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Coherently controlled emission from two atoms dressed via a standing wave laser field

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Abstract
The influence of the standing wave amplitude on the resonance fluorescence of a three-level system of radiators is discussed. Great attention is devoted to the peculiarities of dressed states in the standing wave and the exchange integrals between two atoms situated in the anti-nodes or the nodes. The correlation functions of emitted photons at two dressed frequencies were obtained. It is shown that in the case when the distance between two atoms is smaller than the wavelength of the field, the emitted photons are strongly correlated. As was observed, for large values of laser field intensity, the control of spontaneous emission is possible at two frequencies as well as the atom–atom interaction process. The dependence of the fluorescent light spectrum as a function of the position of two atoms dressed in the standing wave is investigated.

(Some figures in this article are in colour only in the electronic version)

1. Introduction

Recently, great attention in quantum optics is devoted to the study of spontaneous emission and manipulation with the photons from one or more atoms trapped in the standing wave of the external coherent field [1–6] or excited by an intense travelling wave laser [7, 8]. The photons emitted in the cooperative effect can produce entangled states that are important in many areas as quantum information and quantum metrology, the study of strongly correlated systems and more fundamentally in the understanding of quantum physics.

It is known that the measurement of the first- and second-order correlation function of light using the Hanbury-Brown–Twiss interferometer [9] is applied in various fields of physics. For example, the correlation functions from a small number of emitted sources that exhibit photon anti-bunching have been investigated in order to explain the quantum nature of atoms and photons [10]. The resonance fluorescence and the photon correlations from an ensemble of cold atoms were studied in [11]. As is shown, when the photons are emitted in the same direction, the correlation function varies from anti-bunching to bunching with an increasing number of atoms and, when the photons are emitted in the opposite directions, the photons are always anti-bunched regardless of the number of atoms.

Methods proposed for engineering implementation of entanglement between atoms are based on achieving and controlling an effective interaction between the atoms that are to be entangled. Typically, these interactions are mediated by the electromagnetic field or by localization of the atoms. For a system of two cold atoms placed in a vacuum field, the back-action of emitted photons on the wave-packet evolution about the relative position of the two cold atoms was discussed in [12]. It was shown that the photon recoil resulting from the atomic spontaneous emission can induce the localization of the relative position of the two atoms through the entanglement between the spatial motion of individual atoms and their emitted photons.

An interesting problem is connected with the modification of spontaneous emission in a three-level system stimulated by an external coherent field, and such novel effects observed in the lasing without inversion [13] can be used in quantum information processing [14]. For implementing static qubits,

trapped atoms and ions are known to be promising candidates as they provide optimum conditions for quantum information processing. They offer long coherence times for information storage and can be utilized to realize quantum logic gates [15]. On the other hand, such a system may constitute the nodes of a quantum network, while communication between remote nodes may be achieved through photonic channels transmitting quantum states and entanglement [16].

This paper is devoted to the cooperative spontaneous emission from an extended ensemble of Λ type three-level atoms dressed by the standing wave. The cooperative behaviour can generate new effects connected with the non-homogeneity of the dressed field and the position of atoms in the cavity. It is shown that the standing wave modulates the Stark splitting of the atomic levels. Such splitting is directly proportional with the localization of the atoms and achieves the maximal value in the anti-nodes and minimal value in the nodes of the standing wave. Thus, the frequency of the spontaneously emitted photons carries the information about the position of the atoms in the anti-nodes. This is possible due to the dependence of the Rabi frequency on the atomic position in the anti-nodes. The correlation functions of emitted photons at two dressed frequencies (Stokes and anti-Stokes) for various localization distances of the two atoms were obtained. In the case where the distance between the atoms is less than the wavelength of the dressed field, the emitted photons are strongly correlated. As was observed, for large values of the laser field intensity, the control of spontaneous emission is possible at two frequencies and the atom–atom interaction process. The dependence of quasi-energy levels on the atomic positions in the standing wave and the spectrum of emission are investigated.

2. The model Hamiltonian

Let us study the cooperative generation of emitted photons from a system of three-level atoms dressed in a high intensity standing wave that is in resonance with one atomic transition (see figure 1). Assuming that the spontaneous emission rate γ_{32} (between the levels $|2\rangle$ and $|3\rangle$) is smaller than the spontaneous emission rate γ_{31}(transition $|3\rangle \rightarrow |1\rangle$), we will investigate the cooperative emission between the atomic quasi-energy levels of the dressed excited state and the ground state $|1\rangle$. The dressed atomic states are modified due to the strong resonance coupling of the atoms with the standing wave of resonator and in comparison with the similar transition stimulated by the strong travelling wave in free space, the dressed atomic spectrum depends on the position of the localization of the atoms. The excitation scheme represented in figure 1 corresponds to the situation for which the atomic system prepared in the second state $|2\rangle$ is excited by a strong coherent field relative to the transition $|2\rangle \leftrightarrow |3\rangle$. The system is periodically in the excited state $|3\rangle$, so that we can detect the spontaneous emission of dressed atomic states relative to the transition $|3\rangle \leftrightarrow |1\rangle$.

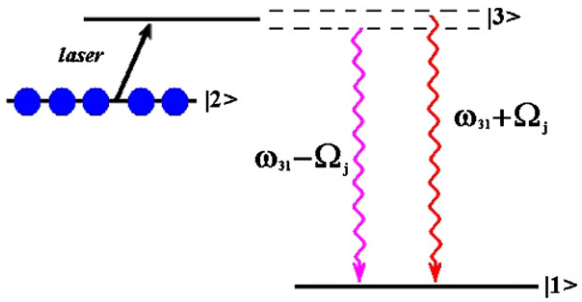

Figure 1. Energetic scheme of a three-level atomic ensemble.

In the dipolar and rotating wave approximation, the interaction of such an atomic ensemble with the dressed standing wave and electromagnetic field vacuum is described by the Hamiltonian

$$H = \sum_{j=1}^{N}\sum_{\alpha=1}^{3}\hbar\omega_{3\alpha}a_{\alpha j}^{\dagger}a_{\alpha j} + \sum_{k}\hbar\omega_k b_k^{\dagger}b_k + \mathrm{i}\hbar\sum_{j=1}^{N}\Omega_j\left(a_{3j}^{\dagger}a_{2j}\mathrm{e}^{\mathrm{i}(\vec{k}_0\cdot\vec{r}_j)(1-\Lambda)-\mathrm{i}\omega_0 t} - H.c.\right) + \frac{\mathrm{i}\omega_{31}d_{31}}{c}\sum_{k}\sum_{j=1}^{N}g_k\left(b_k a_{3j}^{\dagger}a_{1j}\mathrm{e}^{\mathrm{i}(\vec{k}\cdot\vec{r}_j)} - H.c.\right). \quad (1)$$

Here, we consider that the dipole momentum transition d_{31} is larger than d_{32}. In expression (1), $\hbar\omega_{3\alpha}$ is the energy of α level; $a_{\alpha j}^{\dagger}$, $a_{\alpha j}$ are the generation and annihilation operator for level α and atom j; d_{31} and d_{32} represent the dipole momentum transitions between the states $|3\rangle \longleftrightarrow |1\rangle$ and $|3\rangle \longleftrightarrow |2\rangle$, respectively; $b_k^{\dagger}(b_k)$ describes the generation (annihilation) of the photons with energy $\hbar\omega_k$; $\vec{k}_0$, ω_0 are the wave vector and the frequency of external field, respectively; $g_k = \sqrt{2\pi c^2\hbar/V\omega_k}(\vec{e}_\lambda\cdot\vec{d}_{31}/d_{31})$ represents the interaction constant with the fluctuations of electromagnetic vacuum; $\vec{e}_\lambda$ depicts the photon polarization vector; $\vec{d}_{31}$ is the dipole momentum and V is the volume. The atomic energy is measured from the centre of the two upper levels.

