

Vacuum instability by a chromoelectric field in $2 + 1$ dimensionsM. Loewe,^{1,2} F. Marquez,¹ and R. Zamora¹¹*Departamento de Física, Pontificia Universidad Católica de Chile, Casilla 306, Santiago 22, Chile*²*Centre for Theoretical Physics and Mathematical Physics, University of Cape Town, Rondebosch 7700, South Africa*

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The problem of vacuum instability and creation of pairs by external fields $SU(2)$ is studied. The effective mass operator has complex eigenvalues implying decay and pair creation. We consider a constant external chromoelectric field and find a region in the parameter space where the vacuum becomes unstable and decays in pairs. Although the calculation is done in $SU(2)$ color group it is also valid for $SU(3)$.

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I. INTRODUCTION

The prediction of pair creation by a constant background electric field is an important nonperturbative result in quantum electrodynamics (QED). It shows that the vacuum of the quantum field theory (QED) acquires a nontrivial and nonperturbative unstable character in such backgrounds [1–5] and, consequently, decays through pair creation. This instability manifests in the effective action of the theory having an imaginary part. Although the magnitude of this effect is very small for field strengths that can be produced in the laboratory presently, it is expected that in the coming years very strong electric fields may be produced in the laboratory. This would allow for a direct verification of Schwinger's prediction.

Vacuum instability and pair creation in an external field is also an interesting topic in itself to study in non-Abelian theories such as quantum chromodynamics (QCD). However, there are some important differences between an Abelian and a non-Abelian gauge theory. For example, the field strength tensor (in a matrix form) in a non-Abelian theory is defined as

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu + ig[A_\mu, A_\nu], \quad (1)$$

where A_μ denotes the vector potential and g the coupling constant so that it is not merely the curl of the vector potential. Under a non-Abelian gauge transformation

$$A_\mu \rightarrow U(x)A_\mu U^{-1}(x) - \frac{1}{ig}(\partial_\mu U(x))U^{-1}(x), \quad (2)$$

the field strength tensor is not invariant, rather it transforms covariantly as

$$F_{\mu\nu} \rightarrow U(x)F_{\mu\nu}U^{-1}(x). \quad (3)$$

Since the field strength tensor is not gauge invariant (unlike in QED), it is not an observable and an external chromoelectric or chromomagnetic field is not a meaningful concept. It is also well known that in non-Abelian gauge theories

there exists field tensors that are realized in terms of different gauge field configurations that are not gauge equivalent. Also, Wilson loops calculated using these non equivalent gauge fields, produce different results [6]. Nonetheless it is an interesting question to study since, as in any collective phenomenon, such instabilities may arise in the quark-gluon phase [7]. Within the context of effective actions, we note that the effective action of the theory is a gauge invariant quantity since the fermion determinant $\det(i\mathcal{D} - m)$, which leads to the effective action, is gauge invariant.

In this sense one can study the problem of a (color) charged particle moving in the presence of an external chromoelectric or a chromomagnetic field. Furthermore, as in the case of QED, one can assume the external fields to be constant [8]. It was shown many years ago by Brown and Weisberger [9,10] that a constant chromoelectric field (or chromomagnetic) can be obtained from two different classes of gauge potentials, namely, a) $A \propto x$ or, b) A is constant. Basically, the two classes correspond respectively to either the commutator or the curl in (1) vanishing. The first class of potentials is familiar from the study of Abelian theories where a constant field can be associated with a potential of the form (symmetric gauge) $A_\mu = -\frac{1}{2}F_{\mu\nu}x^\nu$. The second class genuinely arises in a non-Abelian theory. The first class of potentials where the potential is linear in the coordinates has been used by many authors [11–15]. In particular Savvidy [16] has used this choice of the potential in discussing the vacuum instability in quantum chromodynamics and Nielsen and Olesen in [17] have employed an analogous gauge to study the instability of $SU(2)$ Yang-Mills theory coupled to Higgs.

