TDR principles is that the imaginary part of the complex dielectric constant is negligible related to the real part, neglecting the effect of the conductive losses. This assumption may be valid only in certain ranges of conductivity (Wyseure et al., 1997).

The real part of permittivity ($\varepsilon'$) and imaginary part ($\varepsilon''$) can be determined using the following relation between $\varepsilon$, $\varepsilon'$ and $\varepsilon''$ (Bittelli et al., 2008):
\[ \varepsilon = \frac{\varepsilon'}{2} \left[ 1 + \sqrt{1 + \left( \frac{\varepsilon''}{\varepsilon'} \right)^2} \right] \]  

[3]

Furthermore, Topp et al. (2000) suggested to quantify the total losses \(\varepsilon''\) as the effective electrical conductivity \((\sigma_e)\), defined as:

\[ \sigma_e = \varepsilon_o \omega \varepsilon'' \]  

[4]

Finally, combining equation [2], [3] and [4] the dielectric constant can be expressed as:

\[ \varepsilon = \varepsilon' + \frac{\sigma_e^2}{4\varepsilon\varepsilon_0^2\omega^2} \]  

[5]

Current TDR devices have incorporated the capability to measure the effective conductivity \(\sigma_e\), in addition to \(\varepsilon\). \(\omega\) is the effective frequency of the sensor, which is defined as the frequency that contains the most energy of the signal (Robinson et al., 2003). This effective frequency can be estimated based on the determination of the rise time at the end of the sensor probe (e.g. Topp et al., 2000; Chung and Lin, 2009).

The equation [5] has enabled to identify both the real and imaginary part of the complex dielectric permittivity, by using the variables measured by the TDR without the need of analyzing the reflected waveform. This has open the possibility to better quantify the overestimation of \(\varepsilon\) when lossy soils are presents.

Bitelli et al. (2008) separate the effect of both parts (real and imaginary) of the dielectric permittivity by quantifying the effective frequency of the soils as a single frequency calculated from the rise time methodology, and then used the real part as an input to Topp et al., (1980) equation and other soils specific curves. Figure 1-1 shows the main results of Bitelli et al., (2008). It is noticeable that the real part of \(\varepsilon\) presents a good fit to the Topp et al., (1980) equation. This analysis could significantly reduce the overestimation of \(\theta\) in the analyzed range. However, this alternative calibration do not
take into account the effect of electrical conductivity related with the dipole moment relaxation, and therefore it is no possible to relate a single relationship to more than a single soil. Besides, the methodology needs the estimation of \( \varepsilon \) before any other procedure, so it also fails when high attenuation levels are present.

Despite the foregoing, this loss analysis can only be performed assuming that the typical waveform algorithm is able to measure the dielectric constant based on the detection of the final rods to estimate to effective travel time (see section 2.2.1). In cases of highly conductive soils, the detection of the final point of the rods becomes impractical (Wyseure et al., 1997).

![Figure 1-1. Real and imaginary part of TDR measured apparent permittivity as function of water content, and Topp et al., (1980) adjustments (Bitelli et al., 2008).](image-url)
1.3.2 TDR measurements in saline soils

Wyseure et al. (1997) was one of the first to analyze the effect of the electrical conductivity of the soil in the dielectric constant. He showed a systematic overestimation of $\varepsilon$ in soils with BEC $> 2$ dSm$^{-1}$, and therefore an overestimation of $\theta$. He also reported an increasing variance of $\varepsilon$ measurements with an increasing bulk electrical conductivity in the soil, making it more difficult to correctly estimate $\theta$. He finally recommend the use of TDR sensor under a saturated BEC lower than 8 - 10 dSm$^{-1}$, always with a need of previous calibration of $\varepsilon - \theta$ relationship for each experiment.

Furthermore, high bulk electrical conductivities can have a more significant effect on the reflection coefficient until the point to fully attenuate it at short distances. Jones et al. (2002) showed that there was not possible to detect the end of the probe under soils with BEC $> 6$ dSm$^{-1}$ due the high levels of waveform attenuation (see Figure 1-2). The impossibility to detect this point makes the traditional TDR technology to fail immediately.
To overcome large levels of waveform attenuation, coating rods with insulation materials have been a good solution to reduce the effect of high bulk electrical conductivity in soils (Majid et al., 1998; Nichol et al., 2002) and then preserve information of the TDR probe necessary to evaluate the dielectric constant. Nonetheless, the insulation material significantly affect the measurements of the dielectric constant of the soil, requiring specific soil calibrated relationships between the insulated $\varepsilon$ and soil permittivity, or directly with moisture content of the soil (Jones et al., 2002). Also, it has been demonstrated that insulating TDR probes affects the frequency dependence on the electric field at the interface between the media and coating material, deteriorating measurements accuracy (Richert, 2009). Moreover, there is a risk to reduce the sensitivity and accuracy of the measurement given the possibility of creating air gaps between the coated material and rods when the sensor is introduced in the soil (Knight et al., 1996).
Schwartz et al. (2013) developed an adaptive-waveform interpretation based on signal noise filters using cascadian Gaussian Kernels using the maximum gradient of the reflection at the end of the transmission line, capable to circumvent measuring problems related to signal noise and signal attenuation when an increase of soil water content and BEC occurred. This analysis reduced the number of parameters needed to develop the waveform interpretation and reduced the sampling error of travel time determination. However, the analysis also need the identification of the endpoint of the TDR probes, limiting the procedure to low conductivities.

