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On the propagation of quantum correlations in a cavity quantum electrodynamics network

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Abstract
In this paper we present the analysis of a quantum direct communication protocol. Based on the propagation of quantum correlations, we analyze a model of two chains of atoms inside cavities, joined by optical fibers, which can be used as a quantum communication protocol. We study the dynamics of the entanglement and the quantum discord, using the generalized master equation. We first study the free evolution of the quantum correlations through the cavity network and compare it with the case where an eavesdropper has performed a measurement on one of the cavities. We also find the quantum discord to be a more robust measure of the quantum correlations and study the optimal initial condition to detect the eavesdropper.

Keywords: correlations, cavity, electrodynamics, propagation

(Some figures may appear in color only in the online journal)

1. Introduction

Over the past few years, quantum correlations has been extensively researched, mainly because of their importance in quantum information and computation. Entanglement [1] and quantum discord (QD) [2] are among the most popular measures of the above mentioned correlations. Entanglement is related to the separability of two subsystems, a concept leading to some interesting applications [3, 4]. On the other hand, QD is defined as a mismatch between quantum analogues of classically equivalent expressions of the mutual information, also leading to important applications [5, 6].

In a realistic physical model, the main system interacts with the surrounding environment, causing, in general, the destruction of the coherence and the entanglement. We recently found that QD seems to be more robust than the entanglement and in some particular cases, may be non zero while the entanglement already suffered from sudden death [7–9]. However, there are other cases, where the opposite may happen [10, 11].

Recently, there has been growing interest in studying atomic systems in cavity quantum electrodynamics, as well as cavity–atom polaritonic excitations [12, 13]. Moreover, much attention has been paid to the possibility of quantum information processing realized via optical fibers between two atoms in distant coupled cavities [7, 9, 14, 15].

This paper is organized as follows: in section 2, we describe the system by an effective Hamiltonian and write a generalized master equation, where the Lindblad terms result from the coupling of each cavity to its own thermal reservoir at zero temperature. In section 3, we give a brief outline of the quantum correlations and present the main results of this paper, related to the optimal propagation of the quantum correlations and discuss our ability to detect third parties trying to gain information, based on the adequate choice of the initial conditions. The last section is devoted to an overall view and discussion of the main results.

2. The model

We have two identical chains of three cavities joined by optical fibers as shown in figure 1, where each cavity interacts with a single atom and its own reservoir. A similar model was used by Zhang *et al* [17] but without losses. We model our system in the short fiber limit $2l\mu/2\pi c \ll 1$, where l is the length of the fiber and μ is the decay rate of the cavity fields into a continuum of fiber modes [18].

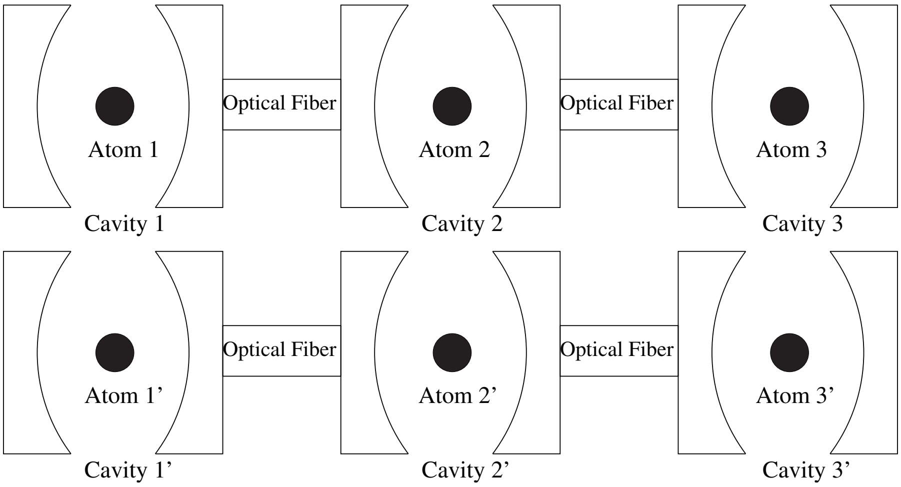

Figure 1. Array of two rows of three cavity–atom systems.