In the case when the atoms are stopped at a given distance, the amplitude of the electromagnetic field in the nodes and loops will differ significantly. Introducing the parameter Λ, we study both situations: when the atoms are dressed in the external field ($\Lambda = 1$), and the Rabi frequency $\Omega_j = \Omega_0\sin(\vec{k}_0\cdot\vec{r}_j)$ or interact with a travelling wave ($\Lambda = 0$) for which $\Omega_j = \Omega_0 = (\vec{d}_{32}\cdot\vec{E})/\hbar$.

As follows from Hamiltonian (1), the third term is time dependent and in order to exclude this dependence we make the transformation $H^{eff} = UHU^{-1} - \mathrm{i}\hbar U\frac{\partial U^{-1}}{\partial t}$, where $U = \exp\{\frac{\mathrm{i}\Delta t}{2}\sum_{j=1}^{N}(a_{3j}^{\dagger}a_{3j} - a_{2j}^{\dagger}a_{2j}) - \mathrm{i}t\sum_{j=1}^{N}\sum_{\alpha=1}^{3}\omega_{3\alpha}a_{\alpha j}^{\dagger}a_{\alpha j} + \mathrm{i}t\left(\omega_{31} + \frac{\Delta}{2}\right)\sum_k b_k^{\dagger}b_k\}$.

After this transformation, the effective Hamiltonian of the system becomes

$$H^{eff} = \frac{\hbar\Delta}{2}\sum_{j=1}^{N}\left(a_{3j}^{\dagger}a_{3j} - a_{2j}^{\dagger}a_{2j}\right) + \hbar\sum_{k}\left(\omega_k - \omega_{31} - \frac{\Delta}{2}\right)b_k^{\dagger}b_k + \mathrm{i}\hbar\sum_{j=1}^{N}\Omega_j\left(a_{3j}^{\dagger}a_{2j}\mathrm{e}^{\mathrm{i}(\vec{k}_0\cdot\vec{r}_j)(1-\Lambda)} - H.c.\right) + \frac{\mathrm{i}\omega_{31}d_{31}}{c}\sum_{k}\sum_{j=1}^{N}g_k\left(b_k a_{3j}^{\dagger}a_{1j}\mathrm{e}^{\mathrm{i}(\vec{k}\cdot\vec{r}_j)} - H.c.\right). \quad (2)$$

In the effective Hamiltonian (2), the parameter Δ represents the detuning between the transition frequency ω_{32} and the frequency of external laser field ω_0; $\Delta = \omega_{32} - \omega_0$. It is supposed that the intensity of the electromagnetic field is larger than the spectral width γ_{32}, γ_{31}, $\hbar\Omega_j >> \hbar\gamma_{32(31)}$. Based on this assumption and considering the last term in expression (2) as perturbed, we diagonalize the non-perturbed effective Hamiltonian using the Bogoliubov transformation. The new operators are defined as $\tilde{a}_{3j} = a_{3j}\mathrm{e}^{-\frac{\mathrm{i}}{2}(\vec{k}_0\cdot\vec{r}_j)(1-\Lambda)-\frac{\mathrm{i}\pi}{4}} = \alpha_j C_{3j} + \beta_j C_{2j}$, $\tilde{a}_{2j} = a_{2j}\mathrm{e}^{\frac{\mathrm{i}}{2}(\vec{k}_0\cdot\vec{r}_j)(1-\Lambda)+\frac{\mathrm{i}\pi}{4}} = \gamma_j C_{3j} + \delta_j C_{2j}$, where α_j^2, β_j^2, γ_j^2 and δ_j^2 describe the probability of population on the atomic levels

$$\alpha_j = \frac{1}{\sqrt{2}}\left(1 + \frac{\Delta}{\sqrt{\Omega_j^2 + \Delta^2}}\right)^{1/2} = -\delta_j,$$

$$\beta_j = \frac{1}{\sqrt{2}}\left(1 - \frac{\Delta}{\sqrt{\Omega_j^2 + \Delta^2}}\right)^{1/2} = \gamma_j.$$

In other words, after this diagonalization, the Hamiltonian (2) can be represented through the new quasi-energy levels with population $U_{3j}^3 = C_{3j}^{\dagger}C_{3j}$ and $U_{2j}^2 = C_{2j}^{\dagger}C_{2j}$ for which the spontaneous emission is possible:

$$H = \hbar\sum_{k}\left(\omega_k - \omega_{31} - \frac{\Delta}{2}\right)b_k^{\dagger}b_k + \hbar\sum_{j=1}^{N}\left(U_{3j}^3 - U_{2j}^2\right)\tilde{\Omega}_j + \frac{\mathrm{i}\omega_{31}d_{31}}{c}\sum_{k}\sum_{j=1}^{N}g_k\{b_k\mathrm{e}^{\mathrm{i}(\vec{k}\cdot\vec{r}_j)-\frac{\mathrm{i}}{2}[(\vec{k}_0\cdot\vec{r}_j)(1-\Lambda)+\pi/2]} \times \left(\alpha_j U_{1j}^3 + \beta_j U_{1j}^2\right) - H.c.\}, \quad (3)$$

where $\tilde{\Omega}_j = \sqrt{\Omega_j^2 + \Delta^2}$. Here the operators $U_{3j}^1 = C_{3j}a_{1j}^{\dagger}$, $U_{2j}^1 = C_{2j}a_{1j}^{\dagger}$ and $U_{1j}^3 = C_{3j}^{\dagger}a_{1j}$, $U_{1j}^2 = C_{2j}^{\dagger}a_{1j}$ are the transition operators between the quasi-energy levels of the excited and ground states, which satisfy the following commutation relation: $[U_{\beta}^{\alpha}, U_{\delta}^{\gamma}] = U_{\delta}^{\alpha}\delta_{\gamma\beta} - U_{\beta}^{\gamma}\delta_{\alpha\delta}$. The above transformation is done after the dressed states of the atoms and can be explained as a new three-level system with two excited quasi-levels from which the spontaneous emission occurs on the ground state $|1\rangle$.