1.4 Future Research

Suggested future work for this thesis is to continue experimentation in natural saline soils from different locations, for example soils from "Salar de Atacama" (Chile, II Región) and "Pampa del Tamarugal" (Chile, II Región) and then allow to use this type of sensors in regions with high relevance to mining processes and water resource management, especially due to current scarcity of water in these regions.

Furthermore, it is recommended to continue the study of new variables detected from the waveform, focusing the analysis in the zone corresponding to the sensor's rods. The aim is to find a unique relationship to estimate moisture content, which could be use in any type of soil without the need of prior calibrations. This new relationship will greatly expand the possibilities to measure moisture content in complex soils, characteristic in many arid regions worldwide, with a fast, efficient and low-cost technology.

2 A NEW TDR-WAVEFORM APPROACH CAPABLE TO ESTIMATE SOIL MOISTURE CONTENTS AT LARGE ELECTRICAL CONDUCTIVITY RANGES.

2.1 Introduction

Soil water content refers to the amount of water stored in the soil’s unsaturated zone, where it becomes a very important parameter in several near-surface geophysical
processes (Robinson et al., 2003). It is crucial in the energy and water balance of a basin as it influences components such as evapotranspiration, runoff and soil infiltration, being a key variable in surface and subsurface hydrology, as well as in land-atmosphere interactions (Entekhabi et al., 1996). Water content also affects several physical properties of the soil that triggers natural hazards such as landslides (Lekshmi et al., 2014), and chemical properties that controls groundwater pollution (Robinson et al., 2008). Therefore, it is of great importance to accurately measure and monitor soil water content at high temporal and spatial scales to improve the knowledge, management and control of hydrological processes that are directly related with various disciplines such as hydrogeology, mining, agriculture, geology and environmental engineering, among others.

Nowadays there are many methods to measure volumetric water content. These methods are typically classified as direct or indirect, where these latter are based on different soil properties, e.g., electromagnetic (EM), electrical, or thermal, which can then be related with \( \theta \). Detailed descriptions of current measurement methods are reviewed elsewhere (Robinson et al., 2008; Seneviratne et al., 2010). Within indirect methods, time domain reflectometry (TDR) has been widely utilized to measure \( \theta \) because it provides good accuracy in a variety of soils, is easy to implement and to calibrate (typically not needed in many soils), provides excellent temporal resolution, and is a non-invasive and non-polluting method (Jones et al., 2002).

In TDR, an EM pulse is launched through a coaxial cable that ends in a metallic probe embedded in the soil. The amplitude of the reflected signal is analyzed to determine the bulk dielectric constant of the media, which can be related to \( \theta \) due to the large difference between the dielectric constant of the air (\( \varepsilon_a \approx 1 \)), the solid phase of the soil (\( \varepsilon_s \approx 3-5 \)), and the water (\( \varepsilon_w \approx 80 \)). Topp et al. (1980) proposed the following third-order polynomial equation to relate \( \varepsilon \) and \( \theta \):

\[
\theta = -5.3 \times 10^{-2} + 2.92 \times 10^{-2} \varepsilon - 5.5 \times 10^{-4} \varepsilon^2 + 4.3 \times 10^{-6} \varepsilon^3
\]  \[6\]
Equation [6] is commonly known as the "universal equation", having errors smaller than 0.013 m$^3$ m$^{-3}$ for a wide range of soils (Jones et al., 2002). However, subsequent studies to that of Topp et al. (1980) have shown that the measurement of $\varepsilon$ is affected by other factors such as salinity (Nadler et al., 1999; Wyseure et al., 1999; Friedman, 2005), temperature (Or and Wraith, 1999) and mineralogy (Regalado et al., 2003). These studies highlight the need for individual calibrations of the $\varepsilon$–$\theta$ relationship to correct for these factors.