2.1. The effective Hamiltonian

The Hamiltonian of an N-atom–cavity system in the rotating wave approximation is given by

$$H = H^{\rm free} + H^{\rm int}, \tag{1}$$

where

$$H^{\rm free} = \sum_{i=1}^{N} \omega_i^{\rm a} |e\rangle_i\langle e| + \sum_{i=1}^{N} \omega_i^{\rm c} a_i^\dagger a_i + \sum_{i=1}^{N-1} \omega_i^{\rm f} b_i^\dagger b_i \tag{2}$$

and

$$H^{\rm int} = \sum_{i=1}^{N} \nu_i (a_i^\dagger |g\rangle_i\langle e| + a_i |e\rangle_i\langle g|) + \sum_{i=1}^{N-1} J_i [(a_i^\dagger + a_{i+1}^\dagger) b_i + (a_i + a_{i+1}) b_i^\dagger], \tag{3}$$

where $|g\rangle_i$ and $|e\rangle_i$ are the ground and excited states of the two-level atom with transition frequency $\omega^{\rm a}$ and $a_i^\dagger$ (a_i) and $b_i^\dagger$ (b_i) are the creation(annihilation) operators of the cavity and fiber mode, respectively. The first, second and third terms in $H^{\rm free}$ are the free Hamiltonians of the atom, cavity field and fiber field, respectively. In addition, the first term in $H^{\rm int}$ describes the interaction between the cavity mode and the atom inside the cavity with the coupling strength ν_i, and the second term is the interaction between the cavity and the fiber modes with the coupling strength J_i.

The first two terms of $H^{\rm free}$ and the first term of $H^{\rm int}$ can be jointly diagonalized in the basis of polaritons. For simplicity we consider the resonance between atom and cavity $\omega^{\rm a} = \omega^{\rm c} = \omega$, and also that the cavities and the fibers are identical. The total Hamiltonian is now given by

$$H = \sum_{i=1}^{N} (\omega - \nu) |E\rangle_i\langle E| + \sum_{i=1}^{N-1} \omega_i^{\rm f} b_i^\dagger b_i + \sum_{i=1}^{N-1} \frac{J}{\sqrt{2}} [(L_i^\dagger + L_{i+1}^\dagger) b_i + (L_i^- + L_{i+1}^-) b_i^\dagger], \tag{4}$$

where $|E_i\rangle = \frac{1}{\sqrt{2}}(|1, g\rangle_i - |0, e\rangle_i)$ and $|G_i\rangle = |0, g\rangle_i$ are the polaritonic states, corresponding to excited and ground state, respectively. The other operators $L_i^\dagger = |E_i\rangle\langle G_i|$ and $L_i^- = |G_i\rangle\langle E_i|$ are to create or destroy those states. So we can consider polaritons as a two-level system. We can have just one photon, at most, because due to photon blockade, double or higher occupancy of the polaritonic states is prohibited [19, 20].

In the case of a three atom-cavity system, we use perturbation theory [21] to find an effective Hamiltonian, supposing that the total detuning $\delta = (\omega - \nu) - \omega^{\rm f} \gg J$. Finally, we projected the fiber state into the zero photon mode, so we end up with a reduced Hamiltonian given by

$$H_{\rm s} = \lambda(|E_1\rangle\langle E_1| + 2|E_2\rangle\langle E_2| + |E_3\rangle\langle E_3|) + \lambda(L_1^\dagger L_2^- + L_1^- L_2^\dagger + L_2^\dagger L_3^- + L_2^- L_3^\dagger), \tag{5}$$

where $\lambda = \frac{J^2}{2\delta}$.

2.2. The master equation

Until now, we have not considered losses. The main source of dissipation originates from the leakage of the cavity photons due to imperfect reflectivity of the cavity mirrors. A second source of dissipation corresponds to atomic spontaneous emission, that we will neglect assuming long atomic lifetimes.

An approach to model the above mentioned losses, in the presence of single mode quantized cavity fields, is using the microscopic master equation, which goes back to the ideas of Davies [22] on how to describe the system–reservoir interactions in a Markovian master equation. For a three-cavity-system at zero temperature, the master equation is [9, 18]

$$\dot{\rho}(t) = -{\rm i}[H_{\rm s}, \rho(t)] + \sum_{n=1}^{3} \sum_{\omega>0}^{\infty} \gamma_n(\omega) \Big(A_n(\omega)\rho(t)A_n^\dagger(\omega) - \frac{1}{2}\{A_n^\dagger(\omega)A_n(\omega), \rho(t)\} \Big), \tag{6}$$

where A_n correspond to the Davies' operators. The sum on n is over all the dissipation channels and the decay rate $\gamma_n(\omega)$ is the Fourier transform of the correlation functions of the environment [16].