Using the method of elimination of electromagnetic field operators relative to the atomic transitions $|3\rangle \rightarrow |1\rangle$ and $|2\rangle \rightarrow |1\rangle$ let us obtain the equation of the generalized dressed atomic operator $O(t)$. In accord with the procedure used in [7], one can write the following equation for the mean value of the operator

$$\frac{d\langle O(t)\rangle}{\mathrm{d}t} = i\sum_{j=1}^{N}\tilde{\Omega}_j\left\langle\left[U_{3j}^3(t) - U_{2j}^2(t), O(t)\right]\right\rangle + \sum_{l,j=1}^{N}\{I_{jl}\alpha_j\{\alpha_l\left\langle\left[U_{1j}^3(t), O(t)\right]U_{3l}^1(t)\right\rangle\zeta(\omega - \tilde{\Omega}_l) + \beta_l\left\langle\left[U_{1j}^3(t), O(t)\right]U_{2l}^1(t)\right\rangle\zeta(\omega + \tilde{\Omega}_l)\} + I_{jl}\beta_j\{\alpha_l\left\langle\left[U_{1j}^2(t), O(t)\right]U_{3l}^1(t)\right\rangle\zeta(\omega - \tilde{\Omega}_l) + \beta_l\left\langle\left[U_{1j}^2(t), O(t)\right]U_{2l}^1(t)\right\rangle\zeta(\omega + \tilde{\Omega}_l)\} + H.c\}, \quad (4)$$

where $\omega = \omega_k - \omega_{31} - \frac{\Delta}{2}$ and $\zeta(\omega \pm \tilde{\Omega}_l) = \mathrm{i}\frac{P}{\omega \pm \tilde{\Omega}_l} + \pi\delta(\omega \pm \tilde{\Omega}_l)$. In expression (4), I_{jl} represents the exchange integral between the atoms j and l:

$$I_{jl} = \left(\frac{\omega_{31}d_{31}}{c\hbar}\right)^2\sum_{k}g_k^2\mathrm{e}^{\mathrm{i}[(\vec{k}\cdot\vec{r}_{jl})-(\vec{k}_0\cdot\vec{r}_{jl})(1-\Lambda)/2]}. \quad (5)$$

As follows from expressions (3) and (4), the dressed state of the atom situated in the position j differs from the similar state of the atom situated in the point l of the standing wave. More than this, correlations between the Stokes and anti-Stokes scattering processes are realized if the atoms are situated in the non-equivalent place of the standing wave. In order to simplify the problem, we will consider the situation when atoms occupy a similar position in the standing wave so that the amplitudes, α_j and β_j, coincide with similar amplitudes, α_l and β_l, for both atoms situated in points l and j. These peculiarities will be discussed in the next section.

3. The equation of motion for two undistinguished atoms localized at various distances in the standing wave

The atomic localization process is based on the interaction of the ensemble via a standing wave laser field. As a consequence, the light scattered at the interaction of the standing wave field and the atomic ensemble depends on the position of the ensemble relative to the standing wave nodes and anti-nodes. In order to study the behaviour of the atomic system in the standing wave, let us first discuss the dynamics of two indistinguishable atoms situated at the distance r_{ab}.

Considering that the operator $O(t)$ can be one of the operators $U_{\beta a}^{\alpha}$ or $U_{\beta b}^{\alpha}$ belonging to a and b, respectively, from the equation of motion (4), one can obtain the following system of equations:

$$\frac{d\left\langle U_{1a}^2(t)U_{2b}^1(t)\right\rangle}{\mathrm{d}t} = 2I^{\alpha}\left\langle U_{2a}^3(t)U_{3b}^2(t)\right\rangle - 2I_0^{\beta}\left\langle U_{1a}^2(t)U_{2b}^1(t)\right\rangle + 2I^{\beta}\left[\left\langle U_{2a}^2(t)U_{2b}^2(t)\right\rangle - \left\langle U_{2a}^2(t)U_{1b}^1(t)\right\rangle\right],$$

$$\frac{d\left\langle U_{1a}^3(t)U_{3b}^1(t)\right\rangle}{\mathrm{d}t} = 2I^{\beta}\left\langle U_{2a}^3(t)U_{3b}^2(t)\right\rangle - 2I_0^{\alpha}\left\langle U_{1a}^3(t)U_{3b}^1(t)\right\rangle + 2I^{\alpha}\left[\left\langle U_{3a}^3(t)U_{3b}^3(t)\right\rangle - \left\langle U_{3a}^3(t)U_{1b}^1(t)\right\rangle\right],$$

$$\frac{d\langle U^2_{2a}(t)U^1_{1b}(t)\rangle}{dt} = 2I_0^{\alpha}\langle U^2_{2a}(t)U^3_{3b}(t)\rangle - 2I^{\beta}\langle U^2_{1a}(t)U^1_{2b}(t)\rangle + 2I_0^{\beta}\big[\langle U^2_{2a}(t)U^2_{2b}(t)\rangle - \langle U^2_{2a}(t)U^1_{1b}(t)\rangle\big],$$

$$\frac{d\langle U^3_{3a}(t)U^1_{1b}(t)\rangle}{dt} = 2I_0^{\beta}\langle U^2_{2a}(t)U^3_{3b}(t)\rangle - 2I^{\alpha}\langle U^3_{1a}(t)U^1_{3b}(t)\rangle + 2I_0^{\alpha}\big[\langle U^3_{3a}(t)U^3_{3b}(t)\rangle - \langle U^3_{3a}(t)U^1_{1b}(t)\rangle\big],$$

$$\frac{d\langle U^2_{2a}(t)U^3_{3b}(t)\rangle}{dt} = \frac{d\langle U^3_{2a}(t)U^2_{3b}(t)\rangle}{dt} = -2(I_0^{\alpha}+I_0^{\beta})\langle U^2_{2a}(t)U^3_{3b}(t)\rangle,$$

$$\frac{d\langle U^2_{2a(b)}(t)\rangle}{dt} = -2\big[I_0^{\beta}\langle U^2_{2a(b)}(t)\rangle + I^{\beta}\langle U^2_{1a}(t)U^1_{2b}(t)\rangle\big],$$

$$\frac{d\langle U^3_{3a(b)}(t)\rangle}{dt} = -2[I_0^{\alpha}\langle U^3_{3a(b)}(t)\rangle + I^{\alpha}\langle U^3_{1a}(t)U^1_{3b}(t)\rangle],$$

$$\frac{d\langle U^3_{3a}(t)U^3_{3b}(t)\rangle}{dt} = -4I_0^{\alpha}\langle U^3_{3a}(t)U^3_{3b}(t)\rangle,$$

$$\frac{d\langle U^2_{2a}(t)U^2_{2b}(t)\rangle}{dt} = -4I_0^{\beta}\langle U^2_{2a}(t)U^2_{2b}(t)\rangle, \tag{6}$$

where $I_0^{\alpha} = \alpha^2 I_{aa}$, $I_0^{\beta} = \beta^2 I_{bb}$ and $I^{\alpha} = \alpha_a\alpha_b I_{ab}$, $I^{\beta} = \beta_a\beta_b I_{ab}$ describe the exchange integrals between the same and different atoms, respectively. The exchange integrals for the same atoms I_{aa}, I_{bb} correspond to half of the spontaneous emission time $\tau_0^{-1}/2 = 2\omega_{31}^3 d_{31}^2/(3c^3\hbar)$.