Currently, an important limitation when using TDR methods is the difficulty to obtain reliable and accurate estimates of $\theta$ in highly conductive soils (Schwartz et al., 2013; Lekshmi et al., 2014). This limitation is due to large dielectric losses induced by the large electrical conductivity of the medium. To address this limitation, coated rods have been suggested to reduce high attenuation levels of the waveform in saline soils (Mojid et al., 1998; Nichol et al., 2002). Nonetheless, it has been demonstrated that coated rods also affects the frequency dependence on the electric field at the interface between the media and coating material, deteriorating the measurements’ accuracy (Knight et al., 1996; Richert, 2009).

To address the effect of electrical conductivity of the media, current TDR sensors measure simultaneously the bulk electrical conductivity (BEC) and $\varepsilon$, allowing to include the BEC in new models to estimate $\theta$ (Wyseure et al., 1997), as well as to quantify dielectric losses to improve $\varepsilon$ measurements (Bitelli et al., 2007). Recently, Schwartz et al. (2013) developed an adaptive-waveform interpretation capable of circumventing measuring problems related to signal noise and attenuation when $\theta$ or BEC of the soil changed, without the need for user-parameters adjustment. This is useful when analyzing large time series, where the soil conditions may change considerably. Despite this advance, the methodology proposed by Schwartz et al. (2013) did not extend the electrical conductivity range in which the sensor can operate correctly.

Current TDR methods are limited to electrical conductivity values up to ~5 dS m$^{-1}$ in the best case of short and high-frequency sensors (Benor et al., 2013; Chandler et al.,
2004; Blonquist et al., 2005). The main reason for the limited range of operation is that most TDR methods estimate $\theta$ through the determination of $\varepsilon$, which is based on the reflected waveform of an EM pulse that travels through a TDR probe (e.g., see Schwartz et al. (2013)). These methods attempt to determine the beginning and the end of the sensor’s rods to relate them with the travel time of the EM pulse. However, as shown below, the detection of the end of the rods is not possible in highly conductive soils, due to the large attenuation levels of the EM pulse associated to conductive losses, precluding the use of this methodology on these soils (Jones et al., 2002; Majid et al., 2003).

I hypothesize that when high attenuation levels of the EM pulse occur, accurate estimates of $\theta$ can be obtained by identifying new aspects of the TDR waveform without including the point at the end of the sensor’s rods. Thus, the objective of this thesis was to develop a new methodology for determining $\theta$ based on a new waveform interpretation that uses variables other than those commonly considered by current methods. This new methodology can strongly expand monitoring of water content using TDR sensors in conductive soils, typical of arid zones, which represent approximately one third of the earth's surface (Rubel and Kottek, 2010), providing valuable information necessary to improve water resource management.

2.2 Materials and Methods

2.2.1 Traditional analysis of TDR waveform

Traditional TDR analysis utilizes a relationship between $\varepsilon$ and $\theta$ based on the theoretical and effective travel time of an EM pulse through a probe embedded in the soil. The theoretical travel time of a TDR-generated EM pulse to cross the probe can be expressed as (Topp et al., 1980; Jones et al., 2002):

$$ t = \frac{2L\sqrt{\varepsilon}}{c} \quad [7] $$
where \( L \) is the length of the probe’s rods; \( t \) is the travel time for the pulse to traverse the length of the probe (down and back: \( 2L \)); and \( c = 3 \times 10^8 \text{ m s}^{-1} \) is the EM pulse velocity in vacuum. Note however that the travel time of the EM pulse must be evaluated based on the electromagnetic length of the probe, which is also known as the apparent length \( (L_a) \).

To estimate \( L_a \), TDR systems determine a relationship between the amplitude \( (V_1) \) of the EM signal after partial reflection along a transmission line as function of time, and the amplitude \( (V_0) \) of the EM signal emitted by the reflectometer. This relationship is called the reflection coefficient \( (\rho) \), and is defined by:

\[
\rho = \frac{V_1}{V_0} - 1
\]  

From the relationship between reflection coefficient and apparent distance, also known as waveform (Figure 1), it is possible to visualize the beginning \( (x_1) \) and the end \( (x_2) \) of the sensor’s rods. These points allow estimating the apparent length as \( L_a = x_2 - x_1 \), and to calculate the effective travel time as:

\[
t = \frac{2(x_2-x_1)}{c}
\]  

Combining equations [7] and [9] leads to a direct expression to estimate \( \varepsilon \):

\[
\varepsilon = \left(\frac{x_2-x_1}{L}\right)^2
\]  

Traditional double-tangent waveform analysis attempts to identify the \( L_a \) by tracing tangent lines at the local maximum and minimum points that refer to the beginning and end of the rods, respectively (Heimovaara, 1994). Figure 1a presents an example of how traditional TDR interpretation estimates \( \varepsilon \), using four different waveforms obtained at the same \( \theta \) but with different BEC’s. As shown in Figure 1a, it is not possible to correctly