The A_n operators are calculated as follows:

$$A_n(\omega_{\alpha\beta}) = |\phi\rangle_\alpha \langle\phi|a_n|\phi\rangle_\beta \langle\phi|. \quad (7)$$

3. Results

3.1. Quantum correlations

We proceed now to analyze the above mentioned correlations. We will restrict our description to the 'X' structured density operator, which is given by

$$\rho_{AB} = \begin{pmatrix} \rho_{11} & 0 & 0 & \rho_{14} \\ 0 & \rho_{22} & \rho_{23} & 0 \\ 0 & \rho_{23}^* & \rho_{33} & 0 \\ \rho_{14}^* & 0 & 0 & \rho_{44} \end{pmatrix}. \quad (8)$$

The entanglement can be found by means of the well-known entanglement of formation (EF), defined in [1] as

$$\mathrm{EF}(C) = h\left(\frac{1+\sqrt{1-C^2}}{2}\right),$$
$$h(x) = -x\log_2(x) - (1-x)\log_2(1-x), \quad (9)$$

where the concurrence (C) is defined as

$$C_{AB} = \max\{0, \alpha_1 - \alpha_2 - \alpha_3 - \alpha_4\}, \quad (10)$$

here α_1, α_2, α_3, α_4 are the square root of the eigenvalues of the product $\rho_{AB}((\sigma_y \otimes \sigma_y)\rho^*_{AB}(\sigma_y \otimes \sigma_y))$, in decreasing order. Notice that σ_y is the usual Pauli's matrix and ρ^*_{AB} is the complex conjugate of ρ_{AB}.

Now, let us focus on the calculation of the QD. There are several methods [23–26], but some are for particular cases or involve approximations [27]. One way to proceed is to convert ρ into a density matrix with real elements, by performing a local unitary transformation on each qubit, $U = \mathrm{e}^{-\mathrm{i}\frac{\theta_1}{2}\sigma_z} \otimes \mathrm{e}^{-\mathrm{i}\frac{\theta_2}{2}\sigma_z}$, with suitable angles θ_1 and θ_2. Since the transformation is local, it does not affect the QD. Defining the new operator as $\rho' = U^\dagger \rho U$, we get for the off-diagonal elements

$$\langle 11|\rho'|00\rangle = \rho_{14}\,\mathrm{e}^{(\theta_1+\theta_2)\mathrm{i}},$$
$$\langle 10|\rho'|01\rangle = \rho_{23}\,\mathrm{e}^{(\theta_1-\theta_2)\mathrm{i}} \quad (11)$$

and similarly for the other two complex conjugates.

If we choose

$$\theta_1 = -\frac{1}{2}(\arg(\rho_{14}) + \arg(\rho_{23})),$$
$$\theta_2 = -\frac{1}{2}(\arg(\rho_{14}) - \arg(\rho_{23})), \quad (12)$$

we make sure that all elements of ρ' are real, and we proceed with the calculation using [25].

As the initial condition we used a W state for the cavities 1 and $1'$, the rest of the cavities started in the ground state ($|G\rangle$)

$$\rho(0) = \left(\frac{1-a}{4}I + a|\psi\rangle\langle\psi|\right)$$
$$\otimes |G_2G_{2'}G_3G_{3'}\rangle\langle G_2G_{2'}G_3G_{3'}|, \quad (13)$$

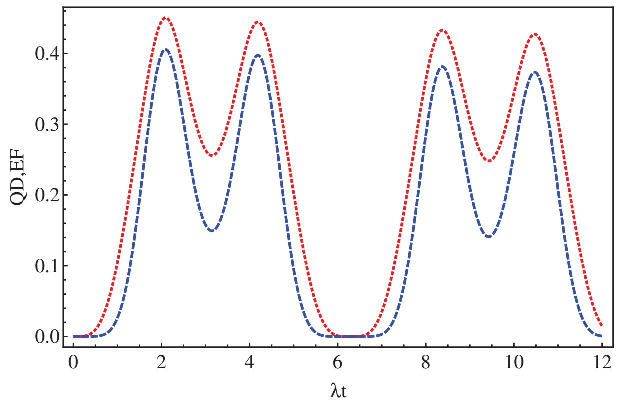

Figure 2. QD (red-dotted); EF (blue-dashed); $a = 0.9$; for the cavities $33'$.

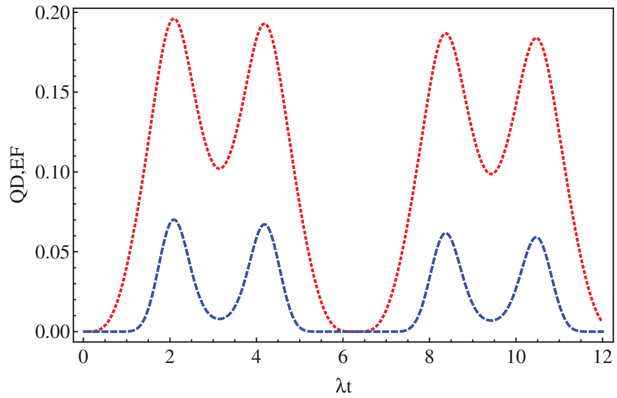

Figure 3. QD (red-dotted); EF (blue-dashed); $a = 0.6$; for the cavities $33'$.

where I is the identity operator for two qubits and $|\psi\rangle = \frac{1}{\sqrt{2}}(|E_1G_{1'}\rangle + |G_1E_{1'}\rangle)$.