In this paper we analyze the problem of vacuum instability for the case of constant gauge field strengths belonging to the non-Abelian $SU(2)$ group in $2 + 1$ dimensions [18–21]. This same kind of potential has been used in exploring temperature and density effects in the frame of a discussion of phase transitions [22,23]. In particular, we calculate the probability density for pair creation by external chromoelectric fields. Our analysis is

valid for the case of a chromoelectric field. In the case of an external chromomagnetic field the effective action does not develop poles and the effect will be only to produce a condensed state $\langle \bar{\psi}\psi \rangle$. So, in this case we will not expect to observe any instability in the vacuum [21]. We have chosen to analyze the $2 + 1$ dimensional problem because it has already been shown that there is no instability for the $3 + 1$ dimensional case [9]. Also, $2 + 1$ dimensional discussion can be of relevance if we are considering quark-gluon plasma at very high temperature, $T \rightarrow \infty$, when one of the dimensions is expected to compactify [24].

II. PROBABILITY DENSITY FOR PAIR EMISSION

As we have already mentioned, a constant non-Abelian field strength can be described by gauge potentials that are either linearly proportional to x^μ or by constant non-commuting gauge potentials. Many people have already used the first class of gauge potentials in studying the instability phenomena. In our calculation, however, we choose the second class of constant potentials since it provides an alternative and physical way of looking at the problem of vacuum decay. Let $A_\mu = A_\mu^a \tau^a$, $a = 1, 2, 3$ denote the $SU(2)$ gauge potentials where a denotes the color index and $\tau^a = \lambda_a/2$ are the $SU(2)$ color generators (in the fundamental representation). We consider a constant chromoelectric field along the x direction (spatial) pointing along the 3 direction in the color space, namely,

$$E_1 = F_{01} = ig[A_0, A_1] = E_1^3 \tau_3. \quad (4)$$

A simple choice of constant gauge potentials that yields such a chromoelectric field is given by $A_0 = -\sqrt{\epsilon_1} \tau_1$, $A_1 = \sqrt{\epsilon_2} \tau_2$ where ϵ_1, ϵ_2 are (dimensional) constants, leading to

$$F_{01} = E_1 = E_1^3 \tau_3 = g\sqrt{\epsilon_1 \epsilon_2} \tau_3 = \epsilon \tau_3. \quad (5)$$

We note here that in $2 + 1$ dimensions both g and A have the dimensions of the square root of a mass. Also, in $2 + 1$ dimensions there are two possible mass terms for the Lagrangian [25]. However one of these terms violates parity and time reversal while the other one violates chiral symmetry. In this article we work within a parity conserving model in $2 + 1$ dimensions [26], where we work in a reducible representation of the gamma matrices [27].

We can now study the motion of a four component Dirac fermion (belonging to a reducible representation) in such a background described by the equation

$$(i\mathcal{D} - m)\psi = (i\partial - g\mathcal{A} - m)\psi = (i\partial - M)\psi = 0. \quad (6)$$

Here $\mathcal{A} = \gamma^\mu A_\mu$, the three gamma matrices are defined in terms of the Pauli matrices (in spinor space) as

$$\begin{aligned} \gamma^0 &= \begin{pmatrix} \sigma_3 & 0 \\ 0 & -\sigma_3 \end{pmatrix}, & \gamma^1 &= \begin{pmatrix} i\sigma_1 & 0 \\ 0 & -i\sigma_1 \end{pmatrix}, \\ \gamma^2 &= \begin{pmatrix} i\sigma_2 & 0 \\ 0 & -i\sigma_2 \end{pmatrix}, \end{aligned} \quad (7)$$

and we have identified the effective mass matrix for this constant gauge potential background as $M = m + g\mathcal{A}$. For later use, we note here that in this reducible spinor representation, it is possible to define two matrices that anticommute with the three gamma matrices in (7), namely,

$$\gamma^3 = i \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \gamma_5 = i\gamma^0\gamma^1\gamma^2\gamma^3 = i \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}, \quad (8)$$

with $\gamma_5^\dagger = \gamma_5$, $\gamma_5^2 = 1$. Note that the fermions belong to the fundamental representation of the color $SU(2)$ group and, therefore, carry both the color index $i = 1, 2$ as well as the spinor index $\alpha = 1, 2, 3, 4$. Therefore, the mass matrix M is a 8×8 matrix in the product space and is not Hermitian, $M \neq M^\dagger$, since the Dirac matrices γ^μ are not Hermitian. As a result, the eigenvalues of the mass matrix become complex. The eight eigenvalues are easily determined as two complex conjugate pairs

$$\begin{aligned} m_1 &= m_5 = m - \frac{g}{2}\sqrt{\epsilon_1} - \frac{i}{2}g\sqrt{\epsilon_2} = m_2^* = m_6^*, \\ m_3 &= m_7 = m + \frac{g}{2}\sqrt{\epsilon_1} - \frac{i}{2}g\sqrt{\epsilon_2} = m_4^* = m_8^*. \end{aligned} \quad (9)$$