...
identify the point $x_2$ when analyzing waveforms with BEC’s higher than 5 dS m$^{-1}$. The main reason for the inability to detect $x_2$ lies in the large attenuation levels of the reflected signal. These attenuation levels are caused by large dielectric losses related to the electrical conductivity of the media. Traditional TDR analysis fails in highly conductive soils because the $L_a$ is not determined properly, yielding large errors when estimating $\varepsilon$. As consequence, an inaccurate estimation of $\theta$ is obtained when equation [6] or other equations based on $\varepsilon$ are used (Ledieu et al., 1986; Schaap et al., 1996; Bitelli et al., 2007).

![TDR waveforms for $\theta_{\text{exp}} = 0.37$ m$^3$m$^{-3}$](image)

Figure 2-1. TDR waveforms, obtained at a fixed moisture content ($\theta_{\text{exp}} = 0.037$ m$^3$m$^{-3}$) but at different bulk electrical conductivities (BEC), reveal that the end of the sensors rods ($x_2$) becomes undetectable above 5 dS m$^{-1}$. This is the reason why traditional TDR analysis fails in highly conductive soils, resulting in inaccurate moisture content estimations.
2.2.2 New TDR-waveform approach

The proposed approach consists in the analysis of three variables obtained from the waveform that do not include the point \( x_2 \). These variables are obtained after the waveform is smoothed by applying a Savitzky-Golay filter with a fourth-order polynomial parameter and a fixed arbitrary frame size (Savitzky and Golay, 1964). The first variable is the slope \((m)\), or the first derivative of the waveform at the first inflection point \( x_m \) after the beginning of the rods. The second variable is the integral \((A)\) of the waveform between the end of the coaxial cable \( (x_0) \) and \( x_m \). The third variable corresponds to the reflection coefficient \((\rho_{end})\), obtained at the end of the wave that is defined by the window length recommended by TDR manufacturers. Figure 1b depicts these three variables for a waveform acquired in a soil with BEC of 0.93 dS m\(^{-1}\) and \( \theta \) of 0.405 m\(^3\) m\(^{-3}\). These three variables were combined into the following single dimensionless variable, \( \mu \):

\[
\mu = \frac{mA}{(1+\rho_{end})^2}
\]  

[11]

Equation [11] was utilized to adjust an empirical fourth-order polynomial function to estimate \( \theta \):

\[
\theta = c_0 + c_1\mu + c_2\mu^2 + c_3\mu^3 + c_4\mu^4
\]

[12]

where \( c_0, c_1, c_2, c_3 \) and \( c_4 \) are soil-specific empirical parameters.
2.2.3 Experimental setup and drainage experiments

To evaluate the proposed methodology, six different sand samples previously oven dried for 48 h at 105°C were put in a vertical column (12.1 cm diameter and 8 cm tall) that was installed in a modified Tempe cell, as shown in Figure 2-3. A 1-bar porous ceramic plate (0600 Series, Soilmoisture, Santa Barbara, CA, USA) was placed at the bottom of the column and connected to a vacuum chamber able to reach -0.8 bar by the action of a vacuum pump (ZA.32, DVP technology, Italy). A 7.5-cm length three-rod TDR probe (CS645, Campbell Scientific Inc., Logan, UT, USA) was inserted vertically at the top of the soil and connected to a time-domain reflectometer (TDR100, Campbell Scientific Inc., Logan, UT, USA) using a 6.5 m length low-loss coaxial cable (LMR-200). The reflectometer was configured with a fixed window length of 3 m, which is the recommended value for this sensor (Campbell, 2010), an offset time of 0.035 ns, and a relative velocity of propagation set at 0.99 (Jones et al., 2002). It was connected to a
datalogger (CR1000, Campbell Scientific Inc., Logan, UT, USA) to store the BEC and the TDR waveform (discretized in 500 points). The installation was placed on a 1-g resolution balance (Midrics 1, Sartorius, Goettingen, Germany) that allowed automatic estimation of the experimental moisture content ($\theta_{\text{exp}}$), as explained below. All the previous data were collected at 60 s time intervals.

To have different conductivities, soils were slowly saturated from above with different NaCl solutions (Table 2-1), and waited ~5 h to achieve a relatively uniform moisture profile. Then, an increasing suction was applied at the bottom of the soil column by the action of the vacuum pump until it was no longer possible to extract more water from the soil (typically at -0.8 bar). During the experiments, the evolution of the $\theta_{\text{exp}}$ was determined by the gravimetric method (Black, 1965), using an algorithm implemented in Matlab (MathWorks, Natick, MA, USA). This method utilized the mass of the water (measured in the balance), the water density, mass of soil, and the soil dry bulk density, which was calculated as the average of four soil samples previously oven-dried.

