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Generation of quantum correlations for two qubits through a common reservoir

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Abstract
Recently, besides entanglement, an entropy-based measurement called quantum discord has been used to describe the quantum correlations in separable and non-separable states. Here we compare the dynamics of entanglement with that of quantum discord, in the case of two qubits in a common thermal or squeezed reservoir. We focus on the generation of quantum correlations, starting from an uncorrelated system.

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(Some figures may appear in colour only in the online journal)

1. Introduction

Until recently, all non-classical correlations in a composite quantum system were regarded as entanglement [1]. The concept of entanglement is fundamental in various tasks such as teleportation [2], cryptography [3], dense coding [4], etc. Furthermore, Yu and Eberly [5] investigated the dynamics of disentanglement of a bipartite qubit system where the qubits were coupled to individual environments. They found that the quantum entanglement may vanish in a finite time while the local decoherence would decay exponentially in time. They called this phenomenon 'entanglement sudden death'. The phenomenon of delayed birth was also found [6], in which entanglement is null during a finite time prior to generation.

However, the discovery that mixed separable states can also have non-classical correlations has opened new perspectives on the study of such correlations [7, 8]. The total quantum correlation can be measured by quantum mutual information that may be divided into classical and quantum parts. The quantum part is called quantum discord, based on the distinction between quantum and classical information theory [9, 10]. Here, we study the generation of correlations, such as entanglement and quantum discord, in a system of two two-level atoms interacting with a common thermal or squeezed reservoir, using some specific initial conditions for the qubits. In some cases, entanglement sudden death may occur in regions where the quantum discord is not null.

2. Correlations

2.1. Entanglement of formation

For a given ensemble of pure states $\{p_i, |\psi_i\rangle\}$, the entanglement of formation is the average entropy of entanglement over a set of states that minimizes this average over all possible decompositions of ρ, see [11].

$$E(\rho) = \min \sum_i p_i E(\psi_i), \tag{1}$$

where the entanglement $E(\psi)$ is defined as the von-Neumann entropy of either of two subsystems $E(\psi) = S(\rho_A) = S(\rho_B)$, where $S(\rho) = -\mathrm{tr}(\rho \log_2 \rho)$, and $\rho_A = \mathrm{Tr}_B(\rho_{AB})$ and $\rho_B = \mathrm{Tr}_A(\rho_{AB})$ are the reduced density matrix of the subsystems A and B, respectively. However, it is very difficult to know which ensemble $\{p_i, \Psi_i\}$ is the one that minimizes the average.

Concurrence is [12, 13] a concept closely related to the entanglement of formation. For a general mixed state ρ_{AB} of two qubits, we define $\widetilde{\rho}$ to be the spin-flipped state $\widetilde{\rho}_{AB} = (\sigma_y \otimes \sigma_y)\rho^*_{AB}(\sigma_y \otimes \sigma_y)$, where ρ^* is the complex conjugate of ρ, and σ_y is the Pauli matrix. The concurrence is defined as

$$C'(\rho) = \max\{0, \lambda_1 - \lambda_2 - \lambda_3 - \lambda_4\}, \tag{2}$$

where $\{\lambda_i\}$ are the square roots, in decreasing order of the eigenvalues of the non-Hermitian matrix $\rho\widetilde{\rho}$.

Finally, the entanglement of formation is related to concurrence, via

$$E(\rho) = H\left[\frac{1}{2} + \frac{1}{2}\sqrt{1 - C'^2}\right] \quad (3)$$

with $H(x) = -x \log_2 x - (1-x)\log_2(1-x)$.

Entanglement vanishes for a *separable* state, defined as $\rho = \sum_i p_i \rho_i^A \otimes \rho_i^B$, and it is equal to one for maximally entangled states.

2.2. Quantum discord

The total correlation of a quantum system is quantified by the quantum mutual information $I(\rho) = S(\rho^A) + S(\rho^B) - S(\rho)$, which can be separated into classical and quantum correlations $I(\rho) = C(\rho) + Q(\rho)$.

In search of a formula for classical correlation, Vedral proposes a list of conditions that a classical correlation must satisfy [10]. The obtained expression that fulfills all the conditions is

$$C(\rho^{AB}) = \max_{\{B_k\}} [S(\rho^A) - S(\rho|\{B_k\})], \quad (4)$$

where the quantum conditional entropy is defined as: $S(\rho|\{B_k\}) = \sum_k p_k S(\rho_k)$ where $\{\rho_k, p_k\}$ is the ensemble of possible results after a set of von Neumann measurements which consists of a complete set of one-dimensional projectors $\{B_k\}$, in this case acting on subsystem B. Also $\rho_k = \frac{1}{p_k}(I \otimes B_k)\rho(I \otimes B_k)$ is the system state after a measurement, where $p_k = \mathrm{tr}(I \otimes \Pi_k)\rho(I \otimes \Pi_k)$ is the probability for obtaining the outcome ρ_k after the measurement. The maximization in equation (4) is done over all possible measurements of B, although we can choose to measure in A obtaining a different result due to the asymmetry of quantum discord (QD). However, this problem disappears for systems where $S(\rho^A) = S(\rho^B)$.