In the case when the atoms a and b are situated at a distance comparable with the wavelength of the cavity $r_{ab} \geqslant \lambda$, the solution of the system of equation (6) can be represented in the following form:

$$\langle U^2_{2a(b)}(t)\rangle = \frac{1}{4}\frac{\Omega^2}{\tilde{\Omega}^2}\left[\frac{(I^{\alpha}+I_0^{\beta})E_{\beta}^{+}}{2(I_0^{\alpha}-I^{\beta})} - \frac{(I^{\alpha}+I^{\beta})I^{\beta}E}{I_0^{2\alpha}-I^{2\beta}} - \frac{(I^{\alpha}-I_0^{\alpha})E_{\beta}^{-}}{2(I_0^{\alpha}+I^{\beta})} - e^{-2I_0^{\beta}t}\right] + \alpha^4\left[\frac{(I^{\beta}+I_0^{\beta})E_{\beta}^{+}}{2(I_0^{\beta}-I^{\beta})} - \frac{2I^{2\beta}e^{-4I_0^{\beta}t}}{I_0^{2\beta}-I^{2\beta}} - \frac{(I^{\beta}-I_0^{\beta})E_{\beta}^{-}}{2(I_0^{\beta}+I^{\beta})} - e^{-2I_0^{\beta}t}\right] + \alpha^2 e^{-2I_0^{\beta}t},$$

$$\langle U^3_{3a(b)}(t)\rangle = \frac{1}{4}\frac{\Omega^2}{\tilde{\Omega}^2}\left[\frac{(I_0^{\beta}-I^{\beta})E_{\alpha}^{-}}{2(I_0^{\beta}+I^{\alpha})} - \frac{(I^{\alpha}+I^{\beta})I^{\alpha}E}{I_0^{2\beta}-I^{2\alpha}} + \frac{(I^{\beta}+I_0^{\beta})E_{\alpha}^{+}}{2(I_0^{\beta}-I^{\alpha})} - e^{-2I_0^{\alpha}t}\right] + \beta^4\left[\frac{(I_0^{\alpha}-I^{\alpha})E_{\alpha}^{-}}{2(I_0^{\alpha}+I^{\alpha})} + \frac{(I_0^{\alpha}+I^{\alpha})E_{\alpha}^{+}}{2(I_0^{\alpha}-I^{\alpha})} - \frac{2I^{2\alpha}e^{-4I_0^{\alpha}t}}{I_0^{2\alpha}-I^{2\alpha}} - e^{-2I_0^{\alpha}t}\right] + \beta^2 e^{-2I_0^{\alpha}t},$$

$$\langle U^2_{1a}(t)U^1_{2b}(t)\rangle = \frac{1}{4}\frac{\Omega^2}{\tilde{\Omega}^2}\left[\frac{I^{\alpha}+I_0^{\alpha}}{2(I_0^{\alpha}-I^{\beta})}E_{\beta}^{+} - I_0^{\alpha}\frac{I^{\alpha}+I^{\beta}}{I_0^{2\alpha}-I^{2\beta}}E + \frac{I^{\alpha}-I_0^{\alpha}}{2(I_0^{\alpha}+I^{\beta})}E_{\beta}^{-}\right] + \alpha^4\left[\frac{I^{\beta}+I_0^{\beta}}{2(I_0^{\beta}-I^{\beta})}E_{\beta}^{+} - \frac{2I_0^{\beta}I^{\beta}}{I_0^{2\beta}-I^{2\beta}}e^{-4I_0^{\beta}t} + \frac{I^{\beta}-I_0^{\beta}}{2(I_0^{\beta}+I^{\beta})}E_{\beta}^{-}\right],$$

$$\langle U^3_{1a}(t)U^1_{3b}(t)\rangle = \frac{1}{4}\frac{\Omega^2}{\tilde{\Omega}^2}\left[\frac{I^{\beta}-I_0^{\beta}}{2(I_0^{\beta}+I^{\alpha})}E_{\alpha}^{-} - I_0^{\beta}\frac{I^{\alpha}+I^{\beta}}{I_0^{2\beta}-I^{2\alpha}}E + \frac{I^{\beta}+I_0^{\beta}}{2(I_0^{\beta}-I^{\alpha})}E_{\alpha}^{+}\right] + \beta^4\left[\frac{I^{\alpha}-I_0^{\alpha}}{2(I_0^{\alpha}+I^{\alpha})}E_{\alpha}^{-} - \frac{2I_0^{\alpha}I^{\alpha}}{I_0^{2\alpha}-I^{2\alpha}}e^{-4I_0^{\alpha}t} + \frac{I^{\alpha}+I_0^{\alpha}}{2(I_0^{\alpha}-I^{\alpha})}E_{\alpha}^{+}\right],$$

$$\langle U^2_{2a}(t)U^1_{1b}(t)\rangle = \frac{1}{4}\frac{\Omega^2}{\tilde{\Omega}^2}\left[\frac{I^{\alpha}+I_0^{\alpha}}{2(I_0^{\alpha}-I^{\beta})}E_{\beta}^{+} - \frac{I_0^{2\alpha}+I^{\beta}I^{\alpha}}{I_0^{2\alpha}-I^{2\beta}}E - \frac{I^{\alpha}-I_0^{\alpha}}{2(I_0^{\alpha}+I^{\beta})}E_{\beta}^{-}\right] + \alpha^4\left[\frac{I^{\beta}+I_0^{\beta}}{2(I_0^{\beta}-I^{\beta})}E_{\beta}^{+} - \frac{2I_0^{2\beta}}{I_0^{2\beta}-I^{2\beta}}e^{-4I_0^{\beta}t} - \frac{I^{\beta}-I_0^{\beta}}{2(I_0^{\beta}+I^{\beta})}E_{\beta}^{-}\right],$$

$$\langle U^3_{3a}(t)U^1_{1b}(t)\rangle = \frac{1}{4}\frac{\Omega^2}{\tilde{\Omega}^2}\left[\frac{I_0^{\beta}-I^{\beta}}{2(I_0^{\beta}+I^{\alpha})}E_{\alpha}^{-} - \frac{I^{\alpha}I^{\beta}+I_0^{2\beta}}{I_0^{2\beta}-I^{2\alpha}}E + \frac{I^{\beta}+I_0^{\beta}}{2(I_0^{\beta}-I^{\alpha})}E_{\alpha}^{+}\right] + \beta^4\left[\frac{I_0^{\alpha}-I^{\alpha}}{2(I_0^{\alpha}+I^{\alpha})}E_{\alpha}^{-} - \frac{2I_0^{2\alpha}}{I_0^{2\alpha}-I^{2\alpha}}e^{-4I_0^{\alpha}t} + \frac{I^{\alpha}+I_0^{\alpha}}{2(I_0^{\alpha}-I^{\alpha})}E_{\alpha}^{+}\right],$$

$$\langle U^2_{2a}(t)U^2_{2b}(t)\rangle = \alpha^4 e^{-4I_0^{\beta}t}, \langle U^3_{3a}(t)U^3_{3b}(t)\rangle = \beta^4 e^{-4I_0^{\alpha}t},$$

$$\langle U^2_{3a}(t)U^3_{2b}(t)\rangle = \langle U^2_{2a}(t)U^3_{3b}(t)\rangle = \frac{1}{4}\frac{\Omega^2}{\tilde{\Omega}^2}E, \tag{7}$$

where $E_{\alpha}^{\pm} = \exp[-2(I_0^{\alpha} \pm I^{\alpha})t]$, $E_{\beta}^{\pm} = \exp[-2(I_0^{\beta} \pm I^{\beta})t]$, $E = \exp[-2(I_0^{\alpha}+I_0^{\beta})t]$. As the atoms are indistinguished and occupy a similar position in the standing wave, we assume that $|\alpha_a| = |\alpha_b| = \alpha$ and $|\beta_a| = |\beta_b| = \beta$.