Soil moisture contents were estimated using the methodology proposed by Topp et al. (1980) (referred to as $\theta_{\text{Topp}}$). Additionally, I performed a soil-specific calibration of moisture estimations by adjusting the coefficients of equation [6] using the least squares approach (this soil moisture estimation is referred to as $\theta_{\text{Topp, cal}}$). Furthermore, soil water contents were estimated using the new approach proposed in this work (referred to as $\theta_{\text{model}}$), and adjusting the coefficients of equation [12] with the same least squares method. The goodness of fit of the three models was quantified using the root mean square error (RMSE) and the Nash-Sutcliffe efficiency (E) (Legates and McCable, 1999).

The measurement of water mass using the automated balance was carried out carefully, avoiding any variation in the weight of the installation that was not related to the soil water. An example of the raw estimations of $\theta_{\text{exp}}$ and $\theta_{\text{Topp}}$ in a sandy soil saturated with distilled water is shown below.
Figure 2-3. Experimental setup utilized to assess the new TDR-waveform approach.

Table 2-1. Experimental conditions utilized in the experiments

<table>
<thead>
<tr>
<th>ID</th>
<th>Dry bulk density</th>
<th>Solution</th>
<th>Saturated BEC</th>
<th>Critical BEC</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0 M</td>
<td>1.533</td>
<td>Distilled water</td>
<td>0.93</td>
<td>-</td>
</tr>
<tr>
<td>0.1 M</td>
<td>1.548</td>
<td>NaCl 0.1 M</td>
<td>3.06</td>
<td>-</td>
</tr>
<tr>
<td>0.2 M</td>
<td>1.541</td>
<td>NaCl 0.2 M</td>
<td>5.47</td>
<td>3.56</td>
</tr>
<tr>
<td>0.3 M</td>
<td>1.593</td>
<td>NaCl 0.3 M</td>
<td>7.22</td>
<td>3.08</td>
</tr>
<tr>
<td>0.4 M</td>
<td>1.523</td>
<td>NaCl 0.4 M</td>
<td>8.26</td>
<td>2.18</td>
</tr>
<tr>
<td>0.6 M</td>
<td>1.620</td>
<td>NaCl 0.6 M</td>
<td>11.32</td>
<td>2.03</td>
</tr>
</tbody>
</table>
2.3 Results

2.3.1 Experimental setup validation

Figure 2-4 shows the raw measurements of volumetric moisture content determined with the gravimetric method ($\theta_{\text{exp}}$) (Black, 1965) and with the Topp et al. (1980) “universal equation”. The experiment was performed in a sand sample that was saturated with distilled water. A pressure of $-0.2$ bar was applied with the suction pump for $\approx 20$ h. Then, the pressure was reduced to $-0.7$ bar until the soil reached $\theta = 0.11$ m$^3$ m$^{-3}$. Both measurement methodologies yielded very similar results for the entire range of $\theta$ (E $= 0.9952$ and RMSE $= 0.0055$ m$^3$ m$^{-3}$). These goodness-of-fit indicators confirm that errors in the experimental setting are minimal.
2.3.2 Moisture contents estimations

Figures 2-5 to Figure 2-7 compares moisture contents obtained using the three methodologies, the traditional TDR estimation ($\theta_{\text{Topp}}$), the calibrated estimation ($\theta_{\text{Topp, cal}}$) and the methodology proposed in this work ($\theta_{\text{model}}$), with the experimental moisture contents ($\theta_{\text{exp}}$). Additionally, the BEC (right y-axis) is shown as function of $\theta_{\text{exp}}$.

In soils with saturated BEC lower than ~3 dS m$^{-1}$ (Figure 2-5a-b), the conventional methodology ($\theta_{\text{Topp}}$) correctly estimates moisture contents in all ranges of $\theta$ with excellent goodness-of-fit indicators ($E \geq 0.953$ and $\text{RMSE} \leq 0.01$ m$^3$ m$^{-3}$). The calibrated estimation ($\theta_{\text{Topp, cal}}$) for the two experiments results in a slight improvement, resulting in $E \geq 0.993$ and $\text{RMSE} \leq 0.004$ m$^3$ m$^{-3}$. In both experiments, the BEC was always smaller than 2.5 dS.
m⁻¹, which is below the maximum BEC specified by the manufacturer (5.0 dS m⁻¹). For the same experiments, the proposed new methodology only improves the accuracy in the estimates of θ versus the not calibrated model (E ≥ 0.989 and RMSE ≤ 0.006 m³ m⁻³ for both experiments).