With this definition for classical correlation, the QD is

$$Q(\rho) = I(\rho) - C(\rho). \quad (5)$$

QD is null, if measuring in B, for a semi-classical state of the form

$$\rho = \sum_{ij} p_{ij} \rho_i^A \otimes \Pi_j^B, \quad (6)$$

where the Π_j are orthogonal projectors. Thus, for this state we can always find a measurement $\{B_k = \Pi_k\}$ which does not affect the initial quantum state. For pure states, the calculation of quantum correlations is simple [14]. The density matrix of any bipartite pure state $\rho = |\phi\rangle\langle\phi|$ can be written in the Schmidt decomposition, where the state is $|\phi\rangle = \sum_j \alpha_j |j\rangle \otimes |j\rangle$. Thus the distribution of quantum information corresponds to

$$I(\rho) = 2S, \quad C(\rho) = S, \quad Q(\rho) = S, \quad E(\rho) = S, \quad (7)$$

where $S = -\sum_j |\alpha_j|^2 \log_2 |\alpha_j|^2$.

In the case of a product state $S = 0$, and for a maximally entangled state $S = 1$.

3. The model

We consider two two-level atoms that interact with a *common* thermal or squeezed reservoir, as [15, 16]. We present a general master equation for the density matrix in the interaction picture, assuming that the correlation time between the atoms and the reservoirs is much shorter than the characteristic time of the dynamical evolution of the atoms, so that the Markov approximation is valid:

$$\begin{aligned}\frac{\partial\hat{\rho}}{\partial t} = \frac{\Gamma}{2}\sum_{i,j=1}^{2} &[(N+1)(2\sigma_i\hat{\rho}\sigma_j^\dagger - \sigma_i^\dagger\sigma_j\hat{\rho} - \hat{\rho}\sigma_i^\dagger\sigma_j) \\ &+ N(2\sigma_i^\dagger\hat{\rho}\sigma_j - \sigma_i\sigma_j^\dagger\hat{\rho} - \hat{\rho}\sigma_i\sigma_j^\dagger) \\ &- M(2\sigma_i^\dagger\hat{\rho}\sigma_j^\dagger - \sigma_i^\dagger\sigma_j^\dagger\hat{\rho} - \hat{\rho}\sigma_i^\dagger\sigma_j^\dagger) \\ &- M^*(2\sigma_i\hat{\rho}\sigma_j - \sigma_i\sigma_j\hat{\rho} - \hat{\rho}\sigma_i\sigma_j),\end{aligned}$$

where Γ is the decay constant of the qubits, and $\sigma_i^+ = |1\rangle_i\langle 0|$ and $\sigma_i^- = |0\rangle_i\langle 1|$ are the raising (+) and lowering (−) operators of the ith atom. The squeeze parameters are $\Psi = 0$, and $N = \sinh^2 r$. Here we consider $M = \sqrt{N(N+1)}$. For simplicity we will consider $\Gamma = 1$.

For a *thermal reservoir*: $N \to n$ with n the mean number of thermal photons. For a *squeezed reservoir*: $N = \sinh^2 r$ is the number of squeezed photons.

4. Results and conclusions

Here we develop the exact formula of entanglement and QD, for a particular system composed of two two-level atoms, with an X form density matrix for all times. The density matrix written in the base $|1\rangle = |11\rangle$, $|2\rangle = |10\rangle$, $|3\rangle = |01\rangle$, $|4\rangle = |00\rangle$, is

$$\begin{pmatrix} \rho_{11} & 0 & 0 & \rho_{14} \\ 0 & \rho_{22} & \rho_{23} & 0 \\ 0 & \rho_{32} & \rho_{33} & 0 \\ \rho_{41} & 0 & 0 & \rho_{44} \end{pmatrix}. \quad (8)$$

For this kind of density matrix the concurrence can be easily found [17]: $C'(\rho) = \max\{0, C'_1(\rho), C'_2(\rho)\}$ where

$$C'_1(\rho) = 2(\sqrt{\rho_{23}\rho_{32}} - \sqrt{\rho_{11}\rho_{44}}), \quad (9)$$

$$C'_2(\rho) = 2(\sqrt{\rho_{14}\rho_{41}} - \sqrt{\rho_{22}\rho_{33}}). \quad (10)$$

In order to evaluate QD, we follow the procedure of [14, 18]. To compute QD, the maximization of the classical correlation is taken over the measurement of [18] and we find three possible minima

$$\begin{aligned} Q_1(\rho) &= Q(k = 0, 1, m = 0), \\ Q_2(\rho) &= Q(k = 1/2, m = 0), \\ Q_3(\rho) &= Q(k = 1/2, m = 1/4). \end{aligned}$$