In accord with solutions (7), the exchange integral depends on the positions and distance between the atoms dressed in the field of the standing wave, and is given by the relation

$$I_{ab} = \frac{3}{4}\tau_0^{-1}\left\{[1-(\vec{n}_d\cdot\vec{n}_{ab})^2]\frac{\sin(\omega_k r_{ab}/c)}{\omega_k r_{ab}/c} + [1-3(\vec{n}_d\cdot\vec{n}_{ab})^2]\left[\frac{\cos(\omega_k r_{ab}/c)}{(\omega_k r_{ab}/c)^2} - \frac{\sin(\omega_k r_{ab}/c)}{(\omega_k r_{ab}/c)^3}\right]\right\}. \tag{8}$$

Here, the slow part of the Rabi frequency was neglected, ω_k describing the Stokes $\omega_{31} - \Omega_j$ and anti-Stokes $\omega_{31} + \Omega_j$ frequencies, and $\vec{n}_d$ represents the atomic dipolar

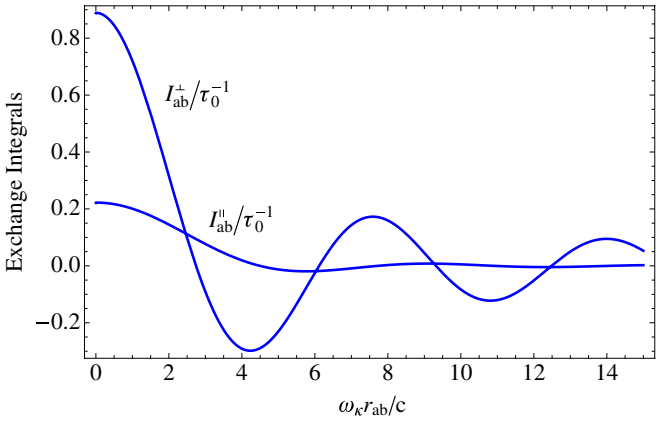

Figure 2. The dependence of the exchange integral on the distance between the atoms when they emit perpendicular and parallel to the direction of propagation of emission.

momentum that emits along the direction $\vec{n}_{ab}$. If $\vec{n}_d \perp \vec{n}_{ab}$, the exchange integral (8) takes the form

$$I^{\perp}_{ab} = \frac{3}{4}\tau_0^{-1}\left[\frac{\sin(\omega_k r_{ab}/c)}{\omega_k r_{ab}/c} - \frac{\sin(\omega_k r_{ab}/c)}{(\omega_k r_{ab}/c)^3} + \frac{\cos(\omega_k r_{ab}/c)}{(\omega_k r_{ab}/c)^2}\right]. \tag{9}$$

In the opposite case, when the dipolar momentum of the atoms is orientated parallel to the direction of propagation of emission $\vec{n}_d||\vec{n}_{ab}$, the exchange integral between the atoms a and b will be described by the expression

$$I^{||}_{ab} = \frac{3}{2}\tau_0^{-1}\left[\frac{\sin(\omega_k r_{ab}/c)}{(\omega_k r_{ab}/c)^3} - \frac{\cos(\omega_k r_{ab}/c)}{(\omega_k r_{ab}/c)^2}\right]. \tag{10}$$

As follows from expressions (9) and (10), the exchange integrals strictly depend on the positions of localization of the atoms and the direction of propagation of emission (see figure 2). In the parallel case, the exchange integral decreases rapidly. If the atoms are situated in anti-nodes and $\omega_k = \omega_{31}$, the exchange integral $I^{\perp}_{ab}$ is maximal. In the opposite situation, when the atoms a and b are placed in nodes and the field state is in resonance with the atomic states, the Rabi frequency is zero and the exchange integral decreases.

In the limit when $I^{\beta} \to I^{\beta}_0$ and $I^{\alpha} \to I^{\alpha}_0$, solutions (7) correspond to the case of atoms localized at a distance less than the wavelength of the cavity, $r_{ab} < \lambda$:

$$\left\langle U^2_{2a(b)}(t)\right\rangle = \alpha^4\left(2\beta^2\frac{t}{\tau_0}+1\right)E^+_\beta + (\alpha^2-\alpha^4)\mathrm{e}^{-2\beta^2 t/\tau_0} + \frac{1}{4}\frac{\Omega^2}{\tilde{\Omega}^2}\left[\frac{\alpha^2 E^+_\beta - \beta^2 E}{\alpha^2-\beta^2} - \mathrm{e}^{-2\beta^2 t/\tau_0}\right],$$

$$\left\langle U^3_{3a(b)}(t)\right\rangle = \beta^4\left(2\alpha^2\frac{t}{\tau_0}+1\right)E^+_\alpha + (\beta^2-\beta^4)\mathrm{e}^{-2\alpha^2 t/\tau_0} + \frac{1}{4}\frac{\Omega^2}{\tilde{\Omega}^2}\left[\frac{\beta^2 E^+_\alpha - \alpha^2 E}{\beta^2-\alpha^2} - \mathrm{e}^{-2\alpha^2 t/\tau_0}\right],$$

$$\left\langle U^2_{1a}(t)U^1_{2b}(t)\right\rangle = \left\langle U^2_{2a}(t)U^1_{1b}(t)\right\rangle = 2\alpha^4\beta^2\frac{t}{\tau_0}E^+_\beta + \frac{1}{4}\frac{\Omega^2}{\tilde{\Omega}^2}\frac{\alpha^2(E^+_\beta - E)}{\alpha^2-\beta^2},$$

$$\left\langle U^3_{1a}(t)U^1_{3b}(t)\right\rangle = \left\langle U^3_{3a}(t)U^1_{1b}(t)\right\rangle = 2\beta^4\alpha^2\frac{t}{\tau_0}E^+_\alpha + \frac{1}{4}\frac{\Omega^2}{\tilde{\Omega}^2}\frac{\beta^2(E^+_\alpha - E)}{\beta^2-\alpha^2},$$

$$\left\langle U^2_{3a}(t)U^3_{2b}(t)\right\rangle = \left\langle U^2_{2a}(t)U^3_{3b}(t)\right\rangle = \frac{1}{4}\frac{\Omega^2}{\tilde{\Omega}^2}E,$$

$$\left\langle U^2_{2a}(t)U^2_{2b}(t)\right\rangle = \alpha^4 E^+_\beta,\ \left\langle U^3_{3a}(t)U^3_{3b}(t)\right\rangle = \beta^4 E^+_\alpha. \tag{11}$$

Here, the exchange integral (8) for the same and different atoms coincides with one-half of the spontaneous emission rate $I_{aa} = I_{ab} = \tau_0^{-1}/2$ and does not depend on the atom positions in the standing wave. As the atoms are closed to each other (r_{ab} is smaller than 0.2λ), they will be situated almost in an equivalent position relative to the standing wave laser field. In this situation, the index j from the expression characterizing the Rabi frequency was omitted $\Omega_j = \Omega$.