In more conductive soils (Figures 2-6 and 2-7), the traditional methodology typically overestimates θ. When the BEC is above a threshold (referred to as the “critical BEC”), which depends on the saturated BEC, the variability in θ_{Topp} becomes enormous. This critical BEC decreases as the saturated BEC increases. For instance, in the 0.2-M experiment (Figure 2-6a), θ_{Topp} overestimates θ_{exp} for BEC’s higher than ~2 dS m⁻¹. This overestimation does not present an important variability in the estimates of θ until the BEC surpasses the threshold of 3.56 dS m⁻¹. For BEC’s higher than 3.56 dS m⁻¹, the variability in θ_{Topp} becomes very large and results in erratic estimations of the moisture content (E ≈ -1.1x10⁵ << 0 and RMSE ≈ 22 m³ m⁻³), even when the BEC is below the maximum BEC specified by the manufacturer. The increasing dispersion in the θ_{Topp} for a defined BEC threshold occurred in all the highly conductive soils experiments. The large variability on the TDR measurements precludes the correct calibration of the traditional methodology when all the data are used in the least squares analysis. For this reason, the calibrated estimations (θ_{Topp, cal}) were developed considering only the data with BEC values lower than the critical BEC determined for each experiment (Table 2-2).

In all the high-conductivity soil experiments (Figure 2-6 and Figure 2-7), the θ_{Topp, cal} did not present an improvement in the goodness-of-fit indicators compared to the θ_{Topp} (Table 2-2). In the same experiments (Figure 2-6 and Figure 2-7) the proposed methodology (θ_{model}) was able to estimate θ accurately for the entire analyzed range (without any critical BEC) with RMSE’s < 0.006 m³ m⁻³ and E’s > 0.989 (Table 2-2), even when the BEC’s were above the maximum value recommended by the TDR manufacturer.
Table 2-2. Goodness-of-fit indicators of the three $\theta$ estimations for each experiment.

<table>
<thead>
<tr>
<th>ID</th>
<th>$E^*<em>{\theta</em>{\text{Topp}}}$</th>
<th>$E^*<em>{\theta</em>{\text{Topp, cal}}}$</th>
<th>$E^*<em>{\theta</em>{\text{model}}}$</th>
<th>RMSE**$<em>{\theta</em>{\text{Topp}}}$</th>
<th>RMSE**$<em>{\theta</em>{\text{Topp, cal}}}$</th>
<th>RMSE**$<em>{\theta</em>{\text{model}}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>m$^3$ m$^{-3}$</td>
<td>m$^3$ m$^{-3}$</td>
<td>m$^3$ m$^{-3}$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.0 M</td>
<td>0.9661</td>
<td>0.9964</td>
<td>0.9892</td>
<td>9.87E-03</td>
<td>4.10E-03</td>
<td>5.57E-03</td>
</tr>
<tr>
<td>0.1 M</td>
<td>0.9537</td>
<td>0.9933</td>
<td>0.9969</td>
<td>8.93E-03</td>
<td>3.38E-03</td>
<td>2.30E-03</td>
</tr>
<tr>
<td>0.2 M</td>
<td>-1.1E+05</td>
<td>-5.3E+06</td>
<td>0.9982</td>
<td>2.22E+01</td>
<td>2.29E+02</td>
<td>4.25E-03</td>
</tr>
<tr>
<td>0.3 M</td>
<td>-2.5E+04</td>
<td>-1.4E+06</td>
<td>0.9983</td>
<td>8.60E+00</td>
<td>6.59E+01</td>
<td>2.21E-03</td>
</tr>
<tr>
<td>0.4 M</td>
<td>-9.0E+05</td>
<td>-5.4E+08</td>
<td>0.9898</td>
<td>5.50E+01</td>
<td>1.34E+03</td>
<td>5.82E-03</td>
</tr>
<tr>
<td>0.6 M</td>
<td>-1.1E+06</td>
<td>-6.7E+07</td>
<td>0.9966</td>
<td>8.15E+02</td>
<td>6.42E+02</td>
<td>4.55E-03</td>
</tr>
</tbody>
</table>

Table 2-3. Sensitivity analysis of the coefficients of equation [12] obtained for the 0.4-M experiment, showing goodness-of-fit indicators and 95% confidence intervals. Values within parentheses represent the variation of the goodness-of-fit indicators from the original 0.4-M experiment.