It is worth noting that the chromoelectric field ϵ is proportional to the real and imaginary parts of the mass eigenvalues, which means that if we had real eigenvalues for the mass matrix, then we would not have a chromoelectric field and, hence, we would not have a decay probability. Since the mass matrix is not self-adjoint, neither is the Hamiltonian of the system, and this is crucial in order to have a nonvanishing decay probability. The decay probability is related to the imaginary part of the effective action and, when fermions are integrated, at one loop level the effective action has the form

$$\Gamma_{\text{eff}} = \text{Tr} \ln (i\mathcal{D} - m). \quad (10)$$

We can use the fact that $\gamma_5^2 = 1$ and γ_5 anticommutes with the three gamma matrices (together with the cyclicity of trace) to obtain

$$\begin{aligned} \Gamma_{\text{eff}} &= \text{Tr} \ln (i\mathcal{D} - m) = \text{Tr} \ln (i\mathcal{D} + m) \\ &= \frac{1}{2} \text{Tr} \ln (-\mathcal{D}^2 - m^2). \end{aligned} \quad (11)$$

Since covariant derivatives D_μ do not commute, we have

$$\mathcal{D}^2 = D_\mu D^\mu + \frac{g}{2} \sigma^{\mu\nu} F_{\mu\nu}, \quad (12)$$

where we have identified

$$\sigma^{\mu\nu} = \frac{i}{2} [\gamma^\mu, \gamma^\nu], \quad (13)$$

and the field strength $F_{\mu\nu}$ is defined in (1).

Using these results as well as a proper time representation for the logarithm in (11), the calculation of the pair decay probability by unit of volume and time can be done as follows: if $w(x)$ is the creation pair probability then

$$w(x) = \text{Re tr}_{aa} \left[\int_0^\infty \frac{ds}{s} \langle x, M | e^{-ism^2} \times \left(\exp \left\{ is \left[(p-gA)^2 - \frac{g}{2} \sigma_{\mu\nu} F^{\mu\nu} \right] \right\} - e^{-isp^2} \right) | x, M \rangle \right]. \quad (14)$$

where the trace runs over color and spinor indices and $|x, M\rangle$ are eigenstates of the mass operator \hat{M} . A sum over the mass eigenvalues is implicit.

The last term on the right-hand side corresponds to the subtraction of the $A = 0$ part of the propagator which is equivalent to remove the zero mode.

In order to compute the trace in the previous equation one should expand the exponential in a Taylor series. The first exponential has the form

$$e^{is[(p-gA)^2 - \frac{g}{2} \sigma_{\mu\nu} F^{\mu\nu}]} = e^{\alpha_0 + \alpha_1 \lambda_1 + \alpha_2 \lambda_2 + \alpha_3 \lambda_3}, \quad (15)$$

where λ_i are the Pauli matrices in color space and α_i are complex matrices in Lorentz space that commute with each other. Recalling that the square of any Pauli matrix is the identity and that the trace of any Pauli matrix vanishes, it is easy to compute the trace on the Taylor series. As a result we are left with another Taylor series

$$\text{tr}_{aa} e^{\alpha_0 + \alpha_1 \lambda_1 + \alpha_2 \lambda_2 + \alpha_3 \lambda_3} = 8 \left[1 + \alpha_0 + \frac{1}{2} (\alpha_0^2 + \alpha_1^2 + \alpha_2^2 + \alpha_3^2) + \frac{1}{3!} (\alpha_0^3 + 3\alpha_0 \alpha_1^2 + 3\alpha_0 \alpha_2^2 + 3\alpha_0 \alpha_3^2) + \dots \right]. \quad (16)$$