4. Coherent control via collective effects

In this section, we discuss the first- and second-order correlation functions in the resonance fluorescence from two indistinguishable atoms localized at different positions in the standing wave of the cavity field. The photon statistics of light describes whether it originates from a classical or quantum source character like a single atom. Using the positive and negative parts of electromagnetic field operators, these functions are defined as [17]

$$G^{(1)}(\vec{r},t,t+\tau) = \langle \vec{E}^{(-)}(\vec{r},t)\vec{E}^{(+)}(\vec{r},t+\tau)\rangle,$$
$$G^{(2)}(\vec{r},t,t+\tau) = \langle \vec{E}^{(-)}(\vec{r},t)\vec{E}^{(-)}(\vec{r},t+\tau)\vec{E}^{(+)}(\vec{r},t+\tau)\vec{E}^{(+)}(\vec{r},t)\rangle, \tag{12}$$

where $\vec{E}^{(+)}(\vec{r},t) = \sum_k q_k \exp(\mathrm{i}\vec{k}\vec{r})b_k$; $\vec{E}^{(-)}(\vec{r},t) = [\vec{E}^{(+)}(\vec{r}, \quad t)]^+$ are the components of the emission electromagnetic field strengths, respectively, and $q_k = \sqrt{2\pi\hbar\omega_k/V}(\vec{e}_\lambda \cdot \vec{n}_d)$. Eliminating the boson operators, the negative part can be expressed through the dressed atomic operators of the Hamiltonian (3):

$$\vec{E}^{(-)}(\vec{r},t) = \frac{\omega_{31}d_{31}}{c\hbar}\sum_{k,j} q_k g_k \exp\left[\mathrm{i}(\vec{k}\cdot(\vec{r}_j - \vec{r})) - i\frac{\pi}{4}\right] \times \int_0^t \exp[\mathrm{i}(\omega_k - \omega_{31})\tau_1][\alpha_j U^1_{3j}(t-\tau_1) + \beta_j U^1_{2j}(t-\tau_1)]\,\mathrm{d}\tau_1.$$

The interference of photons emitted by two sources projects the state of the atom a and b into an entangled state. However, in such kinds of models, the photons involved do not need to be initially entangled and can even be emitted by different sources, but they need to be indistinguishable. In general, it is not easy to realize several independent sources emitting indistinguishable photons. A realizable scheme setup for our model can serve the experiment of observation of collective excitation of two individual atoms in the Rydberg blockade regime [18]. In this scheme, the fluorescence

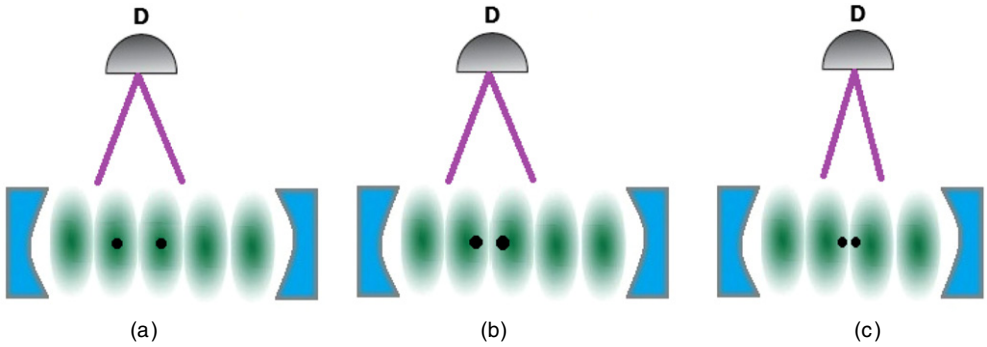

Figure 3. Scheme setup for two atoms stopped at a distance (a) $kr_{ab} = 3.14$, (b) $kr_{ab} = 1.28$ and (c) $kr_{ab} = 0.28$.

emission from two individual atoms held at a distance of $\sim 4\,\mu$m is investigated.

Considering that the spontaneous emission spectrum is spherically distributed and with the same probabilities in all directions, the detector can be situated at a point perpendicular to the direction of the resonator axis, as is represented in figure 3. In this case, the new frequencies $\omega_{31} \pm \Omega$ do not coincide with the frequencies of the cavity modes.

According to solutions (11) at time $\tilde{t} + \tau$, the second-order correlation function of two atoms localized at a distance smaller than the emission wavelength is given by

$$G^{(2)}(\vec{r}, \tilde{t}, \tilde{t} + \tau) = 4K^2[\alpha^4 E^+_\alpha(\tau)\left\langle U^3_{3a}(\tilde{t})U^3_{3b}(\tilde{t})\right\rangle + \beta^4 E^+_\beta(\tau)\left\langle U^2_{2a}(\tilde{t})U^2_{2b}(\tilde{t})\right\rangle + \alpha^2\beta^2[E^+_\alpha(\tau) + E^+_\beta(\tau)]\left\langle U^2_{3a}(\tilde{t})U^3_{2b}(\tilde{t})\right\rangle], \quad (13)$$

where $K = \left(\frac{\omega^2_{31} d_{31}}{c^2 r}\right)^2$ and $E^+_\alpha(\tau)$, $E^+_\beta(\tau)$ coincide with the same notations from (7) for $\tau = t$. Here we considered that the detector is situated perpendicular to the distance $\vec{r}_{ab}$ between the atoms, and a photon being detected in the time $\tilde{t} = t - r/c$ for which the Born–Markov approximation is applicable. We observe that in the point r, the intensity of photons emitted is inversely proportional to the fourth power of the distance between two atoms and the detector. It is well known that the second-order correlation function of the radiation emitted by the source of atoms has a maximum when $\tau \to 0$. As follows from figure 4, this is, of course, a manifestation of the familiar photon-bunching effect. The higher-order correlation properties of the photons emitted may be affected significantly by atomic correlation and by increasing the detuning parameter, the intensity of the correlated function decreases as is shown in the continued dependence.