From now on, in the numerical calculations we used the parameters, $J = 2\pi\ 30\,\mathrm{GHz}$, $\delta = 2\pi\ 300\,\mathrm{GHz}$ and $\gamma = 0.01\,\mathrm{GHz}$.

3.2. Propagation of the quantum correlations

We are interested in sending information through the two chains. This implies starting with two correlated qubits corresponding, for example, to the $11'$ cavities. We study the dynamics of our system such that after some time, the $33'$ pair becomes correlated. In figures 2 and 3, we plot QD and EF, for the $33'$ pair, for an initial Werner state, corresponding to $a = 0.9$ and 0.6, respectively. When the system is nearly a pure state ($a = 0.9$), there is not a big difference between the two curves (figure 2), when $a = 0.6$ there is a substantial difference between QD and EF and obviously the QD is the better option for the propagation of the information (figure 3). This is also true for a variety of initial conditions and loss rates [9].

Next, it will be of interest to study the effect of a measurement on the propagation. As an example, let us perform a projective measurement on cavity 2, such as $\Pi = |G_2\rangle\langle G_2|$. Then we compare the QD after the measurement (QDM) at the $33'$ pair. From figure 4, we can see that for a nearly pure maximally entangled state, the curve corresponding to the QD after the measurement is reduced to

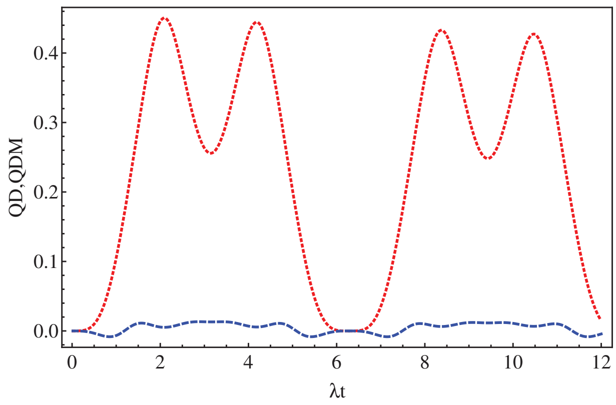

Figure 4. QD (red-dotted); QDM (blue-dashed); $a = 0.9$; for the cavities 33′.

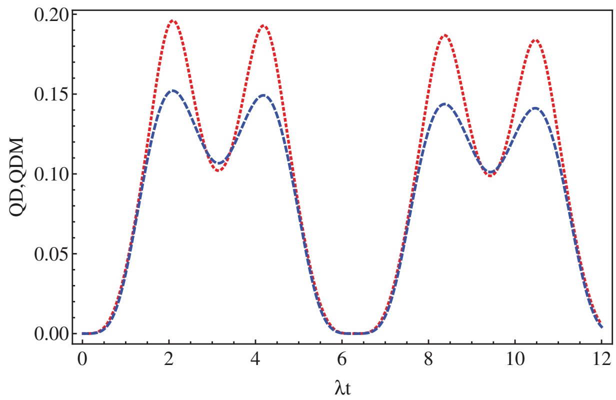

Figure 5. QD (red-dotted); QDM (blue-dashed); $a = 0.6$; for the cavities 33′.

almost zero (blue-dashed) as compared with the undisturbed QD without any measurement (red-dotted). It is quite apparent that in this case we have a very good instrument to detect any external measurement. However if the state becomes more mixed ($a = 0.6$), the discrimination becomes inconclusive, since in figure 5 we do not observe relevant differences anymore between the two curves.

4. Summary and conclusions

In the present work we describe a cavity network to be used as a secure communication protocol. The system consists of two independent channels of cavities coupled by optical fibers, and we send a qubit on each channel. In order to achieve this task, we start with a quantum correlated Werner initial condition for the 11′ pair and study the propagation to the last couple of cavities (33′). After several numerical solutions of the master equation, we found the QD measure to be more robust against decoherence, as compared to the entanglement, specially for the case of mixed initial states.

Next, we analyze the effect on the QD on 33′ if a third person (an eavesdropper) performs a measurement on, say, cavity 2. The main idea behind this is to compute the QD in 33′ after this measurement and compare it with the undisturbed case.

Our results show that this comparison is strongly initial condition dependent. If the initial state is a nearly pure and maximally entangled one, substantial differences show up and we can easily detect the eavesdropper. However, going to a highly mixed initial state, no substantial difference shows up.

The present setup can be realized experimentally [15], and under certain conditions we may detect the presence of an eavesdropper. Thus, it can be seen as a proposal of a security protocol for quantum communications.

Acknowledgments

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