This new Taylor series can be reconstructed and it can be checked that, finally

$$\text{tr}_{aa} e^{\alpha_0 + \alpha_1 \lambda_1 + \alpha_2 \lambda_2 + \alpha_3 \lambda_3} = 8e^{\alpha_0} \cosh \sqrt{\alpha_1^2 + \alpha_2^2 + \alpha_3^2}. \quad (17)$$

Taking the definitions of α_i from (15) we have

$$\text{tr}_{aa} \{ e^{is[(p+gA)^2 - \frac{g}{2} \sigma_{\mu\nu} F^{\mu\nu}]} \} = 8e^{is[p^2 + g^2 A^2]} \times \cosh \sqrt{-s^2 g^2 \epsilon_1 p_0^2 - s^2 g^2 \epsilon_2 p_1^2 + \left(\frac{sg\epsilon}{2} \right)^2}. \quad (18)$$

This result can be confirmed using a calculation software such as Mathematica. Going back to the probability density, we have

$$\omega(x) = 8 \text{Re} \int_0^\infty \frac{ds}{s} \langle x, M | e^{is(p^2 - m^2 + g^2 A^2)} \times \cosh \sqrt{-s^2 g^2 \epsilon_1 p_0^2 - s^2 g^2 \epsilon_2 p_1^2 + \left(\frac{sg\epsilon}{2} \right)^2} - e^{is(p^2 - m^2)} | x, M \rangle. \quad (19)$$

Defining $\tilde{m}^2 \equiv m^2 - g^2 A^2$ and taking the real part

$$\omega(x) = 8 \int_0^\infty \frac{ds}{s} \langle x, M | \cos (s(p^2 - \tilde{m}^2)) \times \cosh \sqrt{-s^2 g^2 \epsilon_1 p_0^2 - s^2 g^2 \epsilon_2 p_1^2 + \left(\frac{sg\epsilon}{2} \right)^2} - \cos (s(p^2 - m^2)) | x, M \rangle. \quad (20)$$

We can insert completeness relations between p 's and compute the momentum integrals to write, for the second term of $\omega(x)$

$$8 \int_0^\infty \frac{ds}{s} \langle x, M | \cos(s(p^2 - m^2)) | x, M \rangle$$

$$= \frac{1}{(2\pi)^{3/2}} \int_0^\infty \frac{ds}{s^{5/2}} [e^{ism^2} + e^{-ism^2} + ie^{ism^2} - ie^{-ism^2}]. \quad (21)$$

The calculation of the integrals appearing in (21) can be performed observing that they can be regularized by considering the following identity:

$$\lim_{\alpha \rightarrow 0} \left[\int_0^\infty ds s^{\gamma-1} e^{-\frac{\alpha}{s} - \beta s} \right] = \frac{2^{1-\gamma}}{\beta^\gamma}, \quad (22)$$

where β now is $\pm im^2$ and $\gamma = -3/2$. With this, the integral in (21) will vanish and the probability density becomes

$$\omega(x) = 8 \int_0^\infty \frac{ds}{s} \int \frac{d^3 p}{(2\pi)^3} \cos(s(p^2 - \tilde{m}^2))$$

$$\times \cosh \sqrt{-s^2 g^2 \epsilon_1 p_0^2 - s^2 g^2 \epsilon_2 p_1^2 + \left(\frac{sg\epsilon}{2}\right)^2}. \quad (23)$$

The integral in (23) cannot be computed exactly. Rather we will look for an upper bound for the probability density. Such an upper bound can be obtained by making the replacement

$$\cosh \sqrt{-s^2 G_1^2 p_0^2 - s^2 G_2^2 p_1^2 + \left(\frac{sg\epsilon}{2}\right)^2} \rightarrow \cosh \left(\frac{sg\epsilon}{2}\right).$$

(24)

The momentum integrals can then be computed and the regularization shown in (22) can be performed. We define

$$R = \sqrt{\left(\frac{g\epsilon}{2}\right)^2 + (\tilde{m}^2)^2} \quad (25)$$

$$\theta = \arctan \left(\frac{\tilde{m}^2}{g\epsilon/2}\right), \quad (26)$$

and obtain for the upper bound of the probability density

$$\omega < \frac{R^{3/2}}{4(2\pi)^{3/2}} \{ (1+i)[e^{\frac{3}{2}i(\theta \mp \pi)} + e^{-\frac{3}{2}i\theta}]$$

$$+ (1-i)[e^{\frac{3}{2}i(-\theta \pm \pi)} + e^{\frac{3}{2}i\theta}] \}, \quad (27)$$

where the upper sign in $(\mp \pi, \pm \pi)$ is for $\tilde{m}^2 \geq 0$ and the lower sign is for $\tilde{m}^2 < 0$. It easily verified that for $\tilde{m}^2 \geq 0$ the previous expression vanishes. So, in order to have a non vanishing probability density $\tilde{m}^2 < 0$, i.e.