Next we investigate the spectrum of fluorescence for the case when the atoms are placed at a distance $r_{ab} \geqslant \lambda$. Suppose that the atoms are situated around the anti-nodes of the standing wave; the first-order correlation function is given by the expression

$$G^{(1)}(\vec{r}, t, t + \tau) = K\left\{\alpha^2\left[\left\langle U^3_{1a}(t)U^1_{3b}(t)\right\rangle + H.c\right]e^{-(i\Omega + 2I^\alpha - i\omega_{31})\tau} + \alpha^2\left[\left\langle U^3_{3a}(t)\right\rangle + \left\langle U^3_{3b}(t)\right\rangle\right]e^{-(i\Omega + 2I^\alpha_0 - i\omega_{31})\tau} + \beta^2\left[\left\langle U^2_{1a}(t)U^1_{2b}(t)\right\rangle + H.c\right]e^{(i\Omega - 2I^\beta + i\omega_{31})\tau} + \beta^2\left[\left\langle U^2_{2a}(t)\right\rangle + \left\langle U^2_{2b}(t)\right\rangle\right]e^{(i\Omega - 2I^\beta_0 + i\omega_{31})\tau}\right\}. \quad (14)$$

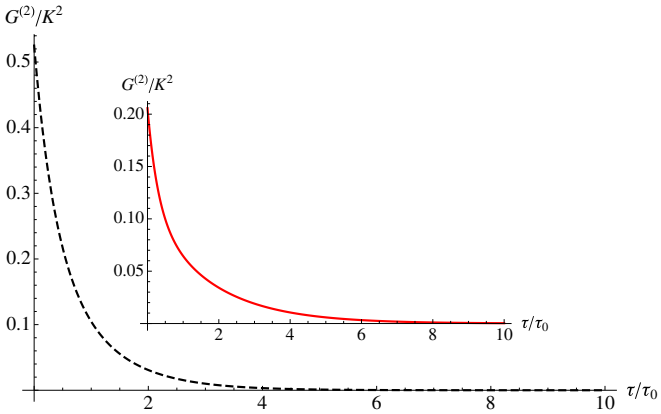

Figure 4. The dependence of the second-order correlation function $G^{(2)}$ as a function of the detection time delay τ/τ_0 for $\triangle/\Omega = 0.5$(dashed), $\triangle/\Omega = 1$(continued) and $\tilde{t}/\tau_0 = 0.1$.

The power spectrum $S(\omega)$ of the fluorescent light spectrum is obtained by taking the Fourier transformation of the normally ordered correlation function (14)

$$S(\omega) = 2K\pi^{-1}\left\{\frac{\alpha^2 I^\alpha_0\left[\left\langle U^3_{3a}(t)\right\rangle + \left\langle U^3_{3b}(t)\right\rangle\right]}{(\omega - \omega_{31} - \Omega)^2 + (2I^\alpha_0)^2} + \frac{\beta^2 I^\beta_0\left[\left\langle U^2_{2a}(t)\right\rangle + \left\langle U^2_{2b}(t)\right\rangle\right]}{(\omega - \omega_{31} + \Omega)^2 + \left(2I^\beta_0\right)^2} + \frac{\alpha^2 I^\alpha\left[\left\langle U^3_{1a}(t)U^1_{3b}(t)\right\rangle + H.c\right]}{(\omega - \omega_{31} - \Omega)^2 + \left(2I^\alpha\right)^2} + \frac{\beta^2 I^\beta\left[\left\langle U^2_{1a}(t)U^1_{2b}(t)\right\rangle + H.c\right]}{(\omega - \omega_{31} + \Omega)^2 + (2I^\beta)^2}\right\}. \quad (15)$$

This expression is correct for $\Omega^{-1} \ll \tau \ll t$ and for a well-fixed time of emission (for example, at $t = 0.1\tau_0$ the spontaneous emission occurs). This problem does not include steady states; for $t \to \infty$ the atomic correlators (7) are zero, i.e. the field correlation function $G^{(1)}(\vec{r}, t, t + \tau)$ depends of the origin of time t and on the delay time τ. As the delay time $\tau \gg \Omega^{-1}$, the integral from the Fourier transformation is valid. In order to obtain a stationary effect, we should restore the atoms in the state $|2\rangle$, by applying a second field to the transition $|1\rangle \leftrightarrow |2\rangle$.

In expression (15), only the incoherent component of the spectrum is considered. The exchange integral is chosen

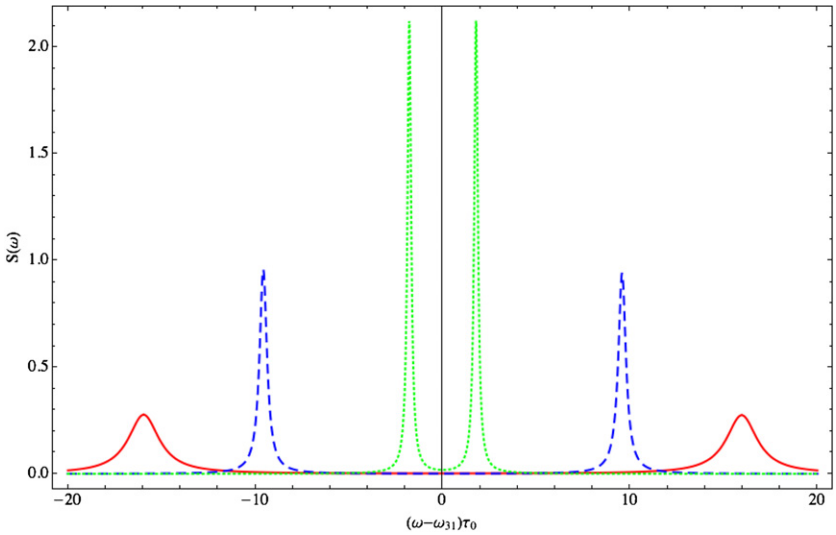

Figure 5. Resonance fluorescence spectrum for $\triangle = 0.1$, $\Omega_0 = 16$, $t = 0.1\ \tau_0$ and $kr_{ab} = 3.14$ (continued), 1.28 (dashed), 0.28 (dotted).

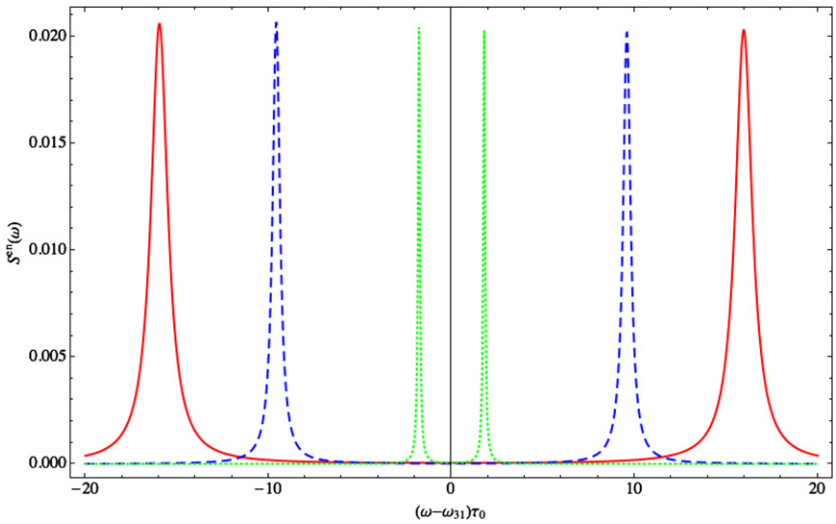

Figure 6. Resonance fluorescence spectrum for the correlated part of two atoms for the same values as in figure 5.

so that atoms a and b are situated in the same position of the field, and as a consequence the terms proportional to $1/r_{ab}^2$ and $1/r_{ab}^3$ from (8) can be neglected. When the Rabi frequency associated with the driving field becomes comparable with the spectral width of the atoms $\Omega \gg \gamma_{23}, \gamma_{13}$, the degenerate level is split into a doublet of dressed states and the spontaneous emission is realized with the frequency $\omega_{31} \pm \Omega$. The appearance of this splitting is represented in figure 5.