Thus

$$Q(\rho) = \min\{Q_1(\rho), Q_2(\rho), Q_3(\rho)\}. \quad (11)$$

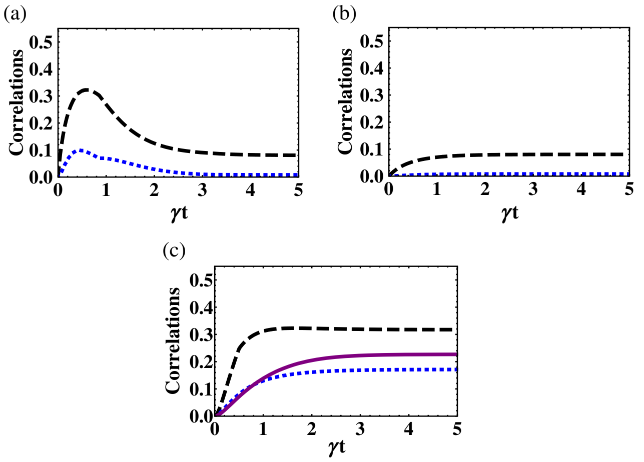

Figure 1. Correlations for a *thermal reservoir* with $n = 0.1$, quantum discord (dashed black), classical correlations (dotted blue), entanglement of formation (solid purple). (a) Initial condition $|11\rangle$, (b) initial condition $|00\rangle$ and (c) initial condition $|10\rangle$.

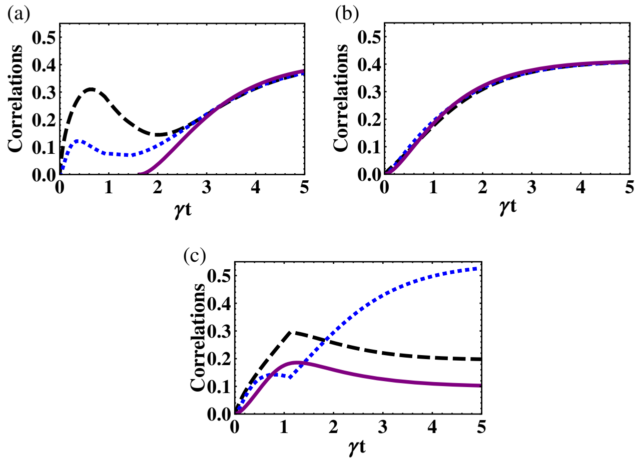

Figure 2. Correlations for a *squeezed reservoir* with $N = 0.1$, quantum discord (dashed black), classical correlations (dotted blue), entanglement of formation (solid purple). (a) Initial condition $|11\rangle$, (b) initial condition $|00\rangle$ and (c) initial condition $|10\rangle$.

The generation of entangled states is an important issue and has been studied in many different systems [19]. There are also some examples of QD [20]. Here we present some results comparing the thermal and squeezed reservoir; measuring correlations with entanglement of formation, QD, and also the classical correlations defined in equation (4). For the thermal reservoir, we take the parameter n as the average number of photons, while for the squeezed reservoir $N = \sinh^2 r$ is the squeezing parameter. We choose three different initial conditions $|\Phi_1\rangle = |11\rangle$, $|\Phi_2\rangle = |00\rangle$ and $|\Phi_3\rangle = |10\rangle$. As $t = 0$, we are in the presence of a product state; the subsystems do not share information at all, but the interaction with the common bath forces them to interact.

For the thermal reservoir it is impossible to create entanglement from two atoms in the excited state $|11\rangle$ or initially in the ground state $|00\rangle$ (figures 1(a) and (b)) and this is independent of the number of photons in the reservoir. However, a thermal bath creates QD for three initial conditions, which increases as we increase n.

As we can see from figure 2, in the case of a squeezed reservoir, entanglement is present for all initial conditions. If the system is initially in the $|10\rangle$ state in a thermal reservoir, we can create entanglement, but the perturbation of photons in the bath will lead to lower values of entanglement as we increase n.

For the initial state $|00\rangle$, since the interaction via squeezed vacuum reservoir is in pairs of photons, the system becomes entangled and we observe the generation of QD (figure 2(b)). For the initial state $|11\rangle$, we have a similar behavior to the $|00\rangle$ case, but with a delay due to the fact that the atoms need to emit the photons and will get entangled later through the bath (figure 2(a)). This time delay becomes shorter as we increase N, as a bigger N increases the probability of absorption of photons. During the delay time, we observe the presence of QD. As we increase N for the squeezed reservoir, both the entanglement and QD increase in the $|00\rangle$ and $|11\rangle$ initial states.

We also observe that the stationary state for the initial conditions $|11\rangle$ and $|00\rangle$ is a pure state given by $|\psi\rangle_{t\to\infty} = \frac{1}{\sqrt{N^2+M^2}}(N|11\rangle + M|00\rangle)$, thus the entanglement and the QD coincide, as shown in equation (7). In the limit $N \to \infty$, we obtain a maximally entangled state $|\psi\rangle_{t,N\to\infty} = \frac{1}{\sqrt{2}}(|11\rangle + |00\rangle)$, for which case in equation (7), $S = 1$.

For an initial state $|10\rangle$ the quantum correlations are favored by thermal reservoir. The opposite is true for the $|11\rangle$ and $|00\rangle$ initial states, whose correlations that are favored by the squeezed reservoir. The classical correlations are also favored by the squeezed reservoir for all initial conditions.

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