<table>
<thead>
<tr>
<th>Eq. [7] Coefficients</th>
<th>Value</th>
<th>St. Deviation $\sigma$</th>
<th>95% Confidence Interval</th>
<th>$E^*<em>{\theta</em>{\text{model}}}$</th>
<th>RMSE**$<em>{\theta</em>{\text{model}}}$ m$^3$ m$^{-3}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$c_0$</td>
<td>0.053</td>
<td>0.001</td>
<td>$\pm6.56E$-05</td>
<td>0.9899 (-0.0001%)</td>
<td>5.82E-03 (0.006%)</td>
</tr>
<tr>
<td>$c_1$</td>
<td>-3.55</td>
<td>0.181</td>
<td>$\pm9.94E$-03</td>
<td>0.9898 (-0.0027%)</td>
<td>5.83E-03 (0.129%)</td>
</tr>
<tr>
<td>$c_2$</td>
<td>4.66</td>
<td>8.475</td>
<td>$\pm4.65E$-01</td>
<td>0.9896 (-0.0229%)</td>
<td>5.89E-03 (1.110%)</td>
</tr>
<tr>
<td>$c_3$</td>
<td>293.7</td>
<td>144.9</td>
<td>$\pm7.95E$+00</td>
<td>0.9895 (-0.0412%)</td>
<td>5.95E-03 (1.995%)</td>
</tr>
<tr>
<td>$c_4$</td>
<td>2,072.1</td>
<td>807.5</td>
<td>$\pm4.43E$+01</td>
<td>0.9898 (-0.0088%)</td>
<td>5.85E-03 (0.429%)</td>
</tr>
</tbody>
</table>

*E: Nash-Sutcliffe efficiency.
**RMSE: root mean square error.
Figure 2-5. Comparison of moisture contents obtained using the traditional TDR interpretation ($\theta_{\text{Topp}}$), the calibrated estimation ($\theta_{\text{Topp, cal}}$) and the methodology proposed in this work ($\theta_{\text{model}}$), with the experimental moisture contents ($\theta_{\text{exp}}$). The bulk electrical conductivity (BEC) associated to the measurements is shown in the right y-axis. Each subfigure presents the results for soils with different electrical conductivities. a) Soil saturated with distilled water and b) 0.1 M NaCl.
Figure 2-6. Comparison of moisture contents obtained using the traditional TDR interpretation ($\theta_{\text{Topp}}$), the calibrated estimation ($\theta_{\text{Topp, cal}}$) and the methodology proposed in this work ($\theta_{\text{model}}$), with the experimental moisture contents ($\theta_{\text{exp}}$). The bulk electrical conductivity (BEC) associated to the measurements is shown in the right y-axis. Each subfigure presents the results for soils with different electrical conductivities. a) Soil saturated with 0.2 M NaCl solution and b) 0.3 M NaCl.
Figure 2-7. Comparison of moisture contents obtained using the traditional TDR interpretation ($\theta_{\text{topp}}$), the calibrated estimation ($\theta_{\text{topp, cal}}$) and the methodology proposed in this work ($\theta_{\text{model}}$), with the experimental moisture contents ($\theta_{\text{exp}}$). The bulk electrical conductivity (BEC) associated to the measurements is shown in the right y-axis. Each subfigure presents the results for soils with different electrical conductivities. a) Soil saturated with 0.4 M NaCl Solution and b) 0.6 M NaCl.
2.3.3 Model Validation

As a validation of the proposed methodology, I carried out a second experiment with the 0.4-M NaCl solution. The empirical coefficients of equation [12] previously calibrated in the first 0.4-M experience were used to validate the volumetric moisture content predicted by the new TDR-waveform approach ($\theta_{\text{predicted}}$). Figure 2-8 compares of the predicted moisture content by both the Topp et al. (1980) ($\theta_{\text{Topp}}$) methodology and the proposed model (left y-axis), with the gravimetric moisture content ($\theta_{\text{exp}}$, x-axis). Additionally, the BEC (right y-axis) is shown as function of $\theta_{\text{exp}}$. The results obtained with the proposed methodology are very acceptable, with an $E = 0.993$ and RMSE = 0.006 m$^3$ m$^{-3}$. Both goodness-of-fit indicators are considerably better than those obtained using the Topp et al. (1980) methodology ($E = -1.3 \times 10^7$ and RMSE = 275.0 m$^3$ m$^{-3}$). Predicted moisture contents show a small increase of dispersion starting at BEC of 6 dS m$^{-1}$, with estimation errors lower than 0.05 m$^3$ m$^{-3}$. 
Figure 2-8. Comparison of the estimated moisture content using the traditional methodology ($\theta_{\text{Topp}}$) and the proposed model ($\theta_{\text{predicted}}$) previously calibrated on a previous 0.4-M experiment, with the experimental moisture content ($\theta_{\text{exp}}$, x-axis). The bulk electrical conductivity (BEC) associated to the measurements is also shown (right y-axis).