$$m^2 < g^2 A^2 = \frac{g^2}{4} (\epsilon_1 - \epsilon_2). \quad (28)$$

With this, the upper bound for our probability density is

$$\omega < \left(\frac{R}{2\pi}\right)^{3/2} \left(\cos \frac{3}{2}\theta + \sin \frac{3}{2}\theta\right). \quad (29)$$

A similar process can be carried out and a lower bound for the probability density can be found through the replacement

$$\cosh \sqrt{-s^2 G_1^2 p_0^2 - s^2 G_2^2 p_1^2 + \left(\frac{sg\epsilon}{2}\right)^2}$$

$$\rightarrow \cos(sG_1 p_0 + sG_2 p_1). \quad (30)$$

The computation of the lower bound can then be performed and it turns out to be null, so we have

$$0 < \omega < \left(\frac{R}{2\pi}\right)^{3/2} \left(\cos \frac{3}{2}\theta + \sin \frac{3}{2}\theta\right). \quad (31)$$

It is important to note that in making the replacements in (24) and (30) we have assumed that the probability density is positive definite. This restricts the region where our bounds are valid and the possible values of θ . Taking this into account, as well as the condition in (28) we can find the region of allowed values for θ

$$-\frac{(8n+1)\pi}{6} < \theta < -\frac{(8n-3)\pi}{6} \quad \text{for } n = 1, 2, 3, \dots \quad (32)$$

or

$$-\frac{\pi}{6} < \theta < 0. \quad (33)$$

With this, we can plot the allowed region within which the probability density has a finite, nonvanishing value. This is shown in Fig. 1.

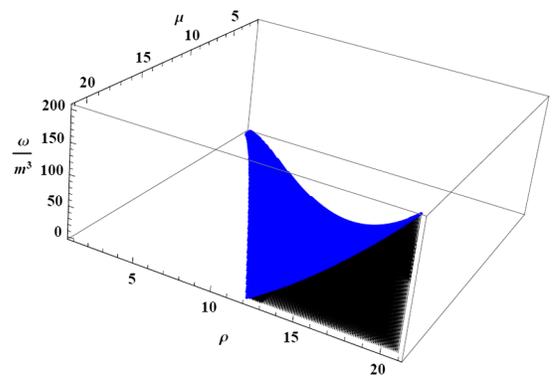


FIG. 1 (color online). Probability density for pair emission in units of m^3 . The blue surface represents the upper bound. The black region is where the exact value of the probability can be. Dimensionless variables $\mu = \frac{g\sqrt{\epsilon_1}}{m}$ and $\rho = \frac{g\sqrt{\epsilon_2}}{m}$ have been defined.

III. COLOR $SU(3)$

This same problem can be solved within the color $SU(3)$ group. The corresponding potential is the same as in $SU(2)$ with $A_\mu^a \neq 0$ only for $a = 1, 2$. Notice that this gauge fixing condition implies that actually this is not a full $SU(3)$ calculation, which would imply the appearance of the eight generators. Our calculation corresponds only to the projection of $SU(3)$ into $SU(2)$. The computation of the trace in (18) is now different

$$\text{tr}_{aa} \{ e^{is[(p+gA)^2 - \frac{g}{2}\sigma_{\mu\nu}F^{\mu\nu}]} \} = 4e^{is[p^2+g^2A^2]} \left(1 + 2 \cosh \sqrt{-s^2g^2\epsilon_1p_0^2 - s^2g^2\epsilon_2p_1^2 + \left(\frac{sg\epsilon}{2}\right)^2} \right). \quad (34)$$

If we compare this with (18) we can see that a new term has been added to our result coming from the first term on the right-hand side. However, because of the same argument following (21) this term will have a vanishing contribution to the probability density. Taking this into account we can say that the probability density in the $SU(3)$ color group yields the same result we have already shown.