The fluorescent spectrum is plotted for three equivalent positions of the atoms dressed in the standing wave of the field as is shown in the scheme setup of figure 3. The first case corresponds to the case where the atom a is placed in the centre of one anti-node $kr_a = 1.5$ and the atom b in the next anti-node centre $kr_b = 4.64$ and the distance between them is $kr_{ab} = 3.14$. In this situation, the Rabi frequency is maximal $|\sin(kr_a)| = |\sin(kr_b)| = 1$, and the intensity of the spectrum is plotted with a continued line. The dashed and dotted peaks correspond to the distances $kr_{ab} = 1.28$ and $kr_{ab} = 0.28$ with atoms located at the left and right sides of the node (the position of the node corresponds to π). In the last case, the atoms are localized near the node and the Rabi frequency tends to zero; thus, the individual terms give the maximal contribution.

According to the above discussion, the amplitude of the spectrum increases with the decreasing of the distance between the atoms, because of the superposition of two Lorentzian lines given by the individual and cooperative terms in $S(\omega)$. For atoms located symmetrically with respect to a node or anti-node, the individual atomic positions determine the distance between the spectral peaks, but the distance kr_{ab} determines the cooperative effects (amplitude of the spectrum).

If the individual atomic spontaneous emission part is not considered, the amplitude spectrum of the photons emitted at the frequency $\omega - \omega_{31} \pm \Omega$ will decrease, the peaks having the same maximum as is shown in figure 6 (for example, at $kr_{ab} = 3$, $S(\omega) \simeq 2$, but $S^{en}(\omega) \simeq 0.02$ for all three values of kr_{ab}).

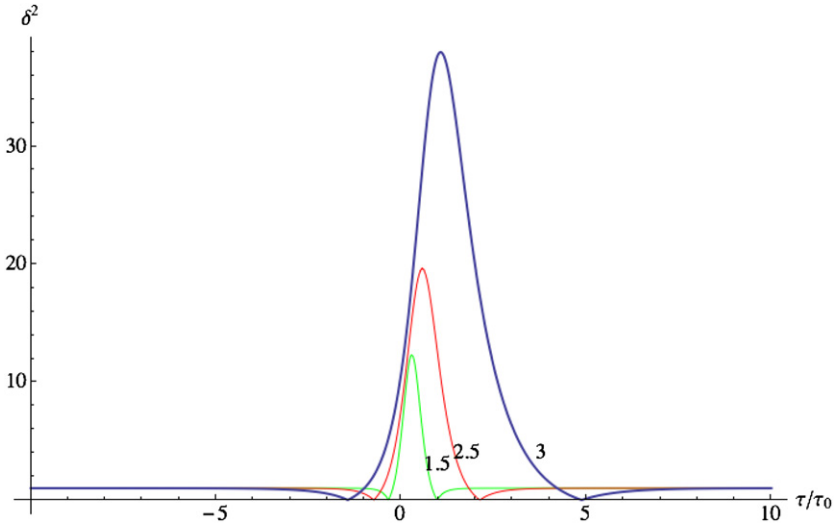

Figure 7. Photon number fluctuation $\delta^2 = G^{(2)} - \left|G^{(1)}\right|^2$ as a function of delay time for the atomic position $kr_a = 1.5$, 2.5 and 3.

The function $S^{en}(\omega) = S(\omega) - S(\omega, \text{un} - \text{correlated})$ describes an entangled state between photons emitted at the Stokes and anti-Stokes frequencies.

The photons statistics for various positions of the atoms is represented in figure 7. As follows, the photon statistics is super-Poissonian and the bunching is shifted from the delay time $\tau = 0$. The slight departure from perfect bunching indicates that one atom emits a photon before the second photon has been emitted.

Photon-bunching generated by two atoms localized at a distance comparable with the wavelength of the cavity for significant atom–atom interactions may instead be entangled via their emitted photons and can be used in quantum processing and transmission in a very crucial way [19]. The correlation function between the atoms a and b describes the entangled state of photons emitted at the interval time τ. By applying laser beams, one manipulates with the atomic positions relatively the anti-nodes, so that the resonance fluorescence spectrum from two atoms can be achieved and described by (13). In our model, the spontaneous emission can occur at a 'well-defined' time; the second correlation function $G^{(2)}$ is not zero and represents an entangled state between the emitted photons.

One approach to scalable quantum computation based on probabilistic entangling gates is to have an array of trapping zones, each containing two atoms: a logic atom a that encodes the quantum information and the transferring atom b that is responsible for the coupling to another atomic pair a_1, b_1 via a probabilistic entanglement protocol. The atoms a and b should be separated at a given distance in the dressed field through the laser action. In order to achieve scalability, we must realize the quantum gates between two arbitrary atoms from the pairs (for example, a and a_1). This can be done realizing an entangled state from the spontaneous emitted photons of the mentioned atoms. After the preparation of the entanglement, we can achieve local deterministic motional controlled-NOT gates on each logic-transferring atoms from the zones (C-NOT for $a-b$ and a_1-b_1 pair). The next step after realizing the gates operation at each pair consists in the measurement of the atomic protocol in the appropriate bases, and the application of single-qubit rotation to the logic atoms based on the measurement results. We consider that good entanglement fidelity can be achieved manipulating with the distance between the logic atom and the atom responsible for the coupling with the next anti-node.

5. Conclusions

The cooperative spontaneous emission from an extended ensemble of Λ type three-level atoms dressed by the standing wave field was investigated. We find that the dressed atomic spectrum depends significantly on the position of atoms localized in the resonator. Using the master equation, we studied the time evolution of fluorescence emission from two indistinguishable atoms dressed with the standing wave of the field. We observed that for large values of the laser field intensity, the excited level splits and the control of spontaneous emission is possible at two frequencies $\omega_{31} \pm \Omega$. The Stark splitting is directly proportional to the atomic localization and achieves the maximal value in the anti-nodes and minimal value in the nodes of the standing wave. Thus, the photons resulting from the atomic spontaneous emission can induce the entangled state through the correlations between the individual trapped atoms. The entanglement of emitted photons at Stokes and anti-Stokes dressed frequencies for various atomic separation was obtained. Our results show that for two atoms situated at a distance less than the wavelength of the dressed field, the emitted photons are strongly correlated.

A realizable model for producing an entangled state that can be controlled via the distance of localization between two atoms in the anti-nodes was discussed. Controlling with the positions of the atoms, we can neglect the cooperative interaction with other atoms of the cavity, and with photon–photon entanglement, good entanglement fidelities can be obtained. The extent to which this can be performed in practice depends on the future development of cavity QED.

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