2.4 Discussion and conclusions

The traditional TDR methodology (e.g., Topp et al., 1980) was unable to estimate $\theta$ for soils with BEC’s higher than 3 dS m$^{-1}$ (a 60% of the maximum BEC specified by the manufacturer). The failure of traditional methods is a consequence of large dielectric losses that occur in highly conductive soils. These losses hinder the end of the sensor’s
rods on the reflected waveform, and result in a large variability of the estimated values as a consequence of a major flaw in the fundamentals of the methodology. Note that for highly conductive soils, traditional measurements cannot be corrected just by fitting the coefficients of the third-order polynomial expressions that are widely used in TDR approaches (e.g., see equation [6], Regalado et al. (2003), or Miyamoto et al. (2001)), because of the large variability that is observed when the critical BEC is surpassed. In cases with BEC lower than the critical BEC, it is possible to recalibrate the \( \varepsilon - \theta \) relationship by fitting the parameters of the universal equation or other mathematical expressions (Schaap et al., 1996; Ledieu et al., 1986; Bitelli et al., 2007).

As explained before, it is possible to infer different ranges of BEC where the traditional methodology can effectively estimate \( \theta \). These ranges are defined by a maximum threshold that I have called the critical BEC (Table 2-1). This critical BEC is the highest BEC at which the sensor can be successfully operated using the traditional TDR methodology. It is not defined for low conductive soils (Figures 2-4) because \( \theta_{\text{Topp}} \) provide good estimates for the \( \theta \) along the entire range of BEC’s and \( \theta \). The critical BEC decreases in a non-linear way as the saturated BEC increases, until a value of \( \approx 2.0 \text{ dS m}^{-1} \) for a saturated BEC of \( 11.3 \text{ dS m}^{-1} \) (Table 2-1). This behavior indicates that the \( \varepsilon \) measures are not only affected by high conductivities, but also by other dielectric phenomena that may be affecting the imaginary part of the dielectric constant (Topp et al., 2000), preventing an accurate estimation of \( \varepsilon \) as the saturated BEC increases. Thus, the use of traditional TDR interpretation is recommended only in soils that have saturated BEC smaller than \( 5 \text{ dS m}^{-1} \). In more conductive soils (saturated BEC \( > 5 \text{ dS m}^{-1} \)), measurements of \( \theta \) show large variability and can be extremely inaccurate, even at low moisture contents or when the BEC is within the range of operation specified by the manufacturer (e.g., see Figures 2-6 where traditional TDR interpretation fails at \( \theta \approx 18 \text{ m}^3 \text{ m}^{-3} \) and BEC \( \approx 2 \text{ dS m}^{-1} \)).

The proposed methodology was able to correctly represent \( \theta \) for saturated BEC in the 0-11 dS m\(^{-1}\) range, with RMSE’s smaller than 0.0022 m\(^3\) m\(^{-3}\) and thus is not limited
by a critical BEC. In this work, I showed that it is possible to relate different parameters of the waveform, which have no direct relation with $\varepsilon$, to estimate $\theta$ accurately in highly conductive soils. The slope ($m$) of the waveform is well related to $\theta$, being steeper with increasing moisture content. However, the $m$-$\theta$ relationship is also influenced by the BEC of each soil, making it impossible to find a unique relationship between these two variables. Furthermore, $\rho_{end}$ has a strong relation with the BEC of the medium, decreasing as BEC increases. The integral of the waveform ($A$) is related to the signal losses along the sensor’s rods. Finally, the dimensionless variable, $\mu$, merge the effects of $m$, $A$ and $\rho_{end}$ to relate them with $\theta$. I selected a fourth-order polynomial for each experiment characterized by its saturated BEC. Yet, other $\mu$ functions could be selected to represent the $\mu$-$\theta$ relationships (e.g., logarithm, power, or other mathematical expressions).

The main limitation of the proposed methodology, however, lies in the need to calibrate the coefficients of equation [12] for each saturation BEC value. This issue might be a restriction when working with soils with salinity conditions that change over time (due to the evaporation, precipitation, dissolution cycles), changing the conductivity of the wetting solution. Nonetheless, the approach presented in this work opens the possibility of expanding the operating range of TDR methods without modifying the sensors. The results presented in this work are promising but more research needs to be carried out to extend the applicability of the proposed methodology. For instance, it is important to study how soil properties (e.g., surface area, porosity, mineralogy, clay contents) can affect the relationship between the proposed dimensionless variable and $\theta$. Therefore, new experiments with multiple soil characteristics are needed.
REFERENCES


Campbell, C. S. (2010), TDR 100 instruction manual, rev. 2/10, Logan, UT, USA.