IV. 3 + 1 DIMENSIONS

So far we have restricted our analysis to the 2 + 1 dimensional case. In this section we will briefly discuss the 3 + 1 dimensional case. In such a case we have an additional phase space integral and then Eq. (21) is changed like

$$\begin{aligned} & 8 \int_0^\infty \frac{ds}{s} \langle x, M | \cos (s(p^2 - m^2)) | x, M \rangle \\ &= \frac{-i}{(2\pi)^2} \int_0^\infty \frac{ds}{s^3} [e^{ism^2} - e^{-ism^2}]. \end{aligned} \quad (35)$$

The extra dimension has increased the exponent of s from 5/2 to 3, meaning that the regularization shown in (22) should now be performed taking $\gamma = -2$. This is key since now, Eq. (27) will look like

$$\omega < \frac{R^2}{2(2\pi)^2} \{ e^{2i(\theta \mp \pi)} - e^{2i\theta} \}, \quad (36)$$

which vanishes. This result coincides with that of Ref. [9]. It is worth noting that the reason for this null decay probability lies in the fact that we have enlarged our phase space. It is the extra dimension in s that makes the integrals vanish.

V. SCALAR FIELDS

Computations of vacuum instability are usually easier to carry out when one is dealing with scalar fields. Given that we could not obtain an exact expression for the probability density for pair emission when working with fermionic fields, one might ask whether or not this is possible when working with colorful bosonic fields. In such a case, the probability density for pair emission is

$$\begin{aligned} w(x) = \text{Re tr}_a \left[\int_0^\infty \frac{ds}{s} \langle x, M | e^{-ism^2} \right. \\ \left. \times (\exp \{ is[(p - gA)^2] \} - e^{-isp^2}) | x, M \rangle \right], \end{aligned} \quad (37)$$

where the trace now runs only over color indices. The trace can be computed in the same manner as before by taking $\alpha_3 = 0$ in Eq. (17)

$$\begin{aligned} \text{tr}_a \{ e^{is[(p+gA)^2]} \} = 2e^{is[p^2+g^2A^2]} \\ \times \cosh \sqrt{-s^2g^2\epsilon_1p_0^2 - s^2g^2\epsilon_2p_1^2}. \end{aligned} \quad (38)$$

Then, after inserting completeness relations between p 's we get

$$\begin{aligned} \omega(x) = 8 \int_0^\infty \frac{ds}{s} \int \frac{d^3p}{(2\pi)^3} \cos (s(p^2 - \tilde{m}^2)) \\ \times \cos \sqrt{s^2G_1^2p_0^2 + s^2G^2p_1^2}. \end{aligned} \quad (39)$$

Once again, this integral cannot be computed exactly. As in the previous sections we will then find an upper and lower bound for it. We can find an upper bound for the probability through the replacement

$$\cos \sqrt{s^2G_1^2p_0^2 + s^2G^2p_1^2} \rightarrow 1, \quad (40)$$

and so we can write

$$\omega(x) < 8 \int_0^\infty \frac{ds}{s} \int \frac{d^3p}{(2\pi)^3} \cos (s(p^2 - \tilde{m}^2)). \quad (41)$$

By the same argument following Eq. (21), it is easy to show that this last integral vanishes. This means then that the probability density itself vanishes and hence, there is no vacuum instability. In this sense, even though the computation is not much easier when dealing with scalar fields, we are able to conclude that there is no vacuum instability.

The same analysis here presented can be carried out in $3 + 1$ dimensions. It is trivial to show that, in the same way the probability vanished when dealing with fermions in $3 + 1$ dimensions, it will also vanish when dealing with scalars.

VI. CONCLUSIONS

We analyzed the problem of pair emission from the vacuum in the presence of a chromoelectric field in $2 + 1$ dimensions. The probability density cannot be computed exactly, but we find an upper and lower bound for it. We also find that there is an allowed region in the parameter space, where the probability will not vanish, shown in Fig. 1. We show that the probability density, however, vanishes in $3 + 1$ dimensions. We also find that for scalar

fields, the probability density vanishes both in $2 + 1$ and $3 + 1$ dimensions.

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